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A computational analysis of Turkish *makam* music based on a probabilistic characterization of segmented phrases

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This study targets automatic analysis of Turkish *makam* music pieces on the phrase level. While *makam* is most simply defined as an organization of melodic phrases, there has been very little effort to computationally study melodic structure in *makam* music pieces. In this work, we propose an automatic analysis algorithm that takes as input symbolic data in the form of machine-readable scores that are segmented into phrases. Using a measure of *makam* membership for phrases, our method outputs for each phrase the most likely *makam* the phrase comes from. The proposed *makam* membership definition is based on Bayesian classification and the algorithm is specifically designed to process the data with overlapping classes. The proposed analysis system is trained and tested on a large data set of phrases obtained by transferring phrase boundaries manually written by experts of *makam* music on printed scores, to machine-readable data. For the task of classifying all phrases, or only the beginning phrases to come from the main *makam* of the piece, the corresponding F-measures are .52 and .60 respectively.

Keywords: *maqam*; *makam*; phraseology; computational musicology; Turkish music

1. Introduction

Makam/*maqam*/*mugam*/ . . . is a key concept in music of a large geographical region (including many north African, middle eastern, and Asian countries), and it is very rarely studied using computational methods. In this study, we consider computational analysis of the traditional *makam* music of Turkey, mainly analysing symbolic data presented in the form of machine-readable scores.

A large number of different descriptions are available for the *makam* concept in Turkish music literature. Some examples are: "*Makam* is a process of occurrence. It is a specific form of a musical scale that characterizes itself by an organization of intervals and various constitutive relations" (Yekta 1924); "the feature that is created by the relation of pitches of a scale or melody and the tonic and/or dominant" (Arel 1968); "a practical melody theory, grouping melodies by families or categories that are distinguished by the use of careful microtonal inflections of certain tones according to custom, together with idealized notions of melodic contour" (Stubbs 1994, 1). A list of definitions and a review of *makam* music theories are available in Can (1993); Elsner and Pennanen (1997); Ayangil (2001); Öztürk (2011); and Yöre (2012), which show that the concept of *makam* has been considered using basically two different conceptualizations: "a scale-centred

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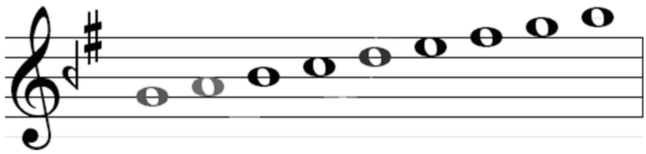

Figure 1. Scale for *makamlar* Hüseyni, Neva, Muhayyer, Gülizar, Gerdaniye, Tahir. Tonic is note Dügah (A) and leading tone is note Rast (G).

approach" and "a melody-centred approach." Powers and Wiering (n.d) interpret the *makam* concept as a mode in the sense of a particularized scale or generalized tune, in the middle of a continuum between scale and tune on its two ends.

While the current "official" theory of Turkish *makam* music as presented by Arel (1968) considers scales and hierarchy of certain tones as a central point for describing *makamlar* (plural of *makam*), this formulation has been subject to criticism by many musicologists (Ay and Akkal 2008). For instance, Öztürk (2011) considers this scale-centred approach as the result of a Westernization process and claims that a melody-centred approach should be preferred. A large part of *makam* literature stresses the importance of melodic progression, "*seyir*." As outlined in (Ayangil 2001; Çelik 2001), most of the historical texts also present the *makamlar* by descriptions of melodic progression rules. There are typically three types/classes of progressions stated in almost all theory books: ascending, ascending–descending (or alternatively "*seyir* in the mid-register"), and descending which describes the average shape of the overall melodic progression.

In today's Turkish *makam* music practice, we find many *makamlar* sharing the same scale and tonic and differing only in melodic progression. Figure 1 presents the scale used in six different *makamlar*, which are mainly differentiated by their melodic progression.

Given the importance of melodic organization, it is interesting to note that computational analysis efforts dedicated to analysis of *makam* music melodies is limited to a few attempts at finding the most repeated motifs in score databases. (We present a review of these studies in Section 2.2. An extensive review of computational studies on Turkish *makam* music can be found in Bozkurt, Ayangil, and Holzapfel 2014.) Finding the most repeated motifs, however, has very little use in analysis of *makam* music: *makamlar* share same/similar paths with other *makamlar* and a piece in one *makam* contains phrases (sometimes even stereotyped phrases) of another *makam*. For instance, a piece in *makam* Muhayyer may contain phrases also used in pieces from *makam* Hüseyni. Musicological studies for analysis of *makam* music melodies mainly target assigning tetra/penta-chord labels or *makam* labels to phrases while there is one main *makam* assigned for the piece as an entity. The piece can contain phrases borrowed from different *makamlar*. However, no agreed methodology is available to date to perform assignment of a phrase to a specific *makam*.

In this paper we address the challenge of studying *makam* membership[1] of a phrase and further propose an analysis method based on a membership measure that outputs a representation of the context change within a piece. To this end, we have designed a score database of manually segmented phrases in 15 different *makamlar*. The phrase boundaries are manually inserted by experts of *makam* music on printed scores and then transferred to machine-readable data using specifically designed editing software. These manually segmented phrases were then assumed to come from overlapping classes. As stated previously, two different *makamlar* may use the same

[1] *Makam* membership refers to the probability of an entity (phrase in our case) being assigned to a specific *makam*. While we consider it here as a measure of *makam* specificity of a phrase, its use in music practice is not well defined. The relevance of *makam* membership in music practice is an open topic for ethnomusicology and needs dedicated field work.

scale. Pieces from these *makamlar* may include very similar phrases and only differ in overall melodic progression and/or emphasis levels of certain scale degrees.

The original contributions of this paper are the *makam* membership definition based on Bayesian classification, the algorithm to process data with overlapping classes, and the method proposed for analysis of a piece. We showed through tests on a database containing 199 pieces from 15 different *makamlar* that an accuracy of 0.5208 is obtained for all phrases and an accuracy of 0.6021 is obtained on the first four phrases of the pieces. Since the pieces involve phrases from different *makamlar* (especially in *makam* transition parts), we do not expect an assignment of each phrase to the *makam* of the piece. Hence, the accuracy values presented should not be considered as a quantitatively exact measure of algorithm efficiency.

Our paper is structured as follows. First, a review of analysis for *makam* music melodies is presented in Section 2. Description of the symbolic *makam* music dataset used in this study is presented in Section 3. Section 4 is dedicated to the proposed computational method: the membership definition, implementation, and the methodology of the resulting phrase recognition algorithm. Section 5 presents results and discussions.

2. Analysis of makam music melodies

2.1. *Analysis methods in musicology resources*

"Analysis of Turkish *makam* music[2] melodies" in musicology literature mainly refers to specifying/listing tetra/penta chords (sorted note sequences that constitute a subdivision of a scale, *cins* (Turkish), *jins*, *ajnas* (Arabic)) used for each melodic segment (e.g. Kılınçarslan 2006; Eroy 2010; Gönül 2010). This approach, though criticized by many scholars[3] for its low usefulness, is considered as a way to represent context change throughout the piece. While almost every scholar agrees that understanding the *makam* concept necessitates studying context changes in melodic progressions in pieces or improvisations, there is no methodology agreed upon for such a representation and ways to interpret it.

Figure 2 shows an example of analysis performed by a *makam* theory teacher from the Istanbul Technical University Turkish Music Conservatory (the oldest and biggest Turkish music conservatory today). Each segment is tagged with the lowest note of the main chord used (Neva, Buselik, Dügah, etc.) and the name of the chord (Buselik, Nişabur, Hicaz, etc.). In our personal communication with various local masters to collect similar data, they refused to perform the task, arguing that listing tetra-chords for each phrase is useless since what matters is the function and it would be different in the context of different *makamlar*. Ederer (2011) proposes similar but a clearer representation (since the link between the chord and the *makam* is also stated for *makam* transitions) in a table form. Again, the aim is to represent context change within the piece or improvisation.

2.2. *Review on computational methods for melodic analysis of* makam *music*

Computational analysis literature on Turkish *makam* music melodic phrases is limited to studies on symbolic data due to the difficulties involved in audio analysis. Heterophony (simultaneous variation of a single melody by several instruments or voices) and frequent use of ornamentations

[2] The term "*makam*" mainly refers to a modality system and is used in many genres (such as folk, art and various popular music genres). Here, we mainly consider a collection including Ottoman/Turkish traditional/classical/art music ("geleneksel/klasik Osmanlı/Türk (sanat) musikisi") and folk music pieces.

