Effect of Levy Type Load Fluctuations on the Stability of Single Machine Infinite Bus Power Systems

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***Abstract*—In this paper, the stochastic excitations in single machine infinite bus power systems have been modeled as alpha-stable Levy processes. Through the simulations of the corresponding stochastic differential equations, we have shown that the impulsiveness and/or asymmetry in the distributions of the load fluctuations can cause the instability of the rotor angle. Hence, the synchronism is lost and the rotor angle although it is stable in the sense of probability, it might not be stable in the mean square sense. However, by properly choosing the parameters of Levy type fluctuations the rotor angle stability can be improved in the sense of probability as the beneficience of noise.**

I. Introduction

Stability in electrical power systems has been an important old problem [1] and the power system stability is still a major problem to prevent the large-scale blackouts in today's complex power grids [2]. The stability problem can be categorized as the rotor angle stability, the voltage stability and the frequency stability. Rotor angle stability is related to the dynamics of generator rotor angles that is the ability of interconnected synchronous machine of a power system to remain in synchronism [3]. Frequency stability is related with the active power balance between the generation and the consumption in the grid and the voltage stability is the ability of a power system subject to a given disturbance to maintain acceptable voltages at all buses [3]. In 2015, the blackout that was the third serious blackout in the Continental European (CE) System within the last 15 years has occurred in Turkey and it has been observed that the tripping of a line caused a loss of angular stability and a loss of synchronism in the Turkish power system [4]. To detect the loss of angular stability is therefore necessary for critical operation conditions. In [5], [6], [7], [8], [9] the variations in the reactive power demands of the loads have been considered as stochastic perturbations and some security measures to indicate the vulnerability to the voltage collapse have been given in [5]. In [10] it has been shown that random noise excitation can cause the single machine infinite bus (SMIB) system to become unstable. The p-moment stability of rotor angle in SMIB system and the influences of noise in the excitation on the dynamical behaviors of a power system have been given in [11]. The Fokker-Planck equation has been developed to model the evolution of the probability density function in stochastic SMIB and the impact of perturbations in the load on the rotor stability have been analysed in [12]. In all the papers [5]-[12], the stochastic fluctuations in electrical power systems either at the loads or at the excitations have been considered as Brownian process (Wiener process).

In this paper, the rotor angle stability phenomena of SMIB, in the case of stochastic fluctuations of Lévy type at the load have been investigated. The simulations show that the rotor angle stability have been effected by the impulsiveness and/or skewness (asymmetry) in the distributions of fluctuations. We have assumed that the stochastic disturbances occurring in power systems could be more realistically modeled by alpha-stable (α-stable) Lévy process compared to the modelling by Wiener process. The main motivation for our assumption is that in [13] the stochastic model of the electricity prices has been proposed as α-stable Lévy process and in which the load has been considered as one of the main factors in determining electricity prices because the sudden demand or supply changes cause sharp spikes in electricity prices, which are characterized by non-Gaussian and heavy-tailed behaviour defined by stable law [17]. In [14] the electricity market data have been also modeled by using the α-stable periodic autoregressive model (PAR).

II. Deterministic Single Machine Infinite Bus Power Systems

The deterministic swing equations in [3] which governs the rotational dynamics of the synchronous machine are given as

$$\begin{aligned} \dot{\delta} &= w \\ M\dot{w} &= -Dw + P_m - P_e \end{aligned} \tag{1}$$

