# Performance of Multicast MISO-OFDM Systems

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***Abstract*—In this paper, we evaluate the performance of multicast OFDM systems with single and multiple transmit antennas. We have shown that it is possible to increase the data rate of multicast OFDM systems by selecting the users with good channel for each subcarrier. For the single transmit antenna case, the resource allocation has been applied using the two-step algorithm where subcarrier allocation and bit loading are performed separately. For the multiple transmit antennas case, we have proposed a suboptimal algorithm using a set of precoding vectors. In this study, we present the simulation results of the complete transmission chain where we have associated the resource allocation algorithms with a powerful erasure code. The results show that the proposed algorithms outperform the classical worst case algorithm for both single and multiple transmit antennas.** [1]

keywords : Multicast OFDM, multiple transmit antennas, precoding vector optimization, adaptive subcarrier allocation, erasure code.

## I. INTRODUCTION

Next generation wireless communication systems will provide a wide range of applications with high and time-varying bandwidth requirements. Up to now, the main wireless applications are broadcast multiuser systems. However, the demand for multimedia services such as video and audio conferencing, online training, news and software distribution and database replication is increasing. For these applications, multicasting offers a significant improvement compared to broadcasting since it allows the transmission of packets to multiple destinations using less resources [1]. However, since in the wireless channel the received signal-to-noise ratio (SNR) of each user is not the same, the data rate of the multicast stream is limited by the data rate of the least capable user. Consequently, this method cannot provide efficient performance when the number of users in the group increases.

The main difficulty in achieving high data rate on the wireless channels is known to be frequency selectivity and fading due to the existence of multiple paths. Orthogonal frequency division multiplexing (OFDM) techniques can significantly alleviate the impacts of frequency selective fading [2] and is attractive for the next generation of wireless systems. Moreover, multiple-input single-output (MISO) systems that use multiple antenna techniques [3] can be combined with OFDM systems to enhance the performance of wireless systems in fading channels.

In [4][5], by assuming that the transmitter knows the instantaneous channel state information (CSI) of all the users, it has been shown that adaptive subcarrier and bit allocation can significantly increase the data rate of broadcast multiuser OFDM scheme. Recently it has been demonstrated that using OFDM with subcarrier and bit allocation, it is also possible to increase the data rate on each subcarrier for multicasting applications by selecting only the users with a good channel condition [6]. In [7] we have shown that multicast OFDM systems with multiple transmit antennas also allow to increase the sum data rate of multicast OFDM systems. Instead of adapting the rate according to the worst user, we have maximized the sum data rate in order to increase the quantity of received data by all users for a given time. Since this optimization problem is complex, we have developed a suboptimal algorithm for the precoding vector, allocation of the subcarriers to the users and loading bits.

In this paper, we propose to evaluate these algorithms in a complete transmission chain and error correcting codes based on erasure codes instead of using automatic repeat request (ARQ). Indeed, it is well known that the ARQ strategy can quickly overwhelm the system as receivers request for retransmission of lost packets. A solution to this problem is to use the fountain concept using erasure codes such as Luby Transform (LT) [8] [9] or Raptor code [10].

This paper is organized as follows. In section II, we provide a general description of the proposed transmission chain for multicast OFDM systems. Then in section III, we describe the two-step suboptimal subcarrier allocation and bit loading for the single transmit antenna case. The extension to the multiple transmit antenna case including the precoding vector optimization, subcarrier allocation and bit loading algorithms is given in section IV. Finally, we give simulation results in section V.

## II. GENERAL DESCRIPTION OF THE PROPOSED TRANSMISSION CHAIN

The multicast OFDM system for $K$ users with a single transmit and receive antenna is shown in Figure 1.

[1] The work of Didier Le Ruyet and Hajer Khiari is supported by the Pidea European project SMART.

