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Comparative dynamic analysis of axially loaded beams on modified Vlasov foundation

Çağlayan Hızal[*1] and Hikmet Hüseyin Çatal[2a]

[1]*Department of Civil Engineering, Engineering Faculty of Izmir Institute of Technology, 35430, Urla, Izmir, Turkey*
[2]*Department of Civil Engineering, Engineering Faculty of Dokuz Eylul University, 35160, Buca, Izmir, Turkey*

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Abstract. Vibration analysis of the beams on elastic foundation has gained the great interest of many researchers. In the literature, there are many studies that focus on the free vibration analysis of the beams on one or two parameter elastic foundations. On the other hand, there are no sufficient studies especially focus on the comparison of dynamic response including the bending moment and shear force of the beams resting on Winkler and two parameter foundations. In this study, dynamic response of the axially loaded Timoshenko beams resting on modified Vlasov type elastic soil was investigated by using the separation of variables method. Governing equations were obtained by assuming that the material had linear elastic behaviour and mass of the beam was distributed along its length. Numerical analysis were provided and presented in figures to find out the differences between the modified Vlasov model and conventional Winkler type foundation. Furthermore, the effect of shear deformation of elastic soil on the dynamic response of the beam was investigated.

Keywords: Vlasov type foundation; Timoshenko beam; forced vibration; separation of variables method

1. Introduction

The dynamic response of the beams on elastic foundation has been investigated by many researchers mostly using Winkler Hypothesis which represents the soil with independent elastic springs resist to transverse displacement. Most of these researchers have studied on free vibration or static response of the beams on Winkler type foundation. Çatal investigated the free vibration of partially embedded piles in Winkler soil with bending moment, axial and shear force effects (Çatal 2002, and 2006). Çatal and Çatal (2006) analyzed a partially embedded pile in elastic soil using differential transform method. Yeşilce and Çatal (2008) obtained the natural circular frequencies of piles embedded in the soil having different subgrade reaction. Çalım and Akkurt (2010) studied on the free vibration and static response of straight and circular beams on elastic foundation. Yaghoobi *et al.* (2014) studied on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loadings using VIM. On the other

∗Corresponding author, Ph.D. Student, E-mail: caglayanhizal@iyte.edu.tr
[a]Professor, E-mail: huseyin.catal@deu.edu.tr

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hand, many foundation models exist to estimate the soil-structure interaction whose best-knowns are two and three parameter elastic foundations. The best-knowns and mostly used of two parameter foundation models are Pasternak and Vlasov type foundations which suppose that Winkler springs are interacted to each other due to the transverse displacement (Pasternak 1954, Vlasov and Leont'ev 1963). Two-parameter foundation models define a second parameter which represents the coefficient of an incompressible shear layer on soil surface to constitute the interaction between the elastic Winkler springs. The first parameter of the two-parameter elastic soil can be evaluated same as the Winkler model according to soil properties. However the second parameter of the soil can be obtained by using different ways for each two-parameter elastic soil model proposed by different researchers. In Pasternak model, the influence of the soil to both sides of foundation beam is ignored differently from the Vlasov Model but despite this difference, the second parameter can be taken as same values in both methods (Morfidis and Avramidis 2002). Many researchers studied about the vibration of beams on two parameter foundations. Arbeloda-Monsalve *et al.* (2008) analyzed a Timoshenko beam-column with generalized end conditions on two parameter elastic foundation. Celep *et al.* (2010) calculated the response of a completely free beam on a tensionless Pasternak foundation subjected to a dynamic load. Malekzadeh and Karami (2008) analyzed free vibration of thick beams on two-parameter elastic foundations using differential quadrature and finite element method. Ma *et al.* (2009) analyzed statically an infinite beam resting on a tensionless Pasternak foundation.

Forced vibration of the beams on one or two parameter elastic foundations are commonly analyzed by considering the effects of lower modes of vibration. Although the lower modes are more effective especially on the displacement and angle of rotation, internal forces including bending moment and shear forces are affected ultimately by higher modes of vibration. This point has a great importance to obtain the maximum internal forces of beams subjected to dynamic loads. Studies about dynamic analysis of the beams which were modeled as distributed parameter system also contribute to solution of the forced vibration equations of beams on elastic soil. In recent years, considerable amount of studies have been carried out by many researchers about dynamic response of uniform, prismatic or composite beams (Attarnejad *et al*. 2010, Gunda *et al*. 2011). Dadfarnia *et al.* (2005) analyzed a Timoshenko beam by selecting different time function for displacement and angle of rotation using the Galerkin method. Demirdağ and Çatal (2007) studied earthquake response of semi-rigid supported single storey frames modeled as continuous system. Demirdağ (2008) investigated the free vibration of elastically supported Timoshenko columns with attached masses by transfer matrix. Yeşilce and Çatal (2009) investigated the free vibration of axially loaded Reddy-Bickford beam on elastic soil by using the differential transform method**.** Çalım (2009) investigated the forced vibration of the beams on viscoelastic foundations. Sapountzakis and Kampitsis (2010) analyzed Timoshenko beam-columns partially supported on tensionless Winkler foundation. Çatal (2012) analyzed the response of a forced Euler-Bernoulli beam using the differential transform method. Yeşilce and Çatal (2011) investigated the free vibration of axially loaded and semi rigid connected Reddy-Bickford beam on elastic soil by using the differential transform method. Yeşilce (2011) analyzed the free vibration of axially loaded and semi rigid connected Reddy-Bickford beam on elastic soil by using the differential transform method and differential quadrature method.

In this study, the dynamic response analysis of the axially loaded beams resting on the Vlasov type foundation was performed by considering Timoshenko beam theory and rotatory inertia of the beam. Free vibration equation of the beam depending on transverse displacement shape function which varies depending on the non-dimensional location coordinate was obtained by using the

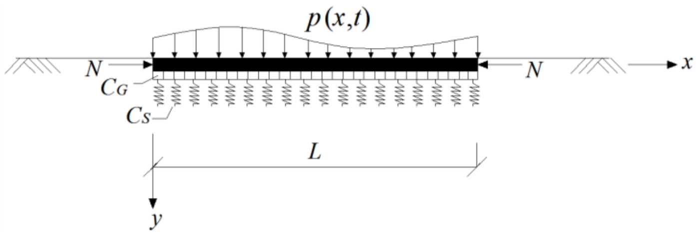

Fig. 1 Timoshenko beam resting on two-parameter elastic foundation

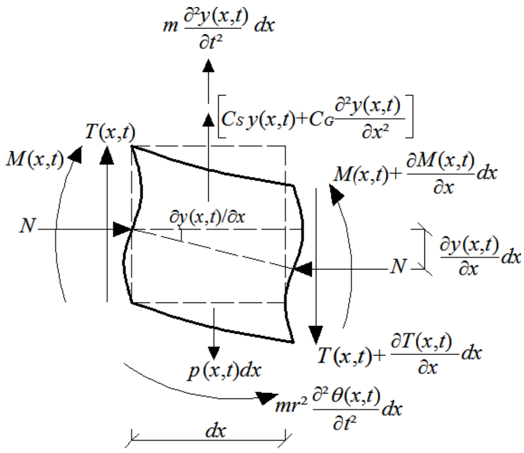

Fig. 2 Free body-diagram of infinitesimal element obtained from the beam shown in Fig. 1

separation of variables method. Natural frequencies of the beam were calculated by using an iterative computer program prepared by the authors. Forced vibration equation of the beam was obtained by using the orthogonality properties of vibration modes and a general solution was obtained according to concentrated dynamic loading case. In numerical examples, dynamic response analysis of simply supported beams resting on elastic soil was analyzed by considering both Winkler and Vlasov type foundation models.

