# On Operational Space Tracking Control of Robotic Manipulators With Uncertain Dynamic and Kinematic Terms

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*In this study, a continuous robust-adaptive operational space controller that ensures asymptotic end-effector tracking, despite the uncertainties in robot dynamics and on the velocity level kinematics of the robot, is proposed. Specifically, a smooth robust controller is applied to compensate the parametric uncertainties related to the robot dynamics while an adaptive update algorithm is used to deal with the kinematic uncertainties. Rather than formulating the tracking problem in the joint space, as most of the previous works on the field have done, the controller formulation is presented in the operational space of the robot where the actual task is performed. Additionally, the robust part of the proposed controller is continuous ensuring the asymptotic tracking and relatively smooth controller effort. The stability of the overall system and boundedness of the closed loop signals are ensured via Lyapunov based arguments. Experimental results are presented to illustrate the feasibility and performance of the proposed method.*
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## 1 Introduction

Most of the proposed past research on control of robot manipulators focused on improving joint space tracking performance, despite the fact that it is the end effector of the robot actually performing the desired task at the operational space (Cartesian space or end-effector space). This is mainly due to the fact that, on most robot manipulators, sensors measure joint variables and actuators are designed to apply the corresponding control input torque to the joints. The operational space variables are then calculated using the forward kinematics of the robot manipulator. Therefore, when the desired task is given in the operational space, one commonly accepted method is to convert the operational space reference/desired trajectory into the joint space trajectory by using inverse kinematics, and then ensure joint space tracking via the controllers applied to the joints. This method requires calculation of inverse kinematics of the manipulator in real time. In an alternative method, the tracking error term is defined in the operational space of the robot manipulator while the control input torque is again designed to be applied to the joints [1]. This technique avoids the calculation of inverse kinematics of the robot manipulator but requires the inverse Jacobian matrix.

Another challenge to be dealt with in control of robot manipulators is the presence of dynamic and/or kinematic uncertainties. From a theoretical point of view, when the dynamic model of the robot manipulator has structured/parametric uncertainties, adaptive control techniques are preferred. In view of this, several works addressed adaptive operational space tracking control to deal with kinematic uncertainties. In Ref. [2], Cheah proposed an adaptive law to estimate the uncertain kinematic model parameters for the approximate Jacobian method. Without requiring the operational space velocity and the inverse of the approximate Jacobian matrix, the authors in Refs. [3] and [4] proposed an approximate Jacobian controller for robot manipulators having uncertainties in kinematics and in Jacobian. In Ref. [5], Cheah presented approximate transpose Jacobian and inverse Jacobian methods for set point control of nonredundant robots with parametric uncertainties in kinematics. Adaptive controller formulations to deal with dynamical uncertainties were also presented in several works. In Refs. [6] and [7], the authors designed an adaptive controller that achieves asymptotic operational space tracking despite parametric uncertainties associated with the dynamic model. In Refs. [8] and [9], the authors developed a quaternion-based adaptive full-state feedback controller for redundant robot manipulators with parametric uncertainties in their dynamic model. In Refs. [10] and [11], the authors designed an adaptive feedback linearizing controller to compensate for the parametric uncertainties in dynamics. The study in Ref. [12] presented an adaptive operational space controller for redundant robots by considering time-varying uncertainties and without knowledge of their bounds. Recent works on adaptive control were realized for robot end-effector motion in Refs. [13] and [14]. While different research problems were tackled in the aforementioned past works, nearly all of them required the robot dynamics to be in a certain form (i.e., the linear parametrization property).

On the other hand, to deal with unstructured dynamical uncertainties, robust control techniques are utilized. In robust control literature, two methods are common: variable structure type controllers and high gain controllers. Variable structure type controllers for operational space tracking of robot manipulators presented in Refs. [15–21] are based on discontinuous control method. Zergeroglu et al. [15] developed a robust controller that achieves uniformly ultimately bounded end-effector and subtask tracking despite the parametric uncertainties associated with the dynamics and additive external disturbances. Braganza et al. [16] and Kapadia et al. [17] developed robust operational space tracking controllers for kinematically redundant robot manipulators in the presence of unstructured uncertainties. Recently, variable structure type controllers were designed for operational space tracking control of robot manipulators [18–21]. To eliminate the discontinuity, in Ref. [18], the authors integrated a fuzzy logic control method with sliding mode controller and achieved the robustness of the robot manipulator. In Ref. [19], to deal with dynamic uncertainties of robot manipulators, Galicki developed a terminal sliding mode algorithm that includes the first-order sliding mode

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method achieving asymptotic convergence and the second-order sliding mode method generating absolutely continuous control, then extended his study by considering the kinematic uncertainties in Ref. [20]. Galicki [21] proposed a Jacobian inverse-free robust controller to achieve asymptotic convergence and utilized a boundary layer control method to eliminate the undesirable chattering effects. Variable structure controllers ensure asymptotic tracking but usually require the use of switching functions, like the signum function, in their design, which results most of them to be discontinuous. Continuous approximations of the signum function such as hyperbolic tangent or saturation functions can be utilized but then asymptotic tracking will be lost. On the other hand, high gain controllers cannot usually ensure asymptotic tracking [22]. In our previous study [23], we proposed a continuous robust controller to ensure an asymptotic operational space tracking despite the presence of unstructured uncertainties associated with the dynamical terms. However, the previous study did not deal with the parametric uncertainties in the kinematic model and only performed on simulation studies. As a subclass of robust controllers, learning controllers also deal with unstructured uncertainties. For robot manipulators with periodic desired end-effector trajectory, Dogan et al. [24] developed an operational space learning controller that ensures asymptotic operational space tracking by learning the uncertainties associated with the robot dynamics.

