BIOKINEMATIC ANALYSIS OF HUMAN ARM

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Anyone who has never made a mistake
has never tried anything new…

<u>Albert Einstein</u>

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ABSTRACT

BIOKINEMATIC ANALYSIS OF HUMAN ARM

Theory of Machines and Mechanisms is one of the main branches of science including many sub-branches such as biomechanics, human machine systems, computational kinematics, mechatronics, robotics, design methodology, dynamics of machinery, gearings and transmissions, cams and linkages, micro machines, nonlinear oscillations, reliability of machines and mechanisms etc. In this large area of interest, this study can be matched with the sub groups biomechanics, robotics, computational kinematics and design methodology. The main concern of the thesis is the biokinematics of the human arm. In the process of design, a suitable tool for the kinematics of human arm is investigated as quaternions along with examples. Moreover, the history of the formulas of Dof is presented as 38 equations with the unique key controlling parameters that are used in the design of new Cartesian and serial platform type robot manipulators. Structural syntheses of new manipulators are considered. Simple serial platform structural groups in subspace $\lambda=3$, and general space $\lambda=6$ are presented along with examples. Furthermore, type synthesis of human arm is accomplished with the new proposed parallel manipulator for the shoulder, elbow and wrist complex. Finally, computational kinematics of the serial human wrist manipulator and the geometrical kinematic analysis of the orientation platforms of the new parallel manipulator design for the human arm are accomplished.

ÖZET

İNSAN KOLUNUN BİOKİNEMATİK ANALİZİ

Ana bilim dallarından biri olan Makina ve Mekanizmalar Teorisi, birçok alanı kapsamaktadır. Bu alanlardan en önemlileri, biomekanik, insan-makina sistemleri, sayısal kinematik, mekatronik, robotik, dizayn metodolojisi, makina dinamiği, dişliler ve aktarma organları, kamlar ve linkler, mikro mekanizmalar, lineer olmayan titreşimler ve mekanizma güvenilirliği olarak özetlenebilir. Bu kadar geniş bir alan içerisinde, ilgili çalışma biomekanik, robotik, sayısal kinematik ve dizayn metodolojisiyle ilişiklendirilebilinir. Tezin ana konusu, insan kolunun biokinematik analizidir. Dizayn süreci içerisinde, insan kolunun kinematik analizinde kullanılacak önemli bir araç olan quaternionlar, örneklerle incelenmiştir. Yapısal sentez formullerinin tarihi araştırılmış, 38 farklı denklemde açıklamalarıyla birlikte gösterilmiştir. Bu formuller yeni Kartezyen ve seri platform tipli robot manipulatörlerin dizaynında kullanılmıştır. Yeni tasarlanan robot manipulatörlerin yapısal sentezi yapılmış, seri platform robot manipulatörlerinin alt uzay $\lambda=3$ te ve uzay $\lambda=6$ daki yapısal grupları gösterilmiştir. İnsan kolunun kategori sentezi tamamlanmış, omuz, dirsek ve bilek kompleksi için yeni bir parallel robot manipulatör önerilmiştir. Son olarak seri insan bilek manipulatörünün sayısal kinematiği ile birlikte, yeni paralel manipulatörün oryantasyon platformlarının geometrik kinematik analizi yapılmıştır.

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CHAPTER 1

INTRODUCTION

Kinematic analysis of robot manipulators can be carried out by using many tools, such as screw theory, quaternions, biquaternions, rotation and transformation matrices etc. Each has its own advantages and disadvantages when compared in different tasks; for instance, pure rotation motions can be easily and precisely analysed by quaternion operators, while rotation matrices yield computational errors and lack computational efficiency. On the other hand, quaternions can not be used solely in the analysis of translation motions, where transformation matrices are capable. As a result of the fact, it is very important to select the right tool for the desired application for the ease of use. From this point of view, the first step of the scientific investigation was assigned to find the best promising tool for the analysis of human arm motion.

