EXTRA TIME-LIKE DIMENSIONS

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ABSTRACT

EXTRA TIME-LIKE DIMENSIONS

The current status of extra dimensions in high energy physics with particular emphasis on extra time-like dimensions is briefly reviewed. Some of the phenomenological problems related to the use of extra time-like dimensions that mentioned in the literature are reconsidered. It is found that the use of extra time-like dimensions is less problematic than it was previously thought.

ÖZET

ZAMAN BENZERİ EKSTRA BOYUTLAR

Yüksek enerji fiziğinde ektra boyutların şu andaki durumu, zaman benzeri ekstra boyutlara vurgu yapılarak gözden geçirildi. Zaman-benzeri ekstra boyutların kullanımı ile ilgili literatürde belirtilen fenomenolojik problemler yeniden değerlendirildi. Zaman benzeri ekstra boyutların kullanımının daha önce düşünüldüğünden daha az problemli olduğu görüldü.

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CHAPTER 1

INTRODUCTION

While the observable world is four dimensional (i.e. three space dimensions plus one time dimension.), the idea of extra dimensions (in addition to the usual four dimensions) has been getting an increasing important role in the understanding of the nature. The idea was introduced by Theodor Kaluza first as early as 1921 to unify electromagnetism with gravity ”(Erlich 2005, Nordström 1914)”. However this idea had been shelved until it was noticed that the string theory, which unify the gravity with the other fundamental interactions (electromagnetism, the weak and strong interactions) requires more than four dimensions. (10 in the case of superstring, 26 in the case of bosonic string ”(Polchinski 1998)”) Since then, the idea of extra dimensions has been thought to have a key role in the study of unification.

The fact that the physics at present energies seems to be four dimensional, requires extra dimensions be small or the physical world be confined to a small region in extra dimensions. In this way, we can have explanations to long-standing problems of physics such as the huge discrepancy between the electroweak and Planck scales in addition to unification of fundamental forces of nature. Moreover, extra dimensions may provide explanations for internal symmetries and particle properties through extended spacetime symmetries.

In this thesis, we are interested in extra time-like dimensions. First we start with a historical background and review the current status of the extra dimensional models. The extra dimensional space in the great majority of extra dimensional models is spacelike although we also briefly mention few studies that use the time-like dimensions. We give a summary of recent developments related to extra dimensions. Then, we consider the studies involving extra time-like dimensions and mention some phenomenological difficulties related to their use. Finally, we find that these difficulties are more moderate than it was claimed in literature.

In chapter 2, we consider the historical background of extra dimensions and the Kaluza-Klein theory, which is the first rigorous model that unified electromagnetism with gravity by introducing a fifth dimension. In this context, the gauge symmetry is explained as a geometric symmetry of the five dimensional spacetime. Also we indicate how these ideas can be used for quantization of the electric charge and an estimate on the scale of this type of extra dimensions

In chapter 3, we consider some recent popular extra dimensional models. We classify these models into two categories; Arkani-Hamed, Dvali, Dimopoulos (ADD) model of large extra dimensions, and the Randall-Sundrum models. We mention their basic features such as the structure of metric, compactification mechanism of extra dimensions and their physical significance. We mention possible modifications due to replacement of the extra space-like dimension with a time-like extra dimension as well. Finally we examine what happens if we take two of dimensions are time-like instead of one.

In chapter 4, we concentrate on the phenomenological difficulties of models involving extra time-like dimensions. We mention the tachyonic modes due to extra time-like dimensions and some properties of these modes. Finally we study the spontaneous decay of proton and the imaginary contribution to fermion self-energies through tachyons induced by extra time-like dimensions in detail. Finally we discuss some ideas or methods to get rid of or to moderate these difficulties. We also find that a careful analysis of the related Feynman diagrams shows that these phenomenological problems are less severe than it was discussed in literature.

CHAPTER 2

KALUZA-KLEIN THEORY

2.1. Historical Background

The concept of dimension goes back to ancient times. It was first introduced by Aristothales. He wrote in "On Heaven" that "The line has magnitude in one way, the plane in two ways and the solid in three ways and beyond these there is no other magnitude because the three all". In modern language, a line is a one dimensional space, a plane is two dimensional and so on. Aristotle concluded directly from physical experience that there is no object described by dimensions grater than three. Although such a primitive description of dimension were introduced as early as the 2th Century BC, the main development was due to the cartesian system introduced by Rene Descartes in the 17th Century. This formalism facilitated the description of arbitrary dimensional spaces through the studies of Bernhard Riemann, Charles Hinton and Edwin Abbot "(Erlich 2005)".

All these developments are background for Einstein's special and general relativity theories. In the early 20th Century, the theoretical understanding of physics had been improved drastically, the theory of electromagnetism was completed and well-understood. In the formulation of special relativity, Einstein mixed time and space coordinates "(Einstein 1905)". After Minkowski suggested that the time was the fourth coordinate as $x^0 = ict$, it was seen that Lorentz transformations could be thought as rotations in a four dimensional space "(Erlich 2005, Minkowski 1909)". These coordinates are collectively denoted by x^μ, where μ runs over 0, 1, 2, 3. In this notation, any vector is described by these four coordinates and called a 4-vector and may be represented by

$$(x^0, x^1, x^2, x^3) = (ict, x, y, z) \tag{2.1}$$

After Einstein showed that Maxwell equations were invariant under the principles of Special relativity in "(Einstein 1905)", it was seen that electromagnetic theory could be understood geometrically. In this context, Maxwell field can be thought to have four components, one of which is the scalar potential and the others are the vector potentials, and denoted by A_μ similar to the position vector in spacetime.

In 1916, Einstein had, using Riemannian differential geometry, just completed the theory of general relativity which has given a relativistic and geometric description of gravity. In the general relativity, gravitational field can be represented by a second rank tensor, $g_{\mu\nu}$, which appears in the expression of the infinitesimal distance between two points in four spacetime in the following way

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu \tag{2.2}$$

where the summation convention (that is, the convention where there is a sum over repeated indices) has been used and also note that the metric itself is a function of the coordinates.

Maxwell's unified theory of electromagnetism and Einstein's theory of special and general relativity inspired works of unification of electromagnetism and gravity. Nordström proposed that the electromagnetism and gravity could be unified in four dimensional spacetime as early as 1914 "(Nordström 1914)". However Nordström could not introduce the correct framework since the general theory of relativity did not exist at that time. After general relativity was formulated, Weyl and Kaluza tried to unify these two interactions by using Einstein's tensor potential but following two distinct methods. While Weyl tried to alter the geometry "(Weyl 1918)", Kaluza kept the Riemannian geometry but extended the number of dimensions to five "(Kaluza 1921, Häggblad 2003)".

In this chapter, we consider the Kaluza's idea which is called Kaluza-Klein Theory after some corrections of Klein "(Klein 1926)".

2.2. Kaluza-Klein Theory: Electromagnetism Through Five Dimensional Gravity

Kaluza's original idea was based on Einstein's general relativity in five dimensional spacetime. The aim of this entirely classical theory is to derive both Maxwell's equations of electromagnetic theory and Einstein's general relativity theory from a single field and so unify electromagnetism with gravity by using Einstein's vacuum gravity in five dimensions "(Klein 1926)". Kaluza introduced "cylinder" condition to be able to explain why there were no evidence for the extra dimension, that is, all partial derivatives with respect to the fifth dimension are zero "(Kaluza 1921)".

Although Kaluza's idea was attractive, it suffered from two obvious drawbacks. First, there is no obvious reason for the dependence of the fields on the extra coordinate θ. Secondly, if there is a fifth dimension why we have not seen it? The resolution of both problems was supplied by Oscar Klein in 1926 "(Klein 1926)". He show that Kaluza's cylinder condition would arise naturally if the fifth dimension has a circular topology so that the fifth coordinate is periodic $0 \leq \theta \leq 2\pi$.

Five dimensional Klauza-Klein theory unifies electromagnetism with gravity by using Einstein's theory of gravity in five dimensions. Thus the initial theory has five-dimensional general coordinate invariance. However, it is assumed that one of the spatial dimensions compactifies so as to have the geometry of a circle S^1 of very small radius. Then, there is a residual four-dimensional general coordinate invariance. In other words, the original five-dimensional general coordinate invariance is spontaneously broken in the ground state. In this way, we arrive at an ordinary theory of gravity in four dimensions, together with a theory of an Abelian gauge field, with connections between the parameters of the two theories because they both are derived from the same initial five-dimensional theory.

The general form of the metric tensor is "(Witten 1981)":

$$\overline{g}_{AB} = \begin{pmatrix} g_{\alpha\beta}(x,\theta) - B_\alpha(x,\theta)B_\beta(x,\theta)\Phi(x,\theta) & -B_\alpha(x,\theta)\Phi(x,\theta) \\ -B_\beta(x,\theta)\Phi(x,\theta) & -\Phi(x,\theta) \end{pmatrix} \tag{2.3}$$

where the capital letters $A, B\ldots$ run over $0, 1, 2, 3, 5$

To extract the graviton and the Abelian gauge field A_α it is necessary to replace $\Phi(x,\theta)$ by its ground-state value $\tilde{g}_{55}$. After using the θ independence of the fields (i.e the cylinder condition) one obtains "(Overduin and Wesson 1997)"

$$\overline{g}_{AB}(x) = \begin{pmatrix} g_{\alpha\beta}(x) - \xi^2 A_\alpha(x)A_\beta(x)\tilde{g}_{55} & \xi A_\alpha(x)\tilde{g}_{55} \\ \xi A_\beta(x)\tilde{g}_{55} & -\tilde{g}_{55} \end{pmatrix} \tag{2.4}$$

and its inverse

$$\overline{g}^{AB}(x) = \begin{pmatrix} g^{\alpha\beta} & -\xi\, A_\beta \\ -\xi\, A_\alpha & -\tilde{g}_{55}^{-1} + \xi^2 A_\mu A^\mu \end{pmatrix} \tag{2.5}$$

The action written in this theory has the same form as the Einstein-Hilbert action.

$$\overline{S} = -\frac{1}{16\pi\overline{G}} \int d^5x \sqrt{-\overline{g}}\ \overline{R} \tag{2.6}$$

where; $\overline{G}$ is the five dimensional gravitational constant, $\overline{R}$ is five dimensional scalar curvature defined as $\overline{R} = \overline{g}^{AB}\overline{R}_{AB}$ and $\overline{R}_{AB}$ is the five dimensional Ricci tensor. The capital letters A, B run over 0, 1, 2, 3, 5 as follows;

$$(x^0, x^1, x^2, x^3, x^5) = (t, x, y, z, \theta) \tag{2.7}$$

and we let $g_{\mu\nu} = \eta_{\mu\nu}$ where $\eta_{\mu\nu} = diag(+ - - -)$

We obtain the five-dimensional Einstein equations in vacuum by the variation of Eq(2.6), which have the same form as the four-dimensional Einstein equations.

$$G_{AB} = \overline{R}_{AB} - \frac{1}{2}\overline{g}_{AB}\overline{R} \tag{2.8}$$

Five dimensional Ricci tensor, $\overline{R}_{AB}$ in terms of 5D Christoffel symbols is given by

$$\overline{R}_{AB} = \partial_C\overline{\Gamma}^C_{AB} - \partial_B\overline{\Gamma}^C_{AC} + \overline{\Gamma}^C_{DC}\overline{\Gamma}^D_{AB} - \overline{\Gamma}^C_{DB}\overline{\Gamma}^D_{AC} \tag{2.9}$$

where

$$\overline{\Gamma}^A_{BC} = \frac{1}{2}\overline{g}^{AD}(\partial_B\overline{g}_{CD} + \partial_C\overline{g}_{BD} - \partial_D\overline{g}_{BC}) \tag{2.10}$$

To evaluate Ricci tensor and the scalar curvature, we need the Christoffel symbols.

