Applied Mathematics and Computation 218 (2012) 9594–9595

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

On the application of the Exp-function method to nonlinear differential-difference equations

İsmail Aslan

Department of Mathematics, Izmir Institute of Technology, Urla, İzmir 35430, Turkey

ARTICLE INFO

Keywords:
Exp-function method
Langmuir lattice
Relativistic Toda lattice system

ABSTRACT

When applying the Exp-function method to nonlinear-differential difference equations, Bekir (2010) [1] reported incorrect results.

Recently, Bekir [1] investigated the Langmuir chains equation

$$\frac{du_n}{dt} - u_n(u_{n+1} - u_{n-1}) = 0 \tag{1}$$

based on the ansatz

$$u_n(\xi_n) = \frac{a_1 \exp(\xi_n) + a_0 + a_{-1} \exp(-\xi_n)}{\exp(\xi_n) + b_0 + b_{-1} \exp(-\xi_n)}, \quad \xi_n = dn + ct + c_0, \tag{2}$$

and claimed that Eq. (1) admits the solutions

$$u_n(\xi_n) = \frac{\frac{2c}{e^{2d} - e^{-2d}} \exp(\xi_n) + a_{-1} \exp(-\xi_n)}{\exp(\xi_n)}, \quad \xi_n = dn + ct + c_0, \tag{3}$$

$$u_n(t) = c\,\mathrm{csch}(2d) + \cosh(2(dn + ct + c_0)) - \sinh(2(dn + ct + c_0)). \tag{4}$$

However, the direct substitution of the expression (3) into (1) yields

$$e^{-2(d+2dn+2ct+2c_0)}(e^{4d} - 1)a_{-1}^2, \tag{5}$$

which is not zero in the general case. Similary, it can be shown that (4) cannot be a solution of Eq. (1).

In fact, using the ansatz (2) in a straightforward manner, one can show that Eq. (1) admits the solution

$$u_n(t) = \frac{c}{2}\mathrm{csch}(d) + \frac{2cb_0 \sinh(d) \exp(dn + ct + c_0)}{2(1 + \cosh(d)) \exp(2(dn + ct + c_0)) + 2b_0(1 + \cosh(d)) \exp(dn + ct + c_0) + b_0^2}, \tag{6}$$

where b_0, d, c, and c_0 are arbitrary parameters.

The same author also considered the relativistic Toda lattice system

$$\frac{du_n}{dt} - (1 + \alpha u_n)(v_n - v_{n-1}) = 0, \qquad \frac{dv_n}{dt} - v_n(u_{n+1} - u_n + \alpha v_{n+1} - \alpha v_{n-1}) = 0. \tag{7}$$

E-mail address: ismailaslan@iyte.edu.tr

0096-3003/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.amc.2012.03.072

Then, via the transformation $v_n = (-1/\alpha)u_n - 1/\alpha^2$, the author reduced the system (7) into the single equation

$$\frac{du_n}{dt} - \left(\frac{1}{\alpha} + u_n\right)(u_{n-1} - u_n) = 0 \tag{8}$$

and claimed that Eq. (8) admits the solutions

$$u_n(t) = \frac{\frac{1-e^{-d}+c\alpha e^{-d}}{\alpha(e^{-d}-1)}\exp(\xi_n) + \frac{(e^d-1+c\alpha e^d)b_0}{1-e^d}}{\exp(\xi_n)+b_0}, \quad \xi_n = dn + ct + c_0, \tag{9}$$

$$u_n(t) = -c\coth(d) + \frac{1}{\alpha}(c\tanh(dn+ct+c_0) - \operatorname{sech}(dn+ct+c_0)), \tag{10}$$

$$u_n(t) = \left(c - \frac{1}{\alpha}\right) - c(\coth(d) + \operatorname{csch}(\mathrm{d})) - \frac{c}{2}(\operatorname{csch}(dn+ct+c_0) + \coth(dn+ct+c_0)). \tag{11}$$

However, the direct substitution of the expression (9) into (8) yields

$$-\frac{e^{ct+c_0}(\alpha-1)(e^{d(n-1)} + e^{d(n+1)}(1+c\alpha) - e^{dn}(2+c\alpha))(1-(1+c)\alpha + e^d(\alpha - 1 + c\alpha^2))b_0^2}{(e^d-1)^2\alpha^2(e^{d(n-1)+ct+c_0}+b_0)(e^{dn+ct+c_0}+b_0)^2}, \tag{12}$$

which is not zero in the general case. Similarly, one can show that the expressions (10) and (11) cannot be solutions of Eq. (8). In fact, based on the ansatz (2), one can show that Eq. (8) admits the solution

$$u_n^{\mp}(t) = \frac{c}{1-\exp(d)} - \frac{1}{\alpha} - \frac{c\left(2b_{-1} + \left(b_0 \mp \sqrt{b_0^2 - 4b_{-1}}\right)\exp(dn+ct+c_0)\right)}{2(\exp(2(dn+ct+c_0)) + b_{-1} + b_0\exp(dn+ct+c_0))}, \tag{13}$$

where b_0, b_{-1}, d, c, and c_0 are arbitrary parameters.

As a result, we can conclude that Bekir made some errors in the application of the Exp-function method to nonlinear-differential difference equations.

Reference

[1] A. Bekir, Application of the Exp-function method for nonlinear differential-difference equations, Appl. Math. Comput. 215 (2010) 4049–4053.