A NEW DESIGN APPROACH FOR PLANAR RETRACTABLE PLATE STRUCTURES BASED ON UNIFORM TESSELLATIONS

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Aylin GAZİ GEZGİN

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İZMİR

We approve the thesis of **Aylin GAZİ GEZGİN**

Assoc. Prof. Dr. Koray KORKMAZ
Department of Architecture, Izmir Institute of Technology

Assoc. Prof. Dr. Tuğçe KAZANASMAZ
Department of Architecture, Izmir Institute of Technology

Assist. Prof. Dr. Fatih Cemal CAN
Department of Mechatronics Engineering, Izmir Katip Celebi University

Assoc. Prof. Dr. Mustafa Emre İLAL
Department of Architecture, Izmir Institute of Technology

Assist. Prof. Dr.Özgür KİLİT
Department of Endustrial Design, Yaşar University

26.12. 2016

Assoc. Prof. Dr. Koray KORKMAZ
Supervisor, Department of Architecture
Izmir Institute of Technology

Assoc. Prof. Dr. Şeniz ÇIKIŞ
Head of the Department of Architecture

Prof. Dr. Bilge KARAÇALI
Dean of the Graduate School of
Engineering and Sciences

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ABSTRACT

A NEW DESIGN APPROACH FOR PLANAR RETRACTABLE PLATE STRUCTURES BASED ON UNIFORM TESSELLATIONS

Designs of the retractable plate structures have started to gain importance after the increase in the application of kinetic roofs, facades and surfaces in architecture since last decade of twentieth century. Thus many researchers try to find the most suitable form of the rigid plates by the help of kinematic and numerical analysis in order to fulfil the task of covered enclosure without any interference, gaps or overlaps between the plates. Considering previous works, this study aims to create a method for designers that transform; the predefined rigid plates into retractable plate structures (RPS) without using any complex kinematic or numerical analysis. Throughout the study, shapes of the rigid plates are selected as regular polygons. Tessellation technique is utilized which shows a proper way of covering a plane by using regular polygons.

In the light of this aim, the detailed investigation of how regular polygons are combined in a plane is being carried out. Also two general conditions for the assembly of rigid regular polygonal plates are discovered so that tessellation can form RPS without any interference, gaps or overlaps between each other in closed and open configurations. Then two distinct methods are proposed to design the extra link for the RPS that do not satisfy these two conditions to make them totally operational with respect to the design constraints. Additionally, another method is proposed for the shape modification of the plates where the tessellation satisfies the conditions. Furthermore, for the multi degrees of freedom retractable structures, another method is proposed to convert them into single degree of freedom RPS by utilizing graph theory and duality. In the last part of the thesis, degrees of freedom calculations of the proposed retractable structures are considered and a theorem is proposed to prove that their degree of freedom is one. Throughout the thesis simulation and modelling technique is utilized for analysis of retraction and expansion.

ÖZET

DÜZENLI TESELLASYONLARA DAYALI DÜZLEMSEL TOPARLANABİLEN PLAKA STRÜKTÜRLER İÇİN YENİ BİR TASARIM YAKLAŞIMI

Rijit plakaların bir araya getirilmesi ile meydana gelen genişleyebilen strüktür tasarımları, hareketli çatı, cephe ve yüzey tasarımlarının kinetik mimarideki uygulama alanlarının artması ile önem kazanmaya başlamıştır. Bu strüktürlerin kinetik mimaride kullanımlarında plakaların tam olarak kapanmaları ve üst üste binmemeleri amaçlanmış, bu nedenle birçok araştırmacı tarafından kinematik ve nümerik analizler yapılarak rijit plakalar için en uygun şekil bulunmaya çalışılmıştır. Bu çalışmanın amacı tasarımcıların herhangi bir nümerik yada kinematik analiz yapmadan daha önceden şekilleri belirli rijit plakaları genişleyebilen strüktürler haline getirebilmek için metot üretmektir. Rijit plakaların şekilleri düzgün geometrik çokgenler olarak belirlenmiş ve bir düzlemde düzgün çokgenlerin nasıl bir araya gelebileceklerini gösteren tesselasyon tekniğinden yararlanılmıştır.

Bu amaçla öncelikle bir düzlemde düzgün geometrik poligonların nasıl bir araya gelebilecekleri konusu detayları ile incelenmiş, daha sonra bu düzgün geometrik poligonel şekillere sahip rijit paltformların hangi düzende birleştiklerinde tam olarak kapanmaları ve üst üste binmeyecek şekilde genişleyebilen strüktür oluşturabilmeleri için gerekli olan genel koşullar bulunmuştur. Daha sonra bu koşulları sağlamayan genişleyebilen strükürlerin tam olarak kapanmaları ve üst üste binmeyen genişleyebilen stürtür oluşturabilmeleri için gerekli olan fazladan link tasarımı için iki adet metot önerisi ve koşulları sağlabilecek şekilde biçim değişimi için metot önerisi sunulmuştur. Bunlara ek olarak elde edilen bu genişleyebilen strüktürlerin serbestlik derecelerinin birden fazla olanların çizge kuramı ve dualite kullanılarak serbestlik derecelerinin bire dönüştürülebilmesine yönelik metot önerilmiştir. Son aşamada elde edilen genişleyebilen strüktürlerin serbestlik derecelerinin hesaplanmaları ele alınmış, serbestlik derecelerinin bir olduğunu kanıtlamaya yönelik teorem önerilmiştir. Yöntem olarak “simulasyon ve modelleme” yöntemi kullanılmıştır.

... to my mother Nuray Gazi and father İrfan GAZİ

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CHAPTER 1

INTRODUCTION

Feasibility and structural functionality in conjunction with aesthetic consideration have always been important in all periods of architecture history. However, as a result of the influencing parameters such as rapidly change weather conditions, and functional requirements even for aesthetic expression, architecture becomes in need of kinetic structures to deal with these conditions in 20th century more than previous times. In the early attempts, lack of information awareness has resulted in less usage of kinetic structures in architectural projects, but by the help of many interdisciplinary developments in mechanism science, architectural computing systems, construction and material technology, many kinetic structures have been developed. These developments have been the major cause of the increase in the number of interdisciplinary studies in many different areas to develop kinetic structures. These researches are generally related with the development of novel designs or understanding them by utilizing kinematical background or structural synthesis.

Kinetic structures can be categorized in many different ways such as considering their morphology and kinematic behaviour. Kinetic structures are divided into four categories with respect to their morphology, as bar structures, plate structures, tensegrity structures and membrane structures (Hanaor and Levy 2001). Also in terms of their kinematical behaviour kinetic structures are categorized as deployable, transformable, expandable, retractable, adaptable structures and etc.

1.1. Problem Statement

This dissertation focuses on retractable plate structure (RPS). RPS is a family of structures that are assembled by utilizing set of plates connected by revolute joints. Also, these structures are a kind of mechanisms. Usages of these structures have usually been seen in roof structures, kinetic facades, kinetic photovoltaic panels and also shelter systems. They have a great advantage of speed in deployment, ease of erection and dismantling as long as the weather conditions are suitable during different seasons.

RPS is evolves from pantographic structures. Once the applications of pantographic structures are started to increase in kinetic architecture, the necessity to cover them by variety of materials has gained importance in time. Initially the pantographic bar structures are covered by fabric materials such as membranes, however; many difficulties exist. Such as, stabilization of the fabric material in all possible configurations of folded or deployed configurations is difficult. Also flapping wind effect and other weather conditions should have also been considered. As a result of these issues instead of fabric materials, covering pantographic bar structures by using rigid plates has gained importance. Later in time, rigid plates have been used not only as covering materials but also as the structure by themselves and it is called RPS.

Covering pantographic bar structures by using rigid plates and generating retractable plate structures have some important requirements. One of the most important issues is to determine the most suitable shape of the plates that form an enclosure without any gaps or overlaps in both closed and open configurations of the structure. Also there should not be any interference between the rigid plates during retraction or expansion phase. In order to find the most suitable shape of the plates, researchers have developed many complicated numerical or kinematical analysis. Thus, so far solutions about revealing the suitable plate forms takes too long time and complex engineering knowledge. As a result, this problem has resulted in less usage of RPS in architectural projects. More importantly, as many of them offer just specific unique solutions, there exists no universal solution to present generalization. In addition to this, prior results of the plate shapes have generally been triangular plates.

1.2. Objectives of the Research

Due to the aforementioned problem, configuring the suitable shape of rigid plates is one of the most important issues. Also aforementioned deficiencies of existing works, this study claims that there exists a need to develop a new design approach in order to help practicing architect and designer to choose the plate form, and necessary requirements in the early stages of their design without needing any complex numerical or kinematical analysis. On the other hand, in this study all structures will be one degree of freedom, in other words only one actuator is required move them. So, the main aim of this study is to develop a new approach for the design of single DoF planer RPSs that are able to be enclosed without any gaps or overlaps in their both closed and open configurations by eliminating the chance of interference between each other during the retraction and expansion phase. Besides, this dissertation focuses to develop some conditions to generalize the fact of which polygonal forms can be retractable plates. Additional aim of the dissertation can be given as developing some theorems and methodologies that will help the designer in the selection and assembly order phases of the plates without any further efforts. It should also be noted that this dissertation proposes an interdisciplinary approach between architecture, mechanism science and mathematics.

This study focus on the planar retractable plate structures. Thus, this dissertation has benefited from mathematical uniform tessellation technique that gives the possibilities of covering plate combinations without any gaps or overlaps in terms of regular polygons. In the light of this technique, the shapes of the plates are determined as regular polygons and plates are connected by only revolute joints. This dissertation tries to find an answer to the questions of what kind of regular polygonal plates can be retractable when assembled with each other, which of them can be closed and opened without any gaps, overlaps or interference between each other, what is their degrees of freedoms and does degrees of freedom can be converted.

1.3. Methodology

In the first phase, the thesis is divided into two parts to present a critical review of previous works of RPSs in order to understand their kinematical and structural background and discusses their design methodologies. In the second part, this study focuses on the mathematical background on the tessellation technique. This part presents the theoretical background of assembling regular polygons on the plane, illustrates 1- uniform tessellations and demonstrates all k-uniform tessellations. In the main phase, this study tackle on the developing novel design methodologies to find the most suitable shape for single DoF RPS that is able to be opened and closed without any gaps or overlaps. In this phase this dissertation will not use any kinematical or numerical analysis. The retraction and expansion capabilities of proposed retractable plate structures are simulated by using computer simulation environments. In the final part of the thesis, one of the structural syntheses mobility formulations is used to prove the degrees of freedom of proposed RPSs are always one.

1.4. Organization of the Thesis

This dissertation is organized by forming six chapters. In Chapter 1, the problem definition, aim of the study, methodology and organization of the dissertation is presented. In Chapter 2, previous works, and literature reviews of pantographic structures as well as retractable plate structures are presented main problem of designing RPSs and different ways of revealing suitable shapes of the plates are discussed. In Chapter 3, the basic principles of tessellations with regular polygons are clarified. Then, importance of the symmetry for tessellations is mentioned. Following these, all the possibilities of k- uniform tessellations and duality of tessellations are explained.

Chapter 4 constitutes the main part of this dissertation. Firstly, two conditions are developed to reach single DoF RPSs based on 1- uniform tessellations that are able to be enclosed without any gaps or overlaps in their both closed and open configurations by eliminating the chance of interference between each other during the deployment motion. Then, the thesis focuses on the generation of RPS by considering the number of edges that meet at one vertex and identify the problem for each of them. For tessellations, which do not satisfy the first condition, two methodologies are developed for revealing the needed extra link. Also, for tessellation which does not satisfy the second condition, another methodology that is called “vertex translation” is introduced. In addition to this, a theorem is developed for the relationship between duality of tessellation and graph representation. Next, by considering this theorem a method for systematic conversion of M-DoF RPS to a single DoF RPS is presented. In the final part of the chapter a brief discussion is carried out about the relation between iteration of the RPS and the planar symmetry groups.

In Chapter 5, mobility analysis of RPS based on 1- uniform tessellation is concerned. A theorem is developed to prove the degrees of freedom of proposed RPS are always equal to one. A systematic approach of the assembly technique between different scales of RPS is proposed. In order to present their mobility calculations, numerous of them are assembled with respect to the proposed approach. In Chapter 6, the study is concluded by summarizing the main achievements and the suggestions for the future works.

CHAPTER 2

REVIEW OF PREVIOUS WORK

This chapter presents a review of the development of retractable plate structures (RPS) in order to emphasise the importance of the proposed novel design method when compared with previous methods.

This review is divided into three main parts. The first part is devoted to the explanation of pantographic bar structures. In this part, main geometric characteristics, transformational capabilities, advantages and disadvantages of translational, polar and angulated pantographic structures are discussed by analysing some-well known works. Second part concentrates on the task of covering pantographic structures with rigid materials. Finally the third part of the review deals with the difficulties of the design processes of retractable plate structures and related proposed methods by analysing some examples.

2.1. Pantographic Bar Structures

Generally, pantographic structures can be defined as structures that can be deployed from a small compact state to another while carrying loads, (Glisic, et al. 2013). The main advantages of the pantographic structures is the fact that they can easily be transported and built comparatively, while they can provide large span and efficient load bearing structure. Thus, pantographic structures have usually been proposed to be used as roofs, façades, decorative structures and even photovoltaic panels in kinetic architecture (Sharif, et al. 2012)

It is important to emphasise that the type of pantographic bars determine the final shape of the structure and its deployment behaviour. Pantographic structures with rigid bars are categorized in many different ways. As a common approach they are divided into three groups as translational, polar and angulated bars in terms of the scissor hinge location and shape of the bar (Figure 2.1). In addition to this categorization, Asefi (2010) categorises the pantographic structures by considering their morphologies as shown in Figure 2.2. Throughout this chapter, considering these two categorizations geometrical and kinematical properties of pantographic structures will be investigated based on the first categorization.

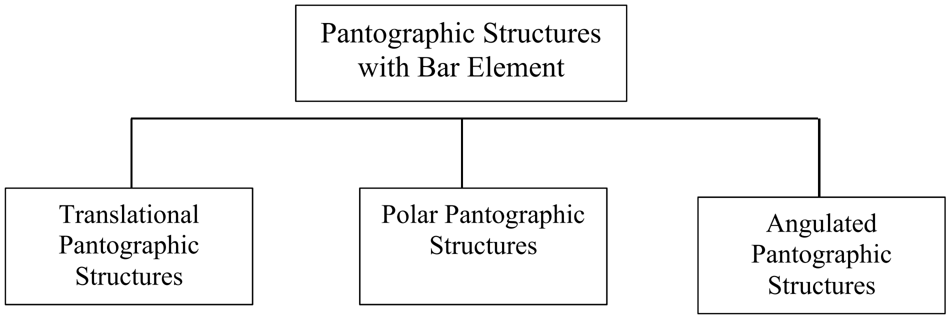


Figure 2.1. Pantographic structures in terms of location of the scissor hinge

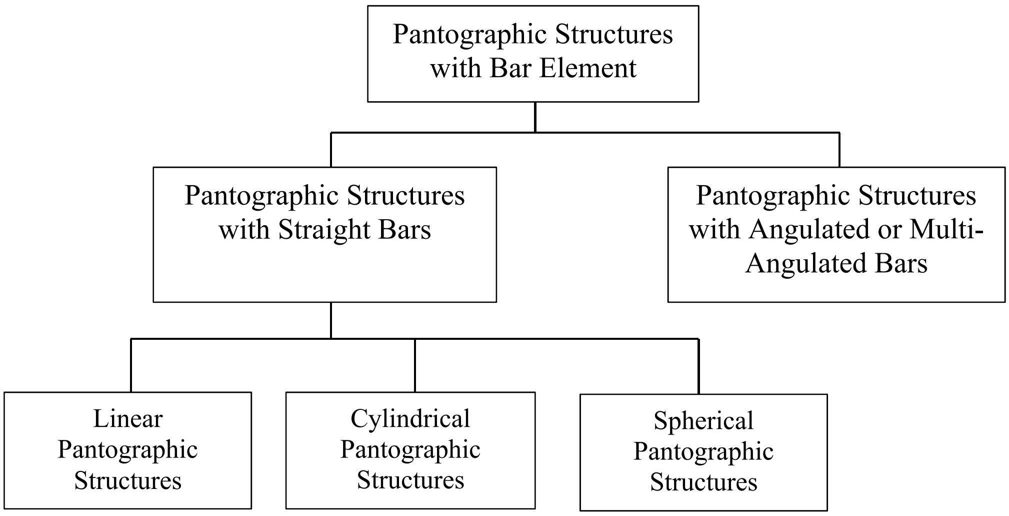


Figure 2.2. Morphology of pantographic structures

2.1.1. Translational Pantographic Structures

Translational pantographic structures consist of translational scissor units that are called lazy-tong mechanisms. These units are connected together by the revolute joints (R) from their ends to form two-dimensional translational structures. In these assemblies the upper and the lower end nodes of the individual scissor units are positioned on unit lines that are parallel to each other (Jensen and Pellegrino 2005;

Doroftei 2014). Translational structures are mostly used in plane or curved scissor grids based on the translational surfaces. Typically there exist two main types of translational pantographic units as plane translation (Figure 2.3) and curved translation (Figure 2.5) pantographic units. According to their morphologies structures formed by utilizing these units are called linear pantographic structures.

The main structural element of the linear pantographic structure is the translational unit in which the links are connected with central or end pivot. Figure 2.3 shows the simplest translational pantographic structure. During its deployment the end joints are always stay on the parallel unit lines. Varying the deployment angle θ the pantographic structure can be deployed from its most compact configuration to its fully deployed position or the reverse of it (Temmerman 2007).

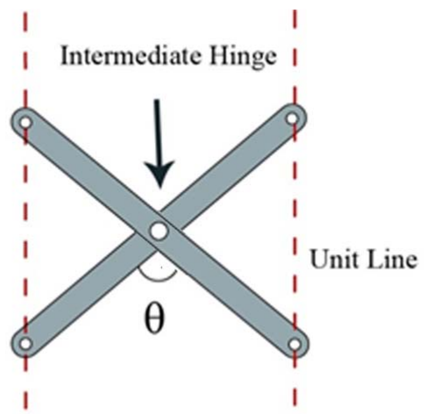


Figure 2.3. Translational plane unit

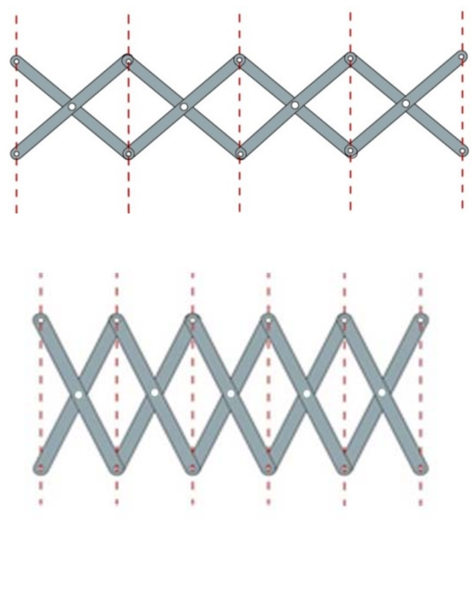


Figure 2.4. Deployment process of the translational pantographic structure

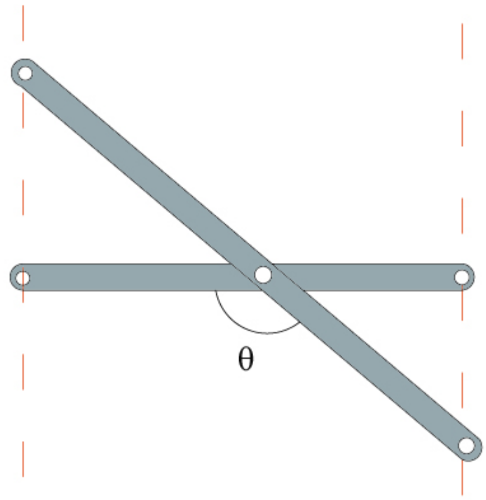


Figure 2.5. Curved translational plane unit

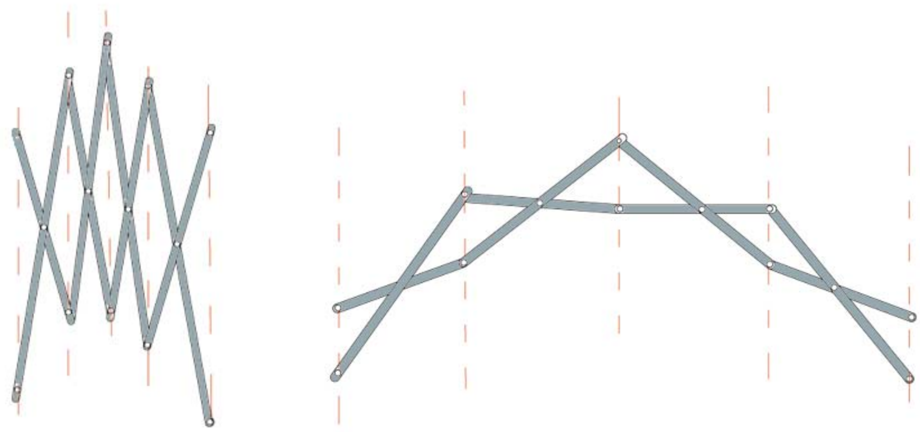


Figure 2.6. Curved translational linkage in its unemployed and deployed position

The early usages of pantographic elements in kinetic architecture are generally based on the translational pantographic structures. Some of the earliest well-known example was the mobile exhibition hall that was designed by Emillo Perez Pineore in Madrid, 1964. The mobile pavilion exhibition hall was the most important breakthrough and the most successful deployable structure of its own time. Its structure consists of rigid bars, cables and joints. Also the whole structure consists of 74 parallel translational units that were manufactured and assembled within 4 months.

Figure 2.7. Mobile exhibition hall designed by Emillo Perez Pineore
(Source: Belda and Carmen 2016)

2.1.2. Polar Pantographic Structures

In kinetic architecture, polar pantographic structures are generally used for generating single-curvature grids or the more basically the scissor arch. Figure 2.8 shows the simple polar pantographic unit. As it can be seen in the figure, unit lines intersect at an angle (γ) which varies continuously during the unit deployment. More importantly as a structural fact, the intermediate hinges are located away from the midpoints of the beams. During transformation, the unit lines rotate around the common intersection point (Mira 2012) (Figure 2.9). From the morphological perspective, polar pantographic elements are called cylindrical or spherical pantographic structures.

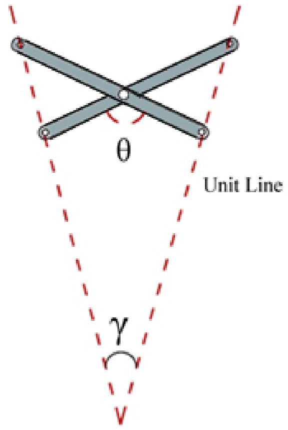


Figure 2.8. Polar pantographic unit

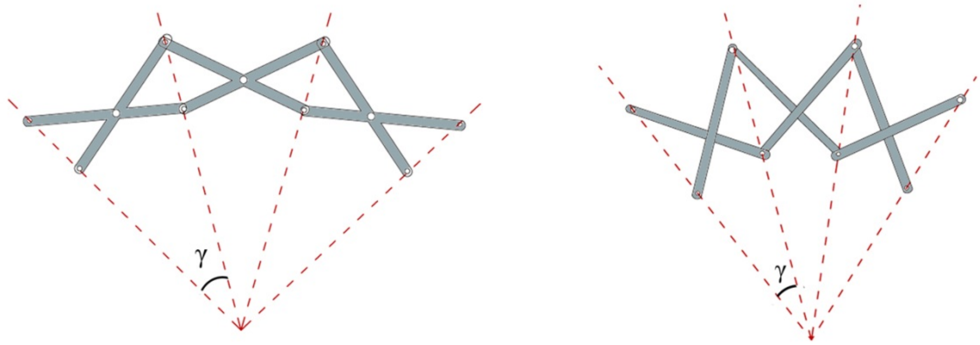


Figure 2.9. Deployment processes of polar pantographic element

One of the most important examples for the polar pantographic structure usage in kinetic architecture is the Sport City Council of Seville in Spain (Figure 2.10) that was designed by Felix Escring and Jose Sanchez. In this structure the major aim is to be able to cover the Olympic swimming pool during cold seasons for heating purposes. Thus, in order to adopt the geometry of the building two square spherical segments (6x6) are supported at the perimeter. Each segment has several polar pantographic elements. In the structure, all the bars have equal lengths and the central joints are placed eccentrically. Thus the final form of the structure becomes spherical to fulfil is main challenge of the task. (Escring et al. 1996; Asefi 2010).

Figure 2.10. Deployable cover for a swimming pool in Seville designed by Escrig & Sanchez (Source: © Performance SL)

2.1.3 Angulated and Multi-Angulated Pantographic Structure

The major advantage of utilizing the angulated pantographic element is being able to achieve various deployment geometries compared with the polar or translational pantographic elements (De Temmerman 2007). Discovery of the simple angulated elements have caused a considerable advance in the design of retractable roof structures by radially allowing the structure to “grow” and “shrink” while maintaining its overall shape.

Simple angulated elements were first introduced by Hoberman (1990). Thus, they are commonly denoted as the Hoberman units (Figure 2.11). Structurally, angulated elements consist two kinked bars with the kink amplitude of β that are assembled with scissor hinges as seen in Figure 2.11. Unlike the polar element, the most important geometric principle of the angulated element is the fact that the angle “γ” will always be constant and $\alpha = \gamma/2$. As a result, the angulated elements are able to be deployed while maintaining the end nodes on radial lines that subtend a constant angle (Hoberman 1990 and Mira 2010).

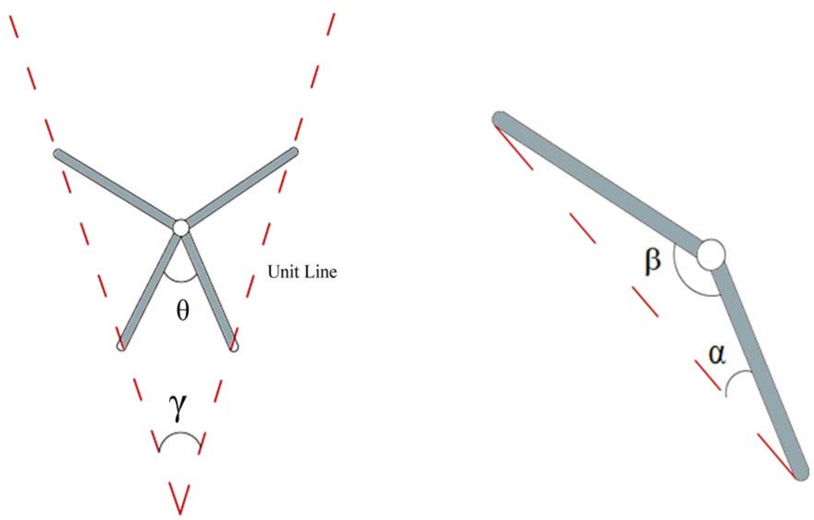


Figure 2.11. Angulated Element

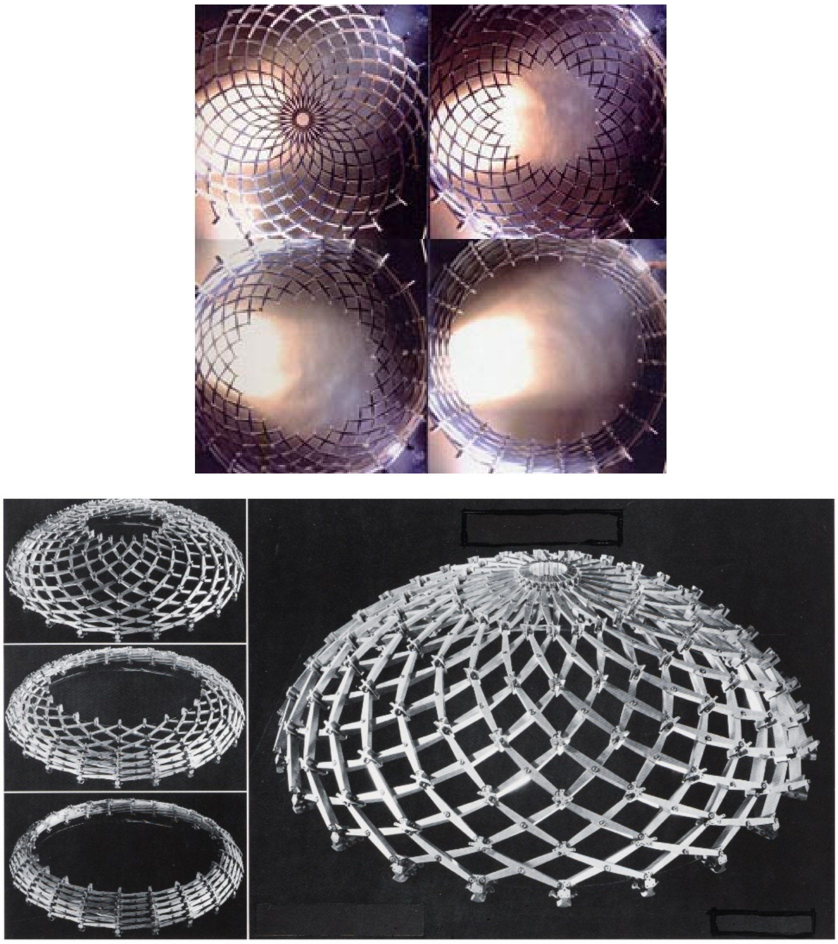


Figure 2.12. Iris Dome, (Kassabian, et al. 1999)
(Source: Hoberman.com)

The most well-known example of angulated unit utilization is the Iris Dome that was designed by Hoberman in 1994. It is the first retractable dome that can be folded along its perimeter. The name of the building was inspired from the iris of the eye due to its expansion and retraction motion. The bottom layer of the Iris dome is connected to the ground that allows small, radial edge expansion and retraction. In addition to this design, another important breakthrough has been achieved by the invention of multi - angulated element by You and Pellegrino (1993; 1997). They found that two or more angulated pantographic structures can be joined together by the scissor hinges from the elements ends. This modified structure have become one of the more general family of structures called foldable bar structure (*FBS*). In this structure each of the multi angulated elements include more than three pivots. (You and Pellegrino (1996),(Pellegrino, et al 1997) noted that consecutive angulated bars in Figure 2.13 maintain a constant angle equal to α when the structure is expanded, thus can be replaced with a single multi-angulated bar. Figure 2.14 shows the retractable structure that consist of 24 bar elements each has four segments with three equal kink angles. This structure also deploys radially in the process of twelve bar element rotating clockwise (black-line) while remaining elements rotate anti-clockwise. In the following years, many researchers dealt with the kinematic analysis of *FBS* with sophisticated analytical tools (Dai and Rees Janes 1999), (Pfister and Agrawal 1999), (Patel and Anonthasuresh, 2005).

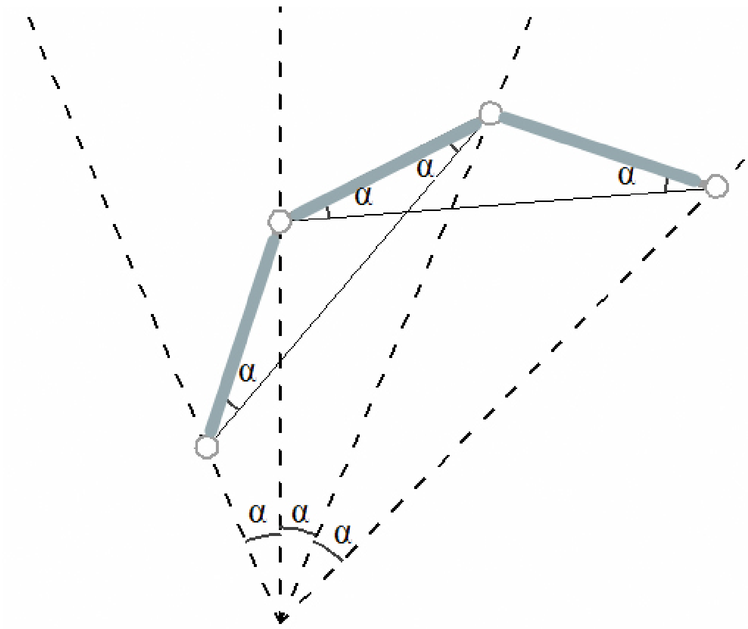


Figure 2.13. Multi angulated element

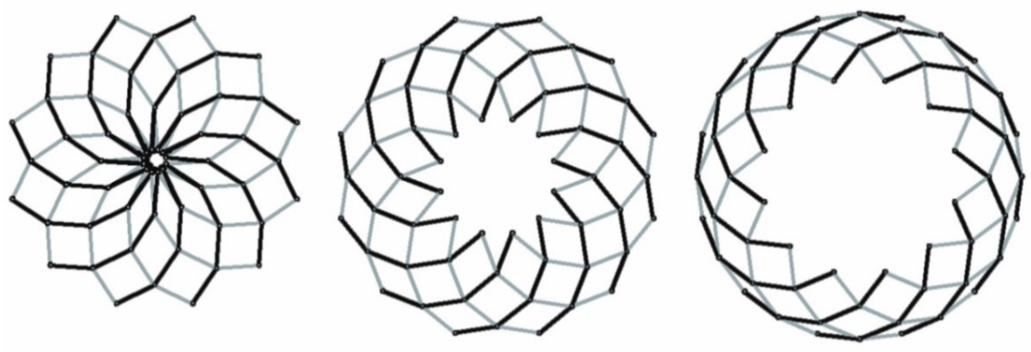


Figure 2.14. Retractable pantograph structure with multi angulated element

2.2. Pantographic Retractable Structures Covered by Rigid Plates

The necessity to cover pantographic structures by using variety of materials has gained importance over time. The task of covering pantographic structures by using fabric materials were already in practice from the ancient times, however it was just at the end of the last century when engineers began to apply the idea for buildings. In practice and research many engineers and designers utilizes textile materials as covering tools. For instance, F. Otto classified the membrane materials as convertible constructions (Otto 1971). On the other hand there exist many difficulties when fabric materials are selected as covering tools. The main difficulty is the stabilization of the fabric material in all possible configurations such as folded, deployed or opened configuration. During these configuration shifts, the structure requires different length patterns, thus the proper fabric pattern should be designed carefully (Mirza 2010 and Friedman 2011). Flapping wind effect and other weather conditions should also be considered as the main issues. In the light of these problems, the idea of covering the pantographic structures by rigid plates instead of the fabric materials is gain importance.

In order to cover pantographic structures by using rigid plates, some important requirements should be met. One of the most important issues is to find the most suitable geometry of the plates so that they can form an enclosure without any gaps or overlaps in the structures closed and open configuration. Also during the deployment motion there should be no interference between the plates. Kassabian , Jensen and Pellegrino are the most important pioneered researchers that tackle these issues.

Kassabian proposed a method to cover the above pantographic bar structure by utilizing rigid panels and also displayed the possibility of employing them as a retractable roof (Kassabian, et al. 1999). The shapes of the covering elements were determined by using kinematic requirements. Limits on the parameters were defined for fully open and closed configurations. Each covering element was attached to a single angulated pantographic structure so that the motion was not inhibited. In the light of this, they found that a general solution for structures with many shapes is to choose covering panels with triangular shapes (Figure 2.15).

Jensen and Pellegrino have developed a method for covering any multi angulated bar structure with plates by finding the extreme positions of pantographic structures. In their study the authors considered that the structure was a combination of interconnected rhombus, and the shape of the rigid plate was determined by analysing the changing rhombus shape during the motion. After this analysis the boundary angle "θ" was determined. ($\theta = \frac{\pi - \beta closed - \beta open}{2}$) (Jensen and Pellegrino 2002).

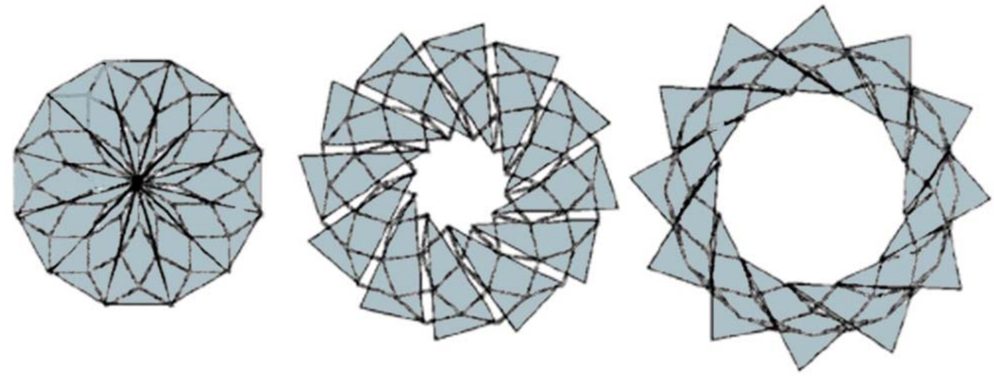


Figure 2.15. Multi-angulated structure with cover elements (Kassabian, et al. 1999)

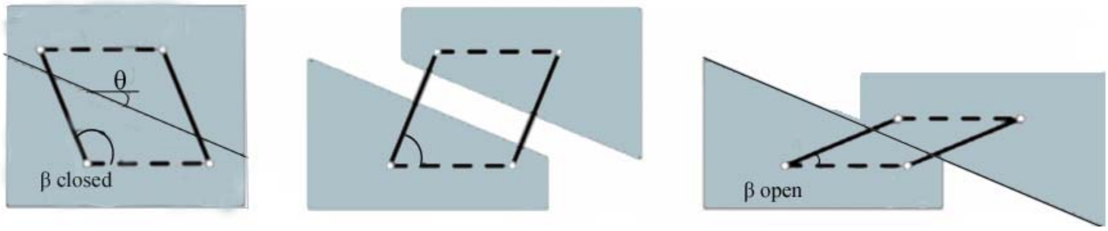


Figure 2.16. Movement of rhombuses with two plates attached.

2.3. Retractable Plate Structures

RPS is a family of structures that is a set of cover plates connected by revolute joints and have been studied deeply in Deployable Structure Laboratory in Cambridge University. A breakthrough in the development of concepts of covering with rigid plates achieved by Jensen and Pellegrino (Jensen and Pellegrino 2002; 2005) by the creation of a family of structures called radially retractable plate structures (RRPS). Jensen and Pellegrino showed that instead of covering angulated structures with rigid elements, angulated elements can be removed and connected directly with scissor hinges.

In the design of the Retractable Plate Structures, there exist mainly two important issues. The first one is related with the rigid materials. The determination of the most suitable shape that should be used to cover without any gaps or overlaps are very important as well as the elimination of the chance of interference between each other during the deployment. The second one is the fact that all of the pivots of the beam must remain within the boundary of its corresponding RPS. To overcome this problem, many designers have developed some empirical, numerical or analytical methods. Considering both issues, this dissertation will focus on the first problem.

In their studies Jensen and Pellegrino have removed the angulated elements and connected the plates directly with revolute joints on exactly the same locations as in the original bar structure. Because of this they claimed that the kinematic behaviour of the new design is unchanged. Also they showed that the relative motion between two adjacent plates are periodic and lay within a certain area, thus they can be fully closed without any overlaps in all configurations. Developing some empirical and analytical approach to find the shapes of hinged plates, they presented a circular and non-circular expandable structure as well as transformable blob structure that can be seen in figure 2.17-2.19 (Jensen and Pellegrino 2005). The same problem has been deal with considering numerical approaches by Buhl, et al. (2004). The important challenge of this study was the suitable shapes for the plates that should be found by utilizing formulations and optimization problems. In their study the overlap and gap area optimization functions were defined and presented.

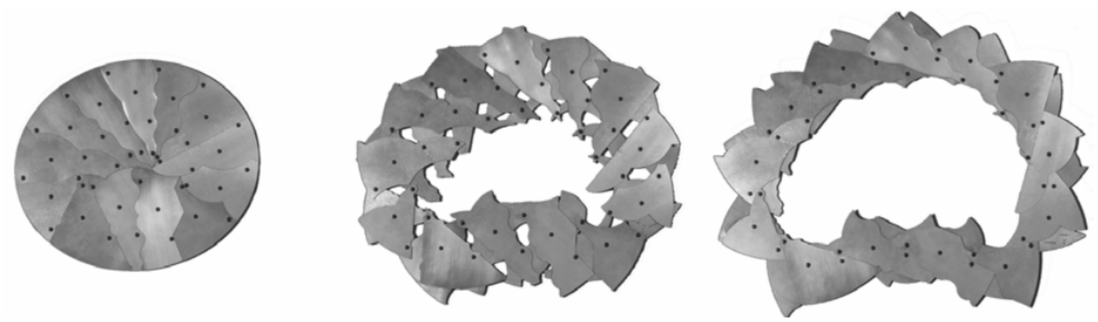


Figure 2.17. Model of a non-circular structure where all boundaries and plates are unique (Jensen 2004)

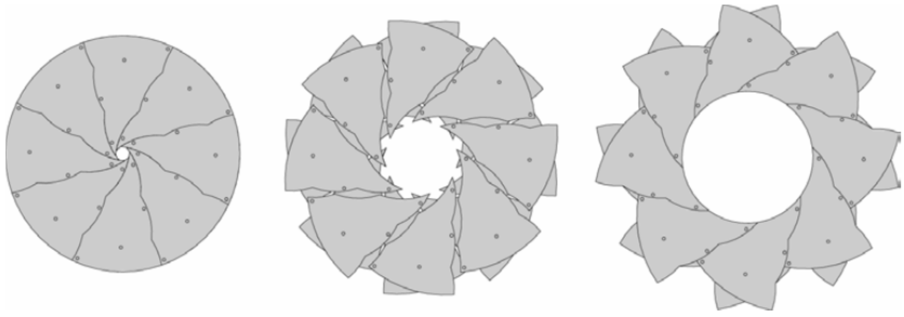


Figure 2.18. Expandable circular plate structures (Jensen and Pellegrino 2005)

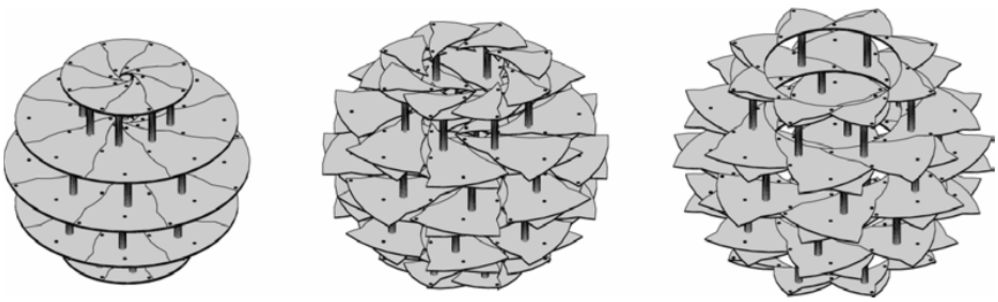


Figure 2.19. Perspective views of expandable spherical structure.
(Jensen and Pellegrino 2005)

Most of the mentioned studies above focus on the design of boundaries of the plates that form enclosures without any gaps or overlaps in both their closed and open configurations. On the other hand another important approach has been developed by Luo et al. (2007). In this research, the authors have focused on the question of whether all of the pivots beam must remain within the boundary of its corresponding plates or not by using an analytical approach and they have derived a set of conditions. One of the most important points of this study is the ability to determine whether all of the pivots of a multi-angulated beam are within its corresponding plate or not without using empirical or numerical ways. By using the results of the analysis, designers can choose and open a profile that suits their needs and then apply appropriate geometrical formulations to determine the edges of the cover plates. This research has a solid contribution on the design processes of retractable plate structures to make them easier.

