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Stochastic Bifurcation in Generalized Chua's Circuit Driven by Skew-Normal Distributed Noise

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In this study, the stochastic phenomenological bifurcations (P-bifurcations) of generalized Chua's circuit (GCC) driven by skew-normal distributed noise have been investigated by numerically obtaining the stationary distributions of the stochastic responses. The noise intensity and/or skewness parameters of skew-normal distributed noise have been chosen as the bifurcation parameters to change the structure of the stochastic attractor. While the number of breakpoints in the piecewise-linear characteristics of the GCC are fixed, it has been observed that the number of scrolls have been changed by tuning the noise intensity and the skewness parameter of the skew-normal distributed noise.

Keywords: Stochastic bifurcation; generalized Chua's circuit; skew-normal distributed noise; P-bifurcation.

1. Introduction

The real-world systems are mostly influenced by inevitable internal or external random perturbations. Although these random perturbations are considered to have negative effects on the performance of a system, in the last three decades, the studies on the stochastic resonance have demonstrated that the presence of an optimal amount of noise can have positive impacts on the system [1–3]. There is still an increasing interest on the problems of stochastic bifurcation occurring in biological, economical, social systems [4–6]. Stochastic bifurcations have been defined as P-bifurcation and dynamical bifurcation (D-bifurcation) in [7]. Phenomenological bifurcation (P-bifurcation) is observed by the qualitative change in the stationary probability distributions while D-bifurcation is observed by the changes of the sign of Lyapunov exponents. In 2004, Ryagin and Ryashko have investigated the attractors of Chua's circuit driven by the Wiener process [8]. In [9, 10], the existence of stationary state densities as a stationary solution of Fokker–Plank equation for the Chua's circuit excited with white noise have been analytically proven. However, existence of stationary densities for Chua's circuit driven by skew-normal distributed noise is not yet available in the literature. Limit cycles of the Lorenz system driven by the Wiener process have been analyzed by P-bifurcation and D-bifurcation in [11], and the effect of time delay on the stochastic bifurcation in Van der Pol oscillator perturbed by white noise have been investigated in [12]. The effect of noise on the dynamical systems which have hidden attractors has been investigated in [13], and the analytical and numerical methods for the problem of localization of hidden attractors in Chua's circuit have been developed in [14, 15].

Stochastic Lévy type dynamical system is a more general class of random dynamical systems [16, 17]. Recently, the applications of stochastic dynamical systems driven by Lévy process have found interest on the stochastic neural networks [18, 19] and on biological systems such as tumor-immune system [5]. Bifurcations for a simple non-linear dynamical system under additive Lévy noises have been analyzed in [20]. The common assumption in the extensively studied stochastic dynamical systems is that the noise has either Gaussian or symmetric α-stable Lévy-type distribution. However, stochastic fluctuations cannot have a perfect symmetry in practice. Since many physical phenomena and observed data exhibit non-symmetrical characteristics, asymmetry (skewness) should also be taken into account in statistical modeling. Basically, asymmetric distributions might have high kurtosis which cause heavy tails in the distributions with infinite higher-order moments [21, 22]. In [23], the changes in the dynamics of generalized Chua's circuit (GCC) have been investigated through α-stable Lévy noise in the framework of the stochastic bifucation concept. By choosing the parameters of the impulsiveness, skewness and the intensity of α-stable noise as bifurcation parameters, the qualitative changes have been observed in the stationary probability distributions and in the structures of the stochastic attractors. However, since the observations are recorded in a certain duration, the effect of the asymmetric behavior of the noise needs to be formulated with a finite variance in practice. Therefore, rather than modeling with impulsive distributions having an infinite variance, the probability density functions (pdfs) are modeled with skew-normal distribution proposed in [24] in order to characterize the real skew distributed data within a finite observation interval. The skew-normal distribution is therefore a proper candidate to fit the histogram of the data having different variances around the mean. In the literature, skew-normal distributions are reported to be utilized in various applications such as modeling the financial data [25], modeling non-line-of-sight (NLOS) channel for wireless communication systems [26], sensing asymmetric signals [27] and modeling the data profile in biomedical imaging [28]. In the presented stochastic generalized Chua's circuit (SGCC), the variations in the structure of the attractors have been observed by changing the intensity and/or skewness parameters of the skew-normal distributed noise while fixing the number of breakpoints in piecewise-linear characteristics of GCC, which indeed determines the number of scroll in the corresponding attractors [29].