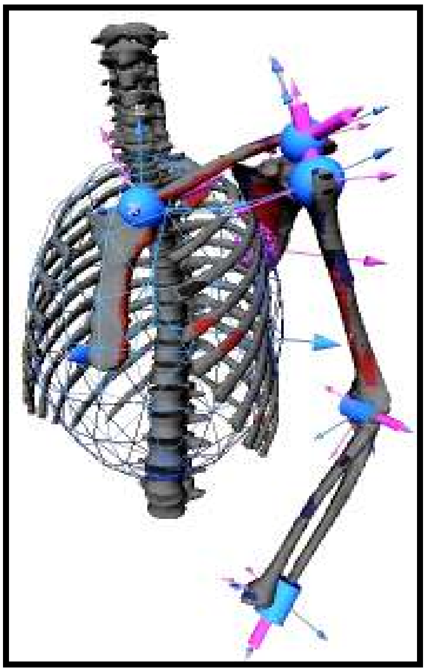


Figure 1.1. Human Shoulder, Elbow, and Wrist Complex
(Source: CMBBE'99)

After a detailed research on human shoulder, elbow, and wrist complex, (*Fig.* 1.1), it was appeared that nearly all of the joints of the complex have limited spherical motions; therefore, analysis of rotations has taken a great priority. So that, due to their precision, computational speed and efficiency in rotations, quaternions were selected as the tool for motion analysis of the human arm, (*Fig.* 1.2).

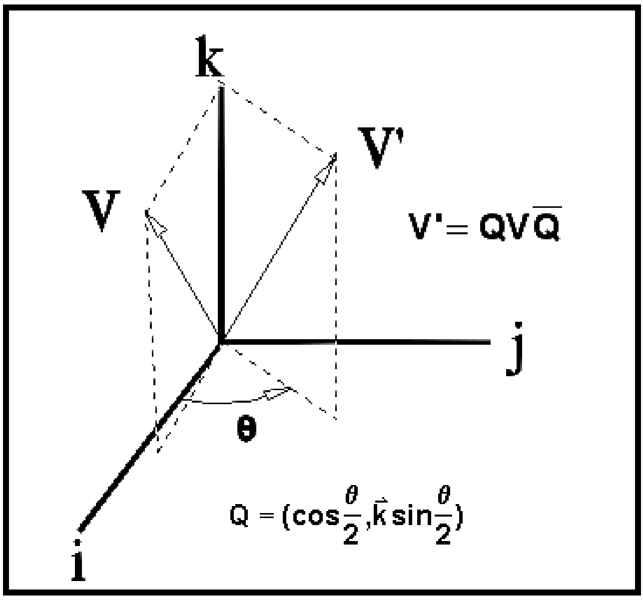


Figure 1.2. Rotation by a Quaternion Operator
(Source: ARTEMMIS)

The investigation was continued by the study of quaternion algebra and operators for rotations. However, it was seen that, regular sequential rotation method of quaternions can not simulate the exact human motion. Thus, for one step ahead to reach the natural motion of human, a new rotation sequence by quaternions was developed and named *modular method*. The new method was applied to the serial 2-DoF, 3-DoF and 4-DoF spherical wrist and compared by the traditional sequential method to prove the results. Including a new representation of quaternion rotation operator, the *modular method* was proposed in *IFToMM International Workshop of Computational Kinematics, CK2005 Italy*.

Later in time, another important concept in the design of robot manipulators was started to be investigated; that is, structural synthesis. Being one of the most important steps in design, structural synthesis provides the calculation of the desired degrees of freedom of the robot manipulators. Over 250 years, starting by Euler, many formulations have been created to fulfil the calculations for different types of robots; also, in each period new parameters have been introduced.

After collecting all of the information from a deep history about structural synthesis, the first time in literature, a compact table including all of the structural formulas of the related years and authors were introduced with the definitions of parameters that are used in the formulations. Using the collected information and table as a guideline, a new structural formula for Cartesian robot manipulators was proposed and nine new Cartesian robot manipulators were constructed with respect to the new formulation. Parallel to this study, structural synthesis of serial platform type manipulators with lower and higher kinematic pairs according to their structures was also examined. Serial platform manipulators were created according to the development of the platforms and closed loops by the new interpretation of the Alizade formula. Also structural groups of serial platform manipulators in subspace $\lambda = 3$ and space $\lambda = 6$ were tabulated in two separate tables, (*Fig.* 1.3). The complete work about structural synthesis was accepted by *IFToMM Journal of Mechanism and Machine Theory (MMT40-129)*.

