On precoding MIMO-FBMC with imperfect channel state information at the transmitter

Didier Le Ruyet
CEDRIC/LAETITIA
CNAM
Paris, France
Email: leruyet@cnam.fr

Rostom Zakaria
CEDRIC/LAETITIA
CNAM
Paris, France
Email: rostom.zakaria@cnam.fr

Berna Özbek
Electrical and Electronics Engineering Dept.
Izmir Institute of Technology
Izmir, Turkey
bernaozbek@iyte.edu.tr

***Abstract*—This paper studies the impact of imperfect channel state information (CSI) due to limited feedback link on the performance of multi-user MIMO system using filter bank based multicarrier (FBMC) modulation. The system is composed of a transmitter performing Zero Forcing (ZF) precoding and single antenna receivers applying decoding techniques. Simulation-based results show that except when the number of users is less than the number of transmit antennas, the BER performance and capacity of FBMC and OFDM modulation are the same. These results are theoretically justified due to the distribution of the interfering terms. As in OFDM, depending on the number of interferers, for a given BER performance target, the required number of feedback bits per channel vector can be rather high. FBMC becomes attractive not only because it relaxes the synchronization with respect to OFDM, but also because it achieves the same performance results as OFDM for multi-user MIMO precoding even in the case of imperfect CSI at the transmitter.**

I. Introduction

Orthogonal frequency division multiplexing with the cyclic prefix insertion (CP-OFDM) is the most widespread modulation among all the multicarrier modulations, and this thanks to its simplicity and its robustness against multipath fading using the cyclic prefix (CP). Nevertheless, this technique causes a loss of spectral efficiency due to the cyclic prefix. Furthermore, the CP-OFDM spectrum is not compact due to the large sidelobe levels resulting from the rectangular pulse. To avoid these drawbacks, filter bank based multicarrier (FBMC) [1] has received a great attention from researchers in recent years. In FBMC, there is no need to insert any guard interval. Furthermore, it uses a frequency well-localized pulse shaping, hence, it provides a higher spectral efficiency [2] [3]. Each subcarrier is modulated with an Offset Quadrature Amplitude Modulation (OQAM) which consists in transmitting real and imaginary samples with a shift of half the symbol period between them.

Multiple-input-multiple-output (MIMO) techniques are playing an important role in wireless communications since they enable to increase the overall system performance. MIMO techniques can be straightforwardly applied to multicarrier modulations when using orthogonal frequency division multiplexing (OFDM). When considering FBMC/OQAM, the extension to MIMO must be carefully addressed due to the presence of the so-called intrinsic interference. FBMC/OQAM systems without channel state information (CSI) at the transmitter has been considered in [4] where the MMSE receiver has been studied, [5] and [6] where improved receivers have been proposed.

When CSI is available at the transmitter, the potential gain increases considerably as shown in [7] and [8] where random vector quantization (RVQ) has been used to compute the achievable data rate of multi input multi output (MISO) point-to-point single user communication and multiuser MIMO broadcast channels with finite rate feedback. Assuming that CSI was perfectly known at the transmitter, a zero forcing (ZF) based approach has been proposed in [9] for multi-stream transmissions in MIMO FBMC/OQAM systems. In [10], the authors have proposed a space division multiple access (SDMA) approach for the MISO broadcast channel based on the Tomlinson-Harashima precoding. Without restriction on the number of transmit antennas and receive antennas, a coordinated beamforming algorithm for point-to-point MIMO FBMC/OQAM systems and multi-user MIMO downlink settings has been recently introduced in [11].

In this paper, we evaluate the impact of imperfect CSI at the transmitter on the performance of ZF based multi-user MIMO precoding for FBMC modulation. For flat fading channels, we will show that as in OFDM, depending on the number of users and consequently the number of interferers, the number of required feedback bits per channel vector can be rather high to cope with the inter-user interference.

The remainder of the paper is organized as follows: Section II reviews the system model of point-to-point FBMC/OQAM transmission. The considered scheme for multi-user MIMO with precoding and imperfect CSI at the transmitter is described in detail in Section III. Performance analysis including bit error rate (BER) and achievable sum rate are addressed in Section IV. Simulation results are presented in Section V and conclusions are drawn in Section VI.

We use lower-case boldface to denote vectors and upper-case boldface for matrices. The conjugate transpose of $\mathbf{a}$ is noted $\mathbf{a}^H$ and the norm of vector $\mathbf{a}$ is denoted $||\mathbf{a}||$.