[3] Private communication with experts including Reha Sağbaş, Ruhi Ayangil, Korkutalp Bilgin, and Cem Çırak, 2013.

Figure 2. Analysis example by an expert in Turkish *makam* music education.

are the main obstacles for reliable estimation of the fundamental frequency, hence analysis on melodies, directly on audio signals.

In most of the computational studies on symbolic data, one of the important deficiencies is representation of microtonal intervals of Turkish *makam* music. Arel theory (Arel, 1968) uses a 24-tone (per octave) Pythagorean system for tuning. Mismatch of the Arel theory with the practice is well known (Bozkurt et al. 2009) but still, it provides the most commonly used notation system today. Large archives containing scanned images of Turkish *makam* music written in Arel notation are publicly available (e.g.: http://notaarsivleri.com/) and commonly used in music circles today.

Until very recently, computational studies had to rely on data collected using notation software designed mainly for equal tempered representations. Popular MIDI (Musical Instrument Digital Interface[4]) editors or other 12-Tet (tone equal temperament) notation software have been used to collect machine-readable symbolic data for Turkish *makam* music. It had been common practice to omit Turkish *makam* music specific accidentals and perform analysis on the resulting data quantized to 12-Tet (e.g. Gedik et al. 2005). In various other studies, software capable of including Turkish *makam* music specific accidentals on staff were used to create score databases and the scores were further exported in XML format for computational analysis. Unfortunately these studies provide very little information about how Turkish music accidentals have been processed (if discarded or not) (e.g. Yener 2004; Yener and Aksu 2004).

It is only very recently that microtonal notation editor software (such as Mus2: http://www.mus2.com.tr/) has been introduced and a few projects on data collection have been launched. Recently, a large collection of machine-readable microtonal scores has been made publicly available (Karaosmanoğlu 2012) and part of our work (Bozkurt, Ayangil, and Holzapfel 2014) is dedicated to contributing to such data collection efforts. The data used in our study are collected using software dedicated to Turkish *makam* music.

It is interesting to note that many computational studies for analysis of Turkish *makam* music use pitch class and/or interval histograms as features of melodic progression (Eroy 2010; Gönül 2010; Sümbüllü and Albuz 2011). Yener (2004) studied automatic detection of stereotyped phrases for *makam* in solo improvisations and in Turkish folk pieces (Yener and Aksu 2004). In these studies, standard statistical measures were obtained (using SPSS) for note sequences and the most frequent three- to eight-note patterns were listed as a result. A similar study was carried out by Müezzinoğlu (2004), this time using SQL (Structured Query Language) for statistical analysis for a specific type of Turkish folk music, *zeybek*. This study also presents note transition probabilities for the

[4] The complete MIDI 1.0 detailed specification is available from www.midi.org

given database. Sümbüllü and Albuz (2011) also used XML data and SQL, SPSS, and Excel to gather statistical information. None of these studies consider studying melodic progression and context change within pieces. The fact that *makamlar* share common phrases/paths and pieces contain phrases from various *makamlar* is simply discarded.

In computational analysis literature on other music traditions, a large group of work is dedicated to melodic pattern extraction with the aim of efficient song retrieval. Comprehensive review of this domain can be found in Rolland (1999), and Meredith, Lemstrom and Wiggins (2002). Most of these studies are based on representations that combine melodic and rhythmic dimensions (Conklin 2001, 2010; Lartillot and Ayari 2009) and pattern discovery leads to capturing melodic patterns, rhythmic patterns, and combinational patterns repeated in large databases. Within the context of analysis of Turkish *makam* music melodies, if the aim is not limited to retrieval of a specific pattern, repeated pattern extraction would only become informative when detected patterns are related to *makam* music specific concepts. The only computational work (we could find) to study the link between *makam* concepts and melodic segments (within Tunisian *maqam* music) is Lartillot and Ayari (2009). However, their work is limited to exposing how melodic boundaries and *maqam*-specific information are related based on listeners' performance of segmentation. Şentürk (2011) used Variable-Length Markov Models (VLMMs) to study the predictability of improvised Turkish folk melodies. Apart from representing the pitch scale by 17 notes in an octave, no *makam* music-specific information is included in the system design. Şentürk's conclusion is that the melodies are highly predictable.

Gündüz and Gündüz (2005) and Tarikci (2010) studied the fractal dimension of notes sequences in Turkish *makam* music pieces. In these studies, symbolic data in the Arel notation system is used and fractal dimension is computed as a measure of complexity of melodies. Tarikci reported that some *makamlar* exhibit more irregular patterns than others and "Turkish art music songs show a fractal behavior." In Gündüz and Gündüz (2005), the authors propose various graphical representations (and the underlying mathematical models) for *makam* music melodies including a radial distribution of notes, animal diagrams, etc. They demonstrate that these representations have some potential in performing structural analysis via visual detection of similarities in different sections of a piece. The design of an algorithm for performing such an analysis in an automatic way is not considered.

In the computational analysis literature, to the best of our knowledge, no study tackles the problem of linking phrases of a piece to a pool of different *makamlar* although it is well-known that a piece in one *makam* contains melodies from other *makamlar*. All studies mentioned above are based on the assumption that the *makam* of the piece defines the *makam* of each phrase in the piece. While the masters of this music emphasize the importance of analysis to study the context change within a piece, no computational study targets this either. As a result, the literature does not provide us any means to perform automatic analysis of melodic phrases of a given Turkish *makam* music piece.

3. The data

Our symbolic database is comprised of scores written using the Mus2 microtonal notation software (http://www.mus2.com.tr/) and further converted to the machine readable text format of SymbTr (Karaosmanoğlu 2012). We have chosen to focus on the more popular 15 *makamlar*, also taking into consideration the sorted list of available scores in each *makam* in Çevikoğlu (2007).

Scores were printed on sheets and given to an expert. The expert was asked to indicate phrase boundaries, as he would do it for analysis based on melodies. There was no time pressure, the

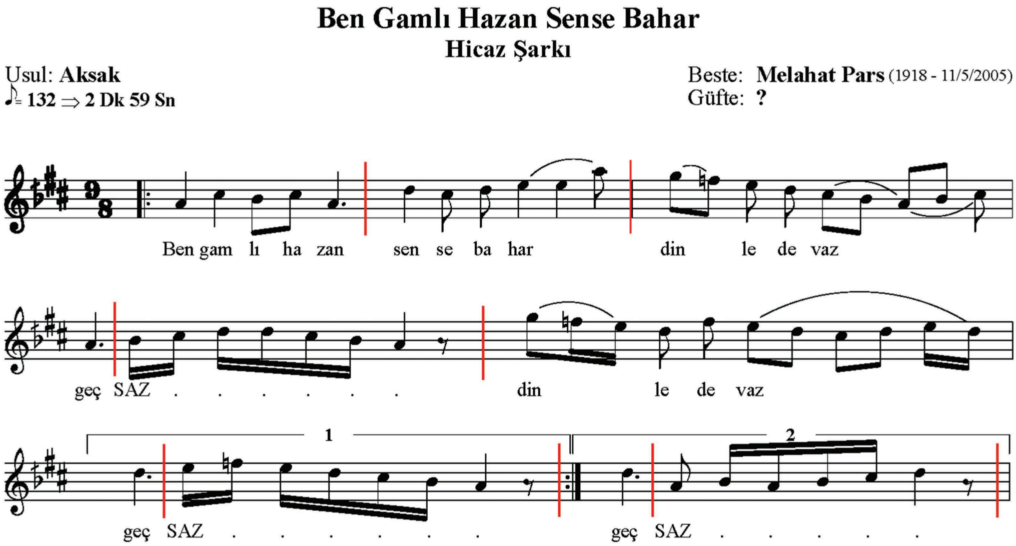

Figure 3. Example excerpt of a score from the database, which includes melodic phrase boundaries labelled by an expert.

Table 1. Machine readable text version of the example in Figure 3.

Index	Code	Note53	NoteAE	Comma53	CommaAE	Num.	Denum.	Ms	LNS	VelOn	Syll
1	9	La4	A4	305	305	1	4	909	95	96	Ben
2	9	Do5#4	C5#4	322	322	1	4	909	95	96	gam
3	9	Si4b4	B4b4	310	310	1	8	455	95	96	li
4	9	Do5#4	C5#4	322	322	1	8	455	95	96	ha
5	9	La4	A4	305	305	3	8	1364	95	96	zan
6	53			0	0	0	0	0	0	0	
7	9	Re5	D5	327	327	1	4	909	95	96	sen
8	9	Do5#4	C5#4	322	322	1	8	455	95	96	se
9	9	Re5	D5	327	327	1	8	455	95	96	ba
10	9	Mi5	E5	336	336	2	4	1818	95	96	har
11	9	La5	A5	358	358	1	8	455	95	96	
12	53			0	0	0	0	0	0	0	
13	9	Sol5	G5	349	349	1	8	455	99	96	din
14	9	Fa5	F5	340	340	1	8	455	95	96	

expert could do segmentation any time within a three-month period, and he could use his instrument when he liked. His segmentations were manually exported to the format in Table 1 using a specially designed interface. In Figure 3 and Table 1, we present an example as tagged by the expert and as shown in the machine-readable format.