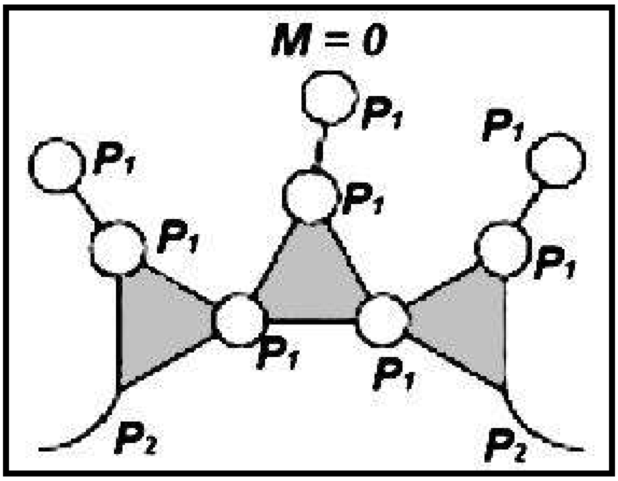


Figure 1.3. One of the Structural Groups of Serial Platform Manipulators in $\lambda = 3$

The last part of the investigation was the design of a new manipulator with variable general constraints that mimics human shoulder, elbow and wrist complex. The manipulator was designed with two orientation platforms in space $\lambda = 6$ and one spherical platform in subspace $\lambda = 3$. After its structural synthesis was completed, and the animations were carried out, geometrical kinematic analysis of its orientation platforms was accomplished.

CHAPTER 2

HISTORY OF DEVELOPMENT

2.1. Background

Hypercomplex numbers, quaternions, give a wide field of applications in the area of computational kinematics. First applications, which quaternions have been found more than 150 years ago, were the description of motion of the rigid body (Hamilton 1866, WEB_1 2005). Hypercomplex numbers allow simplifying the practical calculations in a drastic way. At the same time they are applied to such problems of modern computational kinematics (Porteous 1921, Martinez et al. 2000). From geometric point of view, a quaternion is the quotient of two directed lines in space, or operator, which changes one directed line into another. Sir W.R. Hamilton (Britannica 1886) describes that, if motion in one direction along a line is treated as positive, motion in the opposite direction along the same line is negative.

In the computational kinematics of a rigid system, we have to consider one set of rotations with regard to the axes that are fixed in the system. Using usual methods, we have a problem of complexity. Each quaternion formula is a preposition in spherical trigonometry and the singular quaternion operator $q\ (\ )\ q^{-1}$ turns any directed line, conically, through a definite angle about a definite axis (Hamilton 1866).

The topological geometry in spatial kinematics is discussed in (Porteous 1921), and the representation of spherical displacements and motions are described by the rotation group of unit quaternions.

Angeles (1988) introduced the theory of vector and scalar invariants of a rotation tensor as a function of time of a spherical motion. Nixravesh et al. (1985) introduced the method which is based on a sequence of matrix computation, and identities for relating a representation of spherical motion with their corresponding velocity and acceleration vectors. Larochelle (2000) used planar quaternions to create synthesis equations for planar robots, and created a virtual reality environment that could promote the design of spherical manipulators. Martinez et al. (2000) presented quaternion operators for describing the position, angular velocity and angular acceleration for a spherical motion of a rigid body with respect to the reference frame. Liu et al. (2003) described the physical model of the solution space for the spherical 3-DoF serial wrists, the classification of the reachable and dexterous workspace, and the atlases of the work spaces.

When an end effector of the spherical 3-DoF serial wrist reaches the tools, it will work as a spherical four-bar mechanism with 1-DoF. Several discussion of the design of spherical four-bar mechanisms widely studied in the literature and in the last one was the study of Alizade et al. (2005) that applied superposition method for linearization of nonlinear synthesis equations in the problem of analytical synthesis described by five precision points.

Structural synthesis problem is the first step in the design of new robot manipulators and the fundamental concept in robot design. The mobility of robotic mechanical system describes the number of actuators needed to define the location of end-effectors. It is important that the mobility or the degrees of freedom of robot manipulators (M>1) indicates the number of independent input parameters to solve the problem of all the configuration of robots or a kinematic chain with several actuators. If mobility of the kinematic chain is equal to zero (M=0) and can not be split into several structural groups, we will get a simple structural group. Combining the simple structural groups with actuators, we can get serial or parallel robot manipulators. IFToMM terminology defines “manipulator that controls the motion of its end-effector by means of at least two kinematic chains going from the end-effector towards the frame” as parallel manipulator. In parallel manipulators, two platforms can not be connected by kinematic pairs to each others.

Serial platform manipulators control the motion of the platforms by means of at least two platforms, which are connected by kinematic pairs, and other kinematic chains going from the platforms towards the frame. Several connections of the links in series for gripping and the controlled movement of objects are called serial manipulators. Combination of serial and parallel manipulators gives hybrid robot manipulators. Complex robot manipulators consist of independent loops with variable general constraint (λ=2, 3, 4, 5, 6).