Figure 2.20. Expansion sequence of an RPS with a polygonal opening (Luo, et al. 2007)

On the contrary to the aforementioned studies, Chilton (Chilton, et al. 2003) developed a novel approach for retractable structures that is called swivel diagram with the combination of some features between retractable plate structures and reciprocal structures (Figure 2.21). As opposed to previous examples, the support points are connected directly to the angulated elements. The motion of a swivel diagram is reciprocal in character. Thus this diagram presents some advantages such as including fewer elements compared with RPS and also simpler joint configurations than deployable reciprocal plates. Diagrams are relatively easy to build, yet they have some deficiencies. The most important one is the fact that due to the overlapping problems such as in squares and pentagonal plates, diagrams generally utilizes triangular forms.

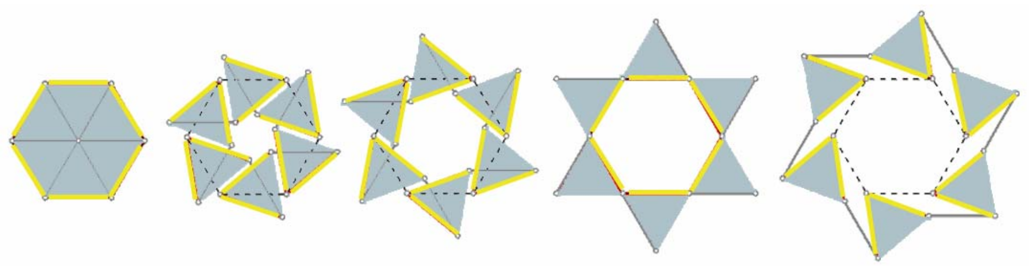


Figure 2.21. Swivel diagram

Retractable plate structures are generally used as roof structures in kinetic architecture. Generally, many researchers have proposed dome shaped structures that can be retracted towards their perimeters. Some well-known examples can be given as “Olympic Arc” designed by Hoberman in 2002, (Figure 2.22), and by Jensen in 2004 the retractable roof made from spherical plates with fixed point of rotation (Figure

2.23). Additionally Qizhang stadium can be mentioned as another important example to the retractable roofs that are not proposed in dome shaped. The roof includes eight petal shaped sections and each of those sections rotates around a fixed shaft. This roof has been inspired from the traditional flower called Magnolia and is a good example for technology, science and art integration.

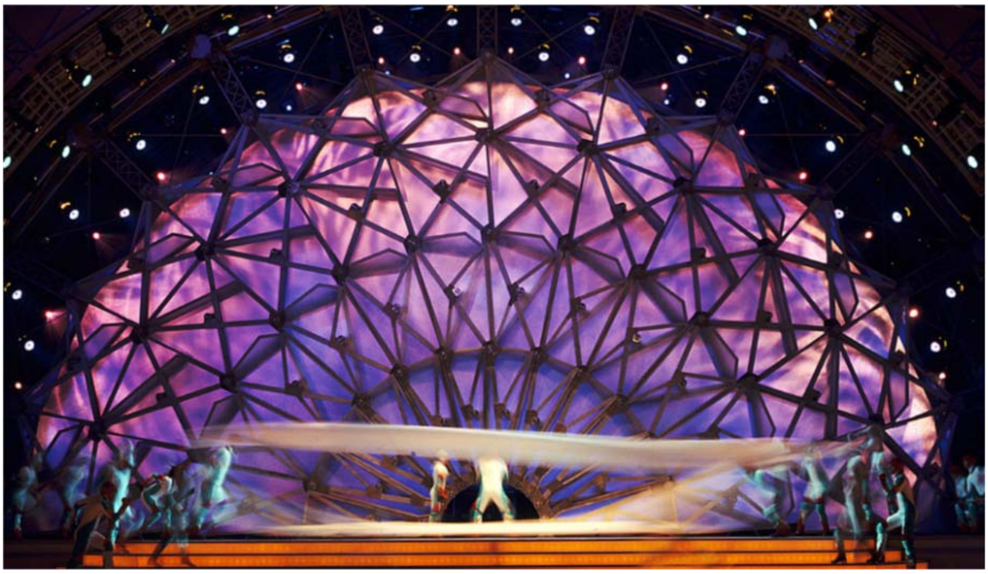


Figure 2.22. Olympic Arch
(Source: www.hoberman.com)

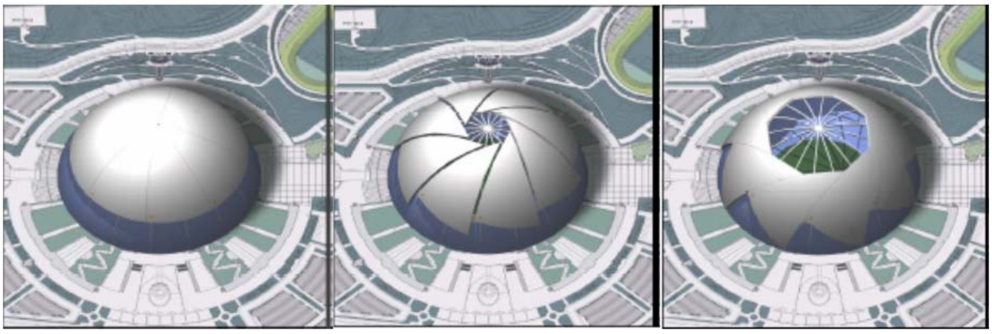


Figure 2.23. Retractable Roof (Jensen 2004)

Figure 2.24. Qizhon's Retractable Roof
(Source: www.isaarchitecture.com)

Throughout this chapter it has been shown that, retractable structures have been used in variable areas in kinetic architecture especially as retractable roofs for in sport facilities. Moreover it has been presented that one of the crucial issues for RPS is the determination of a suitable shape of a rigid material that allows covering without any gaps or overlaps by eliminating the chance of interference between each other during the deployment motion. In the light of this, it has been demonstrated that there exist many important studies to solve this issue. However, all of these researches need to develop many complicated numerical, kinematical or analytical approaches. Furthermore most of the examples that are summarized above have found the final form of the plate as triangular or a geometry that is derived from triangle. Thus, after the investigation of aforementioned studies, it can be seen that none of the existing design proposals consider finding suitable form of the plate based on regular polygons instead of triangle such as square or hexagon.

CHAPTER 3

TESSELLATION TECHNIQUE

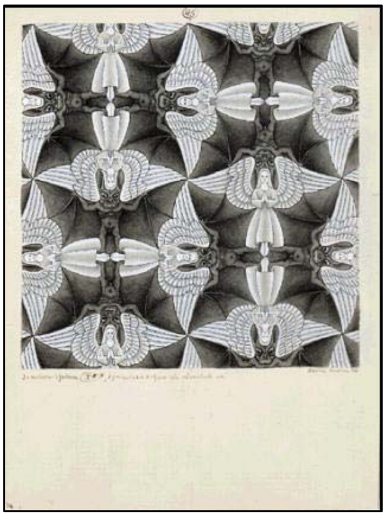
This chapter concentrates on the tessellation technique in mathematics. Firstly, a brief discussion about the meaning of tessellation and some definition presents. Following, this chapter deals with the importance of symmetry for tessellation. In the third part of the chapter, the historical background of tessellation and the issue of tessellating the plane with regular polygons mentioned. Then, all k-uniform tessellation demonstrates. Finally, importance of the duality of tessellation will be introduced.

3.1 Tessellation

Tessellation is an arrangement of closed shapes that completely cover the plane without overlapping or leaving gaps. Throughout the literature tessellation is used in numerous different applications such as geography, cellular biology or crystallography. However, the most common usage is seen in art and architecture. In fact tessellations were discovered in thousands of years ago and since then they have been used in art by many different ancient cultures long before they were studied in mathematics.

The most important usage of the planar tessellations is being seen in the art of ornament. Medieval Islamic artisans developed intricate geometric tessellations to decorate their mosques, mausoleums and shrines. At the same time their culture has spread into Africa, Europe, and Asia to adorn architectural surfaces with geometric patterns. The most well-known usage of tessellation can be seen in the Alhambra at Granada. In twentieth century, tessellations of the Alhambra inspired many architects and artists. Dutch graphic artist Escher was one of them. Escher implemented the real-world objects such as animal forms into systematic mathematical theory of Alhambra. Escher designed each of his interlocking animal forms after a great deal of tinkering and manipulation (Kaplan and Salesin 2000). Another usage of tessellation can be seen in origami art. Origami is the art of folding sheets of paper into various forms without stretching, cutting or gluing other pieces of paper to them (Tachi 2013). In their designs many designers use tessellation as a pattern (Tachi 2013; Miuro 1969; Bateman 2002).

Moreover many designers in art and architecture have been inspired from the origami tessellation. They have used origami tessellation as a main toll to design deployable plate structures (FPS) (Demaine and Rourke 2007).



a) Angel Devil, 1941

b) Lizard, 1942

Figure 3.1. Examples of Escher's works (Source: www.mcescher.com)

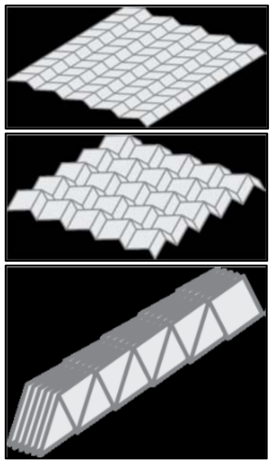


Figure 3.2. a) Moorish stars crease pattern, b) Deployable plate (Source: www.origamitessellations.com)

3.2. Planar Tessellation

Basically tessellation is a technique to cover a plane without any gaps or overlaps. As a result of this property, it has been used in many areas such as science, art and architecture since its first discovery. However, the most well-known usage has been seen in architectural surfaces. Tessellation technique can be applied to the planar, spherical or hyperbolic surfaces. In fact the terminology of the tessellation comes from the Latin word tessella that was the square stone or tile used in ancient Roman planar mosaics. At this point it should be noted that tessellations are also known as tiling, paving or mosaics in the history, and they have appeared since the prehistoric times (Seymour and Britton 1989).

Before focusing on the planar tessellation technique with regular polygons some basic definition should be presented.

3.2.1. Some Basic Definition for Planar Tessellation

In mathematics a plane tessellation *T* means a countable family of closed sets that cover the plane without any gaps or overlaps.

$$T = \{T1, T2, \ldots\}$$

where, T1, T2, … are known as tiles of *T* (Renault 2008). In this family the necessary condition to be a tessellation is being countable, closed and able to cover the target without any gaps and overlaps. Mathematically being countable presents the property where the number of tiles in the tessellation can be counted, being closed assures that each tile is enclosed by its boundary, being able to cover without any gaps means the union of all the sets T_1, T_2, ... are to be the whole plane, and finally being able to cover without overlaps shows that the interior of the sets are pairwise not disjoint (Ling 2003). In Figure 3.3 some of the non-suitable examples are illustrated.

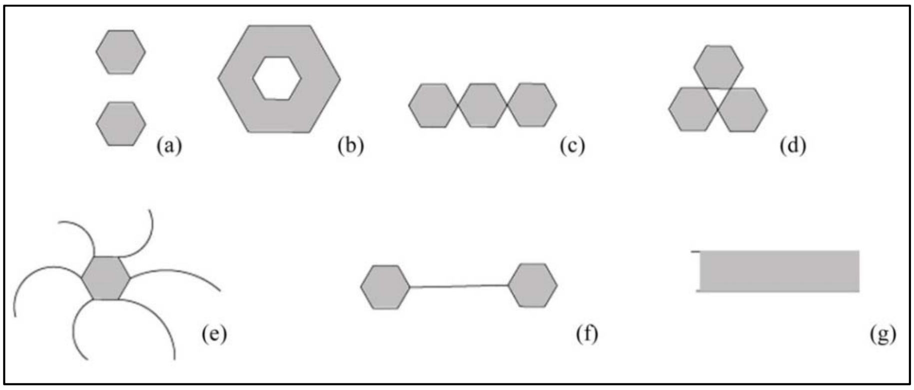


Figure 3.3. Non-suitable examples of plane tiling

Figure 3.3a and Figure 3.3b are not connected because Figure 3.3a and 3.3b have two or more separate pieces. Figure 3.3c and 3.3d are disconnected because the finite set of points has deletion. Figure 3.3e and 3.3f become disconnected upon the deletion of a suitable finite set of points. Also Figure 3.3g is not a closed curve.

Elements of Tessellation: A tessellation of the plane is a family of sets that are called tiles. If any of the two tiles have disjointed interiors, the intersections of a set of tiles are either empty or consists of isolated points and arcs (Ling 2003). These points are called vertices and these arcs are called edges of the tessellation. Vertices, tiles and edges are actually the elements of tessellation (Chavey 1989). Also when an edge connects two distinct points, these points are called endpoints (corners) of the tessellation. It is important to notify that every vertex is the endpoint of some finite number of edges and this finite number is called as the valence of the vertex and it can at least be 3. In some cases, there can be some confusion about the vertices, corners, edges and the sides. In order to avoid the confusion Figure 3.4 display the differences between them.

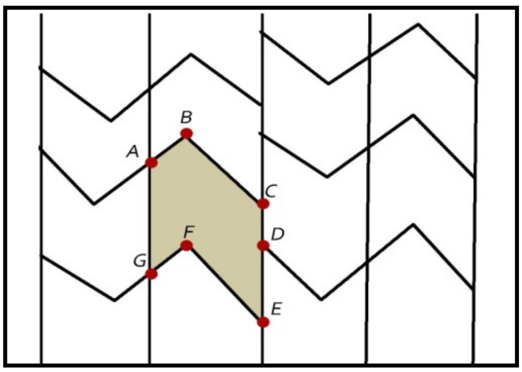


Figure 3.4. Elements of the tessellation

The differences between both the corners and the vertices, and the sides and edges in the case of a polygonal tessellation can be seen in Figure 3.4. The points A, B, C, D, E, F, G are the corners of the tessellation while the vertices of the tessellation are only the points A, C, D, E, and G. Similarly the line segments AB, BC, CE, EF, FG and GA are the sides of the tessellation while the edges are only the line segments of AC, CD, DE, EG and GA. (Grünbaum and Shephard 1984)

Edge to Edge Tessellation: In a planar tessellation with regular polygons, the tessellations are said to be the edge to edge tessellation (Chavey 1984; Grünbaum and Shephard 1984). In the light of this, if the corners and sides of the polygons coincide with vertices and edges of the tiling, the tessellation is called edge-to edge tessellation. Figure 3.5 illustrates the non-edge-to-edge tessellation while Figure 3.6 displays the edge-to edge tessellation.

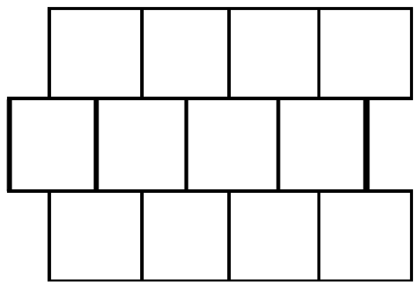


Figure 3.5. Non-edge to edge tessellation

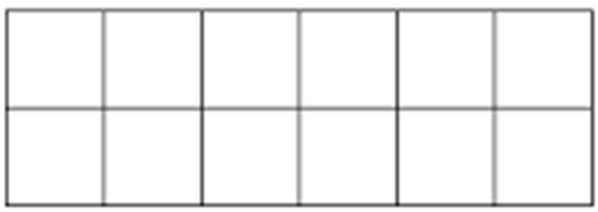


Figure 3.6. Edge-to-edge tessellation

Congruent and Equal Tessellation: Two tessellations are said to be congruent if T1 can be transformed into T2 by a rigid motion. Rigid motions include rotations, translations, and reflections but exclude stretching and shrinking. Furthermore, the tiles are said to be equal if T1 is changed in scale as if magnified equally throughout the plane (Ling 2003). For instance, tessellation in Figure 3.7 is the congruent of the tessellation in Figure 3.8 while it is equal to the tessellation in Figure 3.9.

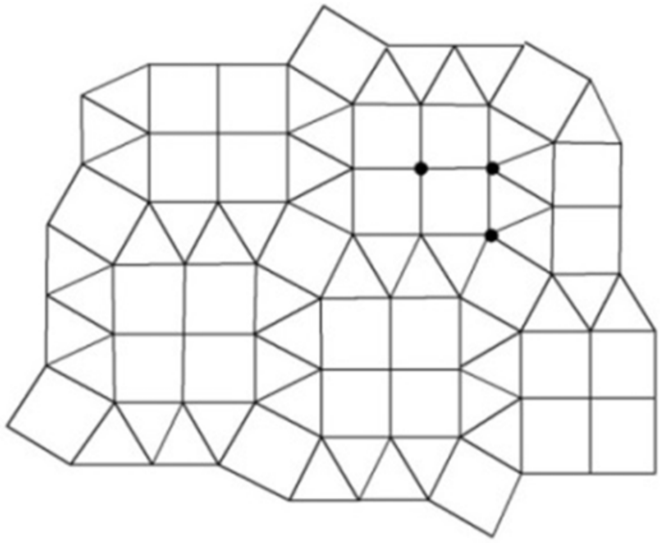


Figure 3.7. 4^4;$4.3.3^2$;$3^3.4^2$ tessellation

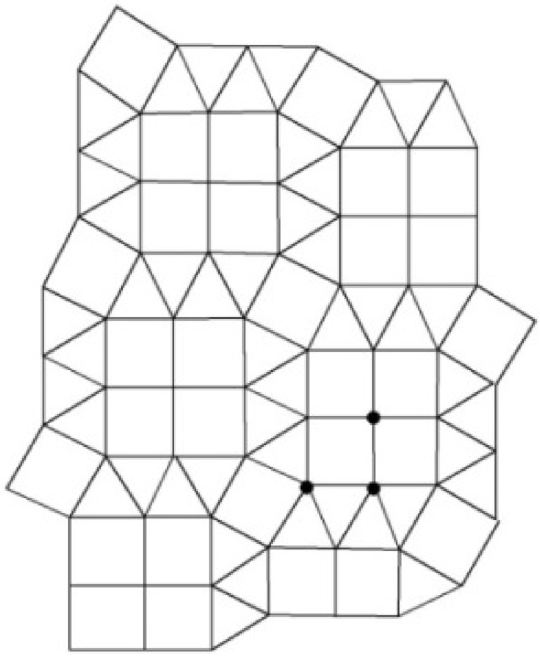


Figure 3.8. Congruent of 4^4; $4.3.3^2$;$3^3.4^2$ tessellation

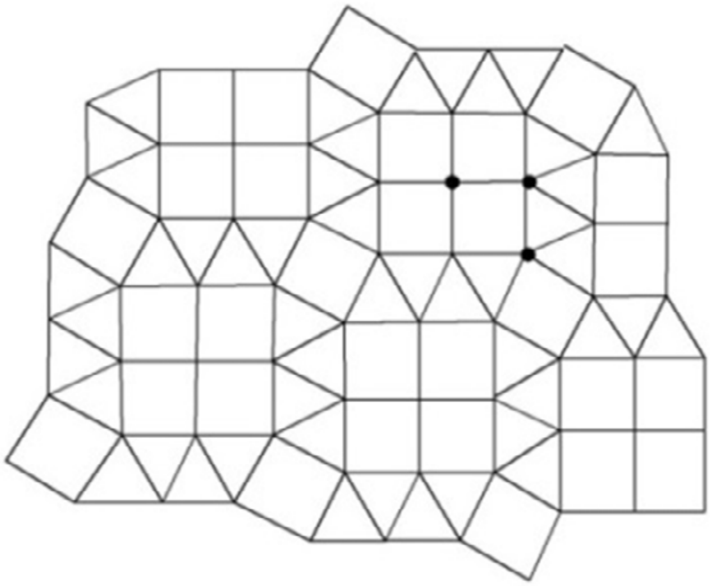


Figure 3.9. Equal of 4^4; $4.3.3^2$;$3^3.4^2$ tessellation

Before advancing further, some terms regarding with the terminology should be mentioned. Throughout the technique it should be noted that ***hedral*** refers to the tiles, ***gonal*** refers to the vertices and the ***toxal*** refers to the edges.

Monohedral Tessellation: Every tile T_i in the tessellation T is congruent to a one fixed set of T. In other words a monohedral is a special type of tessellation in which all the tiles have the same shape (Ling 2003). The examples of monohedral tessellations can be given as regular square, hexagonal and triangular tessellations (Figure 3.10). Additionally, if the tessellations are congruent to two, three, four, ... n distinct prototiles of set T respectively, these are called dihedral, trihedral, 4-hedral,…, n-hedral tilings . As an example dihedral tiling is illustrated in Figure 3.11.

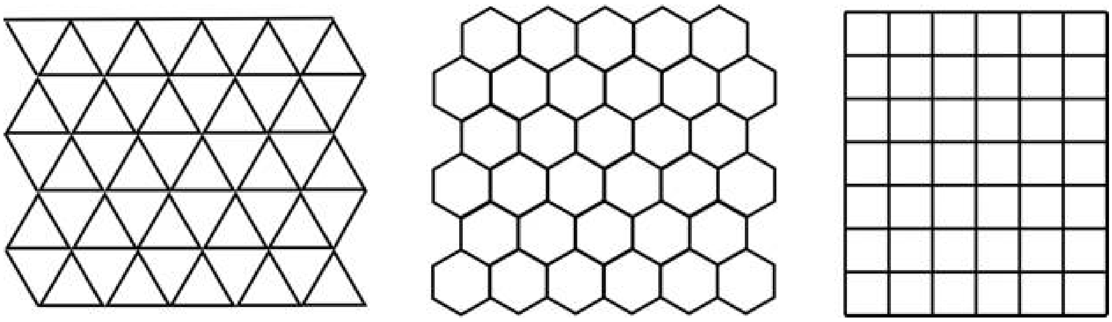


Figure 3.10. Monohedral tessellation

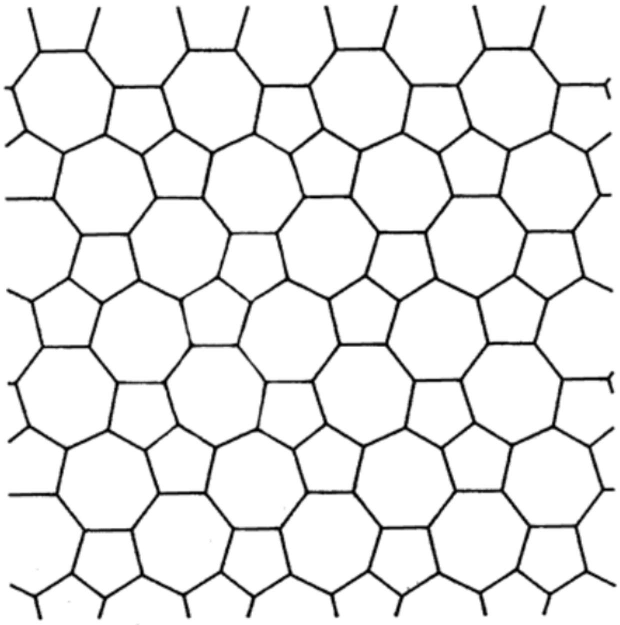


Figure 3.11. Dihedral tessellation

Isohedral Tessellation: Any tessellation *T* is isohedral if there is symmetry between the tiles.

Isotoxal Tessellation: Any tessellation *T* is isotoxal if there is symmetry between the edges.

Isogonal Tessellation: Any tessellation *T* is isogonal if there is symmetry between the vertices. Isogonal is also referred as ***uniform*** by some authors, especially in the case where the tiles consist of all regular polygons.

On the other hand, definition of a tessellation symmetry is very important. Symmetry of a tessellation *T* is an isometry that maps every tile of T to a tessellation of *T*. If the symmetry group of T contains two translations in non-parallel directions, *T* is called periodic tessellation. Patterns that do not repeat in a linear direction are also called non-periodic tessellation (Kinsey and Moore 2002). In addition to this, the term orbit is very important to describe and to understand the symmetry in tessellations. Chavey claims that one of the important criteria to describe regularity of the tessellation is the orbit that is related with the equivalence classes of the elements of tessellations (vertices, edges and tiles). These equivalences are the orbits of tessellations. Tessellations can be symmetric with each other, if they are in the same orbits. If the tessellation is periodic, the number of orbits of the tessellation element must be finite (Chavey 1989).

3.2.2. Symmetry of Tessellation

In math, symmetry is generally defined as the property by which the sides of a figure or an object are reflected each other across to a line or a surface (Britannica 2010). In order to understand symmetry, it is important to define the meaning of isometry. The term isometry comes from the ancient Greek of two words, ***isos*** as equal and ***metron*** as measure. For instance, if P and Q are any two points in plane, and the distance between P and Q are equal to the distance between P_1 and Q_1 after the transformation the point sets are called isometric. In mathematics isometry has four types, as translation, reflection, rotation, and glide reflection. These are called transformation in mathematics. Transformation can be described as moving a figure from one location on a plane to a new location on that same plane.

3.2.2.1. Transformation Types

In translation isometries, a planar figure slides without turning or flipping by a given fixed distance and the direction without changing its size or shape. In this transformation the original figure is called the pre-image and the translated one is called the image. The pre-image and the image have the same shapes, sizes and faces in the same direction (Figure 3.12). Also the distance between any two corresponding points on the original and the translated shape is called the magnitude or size of the translation (Seymour and Britton 1989).

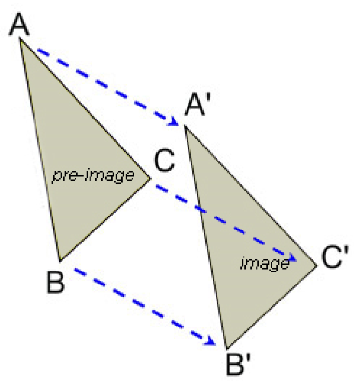


Figure 3.12. Translation

In rotation isometries, a planar figure turns with a given angle around a fixed point. The image has the same size and shape with the pre-image. The fixed point in the rotation plane which a shape is rotated around is called the centre of rotation (Figure 3.13). Moreover, the angle between the original shape and the rotated shape is called the angle of rotation and it defines the magnitude of the rotation.

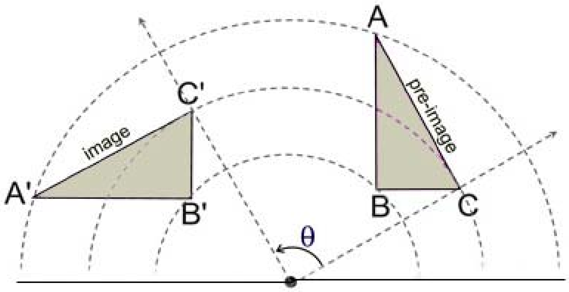


Figure 3.13. Rotation (Source: mathbitsnotebook.com)

A reflection or a flip is the mapping of all points of the original figure onto the other side of an axis so that the perpendicular distances between the points of the image and the axis will not be changed. The original object is called pre-imaged and its reflection is called image. After the reflection both of the figures will have the same shapes and sizes but they will face in opposite directions (Figure 3.14).

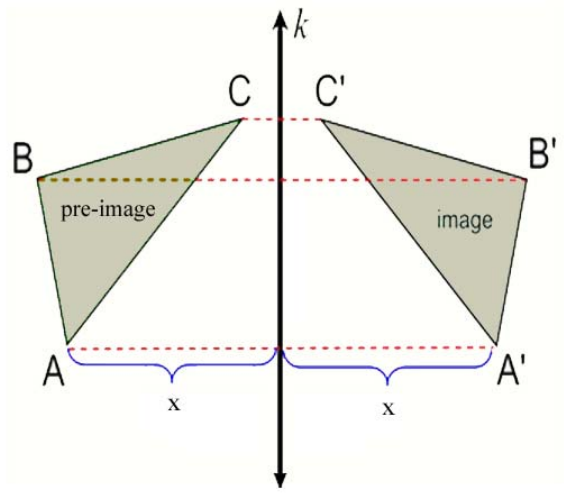


Figure 3.14. Reflection

A glide reflection is a combination of two transformations, a reflection and a translation in the same axis. Similar to the other transformations the original object is called pre-imaged while the reflection is called image. To form an image by utilizing glide reflection pre-image should be filliped around and axis and then translated along a straight line that is parallel to that axis (Figure 3.15).

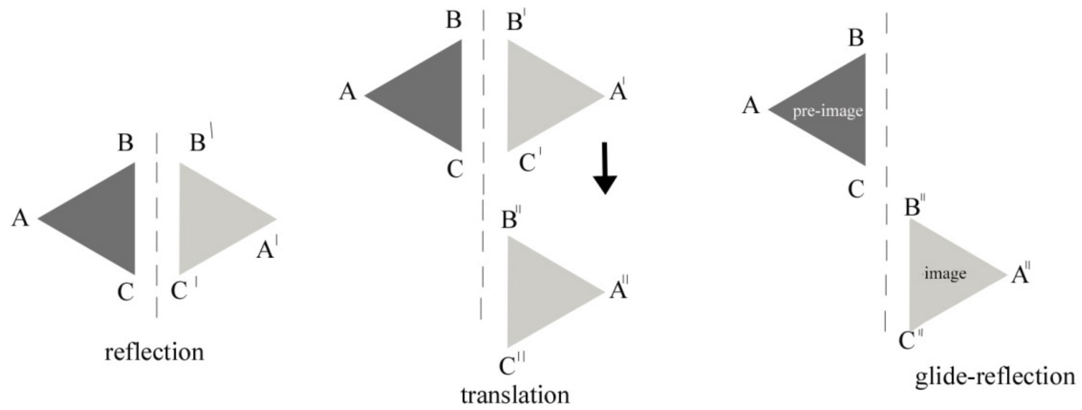


Figure 3.15. Glide reflection

3.3. Tessellations with Regular Polygons

In regular polygon tessellations, only regular convex polygons are used as tiles. Polygons are the planar shapes with the sides formed by the line segments. Also, a line segment is a part of a straight line bounded by endpoints (Seymour and Britton 1989). If such a polygon has n edges (or sides), it is called an n-gon and shown with the symbol of {n}. For instance {3} denotes an equilateral triangle, {4} denotes a square and {5} denotes a regular pentagon.

Tessellations with regular polygons are used widely in wallpapers, fabric designs and also in architecture patterns. Additionally, they were the first kinds of tessellations to be a subject of a mathematical research.

3.3.1. Historical Background

One of the early discoveries about the regular tessellation was known by the early Greek geometries. Pythagoreans combined the regular polygons around one type of vertex which are known today as regular tessellations (square, triangle and hexagonal tessellations, 4^4, 3^6 and 6^3) (Heath 1947). Pappus of Alexandria wrote the classifications of monohedral tessellations and it was seen that he was aware of edge to edge tessellations (Heath 1921; Chavey 1981). One of the pioneering investigations was done by the mathematician and the astronomer Johannes Kepler. Kepler published to describe the regular polygon tessellation of the plane and sphere as the first time in his book of Harmonices Mundi (Kepler 1619). He focused on the 1-uniform tessellation and also mentioned star polygons that can be used in tessellations. Kepler then, enumerates the possible vertex figures and shows 1-uniform tessellations that they can form.

Another important development comes from Badoureau in 1881. Badoureau determines all vertex meeting possibilities of regular polygons, but he interested only in monogonal tessellations. The correct determination about the polygons, and the vertex figures were done by Sommerville (1905) and Chavey (1984). Sommerville is also the first author to rule out all vertex types that cannot form tessellations explicitly. Also, on the contrary to Kepler, Sommerville uses algebra to describe the reasoning and theory behind them. These algebras are also used by modern mathematicians in the literature (Lenngren 2009). Also Grünbaum and Shephard mentioned the possibilities of regular polygons that are meeting at a vertex too (Grünbaum and Shephard 1984).

3.3.2. Tessellating the Plane with Regular Polygons

When covering the surface with regular polygons, one of the most important considerations to be taken is choosing the general polygons that can tessellate the plane. Seymour and Britton explained deeply in their book and conclude that only two types of shapes as triangles and quadrilaterals always tessellate the plane. What about other regular polygons? It is important to define which regular polygons or which combination of regular polygons can tessellate the plane in the design process of an architectural surface. Before discussing this, the importance of the vertex figure should be analysed.

Naming Convention: Tessellations with regular polygons are usually represented by the number of sides of the polygons around any cross point in the clockwise or anti-clockwise in a cyclic order, an a-gon{a}, b-gon {b}, c-gon{c},etc. by a.b.c. For instance, in Figure 3.16 the vertex represented by a red dot is surrounded by six triangular polygons. Thus, this tessellation is represented by 3.3.3.3.3.3, for brevity these can be written in the form of 3^6. In that representation 3 is the number of the sides of a triangle and superscript 6 is the number of triangles around the referred vertex point. This sequence is also called the type of a vertex.

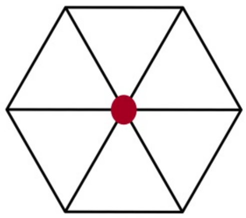


Figure 3.16. 3^6 Tessellation

The possibilities of edge-to-edge tessellation with regular polygons on the plane can be found by using some formulation. The interior angle at each corner of a regular n-gon {n} is (n-2)/n radians (or 180(n-2)/n degrees). In the light of this the formulation, the number of a single type regular polygons that fit a vertex is,

$$360/[(n-2)180/n] = 2n/(n-2) = 2 + 4/(n-2) \tag{3.1}$$

By using this formulation only regular hexagons, squares and equilateral triangles can be tessellated. If three different types of regular polygons should fit around a vertex, the formulation is;

$$[(n_1-2)/n_1 + (n_2-2)/n_2 + (n_3-2)/n_3]180° = 360° \tag{3.2}$$

If four different types of regular polygons should fit around a vertex, the formulation is,

$$1/n_1 + 1/n_2 + 1/n_3 + 1/n_4 = 1 \tag{3.3}$$

If five different types of regular polygons should fit around a vertex, the formulation is,

$$1/n_1 + 1/n_2 + 1/n_3 + 1/n_4 + 1/n_5 = 3/2 \tag{3.4}$$

If six different types of regular polygons should fit around a vertex, the formulation is,

$$1/n_1 + 1/n_2 + 1/n_3 + 1/n_4 + 1/n_5 + 1/n_6 = 2 \tag{3.5}$$

By considering these formulations, there are 21 types of vertices that are also called species fit around a vertex. Due to four types of vertices are two distinct ways to be fitted by the polygons (for instance the combination of 3.3.4.3.4 can be rearranged to reach 3.3.3.4.4) totally 17 solutions exist. Figure 3.17 displays the 21 possible types of vertices (Grünbaum and Shephard 1986).

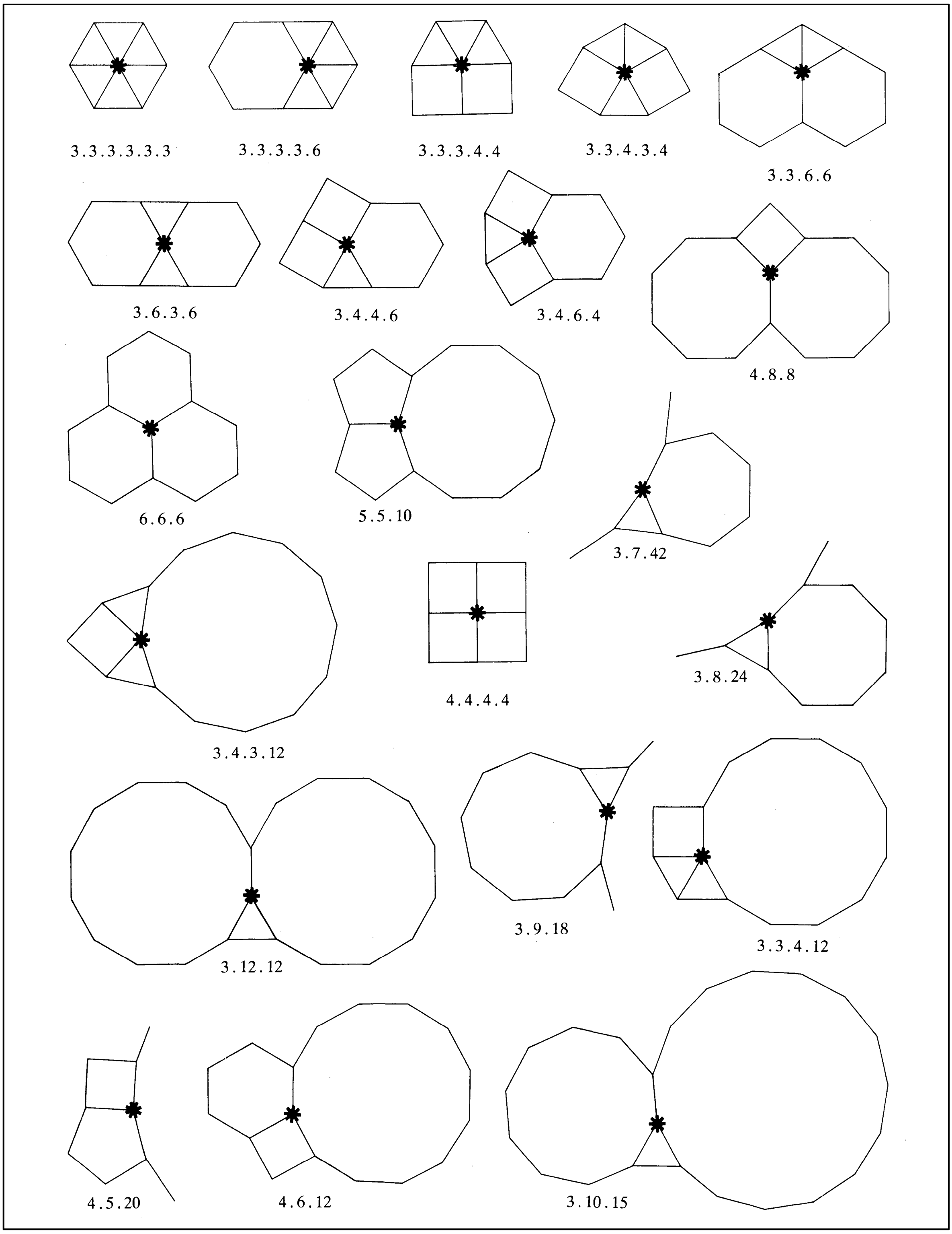


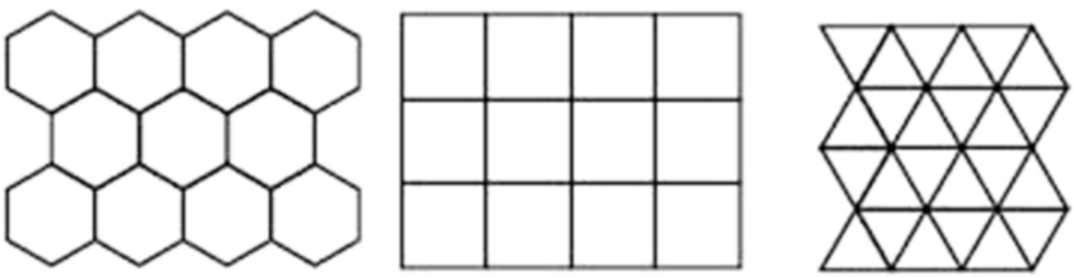
Figure 3.17. 21 Possibilities of vertices

3.3.3. K- Uniform Tessellations

An edge-to-edge tessellation formed by regular polygons is called k-uniform if its vertices form precisely k transitivity classes with respect to the group of symmetries of the tessellation. Shortly, the tessellation is k-uniform if and only if it is k-isogonal and its tiles are regular polygons.

The mathematical literature of k-uniform tessellations is comparatively very new area. 1-uniform tessellations had been completely investigated by Sommerville. The investigation of 2-uniform tessellations is completed by Krötenheerdt by 1969. After fifteen years later, Chavey extended these researches and listed 3 more uniform tessellations. Grünbaum and Shephard also provided extended survey (1987). In addition to these approaches, the 4-5 and 6 uniform tessellations have been found by the computer and published on the web by Galebach (2009), however the procedure of producing 4-5-6 uniform tessellations were not described deeply (Lenngren 2009). Totally, there exist 128 types of k-uniform tessellations.

In a k-uniform tessellation, if k=1, the edge-to-edge tessellation is called uniform tessellation. 1-uniform tessellations are the arrangement of identical shapes and angles at each vertex. Moreover, these tessellations are also called Archimedean tessellations or monohedral tessellations. There are eleven 1-uniform tessellations (Figure 3.18-3.19). Three of them are regular tessellations that are formed by identical polygons and eight of them are semiregular tessellations that are formed by two or more convex regular polygons. Each vertex has the same pattern of polygons around it.



