Basin Stability of Single Machine Infinite Bus Power Systems with Levy Type Load Fluctuations

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Abstract

In this paper, the basin stability of single machine infinite bus power systems with alpha-stable Levy type load fluctuations are investigated over the parameter space of mechanical power and damping parameter. The probabilities of returning to the stable equilibrium point are calculated for different characteristic exponent and skewness parameters of alpha-stable Levy noise to see the effect of impulsive and asymmetric load fluctuations.

1. Introduction

Power system stability is an old problem [1, 2] and the instability in power systems is one of the main reasons for the many major blackouts. The dynamical behavior in power systems is therefore very important for stability in the sense of the frequency and voltage and for synchronization in the sense of rotor angle stability. Frequency stability is related with the active power balance between the generation and the consumption in the grid and the voltage stability is the ability of a power system subject to a given disturbance to maintain acceptable voltages at all buses [3]. Rotor angle stability is related to the dynamics of generator rotor angles that is the ability of interconnected synchronous machine of a power system to remain in synchronism [3]. The detection of the loss of angular stability is necessary for critical operation conditions since the tripping of a line caused a loss of angular stability and a loss of synchronism in the Turkish power system [4]. Today's power systems have a large number of interconnected generators and loads through transmission lines. With the increased of the significant amounts of power from highly variable sources, such as wind turbines and solar cells and a variable electricity consumption due to electric vehicle charging, maintaining the synchrony becomes more important [5]. Since the single machine infinite bus (SMIB) power system, where a synchronous generator connected to an infinitely large node (infinite bus) through a tranmission line, qualitatively exhibits the behavior of multi−machines in a real power system it is well-suited and practically common for stability analysis.

In [6] the relation between the bifurcation parameter and power system stability has been discussed and it has been observed that a small perturbation in the load causes loss of synchronism of the generators with respect to the infinite bus. Global instability in which most of all generators in a system coherently lose synhronism with the remaning generators of the system have been analyzed in [7]. The effects of dynamic loads on the stability of power systems have been investigated in [8] by the analysis of critical parameter. The nonlinear dynamic characteristics of a SMIB power system under a periodic load disturbance have been studied in [9]. The SMIB power system with a synchronous generator modeled by a classical third-order differential equation have been introduced in [10] and the effect of damping parameter on the nonlinear dynamics of third-order SMIB have been investigated. The influences of Gaussian white noise on the stability SMIB power system have been investigated in [11]. In [12] the effects of stochastic excitations in SMIB system have been studied by the p-moment stability of rotor angle. The impact of load perturbations on the rotor stability have been analyzed in [13] by modeling the evolution of the probability density function as the Fokker-Planck equation. In these former studies the stochastic fluctuations in electrical power systems either at the loads or at the excitations have been considered as Brownian process (Wiener process). In [14] the electricity prices have been modelled as α-stable Lévy process and in [15] the electricity market data have been modeled by using the α-stable periodic autoregressive model (PAR). Since the load has been considered as one of the main factors in determining electricity prices because the sudden demand or supply changes cause sharp spikes in electricity prices, then we have assumed that the stochastic disturbances occurring in power systems could be more realistically modeled by alpha-stable (α-stable) Lévy process compared to the modelling by Wiener process [16]. These α-stable Lévy type fluctuations are characterized by non-Gaussian and heavy-tailed behaviour defined by stable law [17].

In [16] we have investigated SMIB with $\alpha-$stable Lévy type load fluctuations and in this paper, we have extended the rotor stability in terms of basin stability. The basin stability is a measure of the basin's volume which allows to quantify the probability to converge to the equilibrium point after being subjected to perturbations. The basin stability in deterministic SMIB systems has been presented in [18, 19] and then the Northern European power grid is considered as a case study. In [20] the basin stability for deterministic SMIB system and four-node network have been investigated. By introducing the notion of stochastic basin of attraction, the basin stability is generalized in [21] and applied to the three-well potential perturbed by two types of noises, Brownian motion and α-stable Lévy motion.

The paper is organized as follows. The stochastic SMIB system with α-stable Lévy type load fluctuations is introduced in Section 2 and the basin stability is analyzed in Section 3.

2. Stochastic Single Machine Infinite Bus Power Systems

The rotational dynamics of the synchronous machine which are called as swing equations in [3] are as:

$$\begin{aligned} \dot{\delta} &= w \\ M\dot{w} &= -Dw + P_m - P_e \end{aligned} \quad (1)$$

where δ is the relative rotor angle of synchronous generator, w is the rotor speed with respect to the synchronous reference, P_m is the mechanical input power, P_e is the electrical power output, M and D are the inertia and the damping coefficients, respectively. $P_e = P_{max}sin(\delta)$ where the maximum output of the synchronous generator is $P_{max} = E^{'}E_B/X_T$ and $E'\angle\delta$ is the internal voltage of generator and $E_B\angle 0$ is the infinite bus voltage; X_T is the total reactance of the transformer and the line.