The history of works about the number of independent loops was done by L. Euler (Courant 1996). Then in the second half of the XIX century, the first structural formulas of mechanisms were created (Chebyshev 1869, Sylvester 1874, Grübler 1883, Somov 1987, Gokhman 1889). As shown in *Table 2.1*, in the mobility equations we can find concepts of the number of independent loops (L), degrees of freedom or mobility of mechanisms (M), the loop motion parameters (λ), the number of joints (j), number of moving links (n), number of mobility of kinematic pairs (f), independent joint constraints (s), number of passive mobilities (j_p), and the number of overclosing constraints (q). To describe and compare the structural formulas and the parameters in structural analysis and synthesis of robotic mechanical system, the unique key controlling parameters are used as shown in *Table 2.1*.

Furthermore, the concepts of the structural formulas and simple structural groups were developed in the first half of the XX century (Koeings 1905, Assur 1952, Muller 1920, Malushev 1923, Kutzbach 1929, Kolchin 1932, Artobolevskii 1939, Dobrovolskii 1939). As shown in *Table2.1,* some new concepts in the problem of structural analysis and synthesis of mechanisms had been reached as number of screw pairs (S_c), simple structural groups with zero mobility (M=0), number of kinematic pairs with i class (p_i, where i is the number of joint constraint), number of links with variable length (n_v), variable general constraint (λ_K), and the family of the elementary closed loop ($d_K = 6 - \lambda_K$).

During the second half of the XX century, the productive results to find general methods for determination of the mobility of any mechanisms had been obtained (Moroshkin 1958, Voinea et al. 1959, Paul 1960, Rössner 1961, Boden 1962, Ozol 1962, Waldron 1966, Manolescu 1968, Bagci 1971, Antonescu 1973, Freudenstein et al.1975, Hunt 1978, Herve 1978, Gronowicz 1981, Davies 1981, Agrawal et al. 1987, Dudita et al. 1987, Angeles et al. 1988, Alizade 1988, McCarthy 2000). In the calculation of mechanism mobility, the following new parameters were used *(Table 2.1) :* rank of linear independent loop equations or the order of the equivalent screw system of the closed loop (r), relative displacements of the joint (m), number of independent, scalar, differential loop-closure equations (λ_K), the rank of the coefficient matrix (r(j)), finite dimensional vector space (d(v)), new formula of the number of independent loops ($L = j_B - B - c_b$, where j_B is the total number of joints on the platforms, and c_B is the total number of branches between moving platforms and B is the number of moving platforms), serial open chains connecting to ground or total number of robot legs (c_l), and the degree of constraint of the platform (U). It should be noted that, branches are the kinematic chains that connects mobile platforms to each other, and legs are the kinematic chains that connects mobile platforms to the fixed frame.

In the beginning of XXI century, further developments of robotic science has arisen the interest in scientific investigations. New parameters in the structural formulas describing the real physical essences should be created in the new investigations and be more suitable for the use in practice in new subjects. In this direction, there are several studies (Huang 2003, Alizade et al. 2004, Gogu 2005, Alizade et al. 2006). In the calculation of degrees of freedom of mechanisms, the new parameters are used in the structural formulas, as a new formulation of the number of independent loops (L=c-B) and new formulation for simple structural groups ($\Sigma f_i=\lambda(c-B)$), where $c=c_b+c_l+c_h$, c_h is the number of hinges between moving platform, and c is the total number of connections. Also note that, hinges are the revolute pairs that connect mobile platforms to each other. For describing and comparing structural formulas and parameters in structural analysis and synthesis of robotic mechanical system the unique key controlling parameters are used as shown in *Table 2.1*.

The basis of structural synthesis of manipulators are based on the principles of truss kinematical unchanging. Determination of indivisible groups as simple structural groups and creating different new manipulators by using their combination had been done by striving to systematize investigation methods of manipulators.

Firstly Assur (1982) developed the concept of the open chain and utilized this concept for plane structure classification. Secondly, the problem of structural synthesis and analysis was investigated by Malushev (1929). The problem of structural synthesis for spatial mechanisms was introduced by Artobolevskii (1939). The task of structural synthesis was solved by using method of developing closed loops. The classes of structural groups are defined by the number of links of the closed loops and the order is equal to the number of legs.

