Role of Fractional Powers in manuevering the Fractional Lower-Order Auto-Covariance of Symmetric alpha-stable noise signals

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***Abstract*—Since, Fractional Lower-Order Auto-Covariance (FLOAC) remains the only technique to quantify the similarity between alpha-stable (α-stable) signals, therefore, the effects of impulsiveness and skewness parameters has also been analyzed before for better generation and detection in various applications. This paper includes the detailed analysis of the FLOAC of symmetric alpha-stable (SαS) noise signals in order to observe the possible involvement of the associated fractional powers. The two associated fractional powers of FLOAC has been maneuvered in three possible ways to observe the probable trend of SαS noise signals in the presence and absence of Gaussian noise. The observation depicts that the fractional powers largely and solely affect the FLOAC when they are maneuvered collaboratively or even individually where the obtained results can be useful in improving many SαS noise signal processing techniques, especially, in the detection of SαS noise carrier signals in Random Communication Systems.**

Keywords—Fractional lower-order auto-covariance; Alpha-stable noise; Gaussian noise; Random communication system

I. Introduction

Alpha-stable (α-stable) noise signal's detection has remained a hot topic in the past where, especially, the Fractional Lower-Order Auto-Covariance (FLOAC) has been exploited to achieve better efficiency. The α-stable noise plays a key role in biomedical signal processing applications which involves α-stable noise filtering and neuroimaging [1, 2]. Especially, molecular communication systems also utilize α-stable noise as a single noise factor to mimic the effects of noise produced by several molecules in the environment [3]. Additionally, the involvement of α-stable noise in manipulating the neural networks has also been investigated [4]. Moreover, Random Communication Systems (RCS) [5] are the example of the on-going investigations related to α-stable noise where these noise signals have been used as carrier signals to perform covert communications. Alpha-stable noise-based communications is an unconventional form of covert communications where the idea was initially propagated in [6].

The FLOAC has helped in proposing the first synchronization for RCSs [7, 8] where the capability of impulsive and skewness parameters to fluctuate the FLOAC was observed [9]. However, no detailed study has been carried out then to explore the role of fractional powers in maneuvering the FLOAC of SαS signals. The obtained results in this paper would benefit different α-stable noise signal detection schemes, especially RCSs8. In the following Section-II and Section III, α-stable noise distribution and FLOAC has been briefly explained, respectively. In Section IV, the observed results are presented where the paper has been concluded in Section-V.

II. Alpha-Stable Distribution

The samples of α-stable noise can be produced via method given in [10] to generate the α-stable distribution. The sequence of α-stable noise, i.e. collection of consecutive α-stable noise samples, is represented by $X \sim S_\alpha(\beta, \gamma, \mu)$ where α has been defined as the impulsiveness parameter in the range $(0 < \alpha \leq 2)$, β has been defined as the skewness parameter defined in the range $\beta(-1 \leq \beta \leq 1)$, γ has been defined as the dispersion parameter in the range $(\gamma \geq 0)$ and μ has been defined as the location parameter which is $\mu \in R$ in [11]. The characteristic function of $X \sim S_\alpha(\beta, \gamma, \mu)$ is expressed in [11] as

$$\phi(\theta) = \begin{cases} \exp\{j\mu\theta - \gamma^\alpha |\theta|^\alpha (1 - j\beta sign(\theta) tan\left(\frac{\alpha\pi}{2}\right))\} & if\ \alpha \neq 1 \\ \exp\{j\mu\theta - \gamma |\theta| (1 + j\beta \frac{2}{\pi} sign(\theta) ln\left(\frac{\alpha\pi}{2}\right))\} & if\ \alpha = 1 \end{cases} \quad (1)$$

Note: $X_G \sim S_{\alpha=2}(\beta=0, \gamma_G, \mu_G)$, $X_C \sim S_{\alpha=1}(\beta=0, \gamma_C, \mu_C)$, $X_L \sim S_{\alpha=0.5}(\beta=1, \gamma_L, \mu_L)$ generates Gaussian, Cauchy and Levy distributed samples, respectively, as these are special cases of α-stable distributions.