6^3 4^4 3^6

Figure 3.18. 1-Uniform tessellations – Regular tessellations

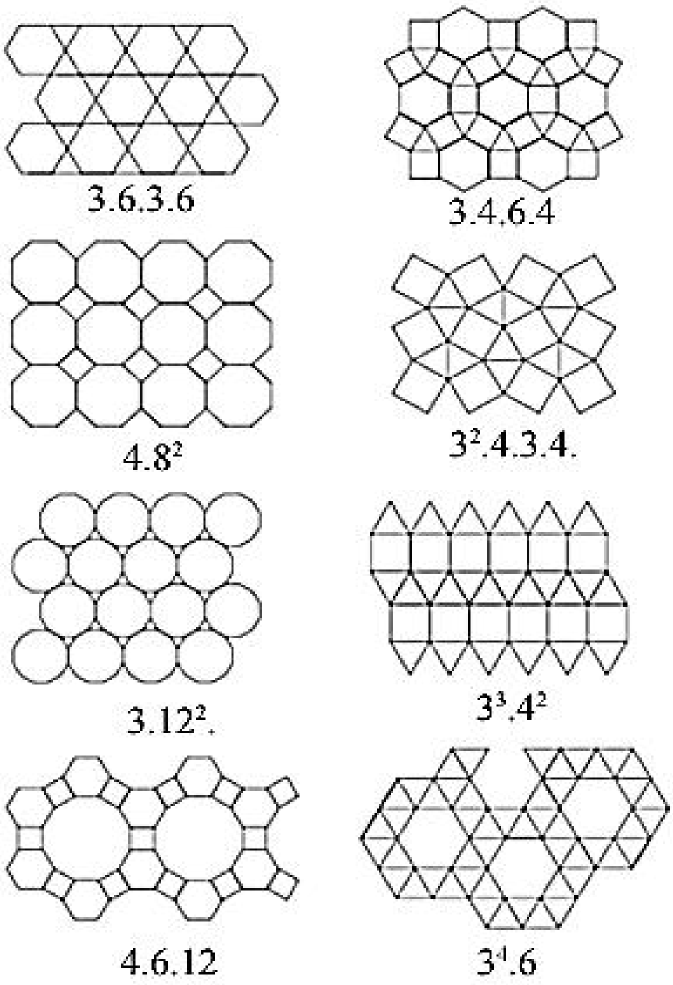
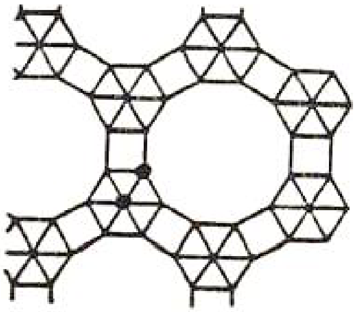


Figure 3.19. 1-Uniform tessellations – Semi-regular tessellations

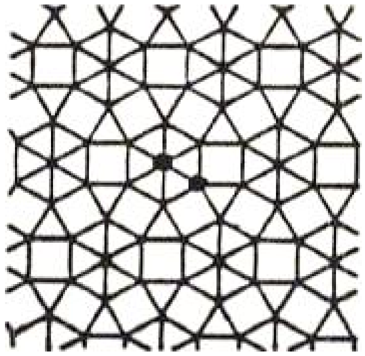
If k=2, the tessellations are called 2-uniform tessellations. 2-uniform tessellations are edge to edge tessellations and there are two different patterns of polygons around the vertices. There exist twenty distinct types of 2-uniform edge-to-edge tessellations by regular polygons, namely:

$(3^6;3^4.6)_1$, $(3^6;3^4.6)_2$, $(3^6;3^3.4^2)_1$, $(3^6;3^3.4^2)_2$, $(3^6;3^2.4.12)$, $(3^6;3^2.4.3.4)$, $(3^6;3^2.6^2)$, $(3^4.6;3^2.6^2)$, $(3^3.4^2; 3^2.4.3.4)_1$, $(3^3.4^2; 3^2.4.3.4)_2$, $(3^3.4^2; 3.4.6.4)$, $(3^3.4^2;4^4)_1$, $(3^3.4^2;4^4)_2$, $(3^2.4.3.4;3.4.6.4)$, $(3^2.6^2;3.6.3.6)$, $(3.4.3.12;3.12^2)$, $(3.4^2.6;3.4.6.4)$, $(3.4^2.6;3.6.3.6)_1$, $(3.4^2.6;3.6.3.6)_2$, and $(3.4.6.4;4.6.12)$.

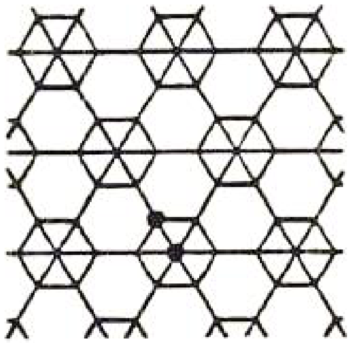
Figure 3.20 illustrates the 2-uniform tessellations.



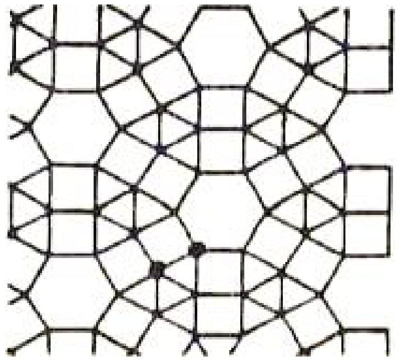
$3^6;3^2.4.12$



$3^6;3^2.4.3.4$



$3^6;3^2.6^2$

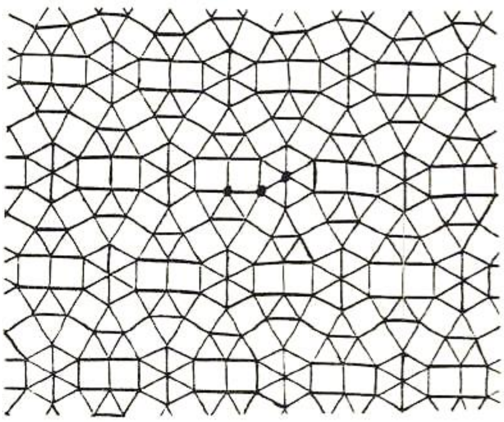


$6.4.3.4;3^3.4^2$

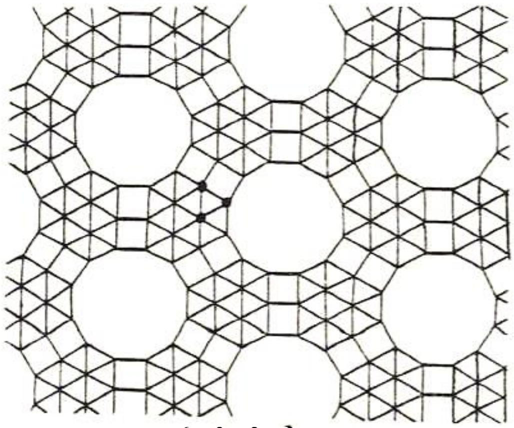
Figure 3.20. Examples of 2-uniform tessellations

3-uniform tessellations are edge to edge tessellations and there are three different orders of polygons around the vertices. The 3-uniform tessellations were enumerated by Chavey (1984) in his Ph.D thesis There exist thirty-nine distinct types of 3-uniform edge-to-edge tessellations by regular polygons which are illustrated in Figure 3.21, namely;

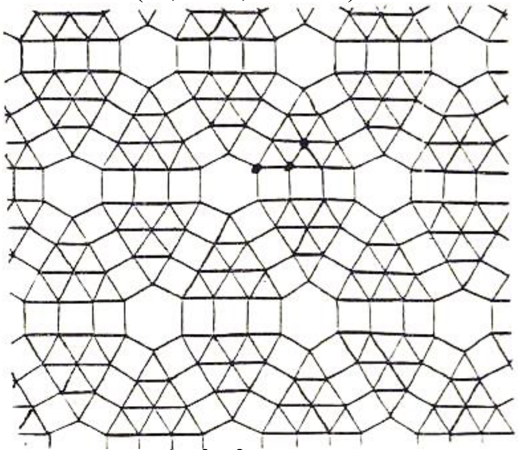
$(3^6;3^4.6;3^2.6^2)_1$, $(3^6;3^4.6;3^2.6^2)_2$ $(3^6;3^4.6;3^2.6^2)_3$, $(3^6;3^4.6;3.6.3.6)_1$, $(3^6;3^4.6;3.6.3.6)_2$, $(3^6;3^4.6;3.6.3.6)_3$, $(3^6;3^3.4^2;3^2.4.3.4)$, $(3^6;3^3.4^2;3^2.4.12)$, $(3^6;3^3.4^2;3.4.6.4)$, $(3^6;3^3.4^2;4^4)_1$, $(3^6;3^3.4^2;4^4)_2$, $(3^6;3^3.4^2;4^4)_3$, $(3^6;3^3.4^2;4^4)_4$, $(3^6;3^2.4.3.4;3^2.4.12)$, $(3^6;3^2.4.3.4;3.4^2.6)$, $(3^6;3^2.4.3.4;3.4.6.4)_1$, $(3^6;3^2.4.3.4;3.4.6.4)_2$, $(3^6;3^2.4.12;4.6.12)$, $(3^6;3^2.6^2;6^3)$, $(3^4.6;3^3.4^2;3^2.4.3.4)$, $(3^4.6;3^3.4^2;3.4^2.6)$, $(3^4.6;3^2.6^2;6^3)$, $(3^3.4^2;3^2.4.3.4;4^4)$, $(3^3.4^2;3^2.4.12;3.4.6.4)$, $(3^3.4^2;3^2.6^2;3.4^2.6)$, $(3^2.4.3.4;3.4^2.6;3.4.6.4)$, $(3^2.4.12;3.4.3.12;3.12^2)$, $(3^2.4.12;3.4.6.4;3.12^2)$, $(3^2.6^2;3.4^2.6;3.6.3.6)_1$, $(3^2.6^2;3.4^2.6;3.6.3.6)_2$, $(3^2.6^2;3.6.3.6;6^3)_1$, $(3^2.6^2;3.6.3.6;6^3)_2$, $(3.4.3.12;3.4.6.4;3.12^2)$, $(3.4^2.6;3.4.6.4;4^4)$, $(3.4^2.6;3.6.3.6;4^4)_1$, $(3.4^2.6;3.6.3.6;4^4)_2$, $(3.4^2.6;3.6.3.6;4^4)_3$, $(3.4^2.6;3.6.3.6;4^4)_4$, $(3.4^2.6;3.6.3.6;4.6.12)$.



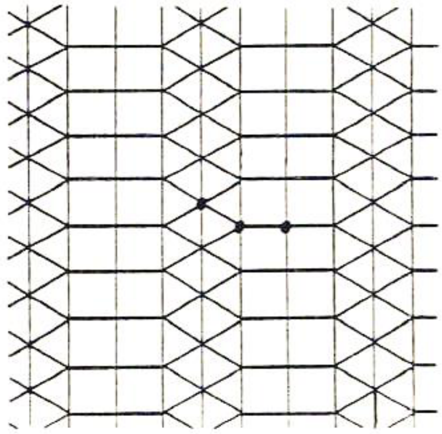
$(3^6;3^3.4^2;3^2.4.3.4)$



$(3^6;3^3.4^2;3^2.4.12)$



$(3^6;3^3.4^2;3.4.6.4)$



$(3^6;3^3.4^2;4^4)$

Figure 3.21. Example of 3-uniform tessellations

The other types of k-uniform tessellations include k=4, k=5, k=6 and k= 7 that are named below.

4-uniform tessellations:

$(3^6;3^4.6;3^3.4^2;3.4^2.6)_1$, $(3^6;3^4.6;3^3.4^2;3.4^2.6)_2$,
$(3^6;3^4.6;3^2.6^2;3.6.3.6)$, $(3^6;3^4.6;3^2.6^2;6^3)_1$, $(3^6;3^4.6;3^2.6^2;6^3)_2$, $(3^6;3^4.6;3^2.6^2;6^3)_3$,
$(3^6;3^4.6;3^2.6^2;6^3)_4$, $(3^6; 3^4.6;3.4^2.6;3.6.3.6)_1$, $(3^6; 3^4.6;3.4^2.6;3.6.3.6)_2$, ,
$(3^6;3^3.4^2;3^2.4.3.4;3^2.4.12)$, $(3^6; 3^3.4^2;3^2.4.3.4;3.4.6.4)_1$, $(3^6;3^3.4^2;3^2.4.3.4;3.4.6.4)_2$,
$(3^6;3^3.4^2;3^2.4.3.4;4^4)$, $(3^6;3^3.4^2;3.4^2.6;3.6.3.6)$, $(3^6;3^2.4.3.4;3^2.4.12;3.12^2)$,
$(3^6;3^2.4.3.4;3.4.3.12;3.12^2)$, $(3^6;3^2.4.3.4;3.4^2.6;3.4.6.4)$, $(3^4.6;3^2.6^2;3.6.3.6;6^3)_1$,
$(3^4.6;3^2.6^2;3.6.3.6;6^3)_2$, $(3^3.4^2;3^2.4.12;3.4.3.12;3.12^2)_1$, $(3^3.4^2;3^2.4.12;3.4.3.12;3.12^2)_2$,
$(3^3.4^2;3^2.4.12;3.4.3.12;4^4)$, $(3^3.4^2;3^2.6^2;3.4^2.6;4.6.12)$, $(3^3.4^2;3^2.6^2;3.4^2.6;6^3)_1$,

$(3^3.4^2;3^2.6^2;3.4^2.6;6^3)_2$, $(3^2.4.3.4;3^2.6^2;3.4^2.6;3.4.6.4)$, $(3^2.4.3.4;3^2.6^2;3.4^2.6;4.6.12)$, $(3^2.4.3.4;3^2.6^2;3.4^2.6;6^3)$, $(3^2.4.12;3.4.3.12;3.4.6.4;4.6.12)$, $(3^2.6^2;3.4^2.6;3.6.3.6;4^4)_1$, $(3^2.6^2;3.4^2.6;3.6.3.6;4^4)_2$, $(3^2.6^2;3.4^2.6;3.6.3.6;4^4)_3$, $(3^2.6^2;3.4^2.6;3.6.3.6;4^4)_4$.

5-uniform tessellations:

$(3^6;3^4.6;3^3.4^2;3.4^2.6;4^4)$, $(3^6;3^4.6;3^3.4^2;3^2.6^2;3.4^2.6)$, $(3^6;3^4.6;3^3.4^2;3.4^2.6;4.6.12)$, $(3^6; 3^4.6; 3^2.6^2; 3.6.3.6; 6^3)$, $(3^6;3^4.6; 3.4^2.6; 3.4.6.4;3.6.3.6)$, $(3^6;3^4.6;3.4^2.6;3.6.3.6;4^4)_1$, $(3^6;3^4.6; 3.4^2.6;3.6.3.6;4^4)_2$, $(3^6;3^4.6; 3.4^2.6;3.6.3.6;4^4)_3$, $(3^6;3^4.6;3.4^2.6;3.6.3.6;4^4)_4$, $(3^6;3^3.4^2;3^2.6^2;3.4^2.6;3.6.3.6)$, $(3^6;3^3.4^2;3.4^2.6;3.6.3.6;4^4)$, $(3^2.4.3.4;3^2.4.12;3.4.6.4;3.12^2;4.6.12)$, $(3^2.4.3.4;3^2.6^2;3.4^2.6;3.4.6.4;6^3)$, $(3^6;3^2.4.3.4;3.4^2.6; 3.4.6.4, 3.6.3.6)$, $(3^4.6; 3^3.4^2;3^2.4.4;3.4^2.6;4^4)$.

6-uniform tessellations:

$(3^6;3^4.6;3^3.4^2;3^2.4.3.4;3^2.6^2;3.4^2.6)$, $(3^6;3^4.6;3^3.4^2;3^2.4.12;3.4^2.6;3.6.3.6)$, $(3^6;3^3.4^2;3^2.4.3.4;3^2.6^2;3.4^2.6;3.4.6.4)$, $(3^6;3^3.4^2;3^2.4.3.4;3.4^2.6;3.4.6.4;4^4)$, $(3^4.6;3^3.4^2;3^2.4.3.4;3^2.6^2;3.4^2.6;3.4.6.4)$, $(3^4.6;3^3.4^2;3^2.4.3.4;3^2.6^2;3.4^2.6;4.6.12)$, $(3^4.6;3^3.4^2;3^2.4.3.4;3.4^2.6;3.4.6.4;3.6.3.6)$, $(3^3.4^2;3^2.4.3.4;3^2.6^2;3.4^2.6;4^4;6^3)$, $(3^3.4^2;3^2.4.12;3.4^2.6;3.4.6.4;3.6.3.6;4^4)$, $(3^4.6; 3^3.4^2;3^2.4.4;3.4^2.6;3.6.3.6;4^4)$.

7-uniform tessellation:

$(3^6;3^4.6;3^3.4^2;3^2.4.3.4;3^2.4.12;3.4^2.6;4.6.12)$,
$(3^6;3^4.6;3^3.4^2;3^2.4.3.4;3.4^2.6;3.4.6.4;3.6.3.6)_1$,
$(3^6;3^4.6;3^3.4^2;3^2.4.3.4.;3.4^2.6;3.4.6.4;3.6.3.6)_2$
$(3^6;3^4.6;3^3.4^2;3^2.4.12;3.4^2.6;3.4.6.4;3.6.3.6)$, $(3^6;3^4.6;3^3.4^2;3.4^2.6;3.4.6.4;3.6.3.6,4^4)$
$(3^6;3^3.4^2;3^2.4.3.4;3^2.6^2;3.4^2.6;4^4;6^3)$, $(3^3.4^2;3^2.4.3.4;3^2.4.12;3^2.6^2;3.4.3.12;3.4^2.6;4.6.12)$.

3.3.4. Duality of Tessellation

The concept of duality occurs in almost every branch of mathematics while isomorphism preserves certain relation. There is a deeper connection between any tessellation begin with placing a vertex at the geometric centre of each polygon in the original tessellation, and then draw a dual edge connecting the new vertices at the centres of those polygons. The processes of finding the dual form of 6^3 tessellation (Figure 3.21) and dual form of 1-uniform tessellation illustrated in Figure 3.22.

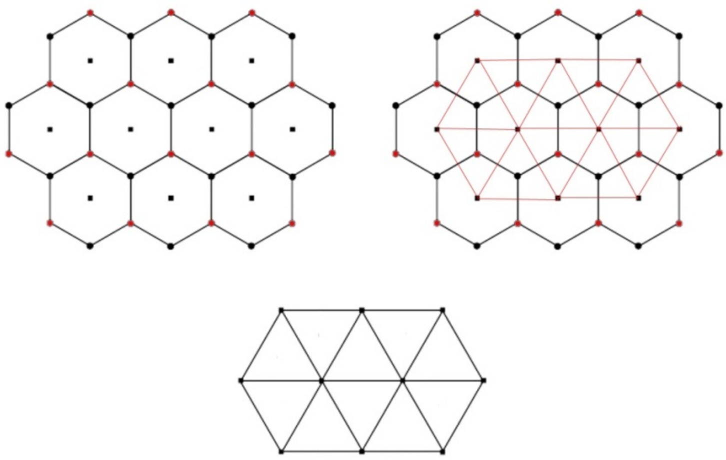


Figure 3.21. Dual Form of 6^3 tessellation

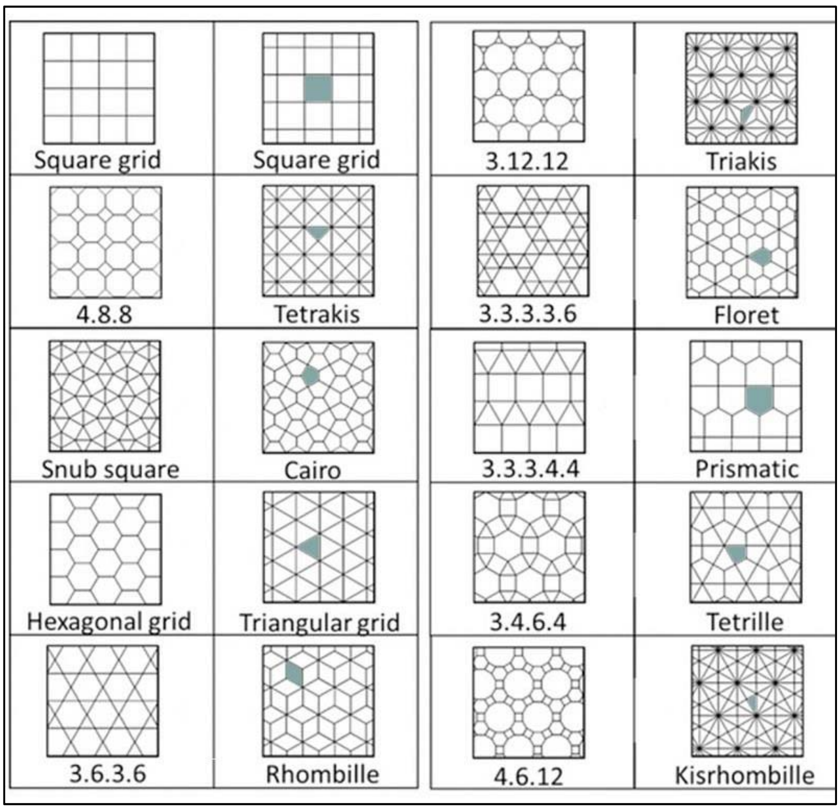


Figure 3.22. Dual forms of 1-Uniform tessellation

CHAPTER 4

DESIGN OF RETRACTABLE PLATE STRCUTURES BASED ON 1-UNIFORM TESSELLATION

This chapter concentrates on developing new approach for design processes of single *DoF* retractable plate structures with 1- uniform tessellation closed without any gaps or overlaps. In the first part of the chapter two general conditions are developed. The first condition state that which tessellation can generate moveable chain or unmoveable chain. The second condition state that which/why tessellation can generate RPSs that are closed without any gaps or overlaps. Then, this chapter handles the issue according to number of edges of 1- uniform tessellation that meet at one vertex. Firstly the RPSs based on a tessellation where four edges meet every vertex is discussed and a method by using vertex translation is developed for finding suitable shape of the plates. Following, this chapter deals with the RPSs based on tessellations where three edges meet at one vertex and it is understood that an extra link is necessary. Hence, in this part of the chapter two different methods are developed for finding the form of extra link. Finally, RPSs based on tessellations where five and six edges meet at one vertex is discussed and an important similarity between graph representation and duality is presented. Additionally according to this similarity a method to convert Multi-*DoF* (M-*DoF*) RPS to a single *DoF* RPS is developed. In all stages of the chapter, kinetic behaviour of the RPSs are analysed by using computer simulation environment.

4.1. Conditions to Design Single *DoF* RPSs based on 1-Uniform Tessellation

In this dissertation, plate shapes are determined according to types of polygons of 1-uniform tessellations. Plates are regular polygons and connected to each other from the nodes with only revolute joints (R). Here the nodes are the vertices of the polygons and the number of edges of polygons gives to type of plates. For instance, if 3^6 tessellation is used, it means the retractable plate structure will consist of ternary plates and the shape of ternary plates will be regular triangle (Figure 4.1 and 4.2) The objective of developing conditions is to constitute knowledge about which 1-uniform tessellation can generate moveable chain or unmoveable chain and which can be closed without any gaps or overlaps by analysing the geometric properties of tessellation.

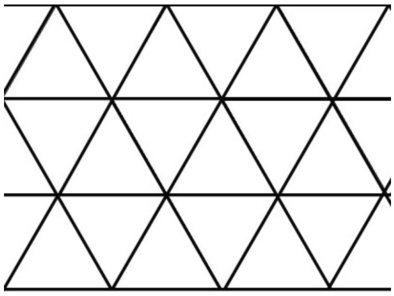


Figure 4.1. 3^6 Tessellation

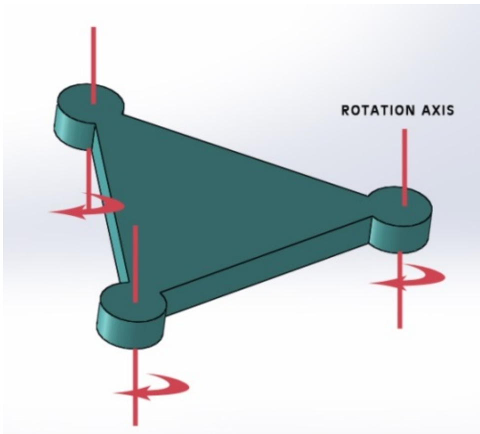
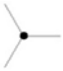
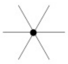
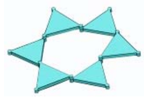
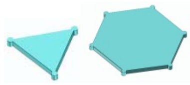
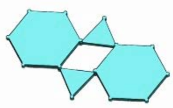
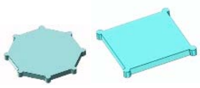
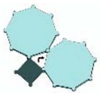
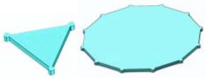
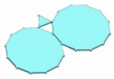
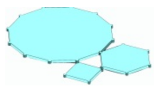
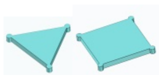
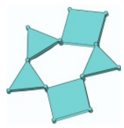
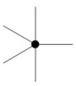
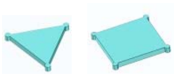
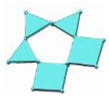
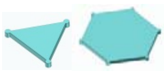
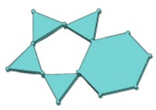


Figure 4.2. Triangular plate

In order to develop conditions; firstly all regular polygonal plates are connected with each other with revolute joint.1-Uniform tessellations have the arrangement of the same number of polygons at the vertices. When these polygonal plates are assembled with each other by considering the combination order around every vertex, they constitute a mechanism or a unmoveable chain For instance, three plates around a vertex connected with revolute joints constitute a unmoveable chain. Four or more plates are necessary to constitute a mechanism. Table 5.1 represents 1-uniform tessellations, number and order of edges at a vertex, type of rigid plates and their assembly order.

Table 4.1. Assembling Regular Polygon According to 1-Uniform Tessellation

Tessellation	*Number and order of edges at one vertex*	*Type of Rigid Plate*	*Assembling Order*	*Chain or unmoveable chain*
4.4.4.4	4 			4 bar chain
6.6.6	3 			Unmoveable Chain
3.3.3.3.3.3	6 			6 bar chain
3.6.3.6	4 			4 bar chain
8.8.4	3 			Unmoveable Chain
3.12.12	3 			Unmoveable Chain
4.6.12	3 			Unmoveable Chain
3.4.6.4	4 			4 bar chain
3.3.4.3.4	5 			5 bar chain
3.3.3.4.4	5 			5 bar chain
3.3.3.3.6	5 			5 bar chain

As it can be seen in table 1, four tessellations (6^3), (8^2.4), (3.12^2), (4.6.12) constitute a structure, while 4^4, 3^6, 3.6.3.6, 3.4.6.4, 3^2.4.3.4, 3^3.4^2, 3^4.6 tessellations generate moveable chains, thus they can be used to design retractable plate structure. By considering these results, it is claimed as a first condition that;

Condition 1: *To reach a retractable plate structure based on 1- uniform tessellations, at least four edges of the selected tessellation must meet at every vertex***.**

Condition 1 displays that if at least four edges meet at every vertex in a tessellation, it is possible to reach RPSs based on those tessellations. RPS is single DoF when four edges meet at every vertex. However every single DoF RPS based on four edges meet at every vertex tessellation has not predictable expansion with gaps or overlaps. Thus, it is necessary to have a second condition to generate RPS without any gaps or overlaps in both closed and open configurations and not interfere with each other during the deployment.

In order to understand the second condition, it is better to look at the kinematic behavior of the RPS based on (4^4), (3.4.6.4), (3.6.3.6) tessellations which have four edges meet at one vertex. As it is mentioned before, every four regular plates constitute parallelogram loops. During the contraction of (4^4), (3.6.3.6) tessellations, loops do not constitute a straight line. Only in fully closed configuration, loops constitute a straight line and closed perfectly where the plates also match perfectly. That is the dead center position of RPS.

In mechanism science, the alternative forms that a linkage can be constructed with the same link and connections are called configurations or assembly modes of the linkages (Atarer 2015). During its motion, the linkage may pass from one assembly mode to another, which is called reconfiguration or assembly mode change. A four bar mechanism in two different configurations is shown in figure 4.3. That four bar mechanism may change the configuration through the dead-center position. In dead-center position, the mechanism loses its mobility and has zero mechanical advantage. The mechanism may pass easily from one configuration to another.

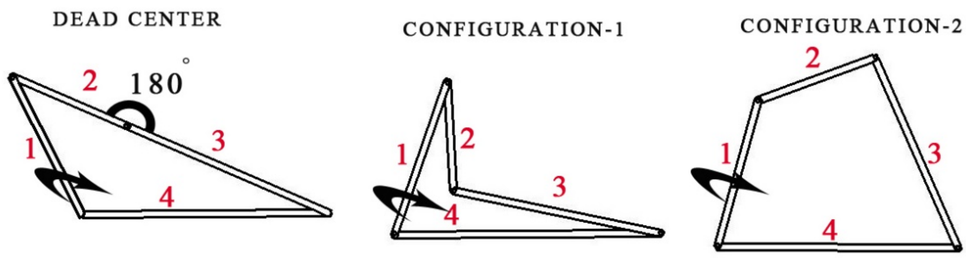


Figure 4.3. Assembly mode change through the dead-center position

Figure 4.4 and 4.5 demonstrate the retraction capability of the RPSs based on (4^4) and (3.6.3.6) tessellations. The loops constitute a straight line and reach dead center position when RPS is closed.

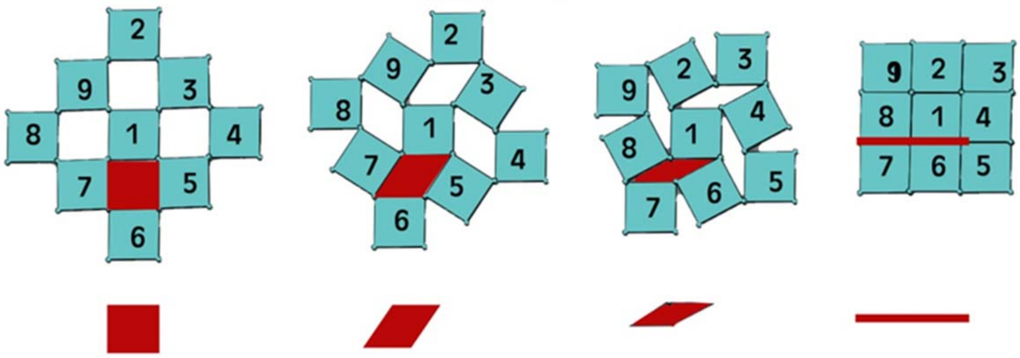


Figure 4.4. Retraction and dead centre position for RPS based on 4^4 tessellation

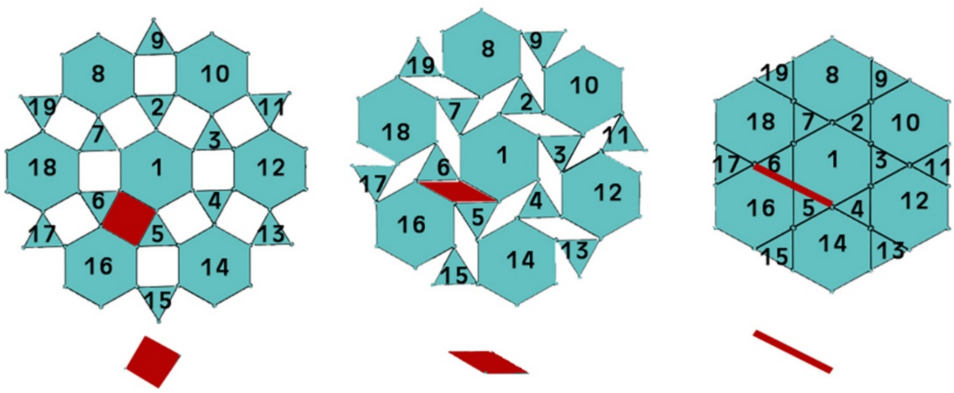


Figure 4.5. Retraction and dead centre position for RPS based on (3.6.3.6) tessellation

On the other hand, RPS based on (3.4.6.4) tessellation reaches dead center position before the closed configuration as in figure 4.6 a. In that position parallelogram loops change the configuration easily. Thus the movement after the dead center position is not predictable and there are overlaps as in Figure 4.6 b. There is a relationship between the neighboring edges and behavior of the loops. Thus it is possible to understand whether the designed RPS will have fully closed configuration or not.

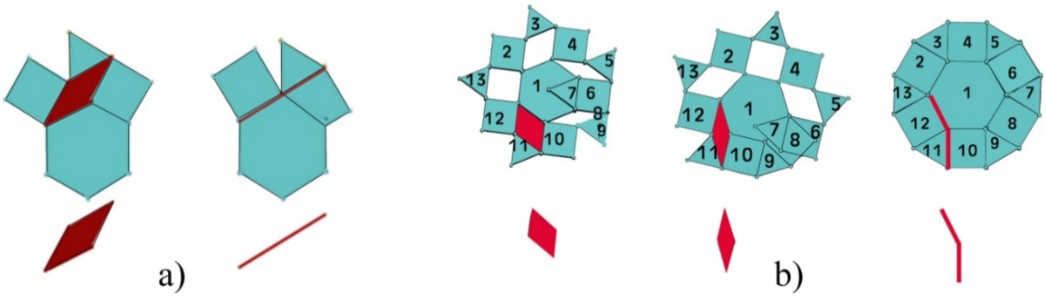


Figure 4.6. a) Sides of single loop reach dead center position before fully closed configuration, b) Unpredictable movement of the parallelogram loops after dead center.

According to aforementioned results, the second condition can be developed that

Condition 2: *To reach a RPS without any gaps or overlaps in closed configuration; minimum two neighboring polygons' edges of the selected tessellation must constitute a straight line*

From now on, design of the RPSs will be carried out based on the conditions mentioned above and by classifying them with respect to the edge numbers meeting at a vertex.

4.2. Design of Single DoF RPSs where Four Edges Meet at One Vertex

Two conditions to design RPSs based on 1-uniform tessellation is mentioned in the first part of this chapter. As understood from the first condition, if a RPS based on a tessellation where 4 edges meet at every vertex, it has one degree of freedom. Thus it expands and retracts in a predictable manner.

There are three 1-uniform tessellations (4^4, 3.6.3.6 and 6.4.3.4) where four edges meet at one vertex. All of them provide the first condition however, only two of them satisfy the second condition (4^4, 3.6.3.6) as shown in Figure 4.4 and Figure 4.5. The neighbouring edges of the 6.4.3.4 tessellation do not constitute a straight line (Figure 4.7). For this reason, there are always gaps or overlaps between the retractable plates.

As it known from the condition two, if minimum two neighbouring edges of selected tessellation generate a continuous line, the RPSs based on selected tessellation does not allow to any overlap or gaps in any configurations. By considering this condition, this part of the dissertation proposed an operation that is called "vertex translation. In order 3.4.6.4 tessellation satisfy the second condition, this operation changes the shape of the polygon by moving the vertices without changing the vertex combination of the tessellation.

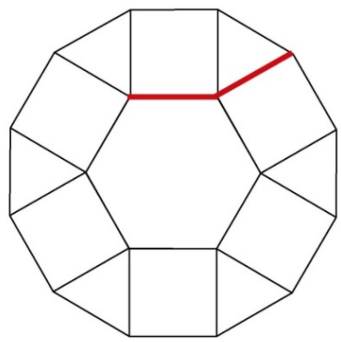


Figure 4.7. Placement of neighbouring edges of 3.4.6.4 tessellation

4.2.1. Vertex Translation Method

This part of the thesis explains the vertex translation on 3.4.6.4 tessellation to satisfy the second condition. Before, the translation, all vertices of the tessellation should be numbered.

In Figure 4.8, the vertices of the tessellation are translated to generate straight lines with neighbouring edges. As it can be seen from the Figure 4.8, the vertex "3" is translated to vertex **3'**, as a result, the vertices of "**5**", "**6**" and "**3'**" and vertices "**2'**", "**3'**" and "**4'**" generate straight lines. After this operation, the shape of the square plate is changed and the tessellation is not regular anymore. On the other hand, the order polygons are still same. In other words, RPS is designed based on irregular 3.4.6.4 tessellation (Figure 4.10). The retraction capability of the RPS is shown in Figure 4.11.

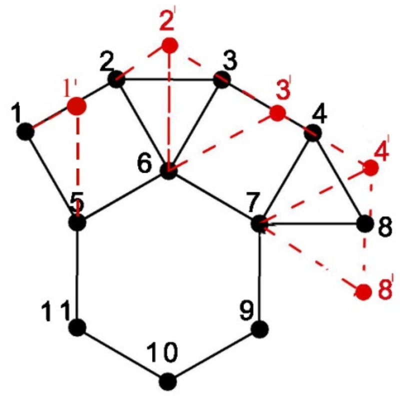


Figure 4.8. Vertex translation of 3.4.6.4 tessellation

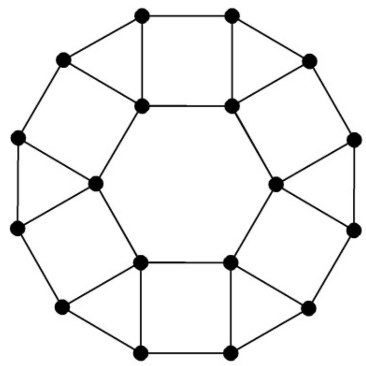


Figure 4.9. 3.4.6.4 tessellation

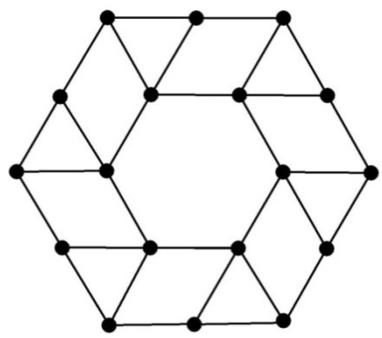


Figure 4.10. 3.4.6.4 Tessellation after vertex translation

Figure 4.11. The Retraction of RPS based on irregular 3.4.6.4 tessellation after vertex translation.

4.3. Design of Single DoF RPSs where 3 Edges Meet at One Vertex

As it is understood from the condition 1; to reach a retractable plate structures based on 1-uniform tessellations, at least four edges meet at each vertex. Thus, the tessellation of (6^3), (8^2.4), (3.12^2), (4.6.12) cannot be moveable. Figure 4.13 shows the assembling order of 6^3 tessellation, as it seen that hexagonal plates generate triangular loops and become a static structure.

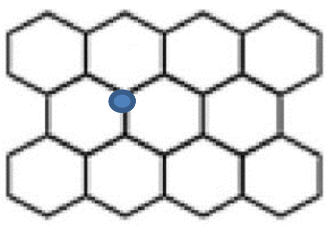


Figure.4.12. 6^3 tessellation

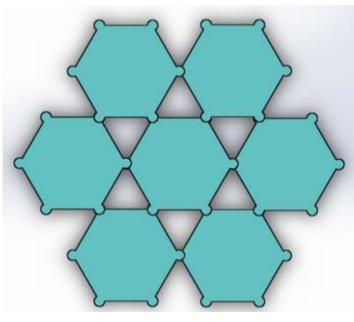


Figure 4.13. Assembling three hexagonal plates

In order to reach a RPS based on a tessellation where 3 edges meet at each vertex, extra links are necessary to create four bar loops. Also, the form of the extra plates becomes important to open and closed states without any gaps or overlaps. Within the scope of this, some sub-questions that are indicated below should be answered to define the design parameters of extra link?

- What is the type of extra link?
- What is the form of the extra link?
- Where are the positions of the joints on the extra link?

In order to find the design parameters of the extra link this thesis presents two different methods. The first method is based on vertex configuration and the other one is based on the duality of tessellation. Firstly, this section explains these methods; following is comparing and discussing their advantages and disadvantages.

4.3.1. Method-1: According To Vertex Configuration

This method consists of only one step and by the help of the vertex configuration, the positions of the joints and type of extra link is determined.

This method is explained with RPS based on 8.8.4 tessellation. Three edges meet at every vertex on the 8.8.4 tessellation. Thus, the type of links are octagonal plates (with eight revolute joints) and tetragonal plate (with four revolute joints).As it known from the table 1, when square and octagonal plates are assembled with each other by considering 8.8.4 tessellation order, triangular loops are generated (Figure 4.17). In addition to this, when this order is iterated, many plates do not be assembled one by one (Figure 4.18).

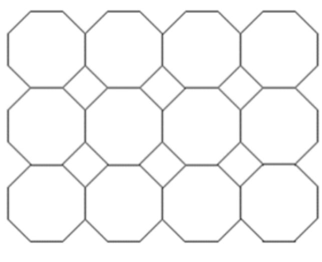


Figure 4.14. 8.8.4 tessellation

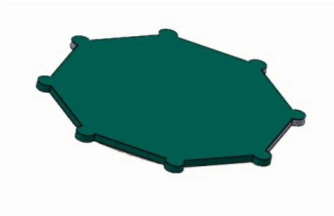


Figure 4.15.Octagonal plate with eight joints

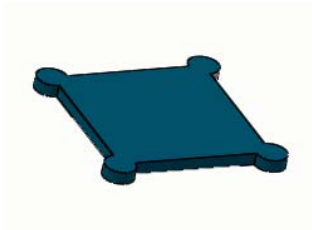


Figure 4.16. Square plate with four joints.

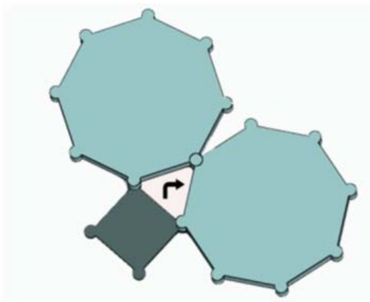


Figure 4.17. Assembling octagonal and square plate

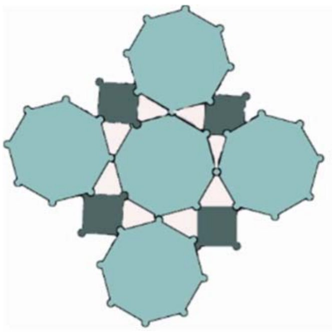


Figure 4.18. Unsuccessful assembly order

In this method, the type of plate, position of the joints and the form of the extra links are determined at the same time. Firstly, again selected tessellation is drawn by considering determined size of the plates. After this, form of the extra links and the position of the joints are obtained by pointing one vertex to the others around.