According to the method of structural synthesis that is given by Baranov (1952), spatial and plane structural groups have been created from corresponding trusses, and class of simple structural groups are defined by the number of closed loops. Kolchin (1960) has introduced concept of passive constraints to account for existence of the paradoxical mechanisms. That concept has not presented any means for identifying the geometric conditions that determine the general constraints.

Table 2.1. Formulas for Structural Analysis and Synthesis

	Equations	***Authors***	***Commentary***
1	$L = j - l + 1$ *l is the* number of links; j is the number of joints	L. Euler, 1752	L is the number of independent loops;
2	$3l_m - 2j - 1 = 0$ $0 < j - j_m < 1 + \frac{1}{2} l$ $j_m > l - 3 \qquad l_m = n = l - 1$	P. L. Chebyshev, 1869	Eq. for planar mech. with 1 DoF j_m is the number of moving joints l_m = n is the number of moving links
3	$3l - 2j - 4 = 0$ $j = n - 1$	J. J. Sylvester, 1874	Eq. for planar mech. with 1 DoF
4	$a)\ M = 3l - 2j - 3$ $b)\ 3l - 2j - 4 + q = 0$ $c)\ 2l - j - 3 = 0$ $d)\ 3l - 2j - 4 + q - C = 0$ $e)\ 5H - 6l + 7 = 0$ $or\ \ M = 6(l-1) - 5p_1$ q is the number of overclosing constraints p_1 is the one mobility joints C is the number of cam pairs H is the number of helical joints	M. Grübler, 1883, 1885	M is mobility of mechanisms. DoF depends from the rank of functional determinant (r=3, 2) a) DoF for planar mech. b) Eq. for kinematic chains with revolute R and prismatic P pairs c) Eq. for plane mech. just with prismatic P pairs d) Eq. for kinematic chains with revolute, prismatic and cam pairs e) DoF of spatial mech. with helical joints
5	$a)\ l - (\lambda - 1)(\nu + 1) = 2$ $b)\ l + q + \sum K_u - (\lambda - 1)(\nu + 1) = 2$ $c)\ M = (l-1) + \sum f_i - j - 5L + q$ $l = 5\nu + 7,\ \ \lambda = 6,\ \ \nu = L - 1,$ $\sum K_u = j_p - 1$ j_p is the passive mobilities in the joints f_i is the mobility of kinematic pairs	P.O. Somov, 1887	a) Eq. for plane (λ=3) and spatial (λ=6) mech. (M=1) b) Eq. for plane and spatial mech. (M=1) c) Somov's universal structural formula λ is the number of independent parameters describing the position of rigid body (general constraint parameter)
6	$a)\ \lambda(l-1) - S = 1$ $S = \sum(\lambda - i) f_i$ is the total number of independent joint constraints $b)\ \sum f_i - \lambda L = 1$ $c)\ \lambda(j - L) - S = 1$	Kh. I. Gokhman, 1889	a) Eq. for plane and spatial mech. (M=1) b) Loop mobility criterion (M=1) c) Eq for mech. (M=1) Eqs. (a) and (c) gives Euler's equation
7	$M = 6n - S$	G. Koeings, 1905	Mobility Eq. for spatial mech. (similar to Gokhman Eq.)
8	$3n - 2j = 0$	L. V. Assur, 1916	Eq. for simple structural groups
9	$(\lambda - 1)S_s - \lambda l + (\lambda + 1) = 0$ $M = \lambda n - (\lambda - 1)S_s$ S_s is the number of screw pairs	R. Muller, 1920	Eq. for kinematic chains with screw pairs (Similar to M. Grubler Eq.)