III. Fractional Lower-order auto-covariance

Covariance and Correlation, i.e. conventional time delay estimation methods, cannot be applied for signal processing of α-stable noise signals for $\alpha < 2$ due to the absence of moments greater than second-order. Therefore, the resemblance among α-stable noise signals can be checked by FLOAC method [9, 12].

If the FLOAC of α-stable noise signal $X \sim S_\alpha(\beta, \gamma, \mu)$ is defined as

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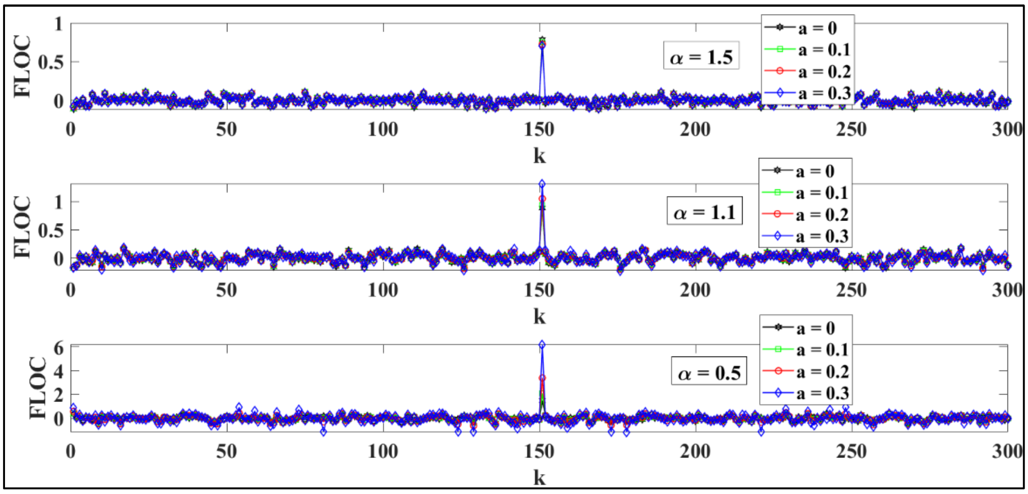

Fig. 1: $\boldsymbol{R_d[k]}$ for $S_{\alpha=0.5,\ 1.1,\ 1.5}$ (β = 0 μ, =0), $\boldsymbol{MSNR_{dB}}$ = -4 dB with various 'a'

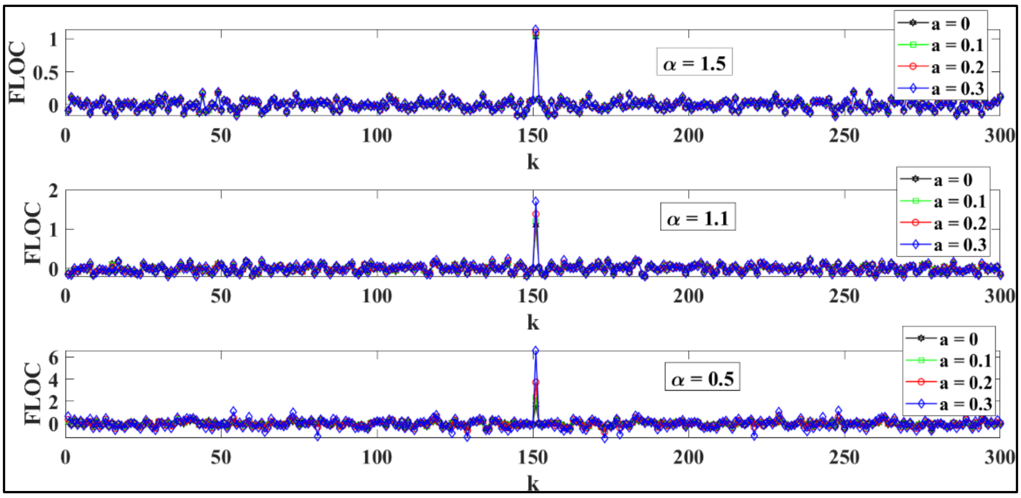

Fig. 2: $\boldsymbol{RG_d[k]}$ for $S_{\alpha=0.5,\ 1.1,\ 1.5}$ (β = 0, μ, =0), $\boldsymbol{MSNR_{dB}}$ = -4 dB with various 'a'

