# VIBRATION ANALYSIS OF ROTATING CURVED BEAMS WITH VARIABLE CROSS-SECTION

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# ABSTRACT


## VIBRATION ANALYSIS OF ROTATING CURVED BEAMS WITH VARIABLE CROSS-SECTION

In this study, in-plane free vibration characteristics of rotating curved beams are investigated by Finite Difference Method and Finite Element Method since the mathematical model of the present problem is based on the differential eigenvalue problem with variable coefficients. A computer program regarding the titled problem is developed in Mathematica and this program is verified by using results available in the literature. The effects of taper parameters of the curved beam and rotation speed on natural frequencies are investigated.

# ÖZET

## DEĞİŞKEN KESİTLİ DÖNEN EĞRİ ÇUBUKLARIN TİTREŞİM ANALİZİ

Bu çalışmada, dönen eğri çubukların düzlem içi titreşimleri, problemin matematiksel modelinin değişken katsayılı diferansiyel özdeğer problemine dayalı olmasından dolayı, Sonlu Farklar ve Sonlu Elemanlar Yöntemleri ile araştırılmıştır.

Başlıktaki problem ile ilgili bir bilgisayar programı Matematica'da geliştirilmiş ve literatüredeki sonuçlar ile doğrulanmıştır. Kalınlık değişim parametreleri ve dönüş hızının doğal frekanslara etkileri araştırılmıştır.

# TABLE OF CONTENTS

# LIST OF FIGURES

**Figure** **Page**

# LIST OF TABLES

**Table** **Page**

# LIST OF SYMBOLS

| | |
|---|---|
| $A$ | cross-section |
| $b$, $b_o$, $b_1$ | width of the beam and its parameters, respectively |
| $b_L$ | width of the beam at the tip |
| $E$ | Young's modulus |
| $h$, $h_o$, $h_1$ | depth of the beam and its parameters, respectively |
| $h_L$ | depth of the beam at the tip |
| $L$ | circumferential length of the beam |
| $L[\,]$ | stiffness operator |
| $M$ | bending moment |
| $M[\,]$ | mass operator |
| $n$ | number of grids |
| $N$ | normal force |
| $N_p$ | normal force due to the rotational speed |
| $Q$ | shear force |
| $r$ | radial direction |
| $R$ | radius of curved beam |
| $s$, $s_L$ | circumferential direction, length of the curved beam |
| $t$ | time |
| $T$ | kinetic energy |
| $u$, $v$ | displacements in $s$ and $r$ axis, respectively |
| $v_p$ | velocity of point p on curved beam axis |
| $V_e$, $V_g$ | elastic and geometric strain energy, respectively |
| $\lambda$ | natural frequency parameter |
| $\omega$ | natural frequency, |
| $\Omega$ | Rotational speed |
| $\theta$ | angle |
| $\rho$ | density of material of beam |
| $(\,')$ | derivative with respect to "s" |
| $(\dot{\,})$ | derivative with respect to "t" |

# CHAPTER 1

# GENERAL INTRODUCTION

Vibration analysis is one of the most important task which is used to avoid collapse of whole structure because of resonance, predict and control possible failures and also protect adjacent structures or machines. The another important issue is preventing machines from spending more energy than actual need therefore engineers and analyzers investigate vibrations of structural elements, structures, and machines.

Curved beams are used in many engineering applications such as aerospace, machinery, and architectural applications. Also they are the very critical parts of machine members and they are found in such as C-clamps, crane hooks, frames of presses, riveters, punches, shears, boring machines, planers, wheels. Curved beams can be classified depending on their geometrical properties. A curved beam can be in the shape of a space curve or a plane curve, have variable curvature and cross-section.

There are some certain differences between curved beams and straight beams. First of all, in straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear, however in curved beams the neutral axis of is shifted towards the center of the curvature of beam which causes a nonlinear which is hyperbolic distribution of stress. Also the neutral axis will always be present in curved beams and the neutral axis lies between the centroidal axis and the center of curvature (Borasi 2002).

In a wide variety of engineering applications rotating components are used such as vehicular propulsion systems, flexible rotating space booms, turbo machinery, automotive cooling systems etc. Rotating beams are modeled in the form of a simple rotating beam because of the complexity of their dynamical behavior.

Because of the wide usage of rotating beams, many investigators have studied rotating beams to investigate the effects of the rotational speed, setting angle, material properties, pre-twist amount, shear deformation and rotary inertia on the bending vibrations of a rotating beam. Some of these studies for rotating straight beam are investigated in the next paragraphs.

Some of the researchers have studied approximation methods to analyze the vibration characteristics of rotating straight beams. For example, Young and Lin (1998) studied stability of coupled bending, bending vibration of pretwisted, tapered beams which rotate at randomly varying speeds. They used stochastic averaging method to derive Ito's equation for an approximation solution and obtained expressions for stability boundaries of the system by the second-moment and sample stability criteria. Hashemi (1999) investigated a new Dynamic Finite Element (DFE) formulation for the vibration analysis of spinning beams. They used frequency dependent trigonometric shape functions to find a simple frequency dependent element stiffness matrix which has both mass and stiffness properties. Also, they used an appropriate bisection method, based on a Sturm sequence root counting technique and studied flexural natural frequencies of cantilevered beams, for a variety of configurations. They compared the results which found by the Dynamic Stiffness Matrix and the classical Finite Elements Method, using ‘Hermite’ beam elements. Banerjee (1999) derived the dynamic stiffness matrix of a uniform rotating Bernoulli-Euler beam by using the Frobenius method of solution in power series. He investigated the free-vibration characteristics of uniform and non-uniform (tapered) rotating beams with particular reference to the Wittrick-Williams algorithm to demonstrate the application of the derived dynamic stiffness matrix.

Exact solutions for the bending vibrations of rotating beams have also been obtained by several researchers, in addition to the approximation methods described above. Lin et al. (2004) investigated to the vibration of a rotating damped blade with an elastically restrained root shown in Figure 1.1. They considered the effects of viscous damping and the translational and rotational damping at the root of blade. They simulated the flow-induced force and moment between the tips of a blade by using the time-dependent boundary conditions. They divided the governing equation into two coupled real differential equations and two coupled equations were uncoupled into an eighth-order characteristic differential equation. The exact steady state solution was obtained by using Green’s function in terms of the eight homogenous solutions. Also they investigated the influence of the translational, rotational and viscous damping constants on the frequency response curves of a rotating beam. They discovered the opposite influence of the transverse viscous damping constant and the root damping constants on the frequency of resonance. Lee and Sheu (2007) developed an exact power series solution for free vibration of a rotating inclined Timoshenko beam. They observed both the extensional deformation and the Coriolis force will have significant influence on the natural frequencies of the rotating beam when the dimensionless rotating extension parameter is large. They discovered several general qualitative relations among the inclination angle, the hub radius and the natural frequencies of the beam without numerical analysis. Also they studied and compared the influence of extensional deformation in the centrifugal stiffening force term on the natural frequencies evaluated by the Timoshenko and the Euler beam theories.

