# Applicability of Sediment Transport Capacity Models for Nonsteady State Erosion from Steep Slopes

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**Abstract:** The physics-based sediment transport equations are derived from the assumption that the sediment transport rate can be determined by a dominant variable such as flow discharge, flow velocity, slope, shear stress, stream power, and unit stream power. In modeling of sheet erosion/sediment transport, many models that determine the transport capacity by one of these dominant variables have been developed. The developed models mostly simulate steady-state sheet erosion. Few models that are based on the shear-stress approach attempt to simulate nonsteady state sheet erosion. This study qualitatively investigates the applicability of the transport capacity models that are based on one of the commonly employed dominant variables—unit stream power, stream power, and shear stress—to simulate nonsteady state sediment loads from steep slopes under different rainfall intensities. The test of the calibrated models with observed data sets shows that the unit stream power model gives better simulation of sediment loads from mild slopes. The stream power and the shear stress models, on the other hand, simulate sediment loads from steep slopes more satisfactorily. The exponent ( $k_i$ ) in the sediment transport capacity formula is found to be 1.2, 1.9, and 1.6 for the stream power model, the shear stress model, and the unit stream power model, respectively.

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## Introduction

Erosion/sediment transport by sheet flow has been experimentally studied. Kilinc and Richardson (1973) carried out intensive rainfall simulations over different slopes from 5.7 to 40% to study the mechanics of soil erosion from overland flows generated by simulated rainfall. Mosley (1972) examined the effect of slope and catchment size and slope on rill morphology and water and sediment transport discharge from interrill areas and rills. In his experimental study, Mosley (1972) had eight different slopes, ranging from 3 to 12%. Moss and Walker (1978), Moss (1979), and Moss et al. (1980, 1982) carried out rainfall simulations over slopes ranging from 0.1 to 4.2% to measure the total sediment concentration of sheet flow and to study the formation of rills. Loch and Donnollan (1983a,b) and Loch (1984) measured sediment discharge under simulated rainfall over 4% tilted slopes after steady-state runoff had been achieved. Govindaraju et al. (1992) carried out rainfall simulations over a steep slope of decomposed granite to assess the performance of cut/fill slopes prone to erosion.

Mathematical models have been developed to study sediment transport by sheet flow too. Some researchers tried to formulate predictive equations based on watershed parameters (Flaxman 1972). Some derived regression equations based on their experimental data (Kilinc and Richardson 1973; Leaf 1974; Megahan

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1974) and some developed black-box type models (Guldal and Muftuoglu 2001). A more physics based modeling approach that provides the spatial and/or temporal distribution of unknown quantities has been attempted by Meyer and Wischmeier (1969), Rowlinson and Martin (1971), Foster and Meyer (1972), Smith (1976), Li (1979), Foster (1982), Woolhiser et al. (1990), and Govindaraju and Kavvas (1991). Most of these physics-based models have a continuity equation for the conservation of the sediment mass and another equation that relates sediment load to the flow transport capacity.

Most physics-based sediment transport equations were derived from the assumption that the sediment transport capacity could be determined by a dominant variable such as flow discharge, flow velocity, slope, shear stress, stream power, and unit stream power. The sediment transport capacity is expressed by the basic form

$$T_c = \eta_i (D - D_c)^{ki} \tag{1}$$

where  $T_c$ =transport capacity (M/L/T);  $\eta_i$  and  $k_i$ =parameters related to flow and sediment conditions; D=dominant variable; and  $D_c$ =critical condition of dominant variable at incipient motion.

In physics-based sheet erosion modeling research the shear stress approach has found a wide application in simulating steady-state sediment transport (Foster and Meyer 1972; McWorter et al. 1979; Foster 1982) and nonsteady state sediment transport (Li 1979; Woolhiser et al. 1990; Govindaraju and Kavvas 1991). The unit stream power approach has also been employed by many researchers to simulate equilibrium sediment loads by sheet flow (Smith 1976; Alonso et al. 1981; Wilson et al. 1982, 1984; Moore and Burch 1986). On the other hand, Rose et al. (1983a,b) employed the stream power approach to simulate sediment loads at steady state.