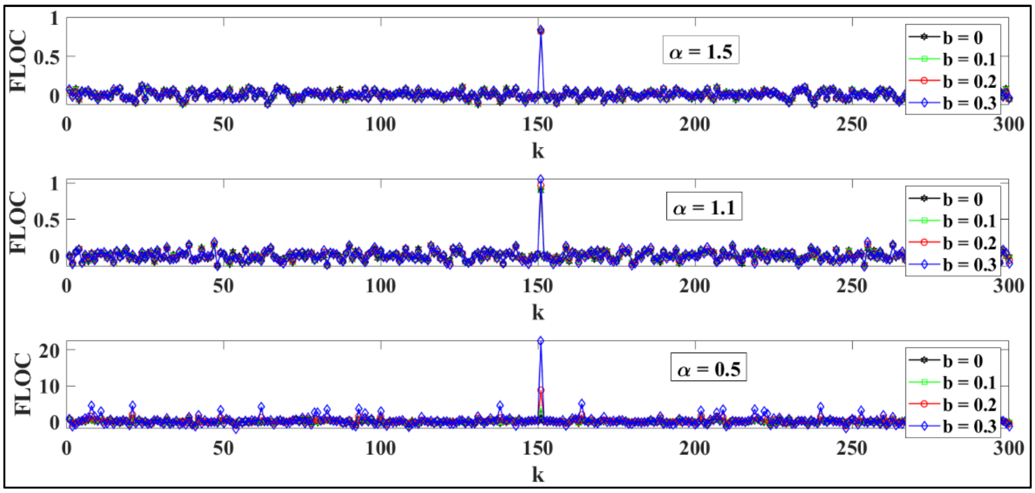

Fig. 3: $\boldsymbol{R_d[k]}$ for $S_{\alpha=0.5,\ 1.1,\ 1.5}$ (β = 0 μ, =0), $\boldsymbol{MSNR_{dB}}$ = -4 dB with various 'b'

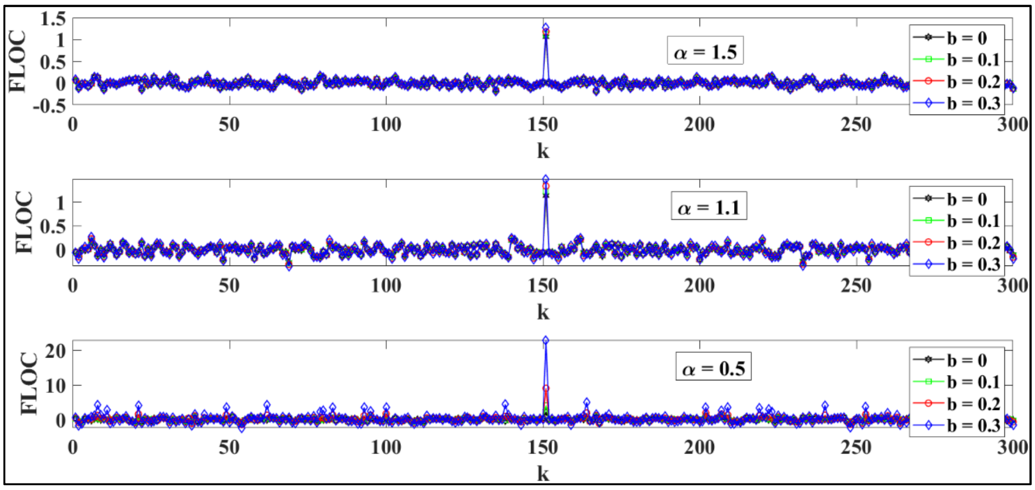

Fig. 4: $\boldsymbol{RG_d[k]}$ for $S_{\alpha=0.5,\ 1.1,\ 1.5}$ (β = 0, μ, =0), $\boldsymbol{MSNR_{dB}}$ = -4 dB with various 'b'