II. FBMC system model

In a baseband discrete time model, we can write at the transmitter side the FBMC signal as follows [2]:

978-1-4799-6422-2/14/$31.00 © 2014 IEEE

$$s[m] = \sum_{k=0}^{M-1} \sum_{n \in \mathbf{Z}} s_{k,n} g_{k,n}[m], \quad (1)$$

where $g_{k,n}[m]$ are the shifted versions of the prototype filter $g[m]$ in time and frequency, M is an even number of subcarriers, and $s_{k,n}$ are the real-valued transmitted symbols.

Assuming that the channel is constant at least over the summation zone $\Omega_{k,n} = \Omega^*_{k,n} \cup \{(k,n)\}$, the signal at the receiver output can be written as [12]:

$$\begin{aligned} y_{k,n} &= h_{k,n} \sum_{m=-\infty}^{+\infty} s[m] g^*_{k,n}[m] + b_{k,n} \\ &= h_{k,n}(s_{k,n} + j u_{k,n}) + b_{k,n} \end{aligned} \quad (2)$$

where $h_{k,n}$ and $b_{k,n}$ are, respectively, the channel coefficient and the noise term at kth subcarrier and nth time index.

The intrinsic interference $I_{k,n} = j u_{k,n}$ is pure imaginary and depends only on symbols transmitted in a restricted set $\Omega^*_{k,n}$ of time-frequency positions around the considered position (k,n). It can be expressed as:

$$I_{k,n} = \sum_{(k',n') \in \Omega^*_{k,n}} a_{k',n'} \Gamma_{\delta k, \delta n}. \quad (3)$$

Table II depicts the main coefficients $\Gamma_{\delta k, \delta n}$ assuming the PHYDYAS prototype filter [15] with an overlapping factor $K = 4$.

III. ZF PRECODING WITH IMPERFECT CSI AT THE TRANSMITTER

We consider a multi-user MIMO FBMC/OQAM downlink scenario where the BS, equipped with N_t transmit antennas, transmits to Q users at the same time and same frequency. We assume here that each user is equipped with only one receive antenna. We can write the received signal at user q on the kth subcarrier and the nth time index as follows (for sake of clarity we omit the index k and n in the remainder of the paper) :

$$y_q = \mathbf{h}_q^H \mathbf{f}_q (s_q + j u_q) + \sum_{j=1; j \neq q}^{Q} \mathbf{h}_q^H \mathbf{f}_j (s_j + j u_j) + n_q \quad (4)$$

where $\mathbf{h}_q$ is the channel vector for the q-th user of size $N_t \times 1$, $\mathbf{f}_q$ represents the precoding vector that maps the data symbols s_q to the transmit antennas and n_q denotes the additive white Gaussian noise vector with variance σ_n^2 and zero mean.

Assuming a finite rate feedback link, each user quantizes the direction of the channel vector using B bits and feeds it back to the base station (BS). The BS must then determine the precoding matrix using the quantized channel direction information (CDI). In this work, we will restrict our analysis to the ZF precoding scheme. Other precoding schemes exist including Dirty Paper Coding strategy or Regularized Zero-Forcing precoding scheme [14].

Considering perfect channel state information at the user side, each user quantizes its CDI, $\mathbf{g}_q = \frac{\mathbf{h}_q}{||\mathbf{h}_q||}$ with a vector $\mathbf{w}_q^*$ that is selected from the codebook $\mathbf{W}$ of size $N = 2^B$ in order to maximize the instantaneous SNR or equivalently to minimize the chordal distance metric.

The optimum precoding vector is selected according to the following criterion:

$$\begin{aligned} \mathbf{w}_q^* &= \arg \max_{\mathbf{w}_i \in \mathbf{W}} |\mathbf{h}_q^H \mathbf{w}_i|^2 \\ &= \arg \min_{\mathbf{w}_i \in \mathbf{W}} (1 - |\mathbf{g}_q^H \mathbf{w}_i|^2) \end{aligned} \quad (5)$$

where $1 - |\mathbf{g}_q^H \mathbf{w}_i|^2$ is the square chordal distance $d^2(\mathbf{g}_q, \mathbf{w}_i)$ between the unit vectors $\mathbf{g}_q$ and $\mathbf{w}_i$.