In the machine readable format, segmentation is represented by lines indicating the code (second column in Table 1) as 53. The data set is composed of 199 pieces manually segmented into 8065 phrases in 15 *makamlar* (number of phrases in each *makam* is available in the confusion matrix presented in Table 5 later and the number of pieces is presented in Table 2).

While a detailed explanation of the SymbTr format can be found in (Karaosmanoğlu 2012), we present a short summary here. "Code" signifies a normal note (#9) or ornamentation or segmentation (#53). The most commonly used ornamentation codes are as follows: #7 for tremolos, #8 for acciaccatura, #12 for trills, and #23 for mordent. "Note53" and "Comma53" include the pitch information in a 53Tet resolution and "NoteAE" and "CommaAE" include the pitches specified as in the Arel theory. The latter is obtained by direct conversion of staff notation into text where

Table 2. Number of pieces in each *makam*.

makam	Acemaşiran	Beyati	Buselik	Hicaz	Hicazkar	Hüseyni	Hüzzam	Mahur
# pieces	9	3	8	34	14	24	18	7
makam	Uşşak	Nihavent	Rast	Saba	Muhayyer	Segah	Kürdilihicazkar	
# pieces	12	7	11	7	11	13	21	

the former is a corrected version of the pitch by master musicians. Here, "Note" stands for "name of the pitch" and "comma" stands for the interval of the pitch with respect to C_1(represented in Holderian/Mercator commas obtained by equal division of an octave by 53). "Num.", "Denum.", and "Ms" columns specify the duration. "Syll." contains the lyrics. "VelOn" is used to specify velocity dynamics. "LNS" (Legato / Normal / Staccato) indicates how tied or detached the notes are. This text representation facilitates viewing the content of the data without need of any specific program but just a text editor and easy data access (compared with, for example, microtonal MIDI files).

4. *Makam* membership of a phrase and its use in melodic progression analysis of individual pieces

In this section, we describe the methodology used to determine the *makam* memberships of individual phrases identified manually by expert musicians on the *makam* music dataset described in the previous section. We begin by elucidating the mathematical details of the membership function that assigns a given phrase to the most likely *makam*. Next, we provide details on the computation of the model-specific parameters that define the membership function from the available phrase data. Finally, we formulate the procedure we have used to determine the *makam* membership of all phrases in our database using the derived membership function along with the performance measures employed to evaluate the accuracy of the resulting *makam* assignments.

4.1. *Membership function*

Phrases can be represented as an ordered sequence of notes at a specific pitch and with a specific duration. This suggests defining a musical phrase s consisting of ℓ_s notes as an ordered collection of pairs

$$s = \{(f_i, d_i)\}$$

where f_i and d_i denote the pitch and the duration of the ith note in the phrase, respectively, for $i = 1, 2, \ldots, \ell_s$. The pitch information is specified in Holderian/Mercator commas and the duration is specified in full notes. As a membership function, we seek a map $\mathbf{M}$ defined by

$$\mathbf{M}(s) = m$$

that links a given phrase s with one of the possible *makamlar*, denoted by m.

In order to derive a membership function, we adopt a Bayesian approach and exploit the probabilistic relationship between the phrases extracted from the musical pieces and the *makamlar* according to which the pieces themselves were written. To this end, we have considered the random variables S and M to represent the musical phrase and the *makam* of the corresponding piece, respectively. In this setting, the optimal membership function can be expressed as

$$\mathbf{M}^{opt}(s) = \arg\max_{m} \Pr\{M = m|S = s\}$$

where $\Pr\{M = m|S = s\}$ denotes the conditional probability[5] of the *makam* m given the phrase s. Using Bayes' rule, we obtain

$$\Pr\{M = m|S = s\} = \frac{\Pr\{S = s|M = m\}p_M(m)}{p_S(s)} \tag{1}$$

with the probability mass functions of the phrases and the *makamlar* expressed by $p_S(s)$ and $p_M(m)$, respectively, along with $\Pr\{S = s|M = m\}$ representing the conditional probability of the phrase s given the *makam* m. Note that the probability mass function $p_M(m)$ refers to the occurrence rate of the different *makamlar* over all phrases, while $p_S(s)$ represents the more esoteric probability of observing a specific phrase s in all of *makam* music, with

$$p_S(s) = \sum_m \Pr\{S = s|M = m\}p_M(m).$$

Since they figure as a constant multiplicative factor in the expression for the conditional probability $\Pr\{M = m|S = s\}$ and can be obtained from the conditional probabilities $\Pr\{S = s|M = m\}$ for the different *makamlar* m along with $p_M(m)$, the exact values of $p_S(s)$ are inconsequential to the *makam* membership assignment of the phrases. The challenge in formulating the membership function thus rests on expressing the conditional probability $\Pr\{S = s|M = m\}$ and its natural logarithm. We propose two *different* quantities for this conditional probability, and consequently two different membership functions. The first quantity, expressed in equation (2), ignores durations. Equations (1) and (2) together give the membership function in equation (3) without durations. The second proposed quantity for the conditional probability, expressed indirectly in equation (4) via the natural logarithm, takes durations into consideration. Equations (1) and (4) together give the membership function in equation (5) which does consider durations. After these, we consider several other membership functions as well.

In this work, we have used pitch distributions of the phrases to characterize the probability structure linking them to different *makamlar*. This allows us to decompose the conditional probability $\Pr\{S = s|M = m\}$ as

$$\Pr\{S = s|M = m\} = \prod_{i=1}^{\ell_s} p_{F|M}(f_i|M = m) \tag{2}$$

where $p_{F|M}(f|M = m)$ represents the probability of observing a note at pitch f in *makam* m, with F denoting the random variable of pitch. This leads to the membership function

$$\mathbf{M}_f(s) = \arg\max_m \left(\log p_M(m) + \sum_{i=1}^{\ell_s} \log p_{F|M}(f_i|M = m) \right) \tag{3}$$

using the natural logarithm function that makes it easier to handle small probability values. The argument to the maximum operator acts as a discriminant function that takes on separate values

[5] The conditional probability Pr{A|B} represents the probability of a chance event A given that the chance event B has occurred already. Note that this deviates from the probability Pr{A} that does not take into account the occurrence of the event B. In this respect, the conditional probability incorporates the additional information from the event B to calculate the probability of A more accurately.

for the different *makamlar*, and the assignment of the phrase s is made to the makam for which this value is maximal.

Note that this decomposition disregards the joint probability structure of the notes (f_i, d_i) as well as the specific order in which they come together to form a phrase, and considers only the number of times each pitch is visited during the phrase. The strong statistical inference from the pitch distributions of musical pieces to the underlying *makam* has been demonstrated in earlier works. Ünal, Bozkurt, and Karaosmanoğlu (2014) have studied the automatic *makam* classification problem using n-grams where they report that even 1-gram information, which corresponds to simply pitch class distributions, is highly discriminative. Gedik and Bozkurt (2010) and Ioannidis, Gómez, and Herrera (2011) have shown that pitch distributions can be effectively used for *makam* classification of audio data.

As an alternative, we also consider the following decomposition for a different log conditional probability

$$\log(\Pr\{S = s|M = m\}) = \frac{\ell_s}{\sum_i d_i} \sum_{i=1}^{\ell_s} d_i \log\left(p_{F|M}(f_i|M = m)\right) \tag{4}$$

which takes into account the duration of the notes as well as their pitch. The multiplicative factor ensures that when all notes have equal duration, the earlier duration-independent expression is obtained.