With the exception of studies by Govindaraju and Kavvas (1991), Woolhiser et al. (1990), and Li (1979), most of the other physics-based mathematical modeling work involved studying

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sediment transport under steady-state conditions from slopes of 0.1-12%. Each study investigated the performance of whichever transport capacity model was employed to study the sheet erosion under steady-state conditions. However, to the knowledge of the writer, there is no study that qualitatively investigated the applicability of all of the commonly employed sediment transport models in estimating sediment loads from steep slopes under non-steady state conditions.

The objective of this study is to investigate the applicability of the most commonly employed sediment transport capacity models to simulate nonsteady state sediment loads from steep slopes under different rainfall intensities. Although Yang (1996) concludes that the sediment transport rate or concentration should be related to the rate of the energy dissipation approach, on which the unit stream power and the stream power models are based, the shear stress approach has been successfully employed by many researchers, as stated above. Therefore, in this study the performance of the unit stream power, the stream power, and the shear stress approaches is extensively investigated to simulate non– steady state sediment transport under different rainfall intensities from steep slopes.

## **Mathematical Development**

There are two parts in modeling rain-induced surface erosion flow dynamics and erosion dynamics. By solving the flow dynamics, one obtains the flow depth and velocity fields on the land surface and the flow discharge from the land surface. The computed flow depth and velocity fields are, in turn, used for the erosion dynamics to predict the sediment concentration field on the land surface and the sediment discharge from the land surface. This approach explicitly assumes that the sediment concentrations in the overland flow regime are sufficiently small so that the suspended sediment does not affect the flow dynamics. Under this assumption, one can simulate these two processes independently. This assumption has been commonly employed by many researchers (Foster and Meyer 1972; Li 1979; Govindaraju and Kavvas 1991).

### Flow Dynamics

Kinematic wave approximation (KWA) is used for modeling nonsteady state flow dynamics in one dimension. Since this study focuses on sediment transport from steep slopes, KWA in one dimension is a fairly good approximation to the full Saint-Venant equations. The KWA equation in one dimension is stated as

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} (Kh^{5/3}) = (r - i)$$
(2)

where

$$K = \frac{\sqrt{S}}{n} \tag{3}$$

and where h= overland flow depth (*L*); r= rainfall intensity (*L*/*T*); *i*=infiltration rate (*L*/*T*); *S*=bed slope; and *n*=Manning's roughness coefficient ( $L^{1/3}/T$ ).

#### **Erosion Dynamics**

The physics-based one-dimensional nonsteady state erosion/ sediment transport equation can be expressed as (Li 1979; Woolhiser et al. 1990)

$$\frac{\partial(hc)}{\partial t} + \frac{\partial}{\partial x}(qc) = \frac{1}{\rho_s}(D_{rd} + D_{fd})$$
(4)

where

$$q = Kh^{5/3} \tag{5}$$

and where *c*=sediment concentration by volume  $(L^3/L^3)$ ;  $\rho_s$ =sediment particle density  $(M/L^3)$ ; *q*=unit flow discharge  $(L^2/T)$ ;  $D_{rd}$ =soil detachment rate by raindrops  $(M/L^2/T)$ ; and  $D_{fd}$ =soil detachment/deposition rate by sheet flow  $(M/L^2/T)$ .

#### Soil Detachment by Raindrops

Soil detachment is a function of the erosivity of rainfall and the erodibility of the soil particles. The erosivity is directly related to the energy produced by raindrop impact and is generally formulated as a power function of rainfall intensity, size of the droplet, cover condition, and terminal velocity of the drop (Meyer and Wischmeier 1969). On a bare soil surface, detachment by raindrops can be expressed as (Li 1979)

$$D_{rd} = \alpha r^{\beta} \left( 1 - \frac{z_w}{z_m} \right) \tag{6}$$

where  $\alpha$ =soil detachability coefficient, which depends on the soil characteristics ( $M/L^2/L$ ). Soil properties known to affect the erodibility include primary particle size distribution, organic matter content, soil structure, content of iron and aluminum oxides, electrochemical bonds, initial moisture content, and aging (Partheniades 1972). Sharma et al. (1993) obtained the range of 0.0006–0.0086 kg/m<sup>2</sup>/mm for  $\alpha$  for easily detachable soils and 0.00012–0.0017 kg/m<sup>2</sup>/mm for less detachable soils. Note that in Eq. (6) *r* is in millimeters per hour,  $\alpha$  is in kilograms per meter squared per millimeter, and  $D_{rd}$  is in kilograms per meter squared per hour. The range for  $\alpha$  obtained by Sharma et al. (1993) is in agreement with Foster (1982).