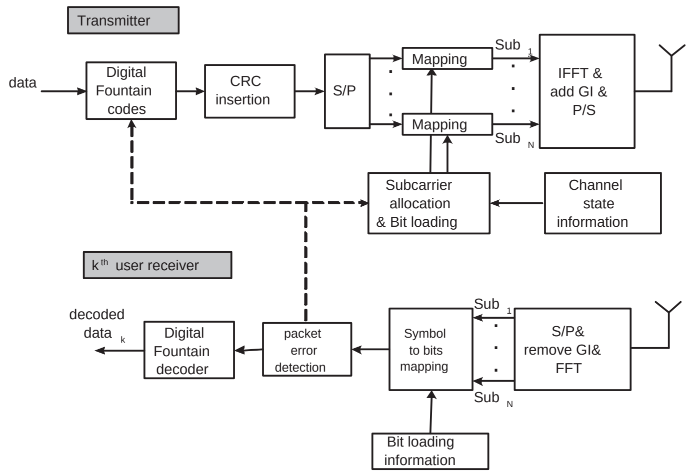

Fig. 1. The transmitter and $k$th receiver structure for multicast OFDM system with one transmit and receive antenna.

### A. Encoding

The original source data is first divided in superframes composed of $K_p$ packets of $K_l$ bits. These packets are transmitted and the users can reconstruct the original source data once they have received a sufficient number of encoded packets. According to the capacity of erasure channels, $K'_p$ correctly received packets should be theoretically sufficient in order to decode the superframe of $K_p$ packets assuming that $K_p$ is long enough. Fountain codes have efficient encoding and decoding algorithms and allow the recovery of the original superframe from any $K'_p$ encoded packet with high probability, where $K'_p$ is just slightly larger than $K_p$. In this work, we will use LT codes [8] that are particularly well adapted to multicast systems since they can generate encoded packets until all the users have correctly received at least $K'_p$ encoded packets. The main benefit of this approach is that different receivers can recover the source data using different encoded packets. When a user has received $K'_p$ packets it sends an acknowledge to the transmitter and consequently the subcarrier allocation eliminates this user from the set of the receivers until the end of the superframe. The LT code can be seen has an irregular density generator matrix codes. Each encoded packet is generated by performing the bitwise modulo 2 sum of $d_n$ source packets. The degree $d_n$ is chosen from a degree distribution $\rho(d)$. In this paper, we have considered the robust soliton distribution in order to avoid failure of the decoding [8].

Each encoded packet is labeled and a cyclic redundancy check (CRC) is added.

### B. OFDM transmission

Each packet is modulated and sent over one subcarrier. The modulated symbols are combined in OFDM symbols with $N$ subcarriers and passed through the IFFT. A guard interval is added at the beginning of each OFDM symbol and the modulated data are transmitted through the frequency selective channels.

The channel is assumed to be constant over one frame and varying between the frames considering Doppler frequency. Moreover, it is assumed that the number of channel taps is equal or smaller than the length of the guard interval (GI) in order to avoid intercarrier and intersymbol interference at the receiver.

Considering that the CSI for all the subcarriers and users are known at the transmitter, the adaptive resource allocation should optimize the allocation of the users to the subcarrier and load bits in a way that maximizes the total number of bits received by all the users. Each subcarrier is assigned to a group of users which receive the same data, and then the number of bits on each subcarrier is determined considering the lowest one among the channel gains of all the users allocated to this subcarrier. It is assumed that the bit allocation information is transmitted to each user through a separate control channel.

Let $R_{p,k}$ be the data rate of the $k$th user and $c_{p,n}$ be the number of bits that are assigned to the $n$th subcarrier. Here, the user index is unnecessary because the users using the subcarrier receive identical data using the same modulation. $c_{p,n}$ is selected from the set of $\{0, 1, 2, ..., M\}$ where $M$ is the maximum number of bits per symbol that can be transmitted by each subcarrier $n = 1, 2, ..., N$.

The data rate $R_{p,k}$ can be expressed as

$$R_{p,k} = \sum_{n=1}^{N} c_{p,n}\rho_{p,k,n} \tag{1}$$

where $\rho_{p,k,n}$ is a binary value indicating whether $k$th user utilizes the $n$th subcarrier or not.

$$\rho_{p,k,n} = \begin{cases} 1 & \text{if the nth subcarrier is used for kth user} \\ 0 & \text{else} \end{cases} \tag{2}$$