$$\begin{aligned}
\overline{\Gamma}^\alpha_{\beta\gamma} &= \Gamma^\alpha_{\beta\gamma} + \frac{\tilde{g}_{55}}{2}\xi^2(A_\beta F^\alpha_{\ \gamma} + A_\gamma F^\alpha_{\ \beta}) \\
\overline{\Gamma}^\alpha_{55} &= 0 \\
\overline{\Gamma}^5_{55} &= 0 \\
\overline{\Gamma}^\alpha_{\beta 5} &= \frac{1}{2}\tilde{g}_{55}\xi F^\alpha_{\ \beta} \\
\overline{\Gamma}^5_{5\beta} &= -\frac{1}{2}\tilde{g}_{55}\xi^2 A^\lambda F_{\lambda\beta} \\
\overline{\Gamma}^5_{\alpha\beta} &= \frac{\xi}{2}(\nabla_\beta A_\alpha + \nabla_\alpha A_\beta) - \frac{1}{2}\tilde{g}_{55}\xi^3 A^\lambda(A_\alpha F_{\lambda\beta} + A_\beta F_{\lambda\alpha})
\end{aligned} \tag{2.11}$$

Using these results, we can find the components of the 5D Ricci tensor:

$$\overline{R}_{\alpha\beta} = R_{\alpha\beta} + \frac{\tilde{g}_{55}}{2}\xi^2 F_{\alpha\lambda}F_\beta^{\ \lambda} + \frac{\tilde{g}_{55}}{2}\xi^2 A_\beta(\nabla_\gamma F^\gamma_{\ \alpha}) + \frac{\tilde{g}^2_{55}}{4}\xi^3 A_\alpha A_\beta F^{\mu\nu}F_{\mu\nu}$$

$$\overline{R}_{5\alpha} = \frac{\tilde{g}_{55}}{2}\xi\nabla_\lambda F^\lambda_{\ \alpha} + \frac{\tilde{g}_{55}^2}{4}\xi^3 A_\alpha F^{\mu\nu}F_{\mu\nu} \tag{2.12}$$

$$\overline{R}_{55} = \frac{\tilde{g}_{55}^2}{4}\xi^2 F^{\mu\nu}F_{\mu\nu}$$

Hence, from the definition of the scalar curvature, $\overline{R} = \overline{g}^{AB}\ \overline{R}_{AB}$;

$$\overline{R} = R + \frac{\tilde{g}_{55}}{4}\xi^2 F^{\mu\nu}F_{\mu\nu} \tag{2.13}$$

As we can easily see from the equations for five dimensional Christoffel symbols, the components of Ricci tensor and Ricci scalar curvature, the four dimensional part is separated from the terms arising from the extra dimensional part of the metric. Similarly the determinant of the metric can be written as a multiplication of these two parts as follows;

$$\overline{g} = -\widetilde{g}_{55} . g$$

By writing $\widetilde{r}^2$ instead of $\widetilde{g}_{55}$ and integrating over the fifth dimension, we can rewrite the Einstein-Hilbert action as follows

$$\overline{S} = -\frac{2\pi\widetilde{r}^2}{16\pi\overline{G}}\int d^4x\sqrt{-g}R - \frac{2\pi\xi^2\widetilde{r}^2}{16\pi\overline{G}}\int d^4x\sqrt{-g}\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{2.14}$$

where the firs part should give the same equations as Einstein's equations and the second part should give Maxwell's equations. From this equivalence, we can write following equations.

$$G = \frac{\overline{G}}{2\pi\widetilde{r}^2}; \qquad \xi^2 = 16\pi G \tag{2.15}$$

Hence we have obtained the four dimensional gravitational constant G and the standard normalization for the gauge field.

2.3. Abelian Gauge Symmetry in Five Dimensions

The key idea how extra dimensions lead to unification of electromagnetism with gravity lies in the fact that the coordinate transformations associated with the fifth dimension can be interpreted as gauge transformations. Consider the following transformation:

$$\theta \rightarrow \theta' = \theta + \xi\varepsilon(x) \tag{2.16}$$

Under general coordinate transformations, the metric transforms as follows;

$$g_{AB} = g_{A'B'} \frac{\partial x^{A'}}{\partial x^{A}} \frac{\partial x^{B'}}{\partial x^{B}} \tag{2.17}$$

Hence under the coordinate transformation given in Eq(2.16), the off-diagonal elements of the metric become;

$$g_{\mu 5} = \xi A_{\mu} \widetilde{g}_{55} \tag{2.18}$$

$$\xi A_{\mu} \tilde{g}_{55} = g_{\mu' 5'} \frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{5'}}{\partial x^{5}} \tag{2.19}$$

$$\xi A_{\mu} \widetilde{g}_{55} = \xi A'_{\mu} \widetilde{g}_{55} - \xi \widetilde{g}_{55} \partial_{\mu} \varepsilon(x) \tag{2.20}$$

Therefore the transformation given in Eq(2.20) associated with the fifth transformation induces an Abelian gauge transformation on A_{μ}

$$A'_{\mu} = A_{\mu} + \partial_{\mu} \varepsilon(x) \tag{2.21}$$

This means that the extra dimension provides an internal dimension for Abelian gauge symmetry and it has to be interpreted as just another spacetime symmetry, but associated with the extra dimension.

2.4. Klein's Idea, Kaluza-Klein Tower, Charge Quantization

In this section we consider the Klein's idea of deriving the Kaluza's cylinder condition hypothesis in a theoretically satisfactory way "(Klein 1926)". Klein brought the theory into the quantum mechanics realms and also gave an explanation to Kaluza's cylinder condition. Klein assumed the fifth dimension to have a circular form with much smaller radius than the observable distance scale. This would mean the cylinder condition would arise naturally and because of the infinitesimal scale, the fifth dimension could not be tested in current experiments "(Häggblad 2003)". Also because of the periodicity, fields could be Fourier expanded in the fifth dimension as follows.

$$g_{\alpha\beta}(x,\theta) = \sum_{-\infty}^{+\infty} g_{\alpha\beta}^{(n)}(x) e^{in\theta/r} \tag{2.22}$$

$$A_{\alpha}(x,\theta) = \sum_{-\infty}^{+\infty} A^{(n)}(x) e^{in\theta/r} \tag{2.23}$$

$$\phi(x,\theta) = \sum_{-\infty}^{+\infty} \phi^{(n)}(x) e^{in\theta/r} \tag{2.24}$$

where the superscript (n) refers to the nth Fourier mode.

We know from the quantum theory that the all Fourier modes carry a momentum of the order of n/r and the sufficiently small radius can explain why the momenta of all modes, except $n = 0$, will be so large that they can not be detected in the current experiments.

Consider the equation of motion for a massless scalar field ϕ in a five dimensional space. The equation of motion for such a field is simply Klein-Gordon equation in five dimensions with $m = 0$.

$$(\Box_x - \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2})\phi(x,\theta) = 0 \tag{2.25}$$

where $\Box_x = \partial_\mu \partial^\mu$ is the four dimensional D'alambert operator. If we substitute the Fourier expansion of $\phi(x,\theta)$ in to the equation of motion, the n we obtain the following equation for the Fourier modes of $\phi^{(n)}(x)$;

$$(\Box_x - \frac{n^2}{r^2})\phi^{(n)}(x) = 0 \tag{2.26}$$

We may take consider $m_n^2 = n^2/r^2$, so that we can regard a massless state in the higher dimensional theory as a massive state in the lower dimensional theory. Here the fields $\phi^{(n)}(x)$, $n > 0$, are massive and only $\phi^{(0)}(x)$ is massless. The mass of $\phi^{(n)}$, $n > 1$ at the order of r^{-1} and we may take it be comparable to the Planck mass.

Moreover, the expansion of the fields into the Fourier modes suggests a possible mechanism to explain the quantization of charge. If we apply the coordinate transformation given in Eq.(2.16) to the $\phi(x,\theta)$, we get

$$\phi^{(n)}(x) \rightarrow e^{in\xi\varepsilon(x)/r}\phi^{(n)}(x) \tag{2.27}$$

The requirement of the covariance of the covariant derivative $\partial_\alpha \to \partial_\alpha + iq_n A_\alpha$, under the $\mathcal{U}(1)$ transformations, (2.21) and (2.27) leads to the following identification

$$q_n = -\frac{n\kappa}{r} = ne, \quad e = \frac{\kappa}{r} \tag{2.28}$$

Thus the charge has been found as quantized in the units of κ/r and allows us to estimate the scale of the radius of the extra dimension.

$$r^2 = \frac{\kappa^2}{e^2} = \frac{16\pi G}{e^2} \tag{2.29}$$

After identifying "e" with the quantum of the electric charge, one finds $r \approx 10^{-33} m$.

2.5. (4+D) Dimensional Kaluza-Klein Theory

After the unification of electromagnetism with the gravity by introducing a fifth dimension, the next question is obviously whether the other forces can be unified with gravity and electromagnetism by the same method. The key to extend the Kaluza-Klein theory to strong and weak nuclear interactions lies in recognizing that electromagnetism has been effectively incorporated into general relativity by adding $U(1)$ local gauge invariance to the theory, in the form of local coordinate invariance with respect to the fifth dimension, θ. To extend the same approach to more complicated symmetry groups, one goes to higher dimensions "(De Witt 1963, Kerner 1968, Trautman 1970, Cho and Freund 1975, Cho 1975)".

The ansatz which is the generalization of (2.4) is the following. Let ϕ_i, ($i = 1, \dots n$) be coordinates for the internal space B, T^a, ($a = 1, \dots N$), be the generators of the symmetry group G of B and the action of the symmetry generator T^a on the ϕ_i be $\phi_i \to \phi_i + \mathbf{K}_i^a(\phi)$, where $\mathbf{K}(\phi)$ is the Killing vector associated with the symmetry T^a. Then the candidate ground-state $M_4 \times B$ with massless gauge fields correspond to ansatz of the following form "(Appelquist *et al.* 1987)";

$$\overline{g}_{AB} = \begin{pmatrix} g_{\mu\nu}(x) & \sum_a A_\mu^a(x) K_i^a(\theta^k) \\ \sum_a A_\mu^a(x) K_i^a(\theta^k) & \gamma_{ij}(\theta^k) \end{pmatrix} \tag{2.30}$$

where γ_{ij} is the metric tensor of the internal space B, the fields $A_\mu^a(x)$ are massless gauge fields of the group G. In this way one may obtain the gauge fields of an arbitrary Abelian or non-Abelian gauge group as components of the gravitational field in $4+n$ dimensions.

One may verify that $4 + n$ dimensional gravitational action really contains the proper kinetic energy term $\sum_a (F^a_{AB})^2$, where $F^a_{AB} = \partial_A A^a_B - \partial_B A^a_A + [A_A, A_B]$ and $A^a_B(x, y) = A^i_B(x) K^i_a(y)$.

An isometry of B is a coordinate transformation $\theta \to \theta'$ which leaves the form of the metric $\gamma_{ij}(\theta)$ for K invariant:

$$\theta \to \theta' \;:\qquad \gamma'_{ij}(\theta') = \gamma_{ij}(\theta') \tag{2.31}$$

Isometries can form a group with generators t_a and structure constant C_{abc}, in the following way. The general infinitesimal isometry is

$$I + i\varepsilon^a t_a \;:\qquad \theta'^n = \theta^n + \varepsilon^a \kappa^n_a(\theta) \tag{2.32}$$

where the infinitesimal parameters ε^a are independent of θ and Killing vectors ξ^n_a, which are associated with the independent infinitesimal isometries, obey the algebra

$$\kappa^m_b{}_m \kappa^n_c - \kappa^m_c \partial_m \kappa^n_b = -C_{abc}\kappa^n_a \tag{2.33}$$

One may consider isometries that satisfy a Lie algebra given by

$$[t_a, t_b] = iC_{abc}t_c \tag{2.34}$$

The N dimensional sphere S^N has isometry group $\mathcal{SO}(N+1)$, and the $2N$ dimensional complex projective plane $\mathcal{CP}^N$ has isometry group $\mathcal{SU}(N+1)$. Hence, it is possible to choose a compact manifold to obtain the isometry group $\mathcal{SU}(3)\times\mathcal{SU}(2)\times\mathcal{U}(1)$, which is the gauge group of electroweak and strong interactions.