The index of the selected vectors $\mathbf{w}_q^*$ are fed back to the transmitter without errors through a finite rate feedback link. Then at the BS, a concatenated quantized CDI matrix $\tilde{\mathbf{W}}$ of size $Q \times N_t$ is built as follows :

$$\tilde{\mathbf{W}} = \begin{bmatrix} \mathbf{w}_1^{*H} \\ \vdots \\ \mathbf{w}_Q^{*H} \end{bmatrix} \quad (6)$$

The beamforming vectors $\mathbf{f}_1, \dots, \mathbf{f}_Q$ are chosen as the normalized columns of the matrix $\mathbf{F}$ given by

$$\mathbf{f}_q = \frac{\mathbf{F}(:,q)}{||\mathbf{F}(:,q)||} \quad (7)$$

where

$$\mathbf{F} = \tilde{\mathbf{W}}^H (\tilde{\mathbf{W}} \tilde{\mathbf{W}}^H)^{-1} \quad (8)$$

At the qth receiver, we first multiply the received signal y_q by $(\mathbf{h}_q^H \mathbf{f}_q)^H$. This treatment is possible assuming that the receiver has been able to perfectly estimate $(\mathbf{h}_q^H \mathbf{f}_q)$ (for example using DM-RS like reference signal). We have :

$$\begin{aligned} \tilde{y}_q &= (\mathbf{h}_q^H \mathbf{f}_q)^H y_q \\ &= |\mathbf{h}_q^H \mathbf{f}_q|^2 (s_q + j u_q) \\ &+ \sum_{j=1; j \neq q}^{Q} (\mathbf{h}_q^H \mathbf{f}_q)^H (\mathbf{h}_q^H \mathbf{f}_j)(s_j + j u_j) \\ &+ (\mathbf{h}_q^H \mathbf{f}_q)^H n_q \end{aligned} \quad (9)$$

Then, by taking only the real part we have

$$\Re(\tilde{y}_q) = |\mathbf{h}_q^H \mathbf{f}_q|^2 s_q + I - J + \Re((\mathbf{h}_q^H \mathbf{f}_q)^H n_q) \quad (10)$$

where

$$I = \sum_{j=1; j \neq q}^{Q} \Re((\mathbf{h}_q^H \mathbf{f}_q)^H (\mathbf{h}_q^H \mathbf{f}_j)) s_j \quad (11)$$

and

	$n-3$	$n-2$	$n-1$	n	$n+1$	$n+2$	$n+3$
$k-1$	$0.043j$	$0.125j$	$0.206j$	$0.239j$	$0.206j$	$0.125j$	$0.043j$
k	$0.067j$	0	$0.564j$	1	$-0.564j$	0	$-0.067j$
$k+1$	$0.043j$	$-0.125j$	$0.206j$	$-0.239j$	$0.206j$	$-0.125j$	$0.043j$

TABLE I. TRANSMULTIPLEXER IMPULSE RESPONSE (MAIN PART) USING THE PHYDYAS FILTER

$$J = \sum_{j=1; j \neq q}^{Q} \Im((\mathbf{h}_q^H \mathbf{f}_q)^H (\mathbf{h}_q^H \mathbf{f}_j)) u_j \tag{12}$$

The quantity $I - J$ is the multi-user interference. In the limited feedback case, since ZF precoding is performed from the selected vectors $\mathbf{w}_q^*$, the multi-user interference is only partially removed.

Interestingly, in the OFDM case, the same treatment is applied but both the real and imaginary part are taken in order to recover the complex symbol. When considering the real part, the only difference between OFDM and FBMC is that the interfering term u_j is a PAM modulation in the OFDM case, while it is a discrete distribution obtained from equation (3) in the FBMC case.

Since $E(|s_q|^2) = E(|u_q|^2) = P/2N_t$ where P is the total transmit power, it can easily be shown that, for both OFDM and FBMC, the SINR can be written as:

$$\gamma_q = \frac{\frac{P}{N_t} \left| \mathbf{h}_q^H \mathbf{f}_q \right|^2}{N_0 B_W + \frac{P}{N_t} \sum_{j=1; j \neq q}^{Q} \left| \mathbf{h}_q^H \mathbf{f}_j \right|^2} \tag{13}$$

where $N_0 B_W$ is the total noise power.