2. Governing equations

The analytical model proposed in this paper consists of an axially loaded uniform beam on two-parameter elastic foundation that has the elastic spring coefficient C_S, and shear layer coefficient C_G as shown in Fig. 1. It was assumed that the beam material was linear elastic with distributed mass m, bending rigidity EI and shear rigidity κAG. According to these assumptions, governing equations of the beam on Vlasov type elastic foundation shown in Fig. 1 can be written as Eq. (1) and (2) by using the equations of moment and transverse force equilibrium of infinitesimal beam element shown in Fig. 2.

$$\frac{\partial T(x,t)}{\partial x} - m\frac{\partial^2 y(x,t)}{\partial t^2} - C_S\, y(x,t) + C_G\frac{\partial^2 y(x,t)}{\partial x^2} + p(x,t) = 0 \qquad (1)$$

$$\frac{\partial M(x,t)}{\partial x} - N\frac{\partial y(x,t)}{\partial x} + mr^2\frac{\partial^2\theta(x,t)}{\partial t^2} - T(x,t) = 0 \tag{2}$$

In Eqs. (1) and (2); N, $\theta(x,t)$, $T(x,t)$, $M(x,t)$ and r denote the angle of rotation, shear force, bending moment functions and the radius of gyration, respectively.

$$M(x,t) = -EI\frac{\partial\theta(x,t)}{\partial x} \tag{3}$$

$$T(x,t) = \gamma(x,t)\kappa AG \tag{4}$$

$$\theta(x,t) = \frac{\partial y(x,t)}{\partial x} - \gamma(x,t) \tag{5}$$

where E, G, I, A, κ and $\gamma(x,t)$ denote the modulus of elasticity, shear modulus, moment of inertia, cross-sectional area, shear correction factor and shear deformation angle of the beam, respectively. Eq. (2) can be written as below by taking $\theta(x,t)$ as $\left[\frac{\partial y(x,t)}{\partial x} - \frac{T(x,t)}{\kappa AG}\right]$.

$$\frac{\partial M(x,t)}{\partial x} - T(x,t) - N\frac{\partial y(x,t)}{\partial x} + mr^2\left[\frac{\partial^4 y(x,t)}{\partial t^2\partial x^2} - \frac{1}{\kappa AG}\frac{\partial^2 T(x,t)}{\partial t^2}\right] = 0 \tag{6}$$

The transverse displacement function, $y(x,t)$ can be written depending on bending moment and shear force functions by using the Eq. (4) and third order derivative of Eq. (5), as follows (Çatal 2006).

$$\frac{\partial^4 y(x,t)}{\partial x^4} = -\frac{1}{EI}\frac{\partial^2 M(x,t)}{\partial x^2} + \frac{1}{\kappa AG}\frac{\partial^3 T(x,t)}{\partial x^3} \tag{7}$$

3. Free vibration analysis

The free vibration equation of motion can be written depending on transverse displacement $y(x,t)$ by substituting the Eqs. (1) and (6) into Eq. (7) for $p(x,t)$=0, as follows.

$$\begin{aligned}\frac{\partial^4 y(x,t)}{\partial x^4} &+ \frac{1}{EI}\left[m\frac{\partial^2 y(x,t)}{\partial t^2} + C_S y(x,t) - C_G\frac{\partial^2 y(x,t)}{\partial x^2} - mr^2\frac{\partial^4 y(x,t)}{\partial t^2\partial x^2} + N\frac{\partial^2 y(x,t)}{\partial x^2}\right] \\ &- \frac{1}{\kappa AG}\left[m\frac{\partial^4 y(x,t)}{\partial t^2\partial x^2} + C_S\frac{\partial^2 y(x,t)}{\partial x^2} - C_G\frac{\partial^4 y(x,t)}{\partial x^4}\right] \\ &+ \frac{mr^2}{EI\kappa AG}\left[m\frac{\partial^4 y(x,t)}{\partial t^4} + C_S\frac{\partial^2 y(x,t)}{\partial t^2} - C_G\frac{\partial^4 y(x,t)}{\partial x^2\partial t^2}\right] = 0\end{aligned} \tag{8}$$

The transverse displacement function, $y(x,t)$ depends on the location and time variables x and t; but it can be written in terms of two independent function by using the separation of variables method such as given below(Çatal 2006).

$$y(x,t)=\sum_{i=1}^{\infty}Y_i(x)\mu_i(t) \tag{9}$$

where, $Y_i(x)$ and $\mu_i(t)$ denote the displacement shape and normal coordinate functions of i^{th} mode, respectively. If the modal coordinate function is taken as $\mu_i(t)=\sin(\omega_i t+\varphi)$ for free vibration case, the equation of motion becomes an ordinary differential equation and can be written as below.

$$\begin{aligned}Y_i^{IV}(x)&+\left[\frac{\kappa AG}{\kappa AG+C_G}\right]\left[\frac{m\omega_i^2-C_s}{\kappa AG}+\frac{\left(m\omega_i^2r^2+N-C_G\right)}{EI}+\frac{m\omega_i^2r^2C_G}{EI\kappa AG}\right]Y_i^{II}(x)\\&+\left[\frac{\kappa AG}{\kappa AG+C_G}\right]\left[\frac{mr^2}{EI\kappa AG}\left(m\omega_i^4-C_s\omega_i^2\right)-\frac{\left(m\omega_i^2-C_s\right)}{EI}\right]Y_i(x)=0\end{aligned} \tag{10}$$

where ω_i denotes the natural angular frequency of i^{th} mode. The free vibration equation of motion becomes as follows for $\xi=x/L$.

$$\begin{aligned}Y_i^{IV}(\xi)&+L^2\left[\frac{\kappa AG}{\kappa AG-C_G}\right]\left[\frac{m\omega_i^2-C_s}{\kappa AG}+\frac{\left(m\omega_i^2r^2+N-C_G\right)}{EI}+\frac{m\omega_i^2r^2C_G}{EI\kappa AG}\right]Y_i^{II}(\xi)\\&+L^4\left[\frac{\kappa AG}{\kappa AG+C_G}\right]\left[\frac{mr^2}{EI\kappa AG}\left(m\omega_i^4-C_s\omega_i^2\right)-\frac{\left(m\omega_i^2-C_s\right)}{EI}\right]Y_i(\xi)=0\end{aligned} \tag{11}$$

where ξ denotes the non-dimensional location variable. Eq. (11) can be written as below.

$$Y_i^{IV}(\xi)+a_iY_i^{II}(\xi)+b_iY_i(\xi)=0 \tag{12}$$

where

$$a_i=L^2\left[\frac{\kappa AG}{\kappa AG-C_G}\right]\left[\frac{m\omega_i^2-C_s}{\kappa AG}+\frac{\left(m\omega_i^2r^2+N-C_G\right)}{EI}+\frac{m\omega_i^2r^2C_G}{EI\kappa AG}\right] \tag{13}$$

$$b_i=L^4\left[\frac{\kappa AG}{\kappa AG+C_G}\right]\left[\frac{mr^2}{EI\kappa AG}\left(m\omega_i^4-C_s\omega_i^2\right)-\frac{\left(m\omega_i^2-C_s\right)}{EI}\right] \tag{14}$$

Finally, the displacement shape function is obtained as follows by solving the Eq. (12).

$$Y_i(\xi)=C_1\cos\lambda_1\xi+C_2\sinh\lambda_1\xi+C_3\cosh\lambda_2\xi+C_4\sinh\lambda_2\xi \tag{15}$$

where

$$\lambda_1=\sqrt{\left|\frac{-a_i-\sqrt{a_i^2-4b_i}}{2}\right|}\qquad \lambda_2=\sqrt{\left|\frac{-a_i+\sqrt{a_i^2-4b_i}}{2}\right|} \tag{16}$$

Angle of rotation, bending moment and shear force functions can be also written by using the separation of variables method as below.

$$\theta(\xi,t)=\sum_{i=1}^{\infty}\Theta_i(\xi)\mu_i(t) \tag{17}$$

$$M(\xi,t)=\sum_{i=1}^{\infty}\overline{M}_i(\xi)\mu_i(t) \tag{18}$$

$$T(\xi,t)=\sum_{i=1}^{\infty}\overline{T}_i(\xi)\mu_i(t) \tag{19}$$

In Eqs. (17), (18) and (19), $\Theta_i(\xi)$, $\overline{M}_i(\xi)$ and $\overline{T}_i(\xi)$ denote shape functions of angle of rotation, bending moment and shear force, respectively. These functions can be obtained as given in Eq. (20) by using Eqs. (1), (6) and (7).

$$\begin{gathered}\overline{M}_i(\xi)=EI\left[\left(\frac{C_s-m\omega_i^{\,2}}{\kappa AG}\right)Y_i(\xi)-\left(\frac{C_G}{\kappa AG}+1\right)\frac{Y_i^{\,II}(\xi)}{L^2}\right]\\ \overline{T}_i(\xi)=\frac{M_i^{\,I}(\xi)-\left(m\omega_{\rm i}^{\,2}r^2+N\right)Y_i^{\,I}(\xi)}{\left(1-\dfrac{m\omega_{\rm i}^{\,2}r^2}{\kappa AG}\right)L},\qquad \Theta_i(\xi)=\frac{Y_i^{\,I}(\xi)}{L}-\frac{\overline{T}_i(\xi)}{\kappa AG}\end{gathered} \tag{20}$$

$$y(\xi,t)=\sum_{i=1}^{\infty}\left(C_1\cos\lambda_1\xi+C_2\sin\lambda_1\xi+C_3\cosh\lambda_2\xi+C_4\sinh\lambda_2\xi\right)\mu_i(t) \tag{21}$$

$$\theta(\xi,t)=\sum_{i=1}^{\infty}\left(C_1K_5\sin\lambda_1\xi+C_2K_6\cos\lambda_1\xi+C_3K_7\sinh\lambda_2\xi+C_4K_7\cosh\lambda_2\xi\right)\mu_i(t) \tag{22}$$