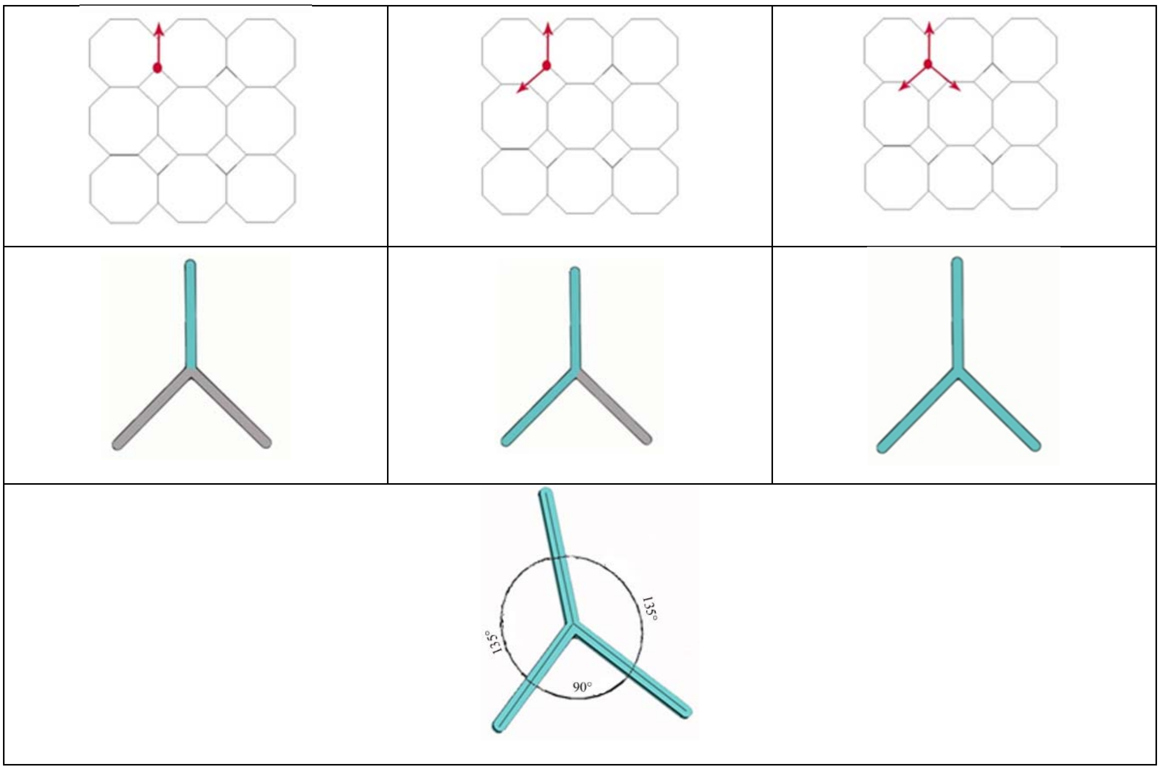


Figure 4.19. Finding position of the joint and form of extra link.

As it shown from the figure, the angles between the members of extra link are 90°, 135° and 135°. Generating parallelogram loops is crucial to reach predictable movement. So, when assembling extra links with plates, the positions of the extra links gain importance.

Figure 4.20 display the one of the assembling order of square and two octagonal plates with an extra link. When two octagonal and one square plates are assembled as seen from the figure 4.20, all extra links move under the plates and RPS reaches closed state without any gap. However, one major drawback of this approach is that when more plate are assembled with this order, all plates do not assemble with each other (Figure 4.21). This is the first assembling order and this order consider to angle of the extra link that match perfectly with each other. On the other hand, the second order considers to be generating a parallelogram. If the second assembling order is used, all of the plates assemble with each other, parallelogram loops generate between them, thus they can move predictable manner; however, there are always overlaps (Figure 4.22).

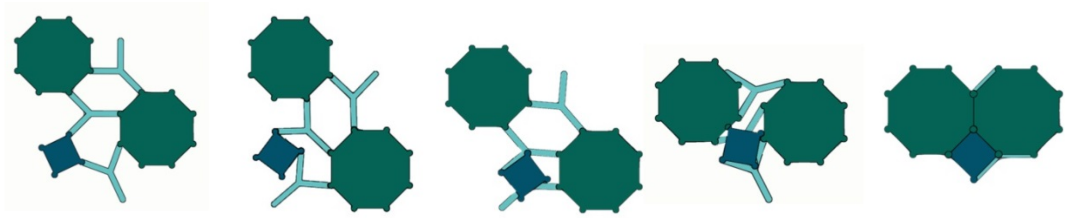


Figure 4.20. Assembly of octagonal and square polygonal plates with extra links

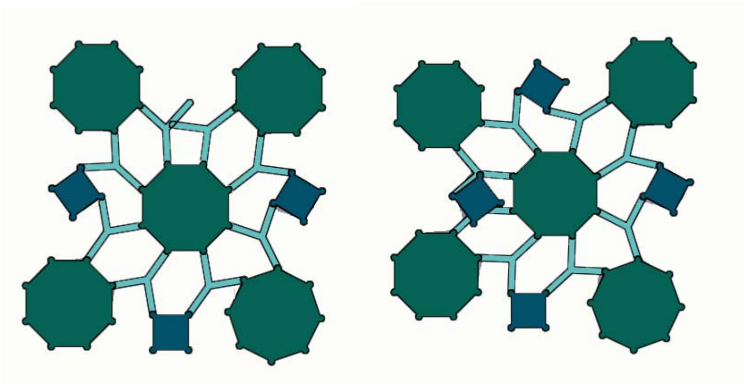


Figure 4.21. Assembly of RPS based 8.8.4 tessellation with extra links by considering first order.

Figure 4.22. Retraction Processes of RPS Based on 8.8.4 Tessellation with Extra Links Considering Second Assembling Order.

The same procedure can applied to the other tessellations that 3 edges meet at one vertex. Below, Figure 4.23-4.26 shows embodiment of the methods for the 12.12.3 tessellation.

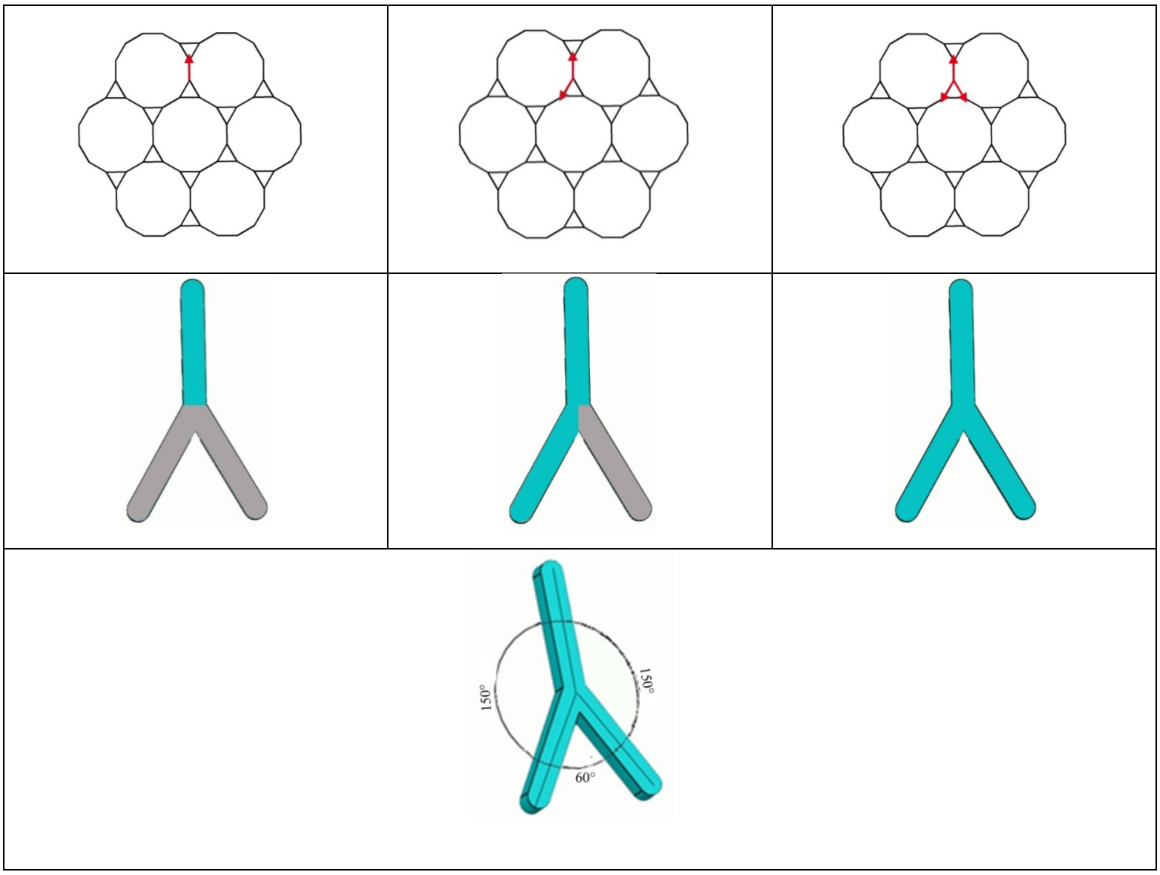


Figure 4.23. Revealing of extra link RPS based on 12.12.3 tessellation

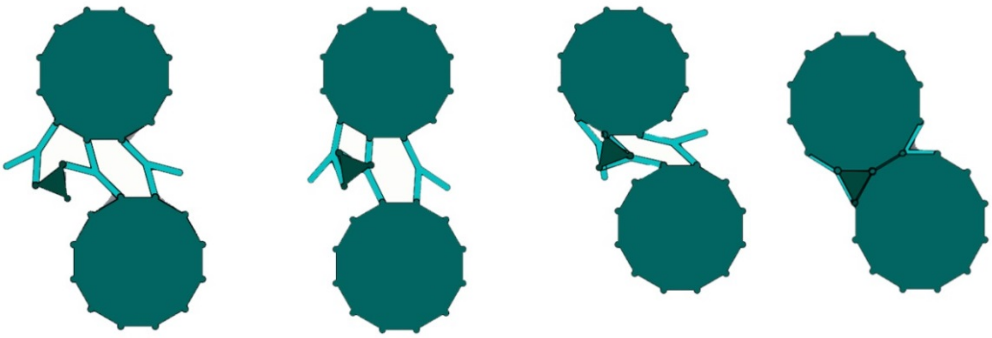


Figure 4.24. Assembling of dodecagon and triangle plate by considering first order.

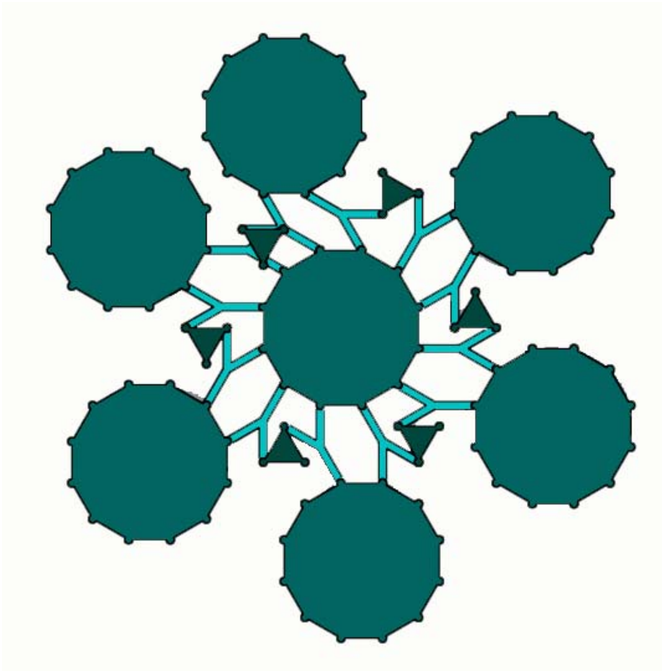


Figure 4.25. RPS based on 12.12.3 tessellation with extra links by considering first order.

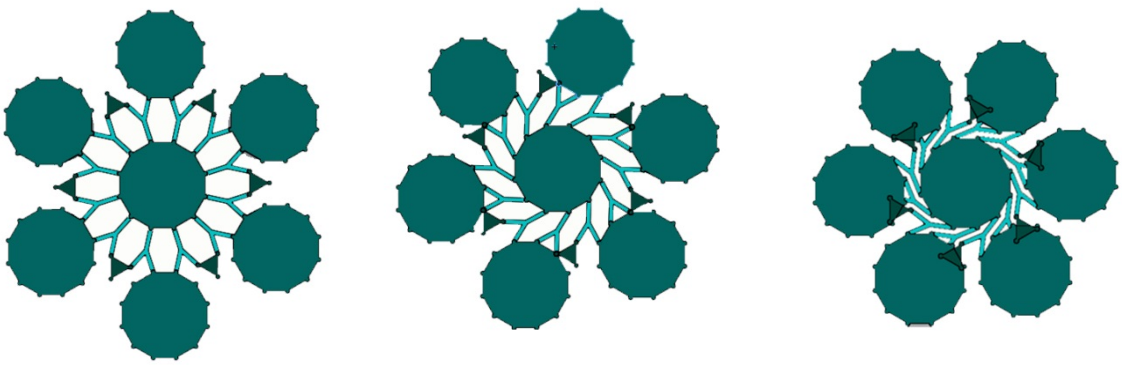


Figure 4.26. Retraction process of RPS based on 12.12.3 tessellation by considering second order

Below, Figure 4.27-4.30 displays the embodiment of the methods for the 12.6.4 tessellation.

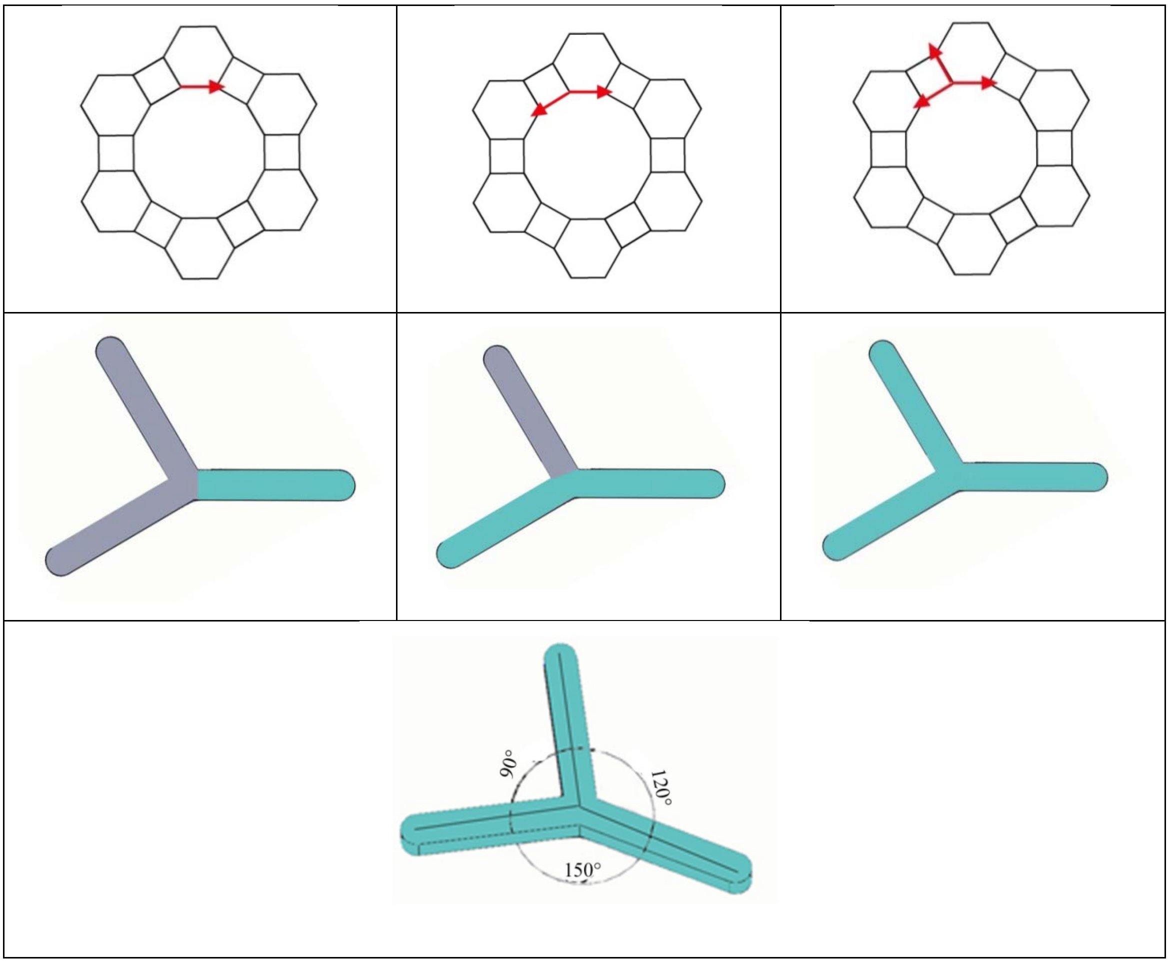


Figure 4.27. Revealing of extra link RPS based on 12.6.4 tessellation

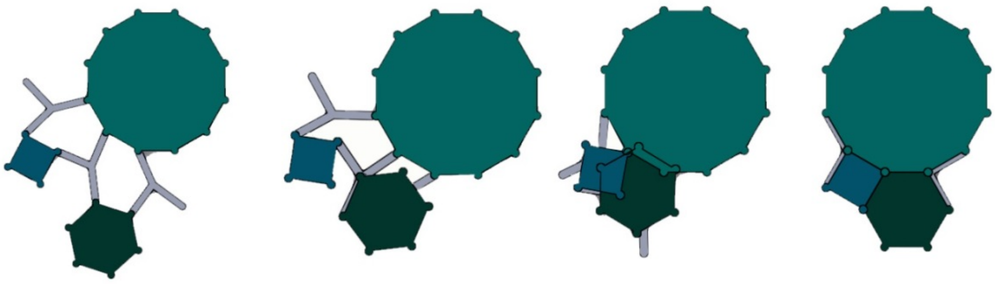


Figure 4.28. Assembly of dodecagon, square and hexagonal polygon based on first order

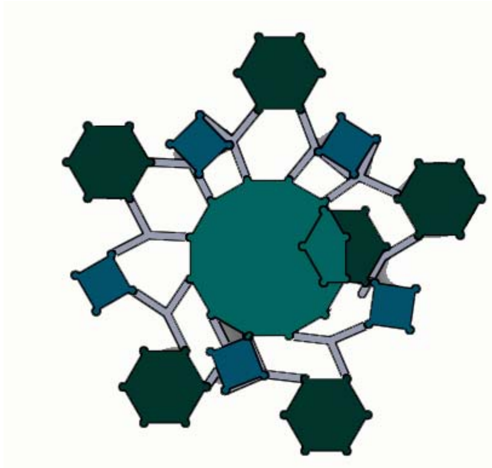


Figure 4.29. RPS based on 12.6.4 tessellation with extra link

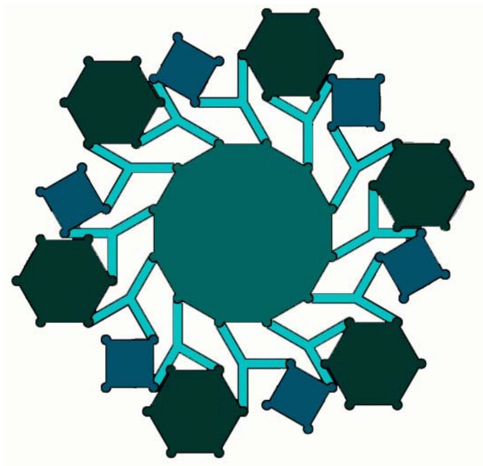


Figure 4.30. Retraction processes of RPS based on 12.6.4 tessellation by considering second order

4.3.2. Methods-2: According to Duality of Tessellation

Like the first method, the second method consists of only one step. By the help of this method type of plate, positions of the joints and form of the extra link are found by the help of duality.

In the first phase dual of tessellation drawn and polygon of dual form found (Figure 4.31 and 4.32).Then, the corner points of the polygonal shape give the positions of the joints (Figure 33).

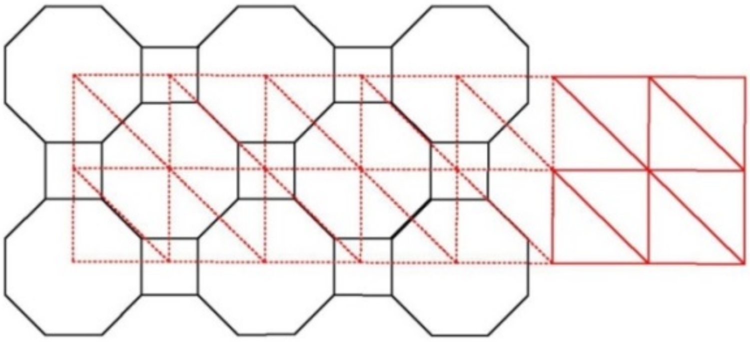


Figure 4.31. Dual form of 8.8.4 tessellation

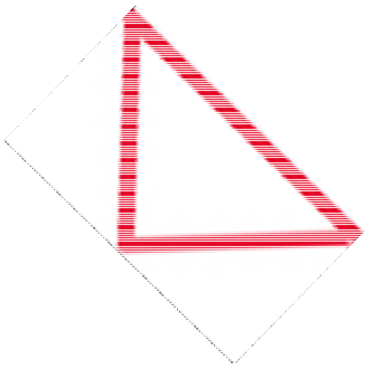


Figure 4.32. The Smallest polygons of dual form

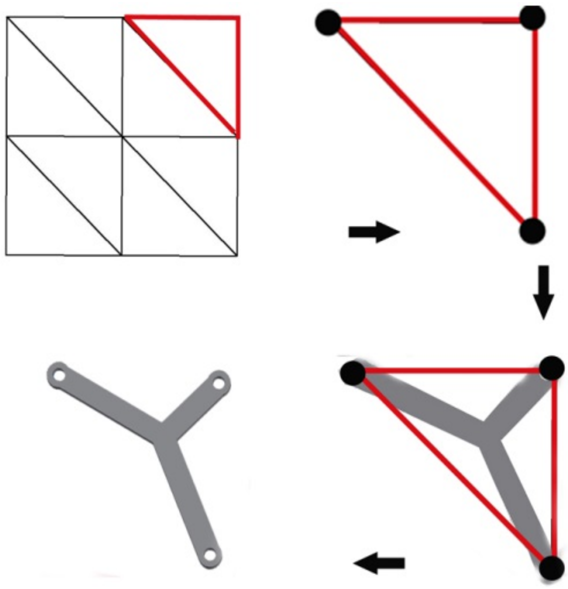


Figure 4.33. Revealing the extra link of RPS based on 8.8.4 tessellation.

There is an important point to find the form of the extra link by considering the joint position. In order to increase the compactness by reducing the change of extra link collision, their forms are selected as shown Figure 4.33. One of the most useful tools is find the midpoint of the polygon and assemble them with the corner point.

As it mentioned above, to generate a parallelogram is very crucial to reach predictable motion. Figure 4.34 displays the assembling of RPS based on 8.8.4 Tessellation with extra link by generating parallelogram loops. It is seen that they can move in a predictable manner, however, there exist gaps between them.

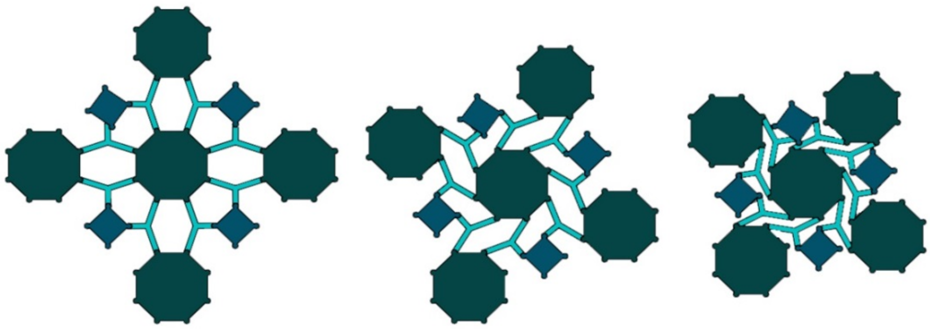


Figure 4.34. Collapsing of RPS based on 8.8.4. tessellation with extra links

The same result can be seen the other tessellations that have 3 edges meet at one vertex. Below, Figure 4.35 shows embodiment of the methods for the 12.12.3 tessellation.

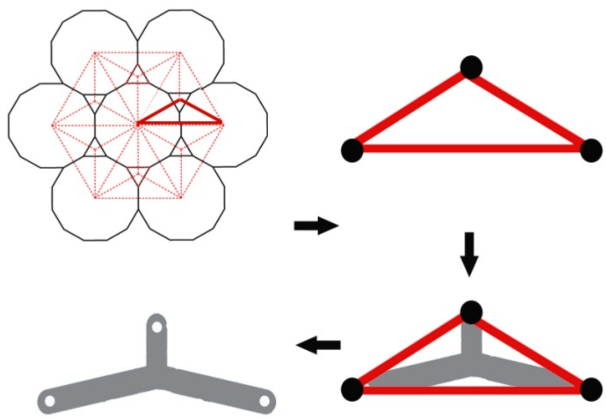


Figure 4.35. Revealing of extra link of RPS based on 12.12.3 tessellation.

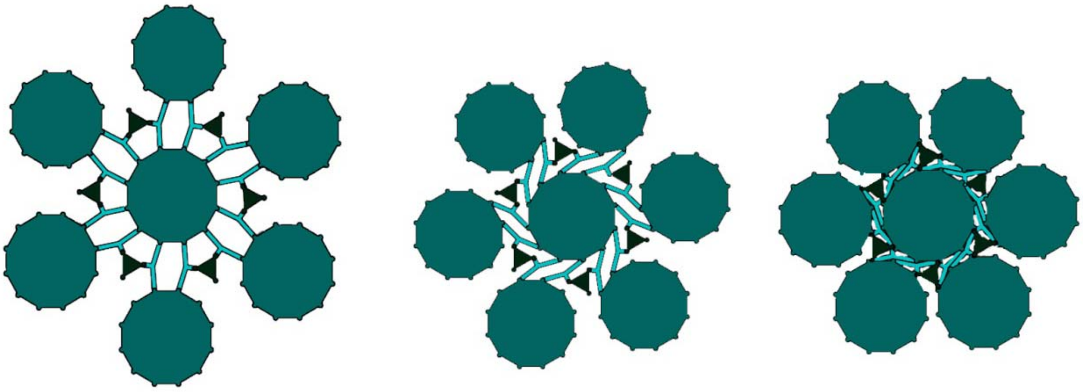


Figure 4.36. Retraction of RPS based on 12.12.3 tessellation with extra link.

Below, Figure 4.37 displays embodiment of the methods for the 12.6.4 tessellation.

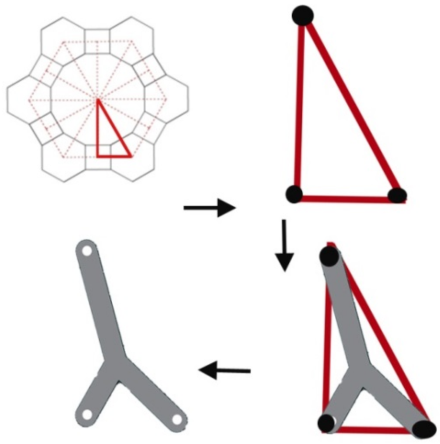


Figure 4.37. Revealing the extra link of RPS based on 12.6.4 tessellation

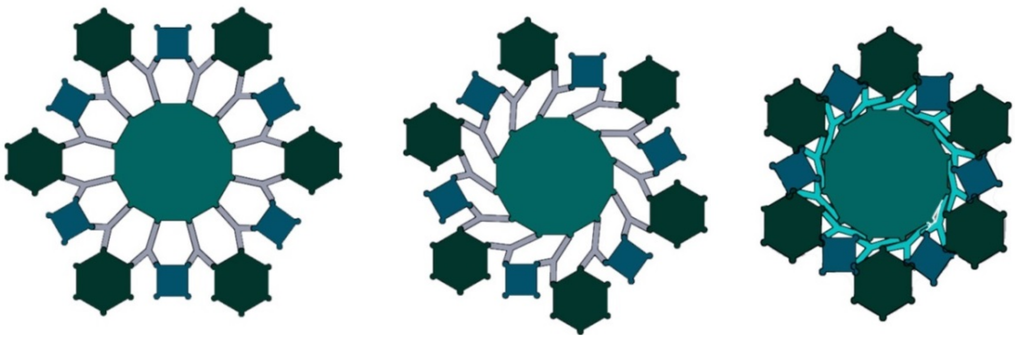


Figure 4.38. Assembling of RPS based on 12.6.4 tessellation with extra link

4.3.3. Special Case: Design of RPS with 6^3 Tessellation

6^3 tessellation is one of the 3 edges meet at one vertex tessellation among the 1–uniform tessellation. As it described above, to design retractable structure with 6^3 tessellation, there need extra link for assemble with the hexagonal plates. On the contrary to other three 3 edges meet at one vertex tessellation (12.12.3, 12.6.4, 8.8.4 tessellation), the form of the extra links are generated exactly same with the both two methods (Figure 4.39-4.44).

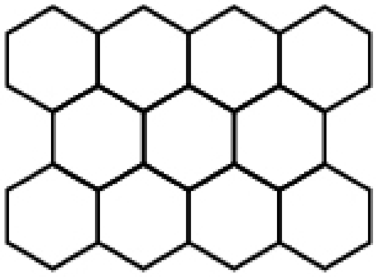


Figure 4.39. 6^3 tessellation

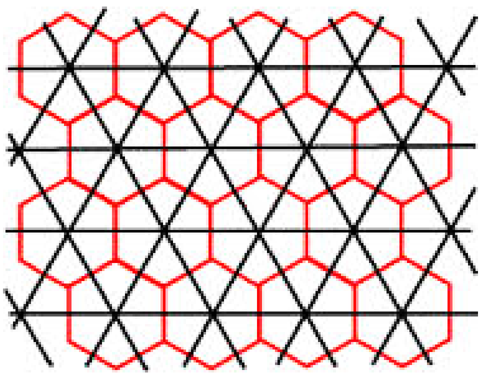


Figure 4.40. Dual of 6^3 tessellation

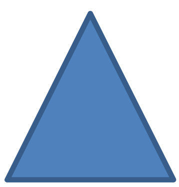


Figure 4.41. The Polygon of dual Form

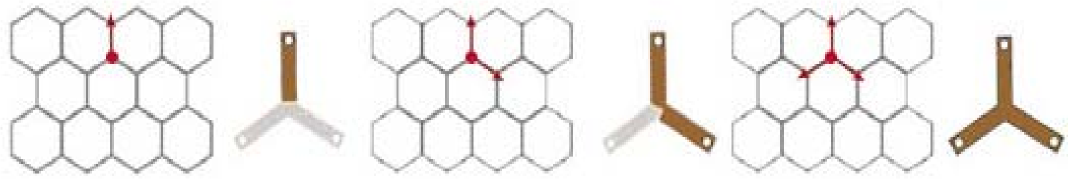


Figure 4.42. Finding the form of extra link according to first method.

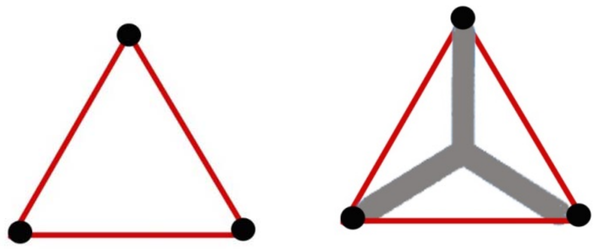


Figure 4.43. Finding the form of extra link according to second method.

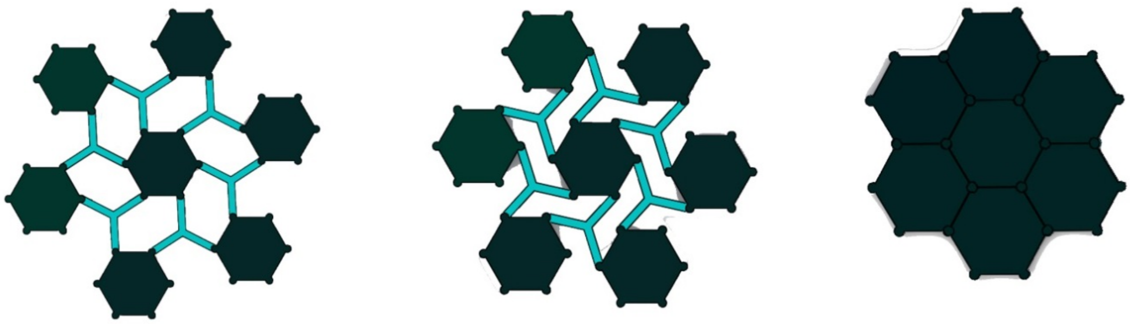


Figure 4.44. RPS based on 6^3 tessellation with extra link according to the Method-1 and Method-2

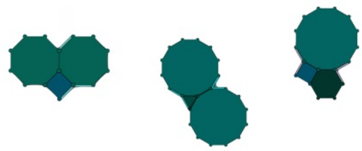
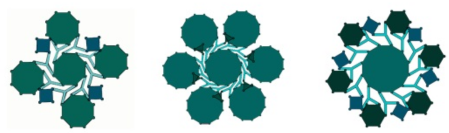
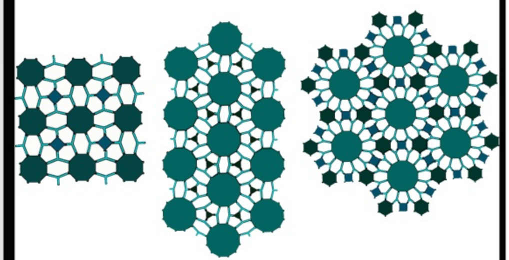
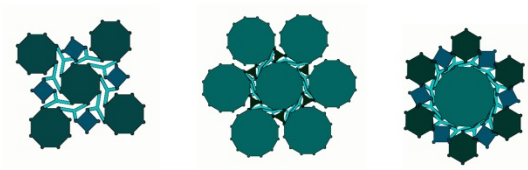
4.3.4. Comparison between Method 1 and Method 2

Both of the two methods focus on finding the type and, position of the joints and form of the extra links. Method "1" uses the vertex configuration of selected tessellation while Method "2" uses the duality of tessellation. Table 4.2 compare the Method 1 and the Method 2 according to their capability of retraction and iteration.

As can be seen from the Table 4.2, if Method 1 is chosen the smallest combination of regular plate with extra link according to tessellation order can be closed without any gaps or overlaps. However, this combination cannot be iterated unrestrictedly. Also, in the closed form of RPS, there is always gaps, and overlaps.

On the contrary to first method, if second method is used, the RPSs can iterate unrestrictedly. In the closed form of RPSs, there are always gaps however, this gap is less than Method 1.

Table 4.2. Comparison of Method 1 and Method 2 according to their capability of contraction and iteration

	Advantage	Disadvantage
Method-1	The smallest assembling combination closed without any gaps or overlaps. 	There is always gaps  Do not iterate 
Method-2	Assembling order can iterate 	There is always gaps 

As it seen that; regardless of the selected methods, there will be space between the plates and extra link. However, knowing size of gap between the plates has an importance for the design processes of RPS.

If method two is selected, the size of the gap can be found before assembling retractable plate with extra link. To find the size of gap; the polygon edge extend by drawing a straight line. Then, the angle between the line and neighbouring plate edges is equal to the angle of gap between the retractable plates in the closed form.

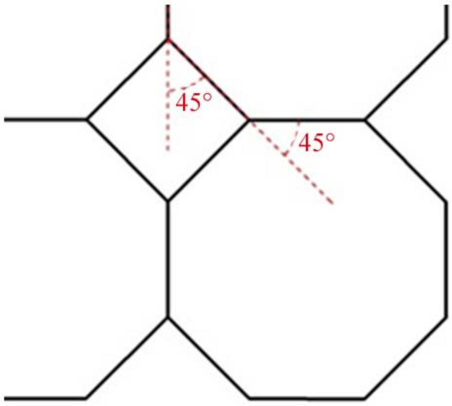


Figure 4.45. Angle between the regular polygons

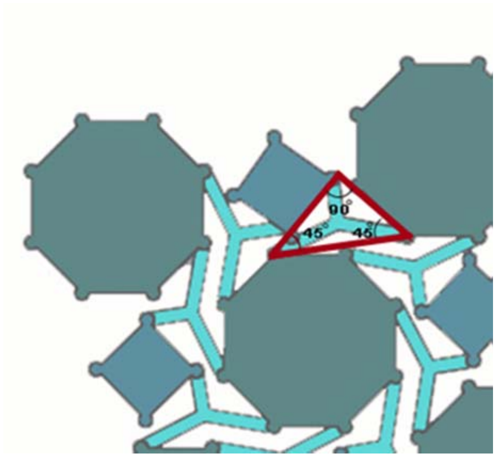


Figure 4.46. Angle between the regular plates

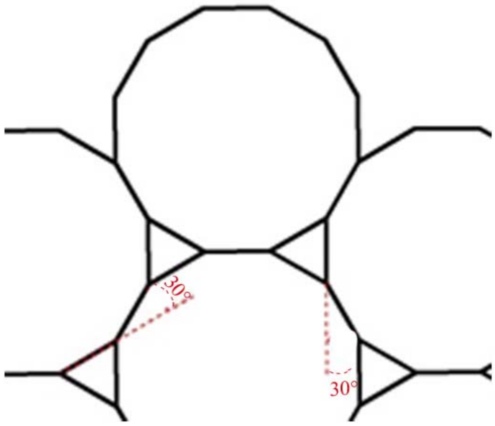


Figure 4.47. Angle between the regular polygon

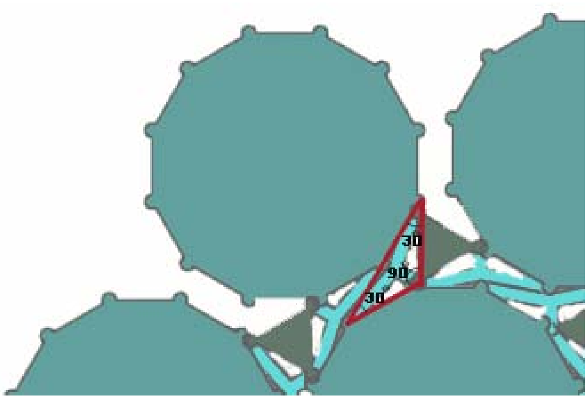


Figure 4.48. Angle between the regular plates

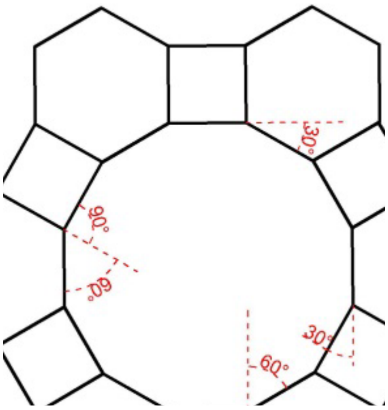


Figure 4.49.Angle between the regular polygon

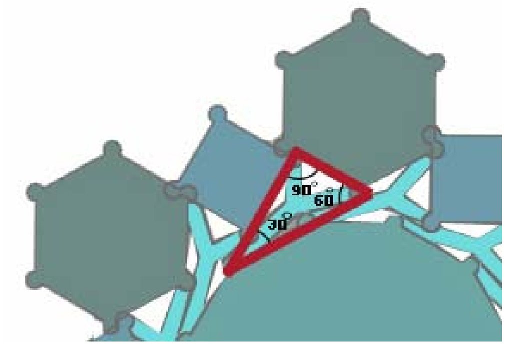


Figure 4.50. Angle between the regular plates

4.3.5. Vertex Translation Method for 3 Edges Meet at One Vertex Tessellation

As explained on the previous pages, it can be reach RPS based on 3 edges meets at one vertex tessellation by adding extra links; however, both of the methods that described above fails to give suitable shapes of extra links to reach RPSs that are not allowed to gaps in any configurations.

The empty space formed after the motion can be eliminated if the assembly of two octagons and a rectangle are modified in such a way that the second condition holds, in other words the angle between two neighbour plates in Figure 4.45 will be zero. To achieve this modification, aforementioned vertex translation operation can be used.

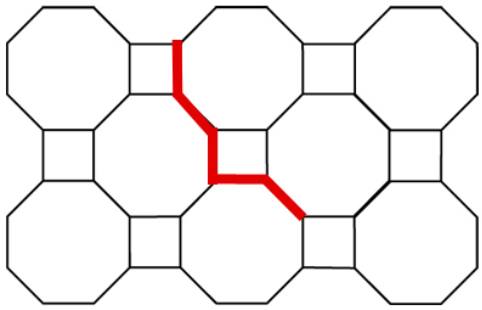


Figure 4.51. The relation between edges of 8.8.4 tessellation

To reach a continuous line between the neighbouring edges firstly, the point of "1", "3", "9", "5" translate to vertex 1', 3 ', 9' and 5'. It should be noted that to create a continuous line between the neighbouring edges of polygons, minimum three neighbouring vertices should be stay on a continuous line.

In Figure 4.52 the vertex of 2, 1' and 10 generate a continuous line. Additionally, the "9" vertex is common vertex of octagon and square. Thus, when the "9" vertex is translate the square polygon needs to rotate as a result the vertex of "8" and "7" should rotate to 8'and 7 ' (Figure 4.53). After this operation, the new forms of polygons become can be seen in the Figure 4.54. After this operation, the shape of the octagonal plate is changed and the tessellation is not regular anymore. On the other hand, the order tessellation is still same. In other words, RPS becomes based on irregular 8.8.4 tessellation. As shown Figure 4.55, new tessellation generates continuous lines between the neighbouring polygons.