The parameter  $\beta$  is an exponent whose range is 1.0–2.0. From experimental studies, it is shown that  $\beta$ =2.0 (Meyer 1971; Foster 1982). Sharma et al. (1993) showed that the value of  $\beta$  is in the range of 1.09–1.44. Foster et al. (1977) used a value of  $\beta$  of 1.0. Tayfur (2001) showed that the change in the value of  $\beta$  in between 1.0 and 1.8 does not affect the sediment discharge significantly. In the present study, the value of  $\beta$  is taken as 1.0.

Parameter  $z_w$  is the flow depth plus the loose soil depth (L), and  $z_m$  is the maximum penetration depth of raindrop splash (L). Eq. (6) is valid when  $z_w < z_m$ ; otherwise, there is no detachment by the raindrops. According to Mutchler and Young (1975),  $z_m$ can be equal to three times the median raindrop size and the median raindrop size can be expressed as a power function of rainfall intensity. According to Li (1979)

$$z_m = 3(2.23r^{0.182}) \tag{7}$$

Note that in Eq. (7), r is in millimeters per hour and  $z_m$  is computed in millimeters.

Eq. (6) expresses the detachment by raindrop impact as a power function of rainfall intensity, flow depth, and loose soil depth. As the sum of the flow depth and loose soil depth increases, the penetration depth decreases and consequently the detachment by raindrops decreases.

#### Soil Detachment/Deposition by Sheet Flow

The soil detachment/deposition rate is proportional to the difference between the sediment transport capacity and the sediment load in the flow. This implies that the flow has the maximum eroding capacity when it is free of suspended sediment. When the

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sediment load is greater than the transporting capacity, deposition occurs. The soil detachment/deposition by sheet flow can be expressed as (Foster 1982; Govindaraju and Kavvas 1991)

$$D_{fd} = \varphi(T_c - q_s) \tag{8}$$

where

$$q_s = \rho_s c q \tag{9}$$

where  $q_s =$  unit sediment discharge (M/L/T). If the transport capacity exceeds the existing unit sediment discharge  $(T_c > q_s)$ , the flow will detach particles; otherwise, it will deposit the particles. Parameter  $\varphi$  is the transfer rate coefficient (1/L), which may vary over a wide range, depending upon the soil type. Foster (1982) gives the range  $\varphi = 3-33 \text{ m}^{-1}$  for sand. In the present study, during detachment  $(T_c > q_s)$ ,  $\varphi$  is taken as 24 m<sup>-1</sup>. During deposition  $(T_c < q_s)$ ,  $\varphi$  is estimated as a function of particle terminal fall velocity  $(V_f)$  and the unit flow discharge (q) as (Foster 1982)

$$\varphi = (0.5V_f)/q \tag{10}$$

The particle terminal fall velocity may be estimated from the particle density and size, assuming that the particles have drag characteristics and terminal fall velocities similar to those of spheres. Yang (1996) expresses the particle terminal fall velocity  $(V_f)$  as a function of the particle diameter and the particle Reynolds number. The particle Reynolds number can be expressed as (Woolhiser et al. 1990)

$$\mathscr{R}_{pn} = \frac{V_f d}{\upsilon} \tag{11}$$

where  $V_f$ =particle terminal fall velocity (L/T);  $\mathcal{R}_{pn}$ =particle Reynolds number; d=particle diameter (L); and v=kinematic viscosity of water  $(L^2/T)$ .