$$M(\xi,t)=\sum_{i=1}^{\infty}\left(C_1K_1\cos\lambda_1\xi+C_2K_1\sin\lambda_1\xi+C_3K_2\cosh\lambda_2\xi+C_4K_2\sinh\lambda_2\xi\right)\mu_i(t) \tag{23}$$

$$T(\xi,t)=\sum_{i=1}^{\infty}\left(C_1K_3\sin\lambda_1\xi-C_2K_3\cos\lambda_1\xi+C_3K_4\sinh\lambda_2\xi+C_4K_4\cosh\lambda_2\xi\right)\mu_i(t) \tag{24}$$

where

$$\begin{gathered}K_1=\left(\frac{\mathrm{EI}\left(\mathrm{C_s}-\mathrm{m}\omega_i^{\,2}\right)}{\kappa\mathrm{AG}}\right)+\left(\frac{C_G}{\kappa AG}+1\right)\frac{EI\lambda_1^{\,2}}{L^2}\ ,\quad K_2=\left(\frac{\mathrm{EI}\left(\mathrm{C_s}-\mathrm{m}\omega_i^{\,2}\right)}{\kappa\mathrm{AG}}\right)-\left(\frac{C_G}{\kappa AG}+1\right)\frac{EI\lambda_2^{\,2}}{L^2}\\ K_3=\frac{\lambda_1\left(-K_1+\left(\mathrm{m}\omega_i^{\,2}r^2+N\right)\right)}{\mathrm{L}(-\mathrm{m}\omega_i^{\,2}r^2/\mathrm{kAG}+1)}\ ,\quad K_4=\frac{\lambda_2\left(K_2-\left(\mathrm{m}\omega_i^{\,2}r^2+N\right)\right)}{\mathrm{L}(-\mathrm{m}\omega_i^{\,2}r^2/\mathrm{kAG}+1)}\ ,\quad K_5=\left(\frac{-\lambda_1}{\mathrm{L}}-\frac{K_3}{\kappa\mathrm{AG}}\right)\\ K_6=\left(\frac{\lambda_1}{\mathrm{L}}-\frac{K_3}{\kappa\mathrm{AG}}\right)\ ,\quad K_7=\left(\frac{\lambda_2}{\mathrm{L}}-\frac{K_4}{\kappa\mathrm{AG}}\right)\end{gathered} \tag{25}$$

Natural angular frequencies of the beam can be calculated by using an iterative procedure based on determination of values of ω_i which give the non-trivial solution of the matrix obtained from boundary conditions of the beam. After obtaining the natural angular frequencies, a normalization is required for the function of $Y_i(\xi)$. After the normalization procedure, the maximum value of $Y_i(\xi)$ should be equal to 1. Thus, normalized shape functions of $\Theta_i(\xi)$, $M_i(\xi)$ and $T_i(\xi)$ can be obtained from the normalized displacement function of $Y_i(\xi)$.

4. Forced vibration analysis

Forced vibration equation of the beam on two parameter elastic foundation can be written depending on the parameters of ξ and t as follows by using the Eqs. (1) and (2).

$$m\frac{\partial^2 y(\xi,t)}{\partial t^2}+C_S\, y(\xi,t)-\frac{1}{L^2}C_G\frac{\partial^2 y(\xi,t)}{\partial \xi^2}-\frac{1}{L}\frac{\partial T(\xi,t)}{\partial \xi}=p(\xi,t) \tag{26}$$

$$\frac{1}{L}\frac{\partial M(\xi,t)}{\partial \xi}+mr^2\frac{\partial^2 \theta(\xi,t)}{\partial t^2}-\frac{1}{L}N\frac{\partial y(\xi,t)}{\partial \xi}-T(\xi,t)=0 \tag{27}$$

Eqs. (26) and (27) become as follows by using the separation of variables method.

$$\sum_{i=1}^{\infty}\left[mY_i(\xi)\,\ddot{\mu}_i(t)+\left(C_S Y_i(\xi)-\frac{1}{L^2}C_G Y_i''(\xi)-\frac{1}{L}\overline{T}_i'(\xi)\right)\mu_i(t)\right]=p(\xi,t) \tag{28}$$

$$\sum_{i=1}^{\infty}\left[mr^2\Theta_i(\xi)\,\ddot{\mu}_i(t)+\left(\frac{1}{L}\overline{M}_i'(\xi)-\frac{1}{L}NY_i'(\xi)-\overline{T}_i(\xi)\right)\mu_i(t)\right]=0 \tag{29}$$

If the ratio of $-\ddot{\mu}_i(t)/\mu_i(t)$ is obtained dividing Eq. (29) by $mr^2\Theta_i(\xi)$, it leads to a constant value and equals to ω_i^2 (Chopra 2007).

$$-\frac{\ddot{\mu}_i(t)}{\mu_i(t)}=\frac{\left(\frac{1}{L}\overline{M}_i'(\xi)-\frac{1}{L}NY_i'(\xi)-\overline{T}_i(\xi)\right)}{mr^2\Theta_i(\xi)}=\omega_i^2 \tag{30}$$

Eq. (30) shows that the ratio of $-\ddot{\mu}_i(t)/\mu_i(t)$ has always same value for both free and forced vibration cases due to the fact that it only depends on shape functions. Thus, the same equality can be also obtained for Eq. (28) (Chopra, 2007).

$$-\frac{\ddot{\mu}_i(t)}{\mu_i(t)}=\frac{\left(C_S Y_i(\xi)-\frac{1}{L^2}C_G Y_i''(\xi)-\frac{1}{L}\overline{T}_i'(\xi)\right)}{mY_i(\xi)}=\omega_i^2 \tag{31}$$

Thus, Eqs. (28) and (29) become as follows by using the Eq. (30) and (31).

$$\sum_{i=1}^{\infty}mY_i(\xi)\left[\ddot{\mu}_i(t)+\omega_i^2\mu_i(t)\right]=p(\xi,t) \tag{32}$$

$$\sum_{i=1}^{\infty}mr^2\Theta_i(\xi)\left[\ddot{\mu}_i(t)+\omega_i^2\mu_i(t)\right]=0 \tag{33}$$

Eqs. (32) and (33) denote that the coupled ordinary differential equations which are including all modes of vibration. Using the orthogonality property of vibration modes, an uncoupled equation of motion can be obtained for each vibration mode. Following equations can be written by multiplying Eqs. (32) and (33) with $Y_i(\xi)$ and $\Theta_i(\xi)$, respectively, and integrating along the beam length provided that *i* and *j* denote different modes.

$$\sum_{i=1}^{\infty}\int_0^1 mLY_i(\xi)Y_j(\xi)d\xi\left[\ddot{\mu}_i(t)+\omega_i^2\mu_i(t)\right]=L\int_0^1 Y_j(\xi)p(\xi,t)d\xi \tag{34}$$

$$\sum_{i=1}^{\infty}\int_0^1 mLr^2\Theta_i(\xi)\Theta_j(\xi)d\xi\left[\ddot{\mu}_i(t)+\omega_i^2\mu_i(t)\right]=0 \tag{35}$$

Finally, a second order differential equation that only depends on time variable *t* can be obtained as follows by combining Eqs. (34) and (35).

$$\ddot{\mu}_j(t)+\omega_j^2\mu_j(t)=\frac{P_j(t)}{M_j} \tag{36}$$

where,

$$M_j=mL\int_0^1\left(Y_j^2(\xi)+r^2\Theta_j^2(\xi)\right)d\xi \tag{37}$$

$$P_j(t)=L\int_0^1 Y_j(\xi)p(\xi,t)d\xi \tag{38}$$

In Eqs.(37) and (38), $P_j(t)$ and M_j can be named as generalized load and mass, respectively. In the case of $p(\xi,t)$ is a concentrated dynamic load function, the generalized dynamic load can be written as follows (Dadfarnia *et al.* 2005).

$$p(\xi,t)=\delta(x-\bar{a}/L)q(t) \tag{39}$$

$$P_j(t)=Y(\xi=\bar{a}/L)q(t) \tag{40}$$

Where δ and $\bar{a}$ denote the Dirac-delta function and distance of the concentrated load from the reference point *y*=0, *x*=0, respectively. Thus, the solution of differential equation given in Eq. (36) can be obtained by using the Duhamel's integral such as below.

$$\mu_j(t)=\mu_0\cos\left(\omega_j t\right)+\frac{\dot{\mu}_0}{\omega_j}\sin\left(\omega_j t\right)+\frac{Yj(\xi=\bar{a}/L)}{M_j\omega_j}\int_0^t q(\tau)\sin\left(\omega_j t-\omega_j\tau\right)d\tau \tag{41}$$

where μ_0 and $\dot{\mu}_0$ are the initial value of modal displacement and velocity, respectively.