If we take the parameter $\epsilon^a(x)$ in (2.33) as a function of x, under non-Abelian gauge transformations;

$$\theta^n \to \theta^n + \kappa^n_a(\theta)\varepsilon^a(x) \tag{2.35}$$

$$A^a_\mu \to A^{a'}_\mu = A^a_\mu + \partial_\mu \varepsilon^a(x) + C_{abc}\varepsilon^b A^c_\mu \tag{2.36}$$

which is just usual Yang-Mills transformation if we display the gauge coupling constant g explicitly by writing

$$C_{abc} = g f_{abc} \tag{2.37}$$

and

$$t_a = gT_a \tag{2.38}$$

so that

$$[T_a, T_b] = if_{abc}T_c \tag{2.39}$$

Thus, non-Abelian gauge transformations are generated by x-dependent infinitesimal isometries of the compact manifold B

2.6. Kaluza-Klein Modes of Photon in Extra Space-like and Time-like Dimensions

Consider the action given in (2.31), the part that describes the photon field;

$$S_{photon} = \frac{e^2}{8\pi} \int d^4x \int d^Dy \sqrt{-g}\, F_{AB}(x,y)F^{AB}(x,y) \tag{2.40}$$

If we take the variation of the action with respect to the coordinates, we get Maxwell's equation in (4+D)-dimensions.

$$\partial_A F^{aAB} = 0 \tag{2.41}$$

If we expand the photon field into Kaluza-Klein modes

$$A_\mu(x,y) = \sum_n A_\mu^{(n)}(x) e^{n_m y^m / L} \tag{2.42}$$

Hence the Maxwell's equation in Lorentz gauge($\partial_A A^A$) can be rewritten as follows;

$$(\Box_x \pm m^2)A^\nu(x) = 0 \tag{2.43}$$

where $m = \sqrt{n^2/L^2}$, $n^2 = n_1^2 + n_2^2 + \ldots$,

The contribution of the extra dimensions to the four dimensional effective equations appear as the mass term as expected. In these equation, the mass terms can be both negative or positive. The sign of the mass term is directly related to whether extra dimensions are time-like or spacelike . If the extra dimensions are spacelike, then the mass has positive sign for the sign of metric $\eta_{\mu\nu} = (+ - - - \ldots)$. Otherwise if the extra dimensions are time-like, then the mass term has negative sign and the KK modes of the photon appear in the equations. Such modes with negative mass are called tachyons and we deal with tachyons in details later.

The Kaluza-Klein picture is purely geometric. Kaluza showed that we can start from a field without matter in five dimensions with some additional part in the metric. The additional part turns out to give Maxwell electromagnetic theory and a scalar field called dilaton field. In such a theory, the photon is only a component of 5D graviton. However, even though Kaluza-Klein theory succeeded to unify electromagnetic theory with gravity, the presence of the scalar field was seen as a problem. Also the theory fails to explain many open questions e.g. why gravity is much weaker than electromagnetism and why the extra dimension must be very small. Despite these open questions in the theory, it inspired many higher dimensional theories, which aim to give probable explanations to some of these open questions "(Sabbata and Schmutzer)". Such models are considered in the next chapter.

CHAPTER 3

SOME RECENT EXTRA DIMENSIONAL MODELS

In the last chapter, we have considered the Kaluza-Klein model for unifying electromagnetism and other gauge interactions with gravity by introducing extra dimensions. Kaluza-Klein approach inspired many extra dimensional models to unify all fundamental interactions (i.e. strong, weak, electromagnetism and gravity). On the other hand recent extra dimensional models mainly try to solve some long-standing problems such as hierarchy between the strength of electromagnetism and gravity,the solution of cosmological constant problem etc. in addition to the unification program may be generated by the geometry of the additional dimensions. In these models, either the extra dimensions are relatively large and the physical particles are localized on a brane (which is a 4-dimensional subspace of the higher dimensional space), or the extra dimensions are small. In the first case, Arkani-Hamed, Dimopoulos and Dvali (ADD-Model) used extra dimensions to generate the hierarchy between the strength of gravity and the other interactions by postulating large volume for extra dimensional space "(Antoniadis *et al.* 1998, Arkani-Hamed *et al.* 1999)". In the second case, Randall and Sundrum tried to olve the hierarchy problem by the large curvature effect of the extra dimension "(Randall and Sundrum 1999)".

Most of the proposed scenarios may be technically divided into two categories. In some models, the metric is in the factorized form(e.g. ADD-Model), where 4D and extra dimensional pieces of the metric are independent. In this case, the metric can be written in the form "(Hewett and Marchal-Russel 2002)";

$$ds^2 = G_{MN}dx^M dx^N = g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(y)dy^i dy^j \tag{3.1}$$

where $M, N = (0, \ldots, 3+\delta)$; $\mu, \nu = (0, 1, 2, 3)$ and $i, j = (4, \ldots, 3+\delta)$; δ is the number of the extra dimensions. In other class of models (Randall-Sundrum Models), the metric is in a non-factorizable form

$$ds^2 = \Lambda(y)g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(y)dy^i dy^j$$

In the next section, we consider the approach of Arkani-Hamed, Dimopoulos, Dvali (ADD-Model) and then in 3.3 we consider Randall, Sundrum (RS-Models)

3.1. ADD Model

First we consider ADD-Model proposed by N. Arkani-Hamed, S. Dimopoulos and G. Dvali. This model includes the SM fields localized on a 3-brane embedded in the $(4 + \delta)$ dimensional spacetime with compact extra dimensions. In this model, the effect of gravity may be neglected. On standard model particles at scales comparable to the size of extra dimensions r since r is not so small; i.e much larger than the scale of SM. While the physics at ordinary scales becomes four dimensional, gravity becomes multi-dimensional at scales just below r. The four dimensional law of gravitational attraction has been established experimentally down to distance about $0.02mm$, so the size of extra dimensions is allowed to be approximately as large as $0.02mm$ "(Hoyle *et al.* 2001)"

This possibility opens up a new way to address the hierarchy problem between the gravity and electroweak scales. In multi-dimensional theories, the four dimensional Planck scale is not a fundamental parameter. Rather the mass scale of multi-dimensional gravity, which we simply denote by M, is fundamental. The full multi-dimensional gravitational action is

$$S = -\frac{1}{16\pi G_{(D)}} \int d^D x \sqrt{g^{(D)}} R^{(D)} \tag{3.2}$$

where $G_{(D)} = 1/M^{D-2} \equiv 1/M^{\delta+2}$, is the fundamental Newton constant, $\delta = D - 4$ is the number of the extra dimensions and $d^D x = d^4 x d^\delta z$, z stands for extra coordinates.

In ADD picture, the long distance four dimensional gravity is mediated by the graviton zero modes whose wave function is constant over extra dimensions. Hence the four dimensional effective action describing the long distance gravity is obtained from Eq(3.2) by taking the metric in the factorized form given by Eq(3.1). In this case, the integration over z is trivial and simply equals to the volume of the extra dimensional space.

$$S_{eff} = -\frac{V_\delta}{16\pi G_{(D)}} \int d^4 x \sqrt{g^{(4)}} R^{(4)} \tag{3.3}$$

where $V_\delta \sim r^\delta$ is the volume of the extra dimensional space. Hence;

$$\frac{1}{16\pi G_{(4)}} = \frac{V_\delta}{16\pi G_{(D)}} \tag{3.4}$$

$$M_{Pl}^2 = r^\delta M^{\delta+2} \tag{3.5}$$

where we have assumed for simplicity that all extra dimensions have the same sizes. Since the size of the extra dimensions is large compared to the fundamental length M^{-1} in this model, the Planck mass is much larger than the fundamental gravity scale M. From Eq(3.5);

$$r \sim M^{-1}(\frac{M_{Pl}}{M})^{\frac{2}{\delta}} \tag{3.6}$$

If $M \sim 1TeV$, then the hierarchy between M_{Pl} and M_{EW} is entirely due to the large size of the extra dimensions. If we identify M by M_{EM}, the hierarchy problem becomes now the problem of explaining why r is large. In the case $M \sim 1TeV$; $r \sim 10^{\frac{32}{\delta}}10^{-17}m$.

Let us analyze various case. In the case $\delta = 1$, $r \sim 10^{13}cm$. In this case, the size of extra dimensions is of the order of the solar distance. This case is obviously excluded.

for $\delta = 2$; $r \sim 0.1mm$, $1/r \sim 10^{-3}eV$

for $\delta = 3$; $r \sim 10^{-7}cm$, $1/r \sim 100eV$

$\ldots\ldots\ldots\ldots\ldots$

for $\delta = 6$; $r \sim 10^{-12}cm$, $1/r \sim 10MeV$

Experimental data coming from astrophysics and cosmology "(Arkani-Hamed *et al.* 1998, Long *et al.* 2002, Beane 1997)" show that the mass scale $M \sim 1TeV$ excludes the case $\delta = 2$. A more realistic value $M \sim 30TeV$ implies $r \sim 1 - 10\mu m$. This motivates search for derivations from Newton's law in a micrometer range, which is difficult but not impossible and that is the main reason why extra dimensions become important. Search for violation of Newton's law at the scales given for $M \sim 1TeV$ and $\delta = 2 - 6$ appear hopeless. For $\delta = 6$ (full dimensionality of space time $D = 10$, as suggested by superstring theory), the value of $r \sim 10^{-12}cm$ is still much larger than the electroweak scale. (Note that these values have been calculated in the case the extra dimensions have the same size. If they have different size, all result are changed).

On the other hand, SM gauge interactions have been accurately measured already at the scale $\sim 100GeV$ and no deviations due to extra dimensions are observed. Hence for the model to be consistent, the fields of SM must be localized on a 3-brane (a four dimensional subspace embedded in higher dimensions with an infinitesimal width in extra directions), in other words with the energy-momentum tensor

$$\hat{T}_{MN}(x,z) = \delta^{\mu}_{M}\delta^{\nu}_{N}T_{\mu\nu}(x)\delta(z) \tag{3.7}$$

According to this model, only gravity can propagate in extra dimensional space. This assumption gives us some idea on why gravity is very weak.

Moreover in this model, The Kaluza-Klein picture can be introduced by considering the Fourier expansions of graviton fields in the compact extra dimensions. The metric can be expanded around the $(4+\delta)$ dimensional Minkowski background as follows;

$$G_{MN} = \eta_{MN} + \frac{2}{M^{1+\delta/2}}h_{MN}(x,z) \tag{3.8}$$

where h_{MN} may be decomposed in a Fourier series (i.e. Kaluza-Klein tower). Each n stands for a graviton mode.

$$h_{MN}(x,z) = \sum_{n} h^{(n)}_{MN}(x)\frac{1}{V_d}e^{-in_m z^m} \tag{3.9}$$

The mass of KK mode gravitons can be obtained by using linearized Einstein equations. Ricci tensor and the scalar curvature in linearized approximation are

$$\begin{aligned} R_{AB} &= \tfrac{1}{2}[\partial_A\partial^C h_{BC} + \partial_B\partial^C h_{AC} - \Box h_{AB} - \partial_A\partial_B h] \\ R &= \partial^A\partial^B h_{AB} - \Box h \end{aligned} \tag{3.10}$$

The corresponding Einstein equation is; $G_{AB} = R_{AB} - 1/2 g_{AB}R$. If we substitute (3.10)into the Einstein equation with $\partial^A h_{AB} = 0$ and $h^A_A = 0$, we obtain an equation for $h_{AB}(x,y)$ that $\Box h_{AB}(x,y) = 0$, where $\Box$ is n-dimensional D'lambert equation. By using Eq(3.9), the mass of KK graviton modes are derived as follows.

$$m_n = \sqrt{n_1^2 + n_2^2 + \ldots + n_n^2} \equiv \frac{|n|}{r} \tag{3.11}$$

The masses of KK modes are given by Eq(3.11) so that the mass splitting of spectrum $\triangle m \propto 1/R$. We see that the KK modes of gravitons behave like massive gravitons. A possible collider signature is an imbalance in the final state momenta and missing mass. Since the mass splitting has the form $\triangle m \propto 1/R$, the inclusive cross-section reflects an almost continuous distribution in mass. This characteristic feature may be enable to distinguish its predictions from high energy phenomenology ”(Csàki 2004)”.