In this study, operational space tracking control of robot manipulators is aimed. The direct method where the tracking error is designed in the operational space is preferred. The control problem is complicated by the presence of parametric uncertainties in the velocity kinematics. To address a realistic problem, the dynamics are considered to have both parametric and unstructured uncertainties. Furthermore, design of a continuous robust controller scheme is aimed. The adaptive portion of the controller formulation uses a gradient-based update to deal with the kinematic uncertainties and then for the robust part, a model free control scheme is utilized to compensate for the dynamical uncertainties where instead of making use of the signum of the error term, integral of the signum of an auxiliary term is utilized to ensure continuous controller structure. When compared to the previous adaptive and learning type operational space controller formulations, the proposed method, due to its robust nature, can compensate for a broader class of uncertainties. In addition, when compared to the robust controller formulations of the same kind, the proposed method ensures a smoother controller effort due to the continuous structure of the overall formulation. The asymptotic stability of the end-effector tracking error and the boundedness of the closed-loop signals are ensured via Lyapunov-type analysis. Experiments conducted on a PHANToM Omni haptic device (Control Laboratory of Izmir Institute of Technology in Turkey, Urla, Turkey) are presented to demonstrate the feasibility and the performance of the proposed controller.

The rest of the paper is organized in the following manner: Dynamic and kinematic models for the robot manipulator are given in Sec. 2. Error system formulation is developed in Sec. 3. Sections 4 and 5 present the design and the stability analysis of the robust operational space controller, respectively. The experimental results are given in Sec. 6. Finally, concluding remarks are given in Sec. 7.

## 2 Robot Manipulator Dynamic and Kinematic Models

The equations of motion describing an $n$-link nonredundant robot manipulator having revolute joints can be given in the following form [25]

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}, t) = \tau(t) \tag{1}$$

where $\theta(t)$, $\dot{\theta}(t)$, $\ddot{\theta}(t) \in \mathbb{R}^n$ are the joint position, velocity, and acceleration vectors, respectively, $M(\theta) \in \mathbb{R}^{n\times n}$ represents the inertia matrix, and the rest of the dynamical terms such as centripetal Coriolis, gravity, and friction effects are combined in $N(\theta, \dot{\theta}, t) \in \mathbb{R}^n$ which also includes additive bounded disturbances, and $\tau(t) \in \mathbb{R}^n$ is the control input torque.

The dynamical terms satisfy the following standard properties and assumptions.

Property 1. *The inertia matrix is positive-definite and symmetric and can be bounded from below and above as* [26,27]

$$m_1\|\xi\|^2 \le \xi^T M(\theta)\xi \le m_2\|\xi\|^2 \; \forall \xi \in \mathbb{R}^n \tag{2}$$

*where $m_1$ and $m_2 \in \mathbb{R}$ are positive bounding constants, $\xi \in \mathbb{R}^n$ is a vector with real entries, $\|\cdot\|$ denotes the standard Euclidean norm, and $(\cdot)^T$ is used to define the transpose of the vector $(\cdot)$.*

Assumption 1. *The dynamical effects combined in $N(\theta, \dot{\theta}, t)$ are assumed to be trigonometric functions of $\theta(t)$ (valid for robots having revolute joints) and all entries of it are bounded when the vector $\dot{\theta}(t)$ is bounded.*

Assumption 2. *The dynamical terms in Eq. (1) are assumed to be at least second-order differentiable. That is, the terms $M(\theta)$, $N(\theta, \dot{\theta}, t) \in C^2$, and $\dot{M}$, $\ddot{M}$, $\dot{N}$, and $\ddot{N} \in \mathcal{L}_\infty$ when their arguments are bounded.*

The end-effector pose, denoted by $x(t) \in \mathbb{R}^n$, is obtained from

$$x = f(\theta) \tag{3}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is the forward kinematics. Differentiating Eq. (3) with respect to time yields

$$\dot{x} = J\dot{\theta} \tag{4}$$

where $\dot{x}(t) \in \mathbb{R}^n$ is the operational space velocity vector, and $J(\theta) \triangleq (\partial f(\theta)/\partial\theta) \in \mathbb{R}^{n\times n}$ is the manipulator Jacobian.

*Remark 1*. During the control development, all kinematic singularities are always avoided and the inverse of the Jacobian matrix is available $\forall\theta$.