When the particle Reynolds number  $(\mathcal{R}_{pn})$  is less than 2.0, the terminal fall velocity of a particle is expressed as (Yang 1996)

$$V_{f} = \begin{cases} \frac{1}{18} \frac{(\gamma_{s} - \gamma)}{\gamma} \frac{gd^{2}}{\upsilon}; & d \leq 0.1 \text{ mm} \\ F \left[ \frac{gd(\gamma_{s} - \gamma)}{\gamma} \right]^{0.5}; & 0.1 \text{ mm} < d \leq 2.0 \text{ mm} \\ 3.32\sqrt{d}; & d > 2.0 \text{ mm} \end{cases}$$
(12)

where g=gravitational acceleration  $(L/T^2)$ ;  $\gamma_s$ =specific weight of sediment  $(M/L^2/T^2)$ ; and  $\gamma$ =specific weight of water  $(M/L^2/T^2)$ ; and

$$F = \begin{cases} \left[\frac{2}{3} + \frac{36v^2\gamma}{gd^3(\gamma_s - \gamma)}\right]^{0.5} - \left[\frac{36v^2\gamma}{gd^3(\gamma_s - \gamma)}\right]^{0.5}; & 0.1 \text{ mm} < d \le 1.0 \text{ mm} \\ 0.79; & 1.0 \text{ mm} < d \le 2.0 \text{ mm} \end{cases}$$
(13)

Note that in Eq. (12),  $V_f$  is in meters per second and d is in meters.

When the particle Reynolds number is greater than 2.0, the terminal fall velocity is determined experimentally. Yang (1996) gives a figure summarizing the fall velocity values depending on the sieve diameter and the shape factor. For most natural sands, the shape factor is 0.7. Rouse (1938) gives  $V_f = 0.024$  m/s for d = 0.2 mm. In the present study, for  $\mathcal{R}_{pn} > 2.0$ , the terminal fall velocity is assumed to be 0.024 m/s.

Transport Capacity Models. Sheet flow transport capacity is a function of several factors that include runoff rate, flow velocity, slope steepness of the surface, transportability of detached soil particles, and the effect of raindrop impact. The basic relationship that does not take into account the effect of raindrop impact on the transport capacity might be a typical sediment transport equation form of Eq. (1). Depending upon the chosen model for the sediment transport capacity of sheet flow, the dominant variable can be shear stress, stream power, and the unit stream power. In the following sections, a brief description of each approach is given.

*Shear Stress Approach*. The transport capacity model that is based on the dominant variable shear stress can be expressed as (Foster 1982; Govindaraju and Kavvas 1991; Yang 1996)

$$T_c = \eta_\tau (\tau - \tau_c)^{k_\tau} \tag{14}$$

$$\tau = \gamma h S \tag{15}$$

and where  $\tau$ =shear stress, which is the tractive force developed

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by the sheet flow to overcome the critical shear stress  $(M/L/T^2)$ ; and  $\eta_i$ =soil erodibility coefficient, which is a function of particle diameter and density. While its value may vary over a wide range, Foster (1982) suggests the value of 0.6 for  $\eta_i$ . Parameter  $k_i$ =exponent whose value varies between 1 and 2.5. Foster (1982) suggests the value of 1.5 for  $k_i$ . Parameter  $\tau_c$ =critical shear stress  $(M/L/T^2)$ , which is a function of the particle diameter and specific weight of the sediment and water. Li (1979) expresses  $\tau_c$  as

$$\tau_c = \delta_s (\gamma_s - \gamma) d \tag{16}$$

where  $\delta_s = a$  constant dependent on flow conditions. Gessler (1965) shows that  $\delta_s$  should be 0.047 for most flow conditions. If rilling develops on the overland flow surface, the value of  $\delta_s$  should be lower (Li 1979). Parameter  $\tau_c$  represents the resistance of the soil against erosion. The critical shear stress is very small for cohesionless soils, and it is often neglected (Foster 1982).