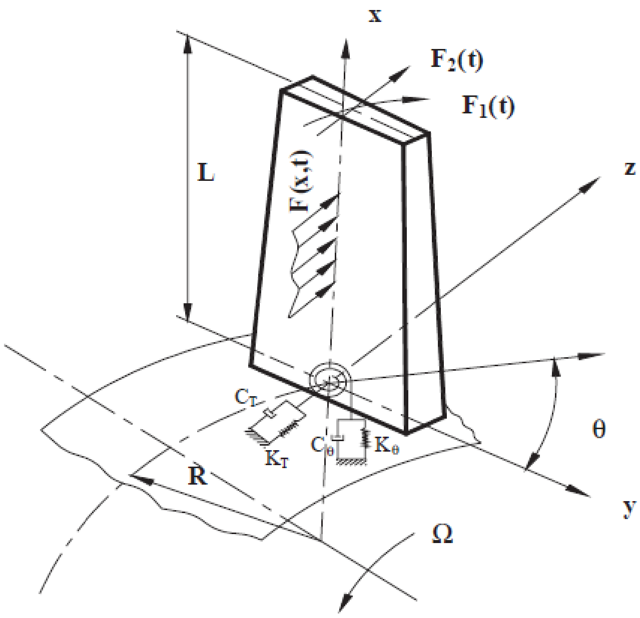



Figure 1.1 Geometry and coordinate system of non-uniform damped beam
(Source: Lin et al. 2004)

All of the studies which mentioned above considered the beam is perfectly straight. There are only a few studies about the vibration characteristics of rotating curved beams because of the complexity of rotating curved beam. Researchers which have investigated the vibration characteristics of rotating curved beams, Wang and Mahrenholtz (1975) investigated bending vibrations of a rotating curved cantilever beam with centroidal axis represented by a circular arc by using Galerkin's method. They used Legendre polynomials to represent the shape functions. They computed and presented various rotating speeds, frequencies accounting for hub size, beam cross-section orientation and curvature by curves. They observed that for rotating beams the frequency generally decreases as the curvature increases, and the reduction from the straight beam frequency increases as the rotating speed increases. It is observed that the effect of the beam cross-section orientation is significant only for the fundamental frequencies. Park and Kim (1999) investigated the dynamic characteristics of a rotating curved beam with a tip mass shown in Figure 1.2. All dynamic effects such as Coriolis force, centrifugal force and acceleration were included for equation of motion. They calculated the time responses for accelerating motion and torque driven motion for dynamic analysis. Then the natural frequencies for curved beams of various radius of curvature were investigated while the rotating speed increases. They mainly focused on the effect of curvature that can change the characteristics of the beam and also studied the effects of tip mass on the dynamic response of the beam. Lee et al (2008) investigated the in-plane free vibration of a rotating curved beam with an elastically restrained root. They neglected the effects of shear deformation and the Coriolis force and derived governing differential equations for the coupled bending extensional vibration of the curved beam using Hamilton's principle and a consistent linearization approach. They used explicit relations to describe the correlation between the axial and radial displacements of the beam. Then they used these relations to transform the coupled governing differential equations into a sixth-order ordinary differential equation expressed in terms of the radial displacement variable only. The respective effects of the arc angle, the rotational speed, the hub radius and the root spring constants on the natural frequencies and divergent instability characteristics of a curved rotating beam are examined and these results are compared with those observed for a straight cantilever beam.

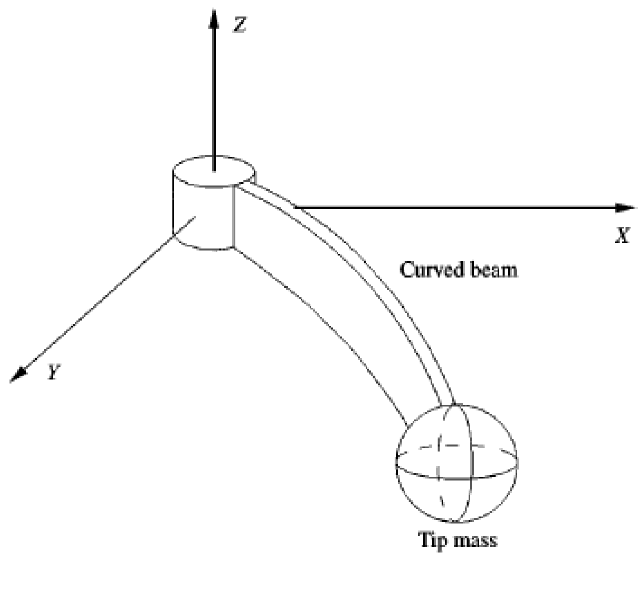



Figure 1.2. Model of the rotating curved beam with a tip mass
(Source: Park and Kim 1999)

In this study, in plane free vibration characteristics of rotating curved beams are studied by Finite Difference Method and Finite Element Method since the mathematical model of the present problem is based on the differential eigenvalue problem with variable coefficients.

A computer program is developed in Mathematica and this program is verified by using results available in the literature. The effects of taper parameters of the curved beam and rotation speed on natural frequencies are investigated.

# CHAPTER 2

# THEORETICAL VIBRATION ANALYSIS

## 2.1. Introduction

In this chapter, first the selected taper geometry of the curved beam is detailed with mathematical formulations. Then, equation of motion of the rotating curved beam with taper is derived for in-plane vibrations. Since the present vibration problem based on *Differential Eigenvalue Problem* with variable coefficients is not solvable exactly, Finite Difference and Finite Element Methods are used to reduce it to *Discrete Eigenvalue Problem*. Finally, aforementioned numerical methods are presented.

## 2.2. Geometry of Curved Beam

A rotating curved beam with variable cross-section is shown in Figure 2.1.

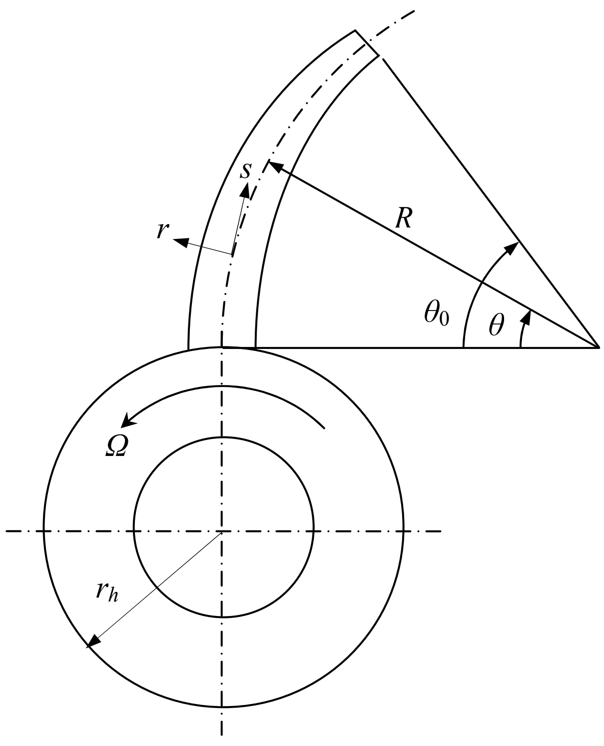



Figure 2.1. Curved beam with variable cross-section

A planar curved beam with variable cross section is also shown inFigure 2.2.

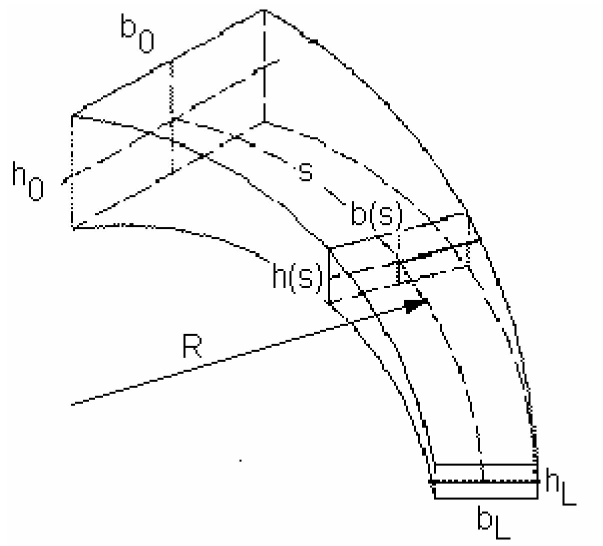



Figure 2.2. A planar curved beam with variable cross section

The breadth and depth functions of the curved beam are selected as follows:

$$b(s) = b_0 - b_1\, s \tag{2.1}$$

$$h(s) = h_0 - h_1\, s \tag{2.2}$$

where $b_0$ and $h_0$ are breadth and depth of curved beam at $s$=0, respectively. Also, $b_1$ and $h_1$ are breadth and depth parameters.

