VIBRATION CHARACTERISTICS OF PORTAL FRAMES

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ABSTRACT

VIBRATION CHARACTERISTICS OF PORTAL FRAMES

This research study deals with the defining of dynamic behaviors of a portal frame structure with tapered members by using Finite Element Method (F.E.M.) and experimental modal analysis.

Portal frame with three beam members is used in this study. Frame structure is composed by two tapered and one uniform cross sectioned beams.

The first part of thesis is about the finite element modelling of the model which is developed in ANSYS. Theoretical modal analysis to find the natural frequencies and mode shapes of the frame is performed by using the finite element model.

The second part of thesis is experimental modal analysis of the frame under study. For this purpose, the experimental modal test setup has been established.

The modal parameters found from both numerical and experimental methods are compared and good agreement is found.

ÖZET

PORTAL ÇERÇEVELERİN TİTREŞİM KARAKTERİSTİKLERİ

Bu araştırma çalışması, daralan kesitleri olan portal çerçevelerin Sonlu Elemanlar Yöntemi (FEM) ve deneysel modal analizi kullanılarak dinamik davranışlarını belirlenmek ile ilgilidir.

Çalışmada üç çubuklu portal çerçeve kullanılmıştır. Çerçeve iki daralan ve bir sabit kesitli çubuktan oluşturulmuştur.

Tezin ilk bölümü ANSYS'de geliştirilen sonlu eleman modeli ile ilgilidir. Sonlu eleman modeli kullanılarak, doğal frekansları ve titreşim biçimlerini bulmak için teorik modal analizi yapılmıştır.

Tezin ikinci bölümünde, çalışmadaki çerçevenin deneysel modal analizi yapılmıştır. Bunun için, deneysel modal analizi test düzeneği oluşturulmuştur.

Sayısal ve deneysel modal analizlerinden bulunan modal parametreler karşılaştırılmış ve iyi uyum bulunmuştur.

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A	cross-sectional area
d_e	displacement vector in the local coordinate system
D_e	displacement vector in the global coordinate system
E	Young's modulus
I_z	area the second moment of area of the cross-section about the z axis
$[k]$	element stiffness matrix
l_e	element length of frame
$[m]$	mass matrix
T	kinetic energy of the element
$[T]$	transformation matrix for the frame element
u	the axial deformation in the x direction
U	the total strain energy
v	the axial deformation in the y direction
ε_x	normal strain in x direction.
θ_z	rotation in the x-y plane and with respect to the z axis
ρ	mass density
η_r	damping loss factor
ζ_r	damping ratio
$(\dot{\ })$	derivative with respect to "t"

CHAPTER 1

GENERAL INTRODUCTION

The understanding of physcal nature of vibration phenomena has always been important for researches and engineers. There are lots of problems such as noise, vibration or failure encountered in practice. These problems direct engineers and researches to investigate vibrations of structural elements, structures and machines.

The role of understanding and predicting dynamic behavior of system is also significant tool for design step. For this reason, analytical and experimental methods are used together. Finite element modeling (F.E.M.) which popular and powerful technique is actually used as analytical methods. Certainly, one of the most important areas of experimental methods is modal analysis. Due to different built-in limitations, assumptions and choices, each approach has its own advantages and disadvantages.

It is obvious that the problem of vibrating frame structures have been examined widely in several fields of engineering i.e. bridge design, building design, space-based antenna and micro frames used in electronic equipment.

Frame structures can be classified as closed or open. Closed frames are developed by chains of beams in which both ends are fixed. Open frames are formed by chains of beams that have one end fixed and the other end free.

Tapered members which generally used in the frame structure have drawn attention .Because, tapered members make the stress in the structure more evenly distributing, so that consumption of material can be reduced.

Because of the wide usage of portal frames in engineered design, many investigators have examined portal frames. Researchers investigating vibration characteristics of portal frames tackle with problems of different points of view aforementioned.

The equations of receptance functions in analyzing the flexural vibration of uniform beams were derived (Bishop 1955). The natural vibration of structures such as portal frames, which may be regarded as being composed of beams that perform flexural vibration was investigated. The technique was based upon tables which were presented by the author (Bishop 1955). The numerical results obtained by the method were compared with the experimental results (Bishop 1956). The vibration analysis of a planar serial frame structure was presented. The transverse and longitudinal motions of each segment were analyzed simultaneously. The eigenvalue problem was solved by using closed-form transfer matrix method (Lin and Ro 2003). Both in plane and out of plane free vibrations of two member open frame structures was investigated. A substructure method was used (Heppler et al. 2003). The vibration of a frame with intermediate constraint and ends elastically restrained against rotation and translation was examined. The separation of variables method was applied for the determination of the exact eigenfrequenciens and mode shapes (Albarracin and Grossi 2005). The finite element modelling and experimental modal testing for the 1/10 scale rig was carried out. A comparison between experimental results and finite element results was presented (Wu 2004). The effects of semi-rigid connections on the responses of steel frame structure were defined by comparing experimental and theoretical modal analysis results (Türker et al. 2009).

Although, the problem to define the vibration characteristics of portal frames is much investigated and a considerable amount of publications have been published so far, it still holds attraction because of wide usage. Therefore, in this study, the vibration characteristics of portal frame were examined. The finite element method and the experimental modal testing technique are employed for this purpose.

This thesis has 6 chapters. First chapter presents the subject and summaries the previous studies on the titled subjects. Second chapter provides the theory about Finite element modelling (F.E.M) and Experimental modal analysis. Finite element analysis and it's results are given in chapter 3. Experimental modal analysis and it's results are given chapter 4. Chapter 5 discusses the vibration characteristics obtained both finite element modelling and experimental modal analysis. Then conclusion is given in Chapter 6.