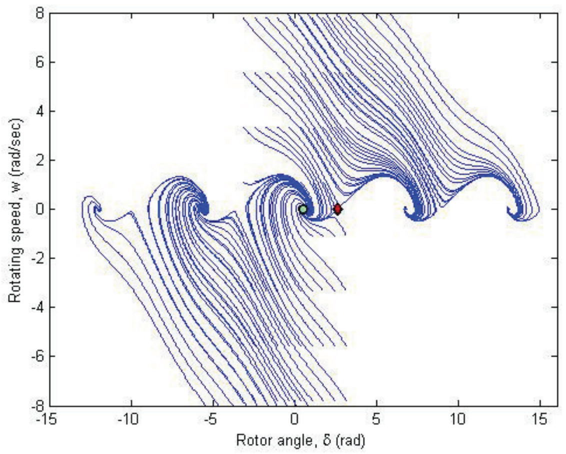

Fig. 1. Phase portraits of deterministic SMIB system for $P_m = 0.5$, $D = 0.8$.

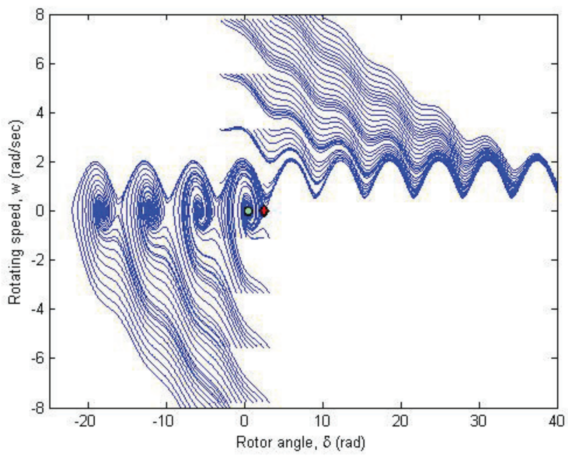

Fig. 2. Phase portraits of deterministic SMIB system for $P_m = 0.5$, $D = 0.36$.

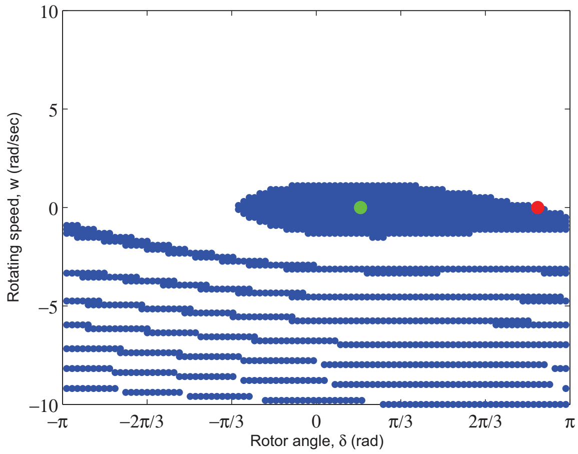

Fig. 3. Basin of attraction of the stable equilibrium point.

The maximum real power that can be transferred to the infinite bus is fixed as P_{max} =1 per unit (p.u.). Then the system (1) has a stable equilibrium point (SEP) located at $[arcsin(P_m) \quad 0]$ and a saddle point located at $[\pi - arcsin(P_m) \quad 0]$. The SEP is indicated by green circle and saddle point is indicated by red circle in Figs. 1- 3. There are no fixed points for $P_m > 1$ and all trajectories converge to the unique rotating orbit. If the mechanical power is kept fixed as $P_m = 0.5$ and when the damping parameter D is greater than the critical damping level D_c all trajectories converge to the SEP as shown in Fig. 1 whereas when the damping parameter D is less than the critical damping level D_c the system has a SEP and a stable limit cycle and depending on the initial condition the trajectories converge either to the SEP or to the stable limit cycle (rotating orbit) as shown in Fig. 2.

Fig. 3 presents the basin of attraction for the parameters $P_m = 0.5$, $D = 0.2$ and $M = 1$. The basin of attraction of SEP is colored in blue while the basin attraction of stable limit cycle is colored white.