This paper is organized as follows: In Sec. 2, skew-normal distributed noise model have been briefly presented and P-bifurcation of SGCC have been numerically analyzed. In the last section, the simulation results have been presented.

2. GCC Driven By Skew-Normal Distributed Noise

The dynamics of GCC [29] are described by the following set of three differential equations:

$$\begin{aligned}\dot{x} &= \alpha[y - h(x)],\\ \dot{y} &= x - y + z,\\ \dot{z} &= -\beta y,\end{aligned} \tag{1}$$

where α and β denote bifurcation parameters, and the piecewise-linear characteristics $h(x)$ is given as

$$h(x) = m_{2q-1}x + \frac{1}{2}\sum_{i=1}^{2q-1}(m_{i-1} - m_i)(|x + c_i| - |x - c_i|), \tag{2}$$

where the number of breakpoints "q" determines the number of scrolls.
The stochastic state equations driven by skew-normal distributed noise can be written in the form as

$$\begin{aligned}dx &= \alpha[y - h(x)]dt,\\ dy &= (x - y + z)dt,\\ dz &= -\beta y dt + \epsilon\xi(t)dt,\end{aligned} \tag{3}$$

where $\zeta(t)$ indicates skew-normal noise introduced in [24] and ϵ is the noise intensity. Equation (3) can be rewritten in the Itô form of SDE,

$$d\mathbf{X} = f(\mathbf{X})dt + \mathbf{g}dM(t), \tag{4}$$

where

$$\mathbf{X} = [x \quad y \quad z]^T,$$
$$f(\mathbf{X}) = \begin{bmatrix}\alpha(y - h(x))\\ x - y + z\\ -\beta y\end{bmatrix}; \quad \mathbf{g} = \begin{bmatrix}0\\0\\ \epsilon\end{bmatrix} \tag{5}$$

and the increment "$dM(t) \doteq \zeta(t)dt$" is a random variable which has a skew-normal distribution.

The approximate numerical solution of Eq. (4) can be obtained by using the Euler–Maruyama method given in [21, 30] as

$$X_{t_i} = X_{t_{i-1}} + f(t_{i-1}, X(t_{i-1}))\tau + g(t_{i-1}, X(t_{i-1})) \triangle M_i, \tag{6}$$

where the skew-normal random variable $\triangle M_i$ is defined by

$$\triangle M_i = M_\lambda([t_{i-1}, t_i]) \sim SN(\xi, \tau, \lambda) \tag{7}$$

is generated using the method given in [31].

2.1. *Skew-normal distributions*

The skew-normal distribution is a generalization of the standard normal distribution, and the density function of the skew-normal distribution is given as

$$\phi(x;\lambda) = 2\phi(x)\Phi(\lambda x) \quad (-\infty < x < \infty), \tag{8}$$

where ϕ and Φ are the standard normal density and distribution functions, respectively. The integral function of $\phi(x;\lambda)$ is denoted by $\Phi(x;\lambda)$. The skewness parameter λ lying in the interval $(-\infty,\infty)$ tunes the asymmetry and it is also called as the slant parameter in [32]. For $\lambda > 0$, the distribution is skewed to the right and, for $\lambda < 0$, the distribution is skewed to the left as shown in Fig. 1. Note that an increase in the absolute value of the skewness parameter results in an increased asymmetric distribution.

The normal distribution is a special case of the skew-normal distribution with $\lambda = 0$. The limit distribution is defined as the half-normal distribution given below.

$$\lim_{\lambda\to\infty} \varphi(x;\lambda) = 2\varphi(x) I_{[0,\infty)}(x), \tag{9}$$

where $I_{[0,\infty)}(\cdot)$ is indicator function of a set $[0,\infty)$. As $\lambda \to \infty$, $\phi(x;\lambda)$ tends to the half-normal density, where an approximate of half-normal density is simulated as shown in Fig. 2.

