THE USE OF NONEXTENSIVE FRAMEWORK IN CONNECTION WITH TRAFFIC FLOW

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ABSTRACT

THE USE OF NONEXTENSIVE FRAMEWORK IN CONNECTION WITH TRAFFIC FLOW

In the analysis of vehicular traffic flow, numerous techniques are utilized in the literature. In this thesis, distinct from the literature, to reveal the complexity of the traffic flow and its connection with the urban and traffic factors, nonextensive thermostatistics is implemented. In real systems, e.g. vehicular traffic flow, the probability distributions would become q-Gaussian and thus the use of nonextensive thermostatistics would be relevant. This approach allows a statistical interpretation to handle the given traffic flow problem. In this thesis, highway traffic flow modeling is in question in the nonextensive framework and two case studies are presented. First is related with lane changing and driver behavior, and the other is related with the superstatistics and traffic flow. In the first case study, scenario-based vehicular interactions are examined and driver behaviors are extracted by virtue of given entropy approaches. Given the configurations, Tsallis entropy approach characterizes safe driving behavior, whereas Boltzmann-Gibbs one describes unsafe driving. In the second case, vehicle speeds on the selected highway are analyzed through superstatistics theory. Two distinct q values are computed as 1.3 and 1.8 out of q-Gaussian and beta parameter distributions, respectively. The q value of 1.3 represents the highway segment with a certain flow, while the q value of 1.8 specifies the history of the traffic flow. As a result, it is revealed that the real vehicular traffic flow would involve the nonadditivity, which mainly stems from vehicular interactions as well as the urban and traffic planning decisions.

ÖZET

GENİŞLETİLEMEZLİK İLKESİNİN TRAFİK AKIŞI ÜZERİNDE KULLANIMI

Literatürde araç trafik akış analizi için pek çok teknik kullanılmaktadır. Bu tezde farklı olarak trafik akışının kompleksiliğini, kentsel ve trafik faktörleriyle ilişkisini ortaya koyabilmek için genişletilemez termoistatistiği uygulanmıştır. Gerçek sistemlerde örneğin araç trafik akışında olasılık dağılımları q-Gaussian çıkmakta ve dolayısıyla genişletilemez termoistatistiğinin kullanımı uygun olmaktadır. Bu yaklaşım trafik akış problemleri ile baş edebilmek için istatistiksel bir çıkarım sağlamaktadır. Bu tezde, genişletilemezlik ilkesinde karayolu trafik akış modellemesi söz konusu olmaktadır ve iki örnek çalışma verilmektedir. İlki şerit değiştirme ve sürücü davranışı, diğeri ise süperistatistik ve trafik akışı ile ilgilidir. İlk örnek çalışmada, söz konusu entropi yaklaşımlarına dayanarak senaryo temelli araç ilişkileri incelenmektedir ve sürücü davranışları ortaya çıkartılmaktadır. Tez kabulleri altında, Tsallis entropisi yaklaşımı güvenli sürüşü, Boltzmann-Gibbs entropisi ise güvensiz sürüşü tarif etmektedir. İkinci örnek çalışmada, seçilen karayolunda araç hızları superistatistik teorisi yoluyla analiz edilmektedir. Beta parametresi ve q-Gaussian dağılımlarından sırasıyla 1.8 ve 1.3 olmak üzere iki farklı q değeri hesaplanmıştır. Bulunan 1.3 q değeri belli bir akışta o karayolu parçasını temsil ederken, 1.8 q değeri trafik akışının geçmişini belirtmektedir. Sonuç olarak, gerçek trafik akışının esas itibarıyle araç etkileşimleri, kent ve trafik planlama kararlarından etkilenen toplanamazlık içereceği gösterilmiştir.

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$C(\nu)$	Correlation function
d	Spatial dimension
$\langle E \rangle$	Mean energy
E_i	Energy states
E_p	Position entropy
$\langle E \rangle_{pot}$	Average potential energy
E_v	Energy entropy
F	Helmholtz free energy
k	Boltzmann constant
m	Microscopic mass
n	Number of degrees of freedom
N	Number of particles, vehicles
p	Probability, momentum
P	Pressure, position
q	Tsallis entropic index
$R_{xx}(\tau)$	Time autocorrelation function
S	Entropy
T	Temperature, long time scale
V	Volume, velocity
W	Total number of configurations
$\langle x \rangle$	Sample mean
$\langle X \rangle$	Statistical average
Z	Partition function
α	Range of interaction parameter
β	Inverse temperature
β_0	Mean inverse temperature
β^o	Inverse temperature
Γ	Gamma function
ε	Gaussian approximation parameter

θ	Angle
κ	Kurtosis
λ	Lyapunov exponent
μ	Chemical potential
ν	Speed data set
ξ	Sensitivity function
Ξ	Grand partition function
τ	Short time scale

CHAPTER 1

INTRODUCTION

1.1. Literature Review

Traffic flow and the highways reflect the ever growing complexity of the human society. This growing complexity demands tools and techniques matching its intricacy. It is surmised that traffic flow is composed of many long-range interacting agents, generating a long-term memory, hence non-Markovian dynamics. Unfortunately the literature lacks research on long-term or long-range memory. Hence, it is safe to say that the traffic flow has never been fully analyzed reflecting its true complex nature. None of the models could fully represent real life and, the true complexity of the vehicular traffic flow.

An ideal model should account for multifarious aspects such as long-range interactions, long-term memory, non-Markovian dynamics, non-Gaussian distributions. These aspects are an interplay of factors composed of for example traffic lights, vehicle overtaking, peripheral elements, speed bumps, bus stops etc. However, writing a hypothetical formula by substituting each and every factor mentioned above would not only be complex, but also would be very difficult to analyze. Instead of dealing with each individual constituting factor, a macroscale solution embodying all of those factors may still reflect the original complexity. This solution is also expected to be more straightforward to deal with. The complexity of vehicular traffic flow is thus described through such a macroscale approach and in the thesis this approach is no other than nonextensive statistical mechanics. The framework allows a statistical interpretation to handle the given traffic flow problem. The main reason to use nonextensivity is that it would cover the aforementioned aspects of the real traffic flow. Specifically, lane changing and driver behavior analyses, and a superstatistical model of the traffic flow are presented in this thesis.

The given literature in the following sections is consistent with the main topics of this thesis. The literature review consists of car following models, lane changing and driver behavior modeling, statistical mechanics and vehicular traffic modeling, and finally superstatistics and vehicular traffic modeling.

1.1.1. Car-following models

Car-following models have been studied for almost more than half of a century in the literature. In a strict sense, the car-following behavior is considered how a vehicle adjusts its own speed and position depending on the vehicle ahead and other surrounding drivers. The car-following behavior can be investigated on a single lane or multi-lane within a short-ranged or long-ranged vehicular distances and interactions under different traffic flow conditions. An extensive number of models are formulated to tackle car-following behavior. During 1950s, the first efforts of researchers establish follow the leader models. Pipes (1953) explains an idealized law of separation, and thus each vehicle adjust the certain distance with preceding vehicle under the safe-distance car following behavior. The study of Chandler, Herman and Montroll (1958) at the General Motors (GM) Research Lab proposes another widely known car following models in fifties. The model developments are carried on by the various studies such as Herman, Montroll, Potts and Rothery (1959), Gazis, Herman and Potts (1959), Edie (1961), and Heyes and Ashworth (1972). The studies deal with finding the optimal parameter combinations for Gazis-Herman-Rothery model and those are examined in the paper of Brackstone and McDonald (1999).

The study of Brackstone and McDonald (1999) gives a thorough overview of the car following models. The authors investigate car following models in five categories. Those are Gazis-Herman-Rothery (GHR), safety distance or collision avoidance (CA), linear (Helly), psychophysical or action point (AP), and fuzzy logic-based models.

Car-following theory can be classified in different groups especially in regard to the oldest examples in traffic literature. For example, Gunay (2007) propose two groups with their examples. Namely, stimulus-based (or psycho-psysical) models, e.g. GM models; and stopping-distance based (or safety distance) models, e.g. Gipps' model. Likewise, Hoogendoorn and Bovy (2001) put forward three types of models that are safe-distance models, stimulus–response models and psycho-spacing. The authors show the widely known examples of each group. For example, the study of Pipes (1953) are attributed to the safe-distance models, whereas the study of Chandler et al. (1958) is given for the stimulus-response models and that models are known as follower response to the stimuli. In those models, the response of the following vehicle may describe the accelerations and decelerations, whereas stimulus may involve relative velocity between the leader and follower, distance-headway and time-headway. Gazis-Herman-Rothery model in the study of Gazis, Herman and Rothery (1961) is proposed as another example for the stimulus-response models. In their model, speed differences between follower and leading vehicles are utilized and driver’s sensitivity coefficient is processed. Hoogendoorn and Bovy (2001) also discuss psycho-spacing models with reference to the first developed examples and in the literature.

Wang, Zhang, Guo, Bubb and Ikeuchi (2011) deal with the simulation modeling of driver's safety approach behavior. Deceleration and acceleration algorithms concerning driver's car following safety behavior is presented. In the light of different modules such as module of leading vehicle and following vehicle, the authors simulate the driver's car following behavior. The performance of their model is validated under three different car-following cases and the simulation results are discussed in their paper. The authors state that since their model is depended on driving human factor analysis, it may be considered as a realistic starting point for the driver's car following models.

1.1.2. Lane Changing and Driver Behavior Modeling

Various kinds of studies on lane changing and driver behavior have evolved in the traffic literature. In the traffic there could be many factors affecting lane changing behavior. Whereas some of those factors are related to road, weather conditions etc., some can be directly connected with driver behaviors and characteristics. Before reviewing recent modeling research, it is important to notice that the classical studies on the development of driver behavior models date back to early 20^{th} century. Hence, one of the classical articles of the time which is considered in psychological perspective belongs to Gibson and Crooks (1938). The study is basically governed by the concepts of the field of safe travel and the field of minimum stopping zone. In the study, for example, factors limiting the field of safe travel are presented and influences of various obstacles on the field of safe travel are discussed. Further, that the minimum stopping could be dependent on the speed, road-surface and brakes is emphasized. In a large time-span Vaa (2014) covers some the other representative classical driver behavioral models in traffic psychology.

In the pursuit of mathematical model building and problem definition for this thesis, there are more relevant recent research on lane changing and driving behavior models. For example, Sun and Elefteriadou (2011) in their study consider a few reasons for lane changing. These reasons are, for example, the existence of heavy vehicle/truck in front of the drive, pavement conditions, passing stopped bus at a bus stop, presence of tailgating vehicle etc. The questionnaire regarding the attributes above is presented to the participants and the authors obtain the analysis of the results. The analyses present the driver type classification, actions of different driver types, factors affecting each lane-changing scenario. The authors state that the results could be applicable for the urban street network. The article of Lv, Song and Fang (2011) points out that there is an interaction among diverse individuals under certain restricts. Since the drivers want to travel at a desired velocity or better driving conditions, they try to change lanes and in the lane changing rule-set of the study, decision to change lane is considered by two criteria i.e. incentive and security. The study also emphasizes the simulation of multi-lane changing behavior to represent the reality. On a three-lane roadway, the authors adopt optimal velocity and full velocity difference single-lane model in their simulations and investigate the effects of lane-changing behavior on several traffic based variables. Li, Jia, Gao and Jiang (2006) extend symmetric two-lane cellular automata (STCA) model to simulate traffic. The authors bring in new set of lane-changing rules to update the STCA model for aggressive lane-changing behavior of fast vehicle and diverse lane-changing behaviors of different types of vehicles. Influences of different lane-changing probabilities and aggressive lane-changing behavior of fast vehicles are discussed, and the results of the default STCA model and their new model are presented in their study.

As a widely known study by Nagel, Wolf, Wagner and Simon (1998) proposes two-lane traffic rules in regard to cellular automata under lane changing behavior. The authors also discuss security and incentive criteria, velocity-based and gap-based rules for the lane changing, and generate space-time plots and fundamental diagrams. Another study by Knospe, Santen, Schadschneider and Schreckenberg (2002) seems representative for the lane changing rules, presenting symmetric and asymmetric models and comparing single-lane and two-lane traffic. Huang (2002) proposes lane changing rules for the two-lane highway configuration. In regard to the lane changing rules, incentive and safety criteria are classified and the effects of the three parameters in the study are investigated. On the work of the lane changing for the two-lane traffic, Nagai, Nagatani and Taniguchi (2005) consider traffic states, jamming transitions and the effect of a bus. The study applies optimal velocity model and the simulation results are discussed. The research by Qian, Luo, Zeng, Shao and Guo (2013) establishes three-lane model and focuses on the overtaking ratio under different control conditions. On four-lane highway, the lane changing and overtaking behavior of the vehicles under mixed traffic conditions are discussed (Chandra & Shukla, 2012).

In various studies, the relationships between driver characteristics and overtaking processes are considered. Mohaymany, Kashani and Ranjbari (2010) find that the younger drivers who have less driving experience, more dangerous maneuvers and more risk-taking behaviors are most likely be at fault in overtaking crashes. The study also recommends that new drivers who are in the first year of driving should be banned to drive on the rural roads. Moreover, driver stress and driving performance are investigated in the study by Matthews et al., 1998. The comparison between young male and female drivers during overtaking on the two-lane highway is addressed, and the Bayesian methodology is proposed (Vlahogianni & Golias, 2012). The statistical analyses are involved by Bar-Gera and Shinar (2005) to provide whether tendency of the drivers to pass is correlated with the driver attributes. The research by Farah and Toledo (2010) models the passing maneuvers in two stages i.e. desire to pass model and gap acceptance model. For example, the probability to desire passing is influenced by the difference between the desired speed of the subject vehicle and the speed of the leading vehicle, and by the following distance. The individual specific error term which captures the driver characteristics is also related with the probabilities of desiring passing and accepting the available gap in their study. Farah, Bekhor and Polus (2009) propose passing risk-prediction model, testing the regression-based models. They utilize some explanatory variables related with the road geometry, traffic conditions, driver’s characteristics.

1.1.3. Statistical Mechanics and Vehicular Traffic Modeling

Wilson (2009) shows the applicability of the methods of statistical mechanics and thermodynamics in the field of urban modeling. The author states the importance of modeling of transport flow in cities and spatial interactions in regard to statistical mechanical analogy. In the light of statistical mechanics terms, the entropy maximization is explained under the suitable constraints for the urban modeling examples i.e. retail and transport flow models. Another recent study i.e. Wilson (2010) presents the importance of entropy maximizing concept and relates its usage to geography, and especially transportation and retail modeling with underlying statistical mechanical concepts. In some parts, the author exhibits retrospective look and indicates the relationship between the entropy maximizing model and widely known examples such as Bayesian methodology, Shannon information theory and cost-benefit analysis in transportation planning and economics. The study presents some analogies between the application fields of entropy maximizing approach and statistical mechanics. For example, in transportation modeling, particularly the expressions of the balancing factors exhibit similarities to the partition functions of Boltzmann thermostatistics. The author implies that the methods of statistical mechanics could safely be applied to a large class of transportation problems. The study also addresses the integration of entropy modeling with potential other approaches e.g. economics, agent-based modeling, activity-based approach, network and spatial analyses.

Mean energy constraint in traffic flow is important for this study. Energy can be given via Hamiltonian mechanics as in some relevant traffic studies in literature. For example, Krbálek (2007) considers N identical vehicles on a circle and describes the energy in the traffic system via Hamiltonian. Hamiltonian function in Krbálek (2007) involves the mean velocity and particle velocity to express the kinetic energy part. Through the same function, spacing between neighboring vehicles that is claimed to be short-range relations is also processed. This is described as short-ranged potential energy in the second part of the Hamiltonian. Chi-square values are provided within the traffic density figures, for varying alpha values ranging from 1 to 5. For the described potential function, alpha values would be larger than 1 to reveal the short-range interactions. However, the best results are presented with alpha values 1 or near it, and therefore the term short-range becomes objectionable. Furthermore, this thesis with an enquiring look doubts what would happen if the alpha values are slightly less than unity, but that would violate the claim that the interactions are short-ranged. Another possible critic is that the values of alpha in the same figures larger than 3 are redundant. The other would be that chi-square value gets larger towards larger densities and thus a potential comment to Krbálek (2007) may be that the alpha values would be varied. That is, alpha values ought to be reduced as density gets larger. The last objection is that the formulations, seemingly, based on a canonical ensemble, if so, it is also violated there. The hunch of the author of the thesis is that it is neither the microcanonical nor canonical. Likewise, the works of Krbálek and Helbing (2004), Abul-Magd (2006), and Weber, Mahnke, Kaupužs and Strömberg (2007) are also representative of using Hamiltonian energy function in traffic flow.