Note that this decomposition captures the relationship between the notes (f_i, d_i) and the phrase s when the phrase is uniformly sampled in time for its total duration. To see this, consider the pitch signal $f(t)$ of a phrase s defined as follows.

$$f(t) = \begin{cases} f_1 & \text{if } 0 \leq t < d_1 \\ f_2 & \text{if } d_1 \leq t < d_1 + d_2 \\ \vdots & \quad \vdots \\ f_{\ell_s} & \text{if } d_1 + d_2 + \ldots + d_{\ell_s - 1} \leq t < d_1 + d_2 + \ldots + d_{\ell_s} \end{cases}$$

The conditional probability $\Pr\{S = s|M = m\}$ can now be expressed as

$$\begin{aligned} \Pr\{S = s|M = m\} &= \prod_{k=1}^{\frac{\sum_i d_i}{\Delta t}} p_{F|M}(f(k\Delta t)|M = m) \\ &= \prod_{i=1}^{\ell_s} \left(\prod_{k=1}^{\frac{d_i}{\Delta t}} p_{F|M}(f_i|M = m) \right) \\ &= \prod_{i=1}^{\ell_s} \left(p_{F|M}(f_i|M = m)\right)^{\frac{d_i}{\Delta t}} \end{aligned}$$

where $\Delta t \ll 1$ denotes the sampling period, which is assumed to be an integer divisor of each d_i. Clearly, then,

$$\log(\Pr\{S = s|M = m\}) \propto \sum_{i=1}^{\ell_s} d_i \log\left(p_{F|M}(f_i|M = m)\right)$$

since Δt becomes a constant dividing factor following the logarithm, and the alternative decomposition for $\log(\Pr\{S = s|M = m\})$ noted in equation (4) follows (up to a factor).

The corresponding membership function then becomes

$$\mathbf{M}_{f,d}(s) = \arg\max_m \left(\log p_M(m) + \frac{\ell_s}{\sum_i d_i} \sum_{i=1}^{\ell_s} d_i \log p_{F|M}(f_i|M=m) \right). \tag{5}$$

Note that the factor before the sum of logarithms serves to achieve equality of the membership functions in equations (3) and (5) when the note durations are equal. Given that the statistical relationship between the phrases and the *makamlar* can be expressed over the notes that make up a phrase, we have derived additional membership functions that capture different aspects of the underlying correlation structure. To this end, we have first expressed the conditional probability $\Pr\{M = m|F = f\}$ as

$$\Pr\{M = m|F = f\} = \frac{p_{F|M}(f|M=m)p_M(m)}{p_F(f)}$$

again using Bayes' rule, where $p_M(m)$ now represents the prior probability of the *makam m* over all pitches observed in *makam* music. Then, the cumulative evidence expressed by the product

$$\prod_{i=1}^{\ell_s} \Pr\{M = m|F = f_i\} = \prod_{i=1}^{\ell_s} \frac{p_{F|M}(f_i|M=m)p_M(m)}{p_F(f_i)}$$

identifies to which *makam m* the phrase $s = \{(f_i, d_i)\}$ should belong, as it is expected to attain larger values for the more plausible *makam*s. This suggests the function

$$\mathbf{M}_f^{note}(s) = \arg\max_m \left(\ell_s \log p_M(m) + \sum_{i=1}^{\ell_s} \log p_{F|M}(f_i|M=m) \right)$$

as an additional membership function, using the natural logarithm to convert the product of probabilities into sums, and leaving the term due to the probabilities $p_F(f_i)$ out as they are the same for all *makamlar* and do not affect the assignment of the phrase into the more likely *makam*. Note that an immediate variation on $\mathbf{M}_f^{note}(s)$ can be obtained by factoring in the note durations to define yet another membership function $\mathbf{M}_{f,d}^{note}(s)$ as

$$\mathbf{M}_{f,d}^{note}(s) = \arg\max_m \left(\ell_s \log p_M(m) + \frac{\ell_s}{\sum_i d_i} \sum_{i=1}^{\ell_s} d_i \log p_{F|M}(f_i|M=m) \right).$$

Following the reasoning that ties the *makam* assignment of a given phrase to the posterior probabilities of the different *makamlar* on the individual notes of the phrase further, we have also defined

$$\mathbf{M}_f^{ave}(s) = \arg\max_m \left(p_M(m) \sum_{i=1}^{\ell_s} p_{F|M}(f_i|M=m) \right)$$

and

$$\mathbf{M}_{f,d}^{ave}(s) = \arg\max_m \left(p_M(m) \sum_{i=1}^{\ell_s} d_i p_{F|M}(f_i|M=m) \right)$$

as two additional membership functions. In essence, $\mathbf{M}_f^{ave}(s)$ and $\mathbf{M}_{f,d}^{ave}(s)$ compute a linear average of the *makam* probabilities conditional on each note of the phrase, weighted equally in the former and according to the note durations in the latter.

Note that computation of the different membership functions described above requires the availability of several conditional probabilities. Details of the estimation of these probability distributions along with the remaining details on the implementation of the proposed phrase classification strategy are given in the next section.

4.2. *Implementation*

The computation of the class membership function derived in the previous section requires the probability mass functions $p_{F|M}(f|M=m)$ and $p_M(m)$ for all pitch levels f and *makamlar* m. In an idealized case, these probabilities would be expected to govern the frequency of occurrence of all notes at the various pitch levels observed across the whole of *makam* music. In reality, however, these probabilities are not available and have to be estimated from sample *makam* music pieces.

In this work, we have estimated the probabilities $p_{F|M}(f|M=m)$ and $p_M(m)$ from the symbolic database of *makam* music pieces described in Section 3. Specifically, for $p_{F|M}(f|M=m)$, we have collected the frequency of the notes at a given pitch f in all pieces in our database composed in the *makam* m via

$$p_{F|M}(f|M=m) \approx \frac{1}{Z} \sum_{(f',d')\,\mathrm{in}\,D_m} \mathbf{1}(f=f')$$

where the summation is carried out over all notes observed in pieces written in *makam* m in the database denoted by D_m, Z is the count of all such notes given by

$$Z = \sum_{(f',d')\,\mathrm{in}\,D_m} 1,$$

and where the function $\mathbf{1}(\cdot)$ returns 1 whenever its argument holds and 0 otherwise.

As for the *makam* prior probability distribution $p_M(m)$, we have used two different methods to be used in the computation of the membership function alternatives. Since $\mathbf{M}_f(s)$ and $\mathbf{M}_{f,d}(s)$ require the *makam* prior probabilities over the phrases, we have computed $p_M(m)$ via

$$p_M(m) \approx \frac{1}{Z'} \sum_{s\,\mathrm{in}\,D_m} 1$$

with

$$Z' = \sum_m \sum_{s\,\mathrm{in}\,D_m} 1$$

to calculate the corresponding discriminant functions. Note that in this way, $p_M(m)$ simply counts the number of phrases in pieces written in the *makam* m divided by the total number of phrases in the database. For $\mathbf{M}_f^{note}(s)$, $\mathbf{M}_{f,d}^{note}(s)$, $\mathbf{M}_f^{ave}(s)$, and $\mathbf{M}_{f,d}^{ave}(s)$, however, we have used

$$p_M(m) \approx \frac{1}{Z''} \sum_{(f',d')\,\mathrm{in}\,D_m} 1$$

with

$$Z'' = \sum_m \sum_{(f',d')\,\mathrm{in}\,D_m} 1$$

as these membership functions require the *makam* prior probabilities over the notes due to their specific formulation.

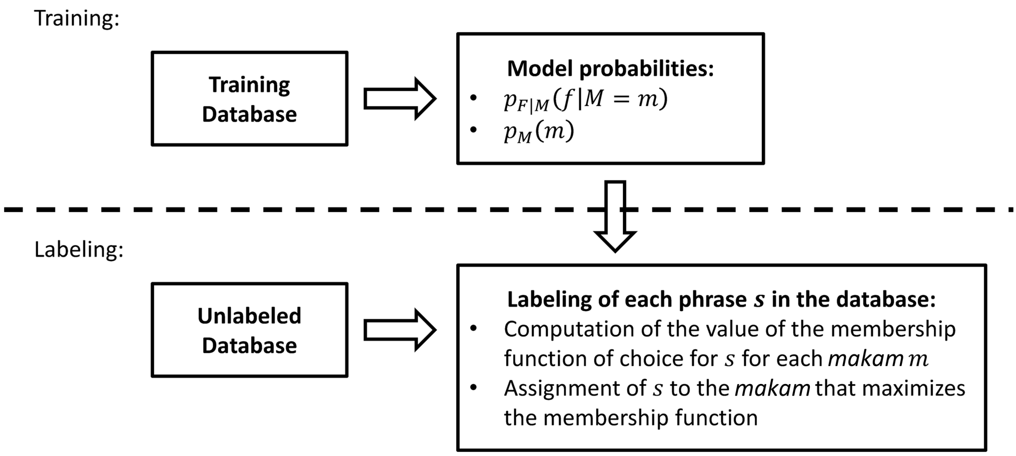

Figure 4. Illustration of the labelling framework. The probability mass functions $p_{F|M}(f|M=m)$ and $p_M(m)$ are calculated using the *makam* associations of the phrases in the symbolic *makam* music database available for training. These probabilities are then used to evaluate the membership function on the phrase in question for labelling, assigned to the *makam* maximizing the membership function.