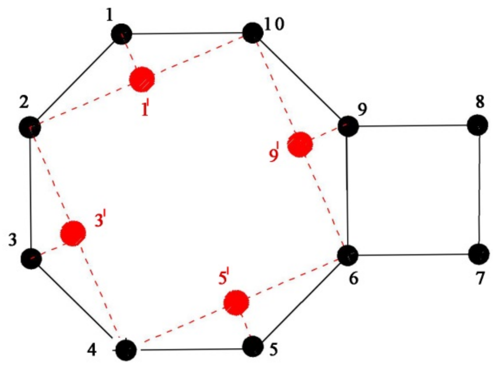


Figure 4.52. First step of vertex translation

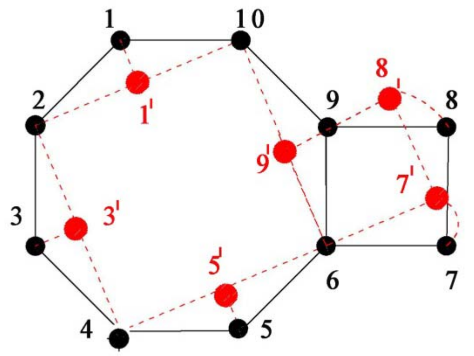


Figure 4.53. Second step of vertex translation

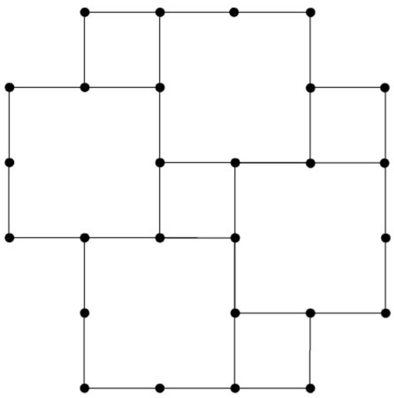


Figure 4.54. After vertex translation

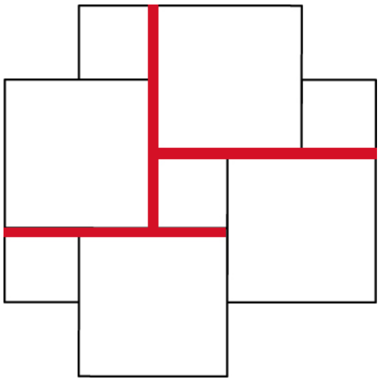


Figure 4.55. Neighbouring polygons generate straight line

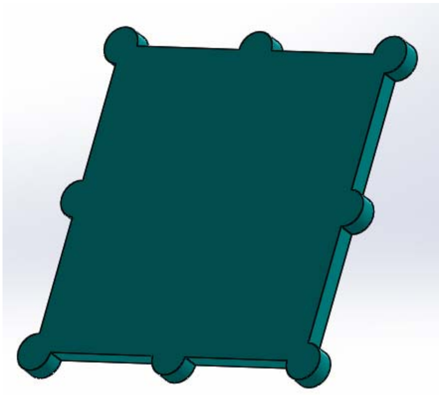


Figure 4.56.Square plate with eight hinges

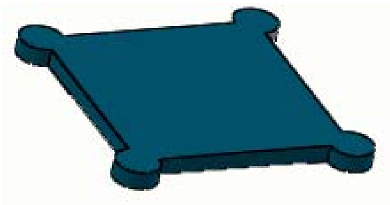


Figure 4.57. Square plate with four hinges

Again three edges meet at one vertex on the new form of tessellation, thus, the RPS based on irregular 8.8.4 tessellation do not allowed any movement and needs an extra links.

The form of the extra links is found by using duality of the tessellation. Firstly the duality of new form of tessellation is drawn (Figure 4.63). Then smallest polygon of dual form is found. The number of edge of this polygon gives the number of elements of the link. Also, the corner point of the polygon gives the joint position of the extra link.

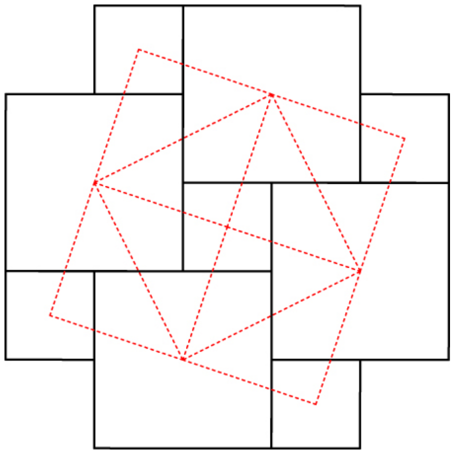


Figure 4.58. Dual of irregular 8.4.8 tessellation

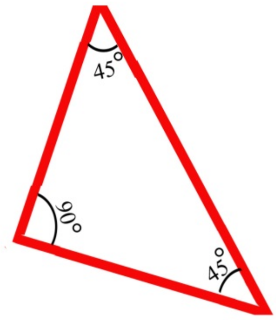


Figure 4.59. Smallest polygon of dual of irregular 8.4.8 tessellation

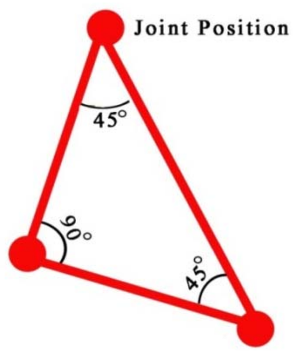


Figure 4.60. Angle between the joint position of extra links

The duality gives the position of the revolute (R) joint; however, it is not enough to determined extra link geometrically. As it mentioned before, in order to increase the compactness by reducing the change of the extra link, the form should be thinner. To define the form of extra link, the additional design parameters that will fully determine the extra link need to be known. The additional design parameter is distances between the revolute (R) joint.

The distance between joint positions of the extra link is depending on the size of plates. As shown Figure 4.61-62, assume that if distance between the joint positions of the plates 50, the shortest length of the members dimension will be 50.

Given the shortest length dimension and the angle between joint position of the extra link, the form of the extra links are determined.

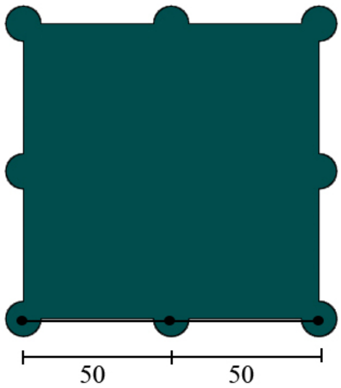


Figure 4.61. Joint position distance

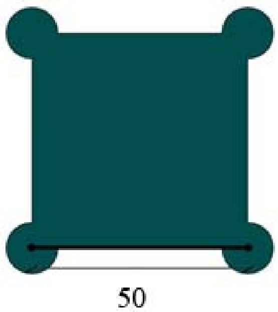


Figure 4.62. Joint position distance

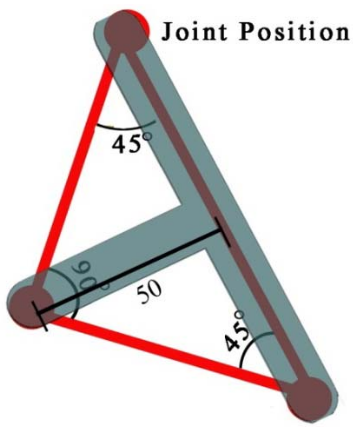


Figure 4.63. Design parameters of extra link

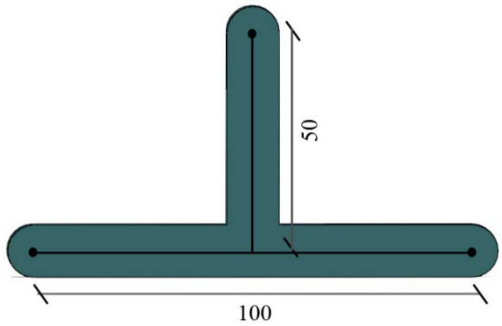


Figure 4.64. Form of extra link for RPS design with irregular 8.4.8 tessellation

To investigate the retraction and expand behaviour, RPS is modelling and simulated by using computer simulation environment. As seen in Figure 4.65, the RPS can construct without any gaps or overlaps and iterated in many times.

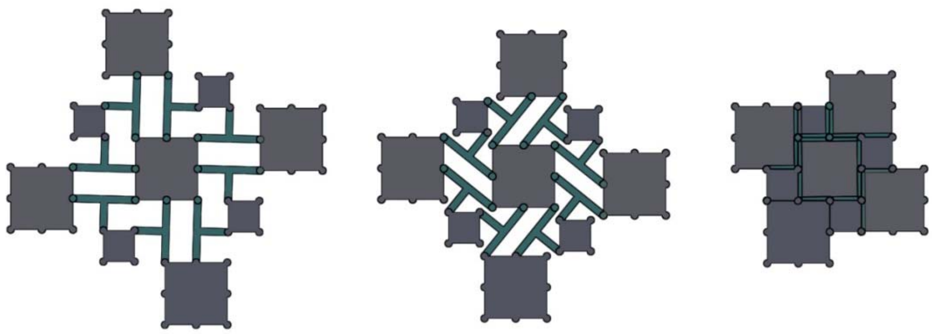


Figure 4.65. Retraction processes of 8.8.4 tessellation after vertex translation

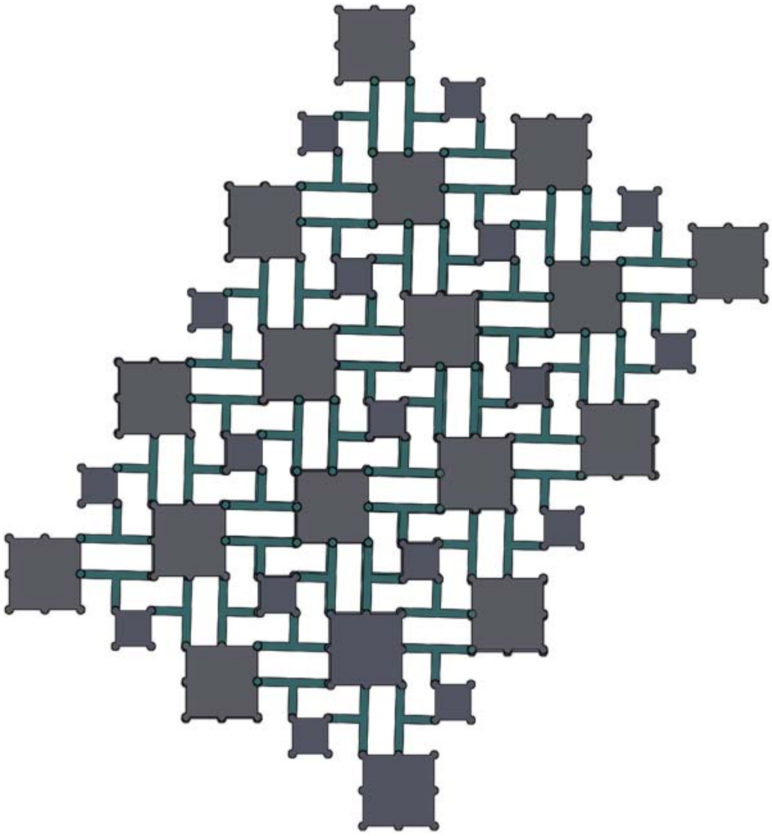


Figure 4.66. Iteration of 8.8.4 tessellation after vertex translation

Aforementioned examples all distance between the joint positions are equal. In this stage according to designer decision the position of the vertices that stay in continues line can be translated on that line. As a result, the size of the plate can be change.

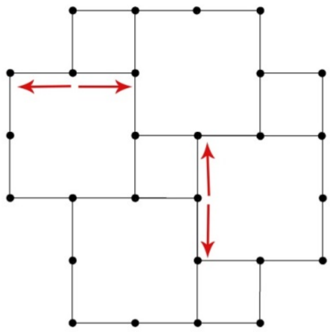


Figure 4.67. Translation of the vertex

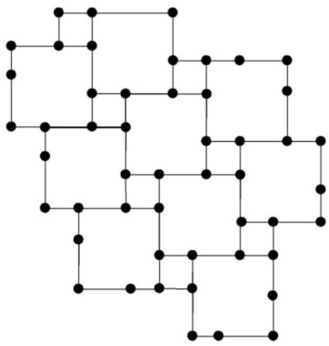


Figure 4.68. Result of the translation

Assume, after translation the position of the vertex, one of the distance between the joint positions is 70 and the other one is 100. By using this design parameter and steps of the methods which is mentioned above, the forms of extra link is found.

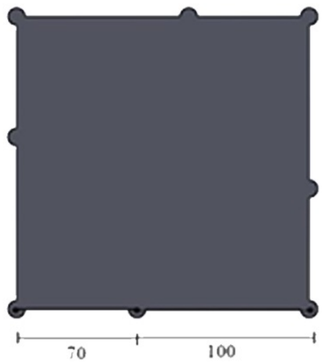


Figure 4.69. Distance between the joint positions of octagonal plate

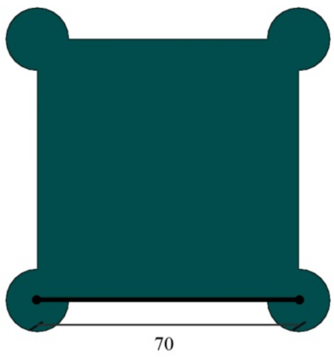


Figure 4.70. Distance between the joint position tetragonal plate

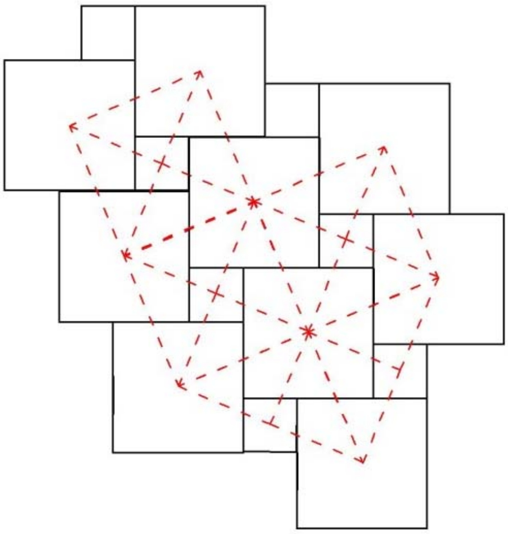


Figure 4.71. The dual form

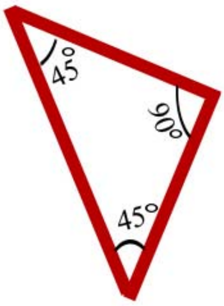


Figure 4.72. The smallest polygon of dual form

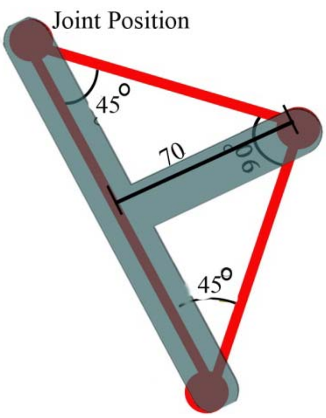


Figure 4.73. Joint position and dimension of extra link

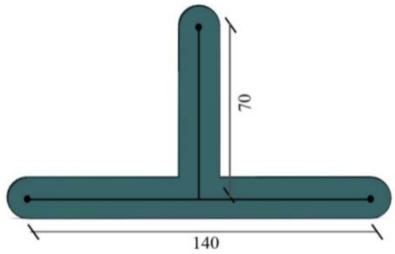


Figure 4.74. Extra link

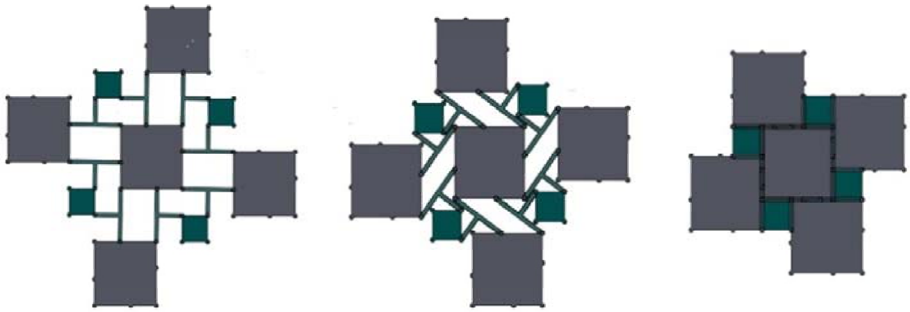


Figure 4.75. Retraction of RPS with design irregular 8.4.8 Tessellation

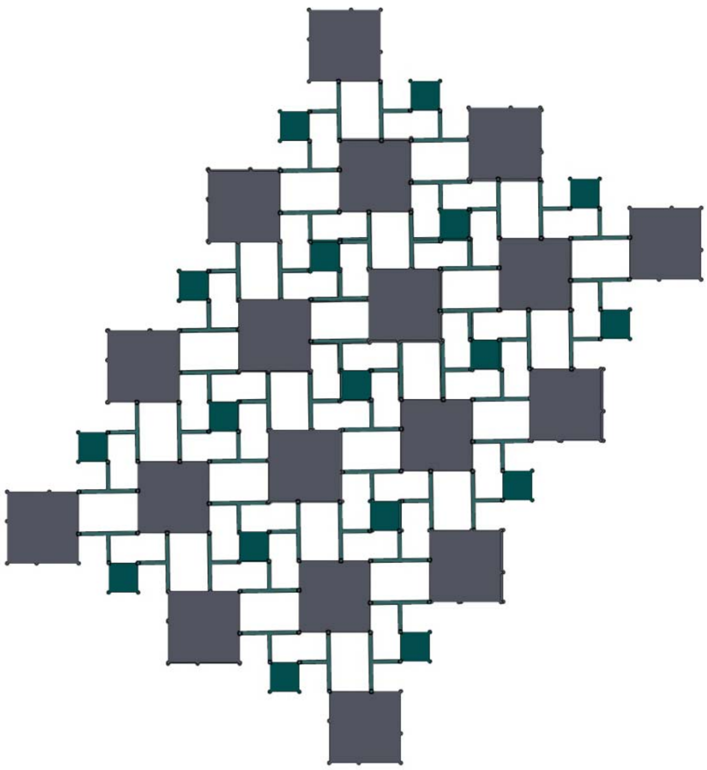


Figure 4.76. Iteration of RPS with design irregular 8.4.8 tessellation

The same procedure can be applied to other 3 edges meet at one vertex irregular tessellation.

For 12.12.3 Tessellation;

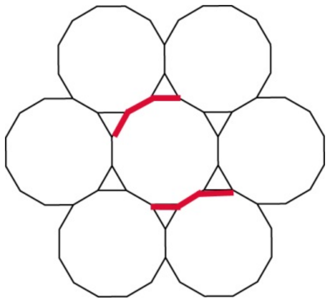


Figure 4.77. The Relation between neighbouring edges of 8.4.8 tessellation

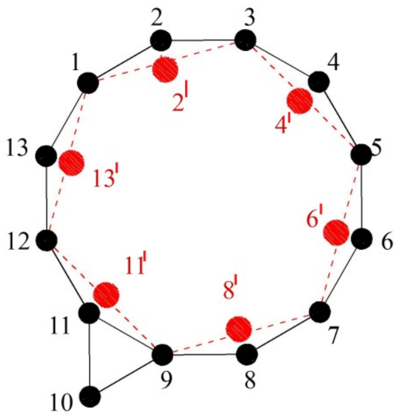


Figure 4.78. The first step of vertex translation

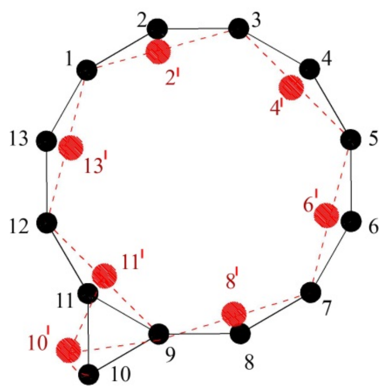


Figure 4.79. The Second step of vertex translation

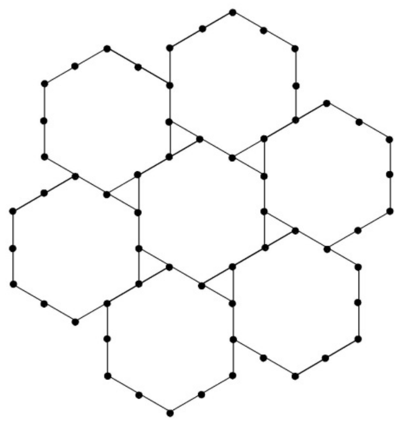


Figure 4.80. After vertex translation

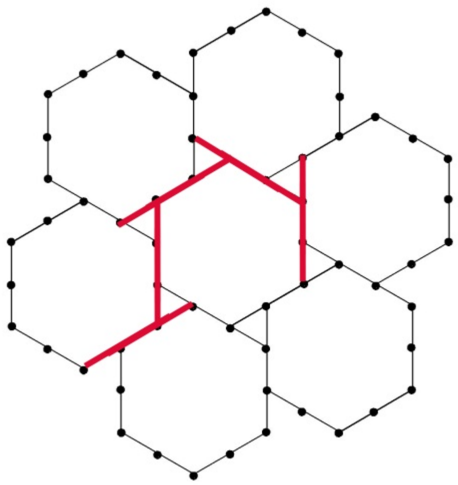


Figure 4.81. Neighbouring polygons generate straight line

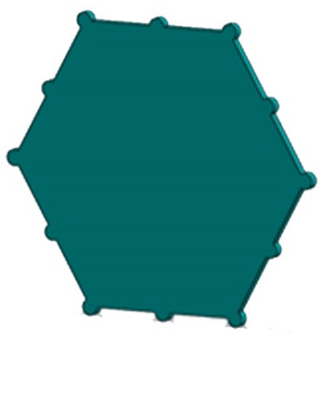


Figure 4.82. Dodecagon plates with twelve hinges

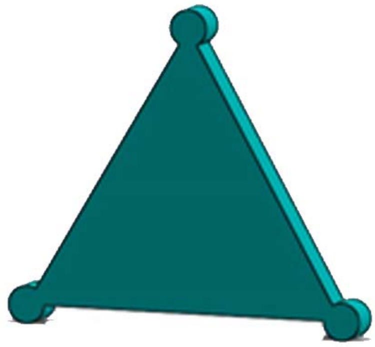


Figure 4.83. Triangle plates with three hinges

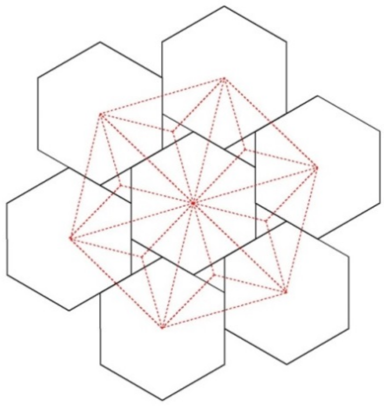


Figure 4.84. The Dual of irregular 12.12.3 tessellation

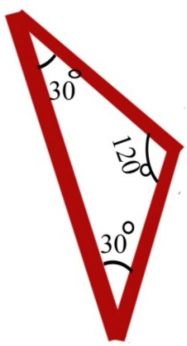


Figure 4.85. The smallest polygon of dual form

To change the position of the joint in other words dimension of the plate edge, the same procedure use below.

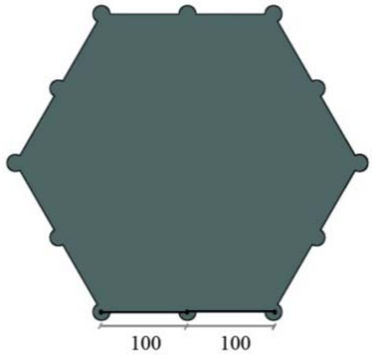


Figure.4.86. Distance between the joint positions of dodecagon plate

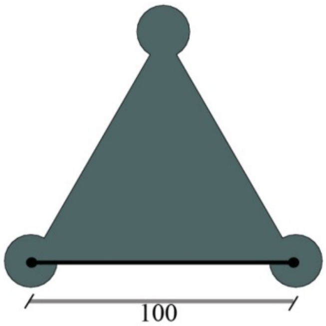


Figure.4.87. Distance between the joint positions of triangular plate

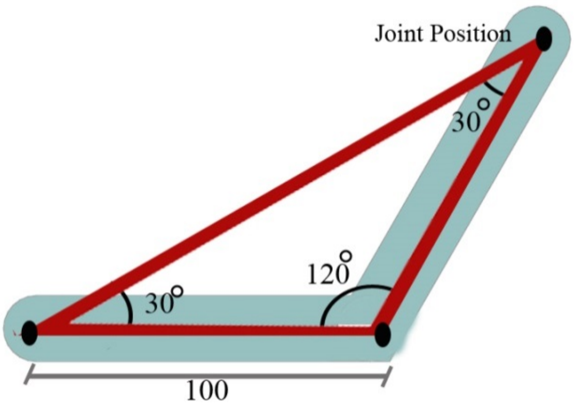


Figure 4.88. Joint positions

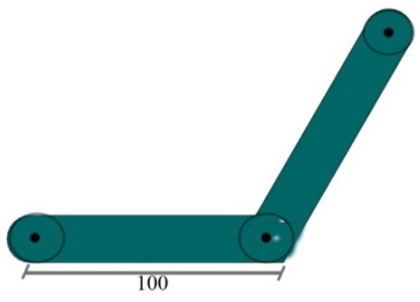


Figure 4.89. Form of extra link

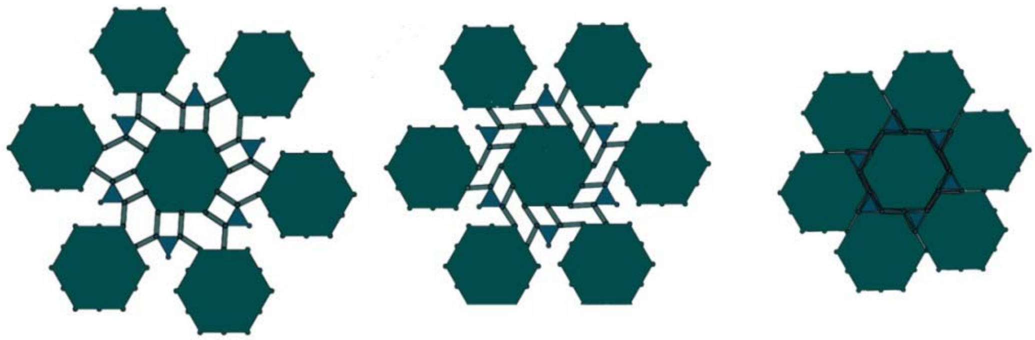


Figure 4.90. Retraction processes of RPS based on 12.12.3 tessellation after vertex translation

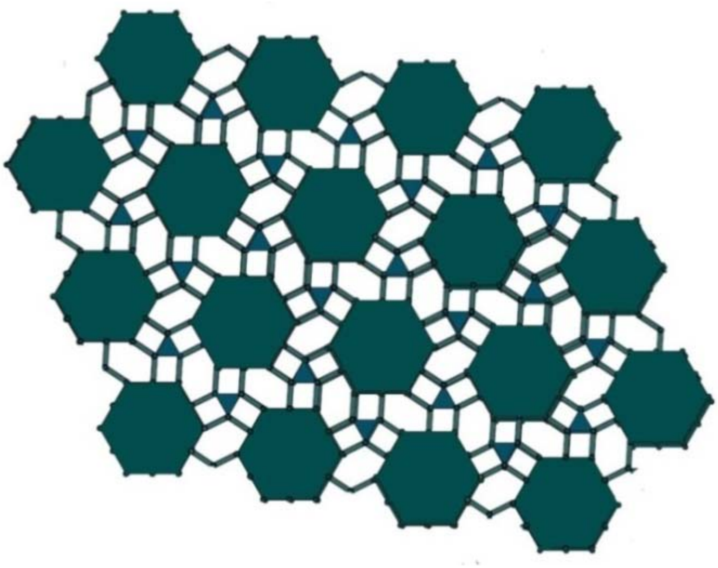


Figure 4.91. Iteration of RPS based on 12.12.3 tessellation after vertex translation

For 12.6.4 Tessellation;

- *Vertex Translation;*

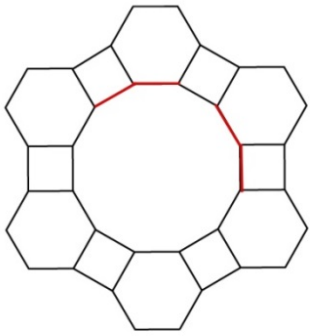


Figure 4.92. The relation between edges of 12.6.4 tessellation

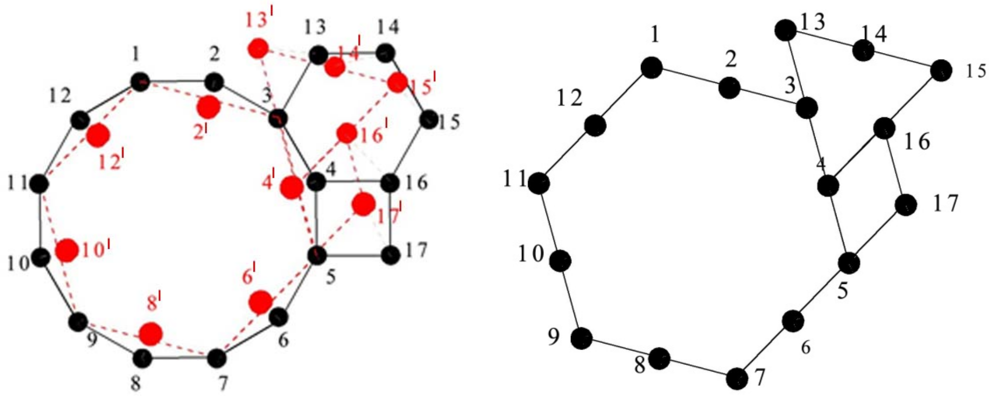


Figure 4.93. Vertex translation

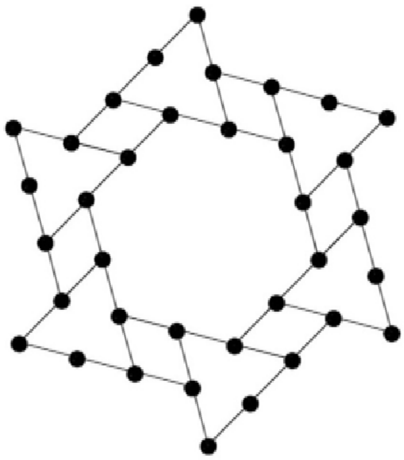


Figure 4.94. After vertex translation

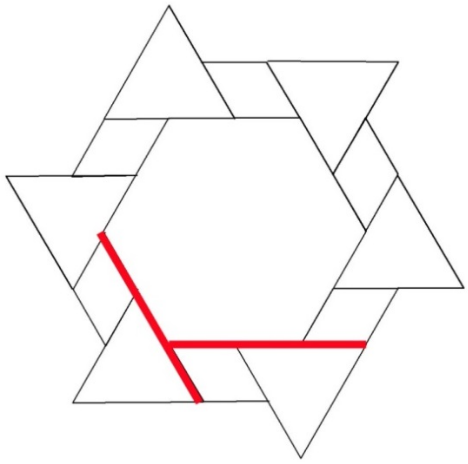


Figure 4.95. Neighbouring polygons generate straight line

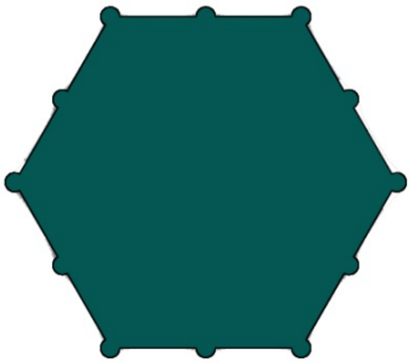


Figure 4.96. Dodecagon plates with twelve hinges

Figure 4.97. Triangle plates with three hinges

- *Revealing the Extra Link ;*

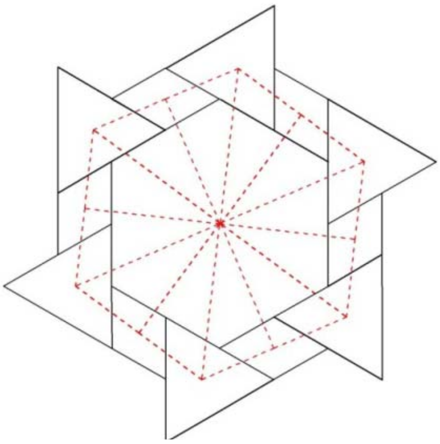


Figure 4.98. The Dual form

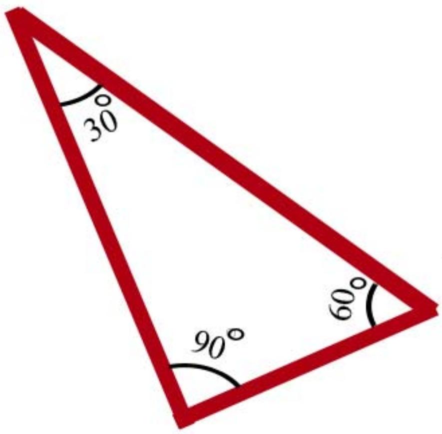


Figure 4.99. The smallest polygon of dual form

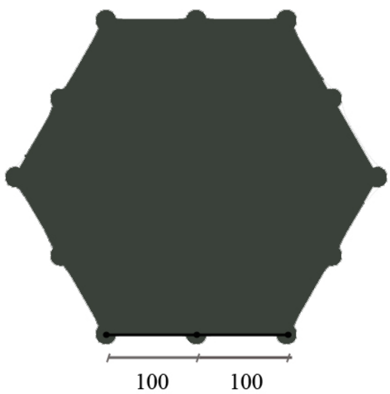


Figure 4.100. Distance between the joint positions

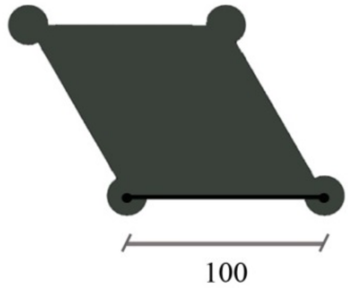


Figure 4.101. Distance between the joint positions

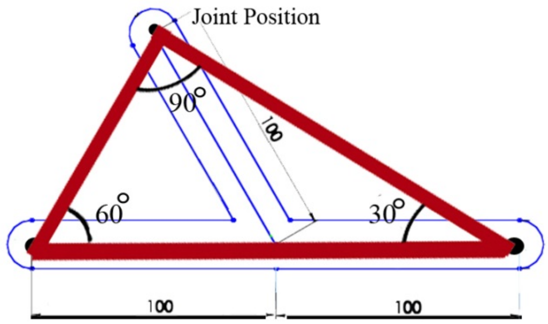


Figure 4.102. Joint position of extra link

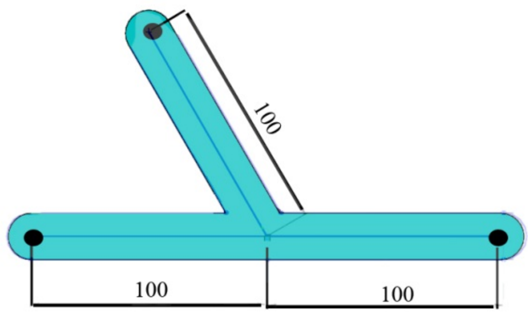


Figure 4.103. Form of extra link

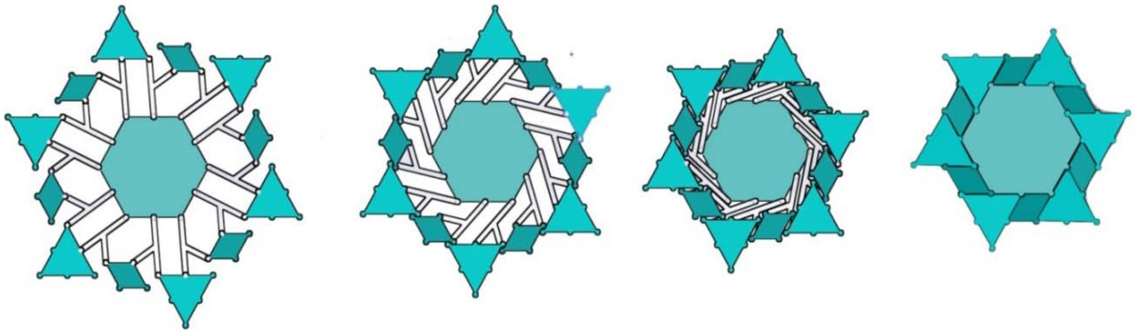


Figure 4.104. Retraction processes of RPS based on 12.6.4 tessellation after vertex translation

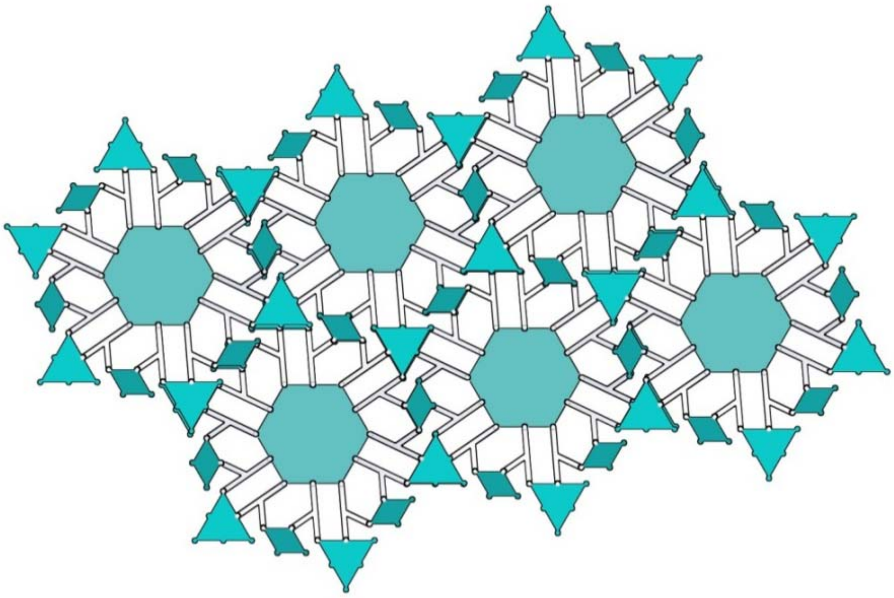


Figure 4.105. Iteration of RPS based on 12.6.4 tessellation after vertex translation

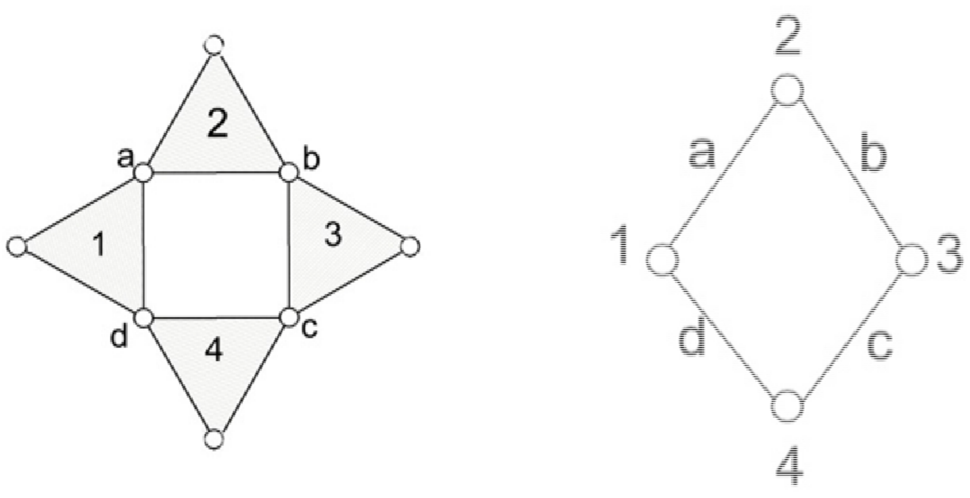
4.4. Design of Single DoF RPSs where Five and Six Edges Meet at One Vertex

Among the eleven 1-uniform tessellation, only four tessellations (4^4, 3^6, $3^4.6$ and 3.6.3.6) satisfy the both first and second conditions at the same time. However, in four of them, just two tessellations (4^4, 3.6.3.6) expand and retract in a predictable manner because of having one degree of freedom. As it is mentioned before, if RPS based on four edges meets at one vertex, they generate parallelogram loops and have single DoF. However, when the numbers of edges around the one vertex increase, the loop form between the plates become pentagonal, hexagonal, etc., as a result, the degrees of freedom of RPS became Multi-DoF (Table 1).

For this reason, 3^6, $3^4.6$ tessellations satisfy the two conditions at the same time, therefore the RPS based on 3^6 and $3^4.6$ tessellations do not expand and contract in a predictable manner. This part of the chapter develops a method for RPS that satisfies the both first and second conditions, but having Multi-DoF. Before mention about the method, a similarity between graph representation and dual tessellation will be discusses in follow.