Šurda (2007) applies a non-Hamiltonian approach to a small system of cars on a single lane road. The nearest neighbor interactions are assumed and the velocities and the coordinates of the cars are considered in the conditional probabilities of the model. The microcanonical and canonical ensembles are utilized in the context of the car system of the study.

There are also some examples of presenting different traffic flow phases in literature. Weber et al. (2007) consider that traffic flow has two separate phases i.e. jam and free-flow and the relationship between those phases is considered within the thermodynamically traffic liquid-gas transitions. The other study, Sopasakis (2004), shows the categorization of traffic regions, free-flow, synchronized traffic, wide moving jams, congested traffic, for example.

In the study of Kosun and Ozdemir (2014), in the given basic car following scenario the distance between adjacent vehicles are analyzed within the entropy framework. Depending on the interaction between the vehicles, the use of Boltzmann-Gibbs and Tsallis entropies is discussed.

1.1.4. Superstatistics and Vehicular Traffic Modeling

The theory of superstatistics is well-presented in the study of Beck and Cohen (2003). The most relevant research papers for both superstatistics and traffic flow belong to Kosun and Ozdemir (2016), Abul-Magd (2007), and Nie, Lu and Shi (2010). In the work of Abul-Magd (2007), the clearance distributions of the traffic flow and the time-headway distributions are studied. Using an analogy, traffic states are considered to be phase changes of matter, as a function of vehicle density. An analytic expression is derived for the spacing distributions that interpolates from the Poisson distribution. The fast dynamics is represented by the vehicle velocity, whereas the slow dynamics by the traffic density. In the study of Nie et al. (2010), characteristics of headway distribution for urban expressway traffic has been analyzed. Kosun et al. (2016) analyze the vehicle speeds, speed differences, and the shuffled speed data through superstatistics. Tsallis q values out of q-Gaussian fit and beta-fit are computed. The authors claim that there ought to be two distinct q values, reflecting the highway or traffic properties, and the interactions.

CHAPTER 2

PROBLEM DEFINITION

Vehicular traffic flow is an important research topic for cities and regions. Traffic flow may consist in a large number vehicles, and is strongly affected by road conditions, weather conditions, interactions among vehicles, environment etc. In the analysis of traffic flow a numerous techniques are developed and implemented in the literature. Some of those techniques are utilized within the framework of thermodynamics and statistical mechanics. This thesis focuses on the vehicular traffic flow phenomenon in the context of nonextensive statistical mechanics. Nonextensive statistical mechanics is the generalization of Boltzmann-Gibbs (BG) thermostatistics. Boltzmann-Gibbs thermostatistics could be described as a special and limiting case of nonextensive thermostatistics. In the traffic flow literature, fingerprints of Boltzmann-Gibbs thermostatistics have been revealed. For example, lane changing and traffic flow models are considered within short-range interacting or the closest neighborhood vehicular flow systems. Thus, in regard to statistical mechanics, the models of the lane changing behaviors and many other traffic flow analysis are handled in the framework of Boltzmann-Gibbs thermostatistics where the total entropy of a system corresponds to the sum of the entropies of the N independent parts of the system i.e. additivity property holds. However, this thesis claims that for a real traffic flow this is generally not so. It is worth emphasizing that the real traffic flow system may involve long-range interaction effects, long-term memory (non-Markovian processes) and (multi)fractal space. Aforementioned properties could make BG statistical mechanics restrictive despite the fact that it can still have relevant position in some standard physical systems where such properties as strong mixing, short-range memory, positive Lyapunov exponent occur. Hence, the author of this thesis states that if lane changing behavior, car following, acceleration, deceleration etc. would be related to the nonextensive (Tsallis) thermostatistics, the traffic flow could be represented more realistically.

One of the main properties of the Tsallis thermostatistics is nonadditivity where the Tsallis entropic index q corresponds to a value except for unity. Nonadditivity or additivity property explicitly implies that subsystems are assumed to be probabilistically independent. The nonadditivity property can be classified in two groups i.e. subadditive (q>1) and superadditive (q<1) according to the value of Tsallis entropic index q, privileges common and rare events, respectively. Here, the q value describes the degree of nonextensivity.

A number of traffic flow variables and mobility conditions may be handled in the nonextensive statistics. In long-range interacting systems, probability distributions e.g. velocities may not be Gaussian and in this situation exponential decay of Boltzmann-Gibbs thermostatistics disappears. Indeed, this thesis asserts that vehicular traffic flow is not typical case of the BG thermostatistics and has not Gaussian form, but it could exhibit power-law decay in the form of q-exponential functions. Also, the q-exponential function may have a cutoff property in some cases.

In this thesis, the lane changing behavior of the vehicles and vehicle speeds are examined as two distinct cases within the nonextensive framework. This yields the questions such as how to relate the vehicular traffic flow to nonextensivity, how to analyze the lane changing behavior and vehicle speed parameter in nonextensive context, how to extract driver behaviors from the lane changing scenarios, how to test the nonextensivity in the traffic flow, and how to connect the nonadditivity and additivity in the traffic flow with the urban and traffic planning decisions.

The first case of the two mentioned above, the lane changing behavior of the vehicles is studied through different hypothetical scenarios in connection with long-range, quasi long-range and short-range interactions. In terms of entropy and energy, lane changing scenarios are linked with traffic characteristics and driver behaviors. The number of accessible and forbidden states are affected through lane changing scenarios with regard to given statistical mechanics formalism. In this case, distinct from the literature, the thesis explores lane changing rules and driver behaviors in the traffic flow within the framework of superadditivity, subadditivity, additivity, extensivity and nonextensivity concepts.

In the second case, the quantitative basis of the research is more pronounced. It is evident that a large number of interacting vehicles may generally exist and in this case their macroscopic dynamics is explained by a statistics, i.e. superstatistics. Certain statistical analyses are carried out to describe the effects of the long-range interactions, long-term memory, environment, etc. on the vehicle speeds at the selected highway. The problem is how to obtain the Tsallis entropic index q indicating the degree of nonextensivity and the number of the entropic indices characterizing the traffic flow.

Notice that the literature still lacks distinctive entropic indices q of a given time series and the thesis extracts two different Tsallis entropic indices from the given traffic flow data. One of them designates the general characteristics of the traffic flow on the highway segment, whereas the other represents how the traffic flow has happened in a time history. The probability distribution of a given parameter is important in extracting the nonextensivity parameters q. For example, the distribution of the local variances of the whole time series is the key to one of the entropic indices. The deviations from the Gaussianity are also investigated by the help of the probability distributions of the given time series.

Let us consider the two cases in more detail to clarify the problem definition of this thesis. In the following sections some of the diagrammatic examples related to the cases are given.

2.1. Lane Changing and Driver Behavior Analyses Case

Lane changing behavior is a crucial part of the vehicular traffic flow and it may involve certain rules especially in real traffic flow. Lane changing behavior can be performed by discretionary or mandatory bases. During lane changing maneuvers, different drivers generally behave in particular manners and some of drivers do not observe the identified rules, whereas the others might do. This reveals that the specific driver characteristics could happen in the traffic flow. Those drivers may be described as aggressive, risky, young, experienced etc. according to the context of the study.

Lane changing is generally required in the traffic flow due to the complexity of the given road segment. Depending on the different road types such as rural ways, arterials and highways, the frequency and behavior of the lane changing could change. Besides, the complexity of the given road could stem from such factors as the environment, vehicular interactions, number of lanes, maintenance, signalization, parking, etc. With respect to the vehicular interactions, nearest neighborhood interactions in lane changing models are well studied in the literature. However, these interactions can be designated short-range and involved in BG statistics. The complexity of the traffic flow and lane changing behavior would embody the long-range vehicular interactions as well. Occurrence of those interactions in the traffic flow is directly related to the lane changing behavior. This thesis states that underlying statistics to examine those long-range interactions is the Tsallis thermostatistics. Energywise and entropywise considerations are provided, and four classes of traffic flow and driver behaviors are extracted, details can be found in Chapter 5. Two of those classes are well within the nonextensive entropy, whereas the others are involved in BG thermostatistics. For example, the entropywise consideration of the traffic flow can be illustrated in Figure 2.1. Four scenarios are generated for this consideration.

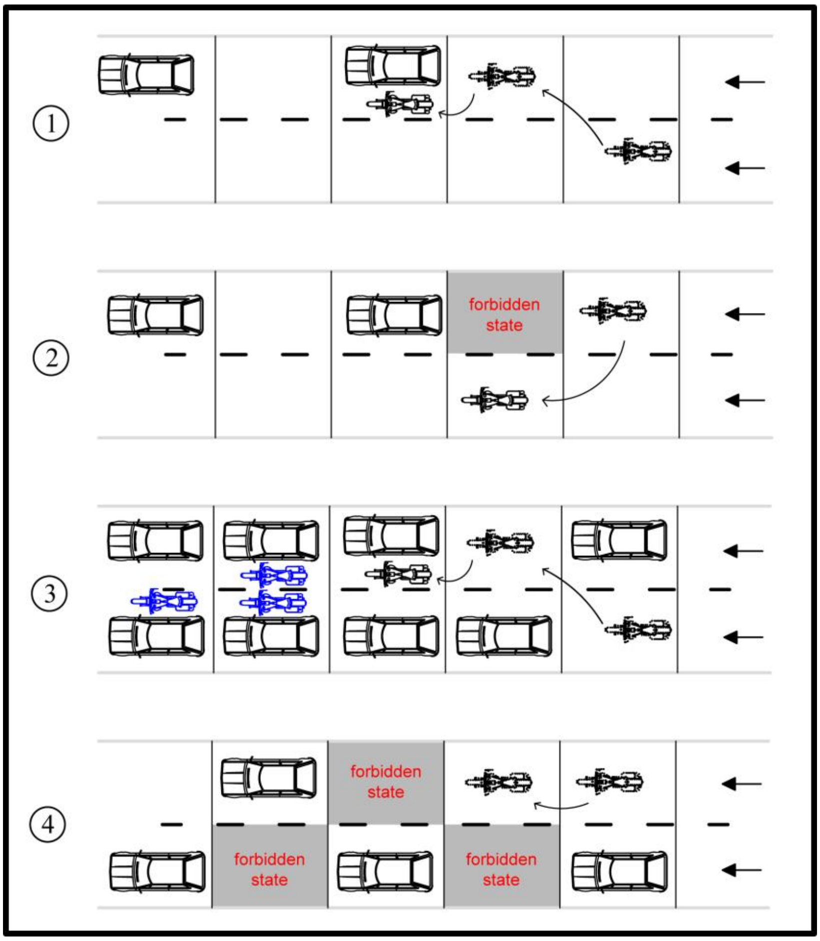


Figure 2.1. Entropywise Consideration of Traffic Flow

Several key points for the scenarios from 1 to 4 are respectively given as follows:

- Drivers may choose all the states and the traffic is disordered.
- Entropic nonadditivity emerges and the prudent drivers leave a safe distance among vehicles around themselves. Traffic flow is ordered.
- Reckless driving behavior generates entropic additivity. Those drivers could choose any unoccupied space. Since allowable states are no longer

proportional to the number of vehicles, the entropic nonextensivity emerges. The disordered traffic flow is present.

- Entropic nonextensivity is observed as in the third case. The traffic flow tends to congest. The traffic flow belongs to subadditive region.

The extracted driving behaviors from the analyses of entropic extensivity and additivity could be shown in Figure 2.2.

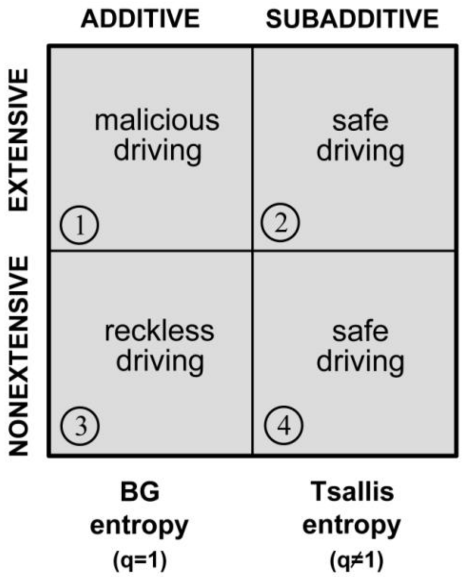


Figure 2.2. Extraction of Driver Behaviors from Entropywise Consideration of Traffic Flow

The scenarios involved in subadditive region exhibit safe driving, whereas the scenarios in additive region exhibit malicious and reckless driving. As a result, safe driving and unsafe driving can be explained by Tsallis and BG statistical mechanics, respectively.

2.2. Superstatistics and Traffic Flow Case

Complex nonequilibrium systems may be considered as the superposition of different dynamics with well separated time scales. In this thesis, real vehicular traffic flow system is described by superimposed dynamics with different time scales and it is designated as a superstatistical complex system. Two different time scales are extracted whose dynamics are fast and slow, details of how to extract the sufficient time scales are given in Chapter 5. By utilizing superstatistics, two entropic indices (*q*) are computed and non-Gaussian behavior of the traffic flow is displayed. To apply the superstatistics theory, vehicle speed time series data are utilized. This thesis contends that the traffic time series data are formed hinging on the factors such as the long-term memory, long-range interactions and environment. In this case, the superstatistics is employed to expound this argument. The results of the analyses substantiate the connection between nonextensivity and traffic flow. Thus, the real traffic flow usually does not manifest a random behavior, but rather ordered characteristics. The inverse local variances of the time series are selected to correspond to the inverse temperature parameters and their fluctuations are considered to demonstrate the complex nonequilibrium traffic flow system in the superstatistics approach. From the probability distributions of the inverse local variances, the Tsallis *q*-statistics is obtained. Hence it may be stated that the probability distribution of the partitioned windows of the time series could exhibit non-Gaussian behavior. Additionally, the probability distribution of the given whole data supports the characteristics of non-Gaussianity and displays the *q*-Gaussian behavior with a specific *q* entropic index. To illustrate the difference between *q*-Gaussian and Gaussian behavior, the semi-log plot of the speed time series can be given as an example below in Figure 2.3. It is seen in Figure 2.3 that the tails of the probability distribution deviate from the Gaussian form. The probability distribution of the given data matches well *q*-Gaussian form. Heavy tails are visible and they are well described by the *q*-Gaussian distribution in this thesis. For more on the issue, please see the discussion under the Section 5.6. The results reveal that the traffic flow displays subadditive property.

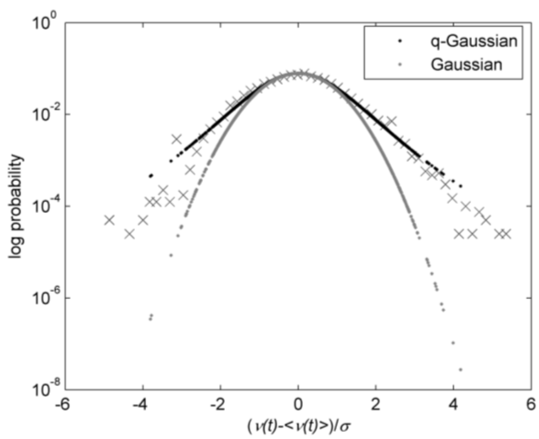


Figure 2.3. Semi-Log Plot of Speed Time Series and Fitting Plots of Gaussian and *q*-Gaussian Forms (Source: Kosun & Ozdemir, 2016)

The deviation from the Gaussianity is obtained by superstatistics theory and that the degree of the nonextensivity differs from the unity accounts for this deviation. In this thesis, the given data of traffic flow fall into the subadditive region in the nonextensive framework and thus the complexity of the traffic flow is endeavored to be formulated by this framework.

CHAPTER 3

EXTENSIVE ENTROPY

In this chapter, the extensive entropy is introduced within the framework of Boltzmann-Gibbs thermostatistics. Entropy is a thermodynamic property of a system and can be basically defined as the measure of disorder. The entropy of a system is the sum of the entropies of each subsystem and in the equilibrium systems the entropy is at a maximum under suitable constraints. The term entropy was coined by Rudolf Clausius in the 19th century. After Clausius, Boltzmann and Gibbs have made great attempt to formulate the entropy for the equilibrium systems.

3.1. Statistical Ensembles

This section explains the ensembles in equilibrium statistical mechanics. The widely known ensembles microcanonical, canonical, and grand-canonical are explained.