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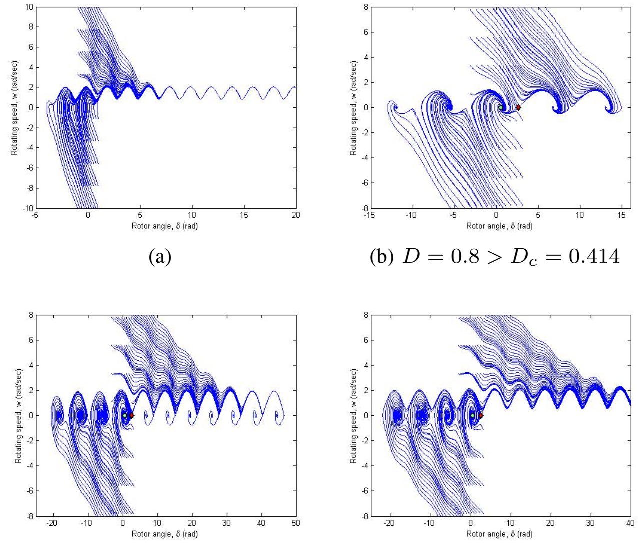
where δ is the relative rotor angle of synchronous generator, w is the rotor speed with respect to the synchronous reference, P_m is the mechanical input power, P_e is the electrical power output, M and D are the inertia and the damping coefficients, respectively. $P_e = P_{max} sin(\delta)$ where the maximum output of the synchronous generator is $P_{max} = E^{'} E_B / X_T$ and $E' \angle \delta$ is the internal voltage of generator and $E_B \angle 0$ is the infinite bus voltage; X_T is the total reactance of the transformer and the line. In our study $M = 1.0$, $E^{'} = 1.0$, $E_B = 1.0$, $X_T = 1.0$ are fixed which are typical in per-unit in power systems. Defining the state variable $\begin{bmatrix} x_1 & x_2 \end{bmatrix}^T = \begin{bmatrix} \delta & w \end{bmatrix}^T$ then (1) becomes as

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -Dx_2 + P_m - sin(x_1) \end{aligned} \quad (2)$$

The fixed points of (2) satisfy $x_2^* = 0$ and $sin x_1^* = P_m$. By linearizing the state equations around the equilibrium points it can be easily seen that $\begin{bmatrix} x_1{}^* & x_2{}^* \end{bmatrix}^T = \begin{bmatrix} \arcsin(P_m) & 0 \end{bmatrix}^T$ corresponds to the stable equilibrium point (SEP) which is indicated by green circle while the state $\begin{bmatrix} x_1{}^* & x_2{}^* \end{bmatrix}^T = \begin{bmatrix} \pi - \arcsin(P_m) & 0 \end{bmatrix}^T$ corresponds to the saddle point which is indicated by red diamond. The phase portraits of deterministic SMIB system shown in Fig. 1 have been obtained for the inital values of "$[-\pi, \pi] \times [-10, 10]$". Since the relative rotor angle δ is periodic with period 2π then there are multiple equilibria in the state space, "$\delta - w$ plane", and therefore in the corresponding cylindrical state space "$[-\pi, \pi] \times R$" there is only one SEP and one saddle.

The similar analysis for Josephson junction which is analogous to the classical, driven pendulum can be found in [15]. It is clearly seen from (2) that there are no fixed points if $P_m > 1$ and all trajectories converge to the unique rotating orbit as shown Fig. 1a. When the mechanical power is fixed as $P_m = 0.5$ then the responses have been obtained by varying the damping parameter D relative to the critical damping level D_c which corresponds to the value satisfying the equation of homoclinic bifurcation curve $P_m = 4D_c/\pi$ which can be obtained by using Melnikov method given in [16]. For $P_m = 0.5$ the critical damping level D_c is theoretically 0.3927 and numerically 0.414. In Fig. 1b the damping parameter D is chosen to be greater than the critical damping level D_c and for this case the trajectories converge to the SEP. In Fig. 1c the damping parameter D is equal to the critical damping level D_c, and all the trajectories converge to the SEP sooner or later. In Fig. 1d the damping parameter D is chosen to be less than the critical damping level D_c and the system has a SEP and a stable limit cycle and in this bistable case depending on the initial condition the trajectories converge either to the SEP or to the stable limit cycle (rotating orbit). In the steady state, there is a balance between the mechanical power input and the electrical power output and the generator runs at a constant speed which leads to a constant rotor angle (i.e., at the equilibrium point $\dot{\delta} = 0$). However, under any disturbance such as random load change, line tripping and loss of generator then an imbalance between the mechanical power input and the electrical power output occurs and the synchronism is lost. As we have mentioned in the introduction, our motivation of choosing L$\acute{e}$vy type fluctuations is that it admits impulsive and asymmetric fluctuations which can be modeled by α-stable random variable [17]. Therefore in the sequel L$\acute{e}$vy type fluctuations in the load have been considered.