Assuming that available total transmit power (in energy per symbol) is limited by $P_T$, in order to maximize the total data rate by all users, the optimization problem can be expressed as

$$\max_{c_p;\rho_p} \sum_{k=1}^{K} R_{p,k} = \max_{c_p;\rho_p} \sum_{k=1}^{K}\sum_{n=1}^{N} c_{p,n}\rho_{p,k,n} \tag{3}$$

subject to

$$\sum_{n=1}^{N} \max_{k} \left( \frac{f(c_{p,n})\rho_{p,k,n}}{\|H_{p,k,n}\|^2} \right) \leq P_T \tag{4}$$

where $f(c_{p,n})$ is the required received power (in energy per symbol) for reliable reception of $c_{p,n}$ bits when the channel gain is equal to unity and $H_{p,k,n}$ is the channel gain from $k$th user for $n$th subcarrier.

For M-ary quadrature amplitude modulation (M-QAM) schemes, the bit error probability is upper bounded by the symbol error probability, which is tightly approximated by $4Q(\sqrt{d^2/(2N_0)})$ [11] where $d$ is the minimum distance between the points in the signal constellation. Since the average energy of a M-QAM symbol is equal to $(M-1)d^2/6$, then the required power (in energy per symbol) $f(c_{p,n})$ for supporting

$c_{p,n}$ bits per symbol at a required bit-error-rate (BER) $p_e$ can be represented by

$$f(c_{p,n}) = \frac{N_0}{3}[Q^{-1}(p_e/4)]^2(2^{c_{p,n}} - 1) \quad (5)$$

where $N_0/2$ denotes the variance of the additive white Gaussian noise (AWGN) and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \mathrm{e}^{-t^2/2}dt$.

## III. Subcarrier Allocation and Bit Loading for Single Transmit Antenna Case

The problem includes the subcarrier allocation and bit loading. Since, it is very difficult to solve (3) subject to (4), we will consider a suboptimal two-step approach where the bit allocation problem and the subcarrier allocation assuming equal power for each subcarrier are separated [6]. Therefore, the user data rate should be maximized for each subcarrier.

### A. Subcarrier allocation

For each subcarrier $n$:

*Step 1) Determination of the number of bits allocated:*

Let $c_{p,k,n}$ be the number of bits that can be received by the $k$th user in the case of $\rho_{p,k,n} = 1$. The number of supportable bits is given by

$$c_{p,k,n} = \min\left(f^{-1}\left(\frac{P_T}{N}H_{p,k,n}\right), M\right) \quad (6)$$

where $f^{-1}(.)$ is the inverse function of $f(.)$ defined in (5).

*Step 2) Selection of the user that maximizes the sum data rate:*

Calculate the tentative total data rate $\bar{R}_{p,k,n}$ when the $k$th user is selected as the user requiring maximum power.

$$\bar{R}_{p,k,n} = u_{p,k,n}c_{p,k,n} \quad (7)$$

where $u_{p,k,n}$ indicates the number of users who have channel gains larger than $H_{p,k,n}$. Select the user index $\kappa$ maximizing $\bar{R}_{p,k,n}$.

$$\kappa = arg\max_k \bar{R}_{p,k,n} \quad (8)$$

*Step 3) Allocation of the users to the subcarrier:*

$$\rho_{p,k,n} = \begin{cases} 1 & \text{if} \quad H_{p,k,n} \geq \mathrm{H}_{p,\kappa,n} \\ 0 & \text{else} \end{cases} \quad (9)$$

### B. Bit Loading Algorithm

Bit loading algorithm is considered using the modified Levin-Campello algorithm [4] [6] under the assumption that the subcarrier allocation is completed. Let $\Delta P_{p,n}(c)$ denote the incremental power needed for the transmission of one additional bit at the subcarrier $n$. When the number of loaded bits for the $n$th subcarrier is $c$, $\Delta P_{p,n}(c)$ is given as

$$\Delta P_{p,n}(c) = \frac{f(c+1) - f(c)}{u_{p,n}H_{p,\kappa,n}} \quad (10)$$

where $u_{p,n}$ is the number of users who share the $n$th subcarrier, which is necessary because the incremental power is shared by the group of users allocated to the subcarrier.

$$u_{p,n} = \sum_{k=1}^{K} \rho_{p,k,n} \quad (11)$$

Then, the bit loading algorithm is summarized as follows:

*Step 1) Initialization:*

$c_{p,n} = 0$ and evaluate $\Delta P_{p,n}(c = 0)$ for all $n$.
Tentative transmit power is $P_T^* = 0$.