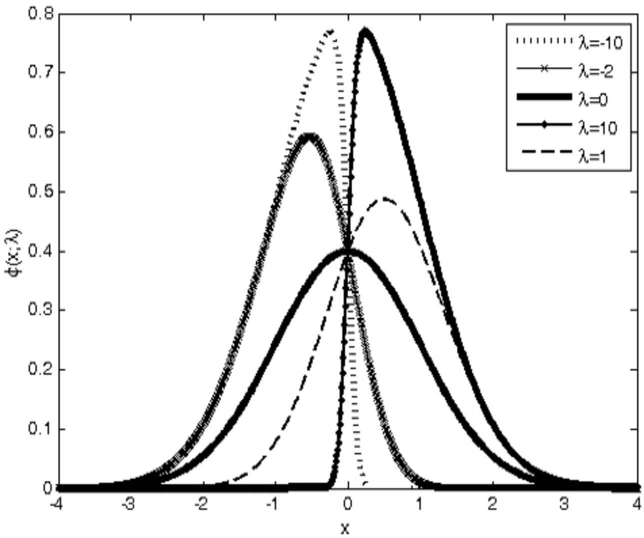
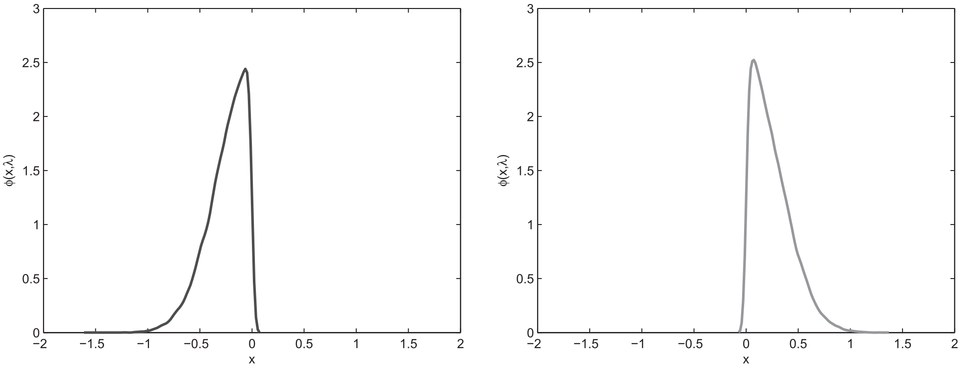


Fig. 1. Skew-normal distribution for different values of λ.



(a) $\lambda = -1000$ (Negative-skewed)   (b) $\lambda = 1000$ (Positive-skewed)

Fig. 2. Half-normal densities.

If Z is a continuous random variable with density function $\phi(x;\lambda)$ given in Eq. (8), then the variable $Y = \xi + wZ$ is called a skew-normal variable with location parameter ξ, scale parameter w and skewness parameter λ, and having the distribution as $Y \sim \mathrm{SN}(\xi, w^2, \lambda)$. The random variable Y at $x \in \mathbb{R}$ satisfies