(a) (b) $D = 0.8 > D_c = 0.414$

(c) $D = D_c = 0.414$ (d) $D = 0.36 < D_c = 0.414$

Fig. 1. Phase portraits of deterministic SMIB system for (a) $P_m > 1$ and (b)-(d) $P_m = 0.5$.

III. Stochastic Single Machine Infinite Bus Power Systems

The imbalance between the mechanical power input and the electrical power output in the SMIB power system given in (1) is modelled by α-stable L$\acute{e}$vy process as $P_L(t) = \sigma L_\alpha(t)$ and $L_\alpha(t)$ is the alpha-stable L$\acute{e}$vy process and σ is the noise intensity and using state variables then the It$\hat{o}$ form of SDE can be written as :

$$d\mathbf{X}(t) = \mathbf{f}(t, \mathbf{X}(t))dt + \mathbf{g} dL_\alpha(t) \quad (3)$$

$$\mathbf{f}(t, \mathbf{X}(t)) = \begin{bmatrix} x_2 \\ -Dx_2 + P_m - sin x_1 \end{bmatrix}; \ \mathbf{g} = \begin{bmatrix} 0 \\ \sigma \end{bmatrix} \quad (4)$$

and the increments of the L$\acute{e}$vy process $dL_\alpha(t)$ is α-stable random variable [17] and its distribution is denoted by α-stable distribution S_α (γ,β,μ) which is characterized by the four parameters: μ denotes the location, γ is scale parameter, the characteristic exponent α ($0 < \alpha \leq 2$) which measures the impulsiveness, and the skewness parameter β which measures the symmetry of the distribution, where $\beta = 0$ refers to symmetric distribution, $\beta < 0$ to left-skewed distribution and $\beta > 0$ to right-skewed distribution. As the value of characteristic exponent "α" decreases then the impulsiveness increases and hence the tails of the corresponding distributions become heavier. Also, the increase in the absolute value of the β results in the more asymmetric (skewed) distribution.

$L_\alpha(t) : t \geq 0$ stands for α-stable Lévy motion in [18], [19]:

- $L_\alpha(0) = 0$ almost surely (a.s.), and L_α(t) has the independent and stationary increments "$dL_\alpha(t)$",
- $dL_\alpha(t) \doteq L_\alpha(t) - L_\alpha(s) \sim S_\alpha((t-s)^{1/\alpha}, \beta, 0)$, $s < t$.

Under the load fluctuations of Gaussian type, (2) have been analyzed in [10] and in [11] where $P_L = \sigma W(t)$.

Remark : Brownian motion is the special case of α-stable Lèvy motion with $\alpha = 2, \ \beta = 0$ " i.e., $S_2(\gamma, 0, \mu) = N(\mu, 2\gamma^2)$ " Normal (Gaussian) distribution with mean μ and variance $2\gamma^2$ [17].

The approximate numerical solution of (3) can be obtained by applying the Euler-Maruyama approximation given in [18], [20] as

$$X_{t_i} = X_{t_{i-1}} + f(t_{i-1}, X(t_{i-1}))\tau + g(t_{i-1}, X(t_{i-1})) \triangle L^{\tau}_{\alpha,i} \quad (5)$$

where the increment of the Lévy process is α-stable random variable $\triangle L^{\tau}_{\alpha,i}$ defined by $\triangle L^{\tau}_{\alpha,i} = L_\alpha([t_{i-1}, t_i]) \sim S_\alpha(\tau^{1/\alpha}, \beta, \mu)$ with $\tau = t_i - t_{i-1}$ have been generated by the method given in [18].