3.2. Randall-Sundrum Models

The discussion in the previous section assumes that the extra dimensions are compact and flat or at least weakly curved. Another possibility is to take the extra dimensions be strongly curved by a large cosmological constant and the metric does not have a factorized form. In other words, the factorized geometry is replaced by the warped geometry. The solutions are done for five dimensional background metric obtained by Lisa Randall and Raman Sundrum at first "(Randall and Sundrum 1999)". The form of the metric is;

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{3.12}$$

where y is the extra dimension and a is any function of y.

Moreover the circular form of the extra dimensions is replaced by an interval in this model. The extra dimension is compactified on an orbifold S^1/Z_2, where Z_2 is defined by the transformation $y \to -y$. An orbifold is defined as the quotient space $\Gamma \equiv M/G$, where M is some manifold and G is a discrete group acting on M "(Sundrum 2005)" (for detailed discussion see "(Nakahara 2003)").

In this way the physical interval extends a length πr. Also the periodicity of $\phi = \phi + 2\pi$ is not dropped out where $y = r\phi$. Moreover there are two branes localized on the orbifold in R-S models. These models can be divide into two classes that will be discussed in the next section.

3.2.1. RS1 Model

The first model proposed by Randall and Sundrum provides a novel and interesting solution to the hierarchy problem. There are two 3-branes located at the fixed points $y = 0$ and $y = \pi r$ of the orbifold S^1/Z_2, where r is the radius of the circle S^1. The brane at $y = 0$ is usually referred to as brane 1 and the brane at $y = \pi r$ is called brane 2 "(Randall and Sundrum 1999)"

Let $\hat{G}_{MN}(x, y)$ be the metric tensor of the multidimensional gravity. Then $g^{(1)}_{\mu\nu}(x) = \hat{G}_{\mu\nu}(x, 0)$ and $g^{(2)}_{\mu\nu}(x) = \hat{G}_{\mu\nu}(x, \pi r)$ describe the metrics induced on the brane

1 and brane 2 respectively. The action of the model is given by

$$S = \int d^4x \int dy \sqrt{-\hat{G}}(2M^3 R^{(5)} + \Lambda) + \int d^4x \sqrt{-g^{(1)}}(L_1 - \tau_1) + \int d^4x \sqrt{-g^{(2)}}(L_2 - \tau_2) \tag{3.13}$$

where $R^{(5)}$ is five dimensional scalar curvature depending on the metric $\hat{G}_{MN}$, M is a mass scale (the five dimensional Planck mass) and Λ is the bulk cosmological constant. L_j are the matter Lagrangians and τ_j are the constant vacuum energy on brane j ($j = 1, 2$).

A background metric solution satisfies the Einstein equation;

$$\sqrt{-\hat{G}}[R^{(5)}_{MN} - \frac{1}{2}\hat{G}_{MN}R^{(5)}] = \frac{1}{2M^3}\Lambda\sqrt{-\hat{G}}\hat{G}_{MN} - \frac{1}{2M^3}T_{MN} \tag{3.14}$$

The RS background solution that describes the spacetime with non-factorizable geometry is given by

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{3.15}$$

$$y = r\phi, \qquad dy = rd\phi$$

one finds;

$$R_{\mu\nu} = \frac{1}{r^2}(\sigma'' - \sigma'^2)g_{\mu\nu} \tag{3.16}$$

$$R_{\phi\phi} = 4(\sigma'' - \sigma'^2) \tag{3.17}$$

$$R = \frac{1}{r^2}(8\sigma'' - 20\sigma'^2) \tag{3.18}$$

If the contributions of matter are neglected (i.e. $L_1 = L_2 = 0$), the energy momentum tensor T_{MN} is determined by the vacuum energy terms.

$$T_{MN} = \tau_1\sqrt{-g^{(1)}}g^{(1)}_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(y) + \tau_2\sqrt{-g^{(2)}}g^{(2)}_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(y - \pi R) \tag{3.19}$$

Substituting this ansatz into the equation of motion yields the following equation

$$6\sigma'^2 = -\frac{\Lambda}{4M^3} \tag{3.20}$$

$$3\sigma'' = \frac{\tau_1}{4M^3}\delta(y) + \frac{\tau_2}{4M^3}\delta(y - \pi r) \tag{3.21}$$

In the interval $-\pi r < y \leq \pi r$, the function $\sigma(y)$ in the warp factor $\exp(-2\sigma)$ is equal to

$$\sigma(y) = k|y| \tag{3.22}$$

where $k > 0$.

Also the parameters must be fine-tuned to satisfy the relations;

$$\tau_1 = -\tau_2 = 24M^3k \tag{3.23}$$

$$\Lambda = 24M^3k^2 \tag{3.24}$$

The equations (3.23) and (3.24) give rise to the constraints on the parameters of the model. These constraints can be thought of as fine-tuning conditions for vanishing effective cosmological constant in four dimensions and this is equivalent to the usual cosmological constant problem. Here k is a dimensional parameter which was introduced for convenience. This fine-tuning is equivalent to the usual cosmological constant problem. If $k > 0$, then the tension on brane 1 is positive, whereas the tension τ_2 on the brane 2 is negative. Hence the background solution is found as follows

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{3.25}$$

The brane 2 is the most interesting from the point of view of the high energy phenomenology. because of the non-trivial suppression factor $e^{-2\sigma(\pi r)}$ on the brane 2. To have the correct interpretation of the effective theory on brane 2, one introduces the Galilean coordinates; $z^\mu = e^{-\pi k r}x^\mu$. Correspondingly, the gravitational field and the energy-momentum tensor should be rewritten in these coordinates.

$$S = \int d^4z e^{4\pi k r}\int dy e^{-4k|y|}\{2M^3e^{-2k(\pi r-|y|)}R^{(4)} + ...\} \tag{3.26}$$

Identifying the coefficient which multiply the four dimensional scalar curvature with M_{Pl}^2, one gets

$$M_{Pl}^2 = e^{2k\pi r}\int_{-\pi r}^{\pi r} dy e^{-2k|y|} = \frac{M^3}{k}(e^{2k\pi r} - 1) \tag{3.27}$$

Approximately;

$$M_{Pl}^2 \approx \frac{M^3}{k}e^{2k\pi r} \tag{3.28}$$

The result in Eq(3.27) is completely general. Any mass parameter on the visible 3-brane in the fundamental higher dimensional theory corresponds to a physical mass. If the quantity kr is chosen to be order of 10, the exponential in Eq(3.22) takes the Planck size input masses to effective masses of the order of a TeV. Hence, in the above model, Planck scale has been obtained from the TeV scale from the without introducing large numbers, provided we live on the brane 2. Some problems with RS1 model are that the fine-tuning between the weak scale and the Planck scale is replaced by the fine-tuning between k and the brane separation r.

To make clearer how the RS models solve the hierarchy problem, we need to study on the properties of gravity in the background given Eq(3.25). First we will focus exclusively on the graviton modes setting the scalar fluctuation to zero and then later we will discuss the relevance of the scalar mode.

In order to find the KK expansion of the graviton modes, we will go to the conformal frame for the metric and parametrize the graviton fluctuation by

$$ds^2 = e^{-A(z)}[(\eta_{\mu\nu} + h_{\mu\nu}(x,z))dx^\mu dx^\nu + dz^2] \tag{3.29}$$

Hence we introduce a new frame where the metric is a conformal form. The relation between y and z can be defined by changing of variable such that $e^{-A(z)}dz = dy$. In the case of conformal metric represented as $\tilde{g} = e^{-A(z)}g$,

$$\tilde{R}_{MN} = R_{MN} + \frac{D-2}{2}[\nabla_M\nabla_N A + \frac{1}{2}\nabla_M A\nabla_N A + \frac{1}{2}g_{MN}\nabla_R A\nabla^R A + g_{MN}\nabla_R\nabla^R\omega]$$

$$\tilde{R} = e^A\{R + (D-1)\nabla_R\nabla^R A - \frac{(D-2)(D-1)}{4}\nabla_R A\nabla^R A$$

Hence the Einstein tensors of two frames can be related to each other as follows;

$$\begin{aligned}\tilde{G}_{MN} = G_{MN} + \frac{D-2}{2}[\frac{1}{2}\nabla_M A\nabla_N A + \nabla_M\nabla_N A \\ -g_{MN}(\nabla_R\nabla^R A - \frac{D-3}{4}\nabla_R A\nabla^R A)]\end{aligned} \tag{3.30}$$

where Eq(3.30) has been written for any function A, in arbitrary D dimensional space.

Next we impose the gauge choice for the perturbations $h^M_M = \partial_M h^M_N = 0$ which is usually called the RS gauge choice.

Although g is the fluctuation of a flat metric, since covariant derivatives ∇ is evaluated with respect to the perturbed metric $\eta_{MN} + h_{MN}$ the Christoffel symbols do not all vanish. Eq(3.30) gives us a correspondence between two frames. G_{MN} in the right hand side of Eq(3.30) is the Einstein tensor of the flat background and it is easy to get the fluctuation of it. After putting all extra terms together coming due to the perturbation, we will yield the linearized Einstein equation in a warped background to be;

$$-\frac{1}{2}\partial_R\partial^R h_{MN} + \frac{D-2}{4}\partial^R A \partial_R h_{MN} = 0 \tag{3.31}$$

In our case $D = 5$ and A depends only on the extra dimension, z. If we rewrite Eq(3.32) for our case;

$$-\frac{1}{2}\partial_R\partial^R h_{MN} + \frac{3}{4}\partial_R A \partial^R h_{MN} = 0 \tag{3.32}$$

By rescaling the perturbation as $h_{MN} = e^{\frac{3}{4}A}\tilde{h}_{MN}$

$$-\frac{1}{2}\partial_R\partial^R \tilde{h}_{MN} + [\frac{9}{32}\partial_R A \partial^R A - \frac{3}{8}\partial_R\partial^R A]\tilde{h}_{MN} = 0 \tag{3.33}$$

This field redefinition was chosen to cancel the term including the first order derivation in Eq(3.34), by this redefinition we have only a differential equation including a second derivative (kinetic energy) term and no potential term.