5. Calculation of coefficients of the two parameter foundation

The calculation of parameters of the elastic soil is also related with the type of the elastic soil model. The first parameter of the elastic soil which represents the modulus of transverse deformation can be evaluated by using the formulas given for Winkler foundation model. However the calculation of the second parameter is directly related with the type of the two-parameter elastic soil model. If the second parameter is taken as a shear layer with coefficient C_G such as Vlasov model, C_S and C_G can be evaluated by using formulas given by Vlasov and Leont'ev for rectangular beams on two parameter foundation (Vlasov and Leont'ev 1966). Formulas given by Vlasov and Leont'ev become as follows for semi-infinite elastic medium as suggested by Zhaohua and Cook for beams on two-parameter elastic foundation (Zhaohua and Cook 1983).

$$C_S = \frac{E_o \bar{b}}{2(1-\upsilon_0^2)} \frac{\bar{\gamma}}{l} \quad , \quad C_G = \frac{E_o \bar{b}}{4(1+\upsilon_0)} \frac{l}{\bar{\gamma}} \tag{42}$$

where, $\bar{b}$ denotes the width of the beam. The parameter of $\bar{\gamma}$ is defined by Vlasov and Leont'ev as a coefficient to characterize the decrease of the deflections with depth and commonly taken as $\bar{\gamma}=1$ (Vallaban and Das, 1991). Parameters of l, E_0, υ_0 are given in following equation.

$$l = \sqrt[3]{\frac{2EI(1-\nu_0^2)}{(1-\nu^2)E_0\bar{b}}} \quad , \quad E_o = \frac{E_s}{1-\upsilon_s^2} \quad , \quad \upsilon_o = \frac{\upsilon_s}{1-\upsilon_s^2} \tag{43}$$

where EI, E_s, υ_s and υ denotes the bending rigidity of beam, the modulus of elasticity and Poisson's ratio of the soil and the beam, respectively.

6. Numerical analyses and discussions

Numerical analysis consists of two parts which were named as numerical example-1 and numerical example-2, respectively. In example-1, numerical results obtained by using the analysis method proposed in this study were compared with earlier studies, and the effect of higher modes to the dynamic response of the beam was highlighted. In numerical example-2, it was aimed to reveal the difference between the Winkler and Vlasov type foundations and effect of the shear deformation of the beam to the vibration. For this reason, numerical analyses were carried out for each case of the Timoshenko beam on the Vlasov and Winkler types of foundation, and Euler beam on the Vlasov and Winkler types of foundation. The solutions obtained for Euler beam theory and Winkler type foundation were provided by taking the shear correction parameter of the beam, and shear parameter of the elastic soil as equal to zero, respectively (κ=0, C_G=0). A computer program was prepared in MATLAB to carry out the dynamic response analysis of the beam. The computer program consists of two parts which are the free and forced vibration analysis cases. In free vibration case, there is an iterative algorithm whose input values are mass, material properties, boundary conditions of the beam, and the number of total modes that will be taken into account. After the free vibration analysis procedure, natural angular frequencies and mode shapes of the beam are obtained. In forced vibration analysis case, maximum values of the displacement and bending moment functions at the midpoint and maximum values of the angle of rotation and shear force functions at supports are obtained according to the dynamic external load function. Maximum values of the displacement, angle of rotation, bending moment and shear force functions are calculated cumulatively for the input value of the total mode number. In this procedure, due to the fact that the effect of higher modes extremely appear on shear force, if the cumulative value of the shear force calculated in the i^{th} mode is close enough to the cumulative shear force calculated in the $(i\text{-}1)^{th}$ mode, the analysis is completed. Otherwise, the analysis procedure goes on until the sufficient convergence is obtained.

6.1 Numerical example-1

In numerical example-1, the dynamic response analysis of a simply supported beam resting on Winkler type foundation was performed and results were compared with those of earlier studies to ensure the accuracy of computer program written by the authors. The proposed beam model was investigated for free vibration by Timoshenko *et al.* (1974), and Friswell *et al.* (2007), and investigated for forced vibration by Çalım (2009), and Sapountzakis and Kampitsis (2010). Properties of proposed beam are; m=0.445 kN.s^2/m^2, I=1.439×10^{-2} m^4, E=24.82×10^6 kN/m^2, κ=0, υ=0.3, r=0 and L=6.096 m, and spring coefficients of the elastic soil are C_S=16550 KN/m^2 and C_G=0. The analysis model of proposed beam and dynamic external load which was applied to this beam was shown in Fig. 3.

In Table 1, natural frequencies of first five modes were obtained for the beam proposed in example-1 and compared with those of previous studies. In Table 2, displacement responses of the proposed beam obtained for the given triangular impulsive load and its comparison with the earlier studies carried out by Çalım (2009), and Sapountzakis and Kampitsis (2010) were presented. In Fig. 3, variation of the displacement and bending moment at midpoint, and angle of rotation and shear force at left support obtained for the increasing values of total considered mode number versus time were presented.

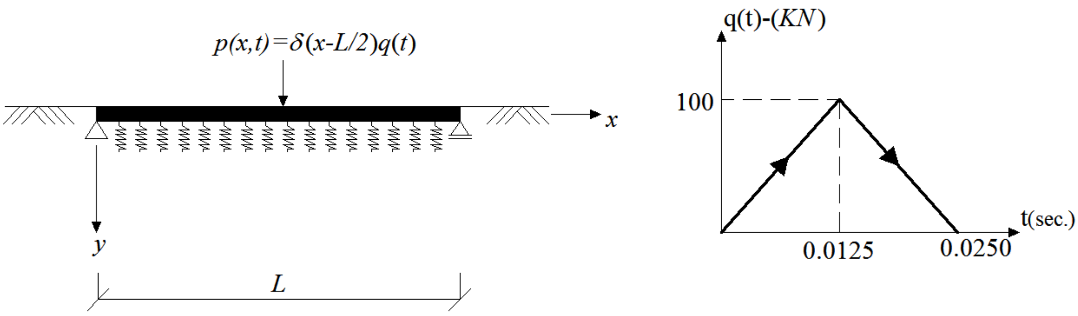

Fig. 3 Simply supported beam on Winkler type foundation

Table 1 Comparison of obtained natural frequencies (Hz.) with those of earlier studies

Mode Number	Timoshenko *et al.* (1974)	Friswell *et al.* (2007)	Çalım (2009)	Sapountzakis and Kampitsis (2010)	Present Study
1	32.9032	32.8980	32.8633	32.7946	32.9464
2	56.8135	56.8080	56.5972	56.5476	56.8905
3	112.908	111.900	110.7390	110.7220	112.0615
4	-	193.760	189.9390	189.4890	194.0449
5	-	-	222.0780	222.0770	300.9472

Table 2 Comparison of displacement responses with those of earlier studies

	Midpoint Displacement(m)		
	Çalım (2009)	Sapountzakis and Kampitsis (2010)	Present Study
Max.	0.002630	0.002630	0.002635
Min.	-0.002500	-0.002500	-0.002482