4.3. *Melodic progression analysis of individual pieces*

Our melodic progression analysis method assumes a scenario where symbolic data of a new piece of *makam* music not in the database is to be analysed using the information gathered from the database. We further assume that the new piece is already divided into a succession of phrases, either manually by an expert or automatically via a melodic segmentation algorithm (such as LBDM (Local Boundary Detection Model), see Cambouropoulos 2006), leaving the phrase segmentation issue beyond the scope of this study. The melodic progression analysis of the new piece is then carried out by first computing the discriminant of the selected membership function for each phrase s in the new piece and assigning s to the *makam* that maximizes it, and returning the sequence of *makam* assignments corresponding to each successive phrase of the piece. The discriminant for the selected membership function is computed according to the definitions of Section 4.1 using the probability measures estimated from the symbolic *makam* music database described in Section 3 according to the formulas given in Section 4.2. This procedure is illustrated in Figure 4. The result of the procedure for a given piece, then, is a time sequence of *makamlar*, indicating which *makamlar* the piece in consideration visits as melodic contour progresses.

The question of which membership function to use is related to the respective accuracies in separating the phrases observed in the pieces written in the different *makamlar*. As a result of a general intuition obtained from *makam* music theory and communication with masters, we expect at least half of the phrases to be mapped to the *makam* of the piece. In addition, it is expected that the *makam* of a piece is introduced specifically in the beginning of the melodic progression (Öztürk 2011); hence the first phrases should generally be mapped to the *makam* of the piece as well. These considerations make it clear that the set of phrases observed in different *makamlar* may contain significant overlaps, requiring a probabilistic characterization of the statistical structure linking the phrases to the different *makamlar* rather than a blind classification strategy. Indeed, the membership functions described in Section 4.1 are designed to capture this statistical structure, allowing us to separate the phrases that are characteristic of a *makam* from the others that have been borrowed from other *makamlar* during the melodic progression of a piece through a decomposition of the *makam* posterior probability. Still, the performance of any recognition method working towards that end must be evaluated by the rate at which the phrases associated with a given *makam* are mapped to that *makam*, while it is clear that the performance cannot be maximal as the borrowed phrases are not expected to be mapped to the *makam* of the piece in which they occur.

We have determined these rates by setting up a leave-one-out cross validation experiment using the pieces in our database as follows.

- For each piece in the database, do:
  - Revise the computation of the probability mass functions $p_{F|M}(f|M=m)$ and $p_M(m)$ to exclude the phrase and the note data of the current piece.
  - For each phrase s in the current piece, do:
    - Compute the discriminant of the selected *makam* membership function for s.
    - Assign s to the *makam* that maximizes the discriminant function and record this assignment.
- Compute the confusion matrix for all *makamlar* by contrasting the *makam* assignments of each phrase to the *makam* of the corresponding piece.

The procedure above was repeated separately for each membership function $\mathbf{M}_f(s)$, $\mathbf{M}_{f,d}(s)$, $\mathbf{M}_f^{note}(s)$, $\mathbf{M}_{f,d}^{note}(s)$, $\mathbf{M}_f^{ave}(s)$, and $\mathbf{M}_{f,d}^{ave}(s)$, and the resulting confusion matrices were calculated that determined the counts of phrases observed in pieces written in *makam* m and assigned to *makam* m' for all *makam* pairs m and m'. The accuracies in *makam* assignment for the different membership functions were then computed from these confusion matrices via F-measures computed using precision and recall rates calculated separately for each *makam* on a one-against-all manner. Specifically, for *makam* m, the precision and recall rates $R^{precision}$ and R^{recall} were calculated via

$$R_m^{precision} = \frac{\sum_s \mathbf{1}(\text{“}s \text{ belonged to } makam\ m \text{ and was assigned to } m\text{”})}{\sum_s \mathbf{1}(\text{“}s \text{ was assigned to } makam\ m\text{”})}$$

and

$$R_m^{recall} = \frac{\sum_s \mathbf{1}(\text{“}s \text{ belonged to } makam\ m \text{ and was assigned to } m\text{”})}{\sum_s \mathbf{1}(\text{“}s \text{ belonged to } makam\ m\text{”})},$$

respectively. The accuracy of the membership function alternatives were then determined by the F-measures calculated by

$$\mathbf{F}_m = 2\frac{R_m^{precision} R_m^{recall}}{R_m^{precision} + R_m^{recall}}$$

separately for each *makam* m.

5. Results and discussion

In this section, we present the experiment results for the proposed phrase recognition strategy on the symbolic database of *makam* music described in Section 3. The experiments entailed first calculating the membership functions $\mathbf{M}_f(s)$, $\mathbf{M}_{f,d}(s)$, $\mathbf{M}_f^{note}(s)$, $\mathbf{M}_{f,d}^{note}(s)$, $\mathbf{M}_f^{ave}(s)$, and $\mathbf{M}_{f,d}^{ave}(s)$ derived in the previous section separately for the phrases in each piece in a leave-one-out formalism. The results of the corresponding phrase assignment rules were then recorded in a global confusion matrix to reflect both the original *makam* of the piece in which the phrase occurred and the *makam* to which the phrase was assigned. The leave-one-out framework was employed by omitting all the phrases of a piece out of the dataset when computing the probability mass

Table 3. F-measures observed for the phrase assignments obtained using the membership functions $\mathbf{M}_f$, $\mathbf{M}_{f,d}$, $\mathbf{M}_f^{note}$, $\mathbf{M}_{f,d}^{note}$, $\mathbf{M}_f^{ave}$, and $\mathbf{M}_{f,d}^{ave}$. The functions $\mathbf{M}_f^{note}$, $\mathbf{M}_{f,d}^{note}$, $\mathbf{M}_f^{ave}$, and $\mathbf{M}_{f,d}^{ave}$ failed to assign any phrase to *makamlar* Beyati and Uşşak, preventing the computation of an F-measure.

	$\mathbf{M}_f$	$\mathbf{M}_{f,d}$	$\mathbf{M}_f^{note}$	$\mathbf{M}_{f,d}^{note}$	$\mathbf{M}_f^{ave}$	$\mathbf{M}_{f,d}^{ave}$
Acemaşiran	0.5863	0.6168	0.3608	0.3834	0.2896	0.3202
Beyati	0.0909	0.0823	–	–	–	–
Buselik	0.6997	0.6889	0.2900	0.1404	0.3514	0.2513
Hicaz	07823	0.7736	0.6732	0.6558	0.5903	0.5615
Hicazkar	0.5826	0.5933	0.5655	0.5403	0.5166	05228
Hüseyni	0.4237	0.4209	0.3951	0.3687	0.2631	0.1631
Hüzzam	0.5927	0.5794	0.5563	0.5373	0.5626	0.5291
Kürdilihicazkar	0.7773	0.7554	0.7063	0.6925	0.6637	0.6690
Mahur	0.6782	0.7057	0.4590	0.3509	0.5850	0.4408
Muhayyer	0.3226	0.3108	0.0630	0.0484	0.0760	0.0286
Nihavent	0.6220	0.5821	0.1948	0.1111	0.0221	0.0294
Rast	0.3366	0.3689	0.0858	0.0862	–	0.0095
Saba	0.6983	0.6610	0.3693	0.1890	0.6821	0.6240
Segah	0.6242	0.5913	0.2065	0.2575	0.3207	0.3401
Uşşak	0.2584	0.2535	–	–	–	–
Average	0.5384	0.5323	0.3789	0.3355	0.4103	0.3453

functions $p_M(m)$ and $p_{F|M}(f|M=m)$ to calculate the membership functions and the subsequent *makam* assignment of each phrase in that piece. This ensured that the knowledge of the piece in question did not affect the learned probability distributions from the music data in any way. As a result, the probability distribution estimates computed separately at each step of the leave-one-out procedure deviated slightly from the global estimates obtained from the entire dataset shown in Figures 5 and 6.