Moreover, we can use separation of variables technique by using $\partial_R\partial^R = \Box_x + \partial_z^2$, where $\Box_x = \eta_{\mu\nu}\partial^\mu\partial^\nu$ is the four dimensional D'lambert operator. Also $\tilde{h}_{MN}(x,z) = \hat{h}_{MN}(x)\Psi(z)$ and requiring that the $\hat{h}$ be a four dimensional mass eigenstate mode $\Box\hat{h}_{MN} = m^2\hat{h}_{MN}$. After separation of variables, we will get the following Schrodinger type equation which Ψ obeys;

$$-\partial_z^2\Psi + (\frac{9}{16}A'^2 - \frac{3}{4}A'')\Psi = m^2\Psi \tag{3.34}$$

Thus the Schrödinger potential is given by

$$V(z) = \frac{9}{16}A'^2 - \frac{3}{4}A'' \tag{3.35}$$

where, $e^{-A} = 1/(k|z|+1)^2$, $A = 2\ln(k|z|+1)$. Hence;

$$\partial_z A = \frac{2k(\Theta(z) - \Theta(-z))}{k|z|+1}$$

$$\partial_z^2 A = \frac{4k(k|z|+1)\delta(z) - 2k^2}{(k|z|+1)^2}$$

where $\Theta(z)$ is heavyside function. Hence the potential is

$$V(z) = \frac{15}{4}\frac{k^2}{(k|z|+1)^2} - \frac{3k\delta(z)}{k|z|+1} \tag{3.36}$$

This potential is commonly referred to as the ”volcano potential” because of its shape. The zero mode solution of Eq(3.35) is;

$$\Psi_0 = e^{-\frac{3}{4}A(z)} = \frac{1}{(k|z|+1)^{\frac{3}{4}}} \tag{3.37}$$

Since we have kept the Lorentz invariance of the action with the RS gauge conditions, we expect a massless 4D graviton to exist. The usual quantum mechanical norm of Ψ

$$\int_0^{z_0} dz|\Psi|^2 = \int_0^{z_0} dz \frac{1}{(k|z|+1)^{\frac{3}{2}}} \tag{3.38}$$

The most important comment about the norm of Ψ is that it is converging in the limit when the size of the extra dimension becomes infinitely large, $z_0 \to \infty$. Since the zero mode graviton has a non-trivial wave function that is peaked around the positive tension brane, one can not find that the zero mode would decouple in the infinitely large extra dimension limit. This implies that gravity itself becomes localized around the positive tension brane in RS model and far away one has only a small tail for the graviton wave function. Since the graviton is confined to the positive tension brane, an observer living on the other end of the interval feels the gravity very weak compared to the other interactions. Hence the solution of the hierarchy problem can be summarized by saying that gravity is so weak compared to particle physics, because we live at a point in extra dimension that is far away from where gravity is localized. Separately, since the interactions amongst particles is localized on the far brane, we feel the fundamental strength of them.

3.2.2. RS2 Model

In this section we send one of the branes in RS1 to infinity so that effectively we have only one brane. The construction the single brane solution is very simple. We do not the impose periodicity condition but still require a Z_2 symmetry under;

$$y \to -y \tag{3.39}$$

If we consider the Schrodinger type equation for Ψ given in Eq(3.35), it can be written for any mode of the graviton as follows;

$$-\Psi_m'' + \frac{\alpha(\alpha+1)}{z^2}\Psi_m = m^2\Psi_m \tag{3.40}$$

where it has been assumed that the volcano potential for large z behaves like $\alpha(\alpha+1)/z^2$ (for RS model, $\alpha = \frac{3}{2}$). Such a differential equation has solutions in terms of the Bessel functions.

$$\Psi_m(z) = \sqrt{|z| + \frac{1}{k}}[a_m Y_2(m(|z| + \frac{1}{k})) + b_m J_2(m(|z| + \frac{1}{k}))] \tag{3.41}$$

where J denotes the first kind of Bessel functions, Y denotes the second kind, a_m and b_m are constants. Because the solution is written as a function of $|z|$, the second derivative of Ψ with respect to z contains a term with $\delta(z)$ (and other terms). We can fix the ratio a_m/b_m by matching the factor in front of $\delta(z)$ in Eq(3.36).

We can consider the Newton's law to test the consistency of the model. We expect the corrections to the Newton's law from gravitons with small m because of carrying interactions over longer distance. Separately, since gravitons are localized around z=0, we can replace the Bessel functions by their asymptotics for small arguments which are;

$$J_2(m(|z| + \frac{1}{k})) \sim \frac{m^2(|z| + \frac{1}{k})^2}{8} \tag{3.42}$$

$$Y_2(m(|z| + \frac{1}{k})) \sim -\frac{4}{\pi m^2(|z| + \frac{1}{k})^2} - \frac{1}{\pi} \tag{3.43}$$

Substituting the asymptotic approximation into Eqs(3.42), we find that overall coefficient in front of $\delta(z)$ vanishes if

$$\frac{a_m}{b_m} = \frac{4k^2}{\pi m^2} \tag{3.44}$$

Hence the general solution is found as;

$$\Psi(m, z) = \sqrt{|z| + \frac{1}{k}} c_2\{Y_2(m(|z| + \frac{1}{k})) + \frac{4k^2}{\pi m^2} J_2(m(|z| + \frac{1}{k}))\} \tag{3.45}$$

Since the extra dimension is not compact, the eigenvalue m is continuous. Therefore we normalize;

$$\int dz \Psi(m, z)\Psi(m', z) = \delta(m - m') \tag{3.46}$$

for $m, m' > 0$. For $m \geq 0$ we impose the normalization condition

$$\int dz \Psi(0, z)\Psi(m, z) = \delta_{m0} \tag{3.47}$$

such that the completeness relation reads

$$\Psi_0(z)\Psi_0(z') + \int_0^\infty dm(m, z)\Psi(m, z') = \delta(z - z') \tag{3.48}$$

We can fix the constant c_2 with the orthonormalization condition. It turns out that the calculation simplifies essentially in the approximation where the arguments of the Bessel functions are large, since the corresponding asymptotics give plane waves. For large mz, the Bessel functions are approximated by

$$\sqrt{z}J_z(mz) \sim \sqrt{\frac{2}{\pi m}}\cos(mz - \frac{5\pi}{4}) \tag{3.49}$$

$$\sqrt{z}Y_2 \sim \sqrt{\frac{2}{\pi m}}\sin(mz - \frac{5\pi}{4}) \tag{3.50}$$

Then the normalization constant is yielded for $m > 0$

$$c_2 \equiv N_m = \frac{\pi m^{\frac{5}{2}}}{4k^2} \tag{3.51}$$

Also for m=0

$$N_0 = \sqrt{k} \tag{3.52}$$

Now if we expand $\tilde{h}(x,z)$ into eigenfunctions $\Psi_0(z)$ and $\Psi_m(z)$ with x depended coefficients $\varphi_m(x)$ as follows

$$\tilde{h}(x,z) = \varphi_0(x)\Psi_0(z) + \int_0^\infty dm\varphi_m(x)\Psi_m(z) \tag{3.53}$$

In the presence of a point particle with mass μ at the origin, the non-relativistic limit of the linearized equation for $\tilde{h}$ reduces to

$$\{\nabla_3^2 - e^{-2k|y|}(\partial_y^2 + 4k\delta(y) - 4k^2)\}\tilde{h}(x,y) = G\mu\delta^3(x)\delta(y) \tag{3.54}$$

where ∇_3^2 is the Laplacian in three dimensions. Substituting the ansatz given in Eq(3.54) into the wave equation for $\tilde{h}$ in Eq(3.55), we find that for $m > 0$ and $r = |x|$;

$$\varphi_m(r) = -\frac{G\mu}{r}e^{-mr}a_m \tag{3.55}$$

where the constants a_m is taken such that

$$a_0\Psi_0(z) + \int dma_m\Psi_m(z) = \delta(z) \tag{3.56}$$

Comparison with the orthonormalization condition for $\Psi_m(z)$

$$a_0 = \Psi_0(0)$$

$$a_m = \Psi_m(z)$$

If we define the four dimensional Newton's constant as $G_4 = Gk$, then we find from Eq(3.55)

$$\tilde{h}(x, 0 \equiv h(x,0) = -\frac{G_4\mu}{r}(1 + \int_0^\infty dm \frac{m}{k^2} e^{-mr}) \tag{3.57}$$

Finally, performing the integral in the last expression, we get

$$h(x,0) = -G_4 \frac{\mu}{r}(1 + \frac{1}{rk^2}) \tag{3.58}$$

Therefore fluctuations around the solution include a state which describes the graviton states. The massless graviton is localized on the brane, hence there is no contradiction with the Newton's law appearing at distances $r \gg k^{-1}$ with the parameter k chosen to be $k \sim M_{Pl}$. The massive KK states are non-localized and form the continuous spectrum starting from m=0 (no mass gap). The RS2 model gives an elegant model of localized gravity with a non-compact extra dimension.

For k being of the order of the Planck mass, result is in very good agreement with the experimental results. Even though the extra dimension is not compact, we obtain the four dimensional Newton potential on the brane at y=0. This non-trivial result finds its explanation in the warped geometry which is responsible for the fact that the amplitude of the zero mode has its maximum at the brane and vanishes rapidly for finite z. On the other hand the massive modes reach their maximum amplitudes asypmtotically far away from the brane. Therefore, there is very small influence on the gravitational interactions on the brane, although the masses of the extra gravitons are arbitrarily small.

Similar results can be obtained by taking $\tau_2 \ll \tau_1$ in the RS1 model. In this case τ_2 does not affect the solution. The second brane is located at $z = z_c$ and the brane separation R can be adjusted in such a way that

$$M_{PL} e^{-k\pi r} \sim M_{Pl}.10^{-15} \sim 1TeV \tag{3.59}$$

This ensures that the hierarchy problem is solved on the second brane. Therefore is is considered to be our plane where SM fields are localized.

Separately, Randall and Sundrum's theory may explain the existence of dark matter which is invisible and makes up 90 percent of the universe. Dark matter emits or absorbs no light and it can be felt only through its gravity. It could simply come from another universe from which we can sense gravitons. In addition the theory can explain why dark matter is usually found in the halos around galaxies. according to theory , large masses on different branes are attracted to each other through hyperspace with the mutual gravitational pulls. Thus a galaxy on our universe may be mirrored by a galaxy from another universe, with only the gravity from its edges apparent.

3.3. RS Model With Extra Time-like Dimension

Common point of all models considered up to there in this chapter is that the extra dimension in all of them is spacelike. In this section, we replace the extra spacelike dimension by a timelike "(Chaichian and Kobakhidze 2000)". Hence the visible brane in RS model is changed into one with positive tension "(Csàki *et al.* 1999, Cline *et al.* 1999)". The replacement of the spacelike dimension by a timelike one, $y \to \tau$; i.e., the change of the signature from $(-++++)$ to $(-+++-)$ leaves the Einstein equations unchanged if it is simultaneously accompanied by the change of the sign of the bulk cosmological constant Λ and the brane tensions τ_1 and τ_2:

$$\begin{aligned} &(-++++) \to (-+++-) \\ &\Lambda \to -\Lambda, \tau_1 \to -\tau_1, \tau_2 \to -\tau_2 \end{aligned} \tag{3.60}$$

Thus in such a scenario, also the AdS_5 space is replaced by the dS_5 one and the solution for the background metric

$$ds^2 = e^{-2k|\tau|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \tag{3.61}$$

Although the similarity of the solutions in the cases of extra spacelike and time-like dimensions, the phenomenological consequences of these two scenarios are drastically different from each other (Extra time-like dimensions have been a subject of interest for some time "(Aref'eva 1985)" and have been review with in the various versions of string and M-theory "(Vafa 1996, Hull 1998, Hull and Khur 1998)"). As it is well known, typical theories with extra timelike dimensions suffer from pathologies such as negative-norm states (ghosts) and tachyons. In fact the KK excitations in case of extra timelike dimensions would be seen by the four dimensional observer as tachyonic states with imaginary masses quantized in units of i/τ_c. The exchange of such KK states induces an imaginary part in the effective low-energy potential between two test "charges".

This complexity is interpreted as violation of casuality and probability in the interaction of two "charged" particles, so they can disappear to nothing. From the experiments dedicated to look for proton or double β decay, the bound of the extra timelike dimension should be $\tau_c \lesssim 10.M_{Pl}^{-1}$ "(Yndurain 1993)" (while a careful study with conservation of the momentum in extra dimensions essentially gives no bound "(Erdem and Ün 2006)")in the case of appearance of tachyonic KK states of photons or gluons. Nevertheless the phenomenological constraints on the extra timelike dimensions are get in the framework of the compact extra dimensions where al KK states may propagate "(Dienes *et al.* 1998, Dienes *et al.* 1999)". On the other hand, in a brane-world scenario, as we have already mentioned, only graviton can spread out through the extra dimensions. If the extra dimensions are timelike, the gravitational potential includes some imaginary part which is interpreted as existence of decays into the unphysical negative energy tachyons and thus the size of the extra timelike dimensions in the factorizable spacetime can be as large as $y_c \sim 1mm$. This scale is large enough to be able to be tested in the collider experiments in principle. Another point is that the KK states of graviton are quite different in warped geometry considered in this section.