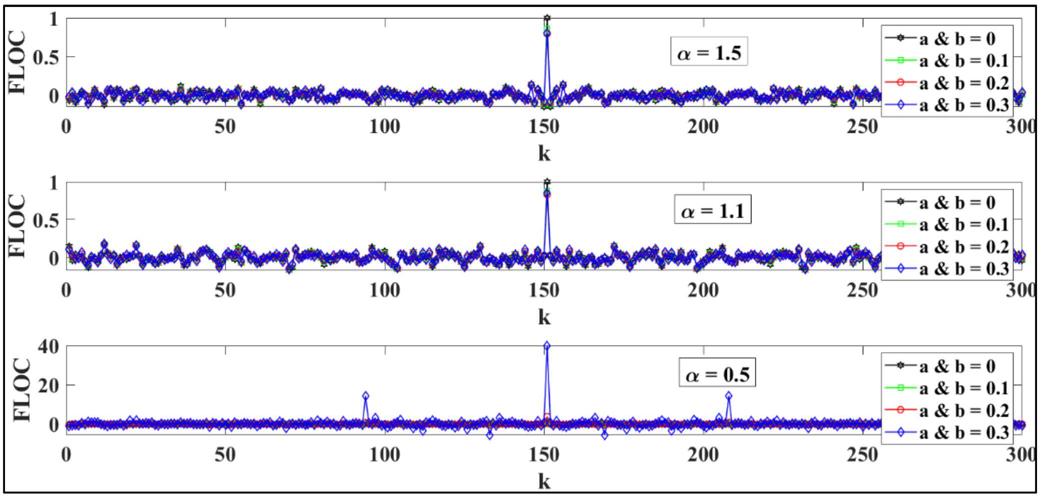

Fig. 5: $\boldsymbol{R_d[k]}$ for $S_{\alpha=0.5,\ 1.1,\ 1.5}$ (β = 0 μ, =0), $\boldsymbol{MSNR_{dB}}$ = -4 dB with various 'a = b'

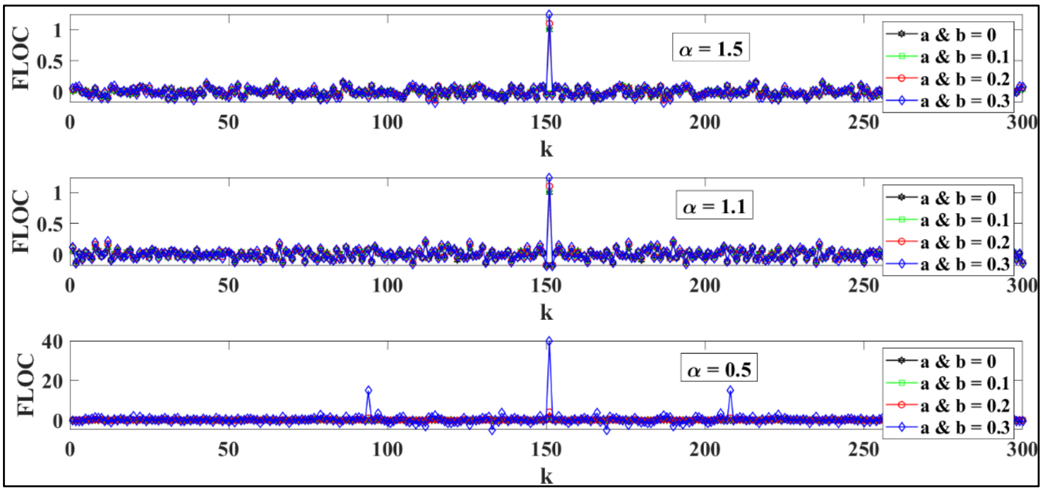

Fig. 6: $\boldsymbol{RG_d[k]}$ for $S_{\alpha=0.5,\ 1.1,\ 1.5}$ (β = 0, μ, =0), $\boldsymbol{MSNR_{dB}}$ = -4 dB with various 'a = b'

$$\boldsymbol{R_d[k]} \triangleq \boldsymbol{E}\ \{(x[i])^a \cdot (x[i+k])^b\} \qquad (2)$$

then the FLOAC, denoted by $R_d[k]$, of X, i.e. X = $\{x[1], \dots, x[n]; n = 1,2, \dots, N\}$, is estimated in [12] as

$$\boldsymbol{R_d[k]} = \frac{\sum_{n=N_1+1}^{N_2} |x[n]|^a . |x[n+k]|^b . sign(x[n] . x[n+k])}{N_2 - N_1} \qquad (3)$$

where N_1 = max (0, - *k*), N_2 = min *(N - k, N)*, $k = (\frac{-N}{2}, \frac{N}{2})$. Similarly, the noisy FLOAC, $RG_d[k]$, of α-stable noise signal X ~ S_α *(β, γ, μ)* added with X_G ~ $S_{\alpha=2}$ ($\beta = 0, \gamma_G, \mu_G$), i.e. X + X_G, has been computed as

$$\boldsymbol{RG_d[k]} = \frac{\sum_{n=N_1+1}^{N_2} |x+x_G[n]|^a . |x+x_G[n+k]|^b . sign(x+x_G[n] . x+x_G[n+k])}{N_2 - N_1} \qquad (4)$$