CHAPTER 2

THEORETICAL BACKGROUND

2.1. Introduction

The finite element method is a numerical method for solving problems of engineering and mathematical physics. The F.E allows users to obtain the evolution in space and/or time of one or more variables representing the behavior of physical system. Nowadays the F.E.M is one of the most popular approaches for the vibration analysis of structures, but accuracy of the F.E.M can be questionable and validation is usually required. For this reason, the experimental modal testing is undertaken to measure the natural frequencies and the corresponding mode shapes of the structures and then a comparison between the experimental results and results obtained the finite element model is made. This chapter presents a brief explanation of the theory of these two important methods.

2.2. Theoretical Modal Analysis

2.2.1. Frequency response functions of an SDoF system

Some mechanical and structural systems can be idealized as SDoF systems. The SDoF system having a mass, a spring and a viscous damper is considered. For a harmonic force $f(t) = F(\omega)\, e^{j\omega t}$, the response of the system is another harmonic function $x(t) = X(\omega)\, e^{j\omega t}$ where $X(\omega)$ is a complex amplitude. The frequency response function (FRF) of the system is given by (He and Fu 2001) as

$$\alpha(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{k - \omega^2 m + j\omega c} \qquad (2.1)$$

or in different forms as

$$\alpha(\omega) = \frac{1/k}{1-(\omega^2/\omega_0^2) + j2(\omega/\omega_0)} = \frac{1/m}{\omega_0^2 - \omega^2 + j2\omega\omega_0\xi} \qquad (2.2)$$

Mobility FRF for this system is given as

$$Y(\omega) = \frac{\dot{X}(\omega)}{F(\omega)} = \frac{j\omega}{k - \omega^2 m + j\omega c} \qquad (2.3)$$

Accelerance FRF for this system is given as

$$A(\omega) = \frac{\ddot{X}(\omega)}{F(\omega)} = \frac{-\omega^2}{k - \omega^2 m + j\omega c} \qquad (2.4)$$

It is known that $\alpha(\omega)$, $Y(\omega)$, and $A(\omega)$ have the following relationships:

$$|A(\omega)| = \omega|Y(\omega)| = \omega^2|\alpha(\omega)| \qquad (2.5)$$

The reciprocals of $\alpha(\omega)$, $Y(\omega)$, and $A(\omega)$ of an SDoF system also used in modal analysis. They are respectively:

$$Dynamic\ stiffness = \frac{1}{\alpha(\omega)} = \frac{force}{displacement} \qquad (2.6)$$

$$Mechanical\ \ impedance = \frac{1}{Y(\omega)} = \frac{force}{velocity} \qquad (2.7)$$

$$Apparent\ \ mass = \frac{1}{A(\omega)} = \frac{force}{acceleration} \qquad (2.8)$$

The FRF of an SDoF system can be presented in different forms to those in previous equations. The receptance FRF can be factorized to become:

$$\alpha(\omega) = \frac{R}{j\omega - \lambda} + \frac{R^*}{j\omega - \lambda^*} \qquad (2.9)$$

where

$$R = \frac{1}{2m\omega_0 j} \tag{2.10}$$

$$\lambda = (-\xi + \sqrt{1-\xi^2}\, j)\omega_0 \tag{2.11}$$

Conjugate coefficients *R* and *R** are called residues of the receptance. λ and λ^* are the complex poles of the SDoF system.

2.2.2. Graphical display of a frequency response function

The possible 2-D graphical display of an FRF are listed as follows:

1. Amplitude–phase plot and log–log plot
2. Real and imaginary plots
3. Nyquist plot
4. Dynamic stiffness plot

Details of the listed items are available in the textbook written by He and Fu (2001). A sample frequency response function with linear-linear plot is shown in Figure 2.1.

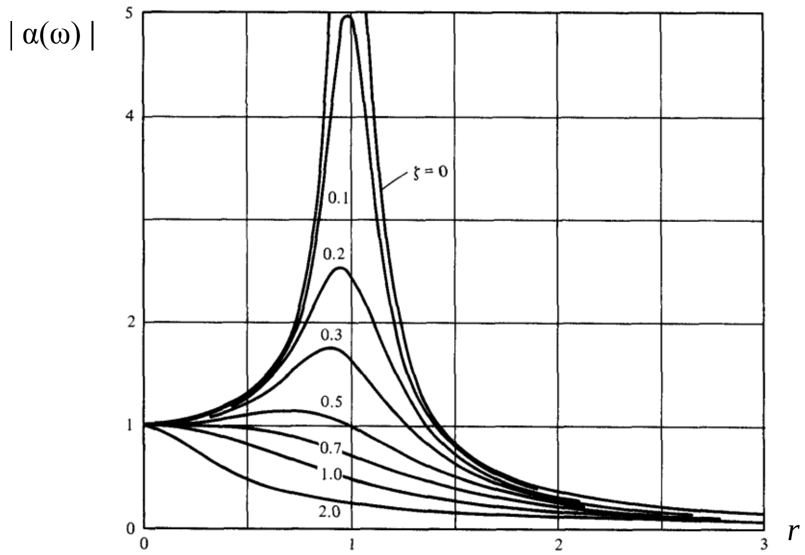

Figure 2.1. A sample for frequency response function

(Source: Tse et al 1978)

2.2.3. Frequency response functions of a damped MDoF system

The equations of motion of a MDoF system is written in matrix form as:

$$[M]\{\ddot{x}(t)\}+[D]\{\dot{x}(t)\}+[K]\{x(t)\}=\{f(t)\} \tag{2.12}$$

where $[M]$, $[D]$ and $[K]$ are mass, damping and stiffness matrices, respectively. Also, $\{x(t)\}$ is displacement vector and $\{f(t)\}$ is force vector.