Let us first determine the mass spectrum of the KK modes of graviton in the effective four dimensional theory. The starting point is the five dimensional Einstein equations as usual. we an write the five dimensional Einstein equations immediately by using the similarity to that obtained in the case of extra spacelike dimension given in Eq(3.14).

$$\sqrt{-G}(R_{MN} - \frac{1}{2}G_{MN}R) =$$

$$-\frac{1}{M_*^3}[\sqrt{G}G_{MN}\Lambda + \sqrt{g^{vis}}g_{\mu\nu}^{vis}\delta_\mu^M\delta_\nu^N\tau_1\delta(y - \pi y_c + \sqrt{g^{hid}}g_{\mu\nu}^{hid}\delta_\mu^M\delta_\nu^N\tau_2\delta(y) \quad (3.62)$$

Similarly to the case of extra spacelike dimension, the background metric given in Eq(3.50) is the solution of Eq(3.51). the perturbation procedure around this solution is exactly the same. The conditions on h$_{MN}(x,y)(\partial_M h_N^M = h_M^M = 0)$ are kept too. If we can expand the graviton field $h_{MN}(x,y)$ upon compactification as follows;

$$h_{MN}(x,y) = \sum_{n=0}^{\infty} h_{MN}^{(n)}(x)\Psi^{(n)}(y) \quad (3.63)$$

the following equation for the graviton field is obtained (assumed that $h_{55} = 0$

$$(\eta_{\alpha\beta}\partial^\alpha\partial^\beta + m_n^2)h_{\mu\nu}^{(n)} = 0 \quad (3.64)$$

where $m_{(n)}$ is the mass of the n. KK state and the sign of $m_{(n)}^2$ is negative in contrast to the case of extra spacelike dimension, thus this equation of motion describes graviton KK states with imaginary masses, i.e tachyonic gravitons. Since the orthogonality condition on $\Psi^{(n)}$ is valid for also here, we have also an equation for Ψ get from the Einstein equation by using the equation for the graviton field. This equation is;

$$\partial_y[e^{-2k|y|}\partial_y\Psi^{(n)}] = -m_{(n)}^2 e^{-2k|y|}\Psi^{(n)} \tag{3.65}$$

This equation is the same as that obtained in RS models except the sign of m_n^2. As we have mentioned in that sectioned, the solutions of such an equation can be written in terms of Bessel functions.

$$\Psi^{(n)}(y) = \frac{e^{k|y|}}{N_n}[J_2(\frac{m_n}{k}e^{k|y|}) + A_n Y_2(\frac{m_n}{k}e^{k|y|})] \tag{3.66}$$

The boundary condition $\partial_y\Psi(y)|_{y=0,y_c} = 0$lead to the following equations:

$$A_n = -\frac{J_1(\frac{m_n}{k})}{Y_1(\frac{m_n}{k})} \tag{3.67}$$

$$A_n = -\frac{J_1(\frac{m_n}{k}e^{k|y_c|})}{Y_1(\frac{m_n}{k}e^{k|y_c|})} \tag{3.68}$$

One can determine A_n and m_n with these equations. In fact, working in the limit of $m_n/k \ll 1$, $A_n \approx 0$ and $J_1(e^{k|y_c|}m_n/k) \approx 0$. Thus the masses of the graviton KK modes can be determined through the roots of $J_1(e^{k|y_c|}$. However in the limit $e^{k|y_c|}m_n/k \gg 1$, $J_1(e^{k|y_c|}m_n/k)$ is approximated by the plane waves of the following form;

$$J_1(e^{k|y_c|}) \approx \sqrt{\frac{2k}{m_n\pi}e^{k|y_c|}}\cos(\frac{3\pi}{4} - \frac{m_n}{k}e^{k|y_c|}) \tag{3.69}$$

and thus $\nabla m = m_{n+1} - m_n \approx \pi k e^{-k\pi y_c}$.

Finally we can determine the normalization constant N_n by substituting Eq(3.55) into the orthogonality condition and it is found as;

$$N_n \approx \frac{e^{k\pi y_c}}{\sqrt{k}}|J_2(e^{k|y_c|})|$$

when $e^{k|y_c|}m_n/k \to \infty$; $N_n \to \frac{e^{k\pi y_c}}{\sqrt{k}}\sqrt{\frac{2k}{m_n\pi}}$. hence the wave function of zero mode can be obtained by the limiting $m_n \to 0$

$$\Psi^{(0)} = \sqrt{\frac{k}{1 - e^{-2\pi k y_c}}} \tag{3.70}$$

At this point, we are ready to discuss the possible influences of extra timelike dimension on the ordinary four dimensional physics by having mass spectrum of KK modes in effective four dimensional theory. Since only gravitational interactions can spread out through the extra dimensions, the effects of extra timelike dimension appears in the gravitational potential of two test particles with mass M_1 and M_2 placed at the points ($x = 0, y = y_c$) and ($x = r, y = y_c$) of the visible 3-brane which interact with each other by exchanging graviton KK modes.This potential can be expressed as;

$$V(r) = \sum_{n=0}^{\infty} G_N^{(5)} \frac{M_1 M_2}{r} |\Psi^{(n)}(\frac{m_n}{k} e^{k|y_c|})|^2 + \delta V(r) \tag{3.71}$$

where;

$$\delta V(r) = \sum_{n=1}^{\infty} G_N^{(5)} \frac{M_1 M_2}{r} |\Psi^{(n)}(e^{k|y_c|} m_n/k)|^2 e^{-im_n r} \tag{3.72}$$

where the five dimensional Newton constant $G_N^{(5)} = 1/M_*^3$ is related to the ordinary four dimensional one $G_N = 1/M_{Pl}^2$ as

$$G_N = G_N^{(5)} k(1 - e^{-2\pi k y_c})^{-1} \tag{3.73}$$

Thus an imaginary part is induced to the Newton's potential as a result of tachyonic nature of the KK modes "(Dvali *et al.* 1999)". Typically such complex contributions to the energy are associated with an instability of the system. Such problems associated with the instability of the system have already considered in "(Yndurain 1993)", and also we will consider them in chapter 4.

CHAPTER 4

EXTRA TIME-LIKE DIMENSIONS

4.1. Classical Tachyons and Their Properties

Einstein concluded in his first paper on the special relativity ”(Einstein 1905)” that nothing can be faster than light in the nature by pointing out that the relativistic formulas of kinetic energy and momentum approach to infinity as $v \to c$, for usual particles

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}, \qquad p = \frac{mv}{\sqrt{1 - v^2/c^2}} \tag{4.1}$$

It can be easily seen that energy and momentum are imaginary (unless $m = 0$) for the velocities grater than c. We can consider the rest mass in the formulas to get rid of imaginary energy and momentum. Since the rest mass is not a directly measurable quantity unless particles are brought to rest, we are free to hypothesize particles for which $v > c$ and m is imaginary ”(Feinberg 1967)”. Such particles are called tachyons.

$$m = i\mu \qquad (\mu \; is \; real) \tag{4.2}$$

In the case of tachyons with imaginary masses, the energy and momentum remain real quantities.

$$E = \frac{\mu c^2}{\sqrt{v^2/c^2 - 1}}, \qquad p = \frac{\mu v}{\sqrt{v^2/c^2 - 1}} \tag{4.3}$$

but;

$$E^2 < p^2 c^2$$

Separately, another problem with tachyons in relativistic quantum mechanics arises from that tachyons are out of the light cone because of the time-like momentum. Out of the light cone, the signs of the scalar components of vectors can be changed to opposite signs by any simple Lorentz transformation. This implies a more direct connections between the positive- and negative-energy solutions of wave functions describing tachyons on contrary to those of usual particles. In fact, the consideration of the transformation properties of fields in arbitrary dimensions can facilitate to understand the properties of tachyons, which arise in particular from the extra time-like dimensions. An example of such tachyonic modes will be considered in the next section by considering the proton decay under a presence of extra time-like dimensions.

Let us consider the solutions to the Klein-Gordon equation for a c-number field $\phi(x)$ with an imaginary mass $m = i\mu$

$$(\Box + \mu^2)\phi = (\nabla^2 - \frac{\partial^2}{\partial t^2})\phi = 0 \tag{4.4}$$

A set of elementary solutions to this equation are clearly;

$$\phi_{+,k}(x) = (2\pi)^{-3/2}\, e^{ik.x}$$
$$\phi_{-,k}(x) = (2\pi)^{-3/2}\, e^{-ik.x} \tag{4.5}$$

If we restrict $\vec{k}$ by $|\vec{k}| \geq \mu$, then the set of functions $\phi^*_{+,k}(\vec{x}, t = 0) = \phi_{-,k}(\vec{x}, t = 0)$ does not form a complete set. Instead of the usual completeness relation;

$$\sum_{all\ k} \phi^*_{+,k}(\vec{x}, t = 0)\phi_{-,k}(\vec{x}, t = 0) = \delta^3(\vec{x} - \vec{y}) \tag{4.6}$$

we have

$$\sum_{|\vec{k}|\geq\mu} \phi^*_{+,k}(\vec{x}, t = 0)\phi_{-,k}(\vec{x}, t = 0) = \bar{\delta}^3(\vec{x} - \vec{y}) \tag{4.7}$$

where

$$\bar{\delta}^3(\vec{x} - \vec{y}) = \int d^3\vec{k}\ \Theta(|\vec{k}| - \mu)e^{i\vec{k}(\vec{x}-\vec{y})}$$

$$= \delta^3(\vec{x} - \vec{y}) - \int_0^{\mu} k^2 dk \int \frac{d\Omega}{(2\pi)^3} e^{i\vec{k}(\vec{x}-\vec{y})}$$

$$= \delta^3(\vec{x} - \vec{y}) + \frac{\lambda cos(\lambda) - sin(\lambda)}{|\vec{x} - \vec{y}|^3\ 2\pi^2} \tag{4.8}$$

where $d\Omega = sin(\theta)d\theta\ d\phi$ and $\lambda = \mu|\vec{x} - \vec{y}|$.

The incompleteness of the allowed set of solutions has several consequences, which can be summarized as follows "(Feinberg 1967)".

- Tachyons can not be localized in space, i.e, a superposition of solutions of the form

$$\psi(x) = \int \phi_{+,k}(x)\, f(k) d^3k, \quad (|\vec{k} \geq \mu|) \tag{4.9}$$

which could be a tachyon wave function, can not be made into $\delta^3(\vec{x})$. In fact, such a superposition can not be made to vanish outside a sphere of finite radius, but rather necessarily has a finite tail. However the tail can be made to decrease with an arbitrary power of x for large x, by choosing the weight function $f(k)$ to have a zero of suitable order at $|\vec{k}| \geq \mu$.