At the equilibrium point $\dot{\delta} = 0$ the generator runs at a constant speed which leads to a constant rotor angle. However, when an imbalance between the mechanical power input and the electrical power output occurs due to the disturbance such as random load change, line tripping and loss of generator, the synchronism is lost. As it was studied in [16] this imbalance between the mechanical power input and the electrical power output in the SMIB power system given in (1) is modeled by $P_L(t) = \sigma L_\alpha(t)$ where $L_\alpha(t)$ is the alpha-stable Lévy process and σ is the noise intensity and by defining the state variable $\begin{bmatrix} x_1 & x_2 \end{bmatrix}^T = \begin{bmatrix} \delta & w \end{bmatrix}^T$ then the Itô form of SDE can be written as :

$$d\mathbf{X}(t) = \mathbf{f}(t, \mathbf{X}(t))dt + \mathbf{g}dL_\alpha(t) \quad (2)$$

$$\mathbf{f}(t, \mathbf{X}(t)) = \begin{bmatrix} x_2 \\ -Dx_2 + P_m - sinx_1 \end{bmatrix}; \ \mathbf{g} = \begin{bmatrix} 0 \\ \sigma \end{bmatrix} \quad (3)$$

and the increments of the Lévy process $dL_\alpha(t)$ is α-stable random variable [17].

There is no closed-form expression for the probability density function of α-stable distributions, however it can be calculated by the inverse Fourier transform of the characteristic function given as

$$\phi(t) = \begin{cases} exp\left\{j\mu t - \gamma^{\alpha}|t|^{\alpha}\left(1 - j\beta sign(t)\tan(\frac{\alpha\pi}{2})\right)\right\} & \text{if } \alpha \neq 1 \\ exp\left\{j\mu t - \gamma|t|\left(1 + j\beta\frac{2}{\pi}sign(t)\ln(\frac{\alpha\pi}{2})\right)\right\} & \text{if } \alpha = 1 \end{cases} \quad (4)$$

The distribution α-stable random variable S_α (γ,β,μ) is characterized by the four parameters: the characteristic exponent α ($0 < \alpha \leq 2$) measures the impulsiveness, and the skewness parameter β measures the symmetry of the distribution, where $\beta = 0$ refers to symmetric distribution, $\beta < 0$ to left-skewed distribution and $\beta > 0$ to right-skewed distribution, μ is location parameter, and γ is scale parameter. The impulsiveness increases with decreasing characteristic exponent "α" and the tails of the corresponding distributions become heavier. As the absolute value of the β increases, asymmetric behavior of the distribution increases.

α-stable Lévy motion $L_\alpha(t)$ has the following properties [22, 23]:

- $L_\alpha(0) = 0$ almost surely (a.s.),
- L_α(t) has the independent and stationary increments "$dL_\alpha(t)$",
- $dL_\alpha(t) \doteq L_\alpha(t) - L_\alpha(s) \sim S_\alpha((t-s)^{1/\alpha}, \beta, 0)$ for any $0 \leq s < t < \infty$.

The Gaussian noise $W(t) \triangleq \frac{dB}{dt}$ is the formal derivative of Wiener process (Brownian motion) $B(t)$ [22] and the increments of the Wiener process "$dB(t)$" is the special case of α-stable Lèvy motion with $\alpha = 2$, $\beta = 0$ " i.e., $S_2(\gamma, 0, \mu) = N(\mu, 2\gamma^2)$ " Normal (Gaussian) distribution with mean μ and variance $2\gamma^2$ [17].

The Euler-Maruyama method given in [23, 24] is applied to approximate the numerical solution of (2) as

$$\mathbf{X}_{t_i} = \mathbf{X}_{t_{i-1}} + \mathbf{f}(t_{i-1}, \mathbf{X}(t_{i-1}))\tau + \mathbf{g} \triangle L^{\tau}_{\alpha,i} \quad (5)$$

where the increment of the Lévy process is α-stable random variable $\triangle L^{\tau}_{\alpha,i}$ defined by $\triangle L^{\tau}_{\alpha,i} = L_\alpha([t_{i-1}, t_i]) \sim S_\alpha(\tau^{1/\alpha}, \beta, \mu)$ with $\tau = t_i - t_{i-1}$ have been generated by the method given in [23].

3. Basin Stability of Stochastic Single Machine Infinite Bus Power System

For different types of disturbance such as short circuits, load fluctuations or renewable generations the possibility of the power system to reach the synchronous state can be easily determined in terms of basin stability.

Basin stability: The criteria for the basin stability is quantified by the percentage of initial values reaching a stable fixed point after a given disturbance.

To estimate the basin stability the definition of return probability can be defined as follows :

Return probability: The probability of the system returning to a stable fixed point is defined as the return probability.