Considering the harmonic motion for displacement and force, the following equations can be written:

$$\{f(t)\}=\begin{Bmatrix} F_1 \\ F_2 \\ \ldots \\ \ldots \\ F_n \end{Bmatrix}\sin\omega t=\{F\}\sin\omega t \tag{2.13}$$

$$\{x(t)\}=\begin{Bmatrix} X_1 \\ X_2 \\ \ldots \\ \ldots \\ X_n \end{Bmatrix}\sin\omega t=\{X\}\sin\omega t \tag{2.14}$$

$$([K]-\omega^2[M])\{X\}=\{F\} \tag{2.15}$$

or

$$[Z(\omega)]\{X\}=\{F\} \tag{2.16}$$

where $[Z(\omega)]$ = $([K] - \omega^2[M])$ is known as the dynamic stiffness matrix of an MDoF system. Similar to SDoF system, the inverse of dynamic stiffness matrix gives the receptance FRF matrix of the system and is denoted by $[\alpha(\omega)]$. It is in open form as:

$$[\alpha(\omega)]=([K]-\omega^2[M])^{-1} \tag{2.17}$$

or

$$[\alpha(\omega)=\begin{bmatrix}\alpha_{11}(\omega) & \alpha_{12}(\omega) & \dots & \alpha_{1n}(\omega)\\ \alpha_{21}(\omega) & \alpha_{22}(\omega) & \dots & \alpha_{2n}(\omega)\\ \dots & \dots & \dots & \dots\\ \alpha_{n1}(\omega) & \alpha_{n2}(\omega) & \dots & \alpha_{nn}(\omega)\end{bmatrix} \quad (2.18)$$

$\alpha_{ij}(\omega)$ is the frequency response function when the system only has one input force applied at coordinate 'j' and the response is measured at coordinate 'i'. A sample plot for receptance $\alpha_{11}(\omega)$ of the 4DoF system is shown in Figure 2.2. It is seen from Figure 2.2 that there are 4 peak due to the degres of freedom of the system.

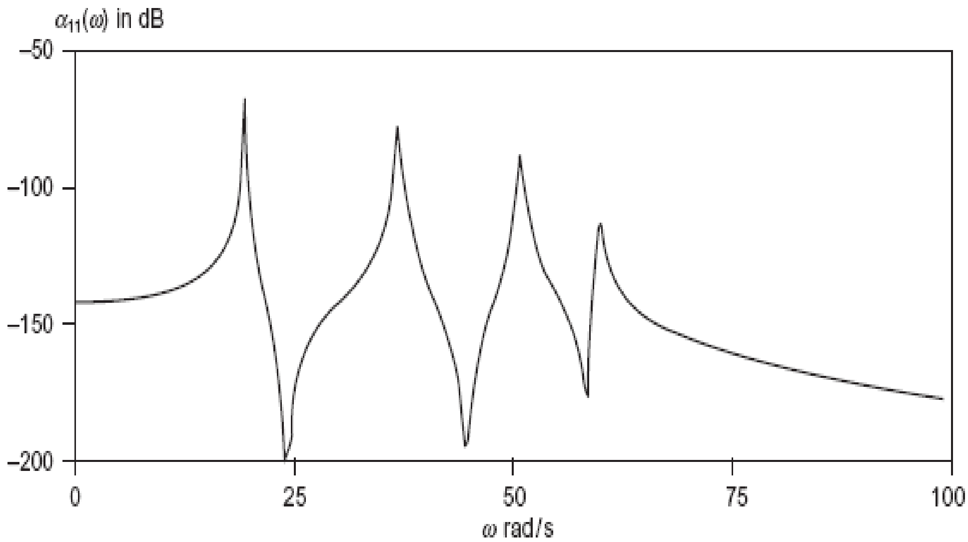

Figure 2.2. A sample for receptance $\alpha_{11}(\omega)$ of the 4DoF system

(Source: He and Fu 2001)

2.3. Finite Element Modeling and Equation of Motion

2.3.1. Frame Element Properties

The bar is capable of carrying axial forces only. On the other hand, the beam is capable of carrying transverse forces, as well as moments. Therefore, a frame element can be obtained by combining bar and beam elements. Consider a frame structure is divided in to frame elements. The elements and nodes are numbered separately in a convenient manner. In a planar frame element, there are three degrees of freedom at one node in its local coordinate system, as shown in Figure 2.1 (Liu 1993).

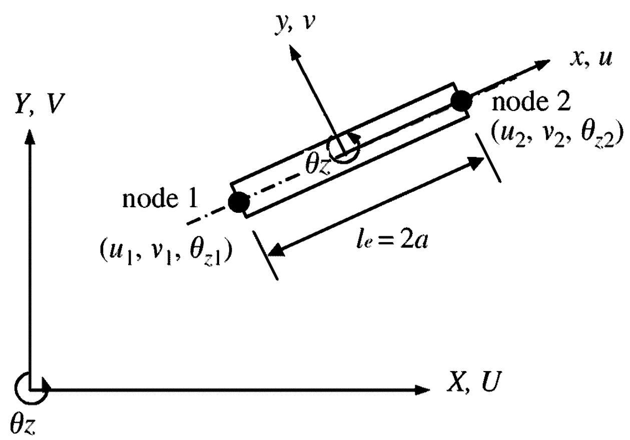

Figure 2.3. A finite element for frame element

(Source: Liu 1993)

The bar element has only one degree of freedom at each node (axial deformation), and the beam element has two degrees of freedom at each node (transverse deformation and rotation). Combining this knowledge gives the degree of freedom and displacements frame elements in Equation 2.5 (Liu, 1993).

$$\{d_e\} = \begin{Bmatrix} u_1 \\ v_1 \\ \theta_{z1} \\ u_2 \\ v_2 \\ \theta_{z2} \end{Bmatrix} \tag{2.19}$$

2.3.2. Element Potential Energy and Stiffness Matrix

A bar element with length 2*a* is shown in Figure 2.2.