3.1.1. Microcanonical Ensemble

The microcanonical ensemble equals to an isolated system composed of N particles which has total energy $\langle E \rangle$ within a constant volume V. The total energy has some precision $\delta\langle E \rangle$ and the number of states i with $\langle E \rangle \le E_i \le \langle E \rangle + \delta\langle E \rangle$ can be designated by W (Tsallis, 2009). All the states which have same energies are equally probable in this ensemble and the entropy is at maximum due to $p_i = 1/W$. The entropy is given by the celebrated Boltzmann expression below.

$$S = k \ln W \tag{3.1}$$

where S is the entropy, k is the Boltzmann constant, and W is the total number of configurations or set of the states.

The thermally isolated N particle system i.e. microcanonical ensemble is shown in Figure 3.1. Circular traffic flow system can be given as an example for the microcanonical ensemble.

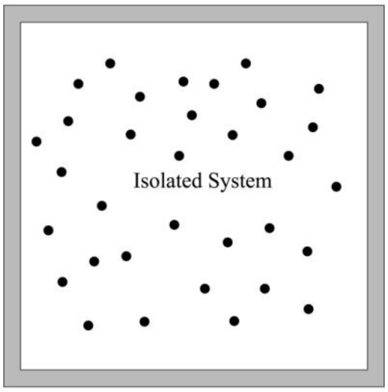


Figure 3.1. Representation of Microcanonical Ensemble

The Boltzmann expression represents that the entropy of the system is related with the probabilities of the states. Indeed increase in state probability i.e. molecular randomness or uncertainty leads to rise of the entropy. Thus, the measure of molecular randomness or disorder can be explained by the entropy. Whenever W increases, the information about the system is lost. When all the states have equal probabilities, full uncertainty or zero information occurs.

The fundamental thermodynamic relation in regard to internal energy can be given below

$$d\langle E\rangle = TdS - PdV + \mu dN \qquad (3.2)$$

where T is the temperature, S is the entropy, P is the pressure, V is the volume, μ is the chemical potential, N is the number of molecules.

From this relation, the temperature T is expressed as

$$\frac{1}{T} \equiv \frac{\partial S_{BG}}{\partial\langle E\rangle} = k\frac{\partial \ln W}{\partial\langle E\rangle} \qquad (3.3)$$

where $T \equiv 1/(k\beta)$ and β is the inverse temperature.

3.1.2. Canonical Ensemble

Canonical ensemble describes *N* particle system that is in long-standing thermal contact with an infinitely large heat bath (thermostat) whose temperature *T* (Figure 3.2). Thermal equilibrium is achieved when the system and heat bath reach the temperature of the heat bath in a fixed volume. The mean energy $\langle E \rangle$ is known through the thermostat (Tsallis, 2009).

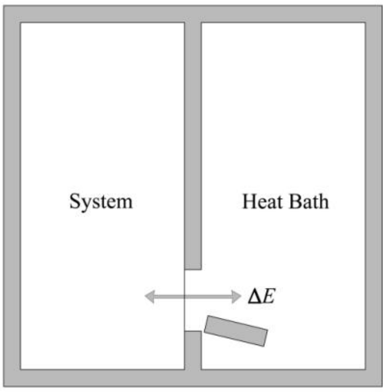


Figure 3.2. Representation of Canonical Ensemble

In the development of the entropic forms J. W. Gibbs supported the ideas of Boltzmann and the Boltzmann-Gibbs (BG) logarithmic formulation for the entropy is emerged as seen in Equation (3.4) (Tsallis, 2009; 2011).

$$S_{BG} = -k \sum_{i}^{W} p_i \ln p_i \tag{3.4}$$

$$\sum_{i}^{W} p_i = 1 \tag{3.5}$$

$$\sum_{i}^{W} p_i E_i = \langle E \rangle \tag{3.6}$$

where E_i are state energies, $\langle E \rangle$ is the mean energy.

BG entropy optimization under the norm and energy constraints, for a system in thermal equilibrium with a thermostat at temperature *T*, provides BG factor or weight (Tsallis, 2009). The obtained BG factor i.e. probability distribution by employing Lagrange multiplier method is shown below.

$$p_i = \frac{e^{-\beta E_i}}{Z} \tag{3.7}$$

And the partition function is given by Equation (3.8)

$$Z \equiv \sum_i^W e^{-\beta E_i} \tag{3.8}$$

where Lagrange parameter β is the inverse temperature and its expression is $\beta \equiv 1/kT$.

In the canonical ensemble, any system in contact with a heat bath has constant number of particles. Therefore, there is no particle exchange with the thermostat, while energy can be transferred between the thermostat and the system due to the thermal contact. All states of the system have any energy, thus they are not degenerate (Bowley & Sánchez, 1999).

The mean energy of the system is expressed with the following formulas (Sethna, 2006).

$$\langle E \rangle = \sum_i^W p_i E_i = \frac{\sum_i E_i e^{-\beta E_i}}{Z} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} \tag{3.9}$$

Therefore, in terms of partition function the mean energy can be expressed as

$$-\frac{\partial \ln Z}{\partial \beta} = \langle E \rangle \tag{3.10}$$

The Helmholtz free energy (F) for a thermodynamic system is also shown in terms of inverse temperature and the partition function,

$$F = \langle E \rangle - TS = -\frac{1}{\beta}\ln Z \tag{3.11}$$

3.1.3. Grand Canonical Ensemble

Suppose a system is coupled with heat bath (Figure 3.3). Particle exchange is allowed bi-directionally, and this case is called the grand canonical ensemble. The particle transition is the distinctive property of the grand canonical ensemble rather than the aforementioned ensembles. Bowley and Sánchez (1999) consider the heat bath as particle reservoir for the grand canonical ensemble and point out that the combination of the system and particle reservoir is closed and isolated from the rest of the universe. The reservoir i.e. heat bath and the system also allow an exchange of energy. The schematic representation of the grand canonical ensemble can be shown below.

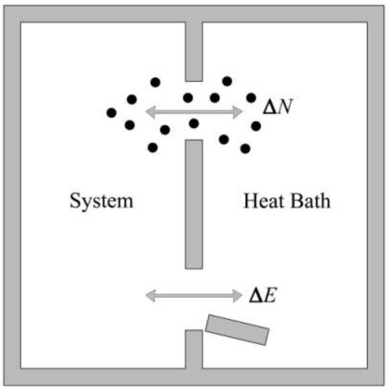


Figure 3.3. Representation of Grand Canonical Ensemble

In the grand canonical ensemble, the number of particles and energy fluctuate, while temperature, volume and chemical potential are fixed. In a vehicular traffic flow, vehicles can flow in and out of a highway system in the framework of grand canonical ensemble. Thus, exit-ramps and entrance-ramps can be seen as vehicular exchange routes on the highway system.

The probability of a state i of the system with N particles and E_i energies is (Sethna, 2006)

$$p_i = \frac{\exp(-(E_i - \mu N)/kT)}{Z} \tag{3.12}$$

where $\mu = -T\frac{\partial S}{\partial N}$ is the chemical potential.

The normalization factor called as grand partition function is defined as (Sethna, 2006)

$$\Xi(T,V,\mu) = \sum_n e^{-(E_n - \mu N_n)/k_B T} \tag{3.13}$$

3.2. Ergodicity, Time and Ensemble Averages

Boltzmann-Gibbs thermodynamics is relied on ergodic theorem and Tsallis (2009a) states that when q entropic index is equal to unity, the system dynamics exhibits positive Lyapunov exponent, hence mixing, hence ergodic, hence leading to an Euclidean, nonfractal geometry.

The equivalence of the time and ensemble averages of the quantities states that the system would be ergodic. The time average of a quantity is defined as (Peebles, 1993)

$$A[.] = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}[.]\,dt \tag{3.14}$$

A denotes time average similar with $\langle X\rangle$ for the statistical average. Specific averages are the mean value $\langle x\rangle = A[x(t)]$ of a sample function and the time autocorrelation function is denoted as $R_{xx}(\tau) = A[x(t)x(t+\tau)]$ (Peebles, 1993). The functions are expressed as

$$\langle x \rangle = A[x(t)] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, dt \tag{3.15}$$

$$R_{xx}(\tau) = A[x(t)x(t+\tau)] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)\, dt \tag{3.16}$$

Suppose that the random variables $\langle x \rangle$ and $R_{xx}(\tau)$ have zero variances i.e. $\langle x \rangle$ and $R_{xx}(\tau)$ are constants (Peebles, 1993). Then, $\langle x \rangle = \langle X \rangle$ and $R_{xx}(\tau) = R_{XX}(\tau)$. The ergodic theorem satisfies their equivalences. In other words, the time averages $\langle x \rangle$ and $R_{xx}(\tau)$ is equal to the statistical averages $\langle X \rangle$ and $R_{XX}(\tau)$, respectively. If a process is called ergodic, it satisfies the ergodic theorem.

3.3. Expressions of Heat and Work in Traffic Flow

The change in the average internal energy is obtained by the summation in Equation (3.17) when the changes are small (Bowley & Sánchez, 1999).

$$d\langle E \rangle = \sum_i dp_i E_i + \sum_i p_i dE_i \tag{3.17}$$

The addition of a small amount of heat would be equivalent to changing the probabilities slightly, leaving the energy levels constant. In contrast, the addition of a small amount of work would correspond to changing the energy levels slightly at constant probabilities (Bowley & Sánchez, 1999).

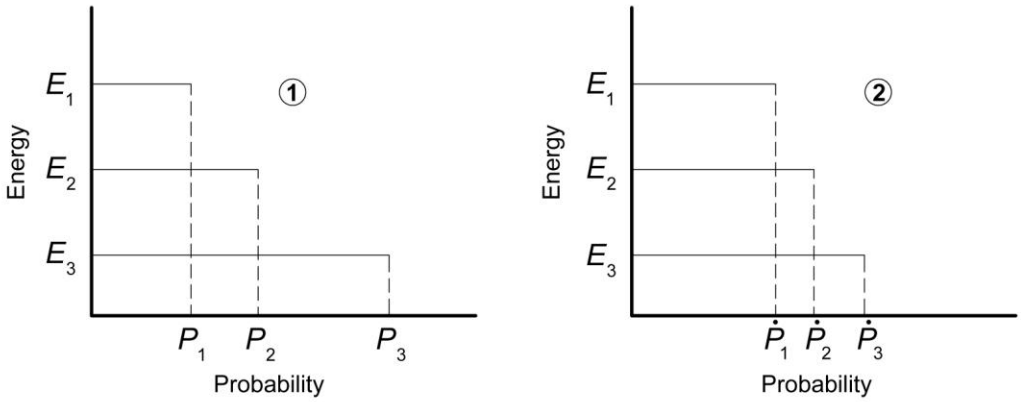


Figure 3.4. Graphical Representation of the Process When Heat is Added

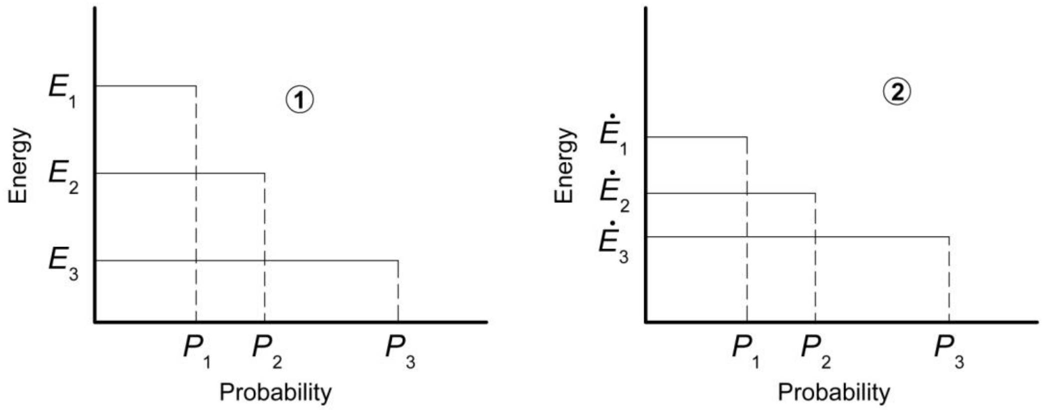


Figure 3.5. Graphical Representation of the Process When Work is Done

Figures 3.4 and 3.5 represent the changes in the cases of heat addition to the system and doing work on the system.

In a traffic flow, suppose that vehicles' velocities correspond to their energies, while probabilities represent the choices related to either lanes or regimes in the traffic. Work done on the system always changes the vehicle energies, keeping the lane-based probabilities fixed. Yet, if the regime-based probabilities are considered when work is done, the probabilities of the regimes e.g. congested, intermediate, free-flow occupied by the vehicles change. These two distinct scenarios are analogies of the expressions of ideal work and ideal heat.

Based on the discussion above, work is done on the system via traffic regulating elements e.g. speed bumps, traffic lights. These elements could affect the number of the regimes by altering the velocities of the vehicles, turning some of the regimes accessible or inaccessible. Hence, this may violate the ergodicity should the number of regimes decrease. As a result, something even a simple as a speed bump could violate the ergodicity principle. Similarly, when the heat transfers into or out of the system, this may generate new regimes or cancel existing ones. For example, adding a new lane on the highway may correspond to adding energy to the system and it changes the regime probabilities at constant energy levels.

The inputs such as traffic signalization, speed bump, grading, warning signs could yield work on the system since lane-based energy levels change but the probabilities are still the same. Regarding changing probability values, the inputs for example adding a new lane and changing the density of the traffic via auxiliary lane connecting an off-ramp would mean the addition of energy to the system.

CHAPTER 4

NONEXTENSIVE ENTROPY

This chapter explains the nonextensive entropy i.e. generalization of Boltzmann-Gibbs entropy that is known as Tsallis entropy. In 1988, Tsallis postulated the nonextensive formalism for the generalization of the Boltzmann-Gibbs thermostatistics (Tsallis, 1988). Boltzmann-Gibbs entropy might be a convenient expression for the particular cases; however it has some limitations when regarding, for example, the systems involving ergodicity breaking and long-range interactions. Such system may not be expounded by Boltzmann-Gibbs entropy.

4.1. Expression of Tsallis Entropy

In the new formalism, Tsallis introduces the q entropic index into the entropy formulation and it is written in the following form

$$S_q = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \qquad \left(\sum_{i}^{W} p_i = 1\,;\, k>0\right) \tag{4.1}$$

where k is the Boltzmann constant, q is the entropic index, and p_i is the state probability.

For the case q=1, Tsallis entropy represents the Boltzmann-Gibbs entropy (S_1 = S_q). For the discrete case, it is obtained as below

$$\lim_{q\to 1} S_q = S_1 = -k\sum_{i=1}^{W} p_i \ln p_i \tag{4.2}$$

If all of the probabilities are equal i.e. $p_i = 1/W \;\; \forall i$, the entropy S_q is obtained as

$$S_q = k \ln_q W \qquad (S_1 = S_{BG}) \tag{4.3}$$

where the q-logarithmic function is defined as

$$\ln_q x \equiv \frac{x^{1-q} - 1}{1-q} \qquad (x > 0; \ln_1 x = \ln x) \tag{4.4}$$

Its inverse, the q-exponential function is shown as below

$$e_q^x \equiv [1 + (1-q)x]^{\frac{1}{1-q}} \tag{4.5}$$

The extremization of S_q under the constraints of norm Equation (4.6) and mean energy Equation (4.7) yields Equation (4.8).

$$\sum_i^W p_i = 1 \tag{4.6}$$

$$\sum_i^W p_i^q E_i = \langle E \rangle_q \tag{4.7}$$

$$p_i \propto [1 + (q-1)\beta E_i]^{\frac{1}{1-q}} = e_q^{-\beta E_i} \tag{4.8}$$

The different forms of constraints for the extremization of S_q are discussed in the studies of (Tsallis, Mendes, & Plastino, 1998; Tsallis, 2009a).

Let us consider the some typical features of systems (Table 4.1.) where standard BG thermostatistics or nonextensive statistical mechanics prevails (Tsallis, 2011).

Table 4.1. Typical Features of Systems under Different Statistical Mechanics Framework

***q*=1 (BG entropy)**	***q*≠1 (Tsallis entropy)**
Short-range interactions	Long-range interactions
Markovian processes	Nonmarkovian processes (long-term memory)
Ergodicity, mixing	Nonergodicity
Gaussian distributions	*q*-Gaussian distributions
Strong chaos (positive maximal Lyapunov exponent)	Weak Chaos

4.2. Subadditivity and Superadditivity

Suppose that A and B are two probabilistically independent subsystems and if the joint probability satisfies $p_{ij}^{A+B} = p_i^A p_j^B (\forall(ij))$, and verified that, for example in Tsallis (2009b)

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k} \tag{4.9}$$

This is also rearranged as

$$S_q(A+B) = S_q(A) + S_q(B) + \frac{(1-q)}{k} S_q(A) S_q(B) \tag{4.10}$$

For *q*=1, the additivity of BG entropy is obtained for any finite value of *k*.

$$S_{BG}(A+B) = S_{BG}(A) + S_{BG}(B) \tag{4.11}$$

4.3. Short and Long-Range Interactions

As provided in Tsallis (2001; 2009a), it is considered that a classical mechanical many-body system is characterized by the Hamiltonian below

$$H = K + V = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i \neq j} V(r_{ij}) \tag{4.12}$$

where m is microscopic mass, p_i and r_i are the d-dimensional linear momenta and positions associated with N particles, and $r_{ij} \equiv r_j - r_i$.