To observe the effect of α-stable Levy type load fluctuations 400 initial conditions of (δ, w) are taken from $[-\pi, \pi] \times [-10, 10]$ and 1000 random realizations are carried out for each initial condition and then the system is integrated long enough and the percentage of the initial values converging to the SEP is calculated.

In the absence load fluctuations the basin stability diagram is obtained by varying the values of mechanical power P_m and damping D as shown in Fig. 4. For the parameters of mechanical power and damping corresponding to the red points,

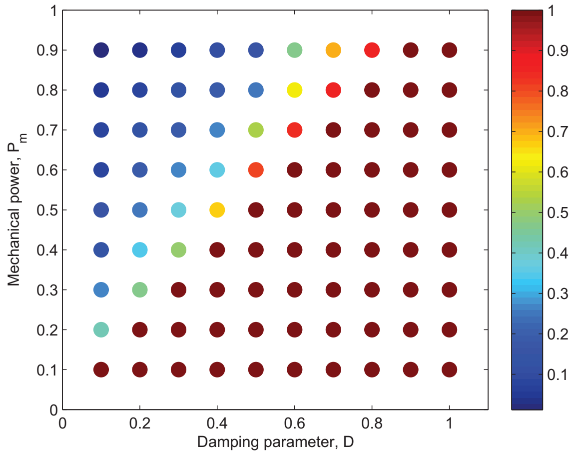

Fig. 4. Basin stability diagram for deterministic case.

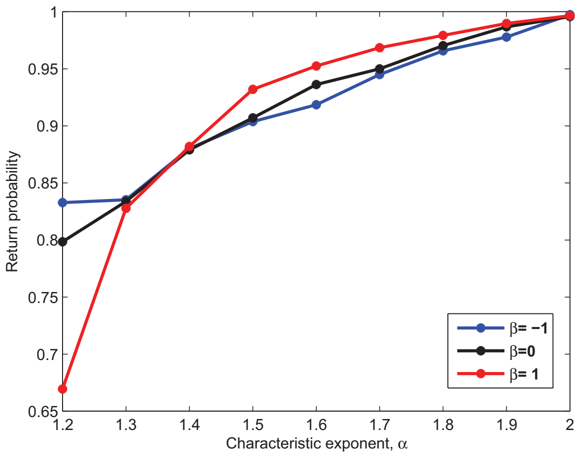

Fig. 5. Return probability for $P_m = 0.5$ and $D = 0.8$.

all trajectories converge to the SEP with the return probability one. For the parameters of mechanical power and damping corresponding to the blue points, the trajectories converge to the stable limit cycle (rotating orbit) with the return probability zero. For example, for the parameters of mechanical and damping corresponding to the yellow points 600 of the 1000 realizations converge to the SEP. Consider the mechanical power P=0.5 and damping $D = 0.8$. In the deterministic case all trajectories converge to the SEP as observed previously. When the power imbalance between the mechanical and electrical power is modelled by Brownian motion ($\alpha = 2, \beta = 0$) the return probability is evaluated as 0.9965. However the return probability decreases with the decrease of characteristic exponent α (increase of impulsiveness) for either symmetric or asymmetric α-stable Lévy motion as shown in Fig. 5.

Fig. 6 shows the return probabilities when the mechanical power $P = 0.5$ and damping $D = 0.2$ are selected. For negative-skewed α-stable Lévy motion the return probability increases with the decrease of characteristic exponent α (increase of impulsiveness) and then for $\alpha = 1.2$ return probability

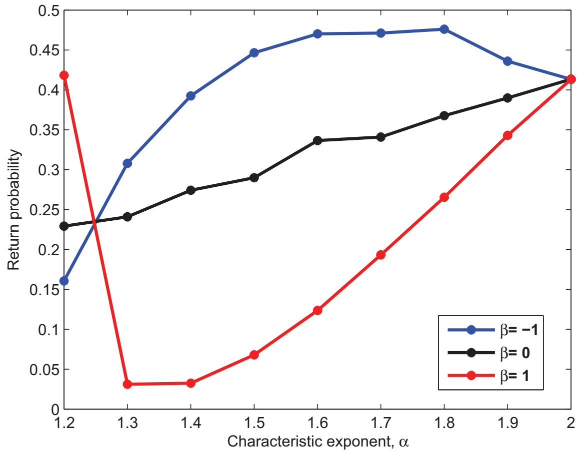

Fig. 6. Return probability for $P_m = 0.5$ and $D = 0.2$.