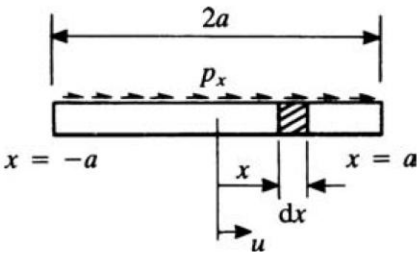

Figure 2.4. A bar element in the local coordinate system

(Source: Petyt 2010)

The total strain energy of bar element is given by as (Petyt 2010)

$$U_{bar} = \frac{1}{2}\int_{-a}^{+a} EA\varepsilon_x^2 dx \tag{2.20}$$

where E is modulus of elasticity and A is cross-sectional area.

The axial strain component can be expressed in terms of the axial displacement $u(x)$, by means of the following relation

$$\varepsilon_x = \frac{\partial u}{\partial x} \tag{2.21}$$

Substituting Equation 2.13 into Equation 2.12 the potential energy becomes

$$U_{bar} = \frac{1}{2}\int_{-a}^{+a} EA\left(\frac{\partial u}{\partial x}\right)^2 dx \tag{2.22}$$

The potential energy stored in the beam element is given by

$$U_{beam} = \frac{1}{2}\int_{-a}^{+a} EI_z\left(\frac{\partial_2 v}{\partial x^2}\right)^2 dx \tag{2.23}$$

where I_z is the second moment of area.

The stiffness matrix for bar element $[k_e^{bar}]$ is expressed by considering the frame displacement vector given by Equation 2.11 as

$$[k_e^{bar}] = \begin{bmatrix} \frac{AE}{2a} & 0 & 0 & \frac{-AE}{2a} & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & \frac{AE}{2a} & 0 & 0 \\ & sym. & & & 0 & 0 \\ & & & & & 0 \end{bmatrix} \tag{2.24}$$

The stiffness matrix for beam element [k_e^{beam}] is expressed by considering the frame displacement vector given by Equation 2.11 as

$$[k_e^{beam}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & \frac{3EI_z}{2a^3} & \frac{-3EI_z}{2a^2} & 0 & \frac{-3EI_z}{2a^3} & \frac{3EI_z}{2a^2} \\ & & \frac{2EI_z}{a} & 0 & \frac{-3EI_z}{2a^2} & \frac{EI_z}{a} \\ & & & 0 & 0 & 0 \\ & sym. & & & \frac{3EI_z}{2a^3} & \frac{-3EI_z}{2a^2} \\ & & & & & \frac{2EI_z}{a} \end{bmatrix} \quad (2.25)$$

The stiffness element matrix for a frame element is obtained by combining [k_e^{truss}] and [k_e^{beam}] as

$$[k_e] = \begin{bmatrix} \frac{AE}{2a} & 0 & 0 & \frac{-AE}{2a} & 0 & 0 \\ & \frac{3EI_z}{2a^3} & \frac{-3EI_z}{2a^2} & 0 & \frac{-3EI_z}{2a^3} & \frac{3EI_z}{2a^2} \\ & & \frac{2EI_z}{a} & 0 & \frac{-3EI_z}{2a^2} & \frac{EI_z}{a} \\ & & & \frac{AE}{2a} & 0 & 0 \\ & sym. & & & \frac{3EI_z}{2a^3} & \frac{-3EI_z}{2a^2} \\ & & & & & \frac{2EI_z}{a} \end{bmatrix} \quad (2.26)$$

2.3.3. Element Kinetic Energy and Mass Matrix

The kinetic energy expressions for the bar and beam elements are given as

$$T_{bar} = \frac{1}{2}\int_{-a}^{+a} \rho A \dot{u}^2 dx \quad (2.27)$$

$$T_{beam} = \frac{1}{2}\int_{-a}^{+a} \rho A \dot{v}^2 dx \quad (2.28)$$

By using the same procedure used for stiffness matrices, the mass matrix of the frame element is obtained as

$$[m_e] = \frac{A\rho a}{105}\begin{bmatrix} 70 & 0 & 0 & 35 & 0 & 0 \\ & 78 & 22a & 0 & 27 & -13a \\ & & 8a^2 & 0 & 13a & -6a^2 \\ & & & 70 & 0 & 0 \\ & sym. & & & 78 & -22a \\ & & & & & 8a^2 \end{bmatrix} \tag{2.29}$$

2.3.4. Global Mass and Stiffness Matrix

[k_e] and [m_e] are expressed in the local co-ordinate system shown in Figure 2.1 as *x*-*y*. To obtain the global element matrices, all the matrices must be expressed in the global coordinate system which is shown in Figure 2.1 as *X*-*Y*. To do this, global displacement vector of which component shown in Figure 2.3 is defined as

$$\{D_e\} = \begin{Bmatrix} D_{3i-2} \\ D_{3i-1} \\ D_{3i} \\ D_{3j-2} \\ D_{3j-1} \\ D_{3j} \end{Bmatrix} \tag{2.30}$$