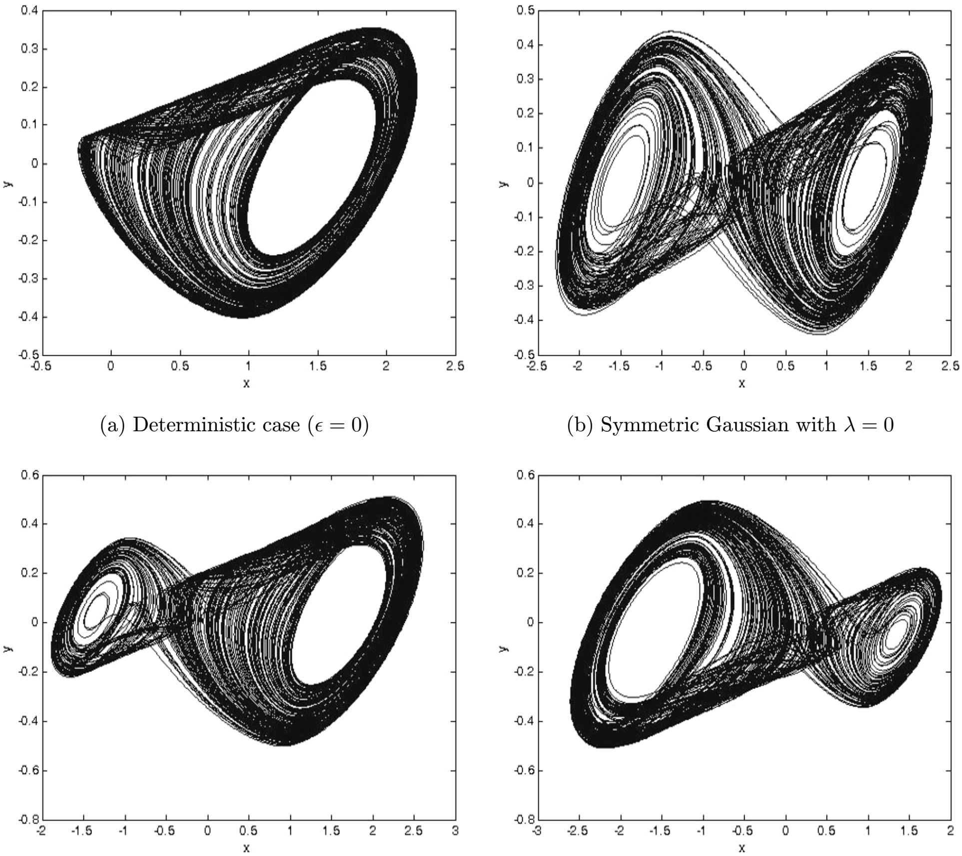
$$\frac{2}{w}\phi\left(\frac{x-\xi}{w}\right)\Phi\left(\lambda\frac{x-\xi}{w}\right) \equiv \frac{1}{w}\phi\left(\frac{x-\xi}{w};\lambda\right). \tag{10}$$

2.2. *P-bifurcation of stochastic GCC*

The dynamical behavior of the system given in Eq. (3) is investigated numerically with the vectors $\mathbf{m} = [m_0; m_1; \ldots; m_{2q-1}]$, and $\mathbf{c} = [c_1; c_2; \ldots; c_{2q-1}]$. The skewness parameter λ and the noise intensity ϵ are considered as the bifurcation parameters.

In the absence of noise ($\epsilon = 0$), by choosing $q = 1$ and the deterministic bifurcation parameters $\alpha = 4.6$, $\beta = 6.02$, the vectors $\mathbf{m} = [-1/7;\ 2/7]$ and $\mathbf{c} = 1$, two single-scroll coexisting attractors are obtained, and a phase trajectory belongs to either one attractor or the other one depending on the initial conditions. Projection of the single scroll attractor on the $x - y$ plane is shown in Fig. 3(a). The addition of a small amount of the symmetric Gaussian noise ($\lambda = 0$) is reported to induce jumps between the attractors as shown in Fig. 3(b), which is called as coherence resonance [33]. Under random disturbances, the trajectories of the stochastically forced Chua's circuit form some bundles.