The two-body potential energy $V(r)$ can be presented near the origin in the $r \to 0$ limit. The potential energy at $r \to \infty$ can be also classified as short-range and long-range interactions (Tsallis, 2002). The potential energy is shown below (Tsallis, 2009a)

$$V(r) \sim -\frac{A}{r^{\alpha}} \qquad (A > 0; \alpha \geq 0) \tag{4.13}$$

Analysis of the average potential energy $\langle E \rangle_{pot}$ per particle

$$\frac{E_{pot}(N)}{N} \propto -A \int_{1}^{\infty} dr\, r^{d-1}\, r^{-\alpha} \tag{4.14}$$

where d is the spatial dimension.

The integral converges for $a/d > 1$ and referred to short-range interactions for classical systems. On the other hand, it diverges for $0 \leq a/d \leq 1$ and referred to long-range interactions (Tsallis, 2009a). In the first case the potential is integrable and energy is extensive i.e. energy is finite in the thermodynamic limit $N \to \infty$. However, in the second case, the system becomes finite and expressed now by Equation (4.15) (Tsallis, 2009a). Energy is also nonextensive for long-range interactions.

$$\frac{E_{pot}(N)}{N} \propto -A \int_{1}^{N^{1/d}} dr\, r^{d-1}\, r^{-\alpha} = -\frac{A}{d} N^{*} \tag{4.15}$$

$$N^{*} \equiv \frac{N^{1-\alpha/d} - 1}{1-\alpha/d} = \ln_{\alpha/d} N \quad \text{and} \tag{4.16}$$

From the expression three cases are extracted as below in the limit $N \to \infty$

$$N^{*} \sim \frac{1}{\alpha/d - 1} \qquad \text{if} \qquad \alpha/d > 1 \tag{4.17}$$

$$N^{*} \sim \ln N \qquad \text{if} \qquad \alpha/d = 1 \tag{4.18}$$

$$N^{*} \sim \frac{N^{1-\alpha/d}}{1-\alpha/d} \qquad \text{if} \qquad 0 < \alpha/d < 1 \tag{4.19}$$

Let us consider that a dynamic system in which the particles interact with all the other particles with no regard to distance where the Hamiltonian model called Hamiltonian Mean Field can be as follows (Tsallis, 2009a):

$$H = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \frac{1}{2N}\sum_{i \neq j} \frac{1-\cos(\theta_i - \theta_j)}{r_{ij}^{\alpha}} \tag{4.20}$$

The first term in the right hand-side of Equation (4.20) corresponds to the kinetic energies of the particles, and the second term describes the interaction potential. The range of interaction parameter a would equal zero in the Hamiltonian mean field model.

4.4. Superstatistics

Superstatistics was introduced by the work of Beck and Cohen (2003), and it covers the q-statistics and BG statistics (Tsallis, 2009a). Superstatistics is the superimposition of two statistics and it is derived from the Boltzmann factor $e^{-\beta E}$ of ordinary statistical mechanics and an intensive parameter β (Beck, 2004). This intensive parameter can be inverse temperature, chemical potential, energy dissipation etc. The main aspect of the superstatistics concept is the fluctuations in the quantity of the selected intensive parameter in a driven nonequilibrium system. Consider that the system is composed of a series of artificially generated windows and in each window the β is approximately constant, thus the system is described by an ordinary Boltzmann-Gibbs statistics. In the long-term, the local intensive quantities in the system would fluctuate.

Suppose that there is a given time series data and it has two different time scales τ and T. T designates the length of the equally partitioned windows of the time series. To reach the local equilibrium, the condition $\tau << T$ should be satisfied where τ determines how fast the local equilibrium is reached in each window of the time series so that when a different dynamics of the following window starts, the dynamics of the previous window has already reached an equilibrium.

Let us consider the function of an averaged Boltzmann factor (BF) in (Beck, 2004)

$$BF(E) = \int_0^\infty f(\beta) e^{-\beta E} d\beta \tag{4.21}$$

where E is the energy, $f(\beta)$ is the probability distribution of β. So that

$$p(E) = \frac{1}{Z} BF(E) \qquad \text{and} \qquad Z = \int_0^\infty BF(E) dE \tag{4.22}$$

where $\frac{1}{Z}$ is the normalization constant of $e^{-\beta E}$.

For a β-dependent normalization constant, the probability distribution becomes

$$p(E) = \int_0^\infty f(\beta) \frac{1}{Z(\beta)} e^{-\beta E} d\beta \tag{4.23}$$

It is timely to point out an interesting fact that if $f(\beta)$ from data analysis turns out to be a chi-square distribution (χ^2), $p(E)$ assumes a Tsallis distribution. For the other classes of $f(\beta)$ distributions, one could investigate such studies as (Beck, 2004), (Beck, Cohen, & Swinney, 2005) and (Rabassa & Beck 2015).

CHAPTER 5

NONEXTENSIVE FRAMEWORK IN TRAFFIC FLOW

In this chapter, some of the properties of the Boltzmann-Gibbs (BG) and Tsallis entropies are presented in connection with traffic flow. Of these properties, extensivity, nonextensivity, additivity and nonadditivity become the focal points in the analyses of the traffic flow.

As Touchette (2002) reveals that additivity and extensivity are two different statements. However they are often confused and utilized to imply one another in the literature. To notice that Touchette (2002) on one hand defines additivity as "a many-body (or joint) physical observable $Q(x^n)$ is said to be additive with respect to two subsystems with states $x^m = x_1 x_2 \ldots x_m$ and $x^{n-m} = x_{m+1} x_{m+2} \ldots x_n$ if $Q(x^n) = Q(x^m) + Q(x^{n-m})$", on the other hand, defines extensivity as "a joint observable $Q(x^n)$ is extensive if the $Q-$density, defined by the ratio $Q(x^n)/n$ reaches a constant in the limit $n \rightarrow \infty$."

Touchette (2002) also points out that if the total energy and entropy of a system are both extensive quantities, the system has a thermodynamic limit.

5.1. Lane Changing and Driver Behavior Analyses in terms of Entropy

The extensivity/nonextensivity and additivity/nonadditivity concepts can be classified into four categories called quadrants within the notion of ordered and disordered traffic flow on a given highway. This classification can be summarized in Figure 5.1. What the important factors in that classification are that the number of vehicles tends to infinity and entropies of the vehicles are assigned.

Quadrant 1: Suppose in a traffic flow that is considered to be in BG domain that is in extensive and additive quadrant in Figure 5.1, all the drivers can choose all the states without taking care of any long-range interactions. This driver behavior can be observed under the disordered traffic flow.

Quadrant 2: In the subadditive and extensive quadrant, the characteristics of the prudent drivers may give raise to entropic nonadditivity. Those drivers take care not to bother other drivers. While the number of vehicles tends to infinity, entropic extensivity exists since allowable states are proportional to *N* (Equation (5.1)) until a certain saturation. In this quadrant, traffic flow is said to be ordered.

Quadrant 3: In the additive and nonextensive quadrant, entropic additivity would exist due to the reckless drivers, in the sense that individual entropies of the drivers are additive due to lack of consideration of others (very short-range interactions). Nonextensivity emerges since allowable states are no longer proportional to *N*. The traffic flow in this quadrant is thought to be disordered. Since a certain saturation is being reached due to the loss of extensivity. At worst traffic jams are expected to occur but the author of the thesis does not involve the case of solid-jam into this quadrant. The evaluation of the solid state of the traffic flow is beyond the scope of the thesis.

Quadrant 4: When a congested flow is imminent, the system entropy is nonextensive due to loss of allowable states, and the entropy is still considered to be subadditive since the drivers are even more prudent. The traffic flow would be inevitably ordered in the system. Quadrant 4 may also lead to initialization of traffic jam, and drivers will gradually consider forbidden states allowable filling up available space among the cars but still observing the usual codes of conduct in traffic flow.

The entropic extensivity could be formulated as below,

$$0 < \lim_{N \to \infty} \frac{S(N)}{N} < \infty \tag{5.1}$$

where *S* is entropy, *N* is the number of vehicles.

When the energy extensivity is in question, the formula can be considered in terms of energy.

$$0 < \lim_{N \to \infty} \frac{E(N)}{N} < \infty \tag{5.2}$$

where *E* is energy, *N* is the number of vehicles.

This classification within the notion of ordered and disordered traffic flow can be tabulated as below,

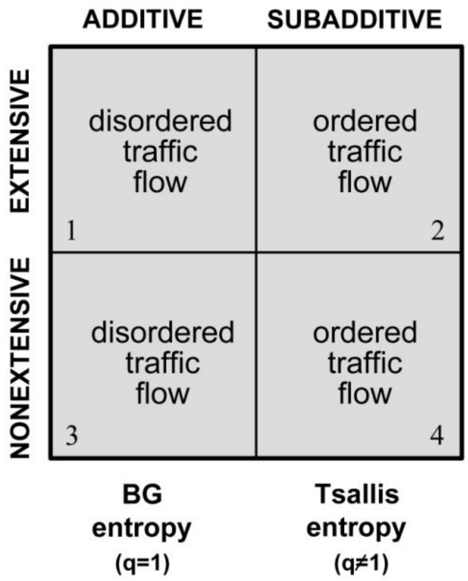


Figure 5.1. Classification of Traffic Flow in terms of Entropic Additivity and Entropic Extensivity

The analyses of this thesis state that there would be transitions among the quadrants of the traffic flow and those are explained below. The possible transitions are also shown in Figure 5.2.

Transition from 2 to 3: From the viewpoint of the transition from 2 to 3, prudent drivers of quadrant 2 do not bother the other drivers especially when the long-range interactions are present. This driver behavior is placed in the extensive and subadditive quadrant. When the traffic flow gets congested, the capacity is slowly reached and the state choices begin to be consumed. Concurrently, the nonextensivity emerges and the drivers could also attempt to occupy the empty spaces. The system remains in nonextensive and additive quadrant, the entropy drops as well.

Transition from 2 to 4, and then 3: If the traffic congestion is beginning to appear where the prudent drivers travel within the subadditive and extensive quadrant, the traffic can tend to nonextensive and subadditive zone. Due to increasing congestion the drivers change their own behaviors and want to occupy the adjacent states ignoring all the interactions. Therefore, the traffic flow is said to be additive and nonextensive until traffic is literally congested, that is to say, nonextensive and additive.

Transition from 1 to 3: Suppose that the traffic flow is in the additive and extensive quadrant, strictly in BG domain. If the traffic is congested and the drivers are positioned bumper to bumper at worst, the traffic flow system is transferred to the additive and nonextensive quadrant.

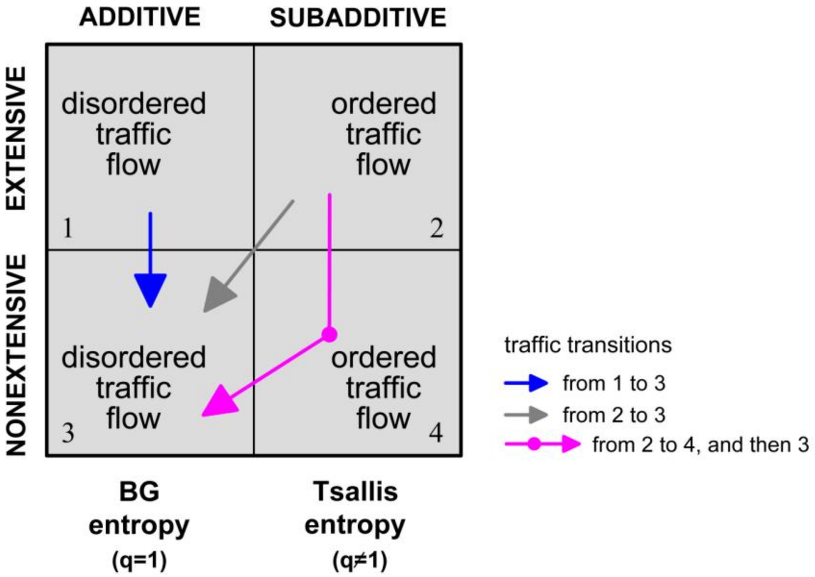


Figure 5.2. Transitions among Traffic Flow Classification in terms of Entropic Additivity and Entropic Extensivity

Through the aforementioned quadrant descriptions, lane changing scenarios can be illustrated as in Figure 5.3. One can describe driver behavior under these classifications. For example, the behavior of the motorbike driver in the traffic can be displayed in Figure 5.3 in terms of entropy.

5.1.1. Short-Range and Quasi Long-Range Interactions in terms of Entropy

In this section, the thesis exhibits the short-range interactions, and quasi long-range interactions among the vehicles in the traffic flow in terms of entropic additivity and entropic extensivity.

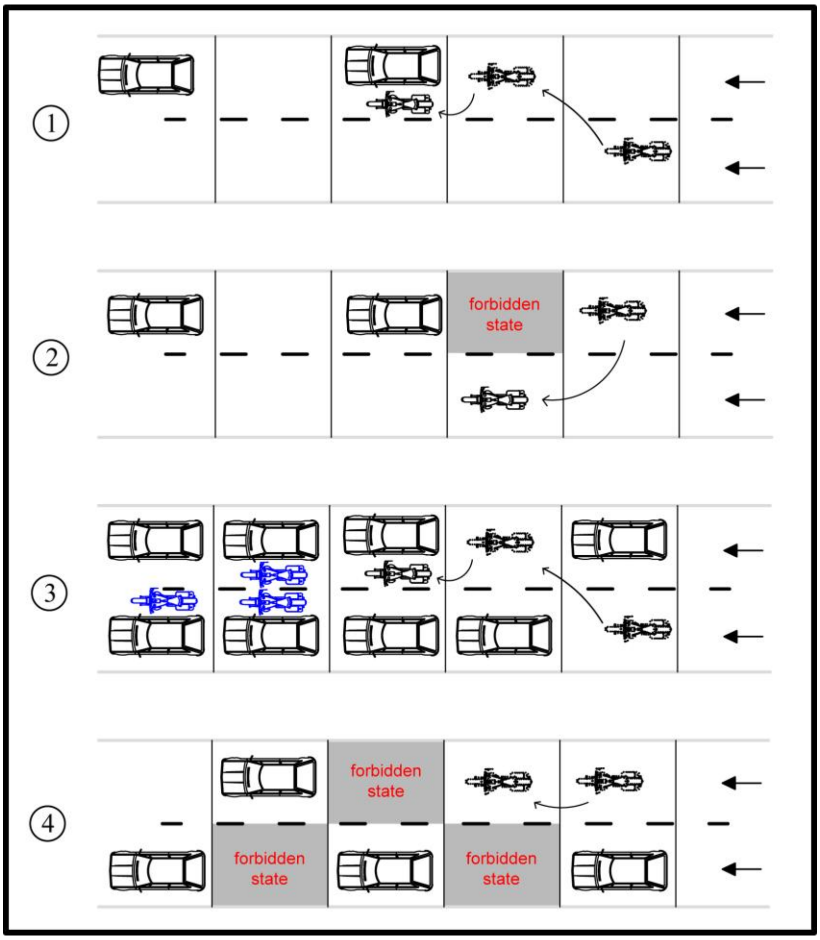


Figure 5.3. Comparison of Four Different Driver Behaviors in the Traffic Flow in terms of Entropic Additivity and Entropic Extensivity

In Figure 5.3 four different driver behaviors are extracted based on the traffic flow classifications. For all the four scenarios depicted in Figure 5.3, suppose that the leading vehicle has a constant velocity and travel on the right lane. However the motorbike, the following vehicle, tries to overtake the car. The driver behavior designation in Figure 5.4 is formed after analyzing the each scenario in Figure 5.3.

For the driver behavior 1 in Figure 5.3, the following vehicle, motorbike, intends to overtake the car with constant velocity. When overtaking, the motorbike does not change lane and prefers to travel on the same state with the car rather than travelling on the adjoining state of the overtaking route. The author of this thesis calls this driving behavior as malicious driving. The driver can choose any possible state with equal probability and the system is defined in additive and extensive quadrant.