The velocity kinematics in Eq. (4) is linearly parameterizable in the sense that

$$J\dot{\theta} = W_j\phi_j \tag{5}$$

where $W_j(\theta, \dot{\theta}) \in \mathbb{R}^{n\times p}$ denotes a known regression matrix and $\phi_j \in \mathbb{R}^p$ denotes an unknown constant parameter vector. The entries of the kinematic parameter vector $\phi_j$ can be lower and upper bounded as

$$\underline{\phi}_{ji} \le \phi_{ji} \le \overline{\phi}_{ji} \tag{6}$$

where $\phi_{ji} \in \mathbb{R}$ denotes the $i$th entry of $\phi_j \in \mathbb{R}^p$ and $\underline{\phi}_{ji}$ and $\overline{\phi}_{ji} \in \mathbb{R}$ denote the $i$th entries of lower and upper bounds $\underline{\phi}_j$ and $\overline{\phi}_j \in \mathbb{R}^p$, respectively.

Assumption 3. *The Jacobian matrix is a function of $\theta(t)$ as arguments of trigonometric functions only, and hence, remains bounded for all possible $\theta(t)$. And $J(\theta)$ and $J^{-1}(\theta)$ are second-order differentiable with $\dot{J}$, $\ddot{J}$, $\dot{J}^{-1}$, and $\ddot{J}^{-1} \in \mathcal{L}_\infty$ provided their arguments are bounded.*

## 3 Error System Formulation

The main control objective is to ensure that the end effector of the robot manipulator tracks a desired operational space trajectory. To quantify the tracking objective, the operational space tracking error, denoted by $e(t) \in \mathbb{R}^n$, is defined as

$$e \triangleq x_d - x \tag{7}$$

where $x_d(t) \in \mathbb{R}^n$ denotes the desired operational space trajectory, which is assumed to be sufficiently smooth. Taking the time derivative of Eq. (7) and substituting Eq. (4), the following expression is obtained

$$\dot{e} = \dot{x}_d + \alpha e - \alpha e - J\dot{\theta} \tag{8}$$

where $\alpha \in \mathbb{R}^{n\times n}$ denotes a diagonal, positive-definite gain matrix. For the subsequent analysis, an auxiliary error-like term, denoted by $r(t) \in \mathbb{R}^n$, is defined as

$$r \triangleq \hat{J}^{-1}(\dot{x}_d + \alpha e) - \dot{\theta} \tag{9}$$

where $\hat{J}(\theta) \triangleq J|_{\phi_j=\hat{\phi}_j} \in \mathbb{R}^{n\times n}$ is the estimated Jacobian matrix with $\hat{\phi}_j(t) \in \mathbb{R}^p$ being the estimated kinematic parameter vector yields (8) to be rewritten as

$$\dot{e} = -\alpha e + \hat{J}r - \tilde{J}\dot{\theta} \tag{10}$$

where $\tilde{J}(\theta) \triangleq J - \hat{J} \in \mathbb{R}^{n\times n}$ is the difference between the manipulator Jacobian and the estimated manipulator Jacobian.

From the above definitions, it is easy to see that $\hat{J}\dot{\theta} = W_j\hat{\phi}_j$ and $\tilde{J}\dot{\theta} = W_j\tilde{\phi}_j$ with $\tilde{\phi}_j = \phi_j - \hat{\phi}_j \in \mathbb{R}^p$ being the parameter estimation error.

Another auxiliary error, denoted by $s(t) \in \mathbb{R}^n$, is defined as[2]

$$s \triangleq \dot{r} + \gamma r \tag{11}$$

where $\gamma \in \mathbb{R}^{n\times n}$ is a diagonal, positive-definite gain matrix.

Taking the time derivative of Eq. (11), substituting the second time derivative of Eq. (9) and then premultiplying by $M(\theta)$ yields

$$M\dot{s} = M\frac{d^2}{dt^2}\left\{\hat{J}^{-1}(\dot{x}_d + \alpha e)\right\} - \dot{\tau} + \dot{M}\ddot{\theta} + \dot{N} + M\gamma\dot{r} \tag{12}$$

where the time derivative of Eq. (1) was also utilized. Via defining an auxiliary vector $Q(x, \dot{x}, \ddot{x}, e, r, s, t) \in \mathbb{R}^n$ as

$$Q \triangleq M\frac{d^2}{dt^2}\left\{\hat{J}^{-1}(\dot{x}_d + \alpha e)\right\} + \dot{M}\ddot{\theta} + \dot{N} + M\gamma\dot{r} + \frac{1}{2}\dot{M}s + r \tag{13}$$

Equation (12) can be rewritten as

$$M\dot{s} = Q - \frac{1}{2}\dot{M}s - r - \dot{\tau} \tag{14}$$

The desired form of $Q$, denoted by $Q_d(x_d, \dot{x}_d, \ddot{x}_d, \dddot{x}_d) \in \mathbb{R}^n$, is defined as

$$Q_d \triangleq Q|_{x=x_d,\dot{x}=\dot{x}_d,\ddot{x}=\ddot{x}_d} \tag{15}$$

after which $\tilde{Q}(x, \dot{x}, \ddot{x}, e, r, s, t) \in \mathbb{R}^n$ is defined as[3]

$$\tilde{Q} \triangleq Q - Q_d. \tag{16}$$

*Remark 2*. The norm of $\tilde{Q}$ can be upper bounded by functions of the error terms in the sense that

$$\|\tilde{Q}\| \le \rho(\|z\|)\|z\| \tag{17}$$

where $\rho$ is a non-negative nondecreasing bounding function of its argument, and $z(t) \in \mathbb{R}^{3n}$ is the combined error vector defined as

$$z \triangleq [\,e^{\mathrm{T}} \quad r^{\mathrm{T}} \quad s^{\mathrm{T}}\,]^{\mathrm{T}} \tag{18}$$

[2]It should be highlighted that the calculation of $s(t)$ requires $\ddot{\theta}(t)$; however, it is not used in control design.

