Compressive Sensing based Low Complexity User Selection for Massive MIMO Systems

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Abstract—**Massive Multiple-input Multiple-output (MIMO) is widely considered as a key enabler of the next-generation networks. In these systems, user selection strategies are important to achieve spatial diversity and maximize spectral efficiency. In this paper, a user selection algorithm is proposed with the reconstruction of the sparse Massive MIMO channel using Compressive Sensing (CS) algorithm. The proposed algorithm eliminates the users based on the channel correlation by employing the CS algorithm which reduces the feedback overhead in the system. The simulation results show that the proposed algorithm outperforms the traditional user selection algorithms in terms of sum data rate and computational complexity. Moreover, the effects of the sparsity level and feedback measurement on the performance are examined.**

Index Terms—**Massive MIMO, user selection, compressive sensing, sparse channel**

I. Introduction

Massive Multiple-input Multiple-output (MIMO) is one of the key candidate technologies in terms of meeting the capacity requirement, supporting high spectrum and energy efficiency for the next-generation of wireless communications. In Massive MIMO systems, the base station (BS) is equipped with hundreds of antennas. The key idea is based on the use of a large number of transmit antennas to serve simultaneously multiple users. The number of antennas at the BS is higher than the number of users. As the number of BS antennas tends to infinity, the effects of fading are eliminated completely [1].

To fully harvest the benefit of excessive BS antennas, the knowledge of channel state information at the transmitter (CSIT) is an essential requirement. However, it is challenging to obtain accurate CSIT. Since the training overhead for CSIT acquisition grows proportionally with the number of BS antennas, it can be very large in such systems. Early works avoid this challenge by adopting a time-division duplexing (TDD) where the CSIT can be obtained by exploiting channel reciprocity and the uplink pilot-aided training overhead is proportional to the number of users.

However, channel reciprocity does not hold for Massive MIMO systems with a frequency-division duplexing (FDD). Pilot based channel estimation and uplink channel feedback are required, which consume spectrum resources. FDD has its own advantages while compared with TDD. It can provide more efficient communications with low latency in terms of delay sensitivity. Therefore, it is also important to consider CSIT acquisition for FDD systems.

Many works have shown that the effective dimension of a Massive MIMO channel is actually much less than its original dimension because of the limited local scattering effect in the propagation environment [2]. Specifically, the Massive MIMO channel has an approximately sparse representation under the Discrete Fourier Transform (DFT) basis if the BS is equipped with a large uniform linear array (ULA). As a consequence, Compressive Sensing (CS) algorithm, which exploits the hidden sparsity under the DFT basis, has been examined for downlink channel estimation and feedback [3].

The other important point is that these systems have many challenges such as pilot contamination, channel modeling and the feedback on user selection. The performance of Massive MIMO systems depends essentially on the user selection approach. In this paper, we present the user selection for Massive MIMO systems with reduced feedback overhead.

In the literature, the user selection has been widely investigated. In the traditional MIMO systems, the semi-orthogonal user selection (SUS) algorithm has been given in [4]. The SUS algorithm iteratively selects the user considering the channel norm and the correlation coefficient. In order to ease the problems of high computational complexity and high feedback, the greedy user selection algorithm has been given in [5] based on the rate allocation in vector perturbation precoding systems, which reduces the computational complexity through removing the insignificant users from the candidate user set.

In [6], the joint antenna selection and user selection problem have been solved in distributed Massive MIMO systems under the back-haul capacity constraint. In [7], the joint strategy has been examined which performs antenna selection and scheduling the users to maximize the sum data rate.

In [8], the user selection scheme for a hybrid architecture based on DFT processing has been provided by considering the achievable rate of the system and guaranteeing the fairness of selection.

The computational complexity of the conventional user selection schemes is too high to be implemented in Massive MIMO systems. In this paper, the main objective is to propose a user selection algorithm that improves the sum data rate and reduces the feedback load in Massive MIMO systems. The user selection algorithm eliminates the users based on the channel correlation and Orthogonal Matching Pursuit (OMP) algorithm is performed to reduce the feedback load. The effects of the sparsity level and feedback measurement are

examined on the sum data rate performance. The zero-forcing (ZF) precoding is employed after.