In the driver behavior 2, the following vehicle has to now change the lane to overtake the car. The motorbike driver has no intention to occupy the state of the leading vehicle. Moreover, the motorbike driver does take into account a safe distance from the leading vehicle and thus the entropy decreases. Under these conditions, the thesis designates this behavior as safe driving and the system is extensive but subadditive. For this scenario, the interactions are described as quasi long-range.

For the driver behavior 3, the motorbike again approaches the leading car to overtake. Similar to the first driving scenario, the motorbike driver chooses the state of the leading vehicle. So, this is called as reckless driving based on the thesis assumptions. The system entropy thus additive but the fact that the states are filled up indicates the nonextensivity of the system.

Let us consider the driver behavior 4, the motorbike driver is more prudent to overtake the car. That is, the driver does not want to choose the state behind the car and has also utmost reluctance to pass into the state of the leading vehicle. Furthermore, the motorbike can not overtake the car because it takes into account the approaching vehicle, thus it can not pass the adjoining state of its own. Therefore, the entropy is subadditive. Moreover, the other states can not compensate for the increase in N (vehicles) and entropic extensivity is violated. All in all, in the fourth behavior, the motorbike driver shows safe driving profile. Like scenario 2, the interactions in the scenario are described as quasi long-range. Therefore, it is safely to say that when the quasi long-range interactions are in question, the nonadditive entropy region may emerge since the state choices are not proportional to *1/W*.

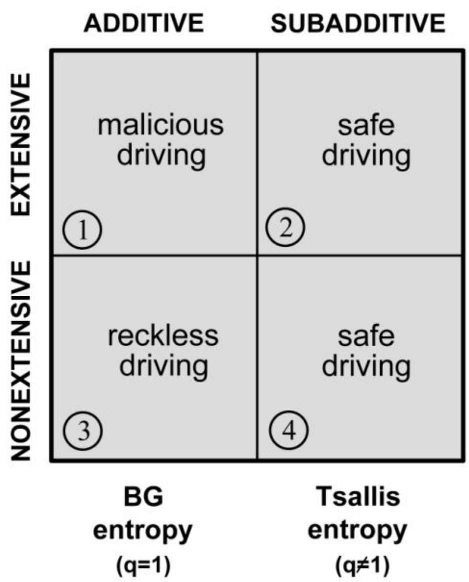


Figure 5.4. Representation of Driver Behaviors in terms of Entropic Additivity and Entropic Extensivity

Figure 5.4 can be interpreted by means of Figure 5.1 and it presents the driver behaviors in terms of entropy. According to Figure 5.4, entropy of the system constituted by N elements is extensive at the first row and it is between $0 < S(N)/N < \infty \lim_{N \to \infty}$, while the entropy is additive at the first column where $q = 1$. In each entropy domain (BG or Tsallis), loss of allowable states results in different driving behavior. As a result, in the light of all the figures i.e. Figure 5.1, 5.2, 5.3, 5.4, it can be pointed out that traffic system is twofold: cooperative and uncooperative traffic system. In the cooperative traffic system, vehicles are travelling in harmony with each other. Some of the states are forbidden automatically, and random movements are decreasing. Cooperation gets traffic system into Tsallis entropy domain and traffic is considered within long-range relationships. In contrast, the traffic system being uncooperative can display random movements provided that no collisions among the vehicles are allowed. The uncooperative traffic system is considered in Boltzmann-Gibbs statistical mechanics where short-range interactions are present.

5.1.2. Short and Long-Range Interactions are Revisited in terms of Entropy

The scenarios given in Figure 5.3 describe the interactions between a motorbike and a car, but not restricted to these two vehicles. The number of alternative scenarios could be increased for the traffic flow example. In Figure 5.3, the short interactions, or quasi long-range interactions, have been emphasized. In this section, the distinction between long-range and short-range interactions is clarified.

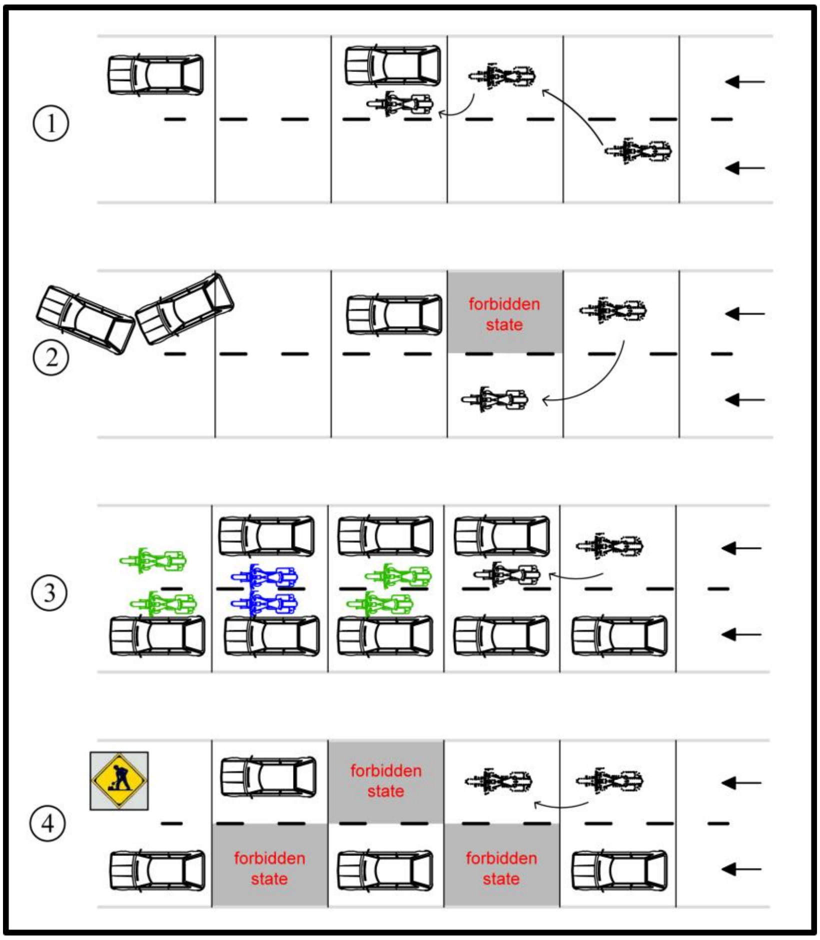


Figure 5.5. Representation of Short and Long-Range Interactions in terms of Entropy

As it is seen in Figure 5.5, the first driving profile is identical with the scenario 1 in Figure 5.3. The author of this thesis refrains from the restating the features of the scenario 1 in this section. Let us consider the driving scenario 2 in Figure 5.5. In this scenario, the long-range interactions are present. The motorbike driver follows the leading vehicle until the driver observes the road block ahead. After observing it, the motorbike passes the adjoining state of its own and follows a new route. The driver behavior could be classified in subadditive and extensive quadrant (Figure 5.6). Here, the safe driving behavior emerges due to the long-range interactions. In the driving scenario 3, the entropy is in nonextensive and additive quadrant, and the reckless driving behavior appears. The traffic flow is congesting and the allowable states are substantially occupied. The scenario 4 in Figure 5.5 exhibits safe conditions in the traffic flow and the motorbike driver is in the subadditive region. Here, the interactions could be designated long-range.

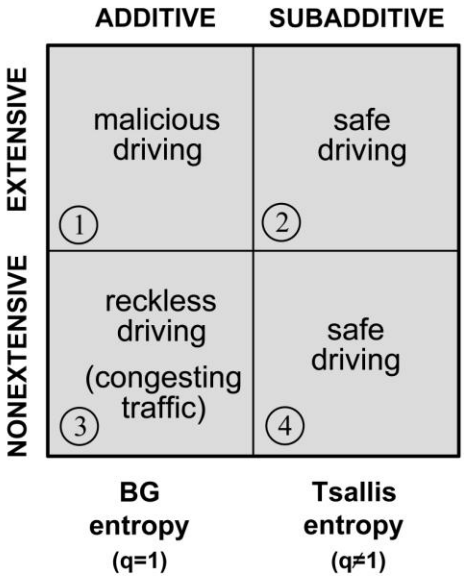


Figure 5.6. Representation of Driver Behaviors in terms of Entropy

5.1.3. Nonextensivity and Nonadditivity of the Traffic Flow

This section emphasizes the importance of nonextensive and nonadditive quadrant in Tsallis statistical mechanics and traffic flow. As it is seen that the traffic flow transitions discussed in Figure 5.2 ultimately visit the last quadrant that is classified as nonextensive and additive. That the traffic flow gets congested indicates the violation of entropic extensivity. The traffic flow may start to move the system dynamics from BG domain to the Tsallis one, and then BG domain again. Short and long-range interactions emerge in this process and the traffic flow also tends to display nonextensive behavior. Consequently, the traffic flow makes a transition into the nonextensive and additive quadrant since the relationship $S(N) \propto N$ can not be maintained for $N >> 1$. This is to say that entropic nonextensivity appears when the number of available traffic states decreases and the traffic capacity saturates where the traffic flow is present in quadrant 3.

5.1.4. Entropy Rates in the Traffic Flow

This part explains the entropy rates which are the distinctive characteristics of each quadrant in Figure 5.2. Entropy rates in the traffic flow can be defined with (non)additive and non(extensive) pairs. Entropy rate in quadrant 1 (extensive and additive quadrant) is the maximum among others. If $S_1(N)/N_1 \cong c_1$ and $S_2(N)/N_2 \cong c_2$ then $c_1 > c_2$. The entropy rates in the quadrants 3 and 4 can be formulated as, $S_3(N)/N_3 \to 0$, $S_4(N)/N_4 \to 0$, respectively. Only that $S_3(N)/N_3$ tends to zero faster than $S_4(N)/N_4$, and a threshold at a small value of $S_4(N)/N_4$, a switch-over is expected from quadrant 3 to 4. However, the main difference is that entropy rate of the quadrant 3 approaches to zero faster.

In the light of these findings, entropy rates can be classified as high, intermediate high, low, and intermediate low for each successive quadrant number.

5.1.5. Lyapunov Exponents and the Traffic Flow

The largest Lyapunov exponent among all the other Lyapunov exponents is the main indicator of the chaotic systems. When the largest Lyapunov exponent (LE) is positive, the strong chaos occurs, hence entropy production is present. Indeed, there is a quick sensitivity to the initial conditions and the entropy is BG, thus extensivity appears with regard to independent systems and velocity distribution is Maxwellian (i.e. Gaussian) (Tsallis, 2001). Contrary to this exponential function realm, slow sensitivity to the initial conditions, which would account for zero Lyapunov exponents, indicates the nonextensivity hence Tsallis entropy. In this statistical mechanics, the energy distribution is power-law and this implies that velocities are not Gaussian but they may display, for example, Levy or Student's t-distributions (Tsallis, 2001).

As Tsallis (2001) discussed, BG statistical mechanics (q=1) involve strongly chaotic systems and disordered systems, and the natural corollary of this is the fact that BG framework is the least predictable thermostatistics. This is where the statistical methods are the most useful. However, the entropy S_q $(q \neq 1)$ would be more predictable that is called intermediate predictable and it should be noted that conventional statistical methods would fail.

Considering $\Delta x(0)$ a small variation in the initial condition x_0 and time evolution $\Delta x(t)$, the sensitivity function $\xi(t)$ is defined as $\xi(t) \equiv \lim_{\Delta x(0)\to 0} \dfrac{\Delta x(t)}{\Delta x(0)}$. And $\xi(t)$ can satisfy $\xi(t) = \exp(\lambda_1 t)$ where λ_1 is the LE.

When $\lambda_1 > 0$, the system is strongly sensitive to the initial conditions i.e. strongly chaotic. However, the case $\lambda_1 < 0$ characterizes the system as strongly insensitive to the initial conditions.

Quadrant 1 in our traffic scenarios, disordered traffic, would exhibit the case $\lambda_1^{\max} > 0$ which implies exponential functional form, mixing, ergodicity and Euclidean geometry. Those properties conform to BG statistical mechanics. However, when $\lambda_1^{\max}$ vanishes i.e. nonergodic and slowly mixing, time dependence of the initial conditions should be evaluated in the nonextensive statistical mechanics framework. Hence, in the traffic quadrant 4, the $\lambda_1^{\max}$ is expected to vanish.

5.2. Cooperative and Uncooperative Traffic Flow in terms of Energy

As discussed before, traffic system has two distinct subdivisions i.e. cooperative and uncooperative systems. Those can be discussed from the energy perspective in this section. In this thesis, energy is appealed to the velocity in traffic flow. To begin with the cooperative traffic flow, if the possible velocities that could be selected by the drivers decrease when the number of vehicles in the traffic flow is increasing and approaching infinity, the system would be nonextensive. Moreover, if the drivers want to travel in harmony with the other vehicles in regard to velocity, the system also becomes nonadditive and this traffic flow is said to be cooperative. The cooperative traffic is also considered in extensive and nonadditive quadrant. That is, in spite of the fact that the vehicles are able to select all possible velocities, they do not and they prefer to be prudent. Those vehicles want to travel in cooperative journey considering long-range interactions and without at risk of collision.

On the contrary, in the uncooperative traffic flow, suppose that possible velocity choices are decreasing due to increasing number of vehicles, resulting in violation of extensivity. However, the drivers try to choose all the existing velocities ignoring the long-range interactions and this could be dangerous in the journey. Such a system stands in the nonextensive and additive quadrant. Probably the worst scenario, which is involved in extensive and additive quadrant, indicates the uncooperative traffic flow as well. In this scenario, the drivers can travel at all the velocities with disregard to safety. These drivers do not want to adjust their velocities because they choose to ignore the traffic rules.

From the discussion above, the driver behaviors for each traffic flow quadrant can be given in Figure 5.7.

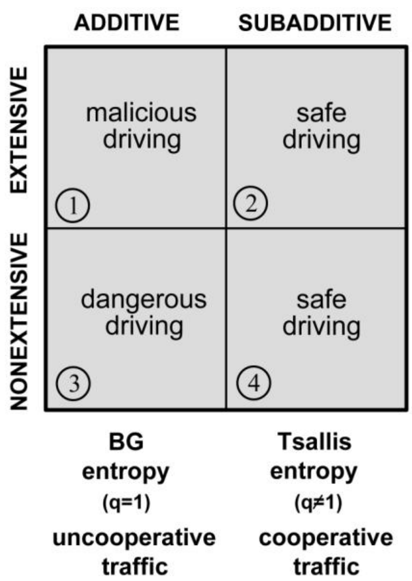


Figure 5.7. Representation of Driver Behaviors in terms of Energy

5.2.1. Representation of Speed Values and States in the Traffic Flow

Speed values in a traffic flow can be attributed to the states. Thus, each state has a specified speed interval. This is where both accessible states and forbidden states can exist. The number of accessible and forbidden states is varied depending on the traffic regime. The representation of accessible and forbidden states in terms of congested, intermediate and free-flow regimes on the slow-lane of a three lane highway is given in Figure 5.8.

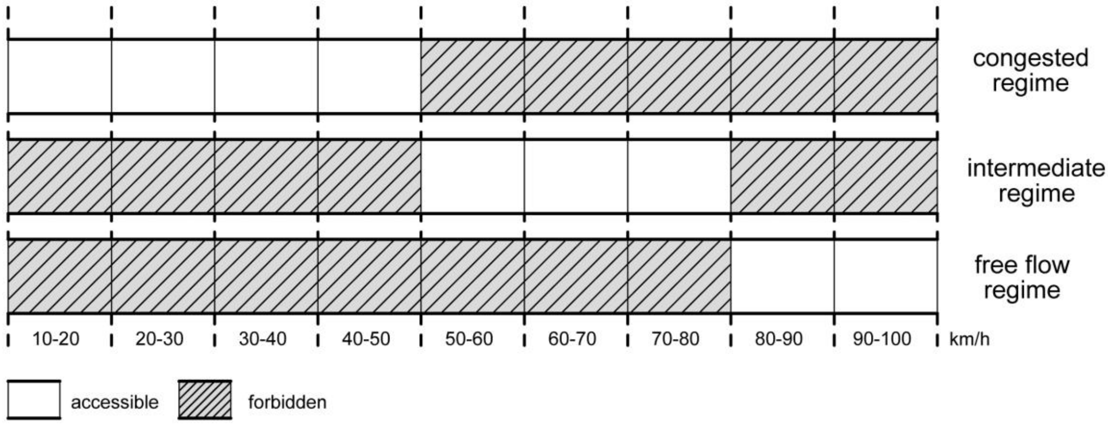


Figure 5.8. Representation of Speed Values and Their States

The author of the thesis considers that the probability distributions of each quadrant would be different from each other (Figure 5.9). Transitions among quadrants imply distinctive probability distributions.