*Step 2) Bit loading iteration:*

Repeat the following unless $P_T^* \geq P_T$
$n^* = \arg\min_n \Delta P_{p,n}(c_{p,n})$
$P_T^* = P_T^* + \Delta P_{p,n^*}(c_{p,n^*})u_{p,n^*}$
$c_{p,n^*} = c_{p,n^*} + 1$
if $c_{p,n^*} = M$, set $\Delta P_{p,n^*}(c_{p,n^*}) = \infty$
else evaluate $\Delta P_{p,n^*}(c_{p,n^*})$.

## IV. Extension to the Multiple Transmit Antenna Case

The problem includes the precoding vector optimization, subcarrier allocation and bit loading. Like previously, we will separate the bit allocation problem and the subcarrier allocation by assuming equal power for each subcarrier. Therefore, the user data rate should be maximized for each subcarrier.

The channel vector for $k$th user and $n$th subcarrier in the frequency domain is written by

$$\mathbf{H}_{p,k,n} = \begin{bmatrix} H_{p,k,n,1} & H_{p,k,n,2} & \ldots & H_{p,k,n,N_t} \end{bmatrix}^T \quad (12)$$

where $H_{p,k,n,j}$ is the channel gain from $j$th transmit antenna to $k$th user for $n$th subcarrier.

Now, the optimization problem can be expressed as

$$\max_{\mathbf{W}_p;c_p;\rho_p} \sum_{k=1}^{K} R_{p,k} = \max_{\mathbf{W}_p;c_p;\rho_p} \sum_{k=1}^{K}\sum_{n=1}^{N} c_{p,n}\rho_{p,k,n} \quad (13)$$

subject to

$$\sum_{n=1}^{N} \max_k \left(\frac{f(c_{p,n})\rho_{p,k,n}}{\|\mathbf{W}_p\mathbf{H}_{p,k,n}\|^2}\right) \leq P_T \quad (14)$$

where $\mathbf{W}_p$ is the $1 \times N_t$ precoding vector.

In [7], we have proposed to choose the precoding vector among a limited set of precoding vectors. This set is composed of the normalized transpose-conjugate of the channel vectors associated to each user. The selected precoding vector and user allocation will maximize the sum data rate.

For each subcarrier $n$:

*Step 1) Initializing of the precoding vector:*

$$\mathbf{W}_{p,i,n}^{\text{init}} = \frac{\begin{bmatrix} H_{p,i,n,1}^* & H_{p,i,n,2}^* & \ldots & H_{p,i,n,N_t}^* \end{bmatrix}}{\sqrt{|H_{p,i,n,1}|^2 + \ldots|H_{p,i,n,N_t}|^2}} \quad (15)$$

where $i = 1, 2, ..., K$.

*Step 2) Calculation of the gains:*

Then, the gain of each user for each initial precoding vector is calculated as

$$\mathrm{G}_{p,k,i,n} = \|\mathbf{W}^{\text{init}}_{p,i,n}\mathbf{H}_{p,k,n}\|^2 \quad (16)$$

Let $c_{p,k,i,n}$ be the number of bits that can be received by $k$th user using $i$th initial precoding vector in the case of $\rho_{p,k,n} = 1$. The number of supportable bits is given by

$$c_{p,k,i,n} = \min\left(f^{-1}\left(\frac{P_T}{N}\mathrm{G}_{p,k,i,n}\right), M\right) \quad (17)$$

*Step 3) Selection of both the precoding vector and the user that maximize the sum data rate:*

Calculate the tentative total data rate $\bar{R}_{p,k,i,n}$ when the $k$th user is selected as the user requiring the maximum power for each initial precoding vector.

$$\bar{R}_{p,k,i,n} = u_{p,k,i,n}c_{p,k,i,n} \quad (18)$$

where $u_{p,k,i,n}$ indicates the number of users who have channel gains larger than $\mathrm{G}_{p,k,i,n}$. Select the user index $\kappa$ and the precoding vector index $\varrho$ maximizing $\bar{R}_{p,k,i,n}$.