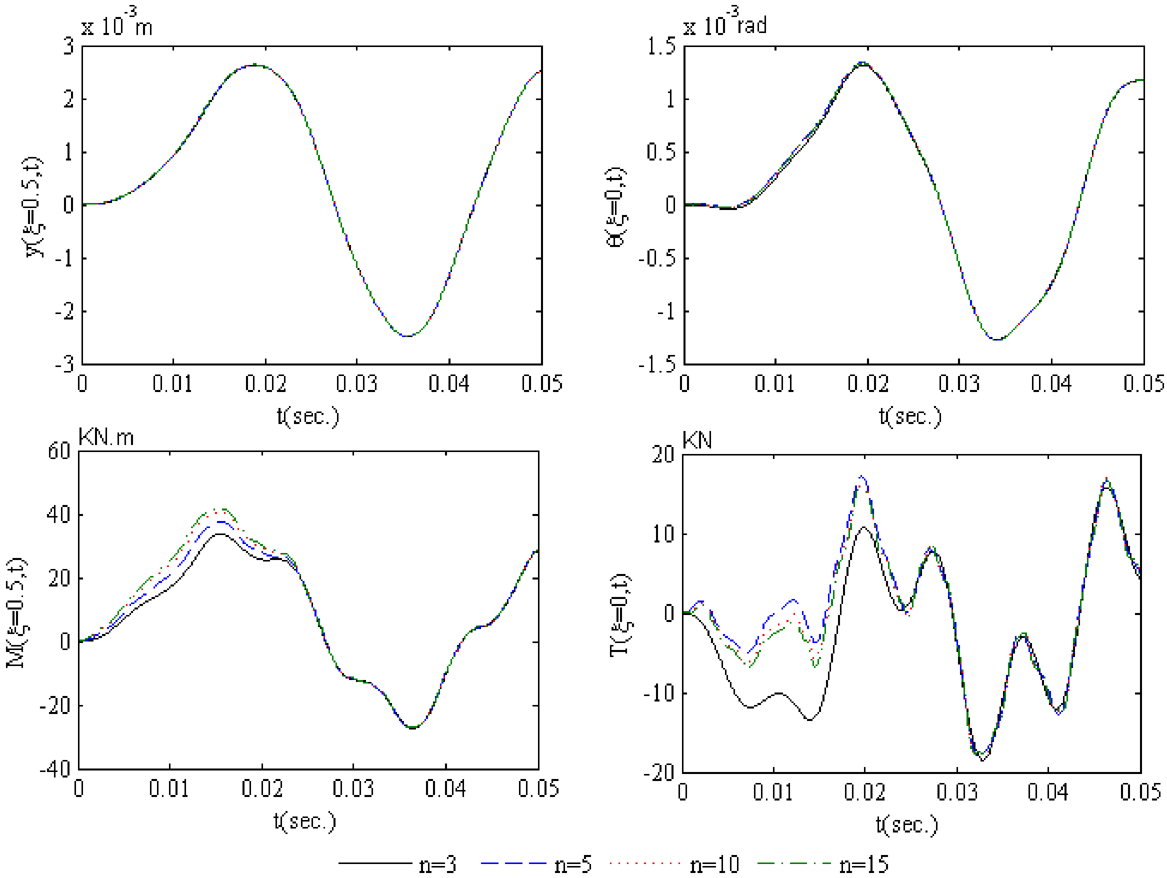

Fig. 4 Variation of the displacement, angle of rotation, bending moment and shear force functions of simply supported beam versus time for the increasing values of total considered mode number, *n*.

In analysis results of example-1, a high degree of consistency was observed between the results of present study and earlier studies. Besides the consistency of analysis results, the influence of higher modes especially on the bending moment and shear force can be seen clearly in Fig. 6. Displacement, angle of rotation, bending moment and shear force functions were obtained by multiplying the normal coordinate function with corresponding mode shape functions. Due the fact that mode shape functions of angle of rotation, bending moment and shear force are derived from normalized displacement mode shape; an increment in the amplitude of mode shapes of those functions is inevitable. For this reason, higher modes of shear force and bending moment functions become extremely effective on the vibration as it is seen in the Fig. 4.

6.2 Numerical example-2

The analysis model presented in numerical example-2 consists of a simply supported reinforced concrete beam on a modified Vlasov type elastic foundation. The beam has distributed mass and elasticity, and it is subjected to a concentrated dynamic load at mid-span as shown in Fig.(7).The characteristics of the reinforced concrete beam are; m=2.555 kN.s^2/m^2, κ=0.667, I=8.333×10^{-2} m^4,

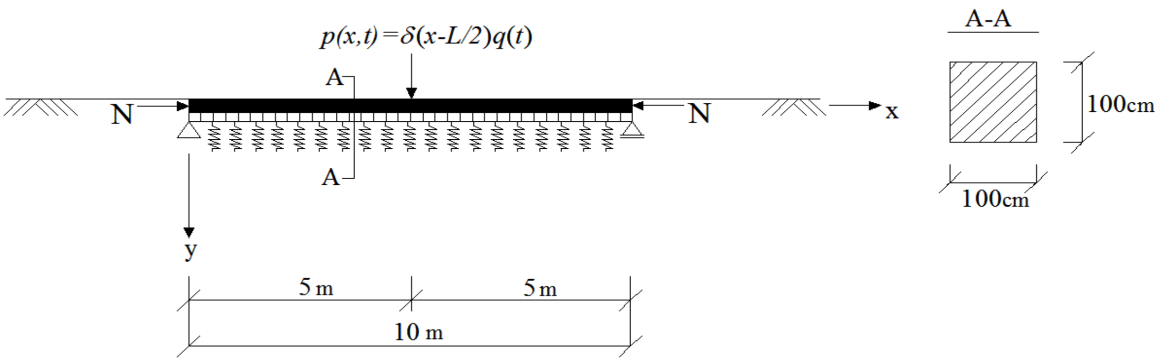

Fig. 5 Simply supported beam on modified Vlasov type foundation

Table 3 The coefficients of C_S and C_G obtained from some values α and β.

$\alpha=\frac{C_S L^4}{EI}$	$\beta=\frac{C_G}{C_S L^2}$	$C_S\,(KN/m^2)$	$C_G\,(KN)$
	0.001		23.333
1	0.010	233.333	233.333
	0.100		2333.333
	0.001		233.333
10	0.010	2333.333	2333.333
	0.100		23333.333
	0.001		2333.333
100	0.010	23333.333	23333.333
	0.100		233333.333
	0.001		23333.333
1000	0.010	233333.333	233333.333
	0.100		2333333.333
	0.001		233333.333
10000	0.010	2333333.333	2333333.333
	0.100		23333333.333

Table 4 Axial compressive loads obtained from some values of N_r

$N_r=\frac{NL^2}{\pi^2 EI}$	N (KN)
0.25	57619
0.50	115238
0.75	172857

E=28×10^6 kN/m^2, G=13.33×10^6 kN/m^2, υ=0.2 and L=10 m. The dynamic external load applied to proposed numerical analysis model is $q(t)$=100sin(10t) KN.

In numerical example-2, the parameters of elastic foundation, C_S and C_G were calculated as depending on the relative stiffness α and relative shear parameter β. Axial compressive load was taken into account according to the relative axial load N_r which indicated the ratio of the axial compressive load to the Euler critical buckling load.

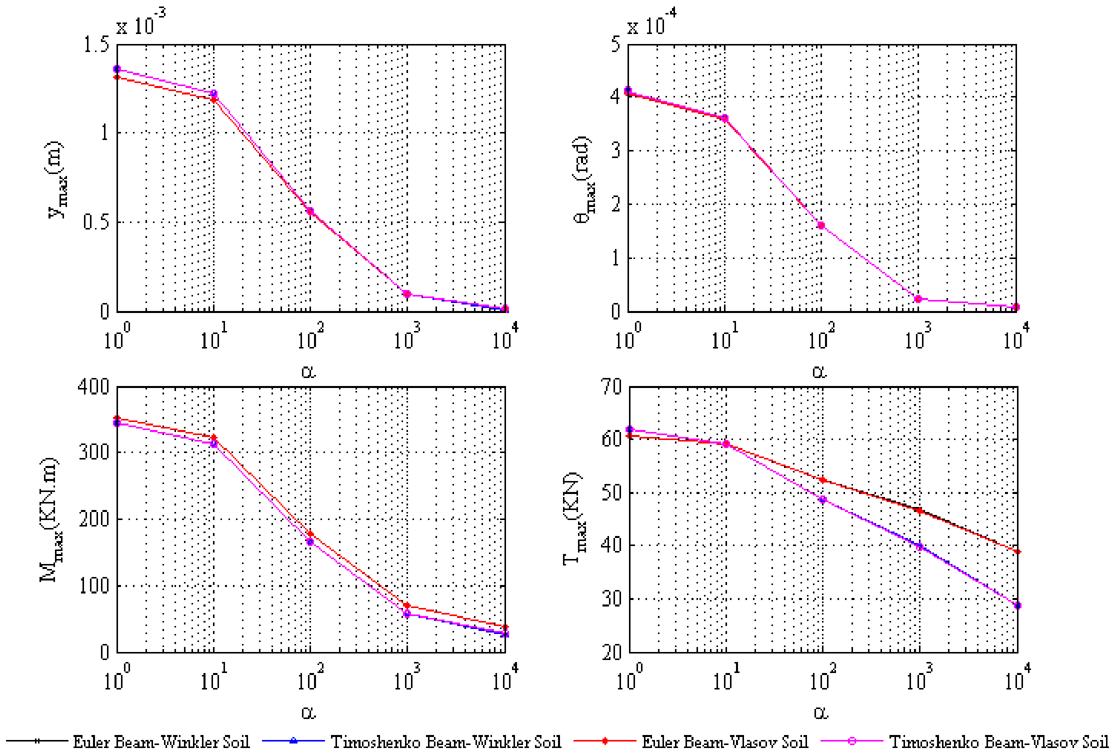

Fig. 6 Variation of maximum values of displacement, angle of rotation, bending moment and shear force of the simply supported beam versus α for N_r=0.25 and β=0.001