[3]It should be noted that $Q$, $Q_d$, and $\tilde{Q}$ will be used only for the subsequent stability analysis, and they are not required to be known for the control design.

## 4 Control Design

From the error system development in Sec. 3 and the subsequent stability analysis, the control input torque $\tau(t)$ is designed as

$$\tau = (K + I_n)\left[r(t) - r(0) + \gamma\int_0^t r(\sigma)d\sigma\right] + \beta\int_0^t \mathrm{Sgn}(r(\sigma))d\sigma \tag{19}$$

where $I_n \in \mathbb{R}^{n\times n}$ is the standard identity matrix, $K \in \mathbb{R}^{n\times n}$ is a constant, diagonal, positive-definite gain matrix, $\beta \in \mathbb{R}^{n\times n}$ is a constant, diagonal, positive-definite gain matrix, and $\mathrm{Sgn}(\cdot) \in \mathbb{R}^n$ is the vector signum function. It is noted that the term $r(0)$ is introduced in the control input to zero initial torque. The controller in Eq. (19) requires only $r(t)$, which has the form $r = \hat{J}(\theta, \hat{\phi}_j)^{-1}(\dot{x}_d + \alpha(x_d - x)) - \dot{\theta}$ and can be calculated via the measurements of $\theta$, $\dot{\theta}$, the parameter estimate $\hat{\phi}_j$ is updated according to[4]

$$\dot{\hat{\phi}}_j = \mathrm{proj}\{\mu\} \tag{20}$$

where the auxiliary term $\mu \in \mathbb{R}^p$ is defined as

$$\mu \triangleq \Gamma_j W_j^{\mathrm{T}} e \tag{21}$$

where $\Gamma_j \in \mathbb{R}^{p\times p}$ is a constant, diagonal, positive-definite adaptation gain matrix and the projection function of $\mu$ is designed as follows:

$$\mathrm{proj}\{\mu_i\} = \begin{cases} \mu_i & \text{if} \quad \hat{\phi}_{ji} > \underline{\phi}_{ji} \\ \mu_i & \text{if} \quad \hat{\phi}_{ji} = \underline{\phi}_{ji} \quad \text{and} \quad \mu_i > 0 \\ 0 & \text{if} \quad \hat{\phi}_{ji} = \underline{\phi}_{ji} \quad \text{and} \quad \mu_i < 0 \\ 0 & \text{if} \quad \hat{\phi}_{ji} = \overline{\phi}_{ji} \quad \text{and} \quad \mu_i > 0 \\ \mu_i & \text{if} \quad \hat{\phi}_{ji} = \overline{\phi}_{ji} \quad \text{and} \quad \mu_i \le 0 \\ \mu_i & \text{if} \quad \hat{\phi}_{ji} < \overline{\phi}_{ji} \end{cases} \tag{22}$$

where $\mu_i$ denotes the $i$th entry of $\mu$, and $\hat{\phi}_{ji}(t)$ denotes the $i$th entry of $\hat{\phi}_j(t)$. Under the projection algorithm, the lower and upper bounds of the estimated parameter vector satisfy that $\underline{\phi}_j \le \hat{\phi}_j(t) \le \overline{\phi}_j$ provided that $\underline{\phi}_{ji} \le \hat{\phi}_{ji}(0) \le \overline{\phi}_{ji}\ \forall i = 1, 2, \ldots, p$ is satisfied [28,29].

After substituting Eq. (16) and the time derivative of Eq. (19) into Eq. (14), the closed-loop error system for $s(t)$ is obtained as

$$M\dot{s} = Q_d + \tilde{Q} - \frac{1}{2}\dot{M}s - r - (K + I_n)s - \beta\mathrm{Sgn}(r) \tag{23}$$

where Eq. (11) was also utilized.

## 5 Stability Analysis

The stability analysis is enframed by the following Theorem:

Theorem 1. *The controller in Eq. (19) and the parameter update rule in Eq. (20) ensure asymptotic operational space tracking in the sense that*

$$\|e(t)\| \to 0 \,\text{as}\, t \to \infty \tag{24}$$

*provided that the controller gains are selected to satisfy the following conditions*

[4]It is highlighted that the projection algorithm is introduced to ensure that $\hat{J}^{-1}$ is available for all $\hat{\phi}_j$.

$$\beta_i \geq |Q_{di}(t)| + \frac{1}{\gamma_i}|\dot{Q}_{di}(t)|\,\forall t \tag{25}$$

$$\frac{\alpha_{\min} > \xi\hat{J}}{2} \tag{26}$$

$$\frac{\gamma_{\min} > \xi\hat{J}}{2} \tag{27}$$

*and the entries of $\boldsymbol{K}$ are chosen sufficiently large compared to the initial conditions of the system. In Eqs. (25), (26), (27), $\beta_i$ and $\gamma_i \in \mathbb{R}$ denote the ith diagonal entries of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, respectively, and $Q_{di}(t)$ and $\dot{Q}_{di}(t)$ denote the ith entries of $Q_d(t)$ and $\dot{Q}_d(t)$, respectively, $\alpha_{\min}$ and $\gamma_{\min}$ denote the minimum eigenvalues of $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$, respectively, and $\xi\hat{J} \in \mathbb{R}$ is a positive bounding constant satisfying $\xi\hat{J} \geq \|\hat{\boldsymbol{J}}(\boldsymbol{\theta})\|\forall\theta$.*