The distributions might be $p(E,A) \neq p(E,NA) \neq p(NE,A) \neq p(NE,NA)$.

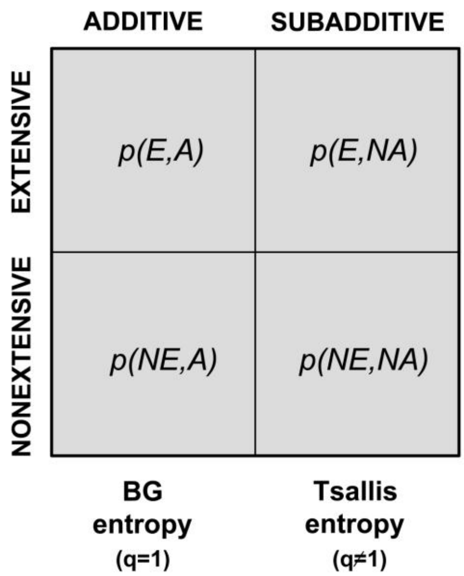


Figure 5.9. Probability Distributions of Each Quadrant

5.3. Lane Changing and Driver Behavior Analyses in terms of Energy and Entropy

Traffic flow classifications are evaluated in a different perspective by the new specific example here. The thesis here puts forward the traffic flow example to be analyzed within both entropy and energy framework as in Figure 5.10.

5.3.1. Short-Range and Quasi Long-Range Interactions in terms of Energy and Entropy

In this section, the thesis exhibits the short-range interactions, and quasi long-range interactions among the vehicles in the traffic flow in terms of entropy and energy.

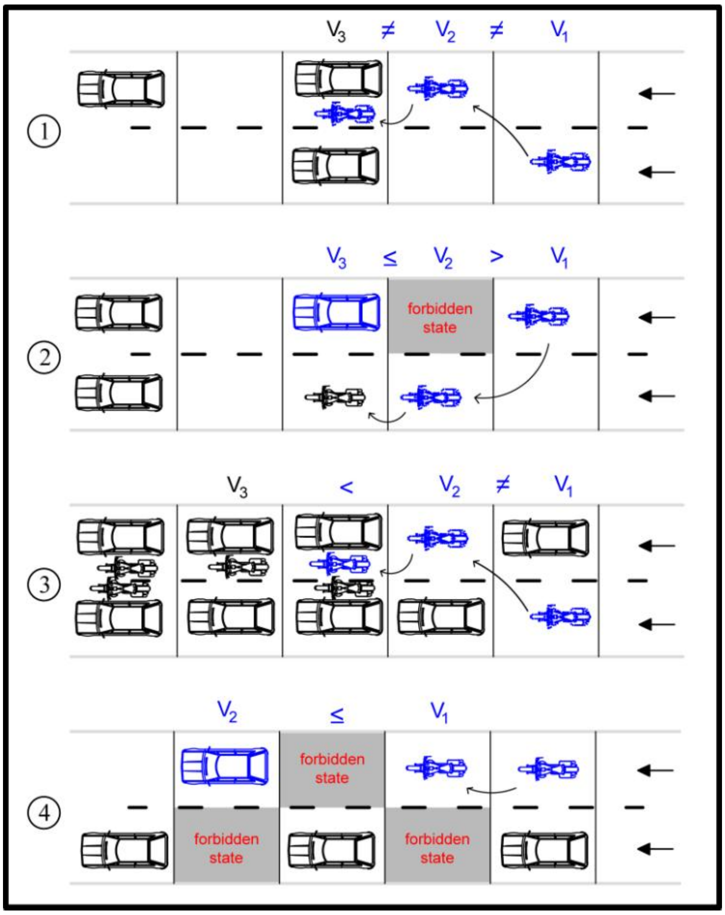


Figure 5.10. Comparison of Four Different Driver Behaviors in the Traffic Flow in terms of Entropy and Energy

The thesis now introduces the traffic flow analysis depending on both energy and entropy together. The entropic additivity and energy extensivity are discussed. The following interpretation for the traffic flow scenario could be included in the consideration of short-range and quasi long-range interactions.

In the first scenario, let us consider a disordered traffic flow with finite capacity, while the number of the vehicles tends to infinity, vehicle energies progressively wane. However, in this scenario, the system energy is extensive, and the entropy could be temporarily additive since the vehicles can choose any state until congestion.

In the second scenario in Figure 5.10, in the ordered traffic flow the entropy is considered as nonadditive and energies progressively diminish and remain extensive. The important point is that drivers can not choose any of the states with equal probability due to the entropic nonadditivity. In this scenario the interactions among the vehicles are described quasi long-range.

In the third scenario, the entropy is additive and the extensivity of energy can be temporarily achieved until the vehicle and the succeeding traffic meet the congestion ahead.

In the fourth scenario, an ordered traffic flow indicates the nonextensivity and nonadditivity with regard to energy and entropy. Since the drivers leave forbidden states among the vehicles, they exhibit safe driving profile.

5.3.2. Short and Long-Range Interactions are Revisited in terms of Entropy and Energy

In this section, this thesis exhibits the short and long-range interactions among the vehicles in the traffic flow in terms of entropy and energy.

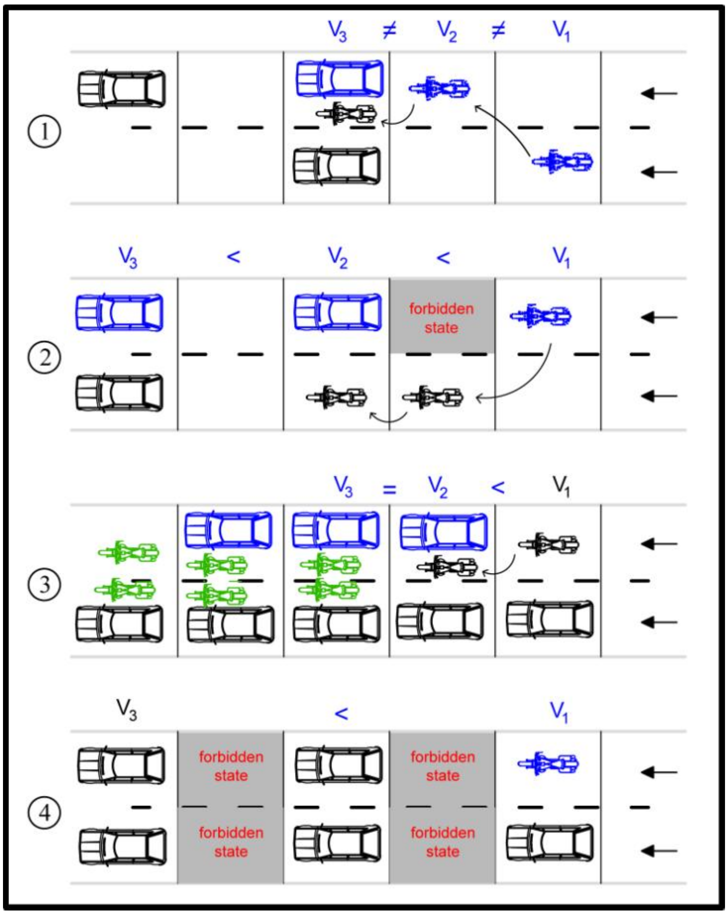


Figure 5.11. Representation of Short and Long-Range Interactions in terms of Entropy and Energy

For the scenario 1 in Figure 5.11, the following vehicle, the motorbike, intends to overtake the car. When overtaking, the motorbike does not change lane and prefers to travel in the state of the car. The author of this thesis calls this driving behavior as malicious driving. Since the driver can choose any possible state with equal probability and may select any velocity, the system can be included in additive and extensive quadrant.

In the scenario 2 in Figure 5.11, the motorbike driver notices the low velocity of the vehicle well ahead i.e. long-range interactions are present and the motorbike passes to the left lane to overtake the car in front. However, the state of the car is not accessible for the motorbike. The entropy decreases and the driver may prefer to choose any velocity in the traffic. In spite of the fact that the motorbike driver takes into account a safe distance with the vehicle in front, the velocity choices are not restricted. Since the motorbike driver opts to drive safe, he may also wish to drive safe energywise. Under these assumptions, the quadrant 2 is assumed to be safe driving. However, it must be noted that quadrant labels are conditional and subject to driver behavior. This behavior is designated safe driving and the system is extensive but nonadditive in terms of energy and position, respectively.

In the scenario 3 in Figure 5.11, the motorbike could have diverse velocities before overtaking the car. Like the first driving behavior, the motorbike driver is free for the state selection and chooses state of the car. When overtaking the car, the motorbike reduces its velocity. Due to the state selection motorbike driver behavior is called reckless driving. The system entropy is thus additive but the fact that the vehicles travel with reduced velocities yields the nonextensivity.

When considering the scenario 4 in Figure 5.11, the motorbike driver perceives the slow vehicle further ahead (long-range interaction) and does not overtake the car in front, leaving a safe distance since the left-lane is also occupied by the other vehicles. The motorbike exhibits nonadditive behavior in terms of entropy. Furthermore the motorbike falls into the nonextensive region in terms of energy since it is constricted to slow speeds. The behavior of the driver could be considered safe driving.

The number of alternatives can be increased and different behaviors may be extracted. For the scenarios in Figure 5.11, the driver behaviors are described as illustrated in Figure 5.12.

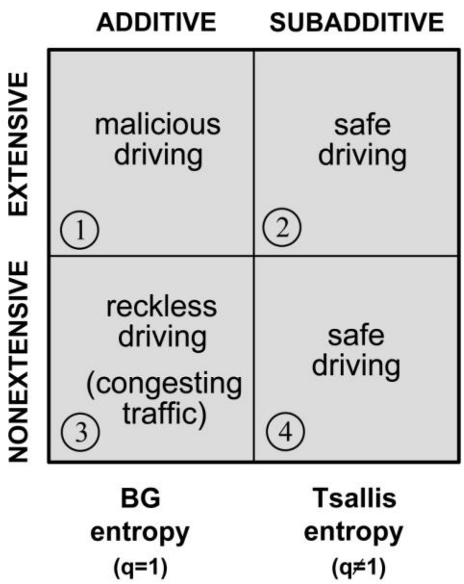


Figure 5.12. Representation of Driver Behaviors in terms of Entropy and Energy

Figure 5.12 presents the driver behaviors in terms of energy and entropy. According to Figure 5.12, energy is conserved at the first row, whereas the entropy is conserved at the first column (Figure 5.13). Exclusive to quadrant 4, it should be noted that neither the energy nor the entropy of the system is conserved, a corollary of an approaching traffic jam. In each entropy domain, loss of energy results in safer driving. Safe driving also suggests an increase in entropic index *q* in subadditive region.

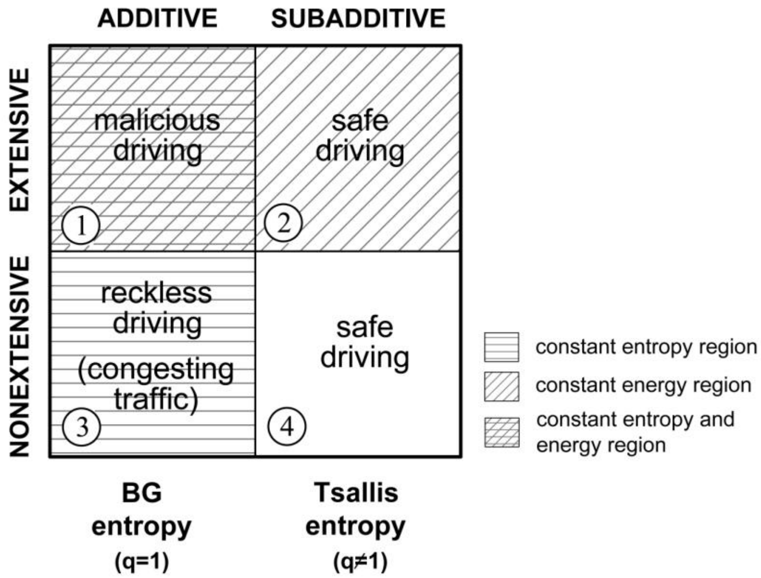


Figure 5.13. Representation of Driver Behaviors in terms of Constant Entropy and Energy

5.4. Energy and Position Planes of the Traffic Flow

If a driver is in extensive and additive quadrant in energy plane, he/she could be in extensive and additive or extensive and nonadditive quadrants of the position plane. The driver may be temporarily present in nonextensive and additive quadrant of the position plane until the capacity saturates and the driver is restrained. However, he/she can not participate in the nonextensive and nonadditive quadrant of the position plane.

If the driver is in extensive and nonadditive quadrant in energy plane, he/she could surely drive in extensive and nonadditive, and nonextensive and nonadditive quadrants of the position plane.

If the driver is in nonextensive and additive quadrant in energy plane, he/she can be present in nonextensive and additive or nonextensive and nonadditive quadrants, whereas can not drive in additive and extensive, and extensive and nonadditive quadrants of the position plane.

In last scenario when the driver travels in the nonextensive and nonadditive quadrant of the energy plane, he/she can only continue to the travel in the nonextensive and nonadditive quadrant of the position plane.

The given statements above in which the link between energy and position plane is established can be illustrated in Figure 5.14.

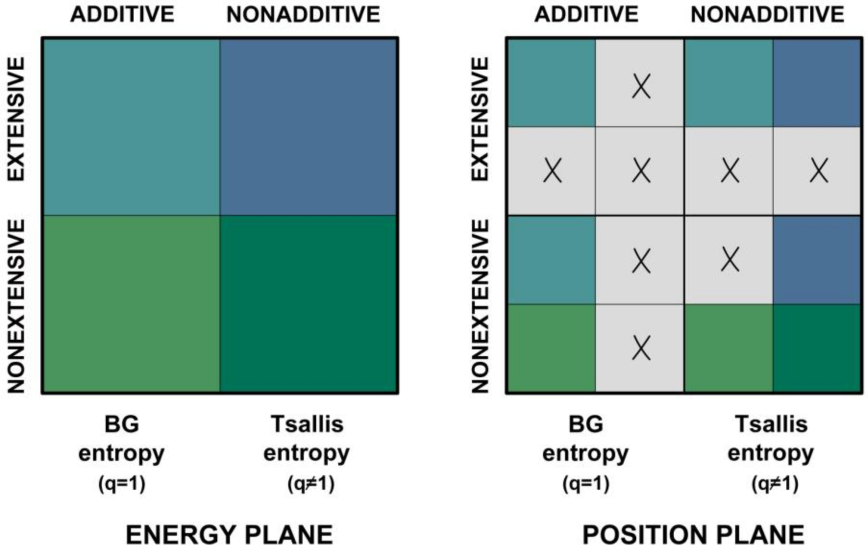


Figure 5.14. A Maximum Allowable Position States in Position Plane for a Given Energy Plane

The thesis therefore suggests that extensivity and additivity concepts would be considered in terms of both positionwise and energywise domain. The configuration space and momentum space under microcanonical ideal gas in the study of Sethna (2006) can be adduced to explain those positionwise and energywise considerations in the thesis.

In the energy and position planes of the traffic flow, there could be different *q* values. On one hand, *q* values will change simultaneously in both energy and position planes. On the other hand, *q* value of any of them remains constant in time, whereas the *q* value of the other one is changing. Thus, a pair of *q* values would emerge within the plane pairs.

This thesis also prescribes finite capacity for the traffic flow on a highway. When referring to extensivity on the energy and position planes, the system is realized to be extensive up until the highway capacity. Otherwise, the system always appears to be extensive. Therefore, it is assumed that the system could be extensive for a while, and then it would be nonextensive.

It is important to expound on the third quadrant (nonextensive and additive) here. That is to say, the third quadrant where all the transitions terminate is subsumed under just the congested category. However, the third quadrant can also represent the moving traffic with constant velocity, where this quadrant may be regarded as heavy but moving traffic on a given roadway.

5.5. Additivity, Superadditivity and Tsallis *q* Index of the Traffic Flow

In the case of $q \neq 1$ the entropy S_q is nonadditive. This case is classified according to q value and total entropy. Let us remind that if $q < 1$ and $S_q(A+B) \geq S_q(A) + S_q(B)$ then the system is called superadditive. If $q > 1$ and $S_q(A+B) \leq S_q(A) + S_q(B)$ then the system is called subadditive.