To indicate the effect of asymmetry of skew-normal distributed noise on the behavior of a single-scroll attractor, the system given in Eq. (4) is perturbed by the skew-normal distributed noise with $\lambda \neq 0$. As an illustrative example, the noise intensity ϵ is kept fixed and the skewness parameter is chosen as $\lambda = 4$ and $\lambda = -4$, respectively. By varying the skewness parameter, it has been observed that one of the scrolls of the attractors enlarges while the other scroll shrinks. The opposite behavior



(a) Deterministic case ($\epsilon = 0$)

(b) Symmetric Gaussian with $\lambda = 0$

(c) SN distributed noise with $\lambda = 4$

(d) SN distributed noise with $\lambda = -4$

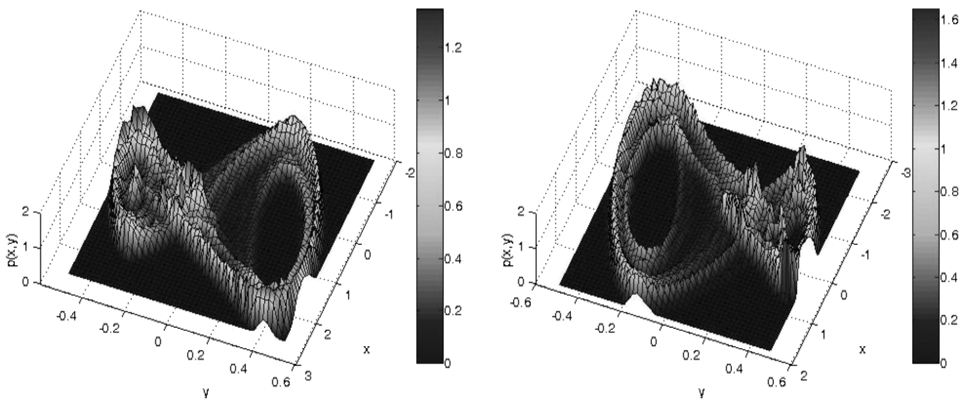
Fig. 3. Noise-induced attractors.

is observed when the sign of the skewness is changed as shown in Figs. 3(c) and 3(d), respectively.

Apart from the changes in the structure of the attractor, the qualitative changes are also observed in the resulting joint normalized histogram, which corresponds to the joint probability densities (stationary densities) $p(x, y)$ with respect to the states x and y as shown in Figs. 4(a) and 4(b) for skewness parameters $\lambda = 4$ and $\lambda = -4$, respectively. The color bar indicates the intensity of the PDF. Depending on the skewness, the trajectory mostly remains at the opposite states which can be observed as shown in Figs. 4(a) and 4(b).

2.3. *P-bifurcation of 7-scroll attractor*

In the following, the deterministic bifurcation parameters are chosen as $\alpha = 9$, $\beta = 14.286$. The piecewise-linear characteristics of the GCC is kept fixed, and the



(a) SN distributed noise with $\lambda = 4$

(b) SN distributed noise with $\lambda = -4$

Fig. 4. Joint normalized histogram $p(x, y)$ of the states $x - y$.

7-scroll attractor in GCC has been specifically chosen since the variations of the 7-scroll attractor can be clearly seen through P-bifurcation by varying the noise intensity. By choosing $q = 4$ and the vectors $\mathbf{m}$ and $\mathbf{c}$ as

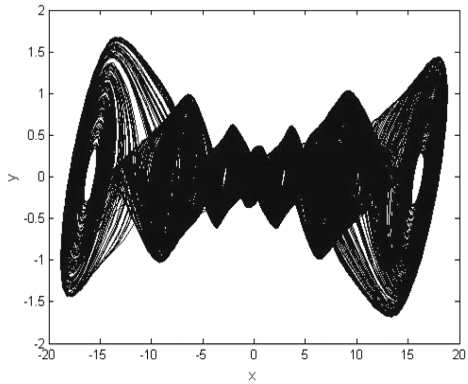
$$\begin{aligned} \mathbf{m} &= [0.9/7;\quad -3/7;\quad 3.5/7;\quad -2.4/7;\quad 2.52/7;\quad -1.68/7;\quad 2.52/7;\quad -1.68/7], \\ \mathbf{c} &= [1; 2.15;\quad 3.6;\quad 6.2;\quad 9;\quad 14; 25], \end{aligned} \tag{11}$$

7-scroll attractors have been obtained from Eq. (3) in the absence of noise and its 2D phase portrait is given in Fig. 5(a).