IV. PERFORMANCE ANALYSIS

A. Bit error rate analysis

RVQ [8] is a practical tool to estimate the performance of MISO point-to-point single user communication and multiuser MIMO broadcast channels with finite rate feedback.

In RVQ, the $N = 2^B$ quantization vectors are independently chosen from the isotropic distribution on the N_t-dimensional unit sphere. From a practical point of view, it is equivalent to build the codebook by selecting randomly 2^B unit vector beamforming vectors. It has been shown that RVQ is very useful for performance analysis and performs close to the optimal quantization when $B \to \infty$.

In [7], under the RVQ assumption, the author has shown that $\mathbf{g}_q$ and $\mathbf{f}_q$ are independent isotropic vectors when using ZF beamforming with $Q = N_t$. Consequently, the inner product $|\mathbf{g}_q^H \mathbf{f}_q|^2$ is beta distributed [1] with parameters 1 and $N_t - 1$. For the case $N_t = 2$, the distribution becomes uniform between -1 and +1.

[1]The beta distribution is given by $f(x; \alpha, \beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ where $B(\alpha, \beta)$ is the beta function to ensure that the probability integrates to 1

In order to derive the BER, we will first evaluate the distribution of the interference terms $\Re((\mathbf{g}_q^H \mathbf{f}_q)^H (\mathbf{g}_q^H \mathbf{f}_j)) s_j$ and $\Im((\mathbf{g}_q^H \mathbf{f}_q)^H (\mathbf{g}_q^H \mathbf{f}_j)) u_j$.

We can write

$$\begin{aligned} \Re((\mathbf{g}_q^H \mathbf{f}_q)^H (\mathbf{g}_q^H \mathbf{f}_j)) &= \Re(\mathbf{g}_q^H \mathbf{f}_q)^H) \Re(\mathbf{g}_q^H \mathbf{f}_j) \\ &\quad - \Im((\mathbf{g}_q^H \mathbf{f}_q)^H) \Im(\mathbf{g}_q^H \mathbf{f}_j) \\ &= A + B \end{aligned} \tag{14}$$

Both terms A and B are distributed Laplace random variables with mean 0 and scale factor s. The parameter s is directly related to the size of the codebook. The scale factor s for different numbers of bits per channel vector and two different multiuser systems ($Q = N_t = 2$ and $Q = N_t = 4$) are given in table II

B (bits)	4	6	8	10	12
$s(Q = N_t = 2)$	0.06	0.03	0.015	0.075	0.037
$s(Q = N_t = 4)$	0.061	0.048	0.036	0.029	0.023

TABLE II. SCALE FACTOR s VERSUS THE NUMBER OF BITS PER CHANNEL VECTOR B FOR $Q = N_t = 2$ AND $Q = N_t = 4$.

The distribution of the sum $A + B$ can be computed as [13]

$$f_{A+B}(x) = \frac{1}{4s} \Big(1 + \frac{|x|}{s}\Big) \exp\Big(-\frac{|x|}{s}\Big) \tag{15}$$

Then the distribution of $\Re((\mathbf{g}_q^H \mathbf{f}_q)^H (\mathbf{g}_q^H \mathbf{f}_j)) s_j$ can be given by

$$f_Z^{(s)}(z) = \sum_{x \in \mathbb{S}} \frac{1}{|x|} f_{A+B}\left(\frac{z}{x}\right) f_X(x) \tag{16}$$

where $\mathbb{S}$ is the constellation set, and $f_X(x)$ is the distribution of s_j (PAM symbols). On the other hand, the distribution of $\Im((\mathbf{g}_q^H \mathbf{f}_q)^H (\mathbf{g}_q^H \mathbf{f}_j)) u_j$ can be given by

$$f_Z^{(u)}(z) = \sum_{x \in \mathbb{U}} \frac{1}{|u|} f_{A+B}\left(\frac{z}{u}\right) f_U(u) \tag{17}$$

where $\mathbb{U}$ is the set of all possible values of the intrinsic interference u, and its distribution is given by $f_U(u)$.

According to (11) and (12), we can deduce that the distribution of the interference terms I and J can be, respectively, given by

$$f_I(z) = \underbrace{f_Z^{(s)}(z) * \ldots * f_Z^{(s)}(z)}_{Q-1 \text{ times}} \tag{18}$$

and

$$f_J(z) = \underbrace{f_Z^{(u)}(z) * \ldots * f_Z^{(u)}(z)}_{Q-1 \text{ times}} \tag{19}$$

where $*$ stands for the convolution product.