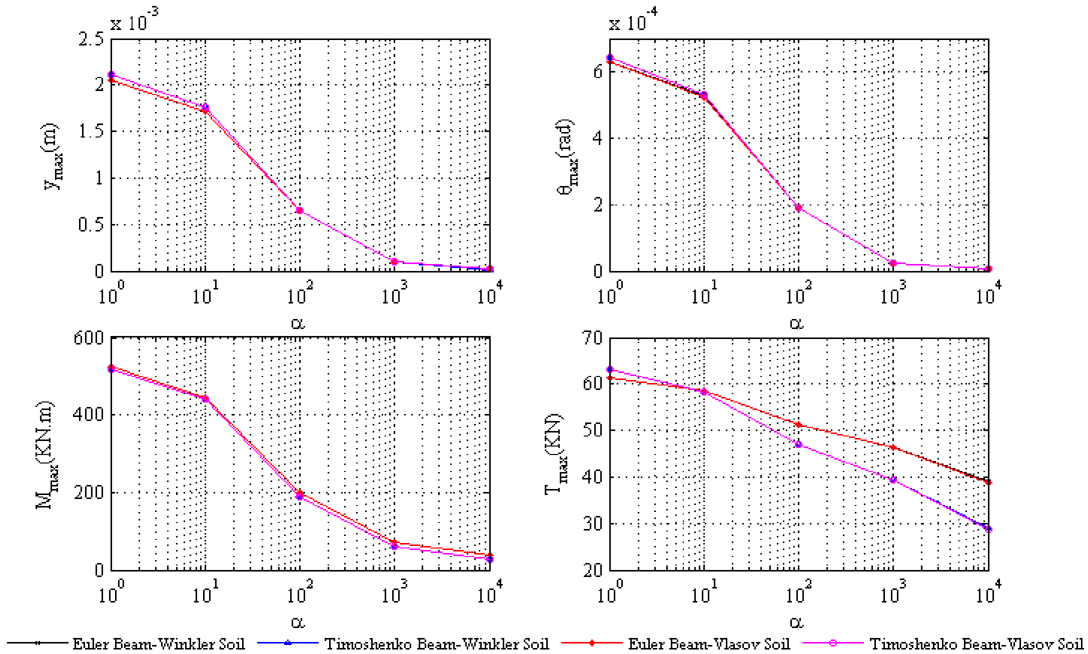

Fig. 7 Variation of maximum values of displacement, angle of rotation, bending moment and shear force of the simply supported beam versus α for N_r=0.50 and β=0.001

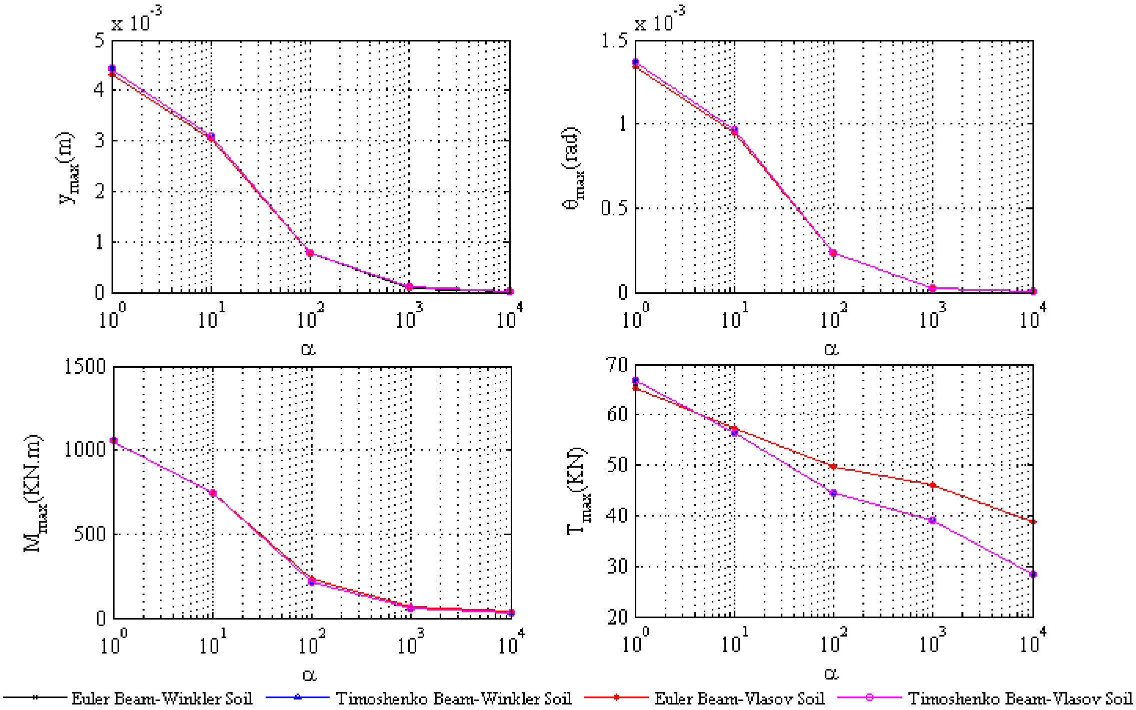

Fig. 8 Variation of maximum values of displacement, angle of rotation, bending moment and shear force of the simply supported beam versus α for N_r=0.75 and β=0.001

Analysis results of numerical example-2 were presented graphically in Figs.6-14 which indicate the variation of maximum values of displacement, angle rotation, bending moment and shear forces of the beam according to the relative stiffness, α. Figs. 6, 7 and 8 indicate the maximum dynamic responses of the beam obtained for N_r=0.25, 0.5, 0.75 and β=0.001, respectively. Figs. 9, 10 and 11 show the maximum dynamic responses of the beam obtained for N_r=0.25, 0.5, 0.75 and β=0.010, respectively. Finally in Figs.12, 13 and 14, the maximum dynamic responses of the beam obtained for N_r=0.25, 0.5, 0.75 and β=0.100 were presented, respectively.

Numerical analysis results of example-2 indicate the influence of shear parameter of the elastic soil, shear deformation of the beam and axial compressive load applied to beam on the amplitude of dynamic response of the beam on Vlasov type foundation, respectively. In Figs. 6-8, 9-11 and 12-14, it was observed that a difference between the amplitude of dynamic responses of beams on Vlasov and Winkler type foundations and this difference was observed to be increasing proportionally with the relative shear parameter, β.

As the effect of the second parameter of Vlasov type soil, the amplitudes of displacement, angle of rotation, bending moment and shear force functions of the Euler beam on Winkler type foundation were observed to be greater than that of the beam on Vlasov type foundation. The differences occurred in displacement, angle of rotation and bending moment functions were found to be increasing numerically for α values between the 1-10 due to the fact that the shear parameter of β was increasing with α.However it was observed to be decreasing for higher values of α. The difference between the amplitudes of shear forces was observed to be increasing proportionally with α. On the other hand, the difference between the amplitudes of displacement, angle of

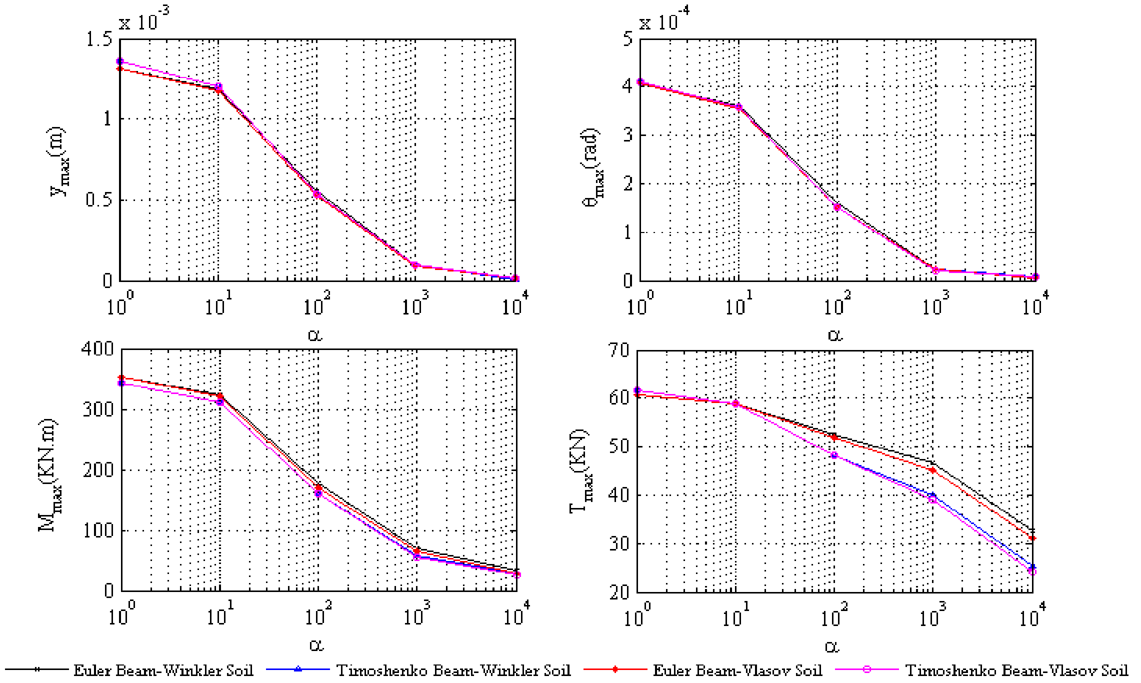

Fig. 9 Variation of maximum values of displacement, angle of rotation, bending moment and shear force of the simply supported beam versus α for N_r=0.25 and β=0.010