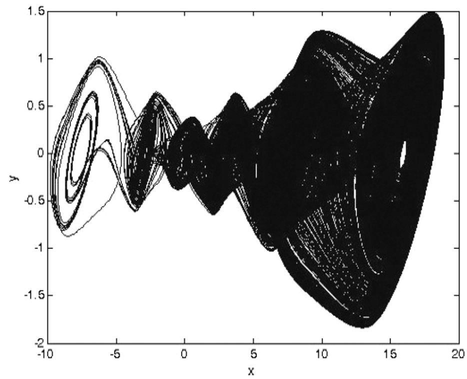
The system given in Eq. (4) with the parameters given in Eq. (11) is perturbed by the skew-normal distributed noise with various values of the noise intensity ϵ. Note that the skewness parameter is chosen as $\lambda = 4$ to maintain the same asymmetric behavior of the skew-normal noise. Figures 5(b)–5(f) present the results of phase plots of the stochastically perturbed 7-scroll attractor for the fixed piecewise-linear characteristic of the GCC. It is observed that an increase in the value of the noise intensity exceeding a critical value ϵ^* of the skew-normal distributed noise has the effect of reducing the number of scrolls. Thus, the multi-scroll attractors of GCC can be adjusted by tuning the intensity of the skew-normal distributed noise.

The bifurcation of GCC have been investigated numerically through a qualitative change of normalized histogram $p(x)$ of the state x to show P-bifurcation.

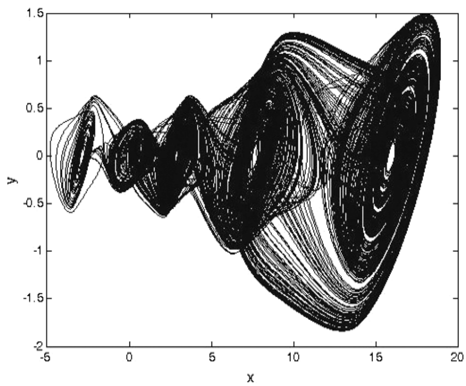
The normalized histograms can also be modeled by the finite mixture of α-stable (FMαS) distributions presented in [34]. Qualitative changes of histograms with respect to the critical noise intensity ϵ^* are shown in Figs. 6(a)–6(f). According to these results, it can be observed that one of the maxima of the corresponding pdf $p(x)$ disappears and qualitative changes in the histograms occur. Correspondingly, the stationary histograms evolve from multi-peaks to less-peaks which indicate P-bifurcation depending on the noise intensity ϵ^*.



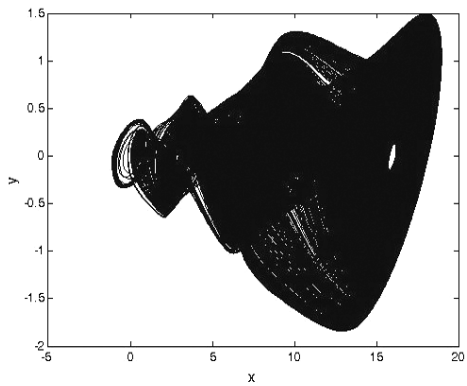
(a) Deterministic case ($\epsilon = 0$)



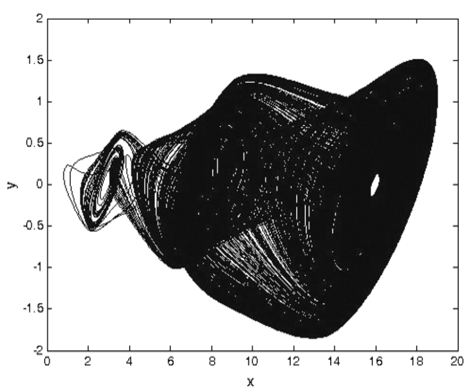
(b) SN distributed noise with $\epsilon^* = 0.001$