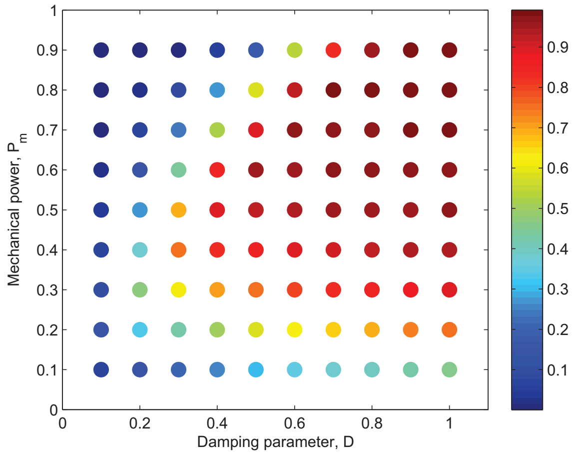

Fig. 8. Basin stability diagram for $\alpha = 1.7$, $\beta = -1$.

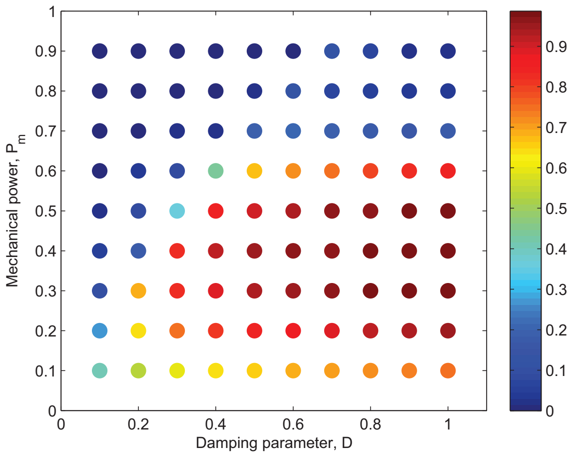

Fig. 7. Basin stability diagram for $\alpha = 1.7$, $\beta = 1$.

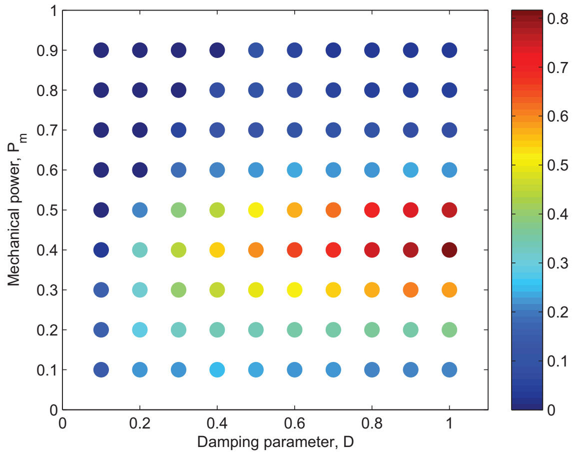

Fig. 9. Basin stability diagram for $\alpha = 1.2$, $\beta = 1$.

decreases. For positive-skewed α-stable L$e$vy motion the return probability decreases with the decrease of characteristic exponent α and then for $\alpha = 1.2$ the return probability increases.

Figs. 7- 9 present the basin stability diagram over the parameter space $P_m - D$ with the parameter changes of characteristic exponent α and skewness β. It can be seen from Figs. 7- 8 that asymmetric α-stable L$e$vy motion with $\alpha = 1.7$ provides a change in the basin stability diagram compared to the deterministic basin stability diagram. The return probability increases for some specific parameter pair value of (P_m, D) and hence the stability of the rotor angle is improved while for the other parameter pair values of (P_m, D) the return probability decreases. Furthermore how the location of the basin stability over parameter space changes according to the skewness parameter $\beta = 1$ and $\beta = -1$ can be clearly seen from Figs. 7- 8. For $\alpha = 1.2$ and $\beta = 1$ the region of stability for SEP becomes small, hence the system is not able to withstand to the perturbations as shown in Fig. 9.

4. Conclusion

In this paper, the fluctuations in the load of SMIB systems have been modeled as α-stable Lévy process and the basin stability of the stable equilibrium point over parameter space of mechanical power and damping have been investigated numerically. For some parameter pair of mechanical power and damping (P_m, D) the return probability decreases with the decrease of characteristic exponent α (increase of impulsiveness) hence it becomes more difficult to converge to the SEP. The synchronous state's stability deteriorates when α decreases. However for some specific parameter pair value of mechanical power and damping (P_m, D), the return probability can be improved by adjusting the impulsiveness or asymmetry of fluctuations which can be considered as the benefit of noise. As a future work, the basin stability of multi-machine systems under Lévy type perturbations will be investigated.

5. References

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