The remainder of this paper is organized as follows. The system model, and the sparse Massive MIMO channels are described in Section II. The proposed algorithm method is presented in detail in Section III. The simulation parameters and the simulation results are provided in Section IV, followed by conclusions in Section V.

II. SYSTEM MODEL

We consider the Massive MISO system model with M antennas at the BS and K single-antenna users, under the assumption of $M >> K >> 1$ as shown in Figure 1. By employing a ULA antenna model, neighboring antennas are spaced by $d = \lambda/2$ with λ which is the wavelength of the carrier frequency.
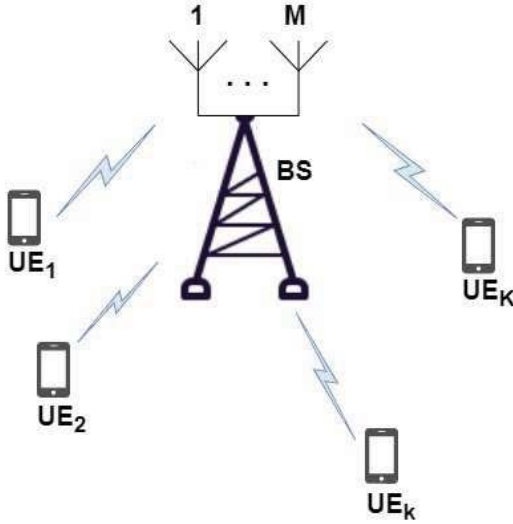

Fig. 1. Massive MIMO System Model.

The received signal at the kth user equipment (UE) is:

$$y_k = (\mathbf{h_k})^H \left(\mathbf{w_k} x_k + \sum_{t=1, t \neq k}^{K} \mathbf{w_t} x_t \right) + n_k \tag{1}$$

where $\mathbf{h_k} \in \mathbb{C}^{Mx1}$ is the channel vector, $\mathbf{w_k} \in \mathbb{C}^{Mx1}$ is the kth UE precoder, the transmitted symbol vector as $\mathbf{x} = [x_1, \ldots, x_K]^T \in \mathbb{C}^{Kx1}$, and n_k is the complex additive white Gaussian noise (AWGN) term with zero mean and σ_n^2 variance $\mathcal{CN}(0, \sigma_n^2)$. The first term of the right side of Eq. (1) contains the desired signal for the kth user, the second term represents the interference caused by the other users, and the last term is the background noise.

For two-dimensional (2D) channel models, the channel vector $\mathbf{h_k} \in \mathbb{C}^{Mx1}$ for the kth user due to the effect of clusters of scatterers is written as follows:

$$\mathbf{h_k} = \sum_{i=1}^{P} \mathbf{s} \left(\phi_{k,i}^{NLOS} \right) \ g_{k,i}^{NLOS} \tag{2}$$

where P is the number of NLOS paths from the BS to the kth user, $g_{k,i}^{NLOS} \sim \mathcal{CN}(0, 1)$ denotes the complex gain of ith path of the kth user representing independent and identically distributed (i.i.d.) with zero mean and the given variance, $\mathbf{s}$ is a steering vector, and $\phi_{k,i}^{NLOS}$ is the angle of azimuth of the ith path of the kth user. It is expressed as $\phi_{k,i}^{NLOS} \triangleq \phi + \delta_p$ with a deterministic nominal angle ϕ and a random deviation δ_p from the nominal angle with angular standard deviation (ASD) σ_ϕ.

The nominal angle ϕ and the ASD σ_ϕ of the multi-path components are key parameters to model the spatial correlation matrix. The angular deviation δ_p can be modeled as a uniform variate which lies within $\delta_p \sim U \left[-\sqrt{3}\sigma_\phi, \sqrt{3}\sigma_\phi \right]$.