Table 2.1 (cont.). Formulas for Structural Analysis and Synthesis

	Equations	***Authors***	***Commentary***
10	$M = 6(l-1) - \sum_{i=1}^{5} ip_i + q - n_v$ p_i is the kinematic pairs with i class i = number of joint constraint	A. P. Malushev, 1923	Universal Somov-Malushev's mobility Eq. n_v is the number of links with variable length
11	$M = \lambda(l - j - 1) + \sum_{i=1}^{j} f_i$ $M = \lambda(l-1) + \sum_{i=1}^{j} (\lambda - i) f_i$	K. Kutzbach, 1929	Other form of universal mobility Eq.
12	$M = 3(l-1) - 2(P + R + K) - p_2$ P is the number of prismatic pairs R is the number of revolute pairs	N. I. Kolchin, 1932-1934	Structural formula for planar mechanisms. K is the number of higher pairs with pure roll or pure slippage p_2 is the number of higher pair with rolling and slipping
13	$M = 6n - \sum_{i=1}^{j} S_j + \sum_{K=1}^{L} d_K + q$ $d_K = 6 - \lambda_K$ is the family of the elementary closed loop or the number of independent constraints in the loops	I. I. Artobolevskii, 1935	Other form of universal mobility Eq. First time in mobility Eq., it is used variable general constraint as variable number of independent close loops family. λ_K is the variable general constraint
14	$M = \lambda n - \sum_{i=1}^{\lambda-1} (\lambda - i) p_i + q$ $\lambda = 2, \ldots., 6$	V. V. Dobrovolskii, 1939	Other form of universal structural formula
15	$a) M = \sum_{i} ip_i - r$ $b) M = \sum_{i} ip_i - \sum_{\lambda} \lambda L_\lambda$ $i = 1, \ldots, 5 \quad \lambda = 2, \ldots, 6$ $c) L = j - n$	U. F. Moroshkin, 1958	a) Structural Eq. of system with the integrable joining b) Eq. of the DoF with variable general constraint c) Number of independent close loops $r = \lambda$ is the rank of linear independent loop
16	$M = \sum_{i=1}^{j} f_i - \sum_{K=1}^{L} r_K - j_p$ $\sum_{i=1}^{j} f_i$ is the total number DoF of joints with revolute, prismatic and helical joints;	R. Voinea and M. Atanasiu, 1959	Mobility Eq of a complex mechanisms $1 \le r_K \le 6$ is the rank of screw system
17	$L = j - l + 1$	B. Paul, 1960	Using formula #1, it was created topological condition of criterion for the degree of constraint of plane kinematic chains
18	$M = \sum_{i=1}^{j} f_i - 6(j - l + 1)$	W. Rössner , 1961	The mobility Eq. taking into consideration Euler's formula # 1

Table 2.1 (cont.). Formulas for Structural Analysis and Synthesis

	Equations	***Authors***	***Commentary***
19	$M=\sum_{i=1}^{j} f_i-6(j-l+1)-3(j-l+1)$	H. Boden, 1962	Mobility Eq., consisting from the planar and the spatial loops
20	$a)\ M=\sum_{i=1}^{j} f_i-6L+q$ $b)\ M=\sum_{i=1}^{j} f_i-3L+q$ $c)\ M=2(l-1)-j+q$ $d)\ M=j-2L+q$	O. G. Ozol, 1962	a), b), and c) mobility Eq.s for variable general constraint, as λ=6, 3, 2 with excessive constraints d) mobility Eq. for cylindirical mechanisms (λ=2)
21	$M=F-r$ F is the relative freedom between links	K. J. Waldron, 1966	Mobility Eq of closed loop r is the order of the equivalent screw system of the closed loop
22	$M=\sum_{i=r+1}^{5}(6-i)p_i-(6-d)L$	N. Manolescu, 1968	Mobility Eq. with the parameter of the family of the elementary closed loop.
23	$M=6(l-1)-\sum_{i=1}^{5}(6-i)f_i+\sum_{K=1}^{L}d_K+$ $+\sum q-\sum j_p$	C. Bagci, 1971	Mobility Eq. to calculate DoF of motion in a mechanism similar to Eq. # 13 by adding parameter j_p
24	$M=(6-d_a)(l-1)-\sum_{i=1}^{5}(i-d_a)p_i$	P. Antonescu, 1973	Mobility formulas with different values for the motion coefficient λ (formula #14)
25	$a)\ M=\sum_{i=1}^{E} m_i-\sum_{K=1}^{L}\lambda_K$ $b)\ M=\sum_{i=1}^{j} f_i-\sum_{K=1}^{L}\lambda_K$ $c)\ M=\sum_{i=1}^{E} m_i-\lambda L$ $d)\ M=\sum_{i=1}^{j} f_i-\lambda L$ $\lambda=2,3,4,5,6$ E is the total number of independent displacement variable m_i is the relative displacements of the joints f_i is the relative joint motion when m_i correspond in 1:1 with DoF in joints	F. Freudenstein, R. I. Alizade, 1975	Mobility Eq.s without exception a) and b) mobility Eq.s are used for mechanisms which contain mixed independent loops with variable general constraint. c) and d) Mobility equations of mechanisms with the same number of independent, scalar loop closure equations in each independent loop. λ_K is the number of independent, scalar, differential loop closure equations λ is the DoF of space where the mechanism operates
26	$M=\lambda(l-j-1)+\sum_{i=1}^{j} f_i$	K. H. Hunt, 1978	Mobility Eq. coming from Eq. 25d using Eq 1
27	$M=\lambda(l-1)-\sum_{i=1}^{j}(\lambda-f_i)$	J. M. Herve, 1978	Mobility formula based on the algebraic group structure of the displacement set