$$\kappa, \varrho = arg \max_{k,i} \bar{R}_{p,k,i,n} \quad (19)$$

Then, the precoding vector is chosen as

$$\mathbf{W}_{p,n} = \mathbf{W}^{\text{init}}_{p,\varrho,n} \quad (20)$$

*Step 4) Allocation of the users to the subcarrier:*

$$\rho_{p,k,n} = \begin{cases} 1 & \text{if} \quad \mathrm{G}_{p,k,\varrho,n} \geq \mathrm{G}_{p,\kappa,\varrho,n} \\ 0 & \text{else} \end{cases} \quad (21)$$

### A. Bit Loading Algorithm

The bit loading algorithm is the same as previously except that $H_{p,\kappa,n}$ is replaced by $\mathrm{G}_{p,\kappa,\varrho,n}$.

## V. Simulation Results

In this section, we evaluate the data rate and the sum data rate of the multicast OFDM systems with single and multiple transmit antennas using the *the proposed algorithm* and the worst user algorithm which is performed choosing the user of least SNR and bit loading.

The data rate and sum data rate are calculated by:

$$\text{DR} = (K'_p * K_l)/(1024\mu s * F) \quad (22)$$

$$\text{SDR} = K * \text{DR} \quad (23)$$

where $F$ is the number of necessary frames to send enough packets to whole users.

Simulation results are performed using Hiperlan/2 standard [12] using channel model A [13] which corresponds to a typical office environment with $9$ channel taps. The total OFDM symbol duration is $4\mu s$ including $0.8\mu s$ guard interval. It consists of $N = 64$ subcarriers. The required BER $p_e = 10^{-4}$ was fixed in order to maximize the data rate. The noise variance and the maximum number of bits per symbol are fixed as $N_0 = 1$ and $M = 8$ respectively. The transmitted power is chosen as $P_T = 30$ dBW. The packet size and the number of packets of the superframe are fixed to $128$ bits and $K_p = 10000$ respectively. In this work, we choose $K'_p = 11000$ in order to perfectly decode the transmit data thanks to the LT code properties. A $16$ bits CRC is applied to each encoded packet. We assume that the CSI is updated every $256$ OFDM symbols.

In Figure 2 and Figure 3, we compare the performance results using a single transmit antenna and assuming that the users are symmetrically distributed around the base station without path loss. We observe that the sum data rate of the two-step algorithm increases almost linearly with the number of users. Compared to the worst case algorithm, the data rate of the two-step algorithm decreases slowly.

In Figure 4 and Figure 5, the two transmit antenna case is considered. The sum data rate and data rate are plotted versus different numbers of users. According to the results, multiple antenna case outperforms compared to the worst user case. The gain increases when the number of user increases.

The multiple transmit antenna case outperform the single transmit antenna case by $15\%$ that is confirmed on the theoretically results [7]. The proposed algorithm has significantly higher data rate as the numbers of users increases.

## VI. Conclusion

In this paper, we have evaluated different resource allocation algorithms for multicast application. We have considered a complete transmission chain including a powerful erasure code. For single transmit antenna, we have shown that the two-step approach improves significantly the performance compared to the worst case algorithm. We have shown that it is possible to further increase the sum data rate using multiple transmit antennas and resource allocation algorithms. While the proposed solution that includes the precoding vector selection, the subcarrier allocation and the bit loading is suboptimal, nevertheless, it is able to extract almost all the gain. The MISO case outperforms the SISO case, the difference in performance is particularly large when the number of users is high. This algorithm has been developed and performed at a subcarrier level. However, it can be also applied on clusters of subcarriers in order to reduce the feedback load.

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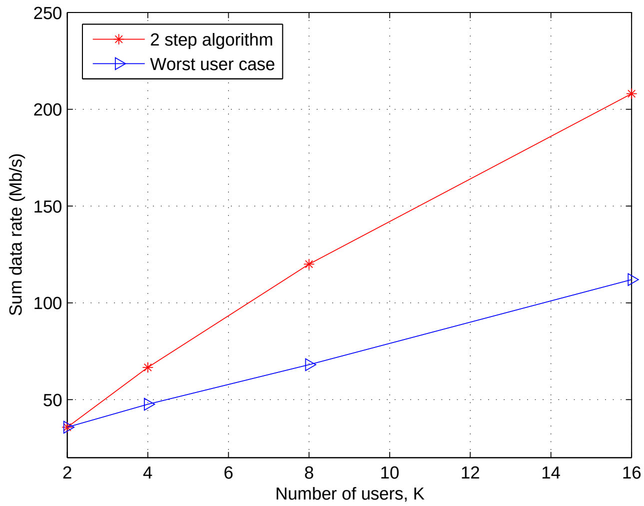


Fig. 2. Sum data rate versus the number of users for $P_T = 30$dBW and $N_t = 1$.