## 2.3. Derivation of the Equation of Motion

The Hamilton's principle is defined as follows (Meirovitch 1967):

$$\delta\int_{t_1}^{t_2} (T - V)\, dt = 0 \tag{2.3}$$

where $T$ and $V$ are the kinetic and strain energies, respectively. For the present problem, kinetic energy $T$ is given as follows (Lee et al 2008);

$$T = \frac{1}{2}\int_0^{\theta_0} \rho AR(\vec{v}_p.\vec{v}_p)\, d\theta \tag{2.4}$$

where $\vec{v}_p$ is the velocity of any point P located on the curved beam axis and written as

$$\begin{aligned} \vec{v}_p &= [\dot{u} - \Omega(r_h \sin\theta + R(1-\cos\theta) + v]\vec{e}_s \\ &+ [\dot{v} + \Omega(R\sin\theta + r_h \cos\theta) + u]\vec{e}_r \end{aligned} \tag{2.5}$$

Potential energy $V$ has two parts as elastic and geometric energies. They are expressed as

$$V = V_e + V_g \tag{2.6}$$

where $V_e$ is elastic stiffness energy and written as

$$\begin{aligned} V_e &= \frac{1}{2}\int_0^L E\,A(s)\left(u' + v/R\right)^2 ds \\ &+ \frac{1}{2}\int_0^L E\,I(s)\left(v'' - u'/R\right)^2 ds \end{aligned} \tag{2.7}$$

and $V_g$ is geometric stiffness energy and written as

$$V_g = \frac{1}{2}\int_0^L N_p(s)\left(v'\right)^2 ds \tag{2.8}$$

where $N_p$ is normal force due to the rotation and found as

$$\begin{aligned} N_p &= \rho A(s) R\Omega^2[(\theta_0 - \theta)(R\sin\theta) \\ &+ r_h \cos\theta) + R(1-\cos(\theta_0 - \theta))] \end{aligned} \tag{2.9}$$

By using Equation 2.4 and Equation 2.6 in Equation 2.3, the following governing equations are obtained:

$$Q' - N/R + (N_p(s)v')' - \rho A(s)\ddot{v} + \rho A(s)\Omega^2 v - 2\rho A(s)\Omega\dot{u} = -\rho A(s)\Omega^2[r_h \sin\theta + R(1-\cos\theta)] \tag{2.10.a}$$

$$Q/R + N' - \rho A(s)\ddot{u} + \rho A(s)\Omega^2 u + 2\rho A(s)\Omega\dot{v} = -\rho A(s)\Omega^2[r_h \cos\theta + R\sin\theta] \tag{2.10.b}$$

with the boundary conditions at $s$=0 (fixed):

$$u = 0 \tag{2.11.a}$$

$$v = 0 \tag{2.11.b}$$

$$v' = 0 \tag{2.11.c}$$

and $s$=$L$ (free):

$$N = 0 \tag{2.12.a}$$

$$Q = 0 \tag{2.12.b}$$

$$M = 0 \tag{2.12.c}$$

In Equations 2.10 to 2.12, $N$, $Q$, $M$ represent the axial normal force, the shear force, and the bending moment, respectively, and are given by

$$N = EA(s)(u' + v/R) \tag{2.13}$$

$$Q = -EI(s)(v''' - u''/R) \tag{2.14}$$

$$M = EI(s)(v'' - u'/R) \tag{2.15}$$

## 2.4. Natural Frequencies by Finite Difference Method

*Differential Eigenvalue Problem* can be written in the following equation:

$$L[V(s)] = \omega^2 M[V(s)] \tag{2.16}$$

where $L[\ ]$ and $M[\ ]$ are linear differential operators having derivatives with respect to $s$, $\omega^2$ is eigenvalues and $V(s)$ is corresponding eigenfunctions. Equation 2.16 can be reduced to *Discrete Eigenvalue Problem* in the form of

$$[K]\{V\} = \omega^2 [M]\{V\} \tag{2.17}$$

by using the FDM (Hildebrand 1987). In Equation 2.17, $[K]$ and $[M]$ are stiffness and mass matrices; $\{V\}$ is the displacement vector which is known as eigenvector. In the FDM, the derivatives of dependent variables in Equation 2.16 are replaced by the finite differences at mesh points shown in Figure 2.3.

There are three types of finite differences: forward, backward, and central. However, the central difference approach provides more accurate results. Accuracy of the solution obtained by FDM is based on truncation error and grid spacing. Grid spacing is selected by testing the convergence of results.

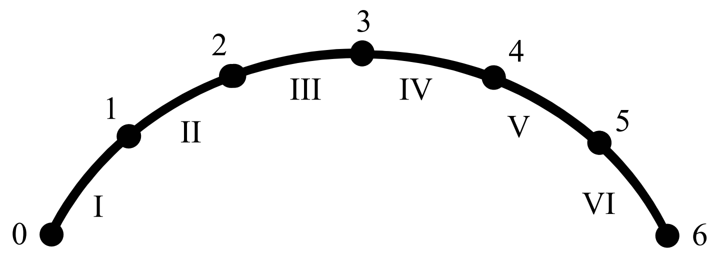



Figure 2.3. A curved domain divided into six sub domains.
(Source: Cangar 2013)

The central difference derivatives are listed in Table A.1. Solutions of the Equation 2.17 can be obtained by mathematical software such as Matlab or Mathematica.

## 2.5. Natural Frequencies by Finite Element Method

The equation of motion of a continuous system is given by

$$M\,[\ddot{v}(s,t)] + L\,[v(s,t)] = 0 \tag{2.18}$$

where $L$[ ] and $M$[ ] are linear differential operators having derivatives with respect to $s$. By using the Finite Element Method (Petyt 2010), Equation 2.18 is reduced to multi-degree-of-freedom system. Therefore equation of motion of the system is given by

$$[M]\{\ddot{x}(t)\} + ([K] + [G])\{x(t)\} = 0 \tag{2.21}$$

where [$M$], [$K$] and [$G$] are mass, elastic stiffness and geometric matrices, respectively. $\{x(t)\}$ is displacement vector. The above equation is reduced to the following generalized eigenvalue equation:

$$([K] + [G] - \omega_i^2 [M])\{u_i\} = \{0\} \tag{2.22}$$

where $\omega_i$ is $i^{th}$ natural frequency and $\{u_i\}$ is the $i^{th}$ vibration mode shape vector.

# CHAPTER 3

# NUMERICAL RESULTS AND DISCUSSION

## 3.1. Introduction

In this chapter, the following studies are presented:

a) Non-rotating and rotating curved beams with constant cross-section,

b) Non-rotating and rotating curved beams with variable cross-section,

In the computer program, rotational effect terms that appeared in Equation 2.5 are neglected. Also, Coriolis terms in Equation 2.10a and 2.10.b are not considered.

The present numerical results obtained for a) are compared with the results available in the existing literature. Material properties for numerical studies are selected as: Young modulus $E$=200 GPa, material density $\rho$=7850 kg/m$^3$.