*Proof.* The proof is started by defining a non-negative scalar function, denoted by $V(\boldsymbol{y}, t) \in \mathbb{R}$, as

$$V \triangleq \frac{1}{2}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \frac{1}{2}\boldsymbol{r}^{\mathrm{T}}\boldsymbol{r} + \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{s} + P + \frac{1}{2}\tilde{\phi}_j^{\mathrm{T}}\Gamma_j^{-1}\tilde{\phi}_j \tag{28}$$

where $P(t) \in \mathbb{R}$ is an auxiliary function defined as

$$P \triangleq \zeta_P - \int_0^t \boldsymbol{s}^{\mathrm{T}}(\sigma)[\boldsymbol{Q}_d(\sigma) - \boldsymbol{\beta}\mathrm{Sgn}(\boldsymbol{r}(\sigma))]d\sigma \tag{29}$$

where $\zeta_P \in \mathbb{R}$ is defined as follows:

$$\zeta_P \triangleq \sum_{i=1}^{n} \beta_i|r_i(0)| - \boldsymbol{r}^{\mathrm{T}}(0)\boldsymbol{Q}_d(0) \tag{30}$$

and $\boldsymbol{y}(t) \in \mathbb{R}^{(3n+1+p)\times 1}$ is defined as

$$\boldsymbol{y}(t) \triangleq [\,\boldsymbol{e}^{\mathrm{T}} \quad \boldsymbol{r}^{\mathrm{T}} \quad \boldsymbol{s}^{\mathrm{T}} \quad \sqrt{P} \quad \tilde{\phi}_j^{\mathrm{T}}\,]^{\mathrm{T}} \tag{31}$$

Based on the proof in Refs. [30] and [31] provided that Eq. (25) is satisfied $P(t) \geq 0$ and thus $V(\boldsymbol{y}, t)$ is a Lyapunov function. Note that Eq. (28) can be lower and upper bounded as follows:

$$\lambda_1\|z\|^2 \leq \lambda_1\|\boldsymbol{y}\|^2 \leq V(\boldsymbol{y}) \leq \lambda_2\|y\|^2 \tag{32}$$

where $\lambda_1$ and $\lambda_2 \in \mathbb{R}$ are positive bounding constants defined as

$$\lambda_1 \triangleq \frac{1}{2}\min\left\{1, m_1, \frac{1}{\Gamma_{j,\max}}\right\}, \quad \lambda_2 \triangleq \max\left\{1, \frac{1}{2}m_2, \frac{1}{\Gamma_{j,\min}}\right\} \tag{33}$$

where $\Gamma_{j,\min}$ and $\Gamma_{j,\max}$ denote the minimum and maximum eigenvalues of $\Gamma_j$, respectively.

Taking the time derivative of Eq. (28) yields

$$\dot{V} = \boldsymbol{e}^{\mathrm{T}}\dot{\boldsymbol{e}} + \boldsymbol{r}^{\mathrm{T}}\dot{\boldsymbol{r}} + \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\dot{\boldsymbol{M}}\boldsymbol{s} + \boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}\dot{\boldsymbol{s}} + \dot{P} + \tilde{\phi}_j^{\mathrm{T}}\Gamma_j^{-1}\dot{\tilde{\phi}}_j \tag{34}$$

Substituting Eqs. (10), (11), (20), and (23), and the time derivative of Eq. (29) into Eq. (34), and canceling common terms, the following expression is obtained:

$$\dot{V} \leq -\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\alpha}\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}\hat{\boldsymbol{J}}\boldsymbol{r} - \boldsymbol{r}^{\mathrm{T}}\boldsymbol{\gamma}\boldsymbol{r} + \boldsymbol{s}^{\mathrm{T}}\tilde{\boldsymbol{Q}} - \boldsymbol{s}^{\mathrm{T}}(\boldsymbol{K} + \boldsymbol{I}_n)\boldsymbol{s} \tag{35}$$

where $-\tilde{\boldsymbol{Q}}^{\mathrm{T}}\Gamma_j^{-1}\mathrm{proj}\{\boldsymbol{\mu}\} \leq -\tilde{\boldsymbol{Q}}^{\mathrm{T}}\Gamma_j^{-1}\boldsymbol{\mu}$ was utilized [28,29]. After considering Assumption 3 along with the boundedness of the output of the projection algorithm, and utilizing Eq. (17), the following upper bound is obtained for the right-hand side of Eq. (35) as

$$\dot{V} \leq -\alpha_{\min}\|\boldsymbol{e}\|^2 + \frac{\xi_{\hat{J}}}{2}\|\boldsymbol{e}\|^2 + \frac{\xi_{\hat{J}}}{2}\|\boldsymbol{r}\|^2 - \gamma_{\min}\|\boldsymbol{r}\|^2 - \|\boldsymbol{s}\|^2 + \rho\|\boldsymbol{s}\|\|\boldsymbol{z}\| - K_{\min}\|\boldsymbol{s}\|^2 \tag{36}$$