For OQAM4 modulation, the asymptotic BER (without thermal noise) can be computed as follows

$$TEB = \int_{-\infty}^{+\infty} f_\gamma(\gamma) \int_{-\gamma}^{+\infty} f_{I-J}(z) dz d\gamma \quad (20)$$

where $f_\gamma(\gamma)$ is the beta distribution of parameters 1 and $N_t - 1$ of $|\mathbf{g}_q^H \mathbf{f}_q|^2$ and $f_{I-J}(z)$ is the distribution of the interference term and can be calculated by convolution using relation (18) and (19).

B. Achievable rate analysis

The instantaneous data rate R_q for the q-th user is calculated as follows :

$$R_q = B_W \log_2(1 + \gamma_q) \quad (21)$$

For the multiuser MIMO broadcast channels, the throughput loss using finite rate feedback with RVQ compared to the sum rate achieved with perfect CSI at the transmitter ZF precoding has been computed by Jindal in [7]. Since the SINR expression γ_q given in (13) for FBMC system is the same, assuming a gaussian distribution of the transmitted symbols s_q, we have the same achievable sum data rate. Particularly, as demonstrated in [7], a finite-rate feedback with B feedback bits per user channel vector incurs a throughput loss compared to full CSI at the transmitter ZF precoding that can be upper bounded by

$$\Delta R < \log_2 \left(1 + \frac{P}{N_0 B_W} 2^{-\frac{B}{N_t - 1}} \right) \quad (22)$$

V. Simulation Results

We first consider both OFDM and FBMC multiuser systems with $N_t = Q = 2$. For sake of simplicity, the elements of channel vector are generated as i.i.d random complex Gaussian variables with unit variance.

In all the simulations, the codebooks for 4 bits and 6 bits per channel vector have been obtained using vector quantization codebook design [14] while the codebooks for 8 and more bits are built based on RVQ.

In Figure 1, give the performance of BER as a function of E_b/N_0 using QPSK/OQAM4 modulation with perfect and imperfect CSI at the transmitter. The number of bits per channel vector B is 4, 6 and 8. We also draw the three asymptotical limits for both OFDM and FBMC computed using Eq. (20) given in the previous section. The performance of finite rate feedback OFDM and FBMC are almost the same despite the fact that the distribution of the interference terms I and J is different. We also provide the performance curve with perfect CSI at the transmitter. In that case, since there is no inter user interference, OFDM and FBMC performance are exactly the same.

In Figure 2, we consider both OFDM and FBMC multiuser system with $N_t = 4$ and $Q = 4$ using QPSK modulation. In this scenario, the performance of FBMC and OFDM is interference limited when $E_b/N_0 > 12dB$ for $B \leq 12$ bits.

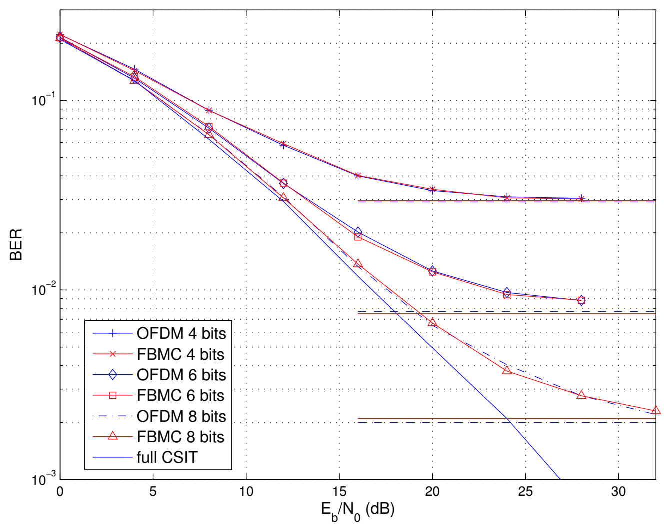

Fig. 1. BER versus E_b/N_0 for $Q = 2$ and $N_t = 2$ using OFDM and FBMC with QPSK/OQAM4 modulation and perfect/ imperfect CSI at the transmitter.