Following the computation of the confusion matrices produced by the membership function alternatives, we have calculated the F-measures to assess the rate of mapping the phrases in each *makam* separately, as well as an average F-measure via the arithmetic average of the individual F-measures obtained for each of the 15 *makamlar* represented in our symbolic *makam* music database. The results in Table 3 show that among the membership functions constructed in the previous section, $\mathbf{M}_f(s)$ resulted in the highest average F-measure across all *makamlar*, attesting to the validity of decomposing the *makam* posterior distributions across the pitch values of the notes making up a phrase. We have also observed that an F-measure could not be computed for the functions $\mathbf{M}_f^{note}$, $\mathbf{M}_{f,d}^{note}$, $\mathbf{M}_f^{ave}$, and $\mathbf{M}_{f,d}^{ave}$ for the *makamlar* Beyati and Uşşak. Further investigations revealed that the assignment rules based on these membership functions failed to assign any phrase to these *makamlar*, preventing the computation of a precision rate as well as a subsequent F-measure. Similar observations were reported by previous *makam* classification studies (such as Ünal, Bozkurt, and Karaosmanoğlu 2014; Gedik and Bozkurt 2010) since the pitch class distributions of Beyati and Uşşak *makamlar* are very similar (as can be seen in Figure 6), and from music theory we know that Rast and Hüseyni pieces may include a large amount of Uşşak *makam* phrases. It is clear that new features are needed to capture the characteristics that differentiate specifically these two *makamlar*.

As the next step, we have repeated the leave-one-out experiments by assuming a uniform prior *makam* distribution $p_M(m)$ instead of the ones computed from the symbolic *makam* music dataset. The resulting F-measures in Table 4 show that a uniform *makam* prior probability allowed assigning more phrases to the less prominent *makamlar* such as Beyati and Uşşak, albeit with no discernible improvement in the best classification performance achieved by the membership function $\mathbf{M}_f$ in the associated F-measures.

The confusion matrix obtained using the best-performing membership function, $\mathbf{M}_f$, shows that a large part of the phrases have been successfully assigned to their actual *makam* (Table 5).

It is also clear that some *makamlar* were recognized relatively easily, such as Kürdilihicazkar, Hicaz, and Saba, while others, such as Beyati and Uşşak were difficult to recognize. This is an

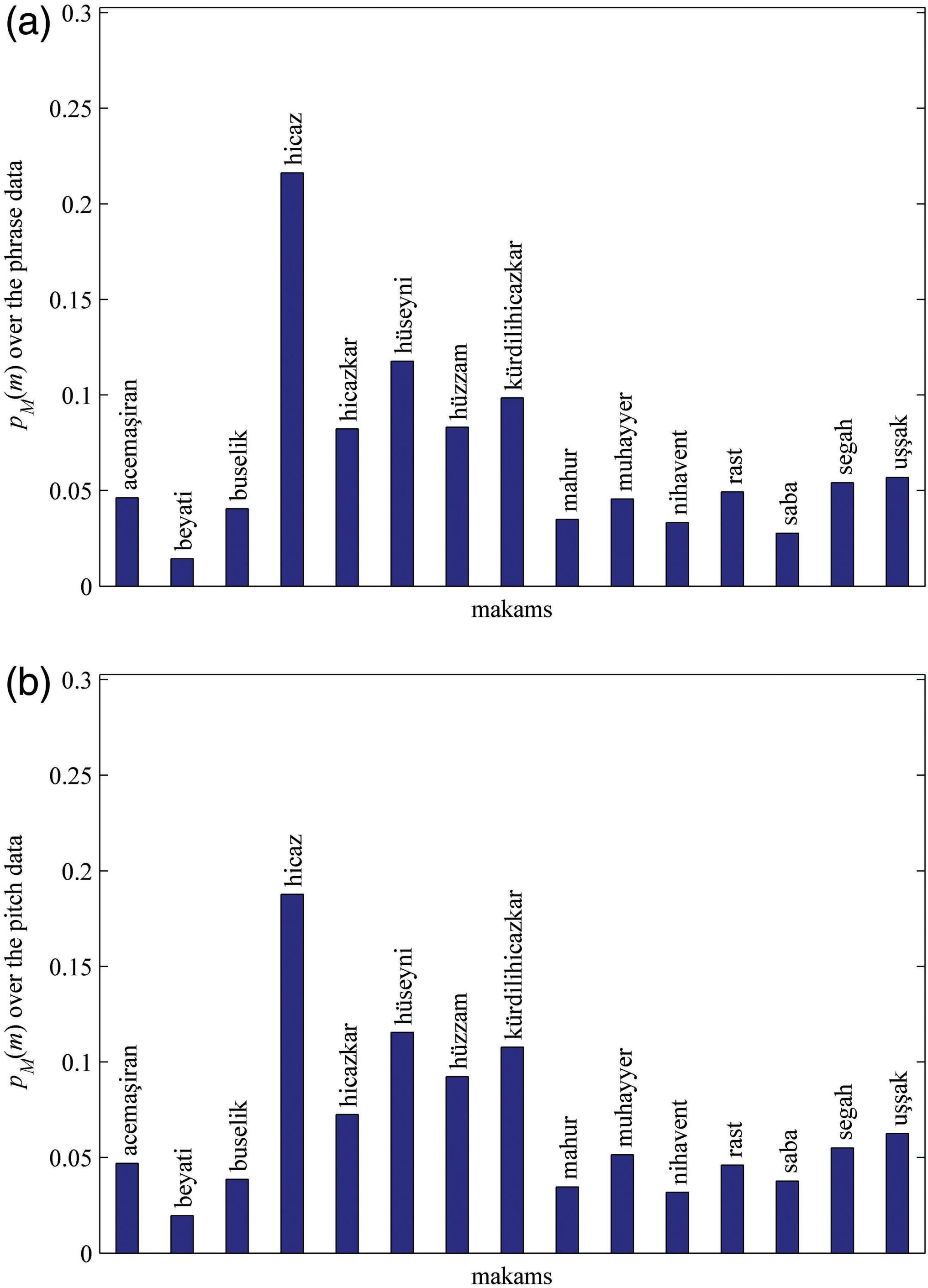

Figure 5. Probability mass function $p_M(m)$ pertaining to the phrase (a) and pitch (b) data estimated from the entire symbolic *makam* music database.