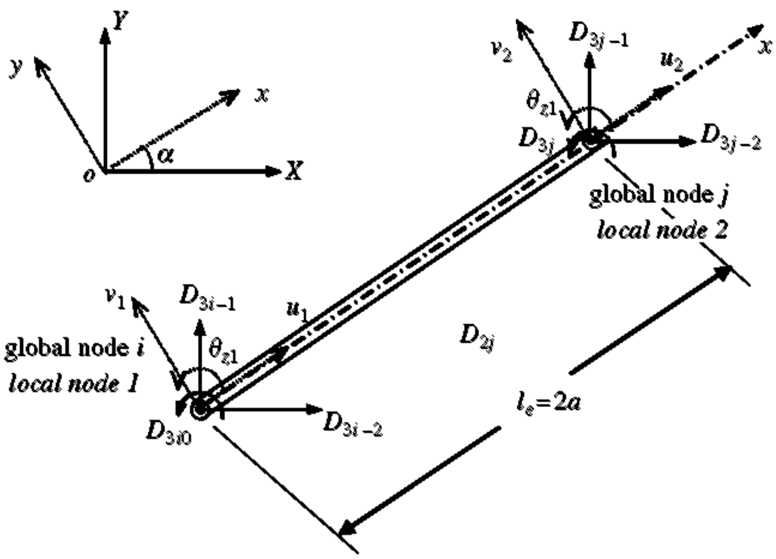

Figure 2.5. Frame element in global coordinates (Source: Liu, 1993)

The coordinate transformation between the displacement vectors are written as

$$\{d_e\} = [T]\{D_e\} \tag{2.31}$$

where [T] is the co-ordinate transformation matrix based on Figure 2.3 and expressed as

$$[T] = \begin{bmatrix} l_X & m_x & 0 & 0 & 0 & 0 \\ l_y & m_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_X & m_x & 0 \\ 0 & 0 & 0 & l_y & m_y & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.32}$$

where l_x=cosα, l_y=-sinα, m_x=sinα and m_y=cosα which are the direction cosines of the axial axis of the element.

Using the transformation matrix [T] the mass and stiffness matrix for frame element in the global coordinate system can be obtained. Therefore, the stiffness and mass matrices of frame in global coordinate system are obtained by the following equations:

$$[K_e] = [T]^T [k_e][T] \tag{2.33}$$

$$[M_e] = [T]^T [m_e][T] \tag{2.34}$$

2.3.5. Equation of Motion

The general differential equation of a forced vibration is given by

$$[M]\{\ddot{x}(t)\} + [D]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \tag{2.35}$$

where [M], [C] and [K] are mass, damping and stiffness matrices, respectively. Also, $\{x(t)\}$ is displacement vector and $\{f(t)\}$ is force vector.

The proportional damping model made a significant contribution at the early development of modal analysis. A structure with proportional damping can be analysed using the theory for an undamped MDoF system. Rayleigh indicated in his work "The Theory of Sound", first published in 1845, that if the viscous damping matrix [C] is proportional to mass and stiffness matrices (or that if the damping forces are proportional to the kinetic and potential energies of the system), then it can be written as (He and Fu 2001) :

$$[C] = \alpha[M] + \beta[K] \tag{2.36}$$

2.3.6. Natural Frequencies and Mode Shapes

Considering the harmonic motion for displacement and force vextors in the general differential equation, the following generalized eigenvalue equation is written:

$$([K] - \omega_i^2[M])\{u_i\} = \{0\} \tag{2.37}$$

where ω_i is i^{th} natural frequency of the system. $\{u_i\}$ in Equation given in Section on "Natural Frequencies" is the i^{th} vibration mode shape vector.

2.4. Experimental Modal Analysis

2.4.1. Measurement Hardware

The experimental modal analysis generally requires several hardware components. The basic setup depends on a few major factors. These include the type of structure to be tested and the level of results desired. The hardware elements required consist of a source of excitation for providing a known or controlled input to the structure, a transducer to convert the mechanical motion of the structure into an electrical signal, a signal conditioning amplifier to match the characteristics of the transducer to the input electronics of the digital data acquisition system, and an analysis system (or analyzer), in which signal processing and modal analysis programs reside. Figure 2.4 shows a diagram of a basic test system configuration.

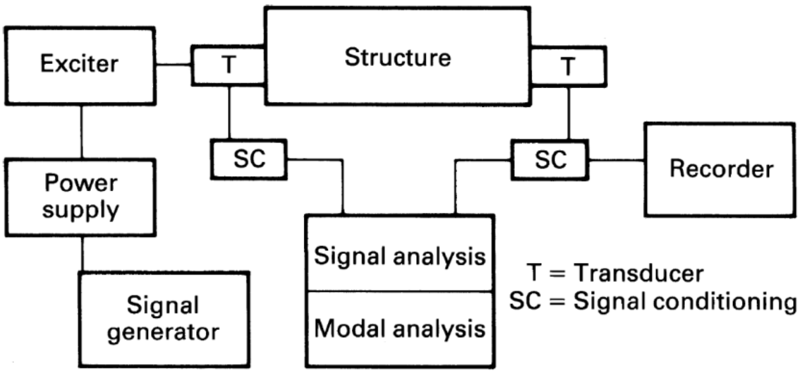

Figure 2.6. General test configuration

(Source: Inman 2006)

2.4.2. Signal Processing

The task of the analyzer is to convert analog time domain signals into digital-frequency-domain information. By using this information, the analyzer performs the required computations. At this point, a Fourier transform is used to alter an analog signal, $x(t)$, into frequency-domain information.

A periodic time signal of period T can be represented by a Fourier series in time as follows:

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_t t + b_n \sin n\omega_t t \right) \tag{2.38}$$

where

$$\omega_t = \frac{2\pi}{T} \tag{2.39}$$

$$a_0 = \frac{2\pi}{T} \int_0^T F(t) dt \tag{2.40}$$

$$a_n = \frac{2}{T}\int_0^T F(t)\cos n\omega_t t dt \qquad n = 1,2... \qquad (2.41)$$

$$b_n = \frac{2}{T}\int_0^T F(t)\sin n\omega_t t dt \qquad n = 1,2... \qquad (2.42)$$

The signals (accelerometer or force transducer outputs) are in the time domain and the desired spectral properties are in the frequency domain. Figure 2.5 shows the various types of time history encountered and their Fourier series or Transforms.