## 3.2. Validation of the Curved Beams Model

### 3.2.1. Non-Rotating Curved Beams with Constant Cross-Section

In plane vibration analysis of circular curved beams with constant cross-section can be solved analytically, but it is considered here to test the FDM algorithm and to show the accuracy and precision of the symbolic program developed in Mathematica.

On the other hand, finite element model of the rotating curved beam with variable cross-section is generated in ANSYS by using APDL (ANSYS Parametric Design Language). BEAM44 which is 3-D Elastic Tapered Unsymmetric Beam is selected to model the curved beam with taper. Real constants of the tapered beam elements are calculated in the APDL code. Spin softening is included for the vibration analysis of the rotating curved beam. More detail about spin softining is available in ANSYS Documentations.

A fixed-free curved beam with the parameters $b_0$=$b_L$=$h_0$=$h_L$=20 mm, $R$=160 mm, $s_L$=251.3 mm is considered as geometric model.

Convergence of the natural frequencies of the geometric model based on FDM (Finite Difference Method) and FEM (Finite Element Method) are given in Table 3.1 and Table 3.2, respectively. It is clear form Tables 3.1 and 3.2 that, the convergences of the natural frequencies are very good.

Table 3.1. Convergence of natural frequencies based on FDM

| n | $f_1$ (Hz) | $f_2$ (Hz) s | $f_3$ (Hz) | $f_4$ (Hz) |
|---|---|---|---|---|
| 21 | 247.9 | 1183.6 | 3773.0 | 7711.1 |
| 41 | 259.1 | 1240.5 | 3936.9 | 8061.2 |
| 81 | 265.1 | 1273.0 | 4030.3 | 8249.5 |
| 121 | 267.3 | 1284.6 | 4063.8 | 8316.0 |
| 161 | 268.2 | 1290.5 | 4081.0 | 8350.1 |
| 201 | 268.9 | 1294.2 | 4091.5 | 8370.9 |
| 241 | 269.7 | 1296.7 | 4098.6 | 8394.9 |

Table 3.2. Convergence of natural frequencies based on FEM

| N | $f_1$ (Hz) | $f_2$ (Hz) s | $f_3$ (Hz) | $f_4$ (Hz) |
|---|---|---|---|---|
| 5 | 274.3 | 1305.9 | 4001.1 | 6633.6 |
| 10 | 271.9 | 1294.6 | 3961.9 | 6567.0 |
| 15 | 271.4 | 1292.5 | 3955.4 | 6556.6 |
| 20 | 271.2 | 1291.7 | 3953.1 | 6553.2 |
| 25 | 271.2 | 1291.4 | 3952.1 | 6551.6 |
| 30 | 271.1 | 1291.2 | 3951.6 | 6550.8 |
| 35 | 271.1 | 1291.1 | 3951.2 | 6550.3 |

Convergence of first natural frequency based on Finite Difference Method is plotted in Figure 3.1. It is seen from Figure 3.1 that number of grid $n$ =201 can be selected.

Also, convergence of first natural frequency based on Finite Element Method is plotted in Figure 3.2. It is seen from Figure 3.2 that number of element $N$=30 can be selected.

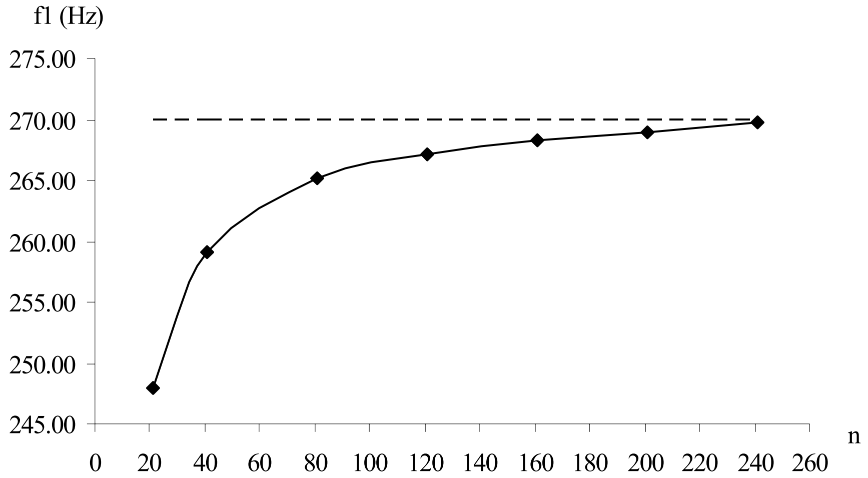



Figure 3.1. Convergence of first natural frequency by FDM

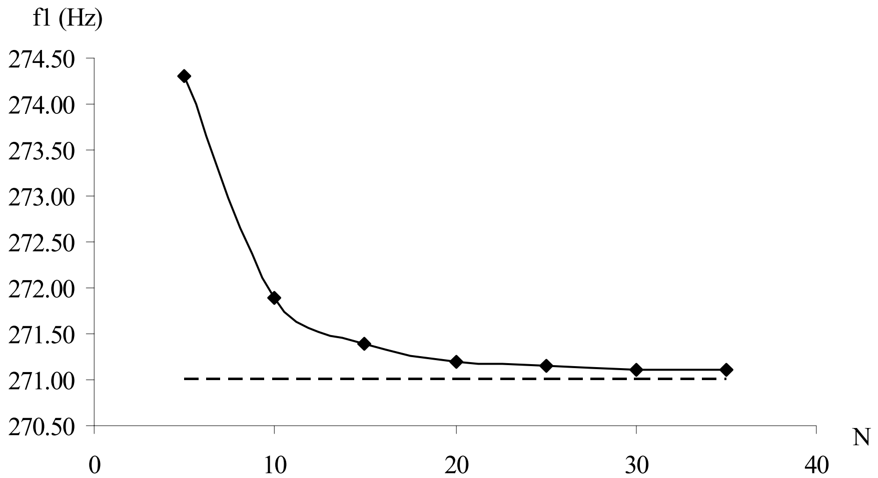
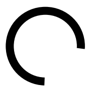



Figure 3.2. Convergence of first natural frequency by FEM

Comparisons of natural frequency parameters $\lambda=\omega^2\rho AR^4/EI$ of fixed-fixed curved beams by FDM with analytical results of Archer (1960) can be done by using the results given in Table 3.3. It can be said that, the present results have good agreement in lower mode with the analytical results given by Archer (1960).

Table 3.3. Comparison of present natural frequency parameters of a curved beams with the analytical results of Archer (1960)

| Mode | Opening Angle | | Present $\lambda$ ($n$ =201) | $\lambda$ (Archer 1960) |
|---|---|---|---|---|
| 1 |  | $\pi$ | 18.3 | 19.22 |
| 2 | | | 89.1 | 93.15 |
| 3 | | | 307.7 | 321.5 |
| 4 | | | 727.0 | 756.3 |
| 1 |  | $3\pi/2$ | 1.833 | 1.946 |
| 2 | | | 12.213 | 12.85 |
| 3 | | | 47.309 | 49.58 |
| 4 | | | 123.033 | 126.6 |

## 3.2.2. Rotating Curved Beams with Constant Cross-Section

In this section, numerical applications are carried out for fixed-free curved beams with different rotation speed to see the effects of rotation on natural frequencies. The same numerical data used in Section 3.2.1 is also used here.