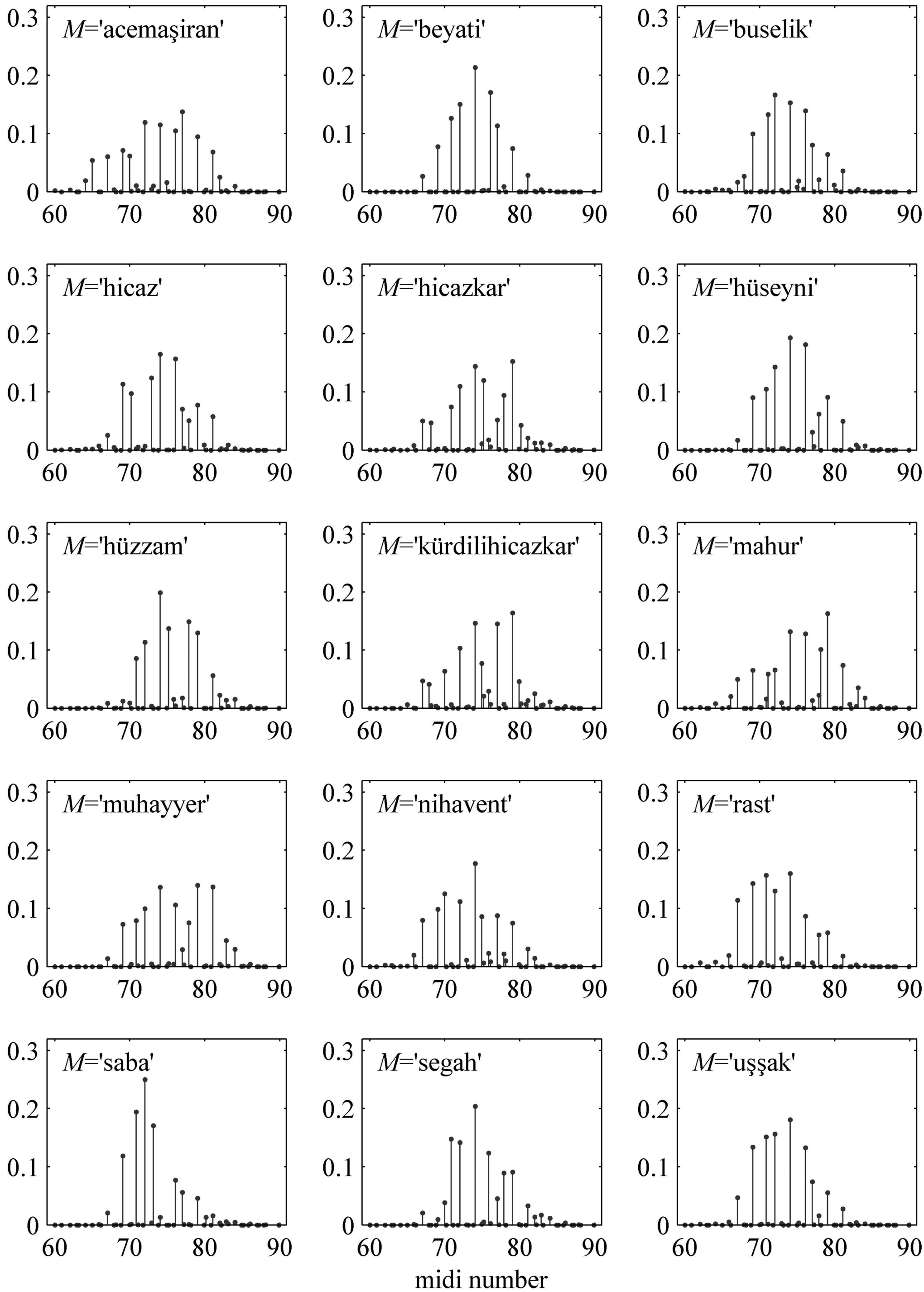

Figure 6. The probability mass functions of the pitch values $p_{F|M}(f|M = m)$ computed from the entire symbolic *makam* music database for each *makam* *m*. The pitch values are displayed in the horizontal axis, expressed by the respective fractional MIDI numbers associated with the *makam* music.

Table 4. F-measures observed for the phrase assignments obtained using the membership functions $\mathbf{M}_f$, $\mathbf{M}_{f,d}$, $\mathbf{M}_f^{note}$, $\mathbf{M}_{f,d}^{note}$, $\mathbf{M}_f^{ave}$, and $\mathbf{M}_{f,d}^{ave}$ under the assumption of a uniform *makam* prior distribution. Note the identity of the first and third as well as the second and fourth columns.

	$\mathbf{M}_f$	$\mathbf{M}_{f,d}$	$\mathbf{M}_f^{note}$	$\mathbf{M}_{f,d}^{note}$	$\mathbf{M}_f^{ave}$	$\mathbf{M}_{f,d}^{ave}$
Acemaşiran	0.5844	0.5544	0.5844	0.5544	0.5074	0.4651
Beyati	0.0891	0.1017	0.0891	0.1017	0.0928	0.1049
Buselik	0.7005	0.8130	0.7005	0.6813	0.6341	0.6443
Hicaz	0.7535	0.7458	0.7535	0.7458	0.7187	0.7047
Hicazkar	0.5858	0.5982	0.5858	0.5982	0.6193	0.5983
Hüseyni	0.2615	0.3026	0.2615	0.3026	0.2309	0.2880
Hüzzam	0.5977	0.5773	0.5977	0.5773	0.5488	0.5283
Kürdilihicazkar	0.7694	0.7751	0.7694	0.7751	0.6378	0.5669
Mahur	0.6752	0.6895	0.6752	0.6895	0.5859	0.6316
Muhayyer	0.3001	0.2984	0.3001	0.2984	0.3004	0.3018
Nihavent	0.6352	0.6232	0.6352	0.6232	0.5078	0.4667
Rast	0.3622	0.3628	0.3622	0.3628	0.3662	0.3226
Saba	0.6600	0.6194	0.6600	0.6194	0.3720	0.4005
Segah	0.6254	0.6089	0.6254	0.6089	0.5614	0.5551
Uşşak	0.2118	0.2362	0.2118	0.2362	0.0965	0.1388
Average	0.5208	0.5183	0.5208	0.5183	0.4520	0.4478

Table 5. The confusion matrix obtained by the membership function $\mathbf{M}_f$ using the probability estimates calculated from the symbolic *makam* music dataset separately at each step of the leave-one-out procedure.

	Predicted *makam*															
True *makam*	Acemaşiran	Beyati	Buselik	Hicaz	Hicazkar	Hüseyni	Hüzzam	Kürdilihicazkar	Mahur	Muhayyer	Nihavent	Rast	Saba	Segah	Uşşak	Total
Acemaşiran	192	36	6	26	4	21	7	7	0	24	25	0	23	0	2	373
Beyati	10	12	0	10	0	11	2	2	0	5	0	6	0	0	57	115
Buselik	9	6	205	42	2	24	15	5	4	7	0	3	0	4	0	326
Hicaz	11	4	15	1328	4	92	28	18	34	132	0	73	3	0	1	1743
Hicazkar	11	0	0	12	32	9	210	68	0	3	10	1	7	8	3	663
Hüseyni	2	21	0	88	0	379	17	0	11	133	0	63	12	0	223	949
Hüzzam	3	4	0	11	19	19	457	8	7	23	0	33	2	63	21	670
Kürdilihicazkar	8	0	1	7	63	1	22	623	0	3	33	11	0	22	0	794
Mahur	2	0	27	14	0	32	1	2	176	14	0	11	0	2	0	281
Muhayyer	0	4	3	25	2	65	18	0	1	150	0	26	5	9	60	368
Nihavent	5	0	0	8	14	1	8	60	4	1	153	3	0	11	0	268
Rast	1	1	1	36	1	96	20	0	1	15	3	156	7	4	55	397
Saba	11	0	0	17	5	7	1	7	0	7	0	4	162	1	1	223
Segah	0	4	0	2	4	14	57	9	0	23	0	31	9	255	28	436
Uşşak	17	57	2	26	0	69	9	0	0	22	0	109	11	2	135	459
Total	282	149	260	1652	439	840	872	80	238	562	224	50	41	381	586	8065

expected result when the scales of the *makamlar* and the distributions in Figure 5 are considered. Beyati and Uşşak share the same scale that is very close to that of Hüseyni, Muhayyer, and Rast.

The time course of the *makam* assignments of the phrases in three selected pieces from the symbolic *makam* music database shown in Figure 7 illustrates the varying use of different *makamlar* throughout a *makam* music piece. These examples illustrate a general characteristic of the *makam* music, to start a piece in its designated *makam*, and visit other *makamlar*. We have chosen representative examples of varying degrees of match with the main *makam* of the piece.