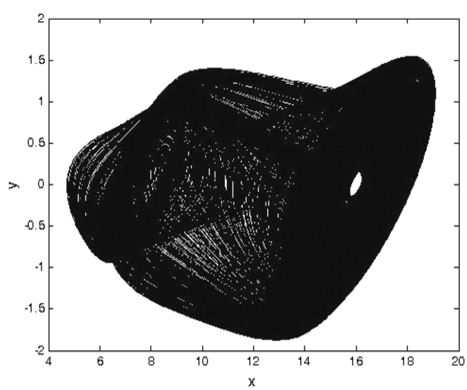
(c) SN distributed noise with $\epsilon^* = 0.0028$



(d) SN distributed noise with $\epsilon^* = 0.0044$

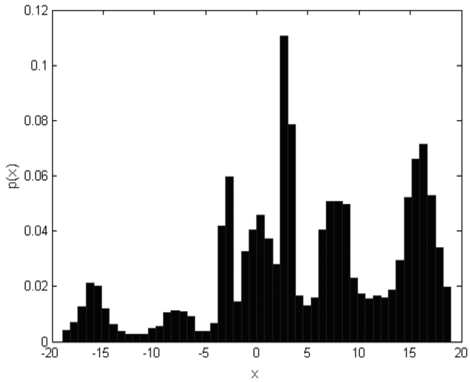


(e) SN distributed noise with $\epsilon^* = 0.014$

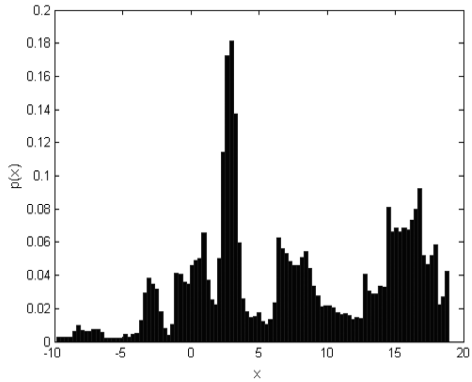


(f) SN distributed noise with $\epsilon^* = 0.03$

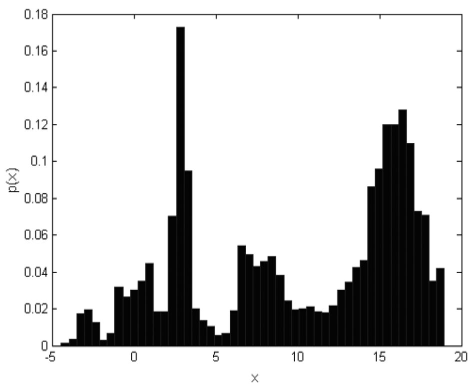
Fig. 5. Variations in 7-scroll attractors induced by skew-normal distributed noise with $\lambda = 4$.



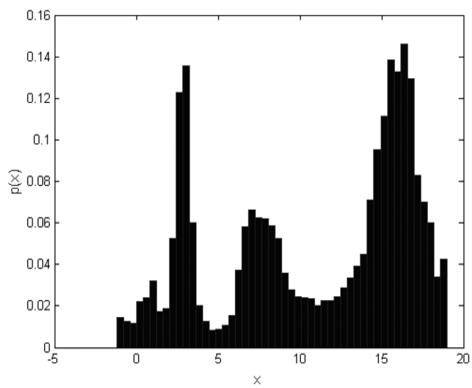
(a) Deterministic case ($\epsilon = 0$)



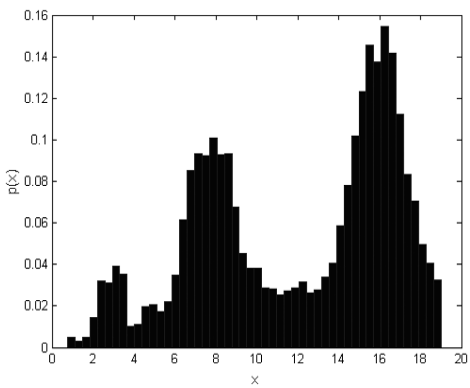
(b) SN distributed noise with intensity $\epsilon^* = 0.001$