where $K_{\min}$ denotes the minimum eigenvalue of $\boldsymbol{K}$. Note that the last two terms of Eq. (36) can be upper bounded as

$$\rho\|\boldsymbol{s}\|\|\boldsymbol{z}\| - K_{\min}\|\boldsymbol{s}\|^2 \leq \frac{\rho^2}{4K_{\min}}\|\boldsymbol{z}\|^2 \tag{37}$$

and in view of this inequality, the right-hand side of Eq. (36) can further be upper bounded as

$$\dot{V} \leq -\left[\min\left\{\left(\alpha_{\min} - \frac{\xi_{\hat{J}}}{2}\right), \left(\gamma_{\min} - \frac{\xi_{\hat{J}}}{2}\right), 1\right\} - \frac{\rho^2}{4K_{\min}}\right]\|\boldsymbol{z}\|^2 \tag{38}$$

Provided that Eqs. (26) and (27) are satisfied, and the entries of $\boldsymbol{K}$ are chosen sufficiently big enough when compared to the initial conditions of the system, the following expression can be obtained as:

$$\dot{V} \leq -\lambda_3\|\boldsymbol{z}\|^2 \tag{39}$$

for some $0 < \lambda_3 < 1$. From Eqs. (28) and (39), $V(\boldsymbol{y}, t) \in \mathcal{L}_\infty$ is ensured. Therefore, $\boldsymbol{y}(t) \in \mathcal{L}_\infty$, and thus, based on its definition in Eq. (31), $\boldsymbol{e}(t)$, $\boldsymbol{r}(t)$, $\boldsymbol{s}(t)$, $\tilde{\phi}(t) \in \mathcal{L}_\infty$. Based on the boundedness of the desired operational space trajectory, from Eq. (7), it is clear that $\boldsymbol{x}(t) \in \mathcal{L}_\infty$. In view of Assumption 3, boundedness of $\boldsymbol{e}(t)$ and $\boldsymbol{r}(t)$ can be utilized along with Eq. (10) to conclude that $\dot{\boldsymbol{x}}(t) \in \mathcal{L}_\infty$. Above boundedness statements can be utilized with Eq. (9) to prove that $\dot{\boldsymbol{\theta}}(t) \in \mathcal{L}_\infty$. The boundedness of the joint velocity can be utilized to prove that $\boldsymbol{W_j}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \in \mathcal{L}_\infty$. Above boundedness statements can be utilized with Eq. (20) to prove that $\hat{\phi}_j(t) \in \mathcal{L}_\infty$. From Eq. (11), it is clear that $\dot{\boldsymbol{r}}(t) \in \mathcal{L}_\infty$, which can be utilized along with the time derivatives of Eqs. (9) and (10) to show that $\ddot{\boldsymbol{\theta}}(t)$, $\ddot{\boldsymbol{e}}(t) \in \mathcal{L}_\infty$, respectively. The above boundedness statements can be utilized along with Eq. (23) to prove that $\dot{\boldsymbol{s}}(t) \in \mathcal{L}_\infty$ where Assumption 1 was utilized. The robot manipulator dynamic model in Eq. (1) can be utilized to demonstrate $\boldsymbol{\tau}(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can then be used to prove that all signals remain bounded under the closed-loop operation. Integrating the inequality in Eq. (39) in time from 0 to $+\infty$ results $\int_0^{+\infty}\|z(\sigma)\|^2 d\sigma \leq (V(0)/\lambda_3)$, from which, $z(t)$ is square integrable. Finally, since $z(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{z}(t) \in \mathcal{L}_\infty$, utilizing Barbalat's Lemma [29] yields $\|z(t)\| \to 0$ as $t \to \infty$ from which the asymptotic tracking result given in Eq. (24) follows.

With the stability proof we have presented in our study, the asymptotic stability of the closed-loop system and the boundedness of the system variables are ensured. However, similar to most adaptive type controllers, the convergence of the parameters to their actual values was not achieved. We can only ensure the convergence of the parameter estimates to some constant values. This may seem like a weakness from a pure theoretical point of view; however, from an engineering point of view, the main objective of the controller, that is tracking an end-effector trajectory, is still achieved.

## 6 Experiment Results

To illustrate the performance of the proposed controller, experiments were performed on a PHANToM Omni haptic device shown in Fig. 1. The Phantom device communicated with a computer via a local area network port. OPENHAPTICS TOOLKIT software used on the computer ensured real-time applications and MATLAB SIMULINK with QUARC library transmitted control input torques to the Phantom device, and read the encoder values with joint positions of the device. During the experimental studies, MATLAB