IV. Numerical Results in the case of Levy type fluctuations at the load

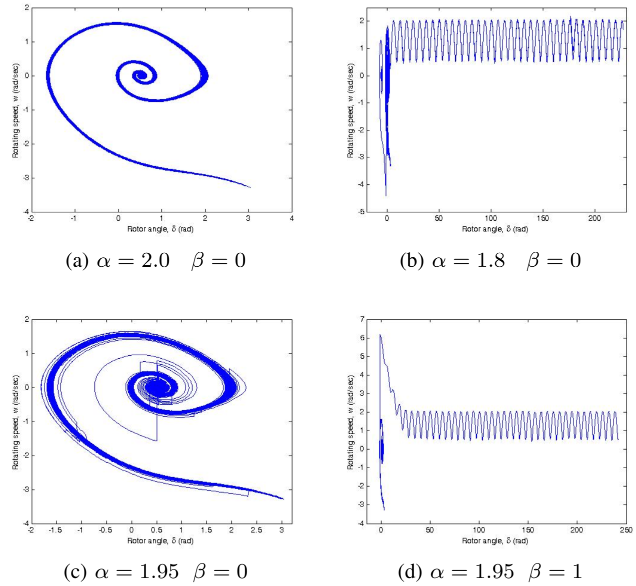
As mentioned in Section II for the deterministic SMIB system when the damping parameter D is less than critical damping level D_c a stable limit fixed point and a limit cycle coexist and in this bistable case, depending on the initial conditions, the trajectories converge either to SEP or to the rotating orbit (limit cycle) which is undesired for the rotor angle stability of SMIB. In this paper, based on a single initial condition, the variation of the basin of attraction of the SEP and the limit cycle under the stochastic load fluctuations have been observed. However, by choosing all initial conditions in the basin of attraction of deterministic SMIB system as in [21], the variation of the basin of attraction over the impulsiveness and skewness parameters will be further investigated. In the sequel, the numerical solutions of the phase portraits of generator angle and speed responses have been obtained for 1000 realizations for 200 seconds with the step size $\tau = 0.01$ and the noise intensity $\sigma = 0.01$.

A. Variation of basin of attraction of SEP by increasing impulsiveness and/or skewness

In this part, we have chosen an initial condition whose trajectory converges to the SEP for the deterministic SMIB and then for this initial condition we have obtained 1000 trajectories which correspond to the realizations of stochastic SMIB.

1) "Gaussian type fluctuations" (Fig. 2a): Under the Gaussian type fluctuations in the load ($\alpha = 2$, $\beta = 0$) the angle responses converge to the SEP and the SEP is stable in the mean square sense.

2) "Increasing the impulsiveness" (Fig. 2b): However, at the load fluctuations by increasing the impulsiveness by choosing $\alpha = 1.8$ while preserving the symmetry at the fluctuations (i.e., $\beta = 0$) and hence aparting from the Gaussian fluctuations although the majority of 1000 realizations converge to the SEP



(a) $\alpha = 2.0 \quad \beta = 0$ (b) $\alpha = 1.8 \quad \beta = 0$

(c) $\alpha = 1.95 \quad \beta = 0$ (d) $\alpha = 1.95 \quad \beta = 1$

Fig. 2. (a) Wiener type fluctuation (b)-(d) Lévy type fluctuations in the load with various α and β parameters.

but for few of realizations sudden jumps occur in the rotor speed and these trajectories converge to the rotating orbit (limit cycle) which is undesired for the rotor angle stability.

3) "Distorting the symmetry" (Fig. 2d): Although the rotor angle responses converge to the SEP for $\alpha = 1.95$, $\beta = 0$ as shown in Fig. 2c by increasing the skewness $\beta = 1$ few of realizations converge to the rotating orbit as in section IV-A2.