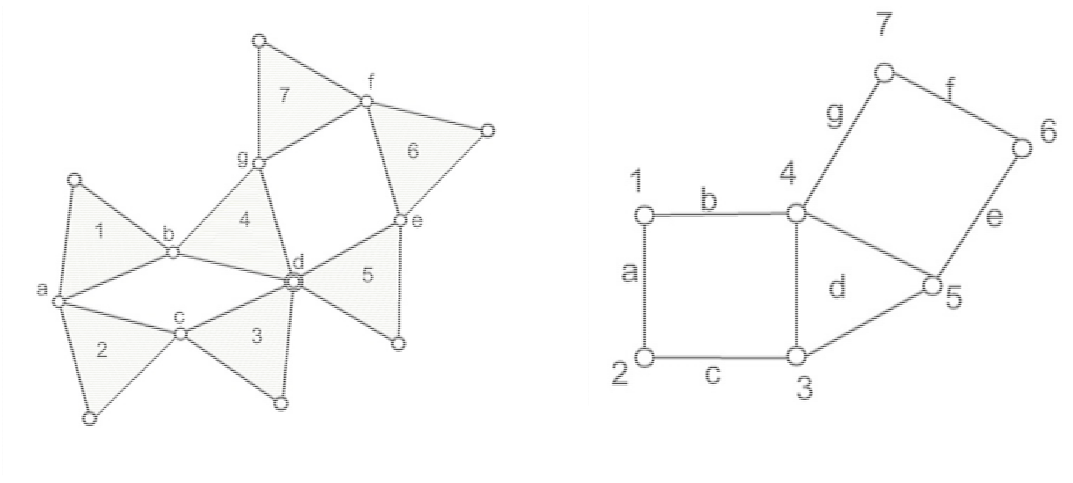
4.4.1 Similarity Between Graph Representation and Dual Tessellation

In many areas from engineering to architecture, graph theory is used for finding communities in networks where it is needed to detect hierarchies of substructures like as mechanism science. In mechanism science generally graph theory is used to employ systematic enumeration, development of computer-aided kinematics and dynamic analysis or systematic classification of mechanisms (Tsai 2000; Feng and Liu 2013). In a graph representation vertices denote links or plates while edges denote joints of a mechanism. In a structural representation, every plate of a RPS is denoted by a polygon whose vertices represent the joints. Figure 4.106 displays structural and graph representations of a four bar planar mechanism that consists of four plates. It is seen that graph representation of the mechanism with four plates consists four edges. In graph representation a loop with four edges represents a four bar mechanism, likewise a loop with five edges represents a five far mechanism. If three plates numbered 3,4,5 are assembled with 2R joint “d” as in Figure 4.107 a, that joint is displayed with a triangle instead of a line.



(a) (b)

Figure 4.106. a) Structural representation, b) Graph representation



(a) (b)

Figure 4.107. a) Structural representation, b) Graph representation

After studying kinematic representation of Retractable Plate Structure and their based tessellation the author realize that the dual of a base tessellation is exactly the same with the graph representation of an expanded RPS. Thus, structural representation of the RPS can be easily drawn from the dual of the base tessellation. This exact similarity is shown in Figures 4.113, 4.114, 4.115, and 4.116 for (4^4), (3.6.3.6), (3^4.6), (3^6) tessellations. In these figures graph representations are drawn from the structural representations and duals are drawn from the tessellations with red colour. If Figure 4.108 and 4.109 are analysed carefully, it can be seen that duals of (4^4), (3.6.3.6) tessellations are composed of tetragonal polygons. If the vertices are numbered and the edges are entitled on a dual, a graph of the RPS can be acquired and the structural representation of the RPS can be easily drawn from that graph. Thus, there is not any necessary to modelling and assembling plate with each other anymore.

Additionally, every vertex represents a plate or link in graph representation. The type of the plate can be determined by counting the number of edges around that vertex. If the six edges meet at a vertex that means that vertex represents a hexagonal plate. Moreover, since an edge on a graph represents a joint between two plates, a loop with four edges represent a four bar mechanism and a loop with five edges represents a multi DoF five bar mechanism. .

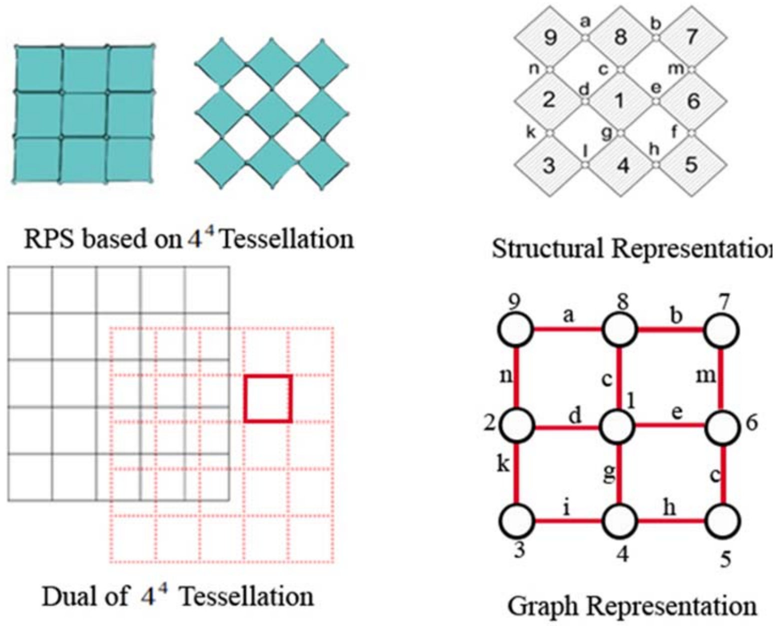


Figure 4.108. Similarity between graph representation of RPS and dual of 4^4Tessellation

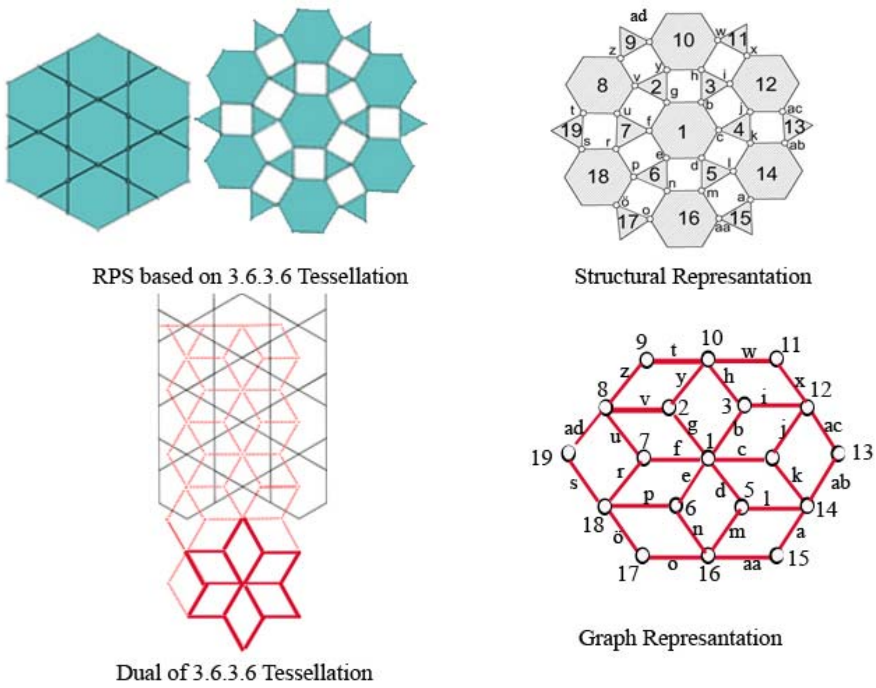


Figure 4.109. Similarity between graph representation of RPS and dual of 3.6.3.6 tessellation

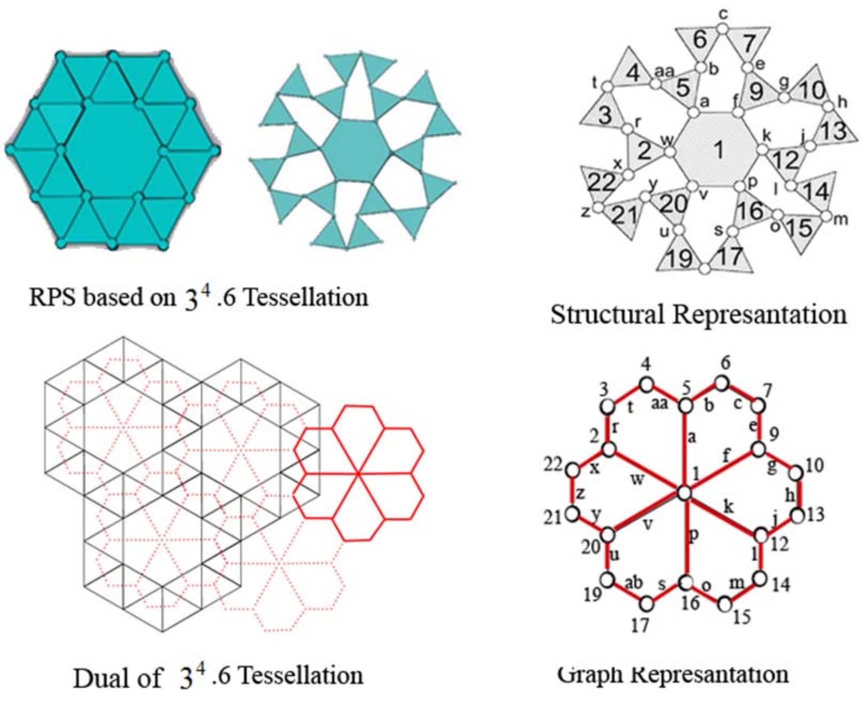


Figure 4.110. Similarity between graph representation of RPS and dual of $3^4.6$ tessellation

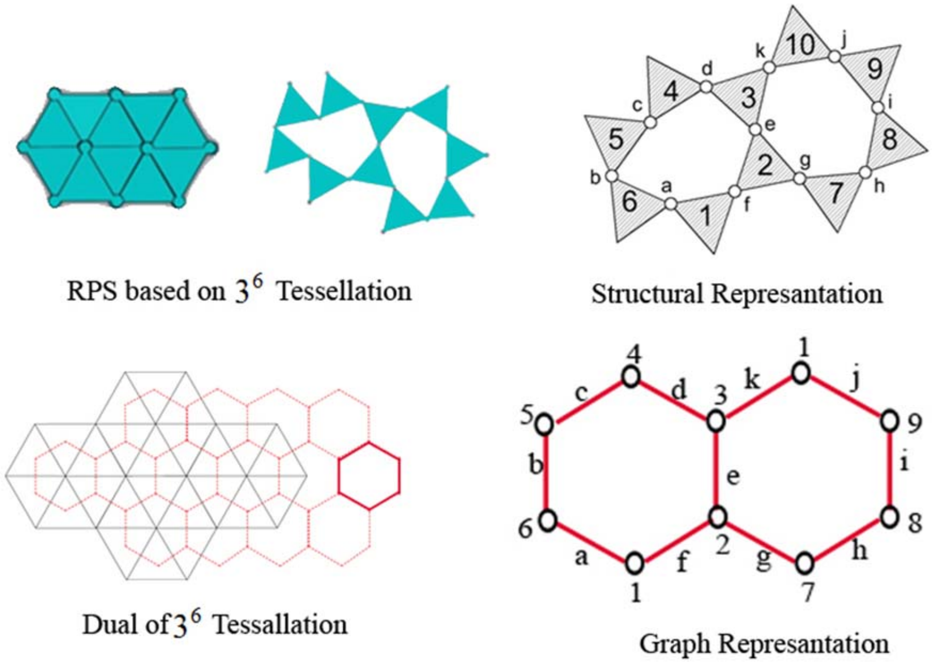


Figure 4.111. Similarity between graph representation of RPS and dual of 3^6 tessellation

According to the results a theorem can be developed from the similarities between duality of the tessellation and the graph representation of the RPS;

Theorem 1: *The dual of tessellation reveals the graph representation of RPS based on that tessellation.*

5.4.2. A Method for Systematic Conversion

$3^4.6$ and 3^6 tessellations satisfy both conditions but RPS based on these tessellations have an unpredictable movement because of their multi DoF nature. In this section, a systematic conversion method is presented in order to reach a single DoF RPS by using similarity between the duality of tessellations and the graph representation of the expanded RPS.

As it understood from the Figure 4.108-4.109 if the form of the dual of selected tessellation has obtain four sides, the RPS will obtain parallelogram loops and due to special geometric condition, it's degrease of freedom will be one. Thus, main purpose of this method is to generate tetragonal polygons on the dual of tessellation. In the light of this, tetragonal loops will be generated by either adding extra edge or point into the graph representation. As it is mentioned before an edge refers to a joint and a point refers to a plate. Indeed, this method will reveal a guide for how the plates should be assembled with each other, is there needing any extra joint or extra link, also if extra link is needed, determined the type of extra link to convert the multi-DoF to single DoF. The method will be explained through the creation of a single DoF RPS based on $3^4.6$ tessellation.

Let's mention the method as an example of RPS based on $3^4.6$ tessellation;

Step 1: *Acquiring the dual of the base tessellation*

The purpose of the first step is to determine whether the RPS will be single DoF or multi DoF with based on selected tessellation. As a first step the dual of $3^4.6$ tessellation is drawn. Black lines show $3^4.6$ tessellation while the red lines show it's dual (Figure 4.112). It can be seen that the dual of $3^4.6$ tessellation consists five-sided polygons. Thus, if the plates are assembled one by one with revolute joints, the graph representation of an expanded RPS based on $3^4.6$ tessellation will consist of five-sided polygons that represent multi DoF loops. As a result, it is revealed that RPS based on $3^4.6$ tessellation will be multi DoF.

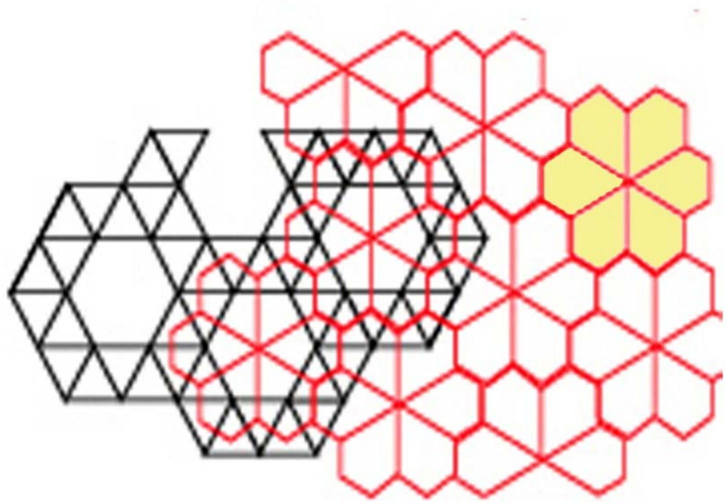


Figure 4.112. $3^4.6$ Tessellation and its dual form

Step 2: *Modification of the dual of base tessellation*

As a second step, extra edges are added to the dual of tessellation in order to generate tetragonal polygons (Figure 4.113). As it seen that there is not needing any extra point. It means that this system do not need extra link. The new form will be the graph representation of the RPS. An extra edge in graph represents the assembly of two plates with an extra joint.

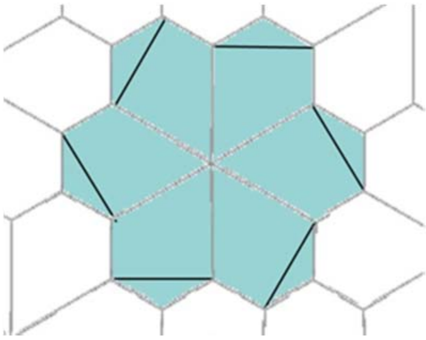


Figure 4.113. Modification of the dual of base tessellation

Step 3: *Focusing on to the key points of graph representation*

In the process of drawing the graph representation from the dual of tessellation, two important points should be considered. The first one is to avoid generating a sub chain or an over constrained sub chain and the second important point is pay attention to the number of edges that meet at every vertex

In the first point, while drawing new edges on the dual to generate tetragonal polygons, it is also possible to generate new triangular polygons. As it is represented in Figure 4.114, a triangle denotes a sub chain in graph representation. The plates numbered 1, 2 and 3 generate a triangle loop when assembled with each other with revolute joints lettered a, b, f. In other words they generate a structure as shown in figure 4.115. Because of this, systems do not move anymore.

In addition to this, if two triangles lettered “a” and “b” are drawn side by side they represent 2R joints in graph representation as in Figure 4.116. Thus plates numbered 2 and 3 generate an over constrained sub chain as in Figure 4.117 and these two plates will move as a single body.

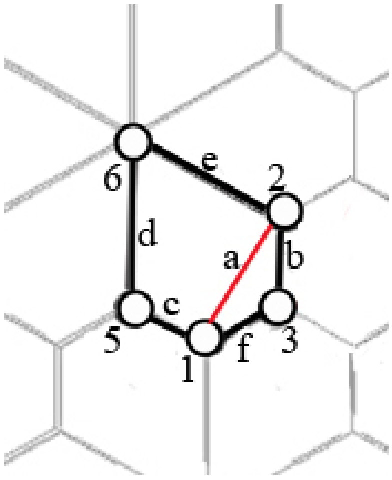


Figure 4.114. Graph representation of a sub-chain creation

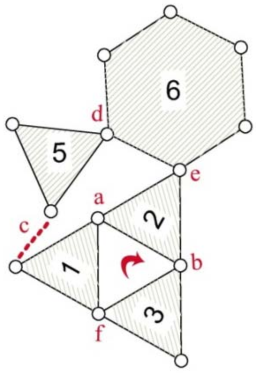


Figure 4.115. Structural representation a sub-chain

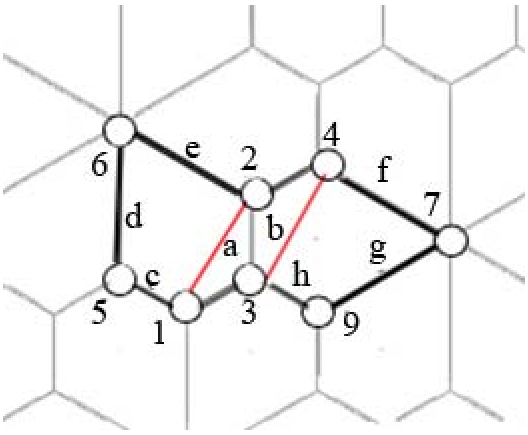


Figure 4.116. Graph representation of an over constraint sub-chain creation

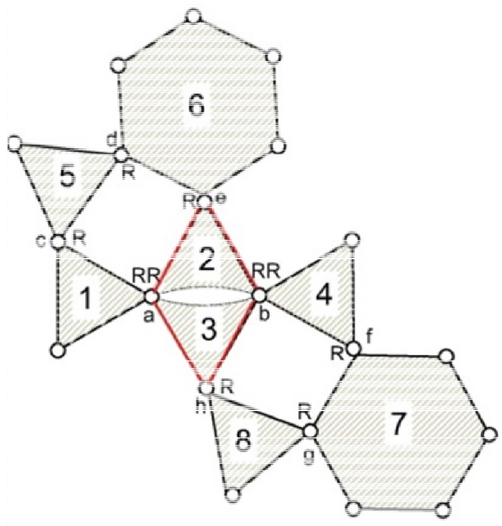


Figure 4.117. Structural representation with over constrained sub-chain

Secondly, on a graph representation if three edges meet at one vertex it is a triangular plate with three joints, if six edges meet at one vertex it is a hexagonal plate with six joints. Thus, when drawing new edges on a dual of tessellation, the number of edges that meet at one vertex should be carefully counted with respect to the chosen tessellation's polygon types. For instance, Figure 4.118 shows a graph representation after drawing new edges to the dual of $3^4.6$ tessellation. The plate "3" has two 2R joints lettered a and b. However, RPS based on $3^4.6$ tessellation should have plates with three joints or with six joints to be connected with neighbouring plates. As it seen in Figure 4.119 the plate numbered 3 is a binary link connected with two 2R joints (a and b) instead of three R joints.

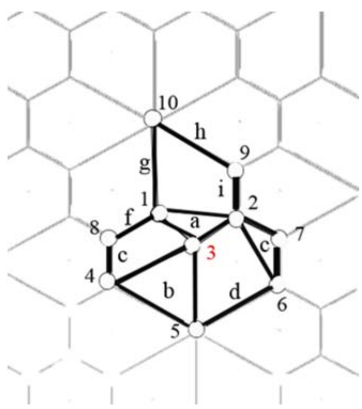


Figure 4.118. Graph representation of generating binary link

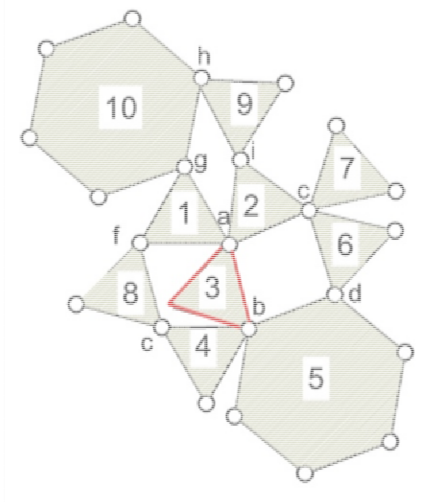


Figure 4.119. Structural representation of generating binary link

By considering focusing on to the key points of graph representation Figure 31 shows a correct graph representation of RPS module where the vertices are numbered and the edges are lettered. Triangular plates are connected to neighbouring plates with 3 joints that are R or 2R joints.

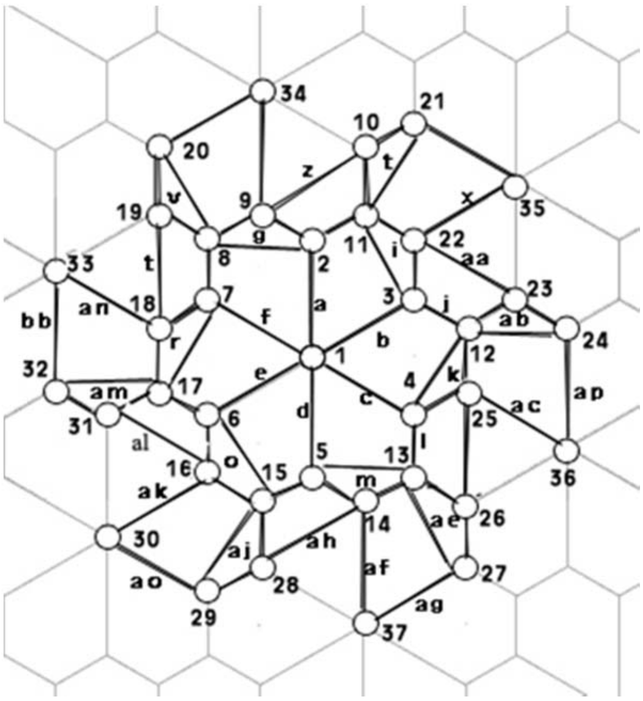


Figure 4.120. Graph representation of the RPS module without any sub chain

Step 4: *Assembly of the plates and RPS construction*

Structural representation of RPS is drawn according to the graph representation. From the graph it is understood that plates numbered 1,30,33,34,35,36,37 are hexagonal, others are triangular plates. Rigid plates are assembled with revolute joints according to the generated graph representation as in Figure 4.121.

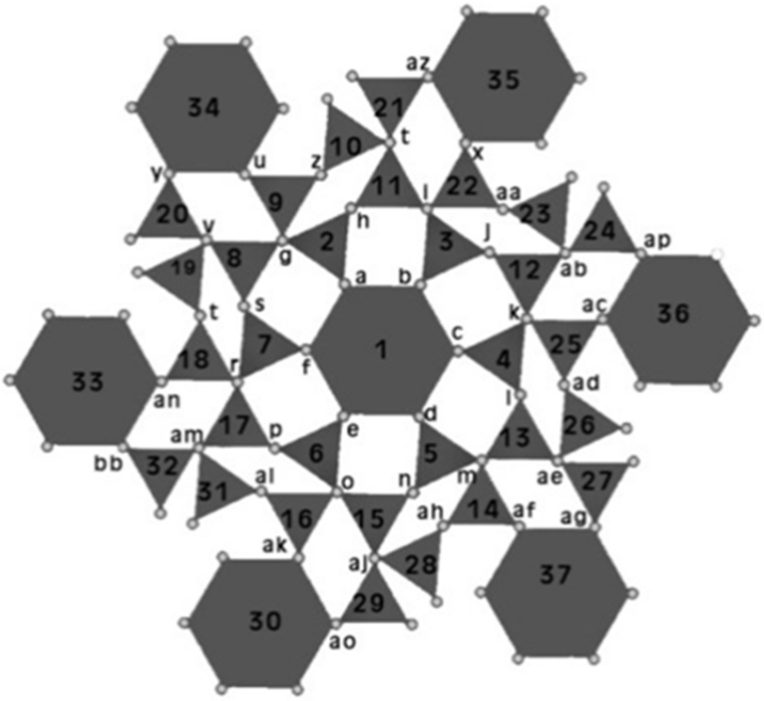


Figure 4.121. Structural representation of RPS module

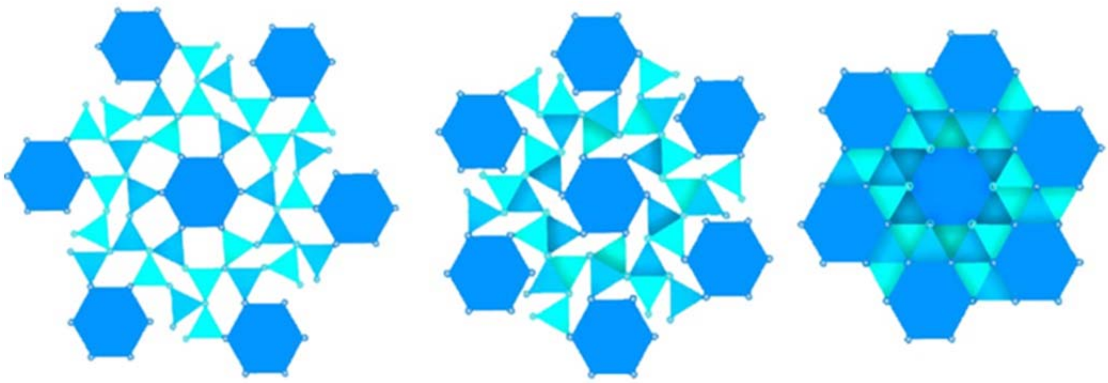


Figure 4.122. Retraction of a RPS module based on $3^4.6$ tessellation

In Figure 4.122, it can be seen that a single DoF RPS module based on (3^4.6) tessellation can be designed by connecting two plates with new joints. It can expand and retract in a predictable manner without any gaps or overlaps.

Among the three five edges meet at one vertices tessellation (3^4.6, 3^2.4.3.4 and $4^{\mathbf{2}}$.$3^{\mathbf{3}}$) only one of them (3^4.6) satisfy the both condition at the same time that is mentioned above. 3^2.4.3.4 and $4^{\mathbf{2}}$.$3^{\mathbf{3}}$ tessellations do not satisfy the second condition. As a result, the RPS based on 3^2.4.3.4 and $4^{\mathbf{2}}$.$3^{\mathbf{3}}$ tessellation have two problems. One problem is having multi degrees of freedom, and the second one is having gaps and overlaps between the retractable plate.

To reach single DoF RPS based on 3^4.6, 3^2.4.3.4 or $4^{\mathbf{2}}$.$3^{\mathbf{3}}$ tessellations that closed without any gaps or overlaps, firstly operation of vertex translation is applied to satisfy the second condition, then, the method of systematic conversion of Multi DoF RPS into a Single DoF RPS applied.

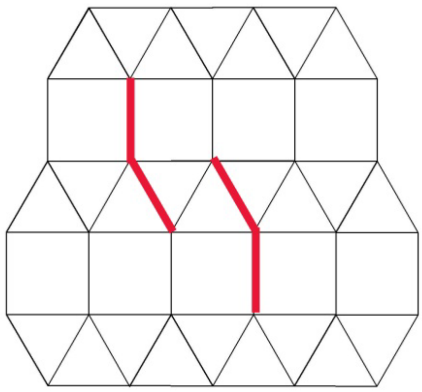


Figure 4.123. The Relation between edges of 3^3.4^2 tessellation

The Figure of 4.123 displays the 3^3.4^2 tessellation and relation between neighbouring edges of the regular polygons with each other. To satisfy the second condition, as shown in the Figure 4.124 the vertex of **1, 2** and **3** is translate to **1'**, **2'** and **3'**. At the end of this processes, at least three vertex of tessellation place in a straight line. (Figure 4.126 and Figure 4.127)

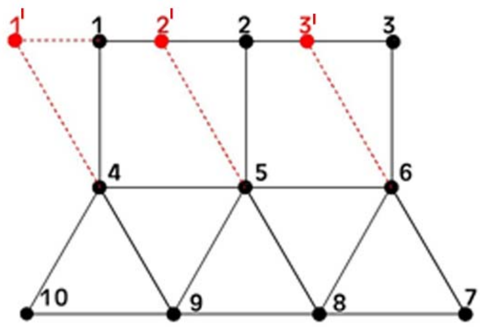


Figure 4.124. Vertex translation on $3^3.4^2$ tessellation

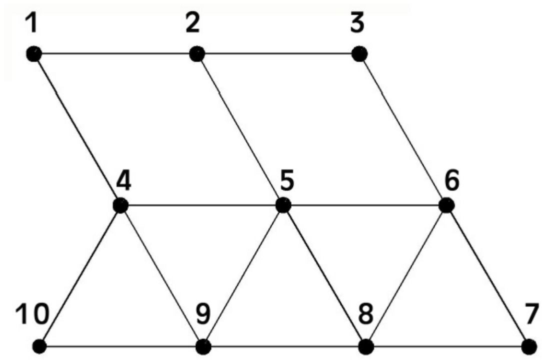


Figure 4.125. Position of the vertex after translation

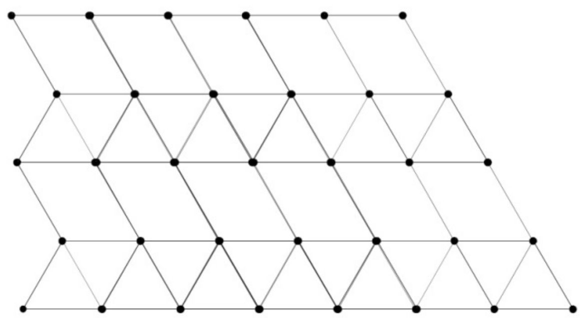


Figure 4.126. After vertex translation

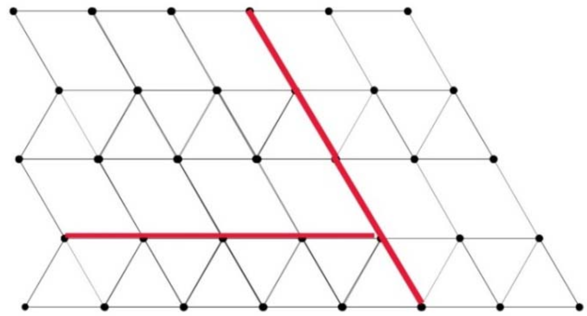
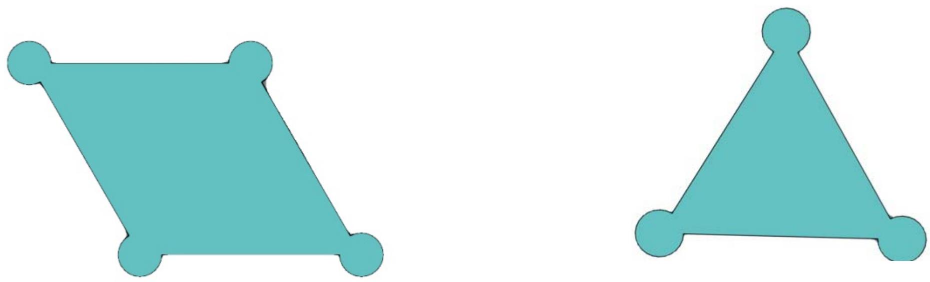


Figure 4.127. To Create straight line

According to new form of the tessellation (Figure 4.127), regular triangular plate with three hinges, irregular tetragonal plate with four hinges are the new forms of plates (Figure 4.128).



(a) (b)

Figure 4.128. The shape of the retractable plate (rhombus and triangle.)

To solve the second problem, all step of the method "the systematic conversion of Multi DoF RPS into a Single DoF RPS" should be applied step by step.

As shown in Figure 4.129, it is started with acquiring the dual of the $3^3.4^2$ tessellation. In the second step some modification should be done by adding extra edge onto dual of $3^3.4^2$ tessellation. In this step the purpose is to generate tetragonal polygon and also should be give attention to focusing on to the points of graph representation that are mentioned above. After achieving graph and structural representation by benefiting the previous step (Figure 4.130 and 4.131) the plates assemble with each other with revolute joint (R) (Figure 4.132.) As can be seen from the this figure, the RPS can expand and retract in a predictable manner without closed any gaps or overlaps however it is iterated within limits because, on the modification of the dual of base tessellation step; over-constrain sub chains are generated or numbers of the joints on the plates have not same with number of edges of polygon of $3^3.4^2$ tessellation. Thus tetragonal polygons cannot be generated infinitely in the dual form of the tessellation.

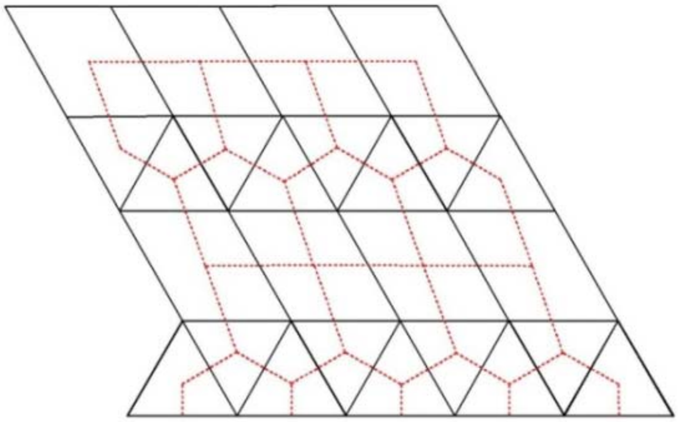


Figure 4.129. Dual form of $3^3.4^2$ tessellation

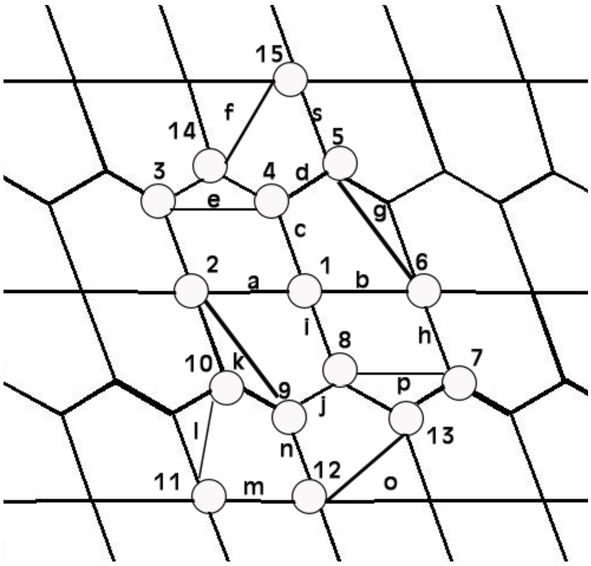


Figure 4.130. Graph representation of RPS based on $3^3.4^2$ tessellation

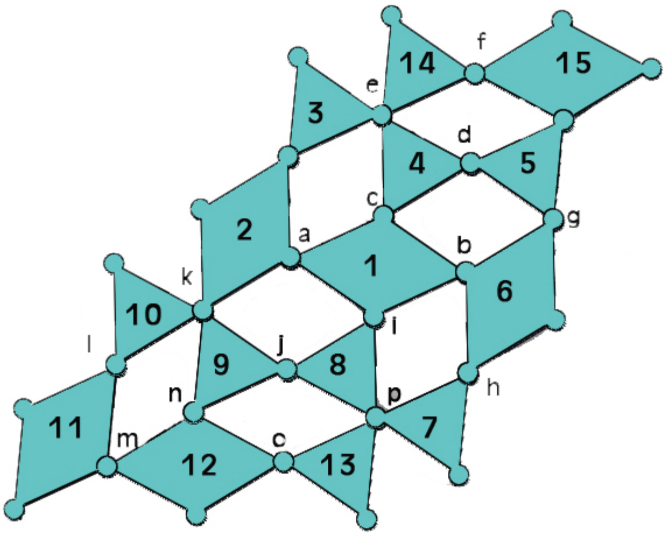


Figure 4.131. Structural representation of RPS based on $3^3.4^2$ tessellation

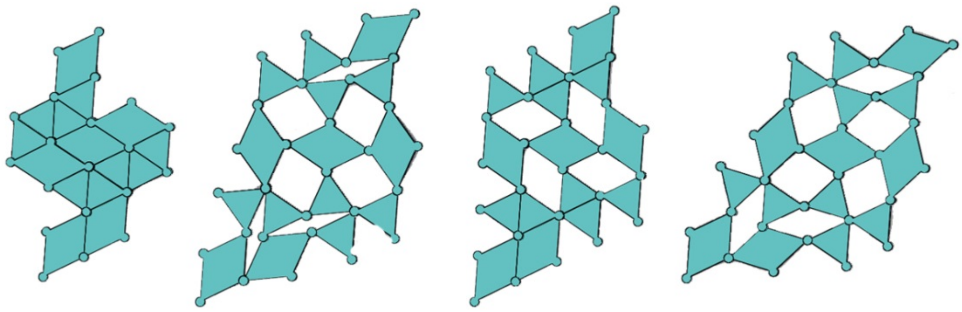


Figure 4.132. Expansion processes of RPS based on $3^3.4^2$ tessellation

The same procedure can apply to the other five edges meet at one vertex tessellation too ($3^2.4.3.4$).

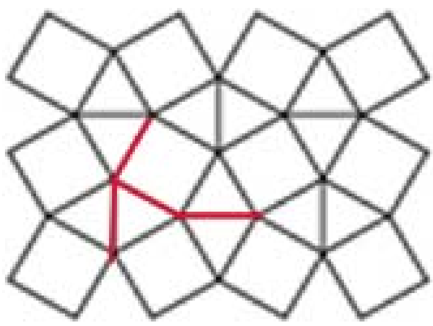


Figure 4.133. $3^2.4.3.4$ tessellation

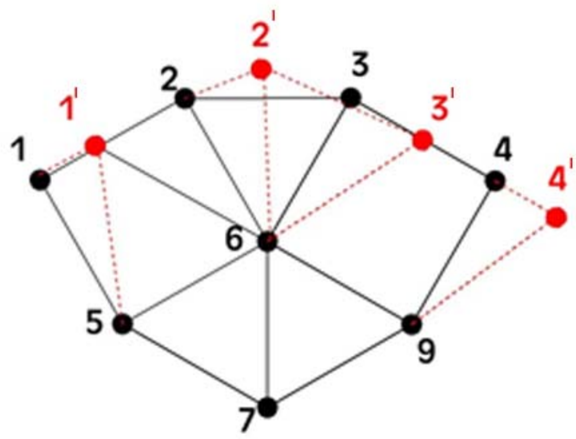


Figure 4.134. Vertex translation of 3^2.4.3.4 combination

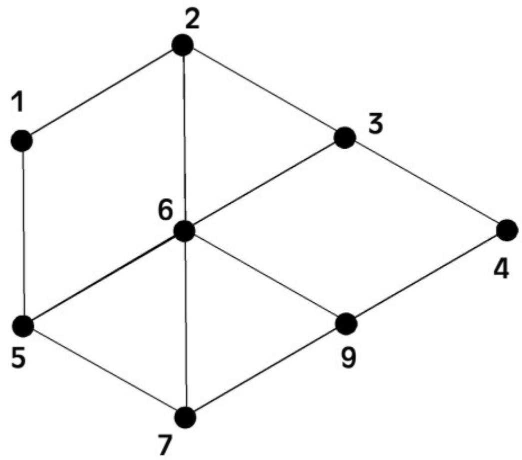


Figure 4.135. Vertex position of 3^2.4.3.4 combination

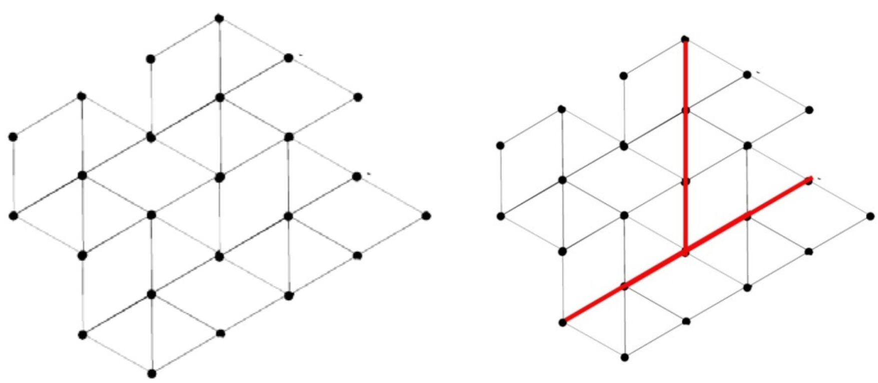


Figure 4.136. New shape and position of neighbouring polygon edge

When the method of systematic conversion apply to the 3^2.4.3.4 tessellation, the same problems happen on the modification of the dual of base tessellation step too. In many place over-constrain sub chains are generated or numbers of the joints on the plates have not same with number of edges of polygon of 3^2.4.3.4tessellation. Thus tetragonal polygon do not generate infinitely in the dual form and the plates do not assemble with each other by using this order.(Figure 4.137-4.138).