Stream Power Approach. Bagnold (1960) was the first person who introduced the stream power concept and defined it as the power per unit area of stream bed (shear stress times flow velocity,  $\tau V$ ). The transport capacity model that is based on the dominant variable stream power can be expressed as (Yang 1996)

$$T_c = \eta_{\tau v} (\tau V - \tau_c V_c)^{k_{\tau v}} \tag{17}$$

where V= flow velocity (L/T); and  $V_c=$  critical flow velocity at incipient sediment motion (L/T). In Eq. (17),  $\tau V=$  stream power and  $\tau_c V_c=$  critical stream power at incipient sediment motion. V is computed from the flow dynamics part of the model as

$$V = Kh^{2/3} \tag{18}$$

Yang (1996) expresses the critical flow velocity as being dependent upon the shear velocity Reynolds number. The shear velocity Reynolds number is expressed as

$$\mathscr{R}^* = \frac{u_* d}{v} \tag{19}$$

where  $u_*$  = shear velocity (*L*/*T*) and is defined as (Yang 1996)

$$u_* = \sqrt{ghS} \tag{20}$$

The critical flow velocity at incipient sediment motion is expressed as (Yang 1996)

$$V_{c} = \begin{cases} \frac{2.5V_{f}}{\log(\mathscr{R}^{*}) - 0.06} + 0.66V_{f}; & 1.2 < \mathscr{R}^{*} < 70\\ 2.05V_{f}; & \mathscr{R}^{*} > 70 \end{cases}$$
(21)

Unit Stream Power Concept. Yang (1972) was the first person who introduced the unit stream power concept. Yang (1973) and Yang and Song (1979) defined unit stream power as the time rate of potential energy dissipation per unit weight of water (flow velocity times energy gradient, which is approximated by the slope of the soil surface or channel bed, VS). The transport capacity model that is based on the dominant variable unit stream power can be expressed as (Yang 1996)

$$T_c = \eta_{vs} (VS - V_c S_c)^{k_{vs}} \tag{22}$$

where S=energy slope, which is assumed to be equal to the bed slope; and  $S_c$ =critical slope at incipient sediment motion. In Eq. (22), VS=unit stream power; and  $V_cS_c$ =critical unit stream power at incipient sediment motion. By utilizing Meyer-Peter and Muller's (1948) bed load equation, the slope at incipient motion ( $S_c$ ) can be obtained as

$$S_c = \frac{0.058 dn^{1.5}}{h d_{00}^{0.25}} \tag{23}$$

where  $d_{90}$ =bed material size, where 90% is finer (*L*). Note that in Eq. (23), *h*, *d*, and  $d_{90}$  are in meters.

#### Solution Procedure

Eqs. (2) and (4) were solved numerically by using the implicit centered finite difference method. The Newton-Raphson iterative technique was used to solve the set of nonlinear equations resulting from the implicit procedure. As upstream boundary conditions, zero flow depth and zero sediment concentration were used. As downstream boundary conditions, zero depth gradient and zero sediment concentration gradient were employed. Since rainfall starts on a dry surface, there is initially no flow and erosion on the hillslope surface. Under the specified initial and boundary conditions, the numerical solutions of Eqs. (2) and (4) are executed simultaneously for each time step. Every time step, Eq. (2) is first solved to obtain flow depths, flow velocities, and unit flow discharges. Then Eq. (4) is solved to compute sediment concentrations and unit sediment discharges. Every time step (j), the loose soil depth  $(l_d)$ , which is required by Eq. (6), is also computed. The loose soil depth at the (j+1) time step is computed as

$$l_d(j+1) = l_d(j) - [D_{rd}(j) + D_{fd}(j)] \frac{\Delta t}{\rho_s}$$
(24)

The details of the numerical scheme can be obtained from Tayfur (1990).

Sediment discharge (1000\*kg/m/ 6 --- Observed data 4 ---- Unit stream power - Stream power 2 Shear stress 0 10 20 30 40 50 60 Time (min)

Calibration Run: (Slope: 20% Rainfall: 57 mm/h)

**Fig. 1.** Simulation of observed data; calibration run (S = 20%, r = 57 mm/h)

## Analysis of Results

The applicability of the unit stream power, stream power, and shear stress approaches is investigated to simulate nonsteady state sediment transport under different rainfall intensities from steep slopes. For this purpose, the experimental data of Kilinc and Richardson (1973) were chosen.