In traffic flow, drivers could travel in additive, subadditive and superadditive domains. As discussed before, drivers commonly tend to travel in additive or subadditive ones. However, the superadditive domain can also be eligible for the drivers. Particularly, the drivers could temporarily utilize the inaccessible speed and position states making the system superadditive. The driver psychology could be construed as a significant factor in the emergence of superadditivity.

Superadditivity in traffic flow is a special case where driver-driven (anger) or emergency-driven (ambulance, fire engine) maneuvering is present. Drivers in superadditivity domain observe the traffic regulations only where there are no interactions among vehicles. Just after the interactions occur, the drivers start to violate the traffic regulations in this domain. For example, in a free-flow traffic an ambulance driver moves on the fixed path e.g. left-lane on a highway (Figure 5.15), however when a densely traffic or interactions are in question, the ambulance driver drops observing the traffic regulations, exhibiting reckless driving. Furthermore, additivity in traffic flow is analogous to the superadditivity in that additivity may be considered as a lower limit of superadditive entropy. However, unlike superadditivity, the drivers in additivity domain disregard the traffic regulations at all times, such as zigzagging in a free-flow traffic, regardless of interactions (Figure 5.15). The reason behind additivity domain driving may be driving under influence (DUI). As for the prudent driver behavior, it is identified with the subadditive domain. So, the prudent drivers' concern is generally safety in all kinds of traffic flow conditions such as free-flow or congested traffic.

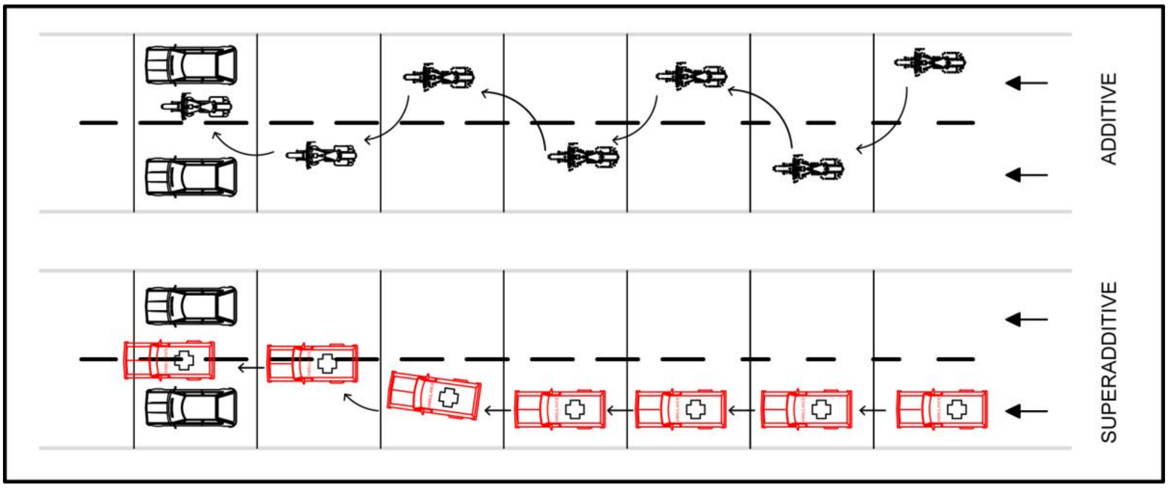


Figure 5.15. Representation of Additivity and Superadditivity in the Traffic Flow

This thesis proposes that the position entropy is the function of velocity $E_p = E_p(V)$. In a densely flowing traffic, the number of accessible states decreases for high velocities, leading eventually to a drop in entropy. On the other hand, it increases for average velocities since there are more accessible states available for the drivers. Similarly, it can be propounded that the energy entropy is the function of position $E_v = E_v(P)$. That is, in free flow traffic, a vehicle travelling on the fast-lane may shift lanes by observing lanes' speed ranges. But, when this speeding vehicle approaches a densely flowing traffic; the driver is forced to decelerate on the fast lane, resulting in a drop in the number of accessible energy states, hence a drop in entropy. From a different perspective, the number of accessible energy states may increase for those travelling on the middle lane with average velocities since they may choose any other velocity states by changing their velocities and lanes easily in a densely flowing traffic and this results in a rise in entropy.

The author of the thesis asserts that *q* value is an index of driver psychology, experience or conformity in the traffic flow. For example, it is here hypothesized that if the driver is prudent, then the mindset of the driver would drive the *q* value from 1, making larger than 1 ($q > 1$). Different drivers in the traffic flow could have dissimilar characteristics. All of those characteristics will affect the *q* value of the system.

For the sake of clarity, physical and psychological accessibility terms have emerged in this study. Accessibility/inaccessibility of one affects the accessibility/inaccessibility of the other. Physical entropy of position stipulates no extra constraints on the accessible states, whereas drivers may or may not impose psychological constraints on those accessible states, rendering some of the states inaccessible. Likewise, physical entropy of velocity (energy) implies that vehicles can attain any speed, yet in terms of the psychological entropy of the velocity, some of those states again may be perceived forbidden.

5.6. Superstatistics and Traffic Flow

Real life dynamics of a certain system e.g. vehicular traffic flow system is often an interwoven dynamics of multiple of those superimposed on one another. If the system demonstrates driven nonequilibrium system characteristics, with multiple time scale separation, then one may suspect of the presence of superstatistics.

In this thesis, firstly the probability distributions of the speed variable at a local point selected from a highway segment are investigated. If the space is Euclidean, the memory is Markovian and the time is continuous, the distribution of a given system would then be considered in a Boltzmann-Gibbs formalism. However, for a real system, such as vehicular traffic flow on the highway segment, this may not be so. Since the systems may exhibit (multi)fractal, non-Markovian properties and the time is discrete, we expect, at least occasionally, that the distributions become *q*-Gaussian and Tsallis formalism could be the underlying statistical mechanics. The vehicular traffic flow is modeled through *q*-Gaussian distributions.

5.6.1. Description of Time Scales of the Superstatistical Model

The dataset is defined as ν in this thesis. Consider $\nu = \{\nu_1, \nu_2 ......\nu_n\}$ and each value in the dataset corresponds to single point speed observations.

Let us assume that the local equilibrium is represented by approximately a Gaussian distribution. When the local characteristics of each partitioned cells (windows) of the time series exhibit Gaussian behavior with zero mean and variance $1/\beta_{T,u}$, the formulae of the local moments are given as in (Van der Straeten & Beck, 2009)

$$\langle \nu \rangle_{T,u} = 0, \quad \langle \nu^2 \rangle_{T,u} = \frac{1}{\beta_{T,u}}, \quad \langle \nu^3 \rangle_{T,u} = 0, \quad \langle \nu^4 \rangle_{T,u} = \frac{3}{\beta^2{}_{T,u}} \qquad (5.3)$$

And the condition for the kurtosis $\kappa_{T,u} = 3$ hints at local Gaussians.

The correlation function of a time series ν can be obtained as in (Van der Straeten & Beck, 2009)

$$C_{s,t}(\nu) = \frac{1}{s-t}\sum_{i=1}^{s-t} \nu_i \nu_{i+t} \tag{5.4}$$

The short time scale τ of the time series is described by the exponential decay of $C_{s,\tau}(\nu)$ and shown as follows (Van der Straeten & Beck, 2009):

$$C_{s,\tau}(\nu) = e^{-1} C_{s,0}(\nu) \tag{5.5}$$

Consider the kurtosis to determine the long time scale (Van der Straeten & Beck, 2009):

$$\kappa_T = \frac{1}{N}\sum_{i=1}^{N} \kappa_{T,i} \quad \text{with} \quad \kappa_{T,i} = \frac{\left\langle u^4 \right\rangle_{T,i}}{\left\langle u^2 \right\rangle_{T,i}^2} \tag{5.6}$$

where N is the total number of identical windows and T is the length of the windows.

Let the kurtosis quantity for a time window of the time series data corresponds roughly to 3. This indicates that the time windows contain data which is approximately normally distributed.

Let us consider that the local behavior of the windows of length T has not perfect Gaussian distribution, then $\varphi_{T,u}$ is introduced to determine the deviations from the fourth moment of 3, and shown as (Van der Straeten & Beck, 2009):

$$\varphi_{T,u} = \left\langle \nu^4 \right\rangle_{T,u} - 3\left\langle \nu^2 \right\rangle_{T,u}^2 \tag{5.7}$$

Gaussian approximation is obtained when $|\varepsilon| << 1$ by

$$\varepsilon = \frac{1}{3}\frac{\sum_{i=1}^{N}\varphi_{T,i}}{\sum_{i=1}^{N}\left\langle v^2\right\rangle_{T,i}^2} \tag{5.8}$$

For a suitable Gaussian approximation, the numerator in the formula would tend to zero. Thus, for the Gaussian approximation of a given time series, the value $|\varepsilon|$ should be small.

5.6.2. *q*-Gaussian Formulations

In superstatistics, the marginal probability $p(v)$ is given as, (Beck, 2004)

$$p(v) = \int f(\beta)\, p(v\,|\,\beta)\, d\beta \tag{5.9}$$

In Equation (5.9), as stated, if $f(\beta)$ is a χ^2 distribution and the $p(v\,|\,\beta)$ is Gaussian one, then the $p(v)$ becomes Tsallis distribution.

Under χ^2 distributed β, the generalized canonical distributions of nonextensive statistical mechanics become, (Beck, 2004)

$$p(v) \sim \left(1 + \frac{1}{2}\beta^{\circ}(q-1)v^2\right)^{\frac{1}{1-q}} \tag{5.10}$$

where β° is the inverse temperature, *q* is the entropic index.

Consider the χ^2 distribution Equation (5.11) in the integration of Equation (5.9),

$$f(\beta) = \frac{1}{\Gamma\left(\frac{n}{2}\right)}\left(\frac{n}{2\beta_0}\right)^{n/2}\beta^{n/2-1}e^{-\frac{n\beta}{2\beta_0}} \tag{5.11}$$

The relation between the parameters q and n is given as, (Beck, 2001), (Beck, 2004)

$$q = 1 + \frac{2}{n+1} \tag{5.12}$$

where n is the number of degrees of freedom.

5.6.3. Superstatistical Analyses of the Vehicle Speed Data

The speed data collected from a surveillance point on Çanakkale-İzmir highway are utilized for the superstatistical analyses. In one direction, several traffic variables were collected for three days, and approximately 37-hours' individual vehicle speed dataset is processed in the analyses. Some descriptive statistics of the sample speed data extracted from the dataset are given below, Table 5.1.

Table 5.1. Descriptive Statistics of the Selected Three Days from 6:00 a.m. to 12:30 p.m.

Days	Mean	Median	Mode	Standard Deviation	Variance	Range
1st	82.18	82.3	84.09	14.58	212.58	126.52
2nd	80.63	80.5	112.16	14.22	202.24	100.45
3rd	81.14	81.4	35.76	14.27	203.69	104.50

The kurtosis diagram for the speed data is obtained as below:

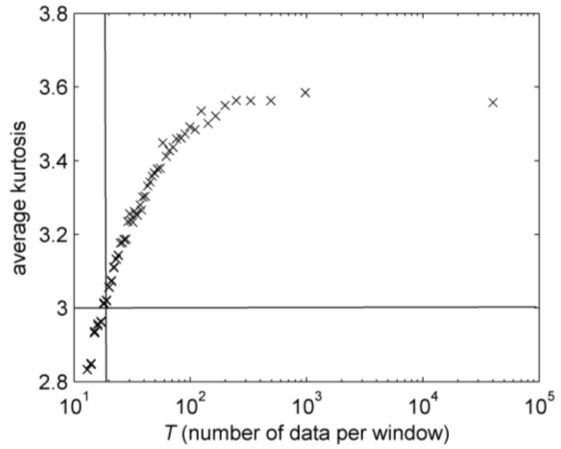


Figure 5.16. Average Kurtosis Diagram of Vehicle Speeds
(Source: Kosun & Ozdemir, 2016)

The kurtosis value corresponds to roughly 3 where the local Gaussian distributions are satisfied. The optimal time scale T of time series data for the vehicle speeds is obtained at around 18 (Figure 5.16).

Let us consider the short time scale τ for the vehicle speed time series data. By virtue of Equations (5.4) and (5.5), the correlation diagram and τ are obtained. The τ value corresponds to 2.4 where the correlation function crosses the point of $1/e$ (Figure 5.17).

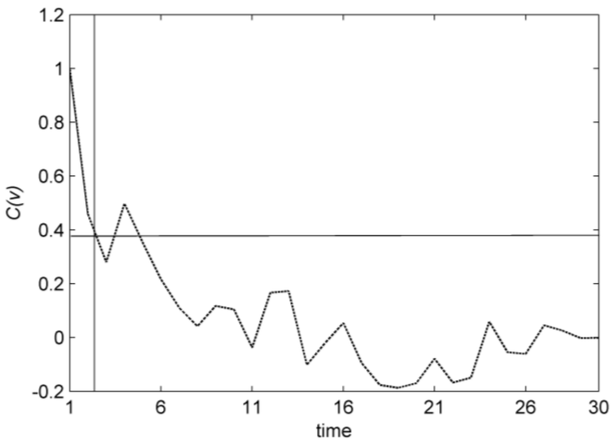


Figure 5.17. Correlation Function of Vehicle Speeds
(Source: Kosun & Ozdemir, 2016)

According to the analyses above, the ratio τ / T equals 0.13. The ratio is relatively small enough and partially verifies that the speed time series is superstatistical. The $|\varepsilon|$ value is also computed as 0.01 and satisfies the Gaussian approximation.

From the vehicle speed time series data, this thesis asserts that two different Tsallis *q* entropic indices specifying the road segment with a certain traffic flow and the interactions would be obtained. The *q* index from the fit out of a *q*-Gaussian reflects the highway or traffic properties with a certain flow and it is time-independent. However, the *q* index from the beta (inverse temperature) distributions indicates the time history of vehicle flow and interactions, and thus it is time-dependent. These distinct *q* values are obtained by considering the probability distribution of beta values and, speed time series and fitting plots of *q*-Gaussian distribution.

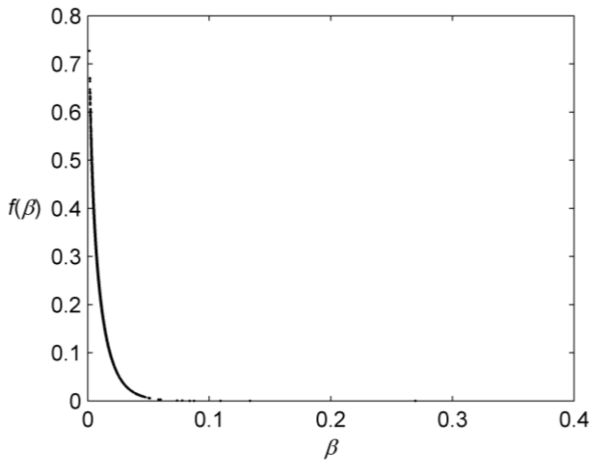


Figure 5.18. Probability Distribution of Beta Values for Vehicle Speeds
(Source: Kosun & Ozdemir, 2016)

The inverse temperature values are distributed with respect to a chi-square distribution (Figure 5.18) and from this distribution *q* value is obtained to be 1.8.

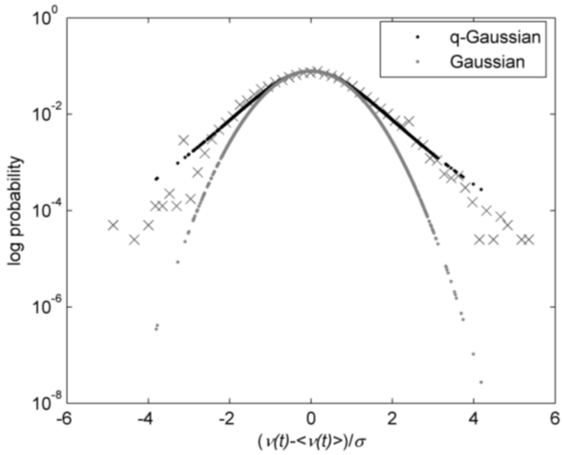


Figure 5.19. Semi-Log Plot of Speed Time Series and Fitting Plots of Gaussian and *q*-Gaussian Forms (Source: Kosun & Ozdemir, 2016)

From Figure 5.19, the deviation from the Gaussian distribution is observed. This confirms that the *q* value of entropic index would be different from the unity. With *q* value of 1.3, the *q*-Gaussian distribution fits well the speed time series where the characteristics of the highway with a certain flow displays non-Gaussianity.