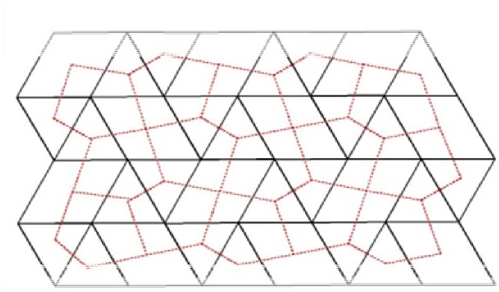


Figure 4.137. Dual form of 3^2.4.3.4 tessellation after translation

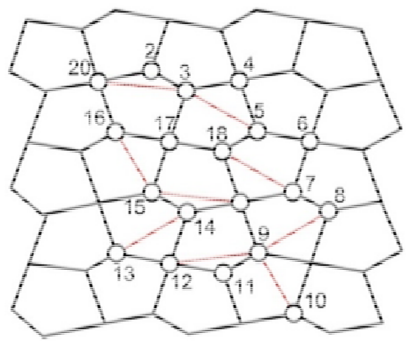


Figure 4.138. Dividing dual polygon in arbitrary

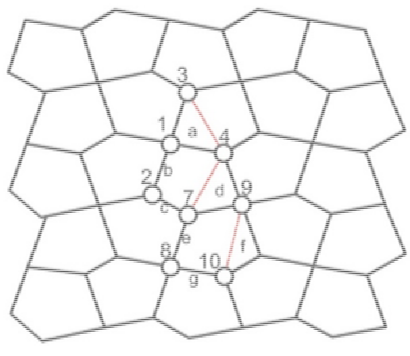


Figure 4.139. Dividing dual polygon in linear direction

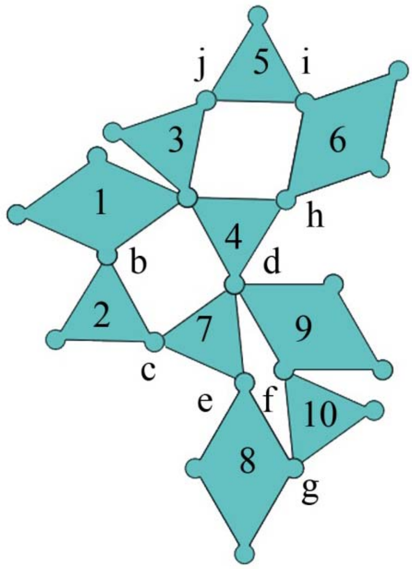


Figure 4.140. Structural representation of RPS based on $3^2.4.3.4$ tessellation with assembling linear direction

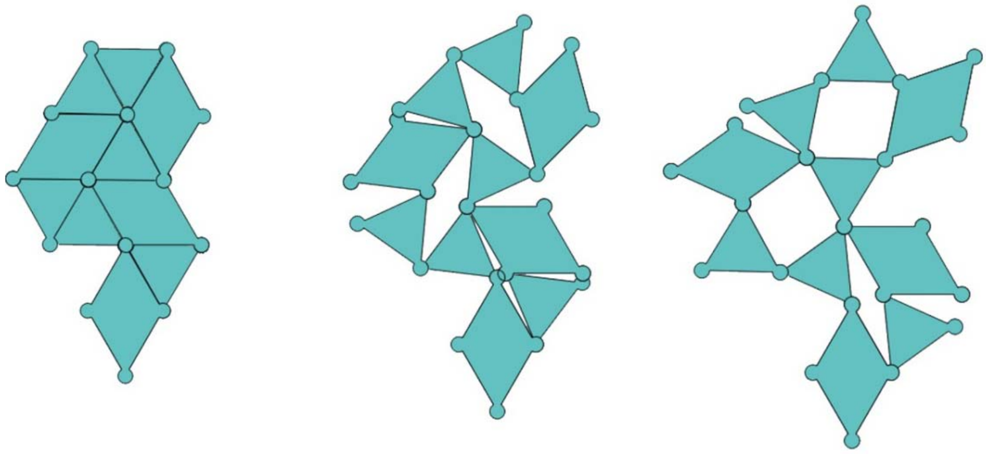


Figure 4.141. Expansion processes of RPS based on $3^3.4^2$ tessellation

On the other hand 3^6 tessellation is the unique tessellation which have six edges meet at one vertex in 1-uniform tessellation. Thus, as it mentioned before the loop form become hexagonal and it have multi DoF. 3^6 tessellation satisfy the second condition, thus to achieve single DoF RPS based on 3^6 tessellation that deploy without any gaps or overlap, needs to convert it to multi DoF to single DoF by finding the true modification of duality of tessellation.

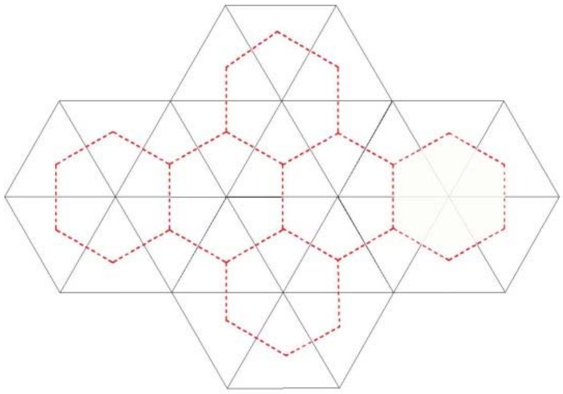


Figure 4.142. Dual form of 3^6 tessellation

When the method of systematic conversion of multi DoF to single DoF is applied to RPS based on 3^6 tessellation by considering the key points, sub-chains or over-constrain sub-chains are always generated by adding extra edges as in Figure 4.143. Besides, the number of joints on the triangular plates will be more or less than three. For example, plate number 42 has four joints and plate numbered 59 has two joints. It is obvious that the graph representation of a single DoF RPS cannot be drawn only by adding extra edges to the dual.

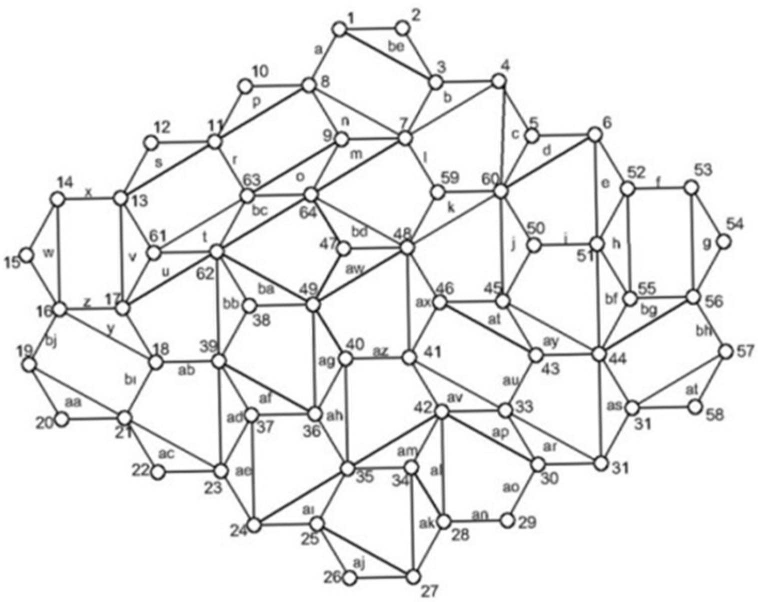


Figure 4.143. Attempt to draw graph representation of RPS based on 3^6 tessellations

As it seen from the Figure 4.143, by adding extra edge in other words by adding extra joints, the mobility of the tessellation do not convert to single degrees of freedom. In order to draw a graph representation consists of tetragonal polygons, both extra vertices and edges should be drawn on the dual. As seen in Figure 4.144, black points display the vertices of the dual while blue points display the newly added vertices. A new graph is generated by drawing the edges between all vertices as in Figure 4.145. The graph is generated after vertices are numbered and edges are lettered as in Figure 4.146. It is seen that six edges meet at new vertices which represent hexagonal extra plates/links with 6 revolute joints (*R*) and three edges meet at existed vertices which represent triangular plates with 3 revolute joints.

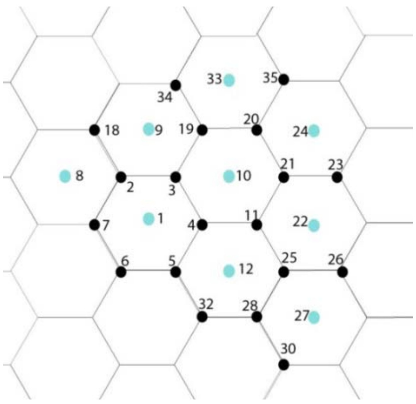


Figure 4.144. Addition of extra points to the dual

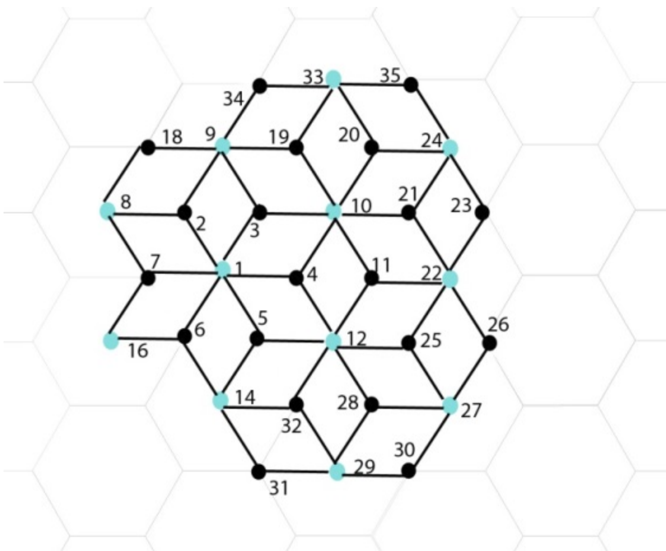


Figure 4.145. Drawing tetragons on the dual

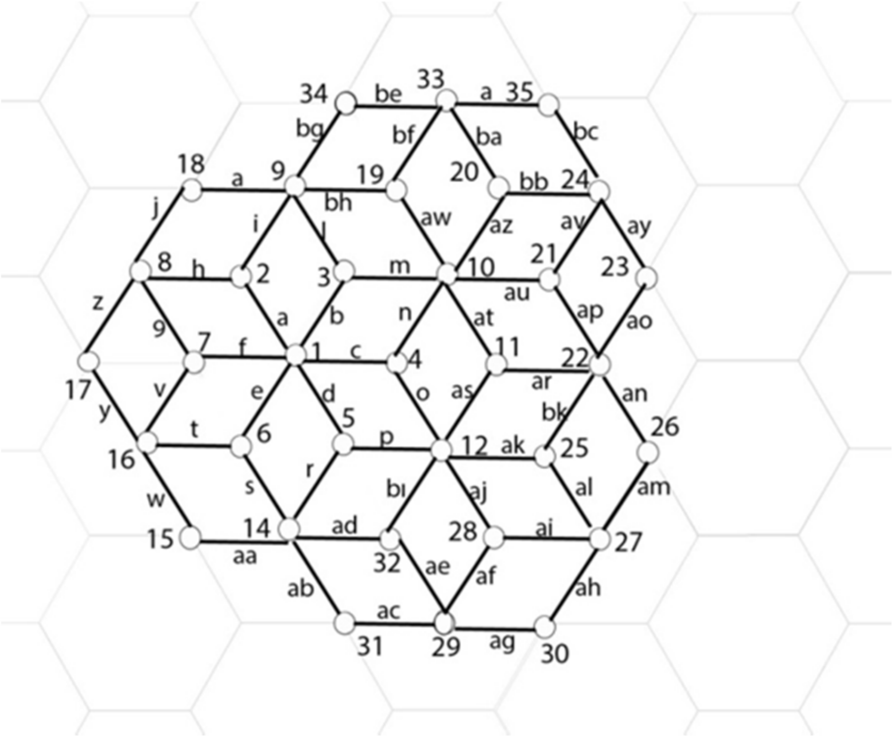


Figure 4.146. Graph representation of RPS based on 3^6 tessellation

As it seen that the system needs to extra links with six revolute joints (*R*) to reach single DoF RPS. At this stage the important question is what is the form of the extra link.

To find the form of the extra link, this thesis returning briefly to the method 2 which is based on according to duality of tessellation.

To summarized the method briefly; firstly the duality of tessellation is drawn (Figure 4.147). Then the dual form is found (Figure 4.148). The number of edge of this polygon gives the number of elements of the link. Also, the corner point of the polygon gives the joint position of the extra link. In order to increase the compactness by reducing the change of extra link collision the midpoint of the polygon is found and assemble them with the corner point. The new single DoF RPS based on 3^6 tessellation consists of triangular plates but connected with hexagonal plates in order to create parallelogram loops as in 4.151.

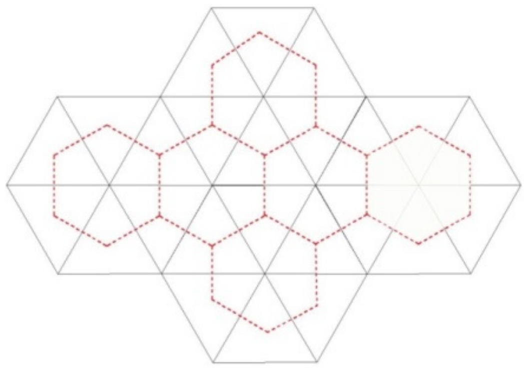


Figure 4.147. Dual of 3^6 tessellation

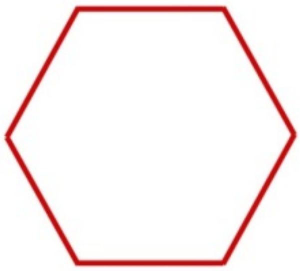


Figure 4.148. The smallest form of dual of 3^6 tessellation

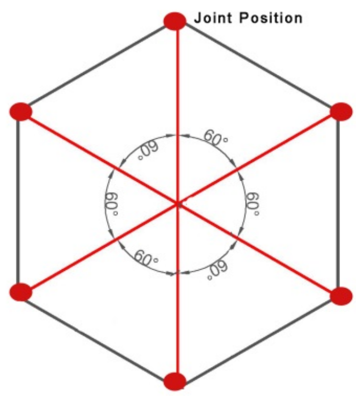


Figure 4.149. Joint position of the extra link

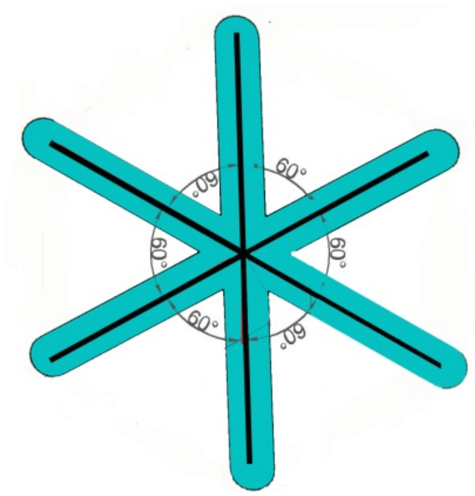


Figure 4.150. Form of the extra link

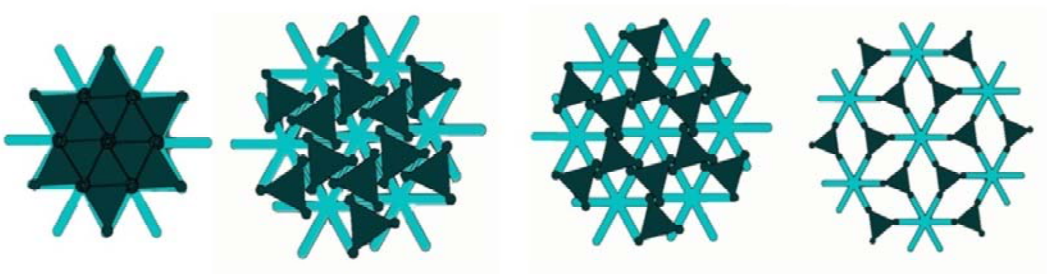
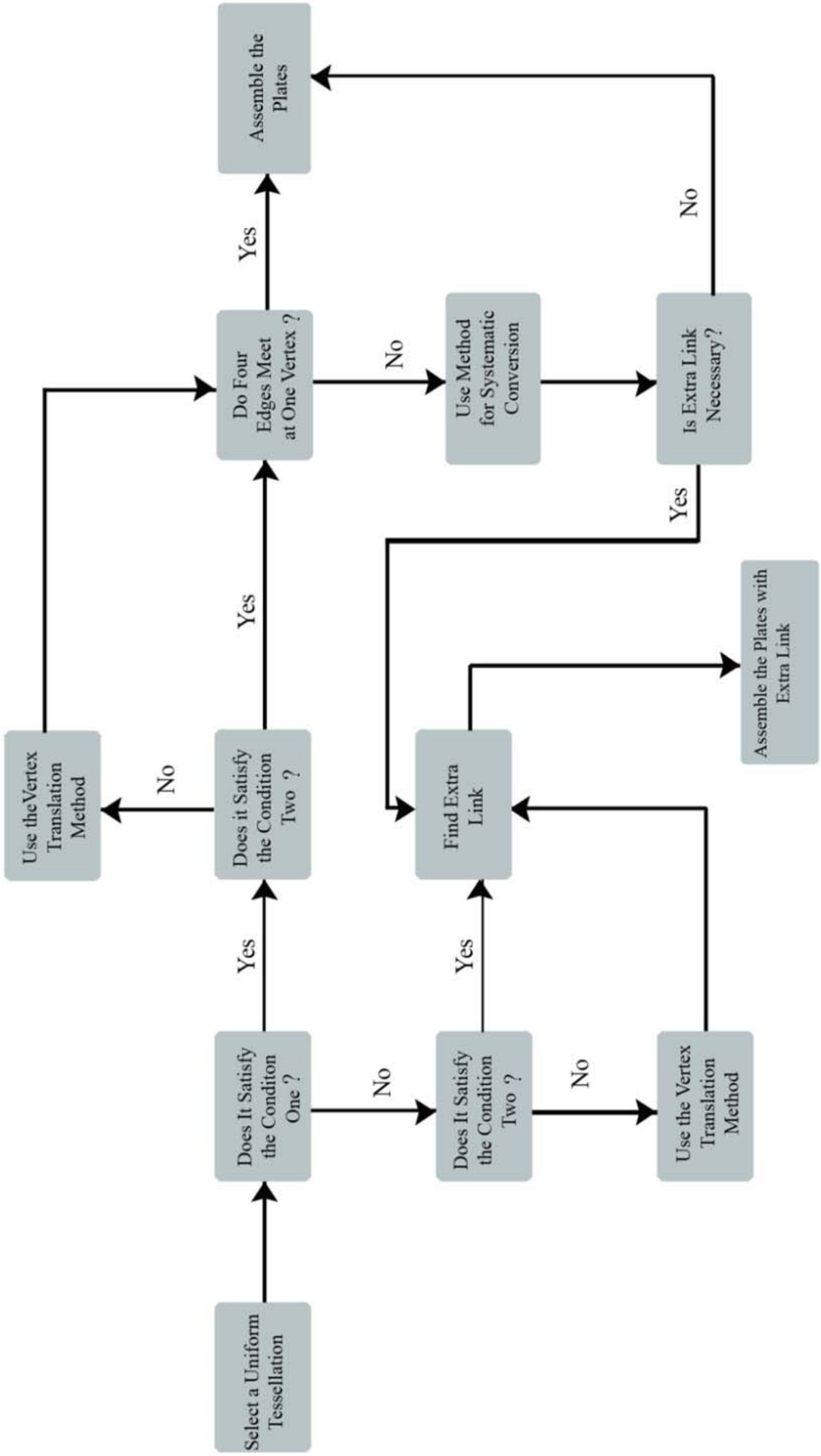


Figure 4.151. The expansion of RPS with design 3^6 tessellation

This chapter presents a new approach for designing processes of RPS without needing any kinematical or numerical analysis. With respect to this, developing some kind of helping tool to the designer is one of the aims of this thesis. In the following a kind of chart is prepared which are shows that needing followed steps briefly, according to selected 1-uniform tessellation when designing retractable plate structures without any gaps or overlaps.

Table 4.3. Flow chart, according to selected 1-uniform tessellation



4.6. Conclusion and Discussions

The first part of the chapter has explained the two conditions to reach retractable plate structures based on 1-uniform tessellation. With respect to the first condition, it was found that if selected tessellation has at least four edges meet at each vertex, RPS based on that tessellation can be retractable. Also, second condition mentioned that If two neighbouring edge of tessellation generate a continuous line, the tessellation can retract and expand without any gaps or overlaps. Thus, following, this chapter handle according to the numbers of vertices of tessellation.

Firstly, this chapter deals with the RPS based on four edges meet at one vertex tessellation which are not satisfy the second condition. The current study found that by using vertex translation operation the 1-uniform tessellation can satisfy the second condition.

Secondly, RPS based on three edges meet at one vertex tessellation is focused on. In contrast to earlier findings, RPS based on three edges meets at one vertex of tessellation need to extra link to be movable. Hence, this part of the chapter presents two different methods to find suitable shape of the extra link and discuss their drawback and benefits. Also, to retract and expand without any gaps or overlaps, the form of tessellation is changed by using vertex translation method too.

Thirdly, this chapter handle the RPS based on five and six edges meet at one vertex tessellation. In this part a strong relationship between graph theory and duality of tessellation has been reported. By help of this relationship, a method is developed that is called systematic conversion to multi DoF RPS to single DoF RPS. On the other hand, some tessellations which do not satisfy the second condition have been changed their form by using vertex translation method. Finally, RPS based on six vertices of tessellation is deal with. Firstly, to find the suitable assembling order, the method which is based on similarities between graph theory and duality of tessellation is used and as a result of this stage, it is understood that there is needing to extra link. The form of the extra link is found by using method: according to duality of tessellation.

Consequently, this chapter shows that, many RPSs can be designed based on eleven 1-uniform tessellation by using aforementioned methods and developed conditions. Therefore among the eleven 1-uniform tessellations, just two tessellations cannot be iterated infinitely.

These tessellations are $3^2.4.3.4$ and $3^3.4^2$ tessellations. A possible explanation for this might be that iteration way of these tessellations.

As can be seen below example, the tessellations (4^4, 4.8^2, 6^3, 3.6.3.6, 3.12^2, 3.4.6.4, 4.6.12, $3^{6,}$ $3^4.6$) iterated radially, and RPSs based on these tessellation expand and retract in a predictable manner, without any gaps or overlaps. In addition to this, position of the plate in graph representation is the same with the position of the plate in structural representation.

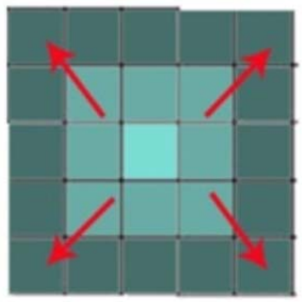


Figure 4.152. Iteration of square tessellation

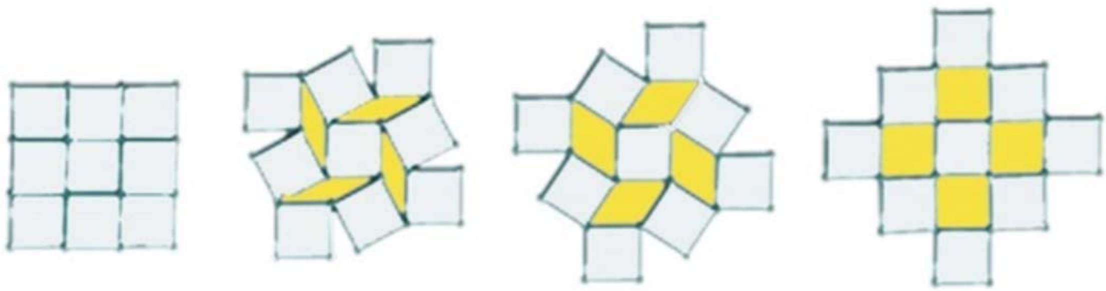


Figure 4.153 Expansion of RPS based on square tessellation

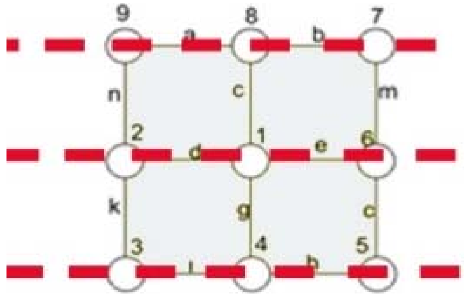


Figure 4.154. Position of the plate in graph representation

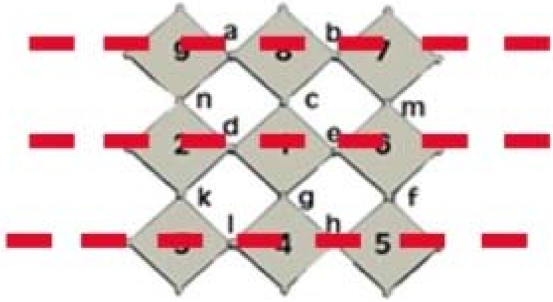


Figure 4.155.Position of the plate in structural representation

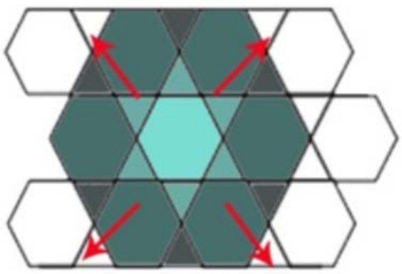


Figure 4.156. Iteration of 3.6.3.6 tessellation

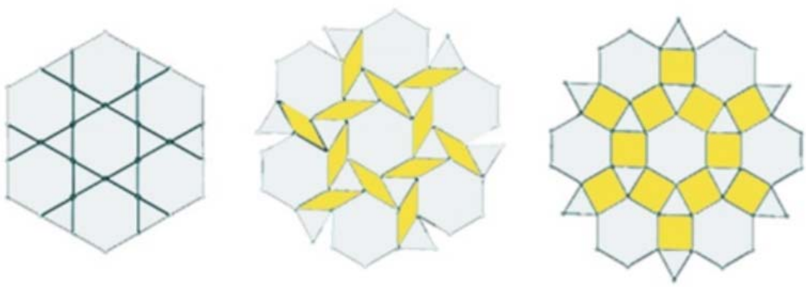


Figure 4.157. Expansion of RPS based on 3.6.3.6 tessellation

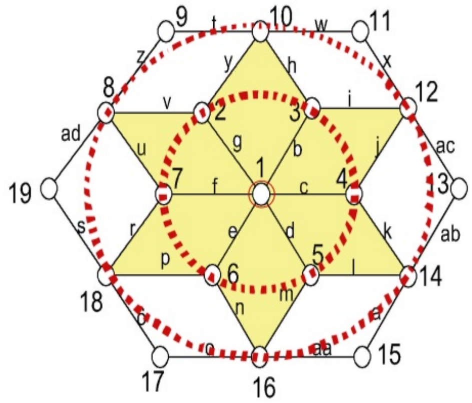


Figure.4.158. Position of the plate in graph representation

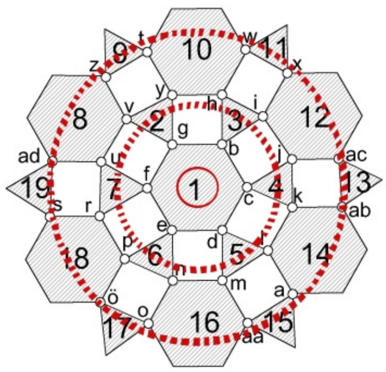


Figure 4.159. Position of the plate in structural representation

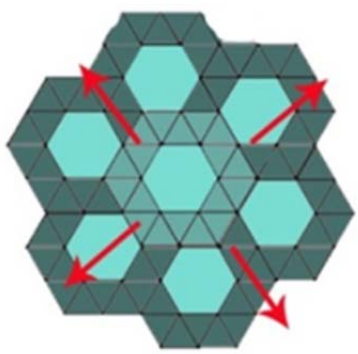


Figure 4.160. Iteration of $3^4.6$ tessellation

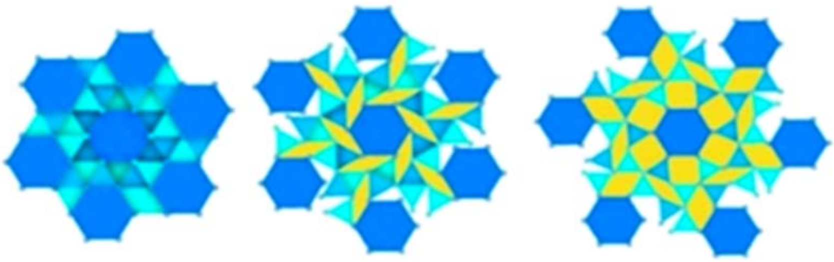


Figure 4.161. Expansion of RPS based on $3^4.6$ tessellation

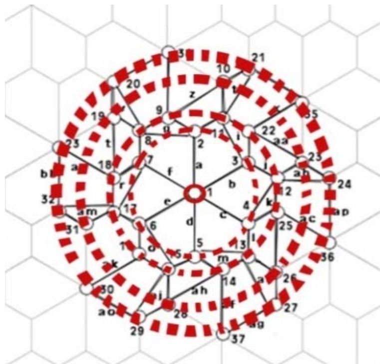


Figure 4.162. Position of the plate in graph representation

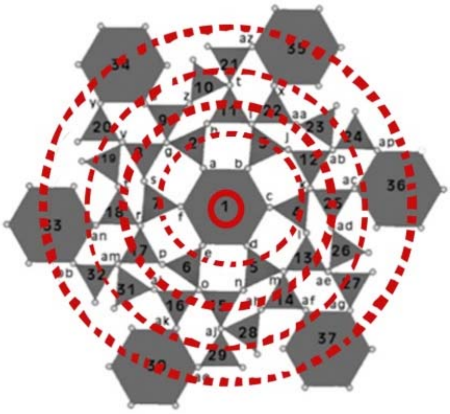


Figure 4.163 Position of the plates in structural representation

On the other hand, as can be seen below Figures, the tessellations $3^3.4^2$ and $3^2.4.3.4$ tessellations does not iterate radially, and the RPSs based on these tessellation do not expand and retract in a predictable manner, without any gaps or overlaps. In addition to this, the position of the plate in graph representation is not same with the position of the plate in structural representation.

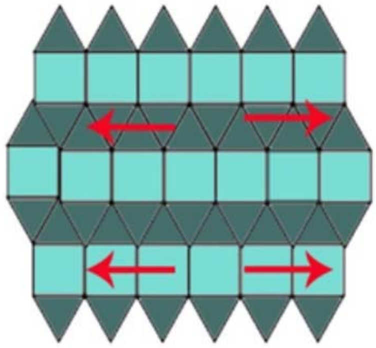


Figure 4.164. Iteration of $3^3.4^2$ tessellation

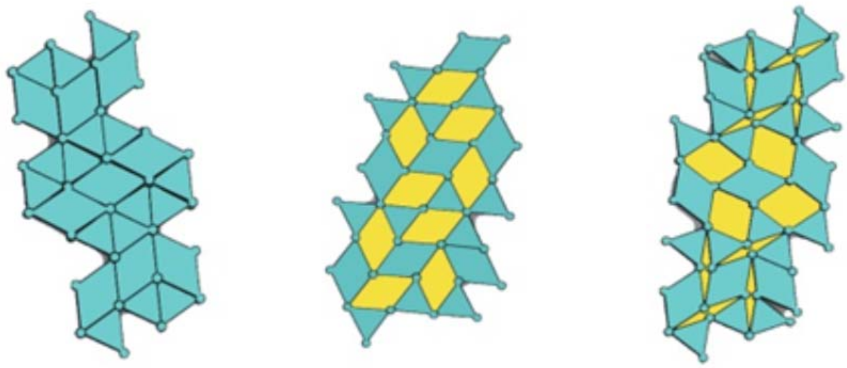


Figure 4.165. Expansion of RPS based on $3^3.4^2$ tessellation

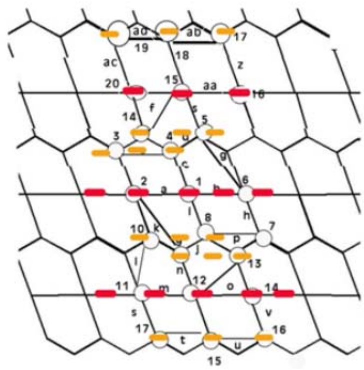


Figure 4.166. Position of the plate in graph representation

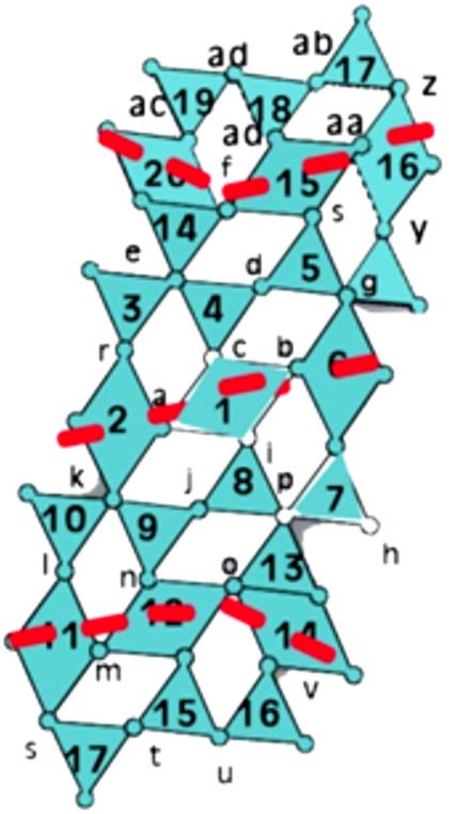


Figure 4.167. Position of the plate in structural representation

These findings might be related to tessellations symmetry group. According to plane symmetry groups; 4^4 and 4.8^2 tessellations have p4m symmetry group (rotation and reflection), 6^3, 3.6.3.6, 3.12^2, 3.4.6.4, 4.6.12 and 3^6 tessellations have p6m symmetry group (rotation and reflection), $3^4.6$ has p6 symmetry group (rotation), $3^2.4.3.4$ has p4g symmetry group (rotation and reflection) and 3^3 $.4^2$ has pgg symmetry group (rotation and glide reflection) (Japlan 1995; Morandi, 2007). It is interesting to note that on the symmetry group (p4m, p6m and p6) of tessellations (4^4, 4.8^2, 6^3, 3.6.3.6, 3.12^2, 3.4.6.4, 4.6.12, $3^{6,}$ $3^4.6$) which is used for can be iterated RPSs, all rotation centres lie on the reflection axes while the other symmetry groups (pgg and p4g) of tessellations (3^3 $.4^2$ and $3^2.4.3.4$) which is used for cannot be iterated RPSs, all rotation centres do not lie on the reflection axes.

Further interdisciplinary research should be undertaken to investigate this relationship as a future work.

CHAPTER 5

MOBILITY ANALYSIS OF RPS BASED ON 1-UNIFORM TESSELLATIONS

This chapter deals with mobility analysis of RPSs based on 1-uniform tessellations. Although it is known that the aforementioned RPSs are over-constrained mechanisms and their degrees of freedom is always one, this chapter proves this fact by using mobility formulations and a theorem that is developed in this study. Besides, a systematic approach of assembly technique between different scales of RPSs is proposed. In order to present their mobility calculations, numerous of them are assembled with respect to the proposed approach.

5.1. Mobility Analysis of RPSs

The degrees of freedom (DoF) or mobility of a mechanical system describes the number of actuators needed to define the location of end-effectors. In mechanisms, the mobility of a mechanism needs to be firstly determined. Traditionally, Chebyshev, Grübler and Kutzbach mobility criterions have been used to study mobility in mechanism science (Alizade, et al. 2007). The formulations provided by these criterions are used to calculate the degrees of freedom or the mobility of mechanical systems. Due to the fact that this dissertation focuses on RPSs where all plates move in planes parallel to one another, they are area kind of planar mechanisms. According to the Grübler's criterion, any planar mechanism's mobility can be calculated by using formulation below,

$$\mathrm{M} = 3(\mathrm{n} - 1) - 2\mathrm{j}_1 - 2\mathrm{j}_2 \qquad (5.1)$$

where, 3 represents the subspace number, M is the total degrees of freedom or mobility of the mechanism, n is the number of total links including the ground link, j_1 is the total number of kinematic pairs with one degree of freedom and j_2 is the total number of higher kinematic pairs that represents the kinematic pairs with two degrees of freedom. It should be noted that in this study higher kinematic pairs are not utilized. After the calculation, if the mobility of the mechanism is found as either equal or smaller than zero ($M \leq 0$) there should not be any movement in the mechanism. In the light of this information, if the square tessellation's (Figure 5.1) mobility parameters are substituted in to the equation 5.1 as n=9, and j_1=12, the mobility is found as zero, $M = 3(9-1) - 2.12 = 0$. However it is known that RPS based on (4^4) tessellation is moveable with a single degree of freedom.

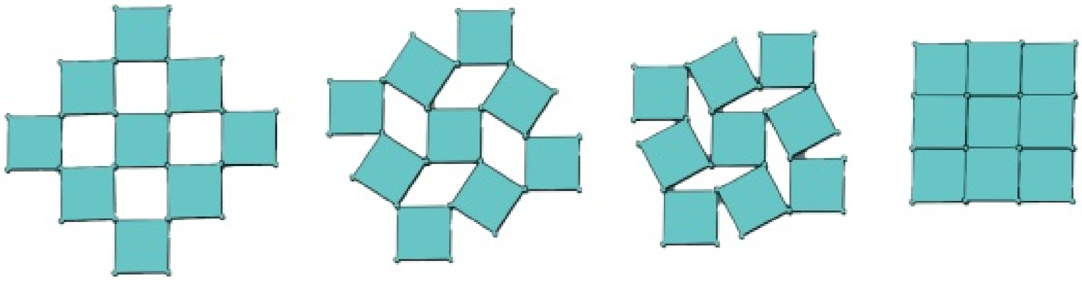


Figure 5.1. Retraction of RPS based on (4^4) tessellation

In literature there are many single degrees of freedom mechanisms that do not meet the mobility criterion. These mechanisms are called over constrained mechanisms. Although they do not meet the mobility criterions, due to the existence of special geometric conditions among the links and joint axes that are referred as over constrained conditions, they are mobile. The RPS mentioned in the example based on square tessellation is one of them. The mobility of this RPS is one due to the existence of parallelogram loops. Same geometric condition can also be seen at (3.6.3.6), (3.4.6.4) tessellations. There are four edges meet at every vertex and again the loops are parallelogram (Figure 5.2).

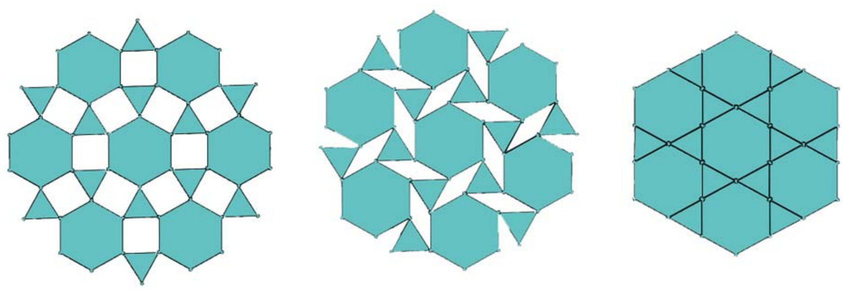


Figure 5.2. Retraction of RPS based on (3.6.3.6) tessellation

Using equation 5.1 with the variables of (3.6.3.6) tessellation as n=19, j_1= 30 the mobility of the RPS is calculated as M= 3(19-1) - 2.30 = -6. As it can be seen RPS seems to be structure but again it has full range of mobility with a single degree of freedom due to the existence of parallelogram loops. It is an over constrained mechanism with respect to the mechanism science terminology. Similarly if 3.4.6.4 tessellation is used to design a RPS (Figure 5.3), using equation 5.1 with the variables of (3.4.6.4) tessellation as n= 13, j_1= 18 the mobility of the RPS is calculated as M= 3(13-1) - 2.18 = 0. As in the previous examples, four edges meet at every vertex and it is moveable due to the existence of parallelogram loops. On the other hand, during the expansion and contraction there is always overlapping plates due to changing the assembly mode. In any mechanism, the alternative forms that can be assembled with the same links, plates and connections are called configurations or assembly modes of that mechanism (Tsai 2000). During its motion, the mechanism may pass from one assembly mode to another, which is called reconfiguration or assembly mode change that is explained in previous chapter.

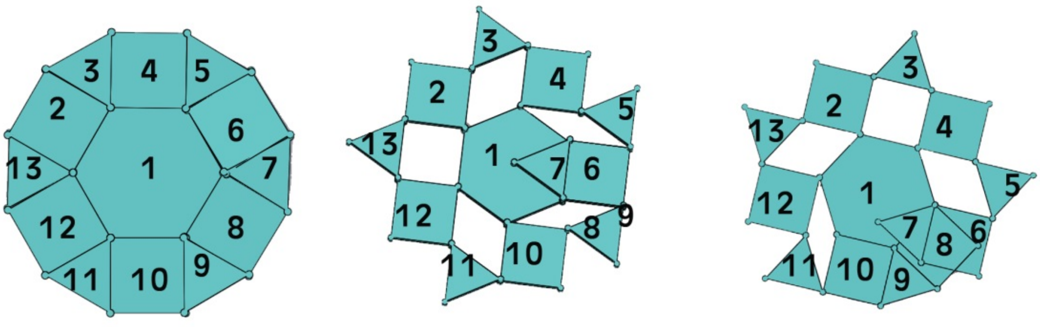


Figure 5.3. Unpredictable expansion of RPS based on 3.4.6.4 tessellation

Above, examples of three 1-uniform tessellations are presented. All of these tessellations have four edges meeting at every vertex and they constitute parallelogram loops with single degree of freedom. Besides them, there are 1-uniform tessellations (3^2.4.3.4), (3^3.4^2), (3^4.6), (3^6) that have more than four edges meeting at every vertex. RPSs based on these constitute five bars or six bars loops. Using equation 5.1 with the variables of (3^4.6) tessellation as n=19, j_1= 24 the mobility of the RPS is calculated as M= 3(19-1) - 2.24 = 6. RPS is moveable but using single actuation causes it to move in an unpredictable manner because of having multi degrees of freedom. It should be noted that this behaviour is because of the five bar loops. Each five bar loop increases the mobility by one.