Kilinc and Richardson (1973) performed experimental studies by using a 1.21 m high×1.52 m wide×4.58 m long flume with an adjustable slope. Commercial sprinklers on 3 m risers, placed 3 m apart along the sides of the flume, simulated rainfall. The flume was filled with compacted sandy soil (90% sand and 10% silt and clay), which was leveled and smoothed before each run. The soil had a nonuniform size distribution with  $d_{50} = 0.35$  mm (the median diameter of the sediment recorded for 50% of the samples having a diameter finer than this size) and  $d_{90} = 1.3$  mm. The compacted sandy soil had a bulk density of 1,500 kg/m<sup>3</sup> and a porosity of 0.43. The major controlled variables were rainfall intensity and soil surface slope. Infiltration and erodibility of the surface were constant. Six bare slopes (5.7, 10, 15, 20, 30, and 40%) were tested with four different rainfall intensities (32, 57, 93, and 117 mm/h). On average, the constant infiltration rate for each run was about 5.3 mm/h. Runoff was recorded continuously and sampled for sediment concentration every 5-10 min during each hour-long run. The details of the experimental setups and experiments can be obtained from Kilinc and Richardson (1973).

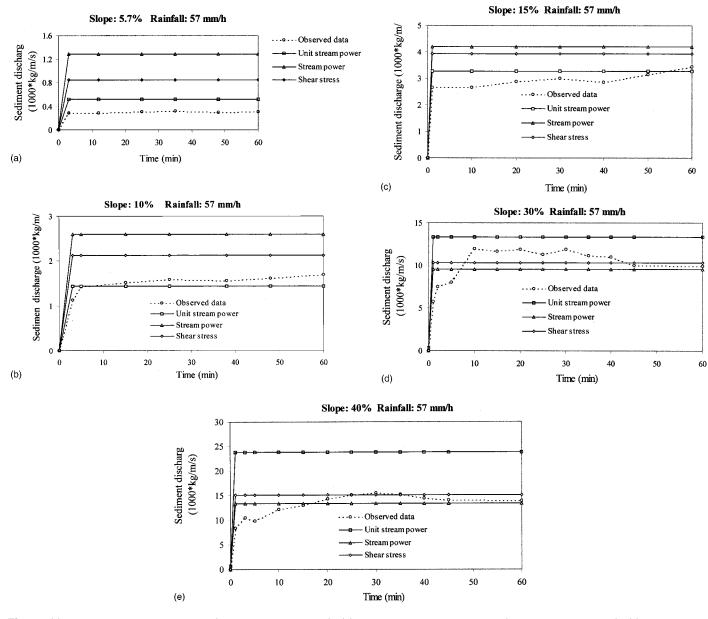
One of the data sets of Kilinc and Richardson (1973) was used for the calibration of the model parameters. Fig. 1 shows the calibration run for the case of 57 mm/h rainfall intensity and 20% slope. The calibrated values of the model parameters that resulted in the best fit for the observed experimental data (Fig. 1) are as follows:

- Manning's roughness coefficient (n): 0.012 (m<sup>1/3</sup>/s),
- Soil detachability coefficient ( $\alpha$ ): 0.0012 (kg/m<sup>2</sup>/mm),
- Soil erodibility coefficient  $(\eta_{\tau} = \eta_{\tau v} = \eta_{vs})$ : 0.10,
- (Unit stream power) exponent  $(k_{vs})$ : 1.56,
- (Stream power) exponent  $(k_{\tau v})$ : 1.18, and
- (Shear stress) exponent  $(k_{\tau})$ : 1.92.

These values are within the ranges suggested in the literature (Foster and Meyer 1972; Woolhiser 1974; Li 1979; Foster 1982; Sharma et al. 1993). Note that the calibrated parameter values are the same for each model. The only difference is that the values of the exponents  $(k_{vs}, k_{\tau v}, k_{\tau})$  are different for each model.