The steering vector $\mathbf{s}$ denotes the antenna array response of the ith path of the kth user in the direction of ϕ^{NLOS}. For ULA antenna models, the array response [9] is defined as:

$$\mathbf{s} \left(\phi_{k,i}^{NLOS} \right) = \frac{1}{\sqrt{M}} \left[1, \ e^{j2\pi \frac{d}{\lambda} sin \left(\phi_{k,i}^{NLOS} \right)}, \ldots \right.$$
$$\left. , e^{j2\pi \frac{d}{\lambda} \left(M-1 \right) sin \left(\phi_{k,i}^{NLOS} \right)} \right]^T \tag{3}$$

We rewrite the channel vector as:

$$\mathbf{h}_k = \mathbf{S}_k \ \mathbf{g}_k \tag{4}$$

with $\mathbf{S}_k = \left[\mathbf{s} \left(\phi_{k,1}^{NLOS} \right), \mathbf{s} \left(\phi_{k,2}^{NLOS} \right), \ldots, \mathbf{s} \left(\phi_{k,P}^{NLOS} \right) \right] \in \mathbb{C}^{MxP}$, $\mathbf{g}_k = [\mathbf{g}_{k,1}, \mathbf{g}_{k,2}, \ldots, \mathbf{g}_{k,P}] \in \mathbb{C}^{Px1}$, and the channel matrix is $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K] \in \mathbb{C}^{MxK}$.

The received signal composed with all UEs is:

$$\mathbf{y} = \mathbf{H}^H \mathbf{W} \mathbf{x} + \mathbf{n}, \tag{5}$$

where $\mathbf{y} = [\mathbf{y_1} \ldots \mathbf{y_K}]^T \in \mathbb{C}^{Kx1}$, and the precoder matrix is $\mathbf{W} = [\mathbf{w_1}, \ldots, \mathbf{w_K}] \in \mathbb{C}^{MxK}$, which is determined with ZF precoding as follows:

$$\mathbf{W} = \eta \mathbf{H} \left(\mathbf{H}^H \mathbf{H} \right)^{-1} \tag{6}$$

In order to keep the short-term power constant, the factor η is calculated as:

$$\eta = \frac{1}{\sqrt{tr \left(\left(\mathbf{H}^H \mathbf{H} \right)^{-1} \right)}}. \tag{7}$$

The signal-to-interference-plus-noise ratio (SINR) for k^{th} user is:

$$\gamma_k = \frac{\left| \mathbf{h_k}^H \mathbf{w_k} \right|^2}{\sum_{j=1, j \neq k}^{K} \left| \mathbf{h_k}^H \mathbf{w_j} \right|^2 + (1/\rho)}, \forall k \in K \tag{8}$$

with $\rho = \frac{P}{\sigma_n^2}$ and, P is the total transmit power.

The average sum data rate is calculated as:

$$R = \sum_{k=1}^{K} \mathbb{E} \left\{ \log_2 \left(1 + \gamma_k \right) \right\} \tag{9}$$

III. Proposed Algorithm

There are several different user selection algorithms such as the capacity based greedy user selection algorithm, the capacity based greedy user selection algorithm with reduced user search space, and the semi-orthogonal user group (SUS) selection. The pair-wise semi-orthogonal user selection algorithm has been presented in [10].

The proposed user selection algorithm eliminates the users based on the channel correlation by employing the CS algorithm which reduces the feedback overhead in the system. The steps of the proposed user selection algorithm are explained in the following three subsections.

A. Sparsity Mapping

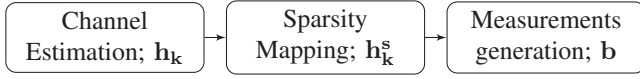The basic procedure of the sparsity mapping is illustrated in Figure 2 representing the user side process.

Fig. 2. User Side Process.

The downlink channel vector $\mathbf{h_k}$ is firstly estimated at the user side. In this paper, we assume that each user estimates its channel vector perfectly.

Due to the antenna correlation at the BS and limited local scattering effects, most of the multi-path energy in each user channel vector tends to be concentrated in a relatively small region within the virtual angular domain. The channel is expected to have a sparse representation in the virtual angular domain, so only a small fraction of components is significant and the others are zero.

Original channel vector $\mathbf{h_k}$ is represented using virtual channel representation with a proper basis,

$$\mathbf{h_k} = \mathbf{U}\mathbf{h_k^s} \tag{10}$$

where $\mathbf{h_k^s} \in \mathbb{C}^{Mx1}$ is the channel representation in the virtual angular domain, and $\mathbf{U}$ is DFT matrix with $\mathbb{C}^{MxM}$.