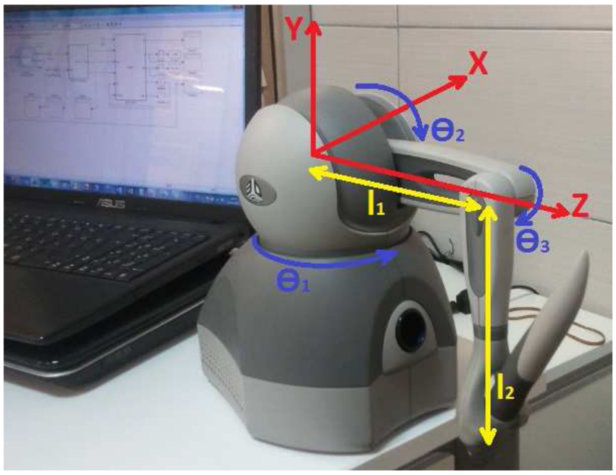


**Fig. 1 PHANToM Omni haptic device**

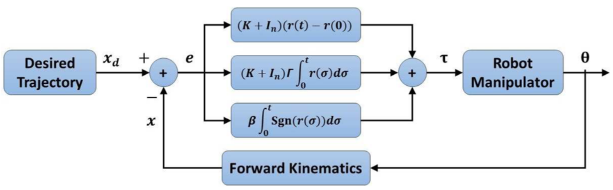


**Fig. 2 The block diagram of the closed-loop system**

SIMULINK was run with a data rate of 100 Hz. The block diagram of the closed-loop system is presented in Fig. 2.

In the experiments, the first three joints of the device were used. Using the standard Denavit–Hartenberg convention on the device, the end-effector position of the device in the operational space can then be calculated in the following form [32–34]

$$\boldsymbol{x}(t)=\begin{bmatrix}X(t)\\Y(t)\\Z(t)\end{bmatrix}=\begin{bmatrix}-s_1(l_1c_2+l_2s_3)\\l_1s_2-l_2c_3+l_y\\c_1(l_1c_2+l_2s_3)-l_z\end{bmatrix} \tag{40}$$

where $l_1=l_2=0.133$ m represent the first and the second link lengths, respectively, $l_y=0.023$ m, $l_z=0.168$ m are the operational space transformation offsets between the origin of the end effector and the first joint. The notations are $c_i=\cos(\theta_i)$ and $s_i=\sin(\theta_i)$ for $i\in\{1,2,3\}$ where $\theta_1$, $\theta_2$, and $\theta_3$ represent joint angles as shown in Fig. 1. In view of Eqs. (5), from Eq. (40), the regression matrix $\boldsymbol{W_j}\in\mathbb{R}^{3\times2}$ is obtained as

$$\boldsymbol{W_j}=\begin{bmatrix}-c_1c_2\dot\theta_1+s_1s_2\dot\theta_2 & -c_1s_3\dot\theta_1-s_1c_3\dot\theta_3\\ c_2\dot\theta_2 & s_3\dot\theta_3\\ -s_1c_2\dot\theta_1-c_1s_2\dot\theta_2 & -s_1s_3\dot\theta_1+c_1c_3\dot\theta_3\end{bmatrix} \tag{41}$$

with $\phi_j=[l_1,l_2]^{\mathrm{T}}\in\mathbb{R}^2$.

The desired operational space trajectory for the experimental studies is selected to ensure that the robot is not close to its kinematic singularities as follows:

$$\boldsymbol{x_d}(t)=\begin{bmatrix}X_d\\Y_d\\Z_d\end{bmatrix}=\begin{bmatrix}0.05(1-\exp(-0.05t))\\-0.05+0.02\cos(0.05t)\\-0.05+0.02\sin(0.05t)\end{bmatrix}(\mathrm{m}) \tag{42}$$

The control gains were selected as $\boldsymbol{\alpha}=\mathrm{diag}\{[40;30;20]\}$, $\boldsymbol{\beta}=\gamma=0.1\boldsymbol{I}_3$, $\boldsymbol{K}=\mathrm{diag}\{[0.12;0.03;0.02]\}$, and $\Gamma_j=2\boldsymbol{I}_2$. The manipulator was initialized to be at rest at joint positions $\boldsymbol{\theta}(0)=[0,-0.26,-0.5]^{\mathrm{T}}$rad. The selection of the controller gains are done by trial and error method. We would like to note that for similar type of controller formulations self-tuning methodologies (as in Refs. [35] and [36]) can be utilized as an add-on strategy, especially for the control gains $\boldsymbol{\beta}$ and $\boldsymbol{K}$; however, specific to the experiments in this study, the tuning process was quite easy.