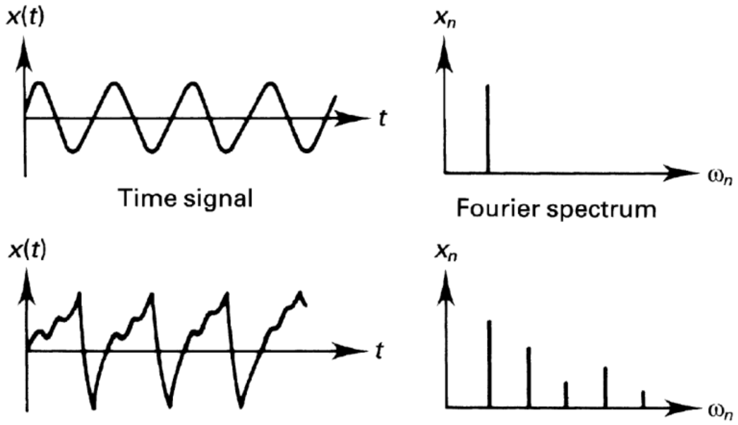

Figure 2.7. Some signals and their Fourier Spectrum

(Source: Inman, 2006)

2.4.3. Modal Data Extraction

After obtaining the FRF (frequency response function) $H(j\omega)$, the task is to compute the natural frequencies and damping ratios associated with each resonant peak of the measured FRF. There are several ways to examine the measured FRF to extract these data. To examine all of them, interested reader should consult well-known textbook written by Ewins (2000). SDOF Method or Peak Amplitude Model is summarized below:

This method works adequately for structures whose FRF exhibit well- separated modes. The method is applied as follows:

(i) First, individual resonance peaks are detected on the FRF plot, and the frequency of one of the maximum responses taken as the natural frequency of that mode (ω_r).

(ii) Second, the local maximum value of the FRF is noted and the frequency bandwidth of 'half-power points' is determined ($\Delta\omega$)).

(iii) The damping of the mode in question can now be estimated from one of the following formulae

$$2\xi_r = \frac{\omega_a^2 - \omega_b^2}{2\omega_r^2} \cong \frac{\Delta\omega}{\omega_r} \tag{2.43}$$

(iv) Last, the estimated modal constant of the mode is calculated as

$$A_r = 2\xi_r\omega_r^2 |H(j\omega)| \tag{2.44}$$

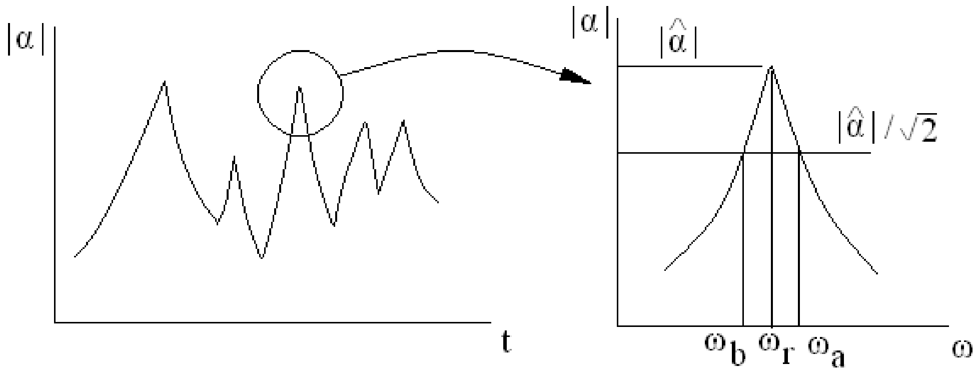

Figure 2.8. FRF (frequency response function)

(Source: Ewins, 2000)

CHAPTER 3

THEORETICAL MODAL ANALYSIS

3.1. Introduction

The portal frame is shown in Figure 3.1 and its properties are given in Table 3.1. It is seen from Figure 3.1 that portal frame has linearly tapered legs. In Table 3.1, horizontal and vertical frame members are numbered with 1, 2 and 3, respectively.

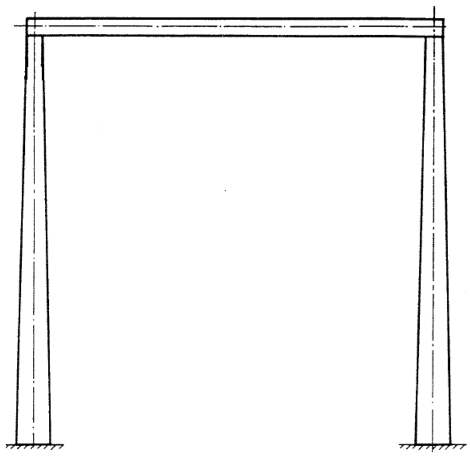

Figure 3.1. Portal Frame

Table 3.1. Geometrical and material properties of the frame structure

Property	Value
b_{01}, h_{01} (mm)	20
b_{02}, h_{02}, b_{03}, h_{03} (mm)	12
L_1, L_2, L_3 (mm)	250
E (MPa)	200000
ρ (tonne/mm^3)	7.85 10^{-9}
υ (-)	0

3.2. Finite Element Model

In ANSYS, 8-node 3D structural solid element (Solid45) is selected and setting the global size of the elements to 5, the solid model is meshed by 2228 elements.

3.3. Natural Frequencies

Natural frequencies found from ANSYS is listed in Table 3.2.

Table 3.2. Theoretical natural frequencies of portal frame

Mode	FEM [ANSYS] *f* (Hz)
1	243.86
2	669.78
3	1212.6
4	1225.2
5	2183.2
6	3057.7

3.4. Mode Shapes

Mode shapes are plotted in Figure 3.2-5.