- The Cauchy initial-wave problem must be restricted somewhat for the ϕ field. In order to determine $\phi(\vec{x}, t)$, it is still possible to prescribe $\phi(\vec{x}, 0)$ and $(\partial\phi/\partial t)(\vec{x}, 0)$ However, these functions can not be prescribed arbitrarily. Instead, they must both e restricted to functions whose Fourier transforms vanish when $|\vec{k}| \leq \mu$.

$$\phi(\vec{x}, t = 0) = \int g(\vec{k})\, e^{ik.x} d^3\vec{k} \tag{4.10}$$

with $g(\vec{k}) = 0$ for $|\vec{k}| \leq \mu$ and similarly for $\partial\phi/\partial t$. These conditions insure that no increasing or decreasing exponentials will occur in $\phi(\vec{x}, t)$. When they are imposed on the initial data, the Cauchy problem may be solved as usual by

$$\phi(\vec{x}, t) = \int d^3\vec{y}\, G_1(\vec{x} - \vec{y}, t)\, \phi(\vec{y}, 0) + \int d^3\vec{y}\, G_2(\vec{x} - \vec{y}, t)\, \frac{\partial\phi}{\partial t}(\vec{y}, 0) \tag{4.11}$$

with;

$$G_2 = -\int d^3\vec{k}(2\pi)^{-3}\, \frac{sin(\omega t)}{\omega}\, e^{i\vec{k}.(\vec{x}-\vec{y})} \tag{4.12}$$

$$G_1 = -\frac{\partial G_2}{\partial t} \tag{4.13}$$

The function $G_2$is the analog for this field of the function $\triangle$ for the ordinary Klein-Gordon field. Unlike that function, it does not vanish for spacelike separation. This may be seen for instance by noting that $G_1 = -\partial G_2/\partial t$ reduces at $t = 0$ to $\bar{\delta}^3(\vec{x}-\vec{y})$ which is not zero for $\vec{x} \neq \vec{y}$. The fact that G_2 does not vanish for spacelike separations is not in contradiction with the fact that $G_2(\vec{x}-\vec{y}, t = 0)$ vanishes. This is because G_2 is not an invariant function. To see this, we can rewrite G_2 as a four dimensional integral as follows;

$$G_2(x) = (2\pi)^{-3} \int d^4k\, \epsilon(k_0)\, e^{ik.x} \delta(k^2 - \mu^2) \tag{4.14}$$

The non-invariance of this integral follows from the fact that the step function $\epsilon(k_0)$ is not an invariant function for spacelike momenta.

- Because of impossibility of localizing $\phi(x)$, the discussion of propagation for for the solutions is more complicated than for the real mass Klein-Gordon equation. In the usual discussion, one chooses $\phi(\vec{x}, t = 0)$ to be a pulse, confined to some region about $x = x_0$ and then the propagation of the pulse determined by the form of the Green functions for large x. In the present case, such a pulse can not be constructed from the allowed solutions and the values of $\phi(\vec{x}, t)$ at large $\vec{x}$ come not only from the fact that G_2 does not vanish at spacelike separation, but also from the fact that $\phi(\vec{x}, 0)$ will not vanish at large $\vec{x}$ It is not hard to see that $G_1(\vec{x}, t)$ is response to a disturbance whose value at $t = 0$ is given by $\bar{\delta}^3(\vec{x})$, which does not vanish at large $\vec{x}$.

4.2. Tachyons in Higher Dimensions

In higher dimensional models, extra dimensions are generally assumed to be space-like as those considered in most of this thesis so far. While the extra dimensions have received more attention in recent years, the scarcity of studies involving extra time-like dimensions arises from the fact that tachyonic modes occur due to the time-like dimensions. These modes are really problematic, because they cause violation of the causality in the case of interaction with the usual particles. Also, there is no an adequate field theoretical description of tachyonic field. From the phenomenological point of view; the most serious problems mentioned in literature are the extremely small lower bound on the size of extra time-like dimensions, the spontaneous decay of stable particles induced by negative energy tachyons with negative energy and imaginary self-energy for charged fermions induced by tachyonic modes, which seems to cause disappearance of fermion into nothing.

Let us consider the propagator of a vector particle expressed in higher dimensions. It is evident that the propagator for Kaluza-Klein modes from Eqs.(2.22) corresponding to an extra dimensions of length L is;

$$\mathcal{D}_{\mu\nu} = -ig_{\mu\nu}\frac{1}{k^2 + \chi_n^2 + i0} \tag{4.15}$$

where k are the momenta conjugate to the ordinary spacetime variables and $\chi_n = n\hat{\chi}$, $n = \pm 1, \pm 2, \ldots$ and $\hat{\chi} = 1/L^2$. If $\hat{\chi}$ is spacelike (in other words, extra dimensions are spacelike), then the denominator of Eq(4.15) is $k^2 - n^2/L^2$ ($\eta = (+, -, -, -)$) and we have the standard tower of particles with masses of order $|n|/L$ (including photon with $n = 0$).

If $\hat{\chi}$ is time-like, on the other hand, the denominator of Eq(4.15) is $k^2 + n^2/L^2$ and has a unphysical pole at $-n^2/L^2$.The main phenomenological difficulties of the extra time-like dimensions are that they cause spontaneous decays of stable particles. Let us consider the potential describing two particles with the same electric charge, e in the non-relativistic limit

$$V(r) = -\frac{1}{2\pi} \int d^3\vec{k}\, e^{i\vec{k}.\vec{x}}\, T_{born} \tag{4.16}$$

where

$$T_{born} = -\frac{a}{\pi} \sum_n \frac{1}{k^2 - n^2/L^2 - iO} \tag{4.17}$$

It is important to notice that unlike the usual non-relativistic particle scattering, the integration is not from minus infinity to plus infinity. Moreover, one must put the correct cut-off momentum (that is the maximum momentum available to the protons) in this case. In the usual non-relativistic scattering, putting a cut-off is also the correct procedure but removing cut-off does not essentially change the result, because the potential is well-localized, in other words higher momenta is less important. First let us see what happen if one does not consider such a cut-off. After integration of Eq.(4.16)

$$V(r) = \frac{a}{r} + \frac{2a}{r} \sum_{n=1}^{\infty} e^{inr/L} \tag{4.18}$$

Now we can calculate the width corresponding to such a complex potential by taking the wave function for two protons inside a nucleus as follows;

$$\Psi(\vec{r}) = \frac{m_\pi^{3/2}}{\sqrt{\pi}} e^{-m_\pi r} \tag{4.19}$$

where m_π is the mass of pion and the width

$$\Gamma = -Im < \Psi|V(\vec{r})|\Psi >= \int_{-\infty}^{\infty} d^3\vec{r}\, \Psi^*(\vec{r}) V(\vec{r}) \Psi(\vec{r}) \tag{4.20}$$

It is found that $\Gamma = 32a\zeta(3)\ (m_\pi L)^3 m_\pi$; where ζ is Riemann's function and $\zeta(3) \simeq 1.2$. Hence the lifetime for decay of a proton in a nucleus can be written down

$$\tau_p = [32a\zeta(3)\ (m_\pi L)^3\ m_\pi]^{-1} \tag{4.21}$$

Measuring L in the units of Planck length $G_N^{1/2}$, we find

$$\tau_p = (G_N^{1/2}/L^3) \times 4 \times 10^{29}\ years \tag{4.22}$$

Eq(4.22) shows that the lifetime of proton depends on the size of the extra timelike dimension. From the requirement of agreement with the experiments, we can determine the size of extra timelike dimension. However since the experiments predict the life time of proton at the order of 10^{-33} years. It implies the upper bound on the extra timelike dimensions of the order of $1/10M_P^{-1}$.We will see in the next section that imposing the cut-off momentum essentially removes the upper bound on the proton lifetime.

4.3. Discussion of Some Phenomenological Problems Related to Extra Dimensional Models

This section is based on our study "(Erdem and Ün 2006)" on the phenomenological problems mentioned in the last section. In this study, we reconsider these difficulties and try to moderate them through a consistent formulation of the field theory for tachyons and their interactions with the usual particles (if they exist at all). The first problem considered here is the extremely small lower bound on the size of extra time-like dimensions deduced from the lifetime of proton. After we re-calculate this process and correct on this problem in the calculation of three level Feynman diagram for scattering of two protons, we consider the imaginary mass contribution to the stable fermions that spontaneously decay.

4.3.1. Spontaneous Decay Of Proton

The scattering cross section corresponding to this diagram may be obtained from the scattering amplitude of elastic fermion-fermion scattering.

$$\frac{d\sigma}{d\Omega} = |T|^2 = \frac{1}{2p_{10}2p_{20}2p'_{10}2p'_{20}}|M|^2 \tag{4.23}$$

where M is the matrix element given by;

$$M = \frac{e^2}{4\pi^2}\bar{u}(p'_1, \lambda'_1)\ \gamma_\mu\ u(p_1, \lambda_1)\frac{1}{k^2 + m_n^2 + iO}\bar{u}(p'_2, \lambda'_2)\ \gamma^\mu\ u(p_2, \lambda_2) \tag{4.24}$$

where $m_n^2 = n^2/L^2$ and u's are 4-component Dirac spinors. In the non-relativistic limit "(Aref'eva 1985)", the zero component of the proton 4-momenta p_{10}, p_{20} and the photon

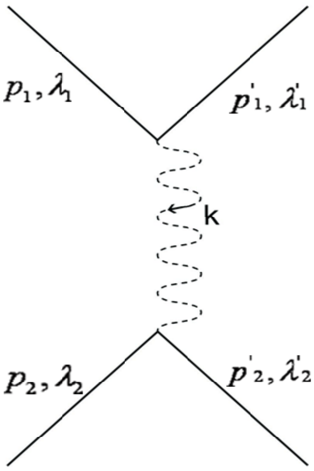

Figure 4.1: The Feynman diagram for the scattering of two protons with the initial 4-momenta and the spins; p_1, p_2 and λ_1, λ_2 and the final 4-momenta and the spins; p_1', p_2' and λ_1, λ_2. The wavy line denotes the tachyonic Kaluza-Klein modes

4-momentum transfer k approximated by

$$p_0 \simeq m + \frac{\vec{p}^2}{2m} - \frac{\vec{p}^4}{8m^3}$$

$$k^2 = (p_1' - p_2')^2 = (p_{10}' - p_{20}')^2 - (\vec{p}_1{}' - \vec{p}_{20}{}')^2 = \frac{(\vec{p}_1{}^{a'} - \vec{p}_2{}')^2}{4m} - \vec{k}^2$$

$$\frac{1}{\sqrt{2p_0}}\, u(p,\lambda) = \sqrt{\frac{m+p_0}{2p_0}} \begin{pmatrix} \chi(\lambda) \\ \\ \frac{\vec{p}.\vec{\sigma}}{m+p_0}\,\chi(\lambda) \end{pmatrix} \tag{4.25}$$

Hence in the non-relativistic limit (i.e. $p_0 = m, 1 - \vec{p}^2/4m^2 = 1$), T becomes;

$$T = \frac{e^2}{4\pi^2}\,\chi^\dagger(\lambda_1')\,\chi(\lambda_1)\,\frac{1}{|\vec{k}|^2 - m_n^2}\,\chi^\dagger(\lambda_2')\,\chi(\lambda_2)\,, \quad |\vec{k}| < |\vec{R}| = R \tag{4.26}$$

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix} \;,\; \gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

In the non-relativistic quantum mechanics,the scattering amplitude for the elastic scattering of a particle from $V(\vec{r})$, in the Born approximation can be written as;

$$T(\vec{k}) = \frac{1}{(2\pi)^2} \int d^3\vec{r}\, e^{i\vec{k}.\vec{r}}\, \chi^\dagger(\lambda_1')\, \chi^{\lambda_2}\, V(\vec{r})\, \chi(\lambda_1)\, \chi\lambda_2 \tag{4.27}$$

If we define a function $f(|\vec{k}|)$

$$f(|\vec{k}|) = \int d^3\vec{r}\, e^{-i\vec{k}.\vec{r}}\, V(|\vec{r}|) \tag{4.28}$$

After comparing (4.17) and (4.26), one notices that

$$f(|\vec{k}|) = \int d^3\vec{x}\, e^{-i\vec{k}.\vec{x}}\, V(|\vec{x}|) \tag{4.29}$$

where

$$f(|\vec{k}|) = \{ \begin{matrix} \frac{e^2}{|\vec{k}|^2 - m_n^2} & for\ \ |\vec{k}| \leq R \\ \\ 0 & elsewhere \end{matrix} \tag{4.30}$$