Table 3.4. Natural frequencies for different $\Omega$ values

| | $\Omega$=0 rad/s | $\Omega$=100 rad/s | $\Omega$=200 rad/s | $\Omega$=300 rad/s |
|---|---|---|---|---|
| $f_1$ (Hz) | 268.9 | 269.1 | 270.1 | 271.5 |
| $f_2$ (Hz) | 1294.2 | 1294.4 | 1295.4 | 1296.9 |
| $f_3$ (Hz) | 4098.6 | 4091.8 | 4092.7 | 4094.2 |
| $f_4$ (Hz) | 8384.9 | 8371.2 | 8372.1 | 8373.6 |

It can be seen from the Tables 3.4 that, when the rotation speed increases, natural frequency parameters also increases.

## 3.3. Applications for Rotating Curved Beams

### 3.3.1. Non-Rotating Curved Beams with Variable Cross-Section

In this section, numerical applications are carried out for fixed-free non-rotating curved beams with variable cross-section. The main numerical data for the selected geometry are as follows: $b_0$=20 mm, $h_0$=20 mm, $R$=160 mm, and $s_L$=251.3 mm. Other data are given in tables. The results are given in Table 3.5 and Figures 3.3 to 3.10.

Table 3.5. Natural frequencies for different $b_L=h_L$ at $\Omega=0$

| | $b_L=h_L$ =18 mm | $b_L=h_L$ =16 mm | $b_L=h_L$ =14 mm | $b_L=h_L$ =12 mm |
|---|---|---|---|---|
| $f_1$ (Hz) | 281.0 | 294.6 | 311.0 | 330.3 |
| $f_2$ (Hz) | 1283.3 | 1272.4 | 1261.7 | 1251.4 |
| $f_3$ (Hz) | 3940.9 | 3786.3 | 3627.1 | 3462.9 |
| $f_4$ (Hz) | 7997.4 | 7614.3 | 7220 | 6812.3 |