The calibrated values of the model parameters were then employed in the simulation of different data sets. Figs. 2(a-e) show

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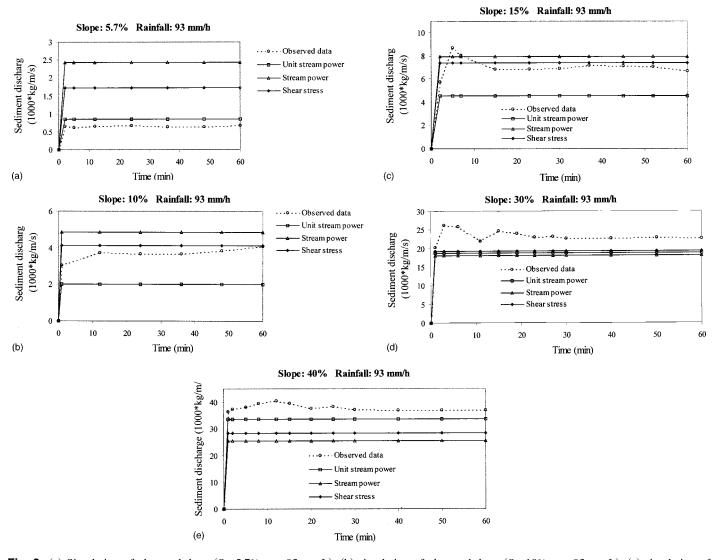


**Fig. 2.** (a) Simulation of observed data; (S = 5.7%, r = 57 mm/h); (b) simulation of observed data; (S = 10%, r = 57 mm/h); (c) simulation of observed data; (S = 15%, r = 57 mm/h); (d) simulation of observed data; (S = 30%, r = 57 mm/h); and (e) simulation of observed data; (S = 40%, r = 57 mm/h)

the simulations of observed sediment loads by the three models from 5.7, 10, 15, 30, and 40% slopes, respectively, under 57 mm/h rainfall intensity. Figs. 3(a-e) show the simulation of observed data by the three models from 5.7, 10, 15, 30, and 40% slopes, respectively, under 93 mm/h rainfall intensity. Under 57 mm/h rainfall intensity, the unit stream power model simulated the observed data from the 5.7, 10, and 15% slopes quite satisfactorily [Figs. 2(a-c)], though it overestimated the loads from steep slopes of 30 and 40% [Figs. 2(d and e)]. On the contrary, under 57 mm/h rainfall intensity, the stream power and the shear stress models simulated the loads from the 30 and 40% slopes satisfactorily [Figs. 2(d and e)], while they overestimated the loads from the 5.7, 10, and 15% slopes [Figs. 2(a-c)]. Under 93 mm/h rainfall intensity, the unit stream power model simulated the loads from the 5.7% slope quite satisfactorily [Fig. 3(a)], though it underestimated the loads from the other slopes [Figs. 3(b-e)]. On the other hand, under 93 mm/h rainfall intensity, the

shear stress and the stream power models simulated the loads from the 10 and 15% slopes satisfactorily [Figs. 3(b and c)], while they underestimated the loads from the 30 and 40% slopes [Figs. 3(d and e)], and overestimated the loads from the 5.7% slope [Fig. 3(a)].

From the analysis of the model simulations of the observed data, it can be concluded that the unit stream power model performs better than the other two models in simulating nonsteady state erosion/sediment transport by sheet flow from bare slopes less than 10%. The stream power and the shear stress models, on the other hand, perform better than the unit stream power model to simulate sediment loads from bare steep slopes greater than 20% under low rainfall intensity. The results also indicate that, in between the 10 and 20% slopes, under low rainfall intensities the unit stream power model gives better simulations; under high rainfall intensities the stream power and the shear stress models give better simulations.



**Fig. 3.** (a) Simulation of observed data; (S = 5.7%, r = 93 mm/h); (b) simulation of observed data; (S = 10%, r = 93 mm/h); (c) simulation of observed data; (S = 15%, r = 93 mm/h); (d) simulation of observed data; (S = 30%, r = 93 mm/h); and (e) simulation of observed data; (S = 40%, r = 93 mm/h)

## **Concluding Remarks**

In this study, the unit stream power, stream power, and shear stress sediment transport capacity models, which had the basic form given by Eq. (1), were investigated to simulate nonsteady state sediment loads by sheet flow from steep bare slopes. For this purpose, the experimental data of Kilinc and Richardson (1973) were employed.