CHAPTER 6

ENTROPY-BASED URBAN AND TRAFFIC PLANNING

6.1. Urban and Traffic Planning under Boltzmann-Gibbs and Tsallis Entropy Formalisms

In this thesis two cases i.e. lane changing and driver behavior analyses, and superstatistics and traffic flow were examined. The first is related with the scenario-based lane changing analyses in terms of energy and position entropy, while the second one depends on obtaining the deviation from the Gaussianity by virtue of vehicle speeds within the framework of superstatistics. Fundamentally, in the first case, the short-range, long-range and quasi long-range interactions in connection with traffic scenarios are investigated. In the second case, the probability distributions from beta parameter and speed values are plotted and as it was explained, they generate different Tsallis q values above unity.

This thesis reveals that the findings from the analyses would be related to the technical aspects of urban and traffic planning. The technical aspects here are directly linked with the design and implementation. In this chapter, the local and potential planning decisions causing the emergence of additive and subadditive regions in the traffic flow are considered. Therefore, distinct from the literature, the influences of the potential planning decisions on the traffic flow are discussed within the framework of both Tsallis and Boltzmann-Gibbs thermostatistics. The lane changing scenarios depict that unsafe driving corresponds to Boltzmann-Gibbs entropy region, whereas safe driving matches the Tsallis entropy domain well. Planners must decide how the road traffic would be safe and thus, from the thermostatistics context in this thesis, how the road traffic remains in Tsallis entropy domain.

To clarify the connection between the entropy and planning, some possible instances of the occurrence of additive and subadditive entropy regions in traffic flow would be given in the following sections.

6.2. Planning and Tsallis Entropy Formalism

The emergence of both long-range interactions and safe driving can hinge on the driver behaviors and the precautions against possible traffic problem in the practice of physical planning. Those precautions may be connected with the technical aspects of the planning such as relevant locations of car parking lots (e.g., avoiding locating them near the intersections), relevant position of the entrance and exit of the school buildings and industrial areas (e.g., avoiding locating them at the side of the urban collectors, arterials etc.), well-arranged vehicle route, leaving sufficient and well-designed paths for pedestrians, bicyclists and the disabled along the urban roads.

Preferably, planners should also refrain from designing sharp curves on roads. For example, pedestrian crossing locating at a short distance ahead would be dangerous for both drivers and pedestrians, making the traffic additive. Further, for example the presence of early warning systems and real-time monitoring in traffic may lead to the generation of long-range interactions, and hence subadditive entropy region. As a result, the traffic flow turns out to be cooperative and safe.

From the superstatistics viewpoint, the deviation from the Gaussian distribution of the speed values could indicate that the traffic does not move randomly, holding nonadditivity. This puts forward that the long-range and quasi long-range interactions mostly occur in the traffic flow, and this stems from the effects of the vehicular interactions, environmental factors, driver behaviors etc. Since the local characteristics of every factor may change, their effects on the traffic flow would be also different.

To achieve safe driving in vehicular traffic flow, the goal is to obtain the Tsallis q index above unity. However, further work needs to be carried out to find how large the proper q value should be. This thesis infers that, by quantitatively interpreting, planning decisions regarding the neighborhoods of the roads ought to keep the q index above unity for safe driving in terms of both positional and energy entropies in the traffic flow.

6.3. Planning and Boltzmann-Gibbs Entropy Formalism

Possible urban and the traffic planning problems could lead to the emergence of Boltzmann-Gibbs entropy formalism. For example, at the bottlenecks, the traffic could remain in additive region, resulting in unsafe driving. This can stem from the behavior of the drivers where some of them could display reckless driving behavior to be ahead throughout the traffic flow. Apart from this, other major and minor factors, such as the presence of parking problems along the roadways (Figure 6.1(a)), pedestrian inflow and outflow of such as administrative buildings, shopping malls and schools (Figure 6.1(c)) along the urban collectors, uncontrolled interruptions on the bicycle routes (Figure 6.1(d)), uncontrolled intersections (Figure 6.1(e)), directly merging the exits of the parking lots with the roadways (Figure 6.1(f)), and even the improper position of waste container along the roadways, may lead to additive manner in the traffic. Of these factors, for example, the parking problem along the roadways may emerge by such factors as land-use decisions, lack of parking lots. The illegal parking along the roadways leads to lane blockages, prompting the reckless driving behavior. For another problem, there could be students plunging into the roadway segment from the exit of school buildings nearby the roadway. This may result in a traffic accident due to the potential short-range interactions, and hence additivity where the states are freely chosen by drivers on the given roadway. Such potential problems can be the outputs of the urban and traffic planning process.

Despite the emergence all of those factors related with planning, driver psychology (Figure 6.1(b)) may also have prominent effect on the occurrence of additivity and subadditivity in traffic flow at both urban areas and uninterrupted highways.

In the real traffic flow, the behaviors of the vehicles (drivers) exposed to all these and related factors could exhibit a distribution approaching a Gaussian where the Tsallis q index corresponds to unity. The reason why the distribution approaches a Gaussian is that the drivers fill up available spaces among the vehicles due to a lack of the quasi long-range or long-range interactions. In this instance, the traffic flow becomes uncooperative and unsafe, and only the very short-range interactions would emerge.

a (Source: Baodanang, 2015)

b (Source: Wikispaces, 2015)

c (Source: Anadoludabugun, 2015)

d (Source: Turkiyebisikletrotalari, 2015)

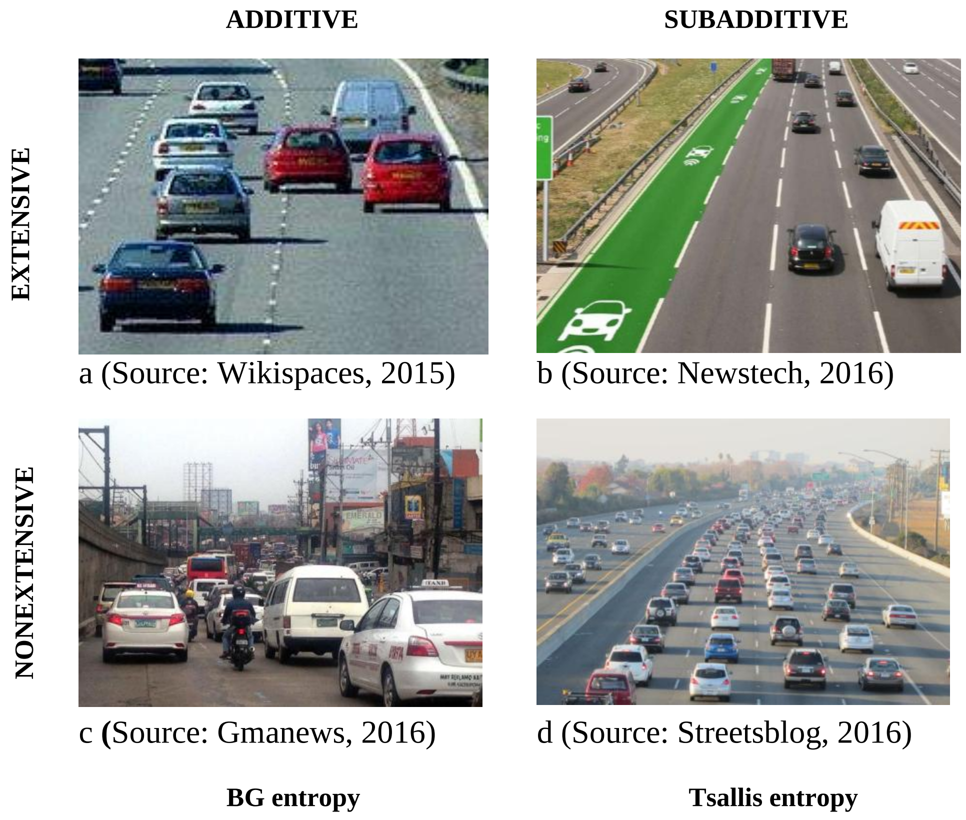
e (Source: Kandarpck, 2015)

f (Source: Fhwa, 2015)

Figure 6.1. Possible Examples Generating Additivity in Traffic Flow

6.4. Revisiting the Traffic Flow Quadrants in terms of Real Traffic Flow Examples

This thesis proposes four quadrants in connection with Tsallis and Boltzmann-Gibbs entropy as stated in Chapter 5. In this section, four real traffic flow examples are presented for each quadrant in the corresponding entropy formalisms. In the first quadrant (Figure 6.2(a)), the vehicle on the right-lane tries to overtake the vehicle in front albeit the middle-lane is occupied by the other vehicles in traffic. This overtaking behavior corresponds to the unsafe driving since the driver can choose any state with equal probability and thus the system is additive. Moreover, the presence of uncongested traffic matches the extensivity well. In the second quadrant (Figure 6.2(b)), the given traffic flow is uncongested, thus the traffic flow system is in extensive region. The drivers leave a forbidden state i.e. safe distance among the vehicles, hence the subadditivity occurs. In the third quadrant (Figure 6.2(c)), the traffic flow is congested and the vehicles e.g. motorbikes try to choose the states of the other vehicles. In this quadrant, because of the occurrence of congestion and reckless driving, the system displays nonextensive and additive properties. In the fourth quadrant (Figure 6.2(d)), it is assumed that the traffic flow is in the congested category; however the drivers maintain a safe distance among the vehicles. Due to the congestion, entropic extensivity is violated. Furthermore, since the safe distance emerges among vehicles, the traffic flow could be involved in Tsallis entropy formalism. Please note that Figure 6.1 and Figure 6.2 only represent the instantaneous traffic, thus the distinction between Boltzmann-Gibbs and Tsallis entropies is determined in the light of these traffic conditions.



a (Source: Wikispaces, 2015)
b (Source: Newstech, 2016)
c **(**Source: Gmanews, 2016)
d (Source: Streetsblog, 2016)

Figure 6.2. Possible Examples in connection with the Proposed Traffic Flow Quadrants

CHAPTER 7

CONCLUSIONS

This thesis has dealt with the analysis of a complex phenomenon known as traffic flow. This complexity stems from not only an interplay of the types of vehicles, volume, driving behavior, the number of lanes, signalization, pedestrian crossing, parking zones, left and right-turns, grade, urban land-use but also more global factors such as an interconnected networks of from the highways to byways. Apart from the traffic elements, one of the possible suspected factors that is the urban land-use has considerable impact on traffic characteristics such as vehicular interactions and speeds etc. It is known that the influence of the urban-land use on vehicular traffic flow is stronger in urban centers. This is because existing roads in urban centers may be insufficient to compensate for the vehicular flow generated by land-use and planning decisions. As much as the traffic flow interruption factors are in question, the complexity of the flow may be much more pronounced. Since this thesis has focused on a highway segment outside the urban centre, it is assumed that urban land-use factor may have slighter local or instantaneous effects on the traffic flow.

In the thesis, the effects of every individual elements or factors on complexity of traffic flow have not been evaluated, and instead its complexity has been described through a macroscale solution. Namely, examining the behavior of each vehicle and their interactions would be involved in many-body problem, making the traffic flow analyses too complex, and this is also far beyond the scope of the thesis. Within the framework of the macroscale solution, the author of this thesis has therefore investigated the statistics of the given problem. Especially the analyses of the vehicle speed time series data display this consideration in the fifth chapter of the thesis.

The deviation from the Boltzmann-Gibbs thermostatistics of the vehicle speed time series data can reveal the complexity of the traffic flow. Concurrently, the signs and effects of non-Markovian memory, long-range interactions, multifractality, environment, traffic elements and such other factors on the vehicle speed behaviors could emerge.

In the light of these, the given problem of this thesis has been investigated under nonextensive thermostatistics. For the unity of the entropic index, the nonextensive thermostatistics exhibits the Boltzmann-Gibbs thermostatistics. Therefore, observing the distinctive characteristics of the nonextensive approach in the traffic flow has become the major task of this thesis. This has been carried out through two cases, namely, lane changing and driver behavior analyses, and superstatistics and the traffic flow. In the lane changing and driver behavior case, short-range, long-range and quasi long-range interactions have been discussed. The short-range and long-range interactions among the particles in physics have been a motivational factor, and the occurrence of those interactions among the vehicles is investigated under hypothetical scenarios in traffic flow. In spite of the fact that the interactions are only represented by the cars and motorbikes, one could conceive the interactions among other vehicle types in the given scenarios.

As it was stated that the long-range interactions implicate nonextensive (Tsallis) thermostatistics, whereas the short-range interactions fall into the Boltzmann-Gibbs one. Apart from these interactions, this thesis also brings forth the new concept called quasi long-range interactions in traffic flow. This new concept emerges since some of the interactions in the traffic flow scenarios have not only been involved in the short-range but also in the long-range interaction categories. However, in this thesis, quasi long-range interactions were subsumed under Tsallis entropy domain due to its close connection with the long-range interactions. All of these interactions were examined in the entropy and energy framework. According to the traffic flow scenarios, distinctive driver behaviors e.g. malicious, reckless, safe are extracted. The analyses are shown that, by virtue of the energy and positional entropy of the vehicles, Tsallis entropy formalism corresponds to safe driving behavior while Boltzmann-Gibbs thermostatistics corresponds to unsafe driving. In the analyses, the objective is to whether the given lane changing behavior fall into Boltzmann-Gibbs or Tsallis formalism i.e. the q entropic index is unity or not. Further research may reveal the exact values of the entropic index q for the lane changing behavior in the real traffic flow. Incidentally, since the driving behavior has a significant role in the traffic flow, the number of alternative scenarios could be expanded and evaluated in terms of Boltzmann-Gibbs and Tsallis entropies in a future work.

Moreover, the solid jam condition in the traffic flow is excluded from the traffic scenarios and this condition would be evaluated in further research. However, in this thesis, it is expected that the beginning of the traffic congestion leads to diminishing of the safe distance among the vehicles. Hence, a number of forbidden states are still allowable in the traffic. For example, let us consider the presence of the reckless drivers in this flow, in such a case it is safe to say that the traffic exhibits additive characteristics. As a result, factors leading to the congestion may hold the traffic in additive region.

In the thesis, additivity in the traffic corresponds to the disordered flow where the sum the individual entropies of the vehicles are equal to the system entropy. In contrast, nonadditivity concurs with the ordered flow. The presence of additivity and nonadditivity in the traffic flow are also investigated through superstatistics. The results in regard to Tsallis q indices in the superstatistics case are given under the probability distribution of beta parameter and vehicle speed time series data. Two different Tsallis q indices are computed out of the traffic flow, which is the distinctive contribution to the literature. One of these indices is time-independent, but the other is time-dependent. The analyses also reveal that the long-range interactions or Tsallis entropy region most likely occur in the traffic flow.

The possible factors affecting the traffic flow are picked out pertaining to Boltzmann-Gibbs and Tsallis entropies. Although the individual influence of each factor on the traffic flow has not been investigated, it turned out that their overall contribution is revealed in the so-called Tsallis q parameter, in an effort to make sense of the traffic complexity. This thesis asserts that nonextensive statistical mechanics framework depicts the traffic flow complexity duly. Consequently, one of the most likely outcomes is that Boltzmann-Gibbs formalism could be replaced by nonextensive framework for a better understanding of the complexity of the traffic flow. Future research may endeavor to utilize this framework in different traffic studies.

As a final point, in this thesis, even though the highway segment is chosen as the case study, possible factors affecting the traffic flow in urban collectors, local roads etc. are given. A future work may focus on the traffic in urban centers and investigate those and other factors. Vehicular traffic flow is in question in this thesis and investigated under the given entropy formalisms. Furthermore, later studies may very well tackle other quantifiable urban phenomena and the proposed macroscale solution here could identify its complexity as well.

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VITA

PERSONAL INFORMATION

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EDUCATION

2010-2016	**Ph.D., Dept. of City and Regional Planning** İzmir Institute of Technology, Turkey
2007-2010	**Master of Science, Dept. of City and Regional Planning** İzmir Institute of Technology, Turkey
2008 (Fall)	**Erasmus Exchange Student** Dept. of Geography, Planning and International Development Studies, Faculty of Social and Behavioural Sciences, Universiteit van Amsterdam, The Netherlands
2006-2010	**Bachelor Degree, Dept. of City and Regional Planning** Dokuz Eylül University, Turkey

PROFESSIONAL EXPERIENCE

Dec. 2010-Jan. 2016	**Research Assistant at Dept. of City and Regional Planning** İzmir Institute of Technology, Turkey
May 2010-Dec. 2010	**City Planner** ESHOT Directorate General, İzmir Metropolitan Municipality, Turkey

PUBLICATIONS

Kosun, C., Ozdemir, S. “A Superstatistical Model of Vehicular Traffic Flow”, *Physica A Statistical Mechanics and Its Applications*, Vol. 444, pp. 466-475, 2016.

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