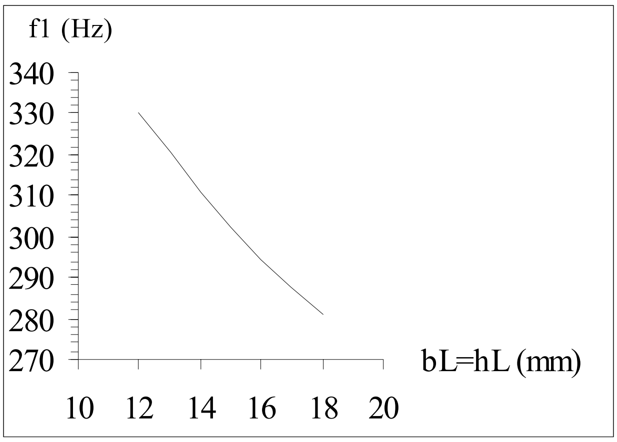



Figure 3.3. First natural frequencies

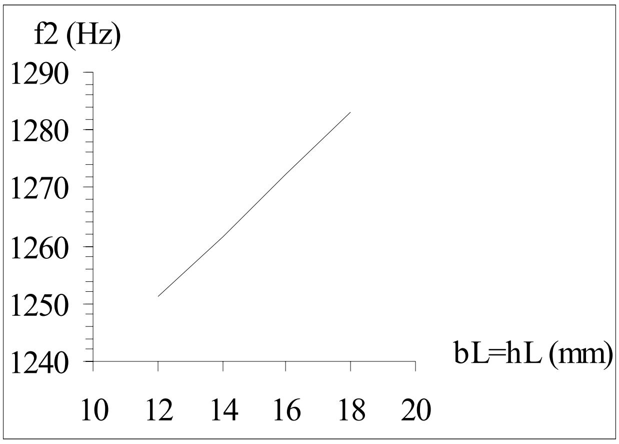



Figure 3.4. Second natural frequencies

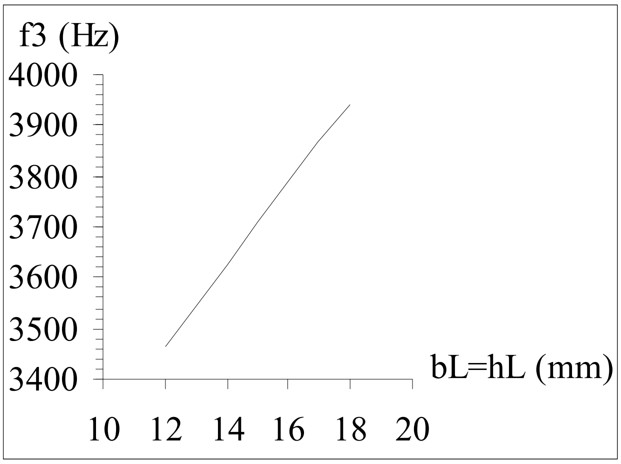



Figure 3.5. Third natural frequencies

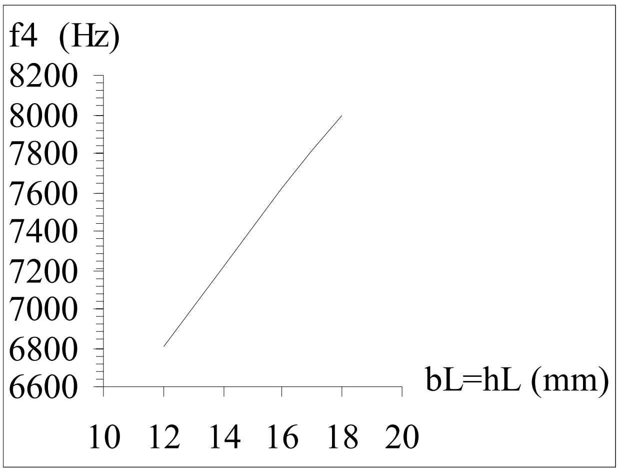



Figure 3.6. Fourth natural frequencies

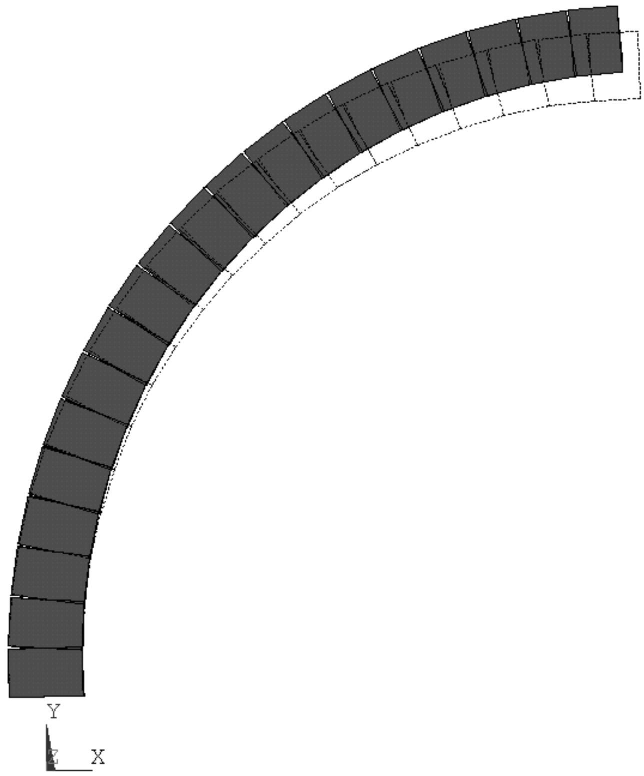



Figure 3.7. First natural mode

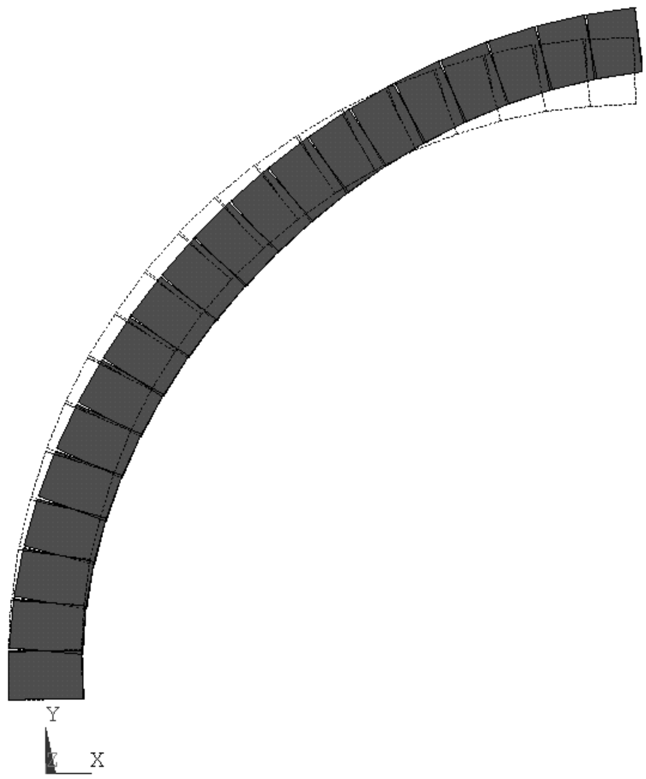



Figure 3.8. Second natural mode

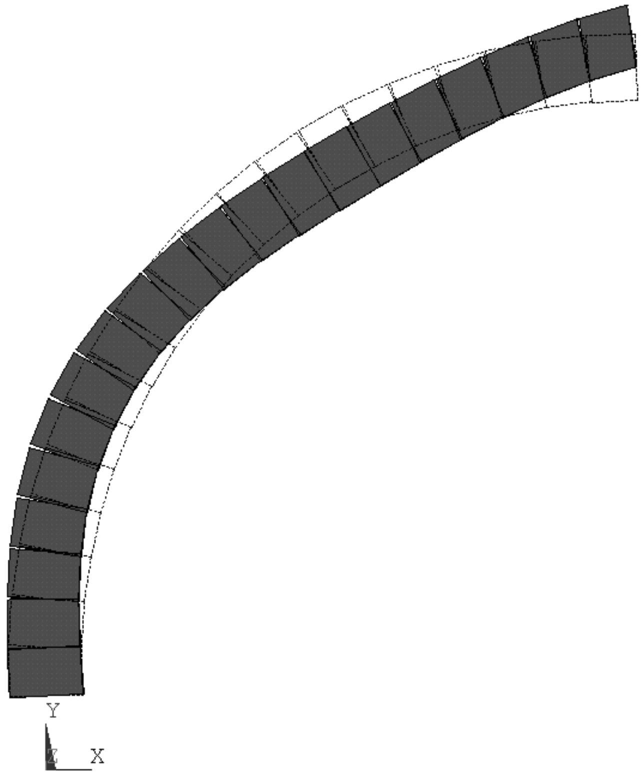



Figure 3.9. Third natural mode

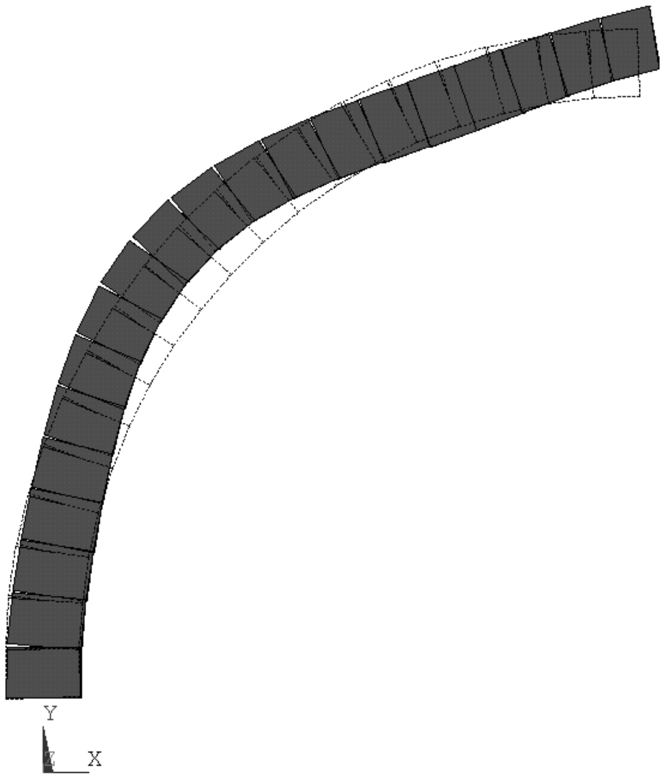



Figure 3.10. Fourth natural mode

### 3.3.2. Rotating Curved Beams with Variable Cross-Section

Numerical applications regarding the title are carried out for different rotational speeds and the results are presented in Tables 3.6-3.8 and Figure 3.11-3.14.

Table 3.6. Natural frequencies for different $b_L$=$h_L$ at $\Omega$=100 rad/s

| | $b_L$=$h_L$ =18 mm | $b_L$=$h_L$ =16 mm | $b_L$=$h_L$ =14 mm | $b_L$=$h_L$ =12 mm |
|---|---|---|---|---|
| $f_1$ (Hz) | 281.1 | 295.1 | 311.3 | 330.6 |
| $f_2$ (Hz) | 1283.6 | 1272.8 | 1262.1 | 1251.8 |
| $f_3$ (Hz) | 3941.2 | 3786.6 | 3627.4 | 3463.2 |
| $f_4$ (Hz) | 7997.8 | 7614.6 | 7220.3 | 6812.7 |

Table 3.7. Natural frequencies for different $b_L$=$h_L$ at $\Omega$=200 rad/s

| | $b_L$=$h_L$ =18 mm | $b_L$=$h_L$ =16 mm | $b_L$=$h_L$ =14 mm | $b_L$=$h_L$ =12 mm |
|---|---|---|---|---|
| $f_1$ (Hz) | 281.9 | 295.8 | 312.2 | 331.6 |
| $f_2$ (Hz) | 1284.6 | 1273.8 | 1263.1 | 1252.9 |
| $f_3$ (Hz) | 3942.2 | 3787.6 | 3628.5 | 3464.3 |
| $f_4$ (Hz) | 7998.7 | 7615.6 | 7221.3 | 6813.8 |

Table 3.8. Natural frequencies for different $b_L$=$h_L$ at $\Omega$=300 rad/s

| | $b_L$=$h_L$ =18 mm | $b_L$=$h_L$ =16 mm | $b_L$=$h_L$ =14 mm | $b_L$=$h_L$ =12 mm |
|---|---|---|---|---|
| $f_1$ (Hz) | 283.3 | 297.2 | 313.7 | 333.1 |
| $f_2$ (Hz) | 1286.2 | 1275.4 | 1264.9 | 1254.7 |
| $f_3$ (Hz) | 3943.7 | 3789.2 | 3630.2 | 3466.1 |
| $f_4$ (Hz) | 8000.3 | 7617.3 | 7223.1 | 6815.6 |