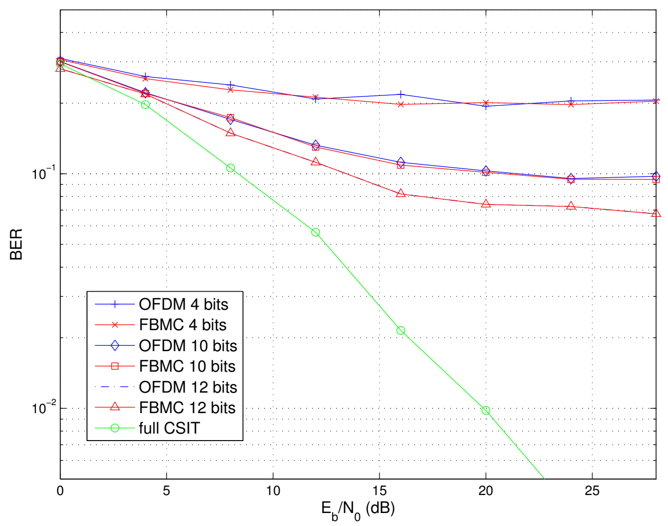

Fig. 2. BER versus E_b/N_0 for $Q = 4$ and $N_t = 4$ using OFDM and FBMC with QPSK/OQAM4 modulation and perfect/ imperfect CSI at the transmitter.

The number of feedback bits should be high in order to mitigate the inter user interference.

Figure 3 we consider the case where $N_t = 4$ and $Q = 2$. In this case, OFDM slightly outperforms FBMC due to the different distribution of the interference terms.

Finally, in Figure 4, we plot the curves BER=$f(E_b/N_0$ using QAM16/OQAM16 modulation for $N_t = 4$ and $Q = 2$. Compared to the QPSK/OQAM case, the performance of FBMC and OFDM are almost the same since the interference distribution of FBMC (that can be approximated by a truncated gaussian distribution) and OFDM (PAM4 modulation close to uniform distribution) are closer.

VI. Conclusion

In this paper, we have evaluated the impact of imperfect CSI at the transmitter on the performance of ZF based MIMO

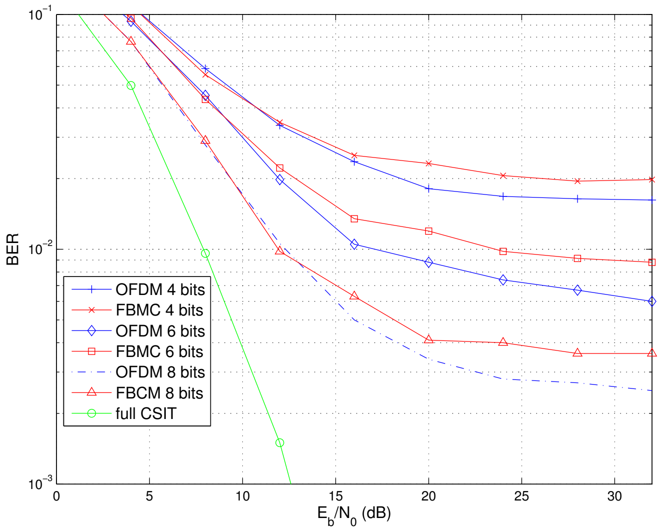

Fig. 3. BER versus E_b/N_0 for $Q = 2$ and $N_t = 4$ using OFDM and FBMC with QPSK/OQAM4 modulation and perfect/ imperfect CSI at the transmitter.

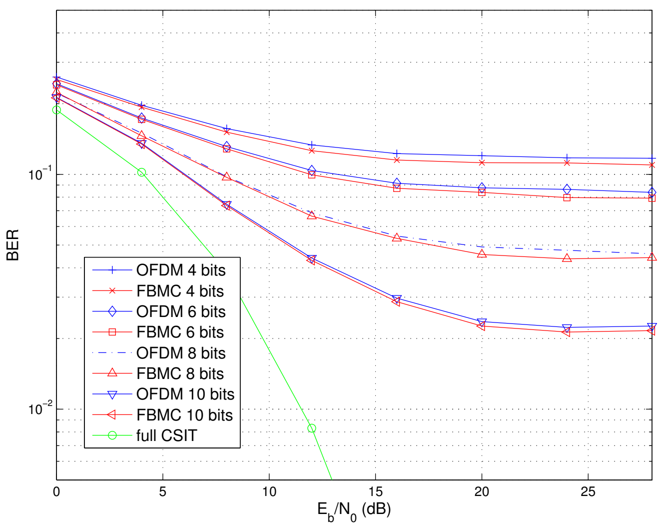

Fig. 4. BER versus E_b/N_0 for $Q = 2$ and $N_t = 4$ using OFDM and FBMC with QAM16/OQAM16 modulation and perfect/ imperfect CSI at the transmitter.

precoding for the FBMC modulation and compared the performance with OFDM in multi-user MIMO downlink scenario where the BS, equipped with multiple transmit antennas, transmits to Q users equipped with a single antenna.

