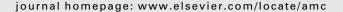
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On the application of the Exp-function method to nonlinear differential-difference equations

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ABSTRACT

Keywords: Exp-function method Langmuir lattice Relativistic Toda lattice system When applying the Exp-function method to nonlinear-differential difference equations, Bekir (2010) [1] reported incorrect results.

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Recently, Bekir [1] investigated the Langmuir chains equation

$$\frac{du_n}{dt} - u_n(u_{n+1} - u_{n-1}) = 0 (1)$$

based on the ansatz

$$u_n(\xi_n) = \frac{a_1 \exp(\xi_n) + a_0 + a_{-1} \exp(-\xi_n)}{\exp(\xi_n) + b_0 + b_{-1} \exp(-\xi_n)}, \quad \xi_n = dn + ct + c_0,$$
(2)

and claimed that Eq. (1) admits the solutions

$$u_n(\xi_n) = \frac{\frac{2c}{e^{2d} - e^{-2d}} \exp(\xi_n) + a_{-1} \exp(-\xi_n)}{\exp(\xi_n)}, \quad \xi_n = dn + ct + c_0,$$
(3)

$$u_n(t) = \operatorname{ccsch}(2d) + \cosh(2(dn + ct + c_0)) - \sinh(2(dn + ct + c_0)). \tag{4}$$

However, the direct substitution of the expression (3) into (1) yields

$$e^{-2(d+2dn+2ct+2c_0)}(e^{4d}-1)a_{-1}^2, (5)$$

which is not zero in the general case. Similary, it can be shown that (4) cannot be a solution of Eq. (1). In fact, using the ansatz (2) in a straightforward manner, one can show that Eq. (1) admits the solution

$$u_n(t) = \frac{c}{2} \operatorname{csch}(d) + \frac{2cb_0 \sinh(d) \exp(dn + ct + c_0)}{2(1 + \cosh(d)) \exp(2(dn + ct + c_0)) + 2b_0(1 + \cosh(d)) \exp(dn + ct + c_0) + b_0^2},$$
(6)

where b_0 , d, c, and c_0 are arbitrary parameters.

The same author also considered the relativistic Toda lattice system

$$\frac{du_n}{dt} - (1 + \alpha u_n)(\nu_n - \nu_{n-1}) = 0, \qquad \frac{d\nu_n}{dt} - \nu_n(u_{n+1} - u_n + \alpha \nu_{n+1} - \alpha \nu_{n-1}) = 0. \tag{7}$$

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Then, via the transformation $v_n = (-1/\alpha)u_n - 1/\alpha^2$, the author reduced the system (7) into the single equation

$$\frac{du_n}{dt} - \left(\frac{1}{\alpha} + u_n\right)(u_{n-1} - u_n) = 0 \tag{8}$$

and claimed that Eq. (8) admits the solutions

$$u_n(t) = \frac{\frac{1 - e^{-d} + c\alpha e^{-d}}{\alpha(e^{-d} - 1)} \exp(\zeta_n) + \frac{(e^d - 1 + c\alpha e^d)b_0}{1 - e^d}}{\exp(\zeta_n) + b_0}, \quad \zeta_n = dn + ct + c_0,$$
(9)

$$u_n(t) = -c \coth(d) + \frac{1}{\alpha}(c \tanh(dn + ct + c_0) - \operatorname{sech}(dn + ct + c_0)), \tag{10}$$

$$u_n(t) = \left(c - \frac{1}{\alpha}\right) - c(\coth(d) + \operatorname{csch}(d)) - \frac{c}{2}(\operatorname{csch}(dn + ct + c_0) + \coth(dn + ct + c_0)). \tag{11}$$

However, the direct substitution of the expression (9) into (8) yields

$$-\frac{e^{ct+c_0}(\alpha-1)(e^{d(n-1)}+e^{d(n+1)}(1+c\alpha)-e^{dn}(2+c\alpha))(1-(1+c)\alpha+e^d(\alpha-1+c\alpha^2))b_0^2}{(e^d-1)^2\alpha^2(e^{d(n-1)+ct+c_0}+b_0)(e^{dn+ct+c_0}+b_0)^2},$$
(12)

which is not zero in the general case. Similarly, one can show that the expressions (10) and (11) cannot be solutions of Eq. (8). In fact, based on the ansatz (2), one can show that Eq. (8) admits the solution

$$u_n^{\pm}(t) = \frac{c}{1 - \exp(d)} - \frac{1}{\alpha} - \frac{c\left(2b_{-1} + \left(b_0 \pm \sqrt{b_0^2 - 4b_{-1}}\right) \exp(dn + ct + c_0)\right)}{2(\exp(2(dn + ct + c_0)) + b_{-1} + b_0 \exp(dn + ct + c_0))},$$
(13)

where b_0 , b_{-1} , d, c, and c_0 are arbitrary parameters.

As a result, we can conclude that Bekir made some errors in the application of the Exp-function method to nonlinear-differential difference equations.

Reference

[1] A. Bekir, Application of the Exp-function method for nonlinear differential-difference equations, Appl. Math. Comput. 215 (2010) 4049-4053.