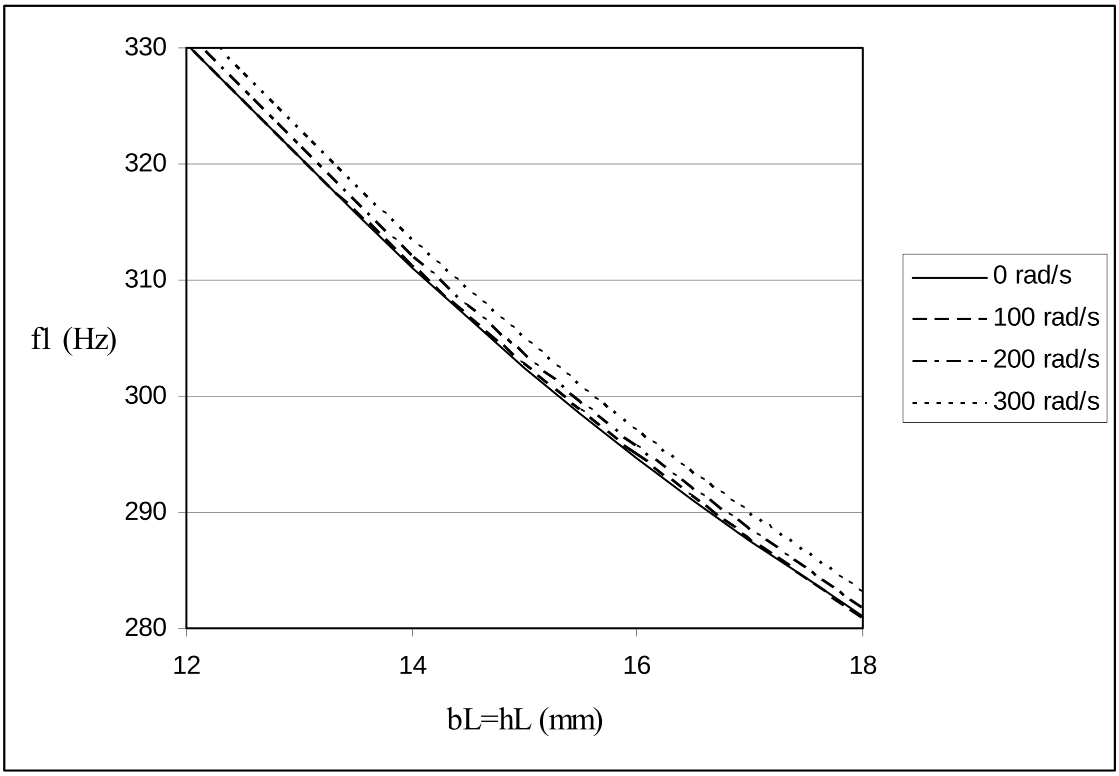



Figure 3.11. First natural frequencies

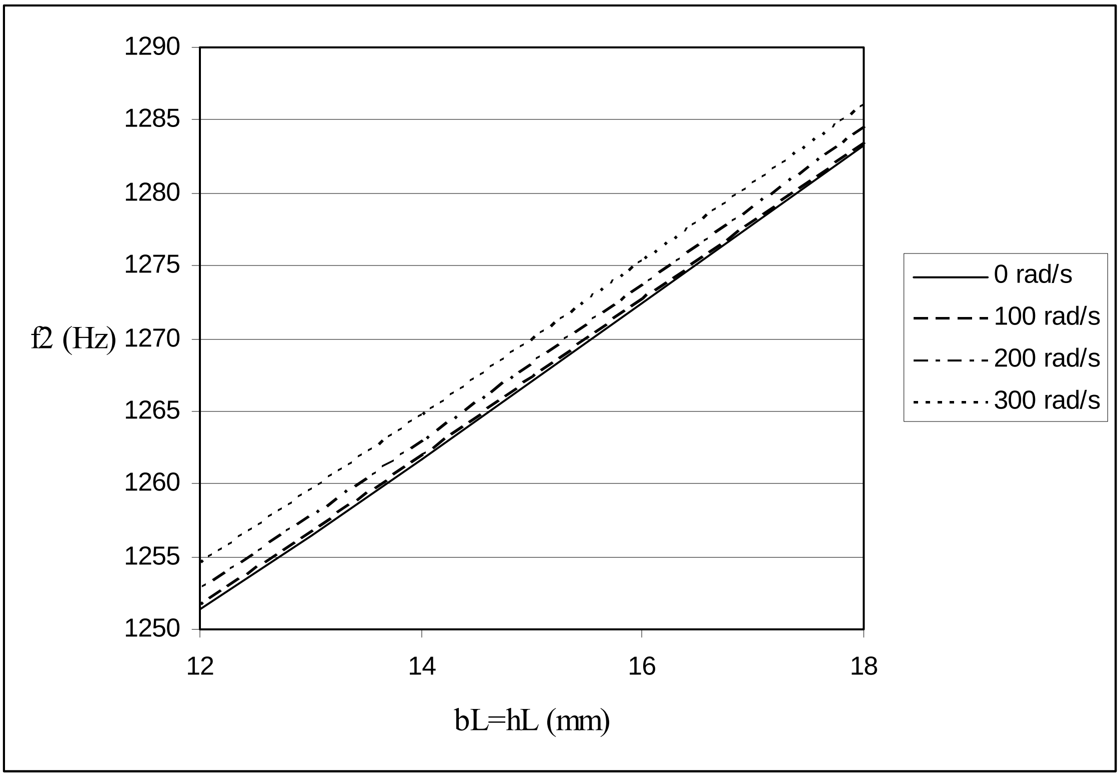



Figure 3.12. Second natural frequencies

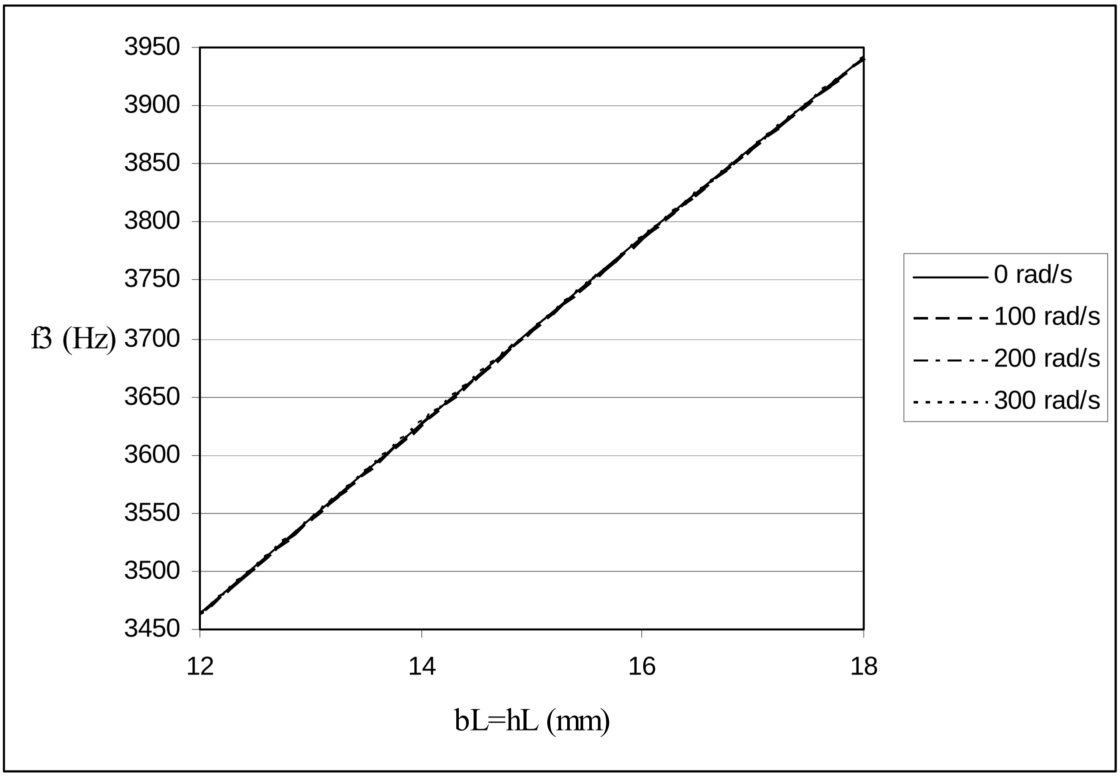



Figure 3.13. Third natural frequencies

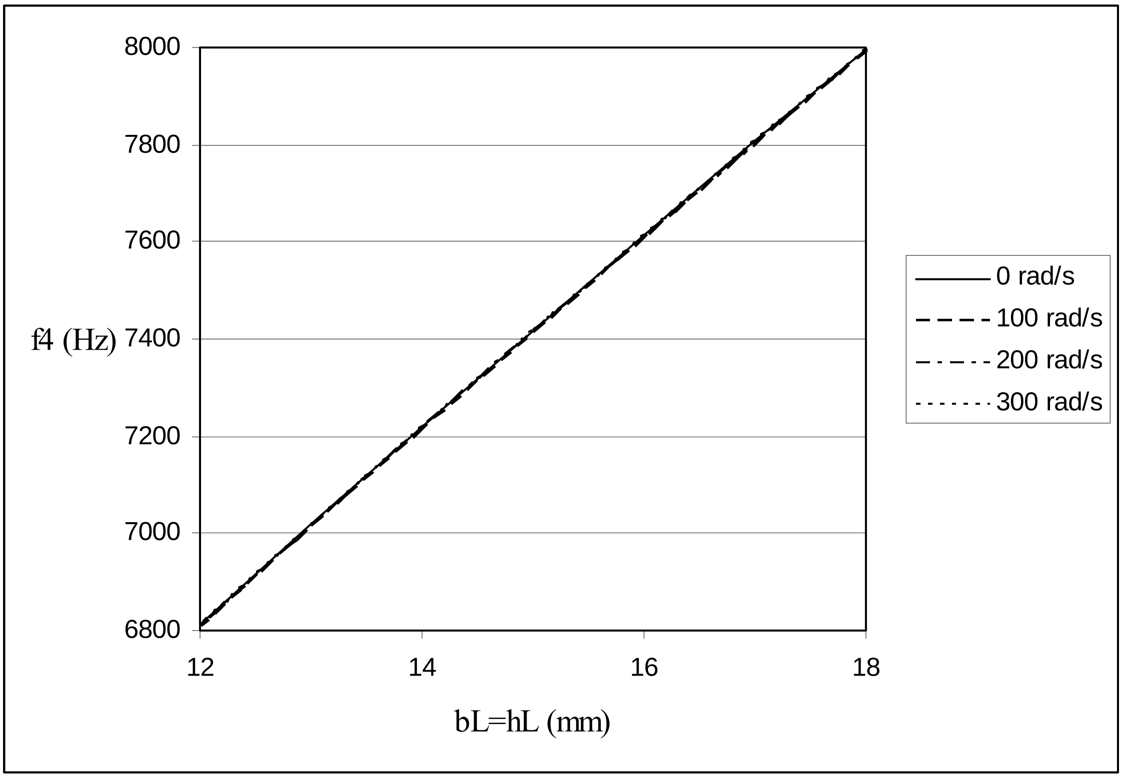



Figure 3.14. Fourth natural frequencies

It can be seen from Table 3.6 to Table 3.8 and Figure 3.11 to 3.14 that rotational speed in the considered range is only effective on first and second natural frequencies, but not on third and fourth ones.

It is very interesting that only first natural frequencies decrease when the taper of the curved beam decrease.

# CHAPTER 4

# CONCLUSIONS

In this study, in plane free vibration characteristics of rotating curved beams are studied by Finite Difference Method and Finite Element Method since the mathematical model of the present problem is based on the coupled differential eigenvalue problem with variable coefficients.

A computer program is developed in Mathematica and this program is verified by using results available in the literature. The effects of taper parameters of the curved beam and rotation speed on natural frequencies are investigated.

It is found that rotational speed in the considered range is only effective on first and second natural frequencies, but not on third and fourth ones.

It is very interesting that only first natural frequencies decrease when the taper of the curved beam decrease.

# REFERENCES


Archer, R.R. 1960. Small vibrations of thin incomplete circular rings. *International Journal of Mechanical Science* 1: 45-56.

Banerjee, J.R. 2000 Free vibration of centrifugally stiffened uniform and tapered beams using the dynamic stiffness method, *Journal of Sound and Vibration* 233 (5):857–875.

Boresi, A.P., Schmidt, R.J. ,2006. Advanced Mechanics of Materials. New Jersey, John Wiley&Sons Press.

Cangar, F. 2013, In-plane free vibration analysis of laminated composite curved beams with variable curvature, M.Sc Thesis, Izmir Institute of Technology.