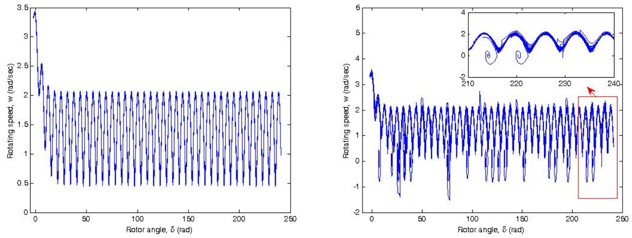
From the above observations we can conclude that if the load fluctuations apart from the Gaussian type fluctuations either by increasing impulsiveness and/or distorting the symmetry then the basin of attraction of the SEP changes and the SEP becomes no more stable in the mean square sense and some of the trajectories converge to the limit cycle. This important observation is distinct from the observation in [11] which states that under the Wiener type fluctuations at the load the SEP is stable in the mean square sense.

B. Variation of basin of attraction of limit cycle by increasing impulsiveness and/or skewness

In this part, we have chosen an initial condition whose trajectory converges to the limit cycle for the deterministic SMIB and then for this initial condition we have obtained 1000 trajectories which correspond to the realizations of stochastic SMIB.

1) "Gaussian type fluctuations" (Fig. 3a): When the random fluctuations in the load are modeled as Wiener process, all realizations converge to the rotating orbit and the rotor angle of the system is unstable both in the mean square sense and in the sense of probability.

2) "Increasing the impulsiveness" (Fig. 3b): When the random fluctuations in the load are modeled as symmetric Lévy process with $\alpha = 1.8$, $\beta = 0$ although a few of the trajectories converge to the SEP as shown in Fig. 3b (i.e.



(a) $\alpha = 2.0 \quad \beta = 0$ (b) $\alpha = 1.8 \quad \beta = 0$

Fig. 3. (a) Wiener type fluctuation (b) Lévy type fluctuations in the load.

TABLE I
PERCENTAGE OF STOCHASTIC TRAJECTORIES CONVERGE TO THE SEP

α / β	1.9	1.8	1.6	1.5	1.4	1.2
0	1.2%	3.3%	10.8%	12.7%	24.5%	43.7%
-1	1.7%	4.8%	19.3%	26.4%	38.1%	61.0%

zoomed version of red area) the majority of trajectories still converge to the rotating orbit (limit cycle) and hence the rotor angle is still unstable both in the sense of probability and in the mean square sense. However, with the increase of impulsiveness the percentage of trajectories which converge to the SEP for 1000 realizations increases as shown in Table I and hence the stability of the rotor angle improves in the sense of probability. This important observation is distinct from the reponse of Wiener type fluctuations where the rotor angle is unstable in the sense of probability which has been observed in section IV-B1.

3) "Distorting the symmetry": The similar behavior can be observed in the case of the deviation from the symmetry in the distributions of the load fluctuations. As shown in Table I under the asymmetric load fluctuations with $\beta = -1$, the percentage of trajectories which converge to the SEP increases with the increase of impulsiveness and hence the stability of the rotor angle improves in the sense of probability. However, for $\beta = 1$ the trajectories do not converge to the SEP, all trajectories converge to the limit cycle. This observation shows us that the tendency of the distribution of load fluctuations is also important for the rotor angle stability in the sense of probability.

V. CONCLUSION

In this paper, the fluctuations in the load of SMIB systems have been modeled as α-stable Lévy process and comparing with Gaussian type fluctuations in the load, the effect of the impulsiveness and/or skewness (asymmetry) in distributions of fluctuations have been analyzed numerically. The aparting from the Gaussianity in distributions of load fluctuations either by increasing impulsiveness and/or distorting symmetry cause the instability of rotor angle in the mean square sense for the basin of attraction of SEP as observed in section IV-A whereas for the basin of attraction of limit cycle as observed in section IV-B, Lévy type fluctuations improve the stability of rotor angle in the sense of probability and this result can be considered as the benefit of noise. As a future work, the variation of the basin of attraction of deterministic SMIB system under Lévy type perturbations will be investigated.