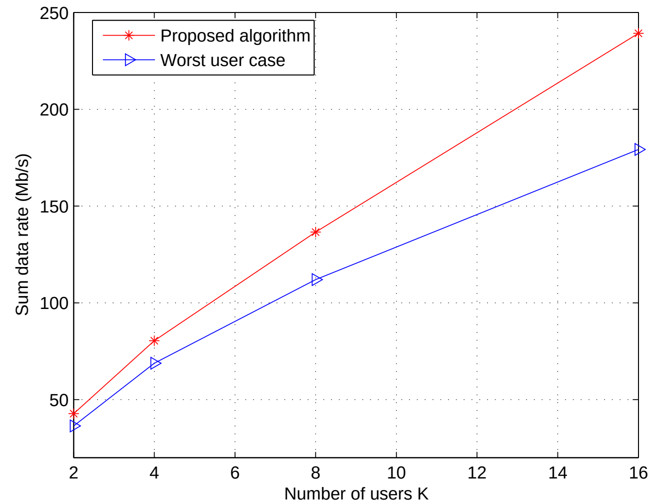


Fig. 4. Sum data rate versus the number of users for $P_T = 30$dBW and $N_t = 2$.

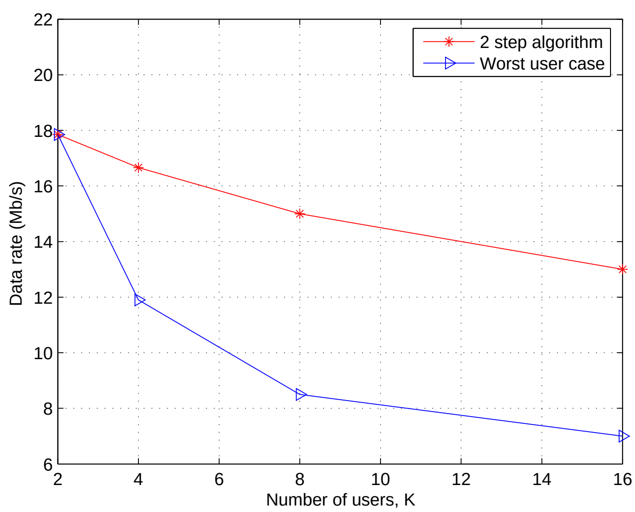


Fig. 3. Data rate versus the number of users for $P_T = 30$dBW and $N_t = 1$.

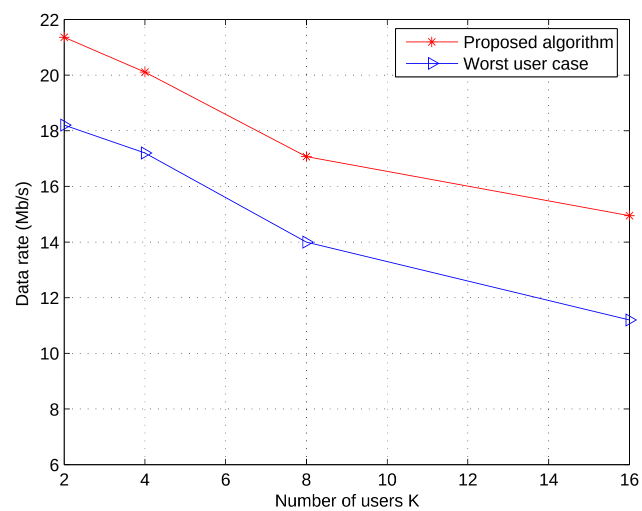


Fig. 5. Data rate versus the number of users for $P_T = 30$dBW and $N_t = 2$.


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