Table 2.1 (cont.). Formulas for Structural Analysis and Synthesis

	Equations	***Authors***	***Commentary***
28	$M = \sum \lambda_K - \sum_{K=1}^{L-1} \sum_{j=K+1}^{L} F_{Kj}$ F_{Kj} is the mobility of the joints that is common between any two loops K and j, and the mobility of the joints in the L loops can be counted once or twice	A. Gronowicz, 1981	Mobility Eq. for multi loop kinematic chains
29	$M = \sum_{i=1}^{j} f_i - r$	T. H. Davies, 1981	Mobility equations similar to Eq. # 15a r is the rank of the coefficient matrix of constraint equations
30	$M = \sum_{K=1}^{L} \lambda_K - \sum_{K=1}^{L-1} \sum_{j=K+1}^{L} F_{Kj} + \sum_{i=1}^{N_1} \frac{1}{2}(\tilde{n}_i^2 + \tilde{n}_i - 2)F_{\tilde{n}i} +$ $+\sum_{i=1}^{N_2} \frac{1}{2}(n_i^2 - 3n_i + 2)F_{ni}$	V. P. Agrawal, J. S. Rao, 1987	Mobility Eq. to any general mechanism with constant or variable general constraints with simple or multiple joints N_1, N_2 is the total number of internal and external multiple joints respectively $\tilde{n}_i$, $F_{\tilde{n}i}$; n_i, F_{ni} is the number of links and the mobility of simple joints forming the i th internal and external multiple joints respectively.
31	$a)\ M = \sum_{i=1}^{j} f_i^e - \sum_{K=1}^{L} \lambda_K^e$ $b)\ M = \sum_{K=1}^{L} \lambda_K - \sum_{j} (L_{comj} - 1) f_{comj}^e$ L_{comj} is the number of loops with common joint j f_{comj}^e is the active degree of mobility of the j th common joint	F. Dudita, D. Diaconescu, 1987	Eq. of a elementary or a complex (multi loop) mechanisms f_i^e is the active mobilities in i th joint λ_K^e is the dimension of the active motion space
32	$M = nullity(J)$ $nullity(J) = d(v) - r(J)$ J is the Jacobian matrix; r(J) is the rank of the Jacobin matrix; d(v) is the finite dimensional vector space v	J. Angeles, C. Gosselin, 1988	The mobility Eq. by using the Jacobian matrix of a simple or multi loop closed kinematic chain without exception
33	$a)\ L = j_B - B - c_b$ $b)\ M = \sum_{i=1}^{E} m_i - \lambda(j_B - B - c_b) + q - j_p$ $c)\ M = \sum_{i=1}^{j} f_i - \lambda(j_B - B - c_b) + q - j_p$ $d)\ \sum_{i=1}^{j} f_i = \lambda(j_B - B - c_b)$ B is the number of mobile platform; j_B is the total number of joints on the mobile platforms	R. I. Alizade, 1988	a) A new formula of number of independent loops b) and c) are structural formulas as a function of number of branches, platforms and sum of mobility of kinematic pairs and other parameters d) Eq. for simple structural groups (λ=6,5,4,3,2) c_b is the total number of branches between mobile platforms