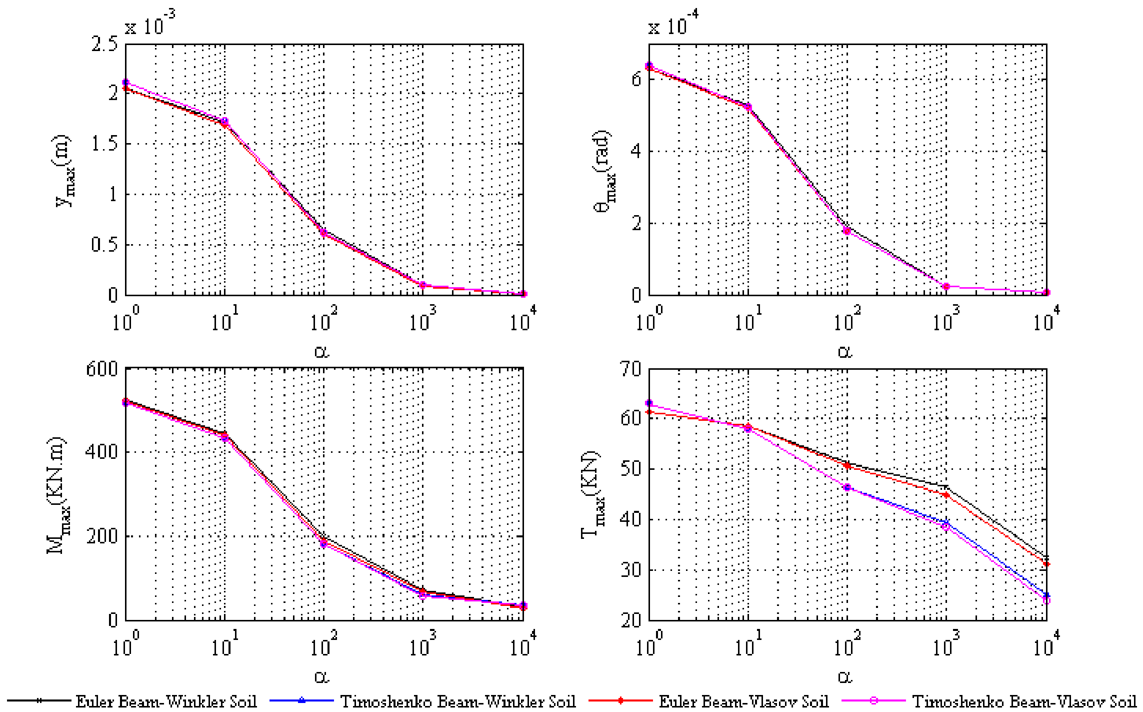

Fig. 10 Variation of maximum values of displacement, angle of rotation, bending moment and shear force of the simply supported beam versus α for N_r=0.50 and β=0.010

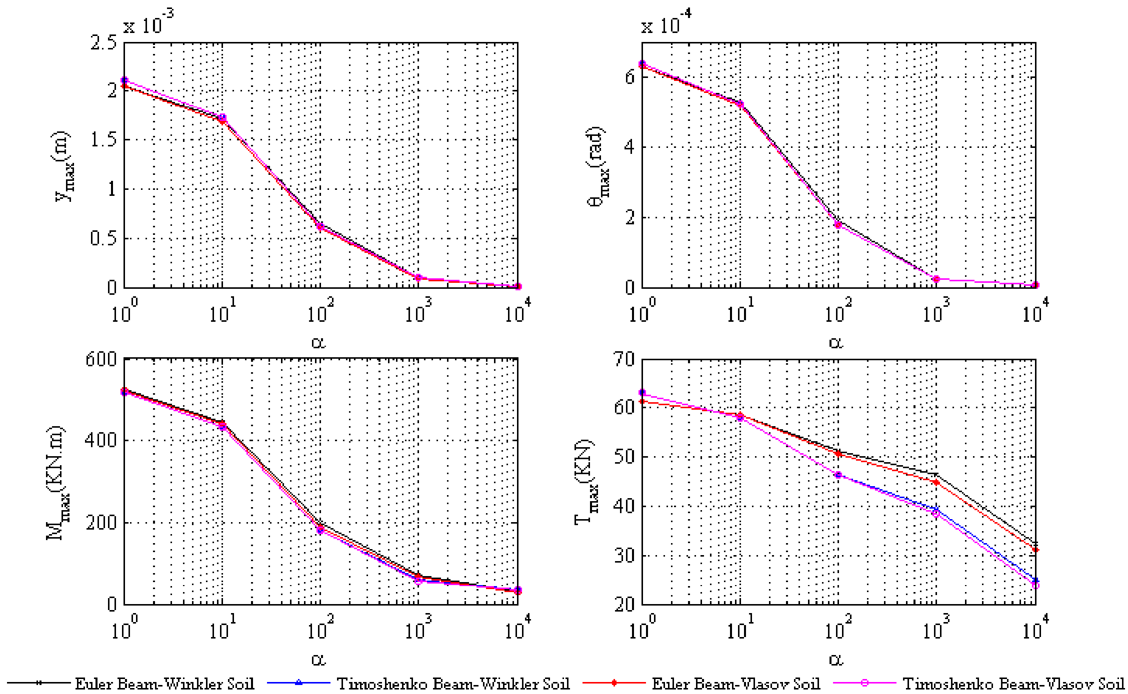

Fig. 11 Variation of maximum values of displacement, angle of rotation, bending moment and shear force of the simply supported beam versus α for N_r=0.75 and β=0.010

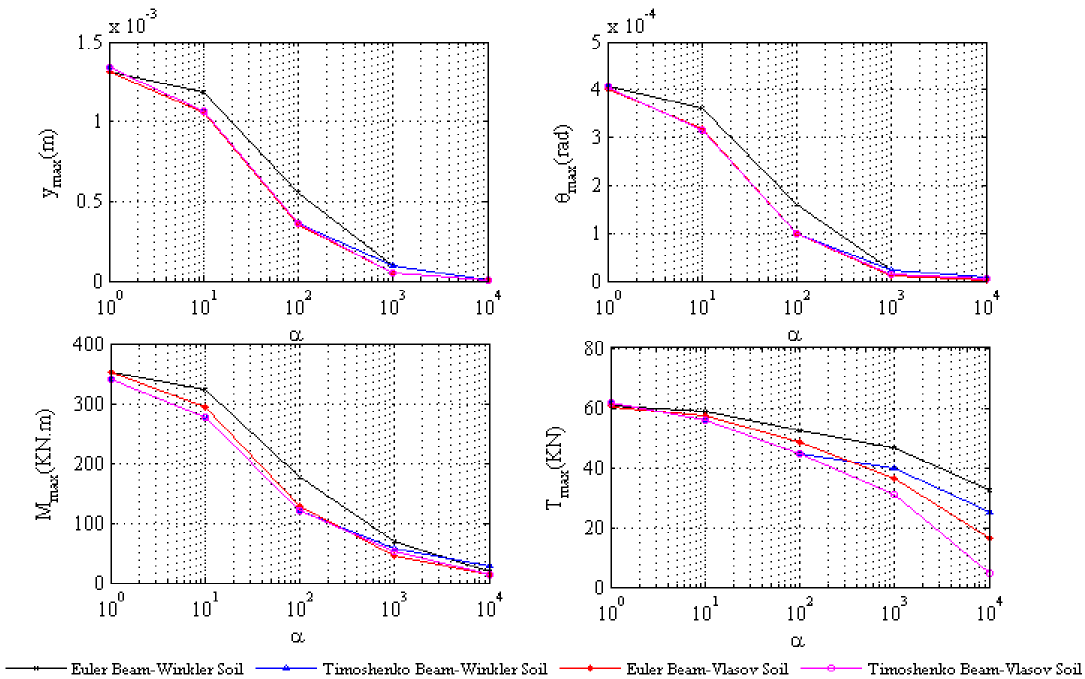

Fig. 12 Variation of maximum values of displacement, angle of rotation, bending moment and shear force of the simply supported beam versus α for N_r=0.25 and β=0.100