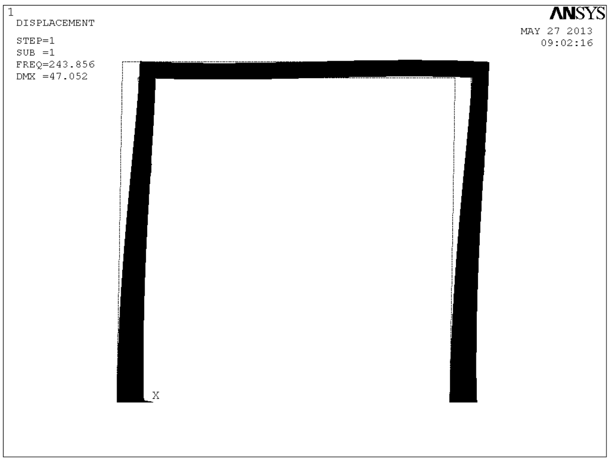

Figure 3.2. First mode shape

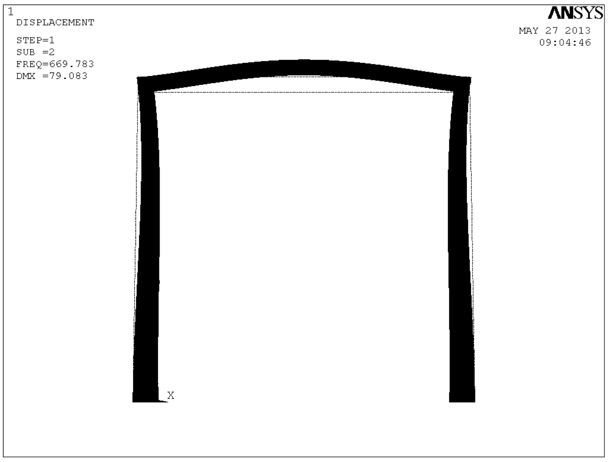

Figure 3.3. Second mode shape

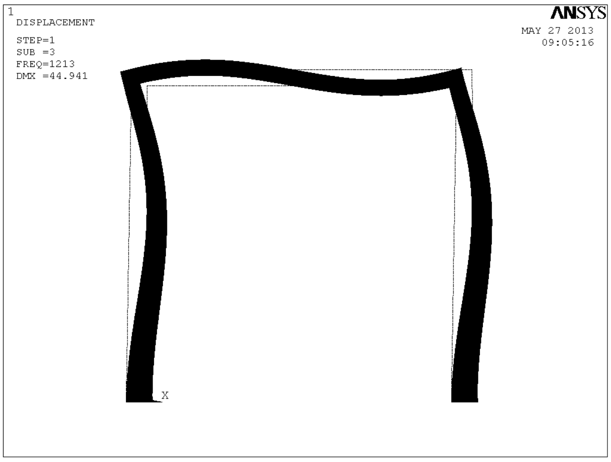

Figure 3.4. Third mode shape

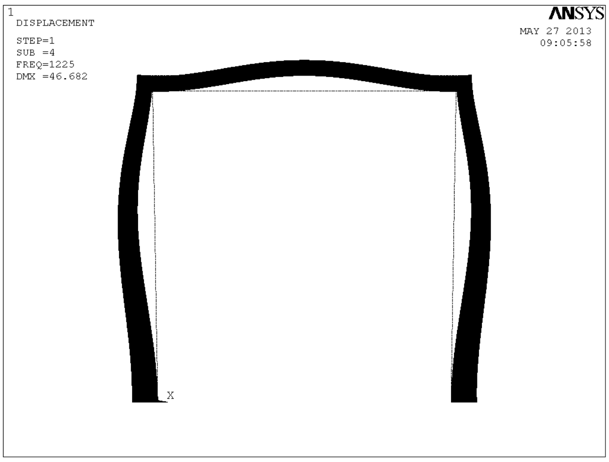

Figure 3.5. Fourth mode shape

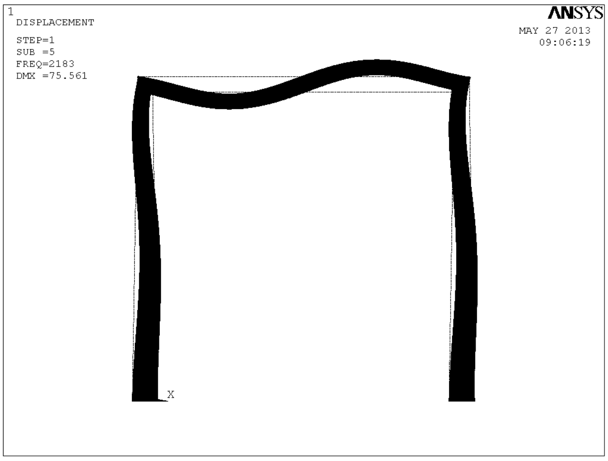

Figure 3.6. Fifth mode shape

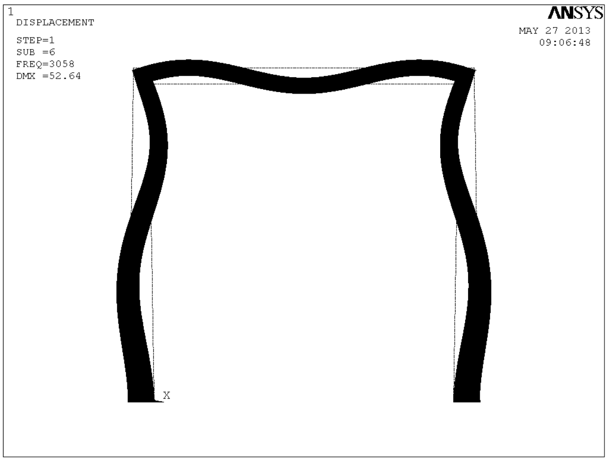

Figure 3.7. Sixth mode shape

CHAPTER 4

EXPERIMENTAL MODAL ANALYSIS

4.1. Experimental Setup

4.1.1. Portal Frame

The portal frame is shown in Figure 4.1 and its properties are given in Table 3.1.