$V(x)$ is obtained as the Fourier transformation of $f(|\vec{k}|)$ as

$$V(x) = \frac{e^2}{(2\pi)^3} \int d^3\vec{k}\, \frac{e^{i\vec{k}.\vec{x}}}{|\vec{k}|^2 - m_n^2}$$

$$= \frac{e^2}{(2\pi)^3} \int_0^R \frac{|\vec{k}|^2 dk}{|\vec{k}|^2 - \frac{n^2}{L^2}} \int_0^\pi exp\{i|\vec{k}|\, r\, cos\theta\}\, sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= \frac{e^2}{2i(2\pi)^2 r} \int_0^R \frac{k\, dk}{k^2 - \frac{n^2}{L^2}} \{exp(ik\, r) - exp(-ik\, r)\}$$

$$= \frac{e^2}{i(2\pi)^2\, r} \int_{-R}^R \frac{k\, exp(ik\, r)}{k^2 - \frac{n^2}{L^2}}\, dk \tag{4.31}$$

The wave function of two protons has been already given as follows

$$\Psi = \frac{\sqrt{m_\pi^3}}{\sqrt{\pi}}\, e^{-m_\pi\, r} \tag{4.32}$$

where m_π denotes the mass of pions. Then the decay width under the potential $V(x)$ is obtained as

$$\Gamma = Im < \Psi|V(r)|\Psi > \tag{4.33}$$

The evolution of $< \Psi|V(r)|\Psi >$ is done in the appendix and found to be

$$< \Psi|V(r)|\Psi >= i\,\frac{e^2\,m_\pi^2}{\pi^2}\,[\frac{2m\beta}{(m^2-\beta^2)^2}\,ln(\frac{\beta+R}{\beta-R})-\frac{m^2+\beta^2}{(m^2-\beta^2)^2}\,ln(\frac{m+R}{m-R})$$

$$-\frac{2mR}{(m^2-\beta^2)(m^2-R^2)}] \tag{4.34}$$

where $m = 2im_\pi$ and $\beta = n/L$. In the case of $\beta > R$, $< \Psi|V(r)|\Psi >$ is real and hence there is decay of proton through the process given in Fig.1. Otherwise, $< \Psi|V(r)|\Psi >$ has an imaginary part, which means that the width is not zero. However, in spite of this non-zero width, tachyonic photon masses are in order of MeV and it means that tachyonic photon modes can not lead to decay of proton.

Hence we have removed the tachyonic photon modes by a cutoff momentum which corresponds to binding energy of nucleus. However, in spite of the real value of $< \Psi|V(r)|\Psi >$, imaginary mass contribution to the stable fermions can exist in spontaneous decay process through the self-energy diagram of a stable fermion, for instance electron, even if the size of extra time-like dimension is taken smaller than the nuclear sizes; which is considered in the next chapter.

4.3.2. Spontaneous Decay of Fermions and Fermion Self-Energies in the Presence of Extra Time-like Dimensions

In this section, we consider the spontaneous decay of a stable fermion into a tachyon and the original particle and the problem of imaginary mass contribution to the fermions through self-energy diagram involving tachyon which can occur even in the case of the real valued $< \Psi|V(r)|\Psi >$. For example, the decay of an electron into another electron and a tachyonic photon is kinematically allowed. In this case it is difficult to identify this negative-energy tachyons with their own anti-tacyons because of the direct connection between the negative- and positive-energy tachyons by a simple Lorentz boost. Therefore, the momentum conservation can be imposed in extra time-like dimension in such cases and once we can identify the tachyon with Kaluza-Klein mode of the photon in the extra time-like dimension , such decays are impossible, since there will be a non-zero momentum flow in the extra time-like dimension due to the tachyon and there is no another momentum to balance it and stabilize the vacuum.

Figure 4.2: The Feynman diagram for the contribution of a photonic tachyon to fermion self-energy. The wavy line denotes the tachyonic Kaluza-Klein modes of photon and the solid line denotes the fermion

The imaginary mass contribution to the self-energy of a stable fermion can be avoided in the same way. Without taking the momentum conservation in the extra time-like dimension into account, the contribution of the self-energy diagram given in Fig.2 to the fermion mass (in the Pauli-Villars regularization) is of the form

$$\delta m \propto \frac{e^2 m}{4\pi^2}\, ln(\frac{\mu^2 - \Lambda 2}{\mu 2}), \quad \mu^2 > 0 \tag{4.35}$$

where $m,\ e,\ \mu\,\Lambda$ stand fort he fermion masses, the electric charge of the fermion, the mass of the tachyonic photon and the Pauli-Villarz regularization cut-off scale respectively and we have the modified the propagator of the tachyonic photon mode by

$$\frac{1}{k^2 - \mu^2} \rightarrow (\frac{1}{k^2 - \mu^2})\ \frac{\Lambda^2 - \mu^2}{k2 - \mu^2 + \Lambda^2} \tag{4.36}$$

By definition, $\Lambda > \mu$, so Eq(4.35) results in an imaginary contribution independent of μ, and Λ, $i(e2m)/(4\pi)$. This contribution is essentially equal to the width of the spontaneous decay of a fermion through release a tachyon. This width is problematic, because it suggests a decay rate for the fermion comparable to that of hadron resonances. This result is extremely problematic, because it predicts a decay rate for the fermion comparable to the decay width of hadronic resonances and moreover the result in Eq(4.36) may be multiplied by a large number, because the number of Kaluza-Klein modes is about Λ/μ_0, where μ_0 is the mass of the first Kaluza-Klein mode and Λ is at most at the order of Planck mass. However if we require the conservation of momentum in extra time-like direction, then usual fermions can only radiate usual photons and the contribution to the fermion self-energies is absent and hence problem is removed.

In this study, we have re-examined some phenomenological difficulties due to the tachyonic photon modes in the study of extra time-like dimensions. We have shown that the lower bound on the size of the extra time dimensions due to the lower bound on the lifetime of proton may be relaxed and moreover the presence of tachyons related to the extra time-like dimensions is not as problematic as the tachyons in the usual 4-dimensional picture. Although we believe that we have made some progress in the phenomenological viability of extra time-like dimensions, there are still some points to be studied further.

CHAPTER 5

CONCLUSİON

We have seen that extra dimensions (and in particular extra timelike dimensions), in particular extra time-like dimensions offer solutions to some long-standing problems of high energy physics such as unification of fundamental forces, quantization of electric charge, gauge hierarchy problem etc.

We have reviewed basic features of Kaluza-Klein compactification and some recent extra dimensional models. Whenever necessary, we have pointed out the modifications introduced into the formulation when the extra spacelike dimensions are replaced by extra time-like dimensions. Then in chapter 4 we have considered the extra time-like dimensions in detail where particular emphasis is given to its phenomenological situation.

First we have repeated the naive calculation of the proton scattering through the tachyonic Kaluza-Klein modes of photon by using a more rigorous approach. Then we have reconsidered the tachyonic contribution to the fermion self energies. We have seen that the spontaneous disappearance in both cases my be avoided by requiring momentum conservation in the extra time-like direction.We have found that tachyonic modes due to extra time-like dimensions do not pose as big problems as stated in the literature. However this does not imply that we have overcome all the difficulties related to tachyonic modes due to the extra time-like dimensions. One needs a rigorous field theory formulation for tachyons for a wholly safe use of extra timelike dimensions.

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APPENDIX A

In this appendix, we give the details of the evolution of the integral given in (4.33).

$$\int_{-R}^{R} \frac{k\ dk}{k^2-\frac{n^2}{L^2}} \int_0^{\infty} r\ e^{(ik-2m_\pi)r} dr \int_0^{\pi} sin\theta\ d\theta \int_0^{2\pi} d\phi = 4\pi \int_{-R}^{R} \frac{k\ dk}{k^2-\frac{n^2}{L^2}} \int_0^{\infty} r\ e^{(ik-2m_\pi)r} dr$$

$$= -4\pi \int_{-R}^{R} \frac{k\ dk}{(k^2-\frac{n^2}{L^2})(k+2im_\pi)} \tag{A.1}$$

If the denominator of the integral is written as;

$$\frac{1}{(k+\beta)(k-\beta)(k+m-\epsilon)(k+m-\epsilon)} = \frac{1}{(x-x_1)(x-x_2)(x-x_3)(x-x_4)} \tag{A.2}$$

where; $m = 2im_\pi$, $\beta = n/L$ and also

$$x = k,\ \ x_1 = -\beta,\ \ x_2 = \beta,\ \ x_3 = -m+\epsilon,\ \ x_4 = -m-\epsilon \tag{A.3}$$

We use the identity;

$$\frac{1}{(x-x_1)(x-x_2)} = \frac{1}{x_1-x_2}[\frac{1}{x-x_1} - \frac{1}{x-x_2}] \tag{A.4}$$

to write (B.2) as;

$$\frac{1}{x_1-x_2}\{\frac{1}{(x_1-x_3)(x_1-x_4)}.\frac{1}{x-x_1} - \frac{1}{(x_2-x_3)(x_2-x_4)}.\frac{1}{x-x_2}\}$$

$$+\frac{1}{x_3-x_4}\{\frac{1}{(x_1-x_3)(x_2-x_3)}.\frac{1}{x-x_3} - \frac{1}{(x_1-x_4)(x_2-x_4)}.\frac{1}{x-x_4}\} \tag{A.5}$$

The second term in (B.5) is

$$\frac{1}{x_3-x_4}\{\frac{1}{(x_1-x_3)(x_2-x_3)}.\frac{1}{x-x_3} - \frac{1}{(x_1-x_4)(x_2-x_4)}.\frac{1}{x-x_4}\}$$

$$= \frac{1}{2\epsilon}\{\frac{a}{x-x_3} - \frac{b}{x-x_4}\} = \frac{1}{2\epsilon}\{\frac{(a-b)x+bx_3-ax_4}{(x-x_3)(x-x_4)}\} \tag{A.6}$$

where

$$a = \frac{1}{(x_1-x_3)(x_2-x_3)}, \quad b = \frac{1}{(x_1-x_4)(x_2-x_4)} \tag{A.7}$$

$$\frac{(a-b)x}{x_3-x_4}=\frac{2mx}{[(m-\epsilon)^2-\beta^2][(m+\epsilon)^2-\beta^2]} \tag{A.8}$$

$$\frac{bx_3-ax_4}{x_3-x_4}=\frac{\beta^2-3m^2-\epsilon^2}{[(m-\epsilon)^2-\beta^2][(m+\epsilon)^2-\beta^2]} \tag{A.9}$$

then (B.6) becomes

$$\frac{1}{x_3-x_4}\{\frac{1}{(x_1-x_3)(x_2-x_3)}.\frac{1}{x-x_3}-\frac{1}{(x_1-x_4)(x_2-x_4)}\frac{1}{x-x_4}\}$$

$$=\frac{1}{(x-x_3)(x-x_4)}\{\frac{2mx}{[(m-\epsilon)^2-\beta^2][(m+\epsilon)^2-\beta^2]}$$
$$-\frac{\beta^2-3m^2-\epsilon^2}{[(m-\epsilon)^2-\beta^2][(m+\epsilon)^2-\beta^2]}\} \tag{A.10}$$

After combining (B.6), (B.10) and using the explicit values of $x_1,\ x_2,\ x_3,\ x_4$ and letting $\epsilon \to 0$, one obtains

$$\frac{k}{(k^2-\beta^2)(k+m)^2}=\frac{1}{2\beta(m-\beta)^2}.\frac{k}{k+\beta}+\frac{1}{2\beta(m+\beta)^2}.\frac{k}{k-\beta}$$

$$+\frac{1}{(m^2-\beta^2)'2}.\frac{k^2}{(k+m)^2}-\frac{\beta^2-3m^2}{(m^2-\beta^2)^2}.\frac{k}{(k+m)^2} \tag{A.11}$$

The evolution of the integral (B.1) by the use of (B.11) gives

$$<\Psi|V(r)|\Psi>=i\frac{e^2\ m_\pi^3}{\pi^2}[\frac{2m\beta}{(m^2-\beta^2)^2}\ ln(\frac{\beta+R}{\beta-R})-\frac{m^2+\beta^2}{(m^2-\beta^2)^2}\ ln(\frac{m+R}{m-R})$$

$$-\frac{2mR}{(m^2-\beta^2)(m^2-\beta^2)}] \tag{A.12}$$