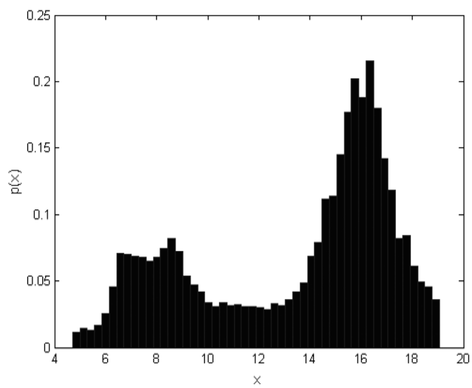
(c) SN distributed noise with intensity $\epsilon^* = 0.0028$



(d) SN distributed noise with intensity $\epsilon^* = 0.0044$



(e) SN distributed noise with intensity $\epsilon^* = 0.014$



(f) SN distributed noise with intensity $\epsilon^* = 0.03$

Fig. 6. Stochastic bifurcations related to 7-scroll attractors with respect to the noise intensity.

Table 1. The variation of entropy with respect to the noise intensity ϵ^* for the skewness parameter $\lambda = 4$.

# of scrolls	Noise intensity ϵ^* for $\lambda = 4$	Entropy
6	0.001	3.9728
5	0.0028	3.9133
4	0.0044	3.8773
3	0.014	3.7518
2	0.03	3.7152

Table 2. Variations in the number of scrolls with respect to the noise intensity ϵ^* for the skewness parameter $\lambda = 4$ and $\lambda = 10$.

# of scrolls	Noise intensity ϵ^* for $\lambda = 4$	Noise intensity ϵ^* for $\lambda = 10$
7	0	0
6	0.001	0.0001
5	0.0028	0.0004
4	0.0044	0.004
3	0.014	0.0075
2	0.03	0.054

The variation of attractor structures depending on the noise intensity and skewness are characterized by the normalized histograms in this paper. These histograms correspond to the distribution $p(x)$ of the time series obtained from the state x.

The entropy as a measure of complexity can be defined as

$$H = -\sum p(x)\ln(p(x), \tag{12}$$

where the distributions $p(x)$'s are as shown in Figs. 6(a)–6(f).

The variation of entropy with respect to the critical noise intensity for skewness parameter $\lambda = 4$ is given in Table 1. It is clearly seen from Table 1 that the entropy decreases while the number of scrolls is decreased for the corresponding critical noise intensity.

The absolute value of λ has also a certain effect on the structure of the multi-scroll attractors, as the signs of the skewness parameter λ and the noise intensity ϵ have. It can be numerically observed that when the skewness parameter is increased then the critical value of the noise intensity to observe P-bifurcation becomes lower. As the numerical example, the critical values of the noise intensity ϵ^* have been presented in Table 2 for the case of skewness parameter $\lambda = 4$ and $\lambda = 10$.

3. Conclusion

The P-bifurcation of the SGCC driven by skew-normal distributed noise have been presented. Stochastic P-bifurcations induced by the skewness parameter and the noise intensity have been observed via the qualitative changes in the normalized histograms. The GCC with 7-scroll attractor has been chosen as the case study, where the skewness parameter λ and noise intensity ϵ have been selected as the bifurcation parameters. The results from numerical simulations have demonstrated that it is possible to decrease the number of scrolls by properly choosing the stochastic excitation which have asymmetric distributions while preserving the chaotic regime. Throughout this paper, the results have been obtained through the simulations. The analytical approach to prove the existence of the stationary densities for stochastic Chua's circuit driven by skew-normal noise is needed as a future study. Furthermore, it has been observed that the entropy decreases while the number of scrolls is decreased with the change of critical noise intensity. It should be noted that an increase in the noise intensity results in a decrease in the number of scrolls if the noise exhibits asymmetric behavior ($\lambda \neq 0$). Another important observation is that if the absolute value of skewness parameter λ increases, the critical value of the noise intensity ϵ^* required for P-bifurcation decreases. The effect of hidden attractors on the stochastic bifurcation phenomenon and the role of the noise type for detecting the hidden attractors are subjects for future investigations.

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