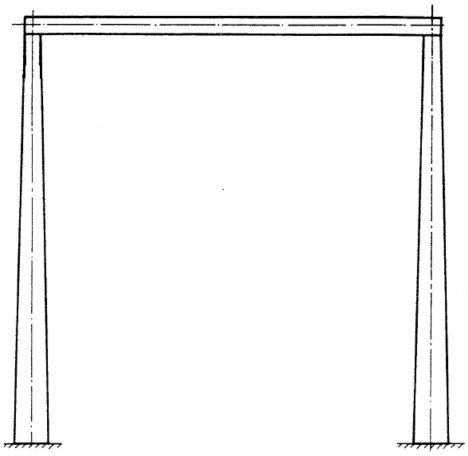

Figure 4.1. Portal Frame

4.1.2. Accelerometers

Kistler PiezoBEAM accelerometer Type:8632C50 ahown in Figure 4.2 is used.

Figure 4.2. Accelerometer

4.1.3. Coupler

Kistler Piezetron Coupler 5108A shown in Figure 4.3 is used.

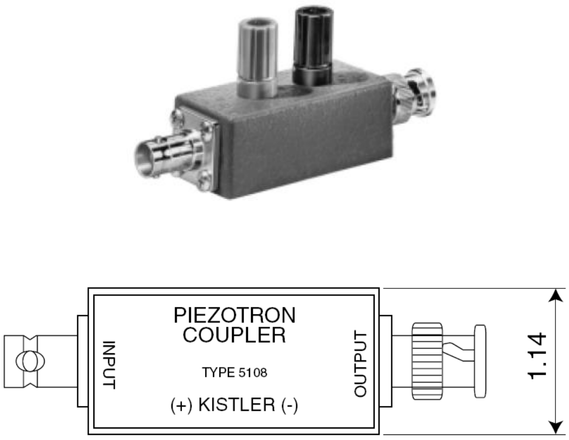

Figure 4.3. Coupler

4.1.4. Power Supply

DC 24 Volt, 2.08 A power supply is used.

4.1.5. Data Acquisition Board

DaqBoard/1000 is a high-speed, multi-function, plug-and-play data acquisition board for PCI bus computers. It features a 16-bit, 200-kHz A/D converter, digital calibration, bus mastering DMA, two 16-bit, 100-kHz D/A converters, 24 digital I/O lines, four counters, and two timers.

4.1.6. Signal Analyzer

DaqView Version 11 is used to see the natural frequencies.

4.2. Natural Frequencies

Natural frequencies found experimentally are listed in Table 4.1.

Table 4.1. Experimental natural frequencies of portal frame

Mode	*f* (Hz)
1	242
2	668
3	1215
4	1225
5	2190
6	3540

CHAPTER 5

DISCUSSION OF RESULTS

Present experimental results, the results of ANSYS, and percentage difference of FEM (% difference = 100*[FEM-Experimental]/ Experimental) are given in Table 5.1.

Table 5.1. Natural frequencies of portal frame

Mode	Experimental *f* (Hz)	FEM [ANSYS] *f* (Hz)	% error
1	242	243.86	0.77
2	668	669.78	0.27
3	1215	1212.6	-0. 2
4	1125	1225.2	0.02
5	2190	2183.2	-0.31
6	3540	3057.7	-13.6

It is seen from Table 5.1 that experimental and numerical results are close to each others.

CHAPTER 6

CONCLUSIONS

In this thesis, firstly vibration of frames are investigated and reported. In the chapter on theoretical background the following details are given: Finite Element Method for vibration analysis of frames, theoretical modal analysis and experimental modal analysis. Vibration analysis of portal frame is presented as natural frequencies and mode shapes by Finite Element Method.

Experimental setup has been prepared for Experimental Modal Analysis for the first time in the Department. Experimental modal tests have been performed. The obtained results are in good agreement.

REFERENCES

Albarracin, C.M. and Grossi, R.O. 2005. Vibration of elastically restrained frames. *Journal of Sound and Vibration* 285: 467-476

Bishop, R.E.D. 1956. The vibration of frames. *Proceedings of the Institution of Mechanical Engineers* 170: 955-968.

Ewins, D.J. 2000. Modal Testing: Theory and Practice. Baldock: Researh Studies Press Ltd

He, J. and Fu, Z.F. 2001. Modal Analysis. Oxford: Butterworth

Heppler, G.R., Oguamanam, D.C.D. and Hansen, J.S. 2003. Vibration of a two-member open frame. *Journal of Sound and Vibration* 263: 299-317.

Inman, D.J.2006. Vibration with control. New Jersey: John Wiley and Sons. Inc.

Liu, G.R. 1993.The Finite Element Method: A Practical Course. Oxford: Butterworth.

Lin, H.P. and Ro, J. 2003. Vibration analysis of planar serial-frame structures. *Journal of Sound and Vibration* 262: 1113-1131.

Maurice P. 1990. Introduction to finite element vibration analysis. Cambridge: Cambridge University Press

Segerlind, L.J. 1984. Applied Finite Element Anlysis. New York: John Wiley and Sons. Inc.

Tse, F.S., Morse I.E. and Hinkle, R.T. 1978. Mechanical Vibrations Theory and Applications. Boston: Allyn and Bacon, Inc.

Türker, T., Kartal, M.E., Bayraktar, A. and Muvafik, M. 2009. Assessment of semi-rigid connections in steel structures by modal testing. *Journal of Constructional Steel Research* 65: 1538-1547.

Wu, J.J. 2004. Finite element modelling and experimental modal testing of a three-dimensional framework. *Journal of Sound and Vibration* 46: 1245-1266.