REFERENCES

[1] S. Abe, Y. Fukunaga, A. Isono, and B. Kondo, "Power system voltage stability," *IEEE Transactions on Power Apparatus and Systems*, no. 10, pp. 3830–3840, 1982.

[2] S. Gupta, F. Kazi, S. Wagh, and R. Kambli, *Neural Network Based Early Warning System for an Emerging Blackout in Smart Grid Power Networks*. Springer, 2015.

[3] P. Kundur, N. J. Balu, and M. G. Lauby, *Power system stability and control*. McGraw-hill New York, 1994, vol. 7.

[4] TEIAS and ENTSO-E, *Report on Blackout in Turkey on 31st March 2015*, 2015.

[5] C. De Marco and A. Bergen, "A security measure for random load disturbances in nonlinear power system models," *IEEE Transactions on Circuits and Systems*, vol. 34, no. 12, pp. 1546–1557, 1987.

[6] F. Milano and R. Zárate-Miñano, "A systematic method to model power systems as stochastic differential algebraic equations," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4537–4544, 2013.

[7] Y. Qiu, J. Zhao, and H. Chiang, "Effects of the stochastic load model on power system voltage stability based on bifurcation theory," *2008 IEEE/PES Transmission and Distribution Conference and Exposition*, pp. 1–6, 2008.

[8] K. Wang and M. L. Crow, "Numerical simulation of stochastic differential algebraic equations for power system transient stability with random loads," *2011 IEEE Power and Energy Society General Meeting*, pp. 1–8, 2011.

[9] D. Podolsky and K. Turitsyn, "Random load fluctuations and collapse probability of a power system operating near codimension 1 saddle-node bifurcation," *arXiv preprint arXiv:1212.1224*, 2012.

[10] D. Wei and X. Luo, "Noise-induced chaos in single-machine infinite-bus power systems," *EPL (Europhysics Letters)*, vol. 86, no. 5, p. 50008, 2009.

[11] Z. Lu, W. Zhao, D. Xie, and G. Li, "P-moment stability of power system under small gauss type random excitation," *Chaos, Solitons & Fractals*, vol. 81, pp. 30–37, 2015.

[12] K. Wang and M. L. Crow, "The fokker-planck equation for power system stability probability density function evolution," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2994–3001, 2013.

[13] R. Weron, *Modeling and forecasting electricity loads and prices: A statistical approach*. John Wiley & Sons, 2007, vol. 403.

[14] P. Kruczek, A. Wyłomańska, M. Teuerle, and J. Gajda, "The modified yule-walker method for α-stable time series models," *Physica A: Statistical Mechanics and its Applications*, vol. 469, pp. 588–603, 2017.

[15] S. H. Strogatz, *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering*, 2014.

[16] J. Guckenheimer and P. J. Holmes, *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields*. Springer Science & Business Media, 2013, vol. 42.

[17] G. Samoradnitsky and M. S. Taqqu, *Stable non-Gaussian random processes: stochastic models with infinite variance*. CRC press, 1994, vol. 1.

[18] A. Janicki and A. Weron, *Simulation and chaotic behavior of alpha-stable stochastic processes*. CRC Press, 1993, vol. 178.

[19] D. Applebaum, *Lévy processes and stochastic calculus*. Cambridge university press, 2009.

[20] P. E. Kloeden and E. Platen, *Numerical solution of stochastic differential equations*, ser. Applications of mathematics. Berlin, New York: Springer, 1999.

[21] P. Ji and J. Kurths, "Basin stability of the kuramoto-like model in small networks," *The European Physical Journal Special Topics*, vol. 223, no. 12, pp. 2483–2491, 2014.