The models were calibrated by one of the data sets, and employed to simulate different sediment loads from different slopes (5.7, 10, 15, 30, and 40%) under two different rainfall intensities (57 and 93 mm/h). The calibrated values of the model parameters were the same for each model, except for exponent  $(k_i)$ . The value of  $(k_i)$  is given in between 1.0 and 2.5 in the literature. The model-calibration results in this study indicate that the value of  $(k_i)$  is around 1.2 for the stream power model, 1.9 for the shear stress model, and 1.6 for the unit stream power model.

The models were found to be very sensitive to the changes in rainfall intensities and slopes. An increase/decrease in rainfall intensity and slope results in an increase/decrease in the sediment yield, and each model is able to capture this behavior. However, the performance of each model in simulating sediment yields from different slopes was found to be very much dependent upon the steepness of the slope, and the intensity of the rainfall. Therefore, the slope steepness and rainfall intensity play a major role in the selection of an appropriate sediment transport capacity model in simulating nonsteady state sediment loads by sheet flow.

The unit stream power model could be selected for simulating nonsteady state erosion/sediment transport from very mild bare hillslopes. Under low rainfall intensities, it could also be employed to simulate loads from mild/steep slopes. For the very steep slopes, the shear stress and stream power models could be employed. Under high rainfall intensities, the stream power and the shear stress models could also be employed to simulate loads from mild/steep slopes.

### Notation

The following symbols are used in this paper:

c = sediment concentration  $(L^3/L^3)$ ;

D = dominant variable;

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- $D_c$  = critical condition of dominant variable at incipient motion;
- $D_{fd}$  = soil detachment/deposition rate by sheet flow  $(M/L^2/T);$
- $D_{rd}$  = soil detachment rate by raindrops  $(M/L^2/T)$ ;
  - d = particle diameter (L);
- $d_{90}$  = bed material size where 90% is finer (L);
- g = gravitational acceleration  $(L/T^2)$ ;
- H = hillslope length (L)
- h = overland flow depth (L);
- i = infiltration rate (L/T);
- j = index for time in numerical scheme;
- $k_i =$ exponent;
  - $l_d$  = loose soil depth (L);
  - n = Manning's roughness coefficient  $(L^{1/3}/T)$ ;
  - q = unit flow discharge  $(L^2/T)$ ;
  - $q_s$  = unit sediment discharge (M/L/T);
  - $\mathcal{P}_{nn}$  = particle Reynolds number;
- $\hat{\mathscr{R}}^*$  = shear velocity Reynolds number;
  - r = rainfall intensity (L/T);
  - S = bed slope;
  - $S_c$  = critical slope at incipient sediment motion;
- $T_c$  = transport capacity of sheet flow (M/L/T);
- t = time(T);
- $u_*$  = shear velocity (L/T);
- V = flow velocity (L/T);
- $V_c$  = critical flow velocity at incipient sediment motion (L/T);
- $V_f$  = particle fall velocity (L/T);
- $z_m$  = maximum penetration depth of raindrop splash (*L*);
- $z_w$  = flow depth plus loose soil depth (L);
- $\alpha$  = soil detachability coefficient ( $M/L^2/L$ );
- $\beta$  = exponent;
- $\gamma$  = specific weight of water  $(M/L^2/T^2)$ ;
- $\gamma_s$  = specific weight of sediment  $(M/L^2/T^2)$ ;
- $\Delta t = \text{time step } (T);$
- $\delta_s = \text{constant};$
- $\eta_i$  = coefficient that represents erodibility of soil;
- $\rho_s$  = mass density of sediment particles  $(M/L^3)$ ;
- $\tau$  = shear stress  $(M/L/T^2)$ ;
- $\tau_c$  = critical shear stress  $(M/L/T^2)$ ;
- v = kinematic viscosity of water  $(L^2/T)$ ; and
- $\varphi$  = transfer rate coefficient (1/L).

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