Simulation-based results have shown that except when the number of users is less than the number of transmit antennas and the order of modulation is limited, the BER performance and achievable sum data rate of FBMC and OFDM modulation are almost the same. These results have been theoretically justified by studying the probability distribution of the interference terms. FBMC becomes attractive not only because it relaxes the synchronization with respect to OFDM, but also because it achieves the same performance results as OFDM for multi-user MIMO precoding even in the case of imperfect CSI at the transmitter. In future work, we will consider the case of user selection in addition to MIMO precoding.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support by the European Union FP7-ICT project EMPhAtiC (http://www.ict-emphatic.eu) under grant agreement ICT-318362.

REFERENCES

[1] B. Le Floch, M. Alard and C. Berrou, "Coded Orthogonal Frequency Division Multiplex", *Proceeding of the IEEE*, Vol. 83, No. 6, Jun. 1995

[2] P. Siohan, C. Siclet, and N. Lacaille. "Analysis and design of OFDM/OQAM systems based on filterbank theory", *IEEE Transactions on Signal Processing,* , 50(5):1170–1183, May 2002.

[3] M. G. Bellanger, "Specification and design of a prototype filter for filter bank based multicarrier transmission", *in Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, Salt Lake City, USA, May 2001, pp. 2417–2420.

[4] M. El Tabach, J.P. Javaudin and M. Hélard, "Spatial data multiplexing over OFDM/OQAM modulations", *Proceedings of IEEE-ICC 2007*, Glasgow, 24–28 June 2007, pp.4201-4206.

[5] R. Zakaria and D. Le Ruyet. "A novel FBMC scheme for Spatial Multiplexing with Maximum Likelihood detection", *in 7th International Symposium on Wireless Communication Systems (ISWCS)*, pages 461–465, Sept. 2010.

[6] Y. Cheng and M. Haardt, "Widely Linear Processing in MIMO FBMC/OQAM Systems", *in 10th International Symposium on Wireless Communication Systems (ISWCS)*, Aug. 2013.

[7] N. Jindal, "MIMO broadcast channels with finite-rate feedback", *in IEEE Transactions on Information Theory*. 52: 5045-5060, 2006.

[8] C. K. Au-Yeung , D. J. Love "Performance Analysis of Random Vector Quantization Limited Feedback Beamforming", *in Proc. of IEEE Asilomar Conf. on Signals, Systems, and Computers*. Pacific Grove, CA, 2005.

[9] M. Caus and A. I. Pérez-Neira, "Multi-stream transmission in MIMO-FBMC systems", *in Proc. ICASSP 2013*, May 2013.

[10] M. Caus and A. I. Pérez-Neira, "SDMA for filterbank with Tomlinson Harashima precoding", *in 2013 IEEE International Conference on Communications (ICC)*, 2013, pp. 4571–4575.

[11] Y. Cheng, P. Li and M. Haardt, "Coordinated beamforming in MIMO FBMC/OQAM systems", *in Proc. ICASSP 2014*, May. 2014.

[12] C. Lele, P. Siohan, R. Legouable, and J.-P. Javaudin. "Preamble-based channel estimation techniques for ofdm/oqam over the powerline", *in IEEE International Symposium onPower Line Communications and Its Applications. ISPLC 07*, pages 59 64, Mar. 2007.

[13] S. Kotz, T. Kozubowski and K. Podgorski, "The Laplace Distribution and Generalizations: A Revisit With Applications to Communications, Exonomics, Engineering, and Finance", *Springer*. No. 183. 2001

[14] B. Ozbek , D. Le Ruyet, "Feedback strategies for wireless communication", *Springer-Engineering Series Book, Springer*, New York, U.S.A, May 2013.

[15] M.G. Bellanger et al., "FBMC physical layer: a primer", PHYDYAS document (Online). Available: http://www.ict-phydyas.org/teamspace/internal-folder/FBMC-Primer_06-2010.pdf.