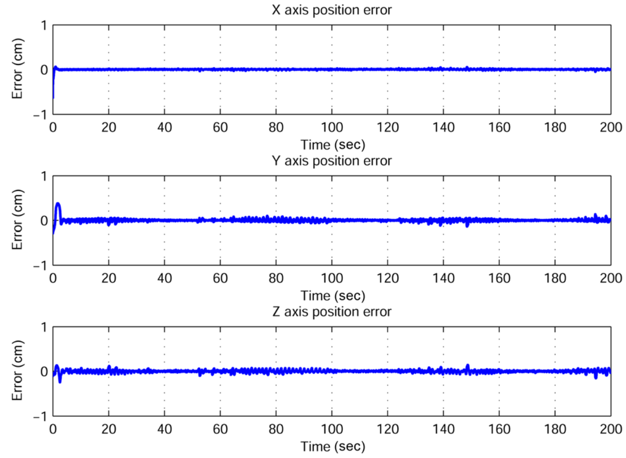


**Fig. 3 Operational space tracking error $e(t)$**

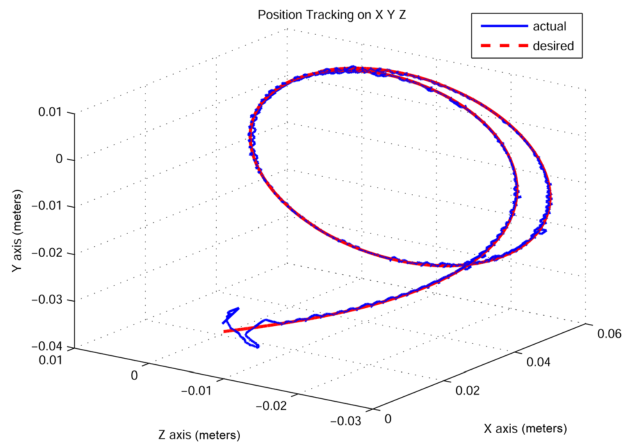


**Fig. 4 Desired and actual operational space trajectories**

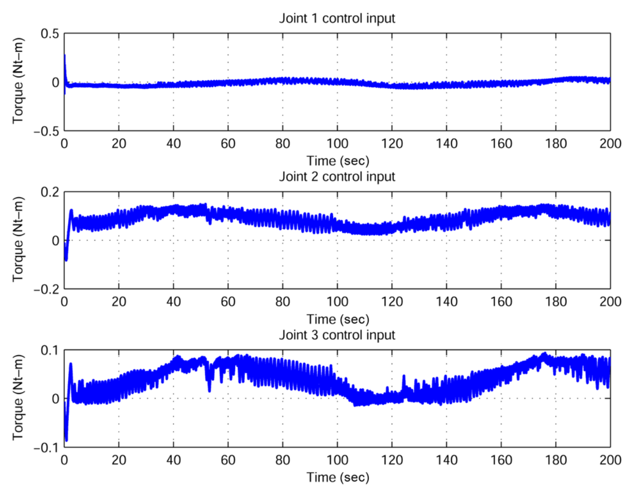


**Fig. 5 The control input torque $\tau(t)$**

The results of the experiments are presented in Figs. 3–6. Figure 3 presents the operational space tracking errors. In Fig. 4, the desired and the actual operational space trajectories are

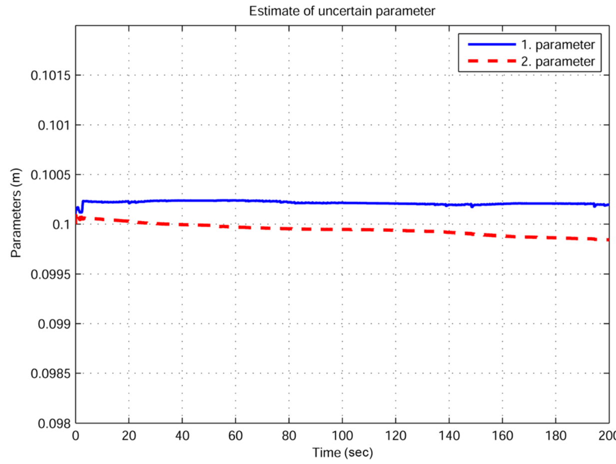


**Fig. 6 Estimates of uncertain parameters $\hat{\phi}_j(t)$**

presented. The actual trajectories presented in Fig. 4 are calculated via the forward kinematic formulations given in Eq. (40). As can be observed from Figs. 3 and 4 at the initial phase of the experiment, there is a slight jerk and after a reasonable amount of time the end-effector position converges to the desired position profile. We would like to note that this initial jerk is mostly due to the initial estimates of the uncertain system parameters. However, when the control effort kicks in and the parameter estimates converging to some values, tracking control objective is already achieved. In Fig. 5, the control input torque values are presented. As it is seen, the control effort is relatively smooth and the values are below the maximum torque limits of each joint (less than 2.2 N·m at nominal position). Finally, the parameter estimates are presented in Fig. 6. As illustrated in Fig. 6, the estimated values of the uncertain kinematic parameters approximately converge to some values in finite time.

## 7 Conclusions

In this study, a new operational space controller formulation for robot manipulators is presented. Under mild assumptions on kinematics and dynamics of the robot manipulator, the proposed robust adaptive operational space controller ensured asymptotic end-effector tracking performance despite unstructured uncertainties in the dynamics and structured uncertainties in the velocity kinematics. We also want to note that the proposed robust-adaptive controller when compared to the adaptive counterparts presented in Refs. [9–14] the literature due to robust part of the controller formulation can compensate for a broader class of uncertainties. As opposed to most variable structure controllers presented in the literature, like the high gain robust controller of Ref. [22], the proposed robust controller is continuous, and asymptotic tracking is ensured via the continuous nature of the controller formulation. Also, when compared to the recent variable structure type controllers of Refs. [17–21,23], the proposed robust-adaptive controller is capable of dealing with both the velocity level kinematic and dynamic uncertainties of robot manipulators. The stability of the proposed controller is ensured via rigorous theoretical analysis based on Lyapunov techniques. Experimental studies performed on a PHANToM Omni haptic device are used to illustrate the feasibility and the performance of the proposed robust-adaptive controller.


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