Hashemi, S.M., Richard M.J., and Dhatt, G. 1999. A new dynamic finite element(DFE) formulation for lateral free vibration of Euler–Bernoulli spinning beams using trigonometric shape function. *Journal of Sound and Vibration* 220(4):601-624.

Hildebrand, Francis B. 1987. Introduction to numerical analysis. New York: Dover Publications.

Lee, S.Y., Sheu, J.J 2007 Free vibration of an extensible rotating inclined Timoshenko beam, *Journal of Sound and Vibration* 304:606-624

Lee, S.Y., Sheu, J.J.,Lin, S.M. 2008 In-plane vibrational analysis of rotating curved beam with elastically restrained root, *Journal of Sound and Vibration* 315:1086–1102

Lin, S.M., Lee, S.Y., Wang, W.R 2004 Dynamic analysis of rotating damped beams with an elastically restrained root, *International Journal of Mechanical Sciences* 46:673–693.

Meirovitch, Leonard 1967. Analytical methods in vibrations. New York: Macmillan Publishing Co.

Park, J.H. Kim, J.H 1999. Dynamic analysis of rotating curved beam with a tip mass, *Journal of Sound and Vibration* 228(5):1017–1034

Petyt, M. 2010. Introduction to finite element vibration analysis. Cambridge: Cambridge University Press.

Wang, J.T.S., Mahrenholtz, O. 1975 Bending frequencies of a rotating curved beam, Archive of Applied Mechanics 44:399–407.

Young, T.H. and Lin, T.M. 1998. Stability of rotating pretwisted, tapered beams with randomly varying speeds. *Journal of Vibration and Acoustics* 120(3):784-790.

# APPENDIX A

# CENTRAL DIFFERENCES

Table A.1. Central differences approximations of O($h^2$)

| Term | Central Difference Expressions |
|---|---|
| $\frac{dw}{ds}$ | $\frac{w(i+1)-w(i-1)}{2h}$ |
| $\frac{d^2w}{ds^2}$ | $\frac{w(i+1)-2w(i)+w(i-1)}{h^2}$ |
| $\frac{d^3w}{ds^3}$ | $\frac{w(i+2)-2w(i+1)+2w(i-1)-w(i-2)}{2h^3}$ |
| $\frac{d^4w}{ds^4}$ | $\frac{w(i+2)-4w(i+1)+6w(i)-4w(i-1)+w(i-2)}{h^4}$ |
| $\frac{d^5w}{ds^5}$ | $\frac{w(i+3)-4w(i+2)+5w(i+1)-5w(i-1)+4w(i-2)-w(i-3)}{h^5}$ |
| $\frac{d^6w}{ds^6}$ | $\frac{w(i+3)-6w(i+2)+15w(i+1)-20w(i)+15w(i-1)-6w(i-2)+w(i-3)}{h^6}$ |