The nth column of $\mathbf{U}$ is given by

$$\mathbf{u}_n \triangleq \frac{1}{\sqrt{M}} \left[1 \ldots e^{-j\frac{2\pi(m-1)n}{M}} \ldots e^{-j\frac{2\pi(M-1)n}{M}} \right]^T \tag{11}$$

for $m, n = 1, 2, \ldots, M$.

After original CSI $\mathbf{h_k}$ is mapped to a sparse signal $\mathbf{h_k^s}$, the measurement matrix is used to reduce the number of samples to be the original signal. For the sparse signal $\mathbf{h_k^s}$, m feedback measurements (FM), $\mathbf{b} \in \mathbb{C}^{mx1}$ is generated as follows:

$$\mathbf{b} = \mathbf{A}^T \mathbf{h_k^s}, \tag{12}$$

where $\mathbf{A} \in \mathbb{C}^{Mxm}$ is a measurement matrix. The measurement matrix, $\mathbf{A}$, is generated off-line and known at both the user and the BS sides. $\mathbf{A}$ is sampled from i.i.d. Gaussian distributed entries with zero mean and $1/m$ variance.

After the random measurement vector, $\mathbf{b}$, is generated, it is fed back to the BS to perform downlink precoding.

B. Channel Vector Reconstruction

The basic procedures of the applying CS algorithm and the proposed user selection are illustrated in Figure 3 representing the BS side process.
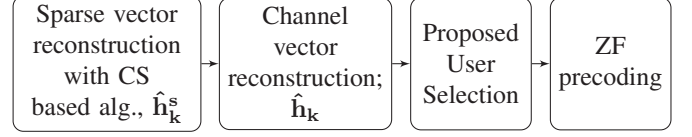

Fig. 3. BS Side Process.

To fully utilize the spatial multiplexing gains and the array gains of Massive MIMO, the CSIT is essential. However, it is inefficient to estimate the entire CSI matrices using long pilot training symbols at the BS. We should exploit the hidden sparsity in the CSIT estimation and feedback process. CS is the efficient reconstruction of a sparse signal from a few samples.

At the BS, CS-based algorithm, OMP is used for reconstructing the sparse signal $\hat{\mathbf{h}}_\mathbf{k}^\mathbf{s}$. In the OMP, all columns of $\mathbf{A}$ are correlated with the $\mathbf{b}$. The algorithm reconstructs the sparse signal iteratively. In each iteration, the algorithm finds the column of $\mathbf{A}$ which is most correlated to the $\mathbf{b}$ and adds its index. The stopping criterion is based on the sparsity level (SL) value.

The reconstructed CSI $\hat{\mathbf{h}}_\mathbf{k}$ is obtained by mapping the reconstructed sparse signal $\hat{\mathbf{h}}_\mathbf{k}^\mathbf{s}$ back based on the same basis used at the user side. After that, the proposed user selection algorithm is applied to the reconstructed CSI. The detail of the proposed algorithm is given in the following section.

Finally, the BS performs the downlink linear precoding scheme called ZF precoding to eliminate inter-user interference and achieve the sum data rate based on the reconstructed channel matrix $\hat{\mathbf{H}}$ instead of $\mathbf{H}$ in Eq. (6).

C. Proposed User Selection Method

The main objective of the proposed algorithm is to improve the sum data rate performance and reduce the complexity.

The steps of the proposed user selection algorithm are as follows:

- **Step 1**: Initialization:

$$S_0 = \{1, \ldots, K\}, \quad \mathcal{S} = \emptyset \tag{13}$$

- **Step 2**: Determine the degrees of orthogonality $\beta_{k,j}$ between all UE channel pairs $j \neq k$:

$$\beta_{k,j} = \frac{\left| \hat{\mathbf{h}}_k^H \hat{\mathbf{h}}_\mathbf{j} \right|}{\left\| \hat{\mathbf{h}}_k \right\| \left\| \hat{\mathbf{h}}_j \right\|} \tag{14}$$

Note that $\beta_{k,j} = \beta_{j,k}$.