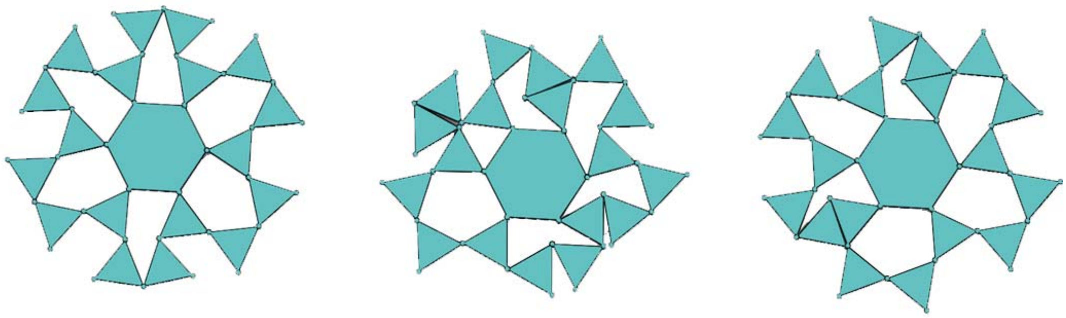


Figure 5.4. Movement of RPS based on (3^4.6) tessellation

As it can be seen from the table 4.1, among the 1-uniform tessellations, only four of them (3^6), (3^2.4.3.4), (3^3.4^2), (3^4.6) have more than four edges meeting at one vertex. RPSs based on these tessellations can be expandable but not in a predictable manner by a single actuation due to the multi degrees of freedom (M-DoF).

5.2. Mobility Relation with Number of Excessive Plates

More than 150 years in kinematics, Grübler-Kutzbach criterion is largely used in mobility calculations but it fails to provide the correct mobility of the RPSs based on 1 uniform tessellations due to the existence of over constraints. In order to develop a valid and general formula for mobility calculation, many scholars have proposed various formulae. In this section, a formula proposed by Alizade and Freudenstein (Alizade, et al. 2006) is used by considering the joint and loop characteristics of the retractable plate structures. Amongst the generated RPSs on the chapter 4, two tessellations are chosen in such a way that one of them will be formed by only utilizing plate assemblies while the other one will be formed by utilizing plate assemblies with extra links as examples to show the mobility calculation procedure.

Before advancing further though, the RPS based on (4^4) tessellation should be recalled for ease of understanding. Considering Grubler's formulation (Equation 5.1), it is known that the mobility of the tessellation is found as zero. However, after many simulations and models with the same number of plates and revolute joints, it is realized that the mobility of the retractable structure is in fact one. As it is mentioned before, the simplest module in this tessellation is an over-constrained multi-loop mechanism that has four parallelogram loops around the fixed plate and the Grübler's formula does not fit for this mechanism. Thus let's try to use Alizade and Freudenstein universal mobility equation below.

$$M = \sum_{i=1}^{j} fi - \sum_{k=1}^{L} \lambda_k + q \tag{5.2}$$

In this equation, j is the total number joints in the mechanism, fi is the total degrees of freedom of the i^{th} joint, L is the total number of independent loops, λ_k is the subspace number of the k^{th} loop, and q is the number of excessive plates. Excessive plate can be described as the links where the removal or addition of them to the system does not affect the overall motion characteristics of the mechanism.

Since only one degree of freedom joints are used in the examples and the subspace of the mechanism's loops are always three as a design constraint of this thesis, the equation becomes,

$$M = J - 3L + q \tag{5.3}$$

where J is the total number of 1-DoF joints.

It is known that the mobility of the simplest retractable module is one and there are totally 4 loops and 12 revolute joints in the system (Figure 5.5). Using equation 5.3 with variables as L=4, J=12 and q=1, the mobility of the module can be calculated as M = 12 - 3.4 + 1 = 1. The excessive plate can be seen as the faded plate in Figure 5.5. Figure 5.6 shows the contraction of the mechanism with and without the excessive plate.

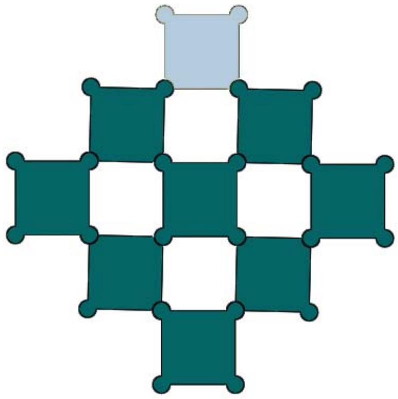


Figure 5.5. Simplest module of the RRP based on square tessellation with an excessive plate

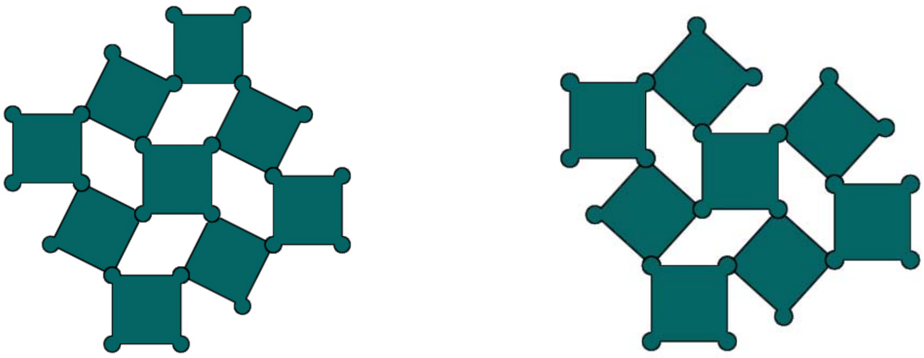


Figure 5.6. Simplest module of the RPS based on square tessellation with and without the excessive plate

For the second case the simplest square retractable module is iterated as in Figure 5.7. In this configuration two modules are sharing the fourth loop and there exist a total of 7 loops and 20 revolute joints. Using equation 5.3 with variables as L=7, J=20 and q=2, the mobility of the module can be calculated as M= 20 - 3.7 + 2 = 1. The excessive plates are again shown in Figure 5.7 as faded plates and Figure 5.8 shows the contraction of the mechanism without the excessive plates.

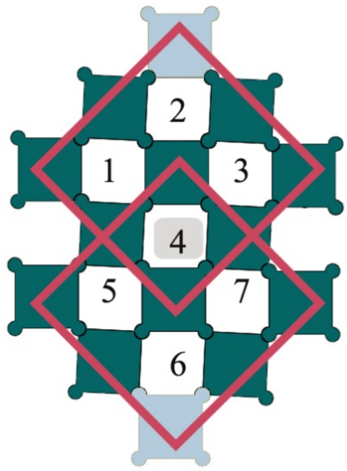


Figure 5.7. Iterated module with a single shared loop 4 and two excessive plates.

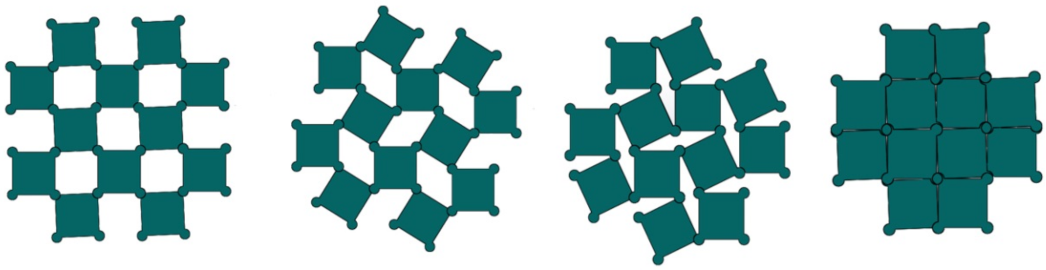


Figure 5.8. Iterated module of the RPS based on square tessellation without two excessive plates

It should be noted that iterations by the new module that creates a single shared loop on the system increases the number of excessive plates (q) by one. For instance, if one more module is iterated as in figure 5.9, there will be two shared loops 4 and 6. Using equation 5.3 with variables as L=10, J=28 and q=3, the mobility of the module can be calculated as M= 28 - 3.10 + 3 = 1. The excessive plates are again shown in Figure 5.9 as faded plates and figure 5.10 shows the contraction of the mechanism with and without the excessive plates.

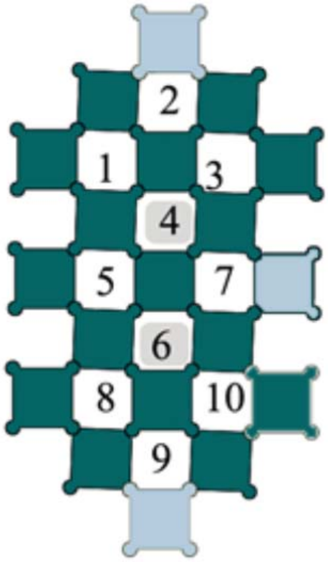


Figure 5.9. Two times iterated modules with two shared loops and three excessive plates

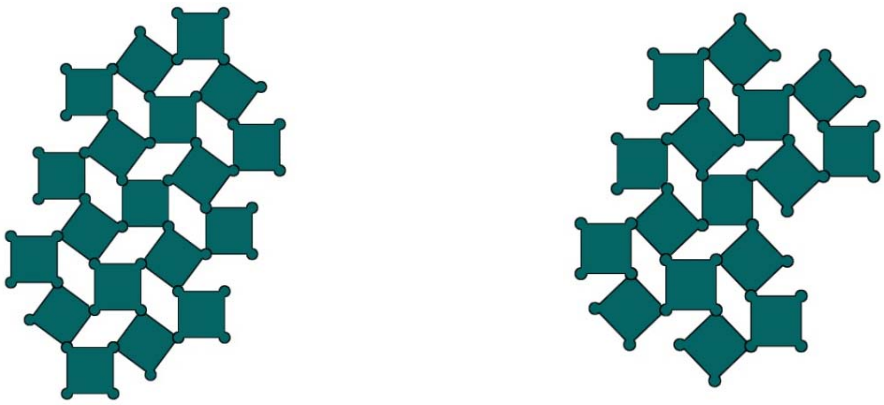


Figure 5.10. Two times iterated modules with and without three excessive plates.

So far the modules were iterated serially, but they can also be iterated in a parallel manner. Consider a single iteration in series that will add a shared loop to the system shown as the 4th loop in Figure 5.11. Following this another iteration but this time in a parallel manner is executed that will add two additional shared loops to the system instead of one as the 3rd and the 10th loop. Finally a third iteration is executed that will add another extra shared loop as the 7th loop. In the final configuration the RPS has four sharing loops numbered 3-4-7-10. Using equation 5.3 with variables as L=12, J=32 and q=5, the mobility of the module can be calculated as M= 32 - 3.12 + 5 = 1. The excessive plates are again shown in figure 5.11 as faded plates and Figure 5.12

shows the RPS with and without five excessive plates. In this example it should be noted that a single parallel iteration with one module will cause two additional sharing loops and adds two excessive plates to the RPS instead of one. In the light of this, it can be said that the number of excessive plates are equal to one plus the number of loops that are shared after the iterations.

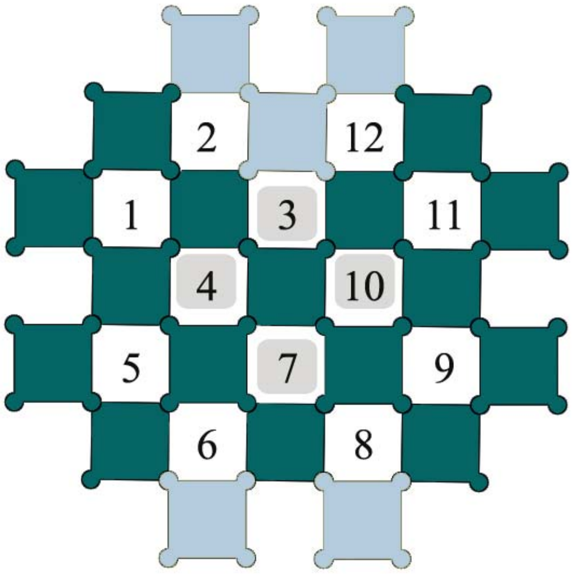


Figure 5.11. Three times iterated module with four shared loops and five excessive plates.

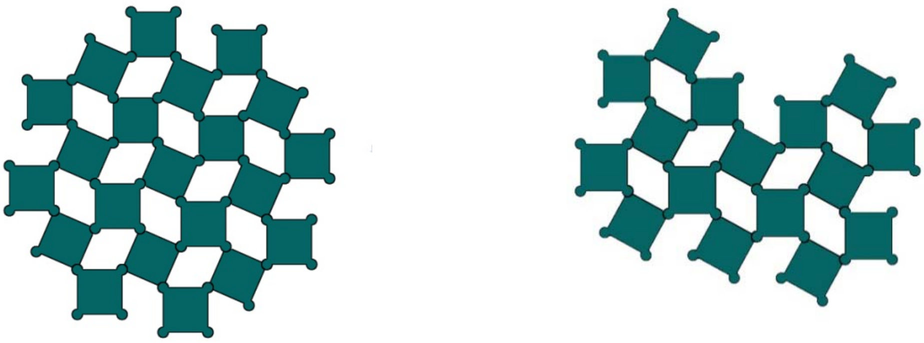


Figure 5.12. Three times iterated module with and without five excessive plates.

Same formulation can be utilized to find the excessive plates in RPS based on (3^6) tessellation. Firstly the simplest module of the RPS should be considered. Using equation 5.3 with variables as L=12, J=30 and q=7, the mobility of the module can be calculated as M= 30 - 3.12 + 7 = 1. The excessive members are shown in Figure 5.13 with blue colors and figure 5.14 shows the retraction of the RPS based on (3^6) tessellation without seven excessive members.

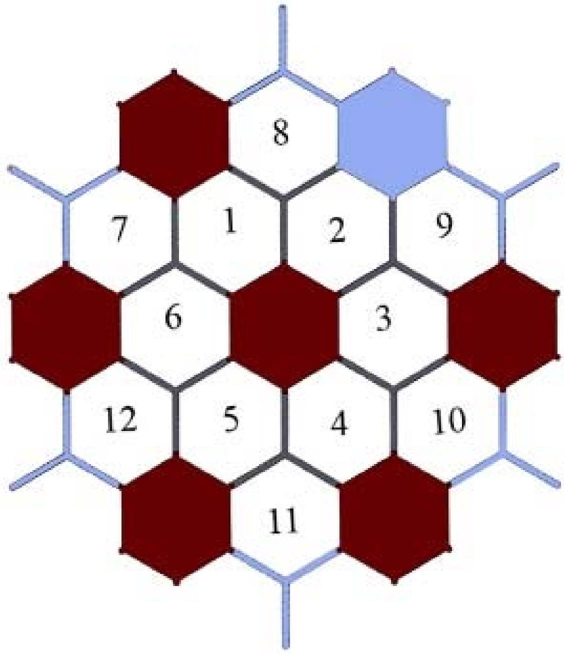


Figure 5.13.Simplest module of the RPS based on (3^6) tessellation

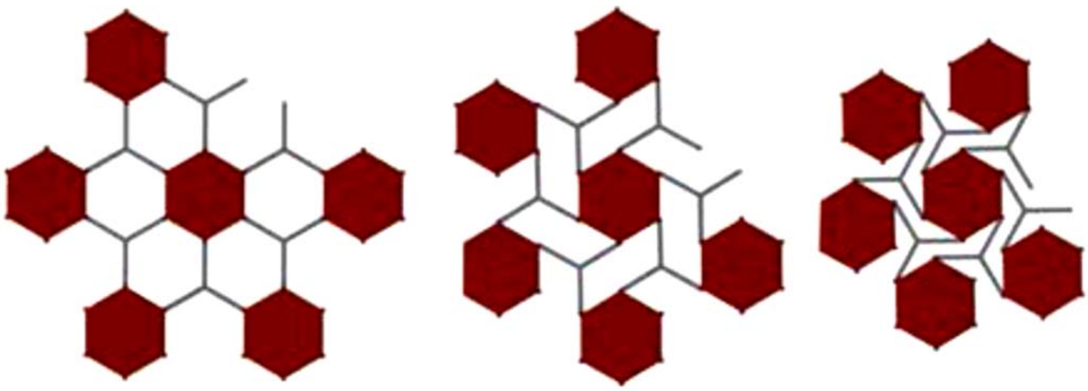


Figure 5.14. Retraction of the RPS based on (3^6) tessellation without seven excessive members

For the second case the simplest module will be iterated as shown in Figure 5.15. As seen in the same Figure the iterated tessellation has five shared loops numbered 1, 7, 6, 5, 12. Using equation 5.3 with variables as L=19, J=46 and q=12, the mobility of the mechanism can be calculated as M= 46 - 3.19 + 12 = 1. The excessive plates are shown in Figure 5.15 with blue colors and figure 5.16 shows the retraction of the mechanism without twelve excessive plates.

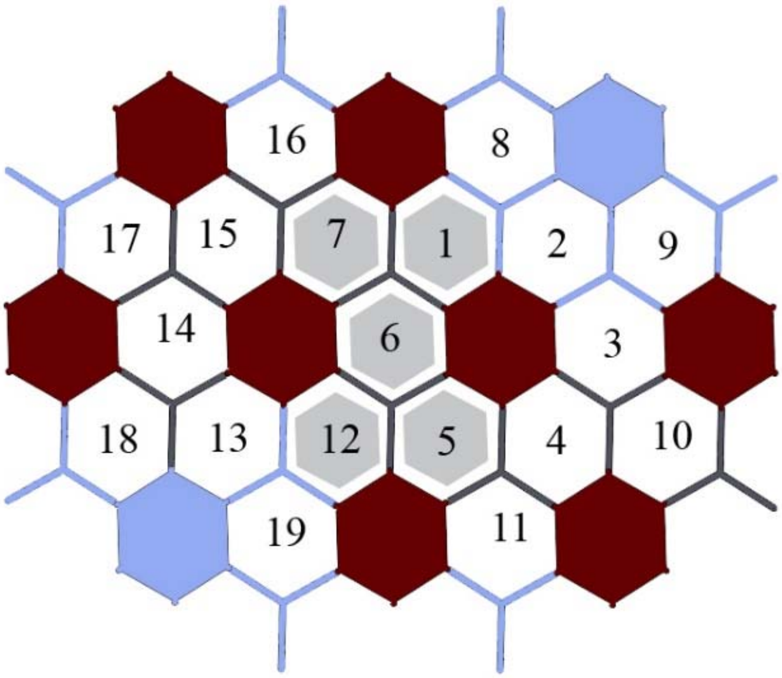


Figure 5.15. Iterated module with five shared loops and twelve excessive members

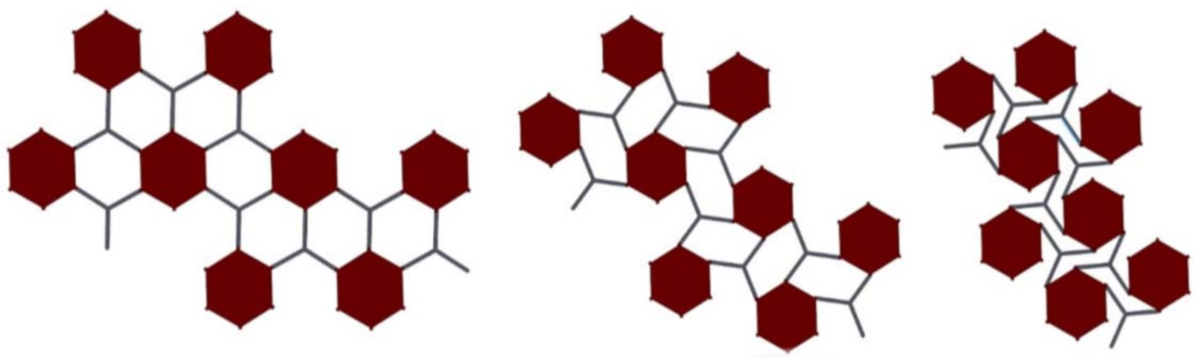


Figure 5.16. Iterated module without twelve excessive plates

For the last case the simplest module is two times iterated as shown in Figure 5.17. As seen in the Figure the iterated module has formed ten shared loops numbered 1,7,6,5,12,13,14,15,17,18. Using equation 5.3 with variables as L=26, J=62 and q=17, the mobility of the mechanism can be calculated as M= 62 - 3.26 + 17 = 1. The excessive plates are shown in Figure 5.18 with blue colors and figure 5.19 shows the contraction of the mechanism without seventeen excessive plates.

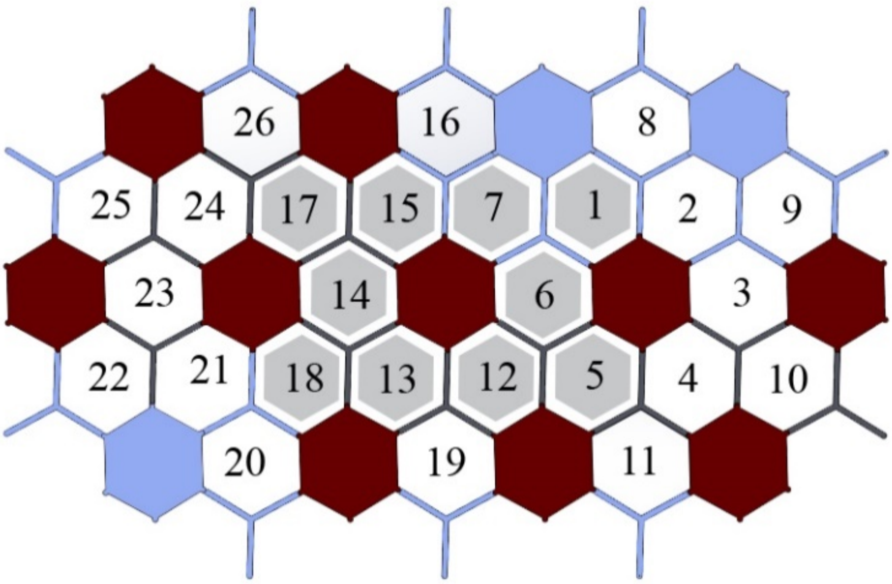


Figure 5.17. Two times iterated module with ten shared loops and seventeen excessive plates

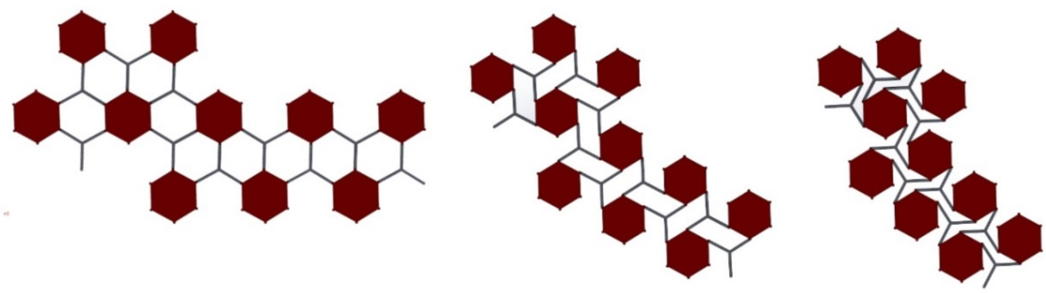


Figure 5.18. Two times iterated module without seventeen excessive plates

Note that in these examples, successive iterations add five additional shared loops and five excessive plates. The number of excessive plates added to the system is equal to the number of shared loops. In the light of this and considering the mobility calculations of (3^6) and (4^4) tessellations, a new theorem can be introduced to the literature related with the mobility calculations of the RPS based on regular tessellations and excessive links as below.

Theorem: *Number of excessive plates is equal to the number of excessive plates of simplest module plus the number of loops that are shared during the whole iteration process of the RPS modules.*

5.3. Retraction Capability of Scaled Modules

So far retractable plate structures were designed by the iteration of identical modules. As a result of this, design alternatives are restricted by using the open and closed forms of these retractable plate structures in the same scale. In order to increase the design alternatives, this section is dedicated to a novel idea where different scaled modules are iterated to form a RPS. Using the proposed idea, retractable modules can get endlessly small while at the same time they still provide the same expansion and contraction motion. In other words, modules of different sizes can be connected to each other without losing their retraction capability. In order to introduce this idea following procedure was applied to a RPS. Firstly an iterated module is selected and it is assembled with its ½ scale module and this procedure is continued as shown in Figure 5.19 and Figure 5.20.

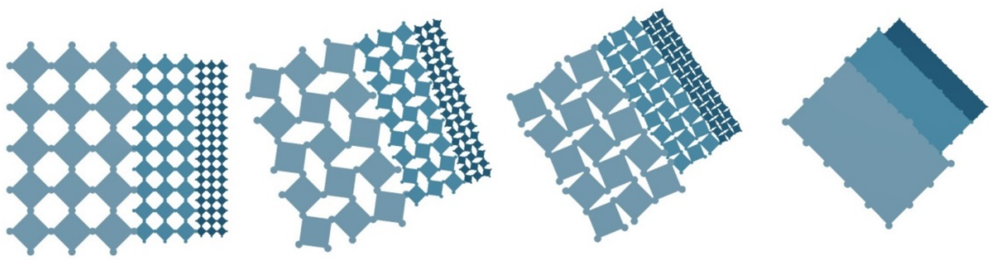


Figure 5.19. Assembly of Different Scale RPSs Based on 4^4 Tessellation

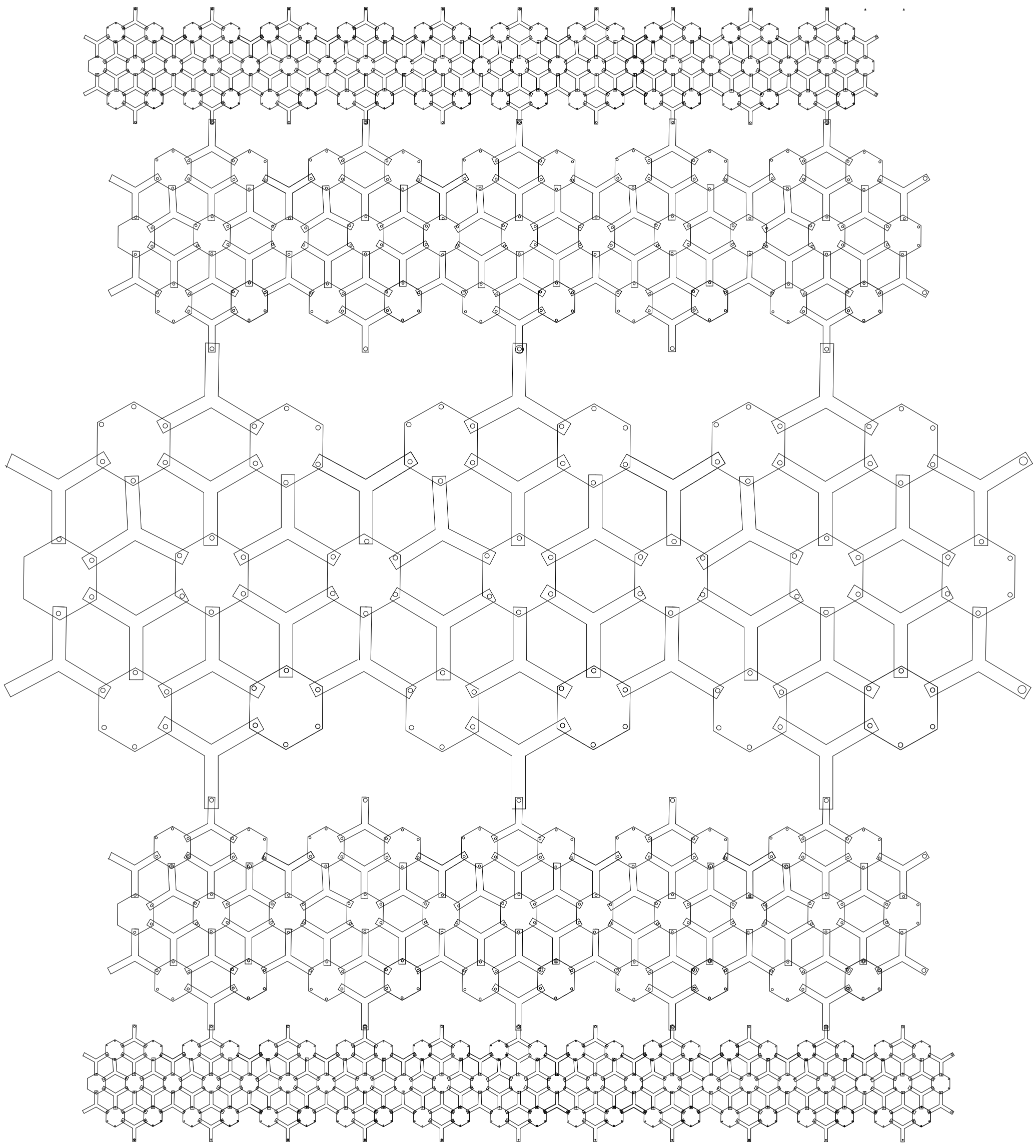


Figure 5.20. Assembly of different scaled RPS based on 3^6 tessellation

By the help of this procedure, selected RPSs can be iterated as many times with different scaled modules. At this point, mobility of the new RPS needs to be determined. For this purpose the mobility calculation of RPS based on 3^6 tessellation will be handled by using 3^6 tessellation and Alizade&Freudenstein's universal mobility equation 5.3.

It can be seen in Figures 5.21a and b that the iterated RPS with respect to the proposed idea is moveable. Figure 5.21c shows the same module with its two excessive plates. Using equation 5.3 with variables as L=11, J=32 and q=2, the mobility of the mechanism can be calculated as M= 32 - 3.11 + 2 = 1. After the removal of the excessive plates the mobility equation can be reformed with the variables L=9, J=28 and q=0. The final result is same again as M= 28 - 3.9 + 0 = 1. Figure 5.21b shows the retraction of the modules.

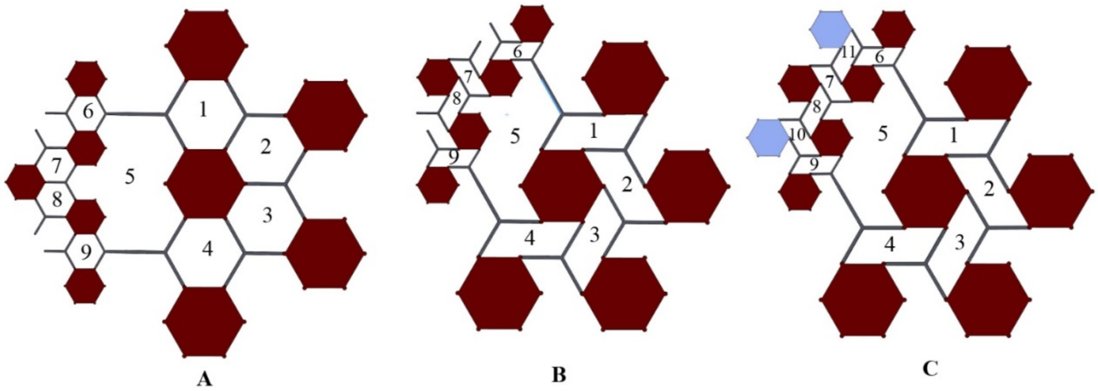


Figure 5.21. Retraction capability of the iterated scaled modules.

If the same analogy is used and the modules are iterated more as in Figure 5.23a, the designed RPS still stays moveable. Figure 5.23b shows the RPS's contraction capability without excessive plates. Using equation 5.3 with variables as L=16, J=47 and q=0, the mobility of the mechanism can be calculated as M= 47 - 3.16 + 0 = -1. Even though RPS does not meet mobility criterion it retracts because there is one passive joint as shown in Figure 5.22a. It is shown in blue circle. Figure 5.23 shows the contraction of the RRPS without passive joint and any excessive plate. Now there are totally 15 loops, 46 revolute joints. If the equation variables L=15, J=46 are inserted to the formula M= 46 - 3.15 = 1.

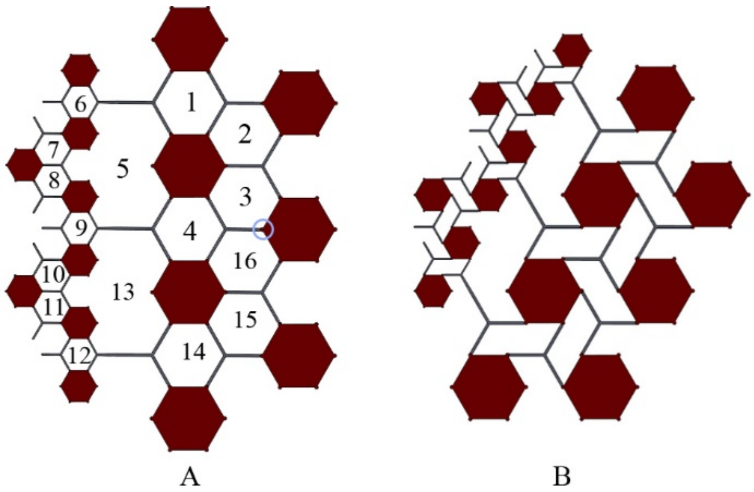


Figure 5.22. Contraction capability of the iterated scaled modules

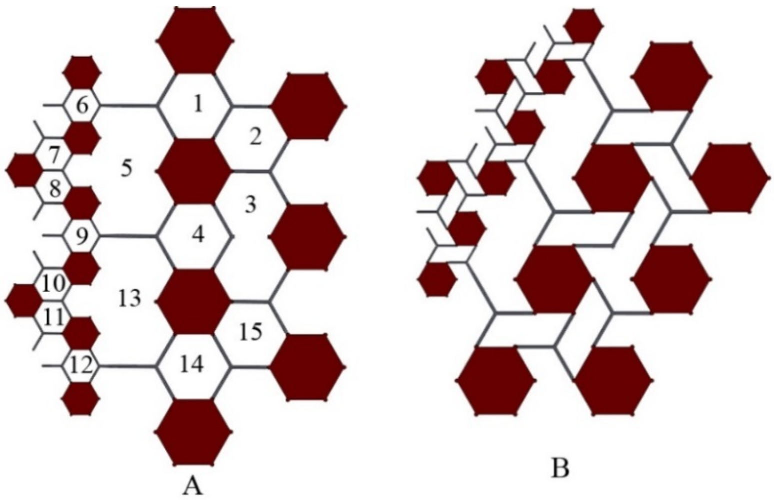


Figure 5.23. Contraction of the iterated scaled modules without passive joint

5.4. Conclusion

This chapter focused on the mobility analysis of RPSs. Mobility analysis was done by using Alizade&Freudenstein's universal mobility equation considering the excessive plates in the system. It was shown that these excessive plates can be eliminated one by one without affecting the retraction capability. Moreover, modules with different scales have been assembled by using a proposed analogy and it was proved that RPSs still achieve the retraction.

CHAPTER 6

CONCLUSION

The aim of this study was to develop a novel approach to design single DoF retractable plate strcutures based on 1- uniform tessellation that are able to be enclosed without any gaps or overlaps in their both closed and open configurations by eliminating the chance of interference between each other during their retraction and expansion phase. To achieve this aim, this thesis was executed as an interdisciplinary study between architecture, mechanism science and mathematics.

In a general point of view, there exist many contributions and achievements that have been acquired by the dissertation. The first achievement of the thesis is the fact that two general conditions were proposed to reveal a single DoF RPS. By considering the first condition architects and designers use less effort to understand RPS based 1-uniform tessellations are moveable or not moveable, single or multi degrees of freedom. Moreover, by considering the second condition, architects and designers use less effort to understand the RPS based on 1-uniform tessellations are able to be enclosed without any gaps or overlaps in their both closed and deployed configurations by eliminating the chance of interference between each other. This thesis proved that if tessellations do not satisfy the second condition, the RPSs retract and expand with gaps and overlaps. Due to the fact that, the issue of matching exactly with each other during the assembly is very crucial for design processes of RPS. This thesis offered another method that is called "vertex translation". Under favour of this easy method, regular polygon form can be transformed with respect to the second condition without changing the assembling order of the tessellation. As a result the generated RPS based on the selected tessellation can be enclosed without any gaps or overlaps in their both closed and open configurations by eliminating the chance of interference between each other during the retraction and expansion phase.

This thesis proved that a tessellation, where three edges meet at one vertex cannot be used to generate retractable plate structures solely. Thus, they need extra links. This thesis has presented two different methodologies to identify the required extra links with its joint positions, plate types and forms. Then a comparison was presented to display the benefits and drawbacks of the two methods with respect to each other.

Additionally this thesis has presented a new theorem for the literature of both architecture and mathematic about the relation between the duality of tessellation and the graph representation of retractable plate structures. This theorem claims that the dual of tessellation reveals the graph representation of RPS based on that tessellation.

Also, this thesis has introduced that any RPS that is based on five or six edges meet at one vertex have multi degrees of freedom. By considering this theorem about the relation between the tessellation and graph theory, this study has developed another methodology for systematic conversion of multi DoF retractable plate structures to a single DoF retractable plate structures. This method has revealed a key map for assembling processes. By the help of this, questions of whether an extra joint or link is needed throughout the design, what kind of plates, extra links or joints should be assembled together and in which order to be converted to a single degrees freedom RPS are answered.

Furthermore, this thesis has presented a discussion part about the relation for the iteration of RPS based on 1- uniform tessellation and symmetry of the selected tessellation. The discussion part claims that the symmetry group affect the iteration capability of tessellation.

Another achievement of this thesis was related with the mobility analysis. The RPS based on 1-uniform tessellations that are proposed throughout this thesis, the degrees of freedom of the related RPSs are always one. In order to prove this by using mobility calculations, a theorem is developed. Besides, a systematic approach of the assembly technique between different scales of RPS is proposed. In order to present their mobility calculations, numerous of them are assembled with respect to the proposed approach. Additionally, assembly of different scales of RPSs give the potential of usage of this system for any kind of formless planar surfaces.

Also one of the important contributions of this study is to combine three different discipline literatures with respect to the architectural point of view. In this thesis, retractable plate structures were investigated by considering their geometrical and kinematical principles. Their deficiencies and benefits were discussed. In addition to this, mathematical tessellation technique was thoroughly investigated by considering their geometrical and theoretical principles by using illustrated tessellation patterns.

In this thesis, two different conditions, two theorems and four methodologies were developed. These efforts were the main contributions to both kinetic architecture literature and practice of kinetic architecture. These developments present a "generalisation result" that has been made available to the architect and any designer to design retractable plate structures without having any complex engineering or mathematical knowledge. Thus, these results present potential advantage for the applications of retractable plate structures in architectural projects more than today. These generalisation results can also be investigated and be available in mechanical and mathematical point of view by the mathematicians and mechanical engineering researchers in the future. Thus, these results offer contributions to three different literatures at the same time.

This thesis has focused on the 1- uniform tessellations. The applications of the acquired results can be applied for the k-uniform tessellations as a future work by analysing the results. Also, the issue of iteration of retractable plate structures can be investigated deeply. Due to the fact that present studies of retractable plate structures are generally roof structures with circular plan shape, after the investigation of the iteration way, RPS can be applied to different parts of the building.

Finally, the investigation of a novel design approach to design retractable plate structures presents a new language for the design processes of kinetic structures. This study opens up many questions and new ideas for the kinetic architure research and applications for the future.

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APPENDIX A

GLOSSARY

Adaptable: Able to adjust to new conditions.

Assemble: To fit together the parts or pieces of

Kinematic Chain: System of interconnected links permitting relative motion of any one link with respect to the remaining links

Change: To make or become different.

Cover: To place something upon or over, so as to protect or conceal

Degree of Freedom: Number of independent variables that must be considered for input motion.

Dead Center Position (Dead Point): Configuration of a mechanism in which the input cannot be moved without assistance to the motion of another link.

Dual Tessellation: The dual of a regular tessellation is formed by taking the center of each polygon as a vertex and joining the centers of adjacent polygons.

Expandable: Having the capacity to be expanded

Expand: To open up or out: spread out.

Excessive Link: If the removal of a link from a mechanism does not change mechanisms mobility, that link is said to be excessive

Joint: Physical representation of a kinematic pair.

Kinematic Pair: Connection between two links restricting their relative motion.

Kinetics: Branch of theoretical mechanics dealing with the motion and equilibrium of bodies and mechanical systems under the action of forces

Link: Solid body as a mechanism element, having one or more kinematic pairs with other bodies.

Linkage: Kinematic chain whose joints are equivalent to lower pairs only.

Loop: Subset of links that forms a closed circuit.

Mechanism: Constrained system of bodies designed to convert motions of, and forces on, one or several bodies into motions of, and forces on, the remaining bodies**.**

Mobility: Degree of freedom

Motion: Changing position of a body relative to a frame of reference.

Passive Joint: If the removal of a joint from a mechanism does not change mechanisms end effector motion, that joint is said to be passive

Parallelogram: Special version of a four bar linkage where the opposite links are equal

Pantographic:

Plane: A two-dimensional area in geometry.

Planar: of or pertaining to a geometric plane.

Revolute Joint: Joint that allows only rotary motion between two links.

Polygon: A plane figure with many sides

Tessellation: To cover a plane without any gaps or overlaps.

Tilling: Synonym meaning of tessellations.

Regular: (of a polygon) having all sides and angles equal.

Static: adj. (physics) (of force) acting by weight without producing movement.

Symmetry: a geometrical or other regularity that is possessed by a mathematical object and is characterized by the operations that leave the object invariant

Transform: ~sth/sb (from sth) (into sth): to change the appearance or character of sth/sb completely.

Translation: Motion (or component of the motion) of a rigid body in which each straight line rigidly connected with the body remains parallel to its initial direction.

VITA

Aylin GAZİ GEZGİN

Education :

2010-2016 Ph.D in Architecture, Izmir Institute of Technology, Department of Architecture
Thesis: " A New Design Approach For Retractable Plate Structures Based On One Unıform Tessellatıons"

2008-2010 Master of Architecture**,** Izmir Institute of Technology, Department of Architecture.
Thesis: **"**A Method To Design Kinetic Planar Surface With Mathematical Tessellation Techniques.

2003-2007 Bachelor of Architecture, Eskişehir Osmangazi University, Department of Architecture

Work Experience:

2010-Present Ministry of Culture and Truism

2008-2010 G-G Research and Development

2006-2006 TAV –Esenboğa Airport

Publication:

Gazi, Aylin and K. Korkmaz, Izmir. "8.8.4 Tesselasyon Kullanarak Genişleyebilen Strüktür Tasarımı." *Proceedings* of *Uluslararası Katılımlı 17.Makina Teorisi Sempozyumu,* 441-447.2015.

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Awards:

2015 Best Student Paper on *Uluslararası Katılımlı 17.Makina Teorisi Sempozyumu,* 8.8.4 Tesselasyon Kullanarak Genişleyebilen Strüktür Tasarımı.