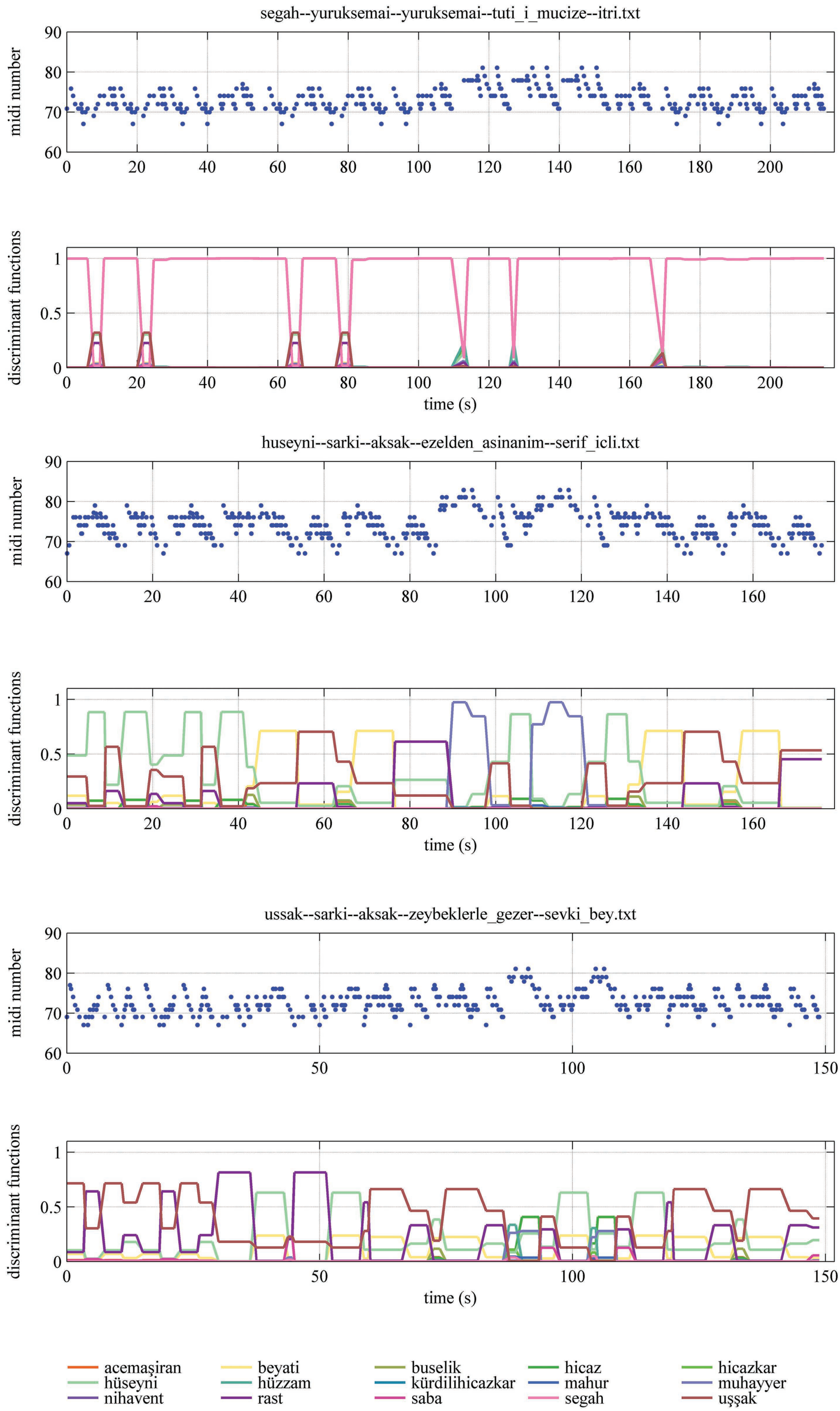

Figure 7. Time course of the *makam* assignments of the phrases for three pieces in the symbolic *makam* music database as determined using the best performing classification rule $\mathbf{M}_f$.

Table 6. F-measures observed for the assignment of the first four phrases of each piece into the different *makamlar*. Only the results obtained using the *makam* priors derived from the symbolic *makam* music database are shown.

	$\mathbf{M}_f$	$\mathbf{M}_{f,d}$	$\mathbf{M}_f^{note}$	$\mathbf{M}_{f,d}^{note}$	$\mathbf{M}_f^{ave}$	$\mathbf{M}_{f,d}^{ave}$
Acemaşiran	0.6182	0.6230	0.2326	0.2667	0.1053	0.1500
Beyati	0.0909	0.2005	–	–	–	–
Buselik	0.8387	0.8387	0.2162	0.2222	0.4091	02162
Hicaz	0.8240	0.8000	06250	0.6164	0.5357	0.5037
Hicazkar	0.6250	0.6383	0.6667	0.5610	0.5000	0.4935
Hüseyni	0.5100	0.5049	0.3571	0.3381	0.2029	0.1354
Hüzzam	0.6867	0.6747	0.6333	0.6129	0.6215	0.6310
Kürdilihicazkar	0.8000	0.7871	0.7732	0.7411	0.7437	0.6860
Mahur	0.6923	0.7200	0.4444	0.4000	0.6977	0.5641
Muhayyer	0.5686	0.5657	0.2041	0.1277	0.2041	0.0870
Nihavent	0.5778	0.5455	0.1290	0.1333	–	–
Rast	0.3495	0.3894	0.1455	0.1455	–	0.0800
Saba	0.8966	0.8475	0.5263	0.3030	0.8333	0.8571
Segah	0.5957	0.5714	0.3188	0.3030	0.3235	0.2857
Uşşak	0.3478	0.3148	–	–	–	–
Average	0.6015	0.6021	0.4056	0.3670	0.4706	0.3908

The first example is a short piece considered to be very representative of *makam* Segah in the music circles. We observe that the progression mainly stays in *makam* Segah as expected.

The second example is in the popular *Şarkı* form in *makam* Hüseyni. Typically, *makam* Hüseyni includes phrases in three main overlapping regions of the pitch space: the lower half of the scale (where some phrases ending on the tonic are considered to be Uşşak phrases), the mid-region of the scale emphasizing the dominant note E (where phrases would be mostly mapped to *makam* Hüseyni) and a higher half of the scale (where phrases emphasizing the upper tonic are mapped to *makam* Muhayyer). We observe on the figure that the *makam* mappings are in line with the above expectation rooted in theory (Özkan 2006) and descriptions by masters (private communication with Ruhi Ayangil and Reha Sağbaş): melodic progression starts in the mid-region where phrases are mainly mapped to *makam* Hüseyni (here the expectation that the initial phrases are mapped to the main *makam* – Öztürk 2011 – is also satisfied). Then, in the 45–90 seconds region, the melody tends towards the tonic and the labelled *makamlar* are Beyati, Uşşak, and Rast (these classes have a large overlap). In the 90–120 seconds region, phrases emphasize the upper tonic and the *makam* is labelled as Muhayyer. Finally, the progression ends with the lower tonic with Beyati and Uşşak labels as before.

The third example is in *makam* Uşşak and the mappings vary among the classes Uşşak, Rast, and Hüseyni. Since the overlap of these classes is very high, this part of the data needs a further detailed study with new features defining the phrases and in-depth musicological research on how strict the boundaries between phrases of these classes can be drawn. A large part of the "confusions" in our results fall into this category. This is clearly observed in the confusion matrices, with phrases from *makamlar* Uşşak, Beyati, Rast, and Hüseyni being cross-mapped within the group of these four *makamlar*.

This temporal organization also suggests a greater accuracy in the mapping of the beginning phrases of a piece into the corresponding *makam*. In order to test this observation, we have calculated the F-measures for each *makam* over the first four phrases of each piece. The results in Table 6 show that the assignment of these phrases into the different *makamlar* is indeed more accurate, with greater F-measures achieved for each *makam*, suggesting that the beginning phrases tend to adhere more strongly to the characteristics of the *makam* within which the piece has been composed. This result is also consistent with the postulates of recent musicological studies (Öztürk 2011).

6. Conclusions

This paper addressed the challenge of designing an automatic analysis method for matching melodic phrases to *makamlar* with an output of a representation of context change within pieces, for the first time in the literature. There is no agreed upon methodology to perform the task by humans, hence we used engineering methods to check the consistency of our results and study a few individual samples to observe if the analysis results match the descriptions in *makam* music studies.

The proposed automatic analysis method is based on Bayesian learning and uses a *makam* membership function also defined in this work. Several versions of the membership function have been tested and the highest accuracy obtained is 0.5208. Considering that the data pool contained 199 pieces from 15 different *makamlar* (including very close *makam* classes such as Uşşak, Beyati, and Hüseyni), the rate of recognition is relatively high. A higher accuracy of 0.6021 is obtained when the system is tested on the first four phrases of the pieces. This result is in line with musicological works (Öztürk 2011) that claim the main *makam* is specifically expressed at the beginning of the piece.

In this work, we used membership functions based on pitch distributions. The system is open to extension to use several additional features to better represent the properties of musical phrases. It is among our future goals to perform an extensive study with the masters on the outputs of our system to collect expert knowledge and define new features that would increase the robustness of the system. We will specifically consider the main difficulty our system faces in recognition, namely mapping of phrases to classes with very close pitch distributions such as Uşşak, Beyati, and Hüseyni. This will require both an in-depth musicological study for these *makamlar* and an experimental study on how experts label phrases taken out of context in these close *makamlar*, and how they draw boundaries between these classes.

Recognition accuracies and analysis of individual examples show that the proposed methodology has a verified potential in automatic analysis of *makam* music pieces on large databases. In our interviews with the experts of this music, they often stated that an analysis of a large repertoire, and gathering more statistical data, could possibly lead to a better understanding of the *makam* concept. Automatic analysis can potentially provide us with the chance to gather information otherwise impossible because of the required amount of manual work.

In order to address this challenge, we developed a segmented phrase database for the first time in the literature, which is publicly available upon request from the authors.

One of the drawbacks of our approach is in processing phrases that come from transposed versions of *makamlar*. It is possible that a piece in *makam* Hicaz can include transposed Hicaz phrases in the development sections as the composer expresses creativity. For these cases, our approach tends to link this phrase with the closest possible *makam* that includes a Hicaz (tetra/penta) chord on that note. Our future work will include analysis with dynamic features to capture such characteristics of transposed *makamlar*.

Part of our future work will be dedicated to exploring tools available for symbolic music analysis such as Humdrum (http://www.musiccog.ohio-state.edu/Humdrum/), music21 (http://web.mit.edu/music21/), Jazzomat (http://lemming.hfm-weimar.de/jazzomat/), and Midi Toolbox (https://www.jyu.fi/hum/laitokset/musiikki/en/research/coe/materials/miditoolbox). Moving in that direction necessitates several format conversions (which are in progress) as well as a study on musical concepts since most of the tools available are developed for Western music.

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