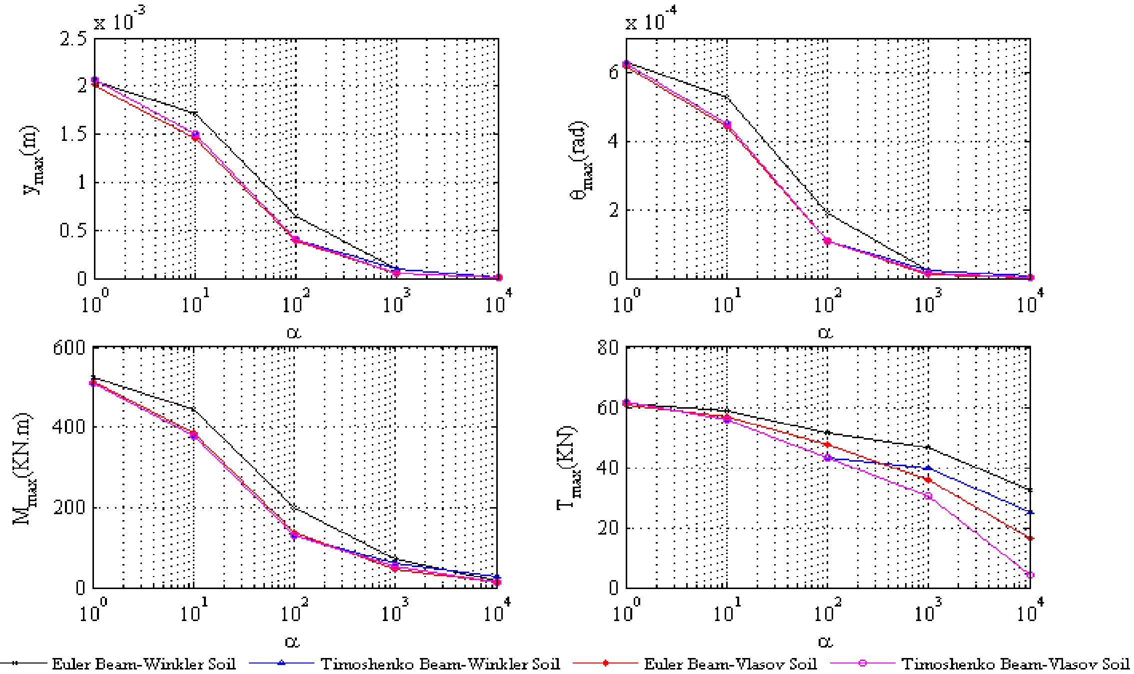

Fig. 13 Variation of the maximum value of displacement, angle of rotation, bending moment and shear force functions of the simply supported beam due to the α for N_r=0.50 and β=0.100

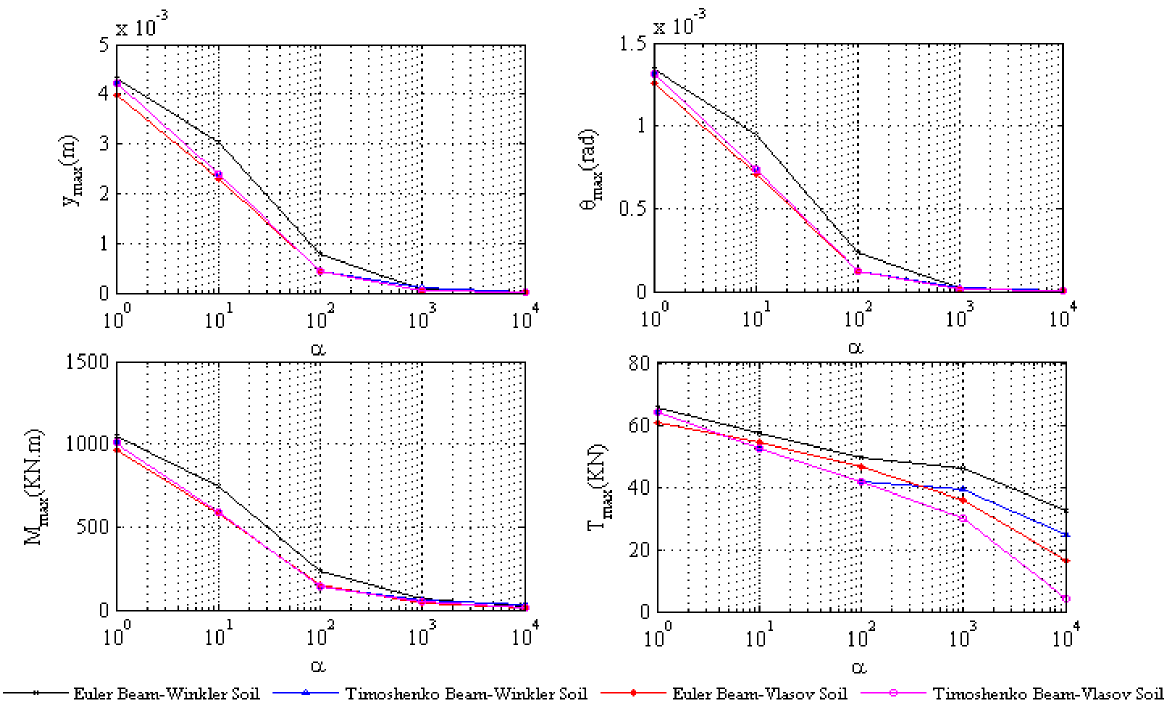

Fig. 14 Variation of maximum values of displacement, angle of rotation, bending moment and shear force of the simply supported beam versus α for N_r=0.75 and β=0.100

rotation, bending moment and shear forces of the beam on Vlasov and Winkler soil was observed to be increasing relatively with proportional to α because of the increasing in β.

Amplitudes of displacement, angle of rotation, bending moment and shear force of the Timoshenko beam on Winkler type foundation were observed to be very close to that of the beam on Vlasov type foundation for α values between the 1-100 due to the fact that the effect of shear deformation of the beam was neutralized the effect of second parameter of the Vlasov type foundation for lower values of α. For the higher values of α, the amplitudes of displacement, angle of rotation, bending moment and shear of the Timoshenko beam on Vlasov type foundation were observed to be greater than that of the beam on Winkler type foundation.

In Figs. 6-14, it was observed that the amplitudes of displacement and angle of rotation of the Timoshenko beam were found to be greater than the Euler beam. However the amplitude of bending moment of the Timoshenko beam was found to be smaller than the Euler beam due to the effect of shear deformation of the beam. It was observed that the difference between Timoshenko and Euler beams did not change so much numerically but it was increasing relatively with proportional to relative stiffness, α. For the minimum value of α, the maximum shear force of Timoshenko beam was observed to be greater than the Euler beam. For higher values of α, the maximum shear force of Timoshenko beam was observed to be smaller than the Euler beam and their difference was found to be increasing both numerically and relatively with relative stiffness, α. This result indicates that the shear deformation of the beam has an enhancing effect on the amplitude of displacement and angle of rotation function for the any value of α ; and shear force function has an enhancing effect for the minimum value of α. On the other hand it was observed the shear deformation had a reducing effect on the amplitude of bending moment function for the any value of α; and it was observed that shear deformation of the beam had a reducing effect on the amplitude of shear force function for higher values of α.

As it seen in the analysis results of numerical example-2, axial compressive load is not effective on the difference between the Winkler and Vlasov type foundations, and between the Timoshenko and Euler beams for the higher values of α. However a remarkable difference was observed between the dynamic response of the beams on Vlasov and Winkler type foundations for the minimum value of α. Nevertheless, axial compressive load is extremely effective on the amplitude of the dynamic displacement, angle of rotation, bending moment and shear force functions.

7. Conclusions

In this study, effects of the second parameter of elastic soil and shear deformation of the beam on the dynamic response were investigated for beams on Vlasov type elastic foundation. Numerical analysis was carried out in two parts. In the first part, dynamic response of a simply supported beam on Winkler type foundation was analyzed and results were compared those of the earlier studies. In second part, an axially loaded simply supported beam resting on modified Vlasov type foundation was investigated, and analysis results were presented in figures which show the variation of the peak values of displacement and bending moment functions at the mid-span, and angle rotation and shear force functions at supports versus the relative stiffness, α. The variation of the maximum value of displacement, angle of rotation and bending moment functions according to the relative stiffness were obtained for the case of relative axial compressive load was 0.25, 0.5 and 0.75 and relative shear parameter was 0.001, 0.01 and 0.1, respectively. Analysis results and conclusions of the presented study can be listed as follows.

- Higher modes of vibration are extremely effective on internal forces especially on the shear force. For this reason, the total number of considered mode should be determined due to the convergence of shear force function obtained in corresponding mode with that of previous mode.
- The influence of second parameter of the Vlasov type foundation is extremely seen in the Euler beams, and Timoshenko beams resting on the soils having higher values of C_S and C_G. For the lower values of C_S and C_G, maximum dynamic response of the Timoshenko beam on Winkler and Vlasov type foundations are found to be very close to each other.
- Shear deformation of the beam enhances the amplitude of displacement and angle of rotation function for the any value of α and shear force function for minimum values of α. Nevertheless, shear deformation reduces the amplitude of bending moment function for the any value of α and shear force function for higher values of α.
- Axial compressive load enhances the amplitude of displacement, bending moment and shear force functions but it does not have an enhancing or reducing effect on the amplitude of dynamic response of the Euler and Timoshenko beams on Vlasov and Winkler type foundations except for the lower values of α.

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