However, $R_d[k]$ and $RG_d[k]$ has been computed for specific values of fractional powers, i.e. $a = b = \frac{\alpha}{2}$ [5, 7-9, 13], and the effects of maneuvering the fractional powers for its complete range on $R_d[k]$ has not been analyzed yet.

IV. Analysis of Fractional powers

Therefore, in this section of the paper, the FLOACs of SαS signals, $\beta = 0$ and $\mu = 0$, i.e. $R_d[k]$ and $RG_d[k]$ in (3) and (4), respectively, has been computed by maneuvering 'a' and 'b' by covering entire range of α. The FLOAC, i.e. $R_d[k]$ in (3), has been analyzed for three possible values of impulsiveness parameter α, i.e. α = 0.5, 1.1, 1.5, to completely analyzed the behavior of FLOAC of SαS signals when 'a' and 'b' are varied in three possible ways, individually and collaboratively. Firstly, in Fig 1 and Fig. 2, the 'b' has been kept constant, i.e. b = 0.15, while fractional power 'a' has been varied, i.e. a = 0, 0.1, 0.2, 0.3. Secondly, in Fig 3 and Fig. 4, the 'a' has been kept constant, i.e. a = 0.15, while only the fractional power 'b' has been varied, i.e. b = 0, 0.1, 0.2, 0.3. Finally, in Fig 5 and Fig. 6, both the fractional power 'a' and 'b' has been varied, i.e. a = b = 0, 0.1, 0.2, 0.3. The Fig. 1 – 6 have been simulated by taking *N=300* and *MSNR* = -4 dB where the MSNR is defined in [12] as $MSNR_{dB} = 10\, log \frac{\gamma}{\gamma_G}$ where γ is the dispersion parameter of the α-stable noise signal X and γ_G is the dispersion parameter of the gaussian noise G.

The FLOAC, i.e. $R_d[k]$ in (3), shows an interesting behavior as 'a' and 'b' are varied in three different ways. In the first case, the computed FLOACs $R_d[k]$ and $RG_d[k]$, in Fig. 1 and Fig. 2, respectively, increases when 'a' increases up to the maximum limit of a, i.e. a ≤ $\frac{\alpha}{2}$ where $R_d[0]$ and $RG_d[0]$ is maximum when α is more impulsive, i.e. α=0.5. In the second case, the computed FLOACs $R_d[k]$ and $RG_d[k]$, in Fig. 3 and Fig. 4, respectively, again increases when 'b' increases up to the maximum limit of b, i.e. b ≤ $\frac{\alpha}{2}$ where $R_d[0]$ and $RG_d[0]$ is maximum when α is more impulsive, i.e. α=0.5. In the third case, the computed FLOACs $R_d[k]$ and $RG_d[k]$, in Fig. 5 and Fig. 6, respectively, increases when 'a' and 'b' increases up to the maximum limit of a and b, i.e. a = b ≤ $\frac{\alpha}{2}$ where $R_d[0]$ and $RG_d[0]$ is maximum when α is more impulsive, i.e. α=0.5.

It has been seen that $R_d[0]$ and $RG_d[0]$ is highest among $R_d[k]$ and $RG_d[k]$ irrelevant which value of fractional power and impulsiveness parameter has been used and it is inversely proportional to α.

V. Conclusion

Fractional lower-order auto-covariance of SαS signals has been analyzed while maneuvering the fractional powers. It has been concluded that FLOAC of SαS results in maximum value if higher values of fractional powers and lower values of impulsiveness parameter are utilized and vice versa. The above results would be useful in enhancing the detection process of α-stable noise signal processing techniques associated with biomedical and communications applications.

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