- **Step 3**: Select the UE to be eliminated according to the degree of orthogonality.

Define $\mathcal{US}_k$ set that holds the users whose degrees of orthogonality higher than β for k^{th} UE.

$$\mathcal{US}_k = \{j \in S_0 : \beta_{k,j} > \beta, \forall k \in S_0\} \tag{15}$$

where β is a small positive constant value. It characterizes the allowed degree of orthogonality between two channel vectors.

- **Step 4**: For k^{th} UE, if the number of $\mathcal{US}_k$ set is different from 0, eliminate this k^{th} UE from the S_0 set. Thus, we obtain the selected user set $\mathcal{S}$ as follows:

$$\mathcal{S} = \mathcal{S} \cup \{k\}, \text{ if Card } (US_k) = 0. \qquad (16)$$

The precoding matrix and the sum data rate are calculated based on the set of selected UEs, $\mathcal{S}$.

IV. SIMULATION RESULTS

In the considered Massive MISO system, there is one BS with M antennas between 20 and 60, and $K = 10$ users with single-antenna. There are only NLOS components in the system; the total path for each UE is 10, with $\sigma_\phi = 2.8284$. The optimal β is determined through the numerical simulations.

We examine the sum data rate versus the number of BS antennas and compare the performance of the proposed algorithm with the algorithm in [10], and all UEs selected. We consider the different β values, the feedback measurement, FM, and the sparsity level, SL, while providing the performance results.

Figure 4 shows the sum data rate performance of the proposed algorithm as a function of different β values for FM= $M/5$ and SL= $M/10$. The β values are taken as $0.3, 0.5$ and 0.8. It is indicated that the user selection becomes more important as the number of BS antennas is reduced. The results show that $\beta = 0.3$ has the best sum data rate performance since it allows to eliminate the higher number of UEs. Thus, we fix $\beta = 0.3$ for the following results.
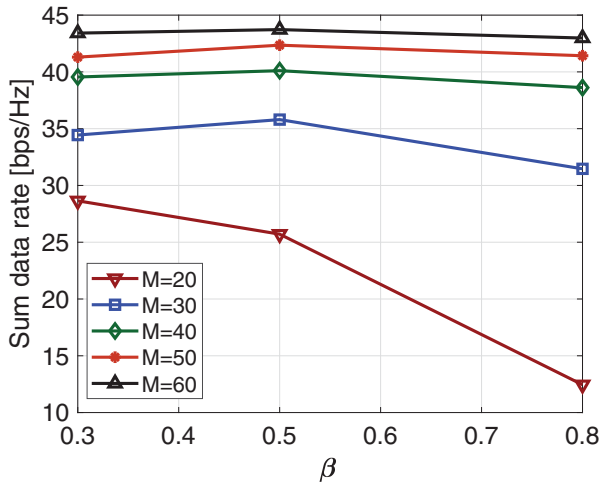

Fig. 4. Comparison of Different β Values in the Proposed Algorithm for K=10, FM= $M/5$ and SL= $M/10$.

Figure 5 shows the sum data rate performance of the proposed algorithm with the $\beta = 0.3$ for FM= $M/2$, and SL changes as $M/2$, $M/5$ and $M/10$. It is seen that higher sparsity level improves the sum data rate performance. This is because the CS algorithm has better performance to reconstruct the sparse signal with higher sparsity levels which

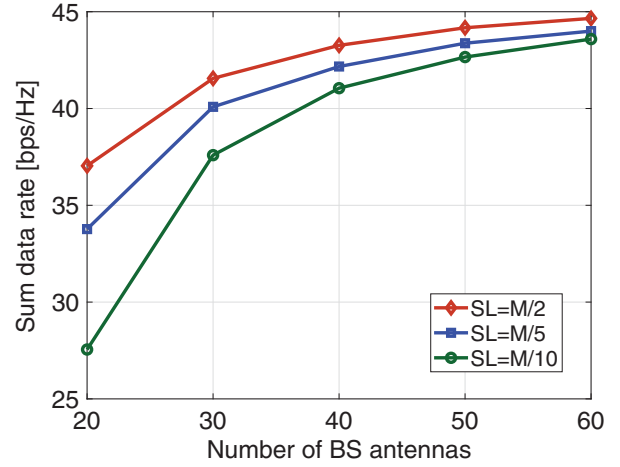indicate the number of the non-zero coefficients of the original signal.

Fig. 5. Sparsity Level Comparison in the Proposed Algorithm for K=10, β=0.3 and FM= $M/2$.

Figure 6 shows the sum data rate performance of the proposed algorithm with the $\beta = 0.3$ for SL= $M/10$ and FM changes as $M/2$, $M/5$ and $M/10$. The numerical results show that when the FM is increased, the sum data rate is also improved. The reason is that increasing the number of measurements that send to the BS side with the feedback link.
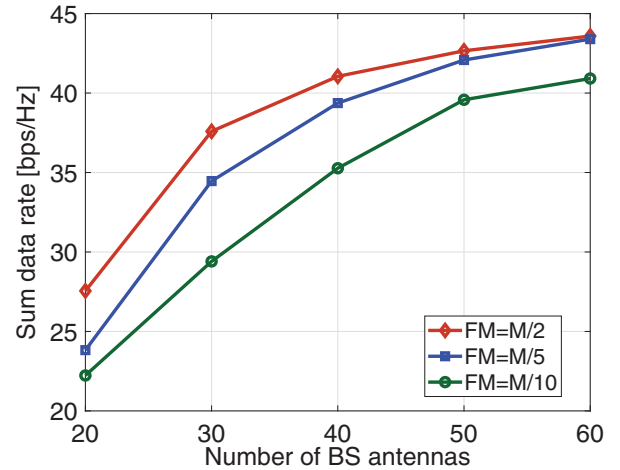

Fig. 6. Feedback Measurement Comparison in the Proposed Algorithm for K=10, β=0.3 and SL= $M/10$.

Figure 7 illustrates the sum data rate performance of all user selection algorithms when the FM, $M/2$, and SL, $M/2$, are the same and also $\beta = 0.3$ for the algorithm in [10] and the proposed algorithm. It is illustrated that for a smaller number of BS antennas, applying user selection is more advantageous. It is because that channel correlation is high for the smaller excess of BS antennas and applying the user

selection algorithm improves the system performance. It is worth noting that the proposed algorithm outperforms in terms of sum data rate performance over all UEs selected and the algorithm in [10].
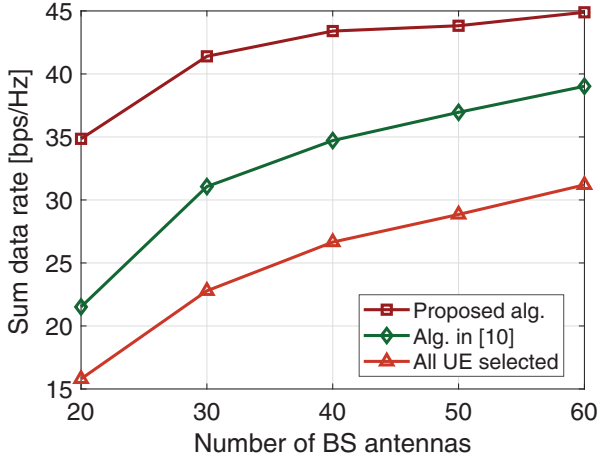

Fig. 7. Comparison of the User Selection Algorithms for K=10, β=0.3, FM= $M/2$ and SL= $M/2$.

V. CONCLUSIONS

In this paper, we have proposed the user selection algorithm by reconstructing the sparse channel of Massive MIMO with the OMP to reduce the feedback overhead. We have shown that for a smaller number of BS antennas, applying user selection is more advantageous.

The proposed algorithm eliminates users according to the channel correlation to improve the sum data rate performance. Also, the effects of feedback measurement and sparsity level parameters have been analyzed through the simulations. Since the feedback measurement and the sparsity level have a severe impact on the sum data rate performance, an optimal choice of these parameters is important. If the sparsity level of the channel is high, the sum data rate performance has been improved for the fixed number of feedback measurements. The simulation results show that the proposed algorithm outperforms the traditional approaches in terms of sum data rate, reduced feedback overhead and low complexity.

ACKNOWLEDGMENT

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