Table 2.1 (cont.). Formulas for Structural Analysis and Synthesis

	Equations	***Authors***	***Commentary***
34	$M = \lambda - \sum_{i=1}^{c_l}(\lambda - f_i)$ $(\lambda - f_i)$ is the degree of constraint of the platform	J. M. McCarthy, 2000	Mobility Eq. of a parallel manipulator
35	$M = (6-d)(l-j-1) + \sum_{i=1}^{j} f_i + q$	Z. Huang, Q .C. Li, 2003	Structural formula for parallel mechanisms
36	$a)\ M = \sum_{i=1}^{j} f_i - \lambda(c-B)$ $b)\ \sum_{i=1}^{j} f_i = \lambda(c-B)$ $c)\ L = c - B$, $c = c_l + c_b$, $c_l = j_B - 2c_b$	Rasim Alizade, Cagdas Bayram, 2003	a) MobilityEq. of mechanisms b) Eq.'s for simple structural groups. c) New formula of the number of independent loops c is the sum of legs and branches, c_l is the total number of legs, connecting mobile platforms to ground
37	$M = \sum_{i=1}^{j} f_i - \sum_{j=1}^{l} S_j + S_p$ S_p and S_j are spatialities of mobile platform and legs respectively	Grigore Gogu, 2005	Mobility Eq. for parallel mechanisms
38	$a)\ M = (B-c)\lambda + \sum_{i=1}^{j} f_i + q - j_p$ $b)\ M = (\lambda+3) + \sum_{l=1}^{c_l}(d_l - D) + \sum_{l=1}^{c_l}(f_l - \lambda_l) + q - j_p$ $c = c_l + c_b + c_h$ D is number of dimensions of vectors in Cartesian space d_l is number of dimensions of vectors in Subspace	Rasim Alizade, Cagdas Bayram, Erkin Gezgin, 2005	a) Mobility Eq. for robotic systems with independent loops with variable general constraint b)A new structural formula of mobility loop-legs equation for parallel Cartesian platform manipulators. λ is the general constraint parameters of simple structural group c_h is the number of hinges

The problem of general constraint parameter was done by Voinea et al. (1960) as the rank of the matrix of coefficients of the unknowns in a system of equations describing the angular velocities of the relative helicoidal movements. Ozol (1963) took a straight point in the theory of structural synthesis by the topological property of mechanisms.

The methods of structural synthesis were based on graph theory to find the set of kinematic chains and mechanisms (Crossley 1966, Dobrjanskyi 1966, Buchsbaym 1967, Freudenstein 1967, Dobrjanskyi et al.1967, Manolescu 1973). The problems of structural analysis and synthesis of plane and spatial structural groups of higher classes were done by Djoldasbekov et al. (1976). Determination of structural groups by using the principles of dividing joints and the method of developing joints were done by Dobrovolskii (1939), and Kojevnikov (1979). The structure theory of parallel mechanisms based on the unit of single-open chains was done by Yang (1983,1985), and the type synthesis of spatial mechanisms on the basis of spatial single loop was introduced by Alizade et al. (1985). The concept of dual graphs and their applications to the automatic generation of kinematic chains was done by Sohn et al. (1986).

A computer-aided method for structural synthesis of spatial manipulators by using method of developing mobile platforms and branches was done by Alizade et al. (2004), and Alizade (1988). Class of the structural group is defined by the number of mobile platforms, kind is defined by the set of joints on the mobile platforms, type of the structural group is determined by the number of branches between mobile platforms, and order describes the number of legs that connect mobile platforms to the ground. A computer-aided method for structural synthesis of planar kinematic chains was introduced by Hwang et al. (1986), and the concept of loop formation which cancels the necessity of the test for isomorphism was also introduced by Rao et al. (1995).

According to the structural synthesis of parallel mechanisms based on the unit of single-open chains, a class of 3 DoF (3 translation motion), 5 DoF (3 translation and 2 rotation), and 6 DoF (3 translation and 3 rotation) parallel robot manipulators were analyzed by Yang et al. (2001, 2002) and Shen et al. (2005).

On the biomechanics side, ISB (*International Society of Biomechanics*) proposed a definition on joint coordinate system for the shoulder, elbow, wrist and hand (Wu et al. 2004). For each joint in the complex, a standard for the local axis system in each movable bone or segment is generated. Stanisic et al. (2001) proposed a dexterious humanoid shoulder mechanism that can be used as a simple shoulder joint. In the investigation, kinematic equations of the mechanism are studied as well as its singular configurations. Okada et al. developed three degrees of freedom cybernetic shoulder that mimics the biological shoulder motion, has high mobility and has sensitive compliance.

Bosscher et al. (2003) proposed a novel mechanism to implement multiple collocated spherical joints that has a large range of motion. Bonev et al. (2005) presented the singularity loci of spherical parallel mechanisms. Gosselin et al. (1994) developed a high performance three degrees of freedom orienting device. When compared to its predecessors, the mechanism is the least singular one.

As mentioned above, the history approaches the same problems in different points of views. Although they have some distinctions, all investigations improve its area of interest one step ahead and provide information for future investigations.

2.2. Research Statement

In the path of computational kinematics, rotation matrices have proved much in many applications that are related to the position analysis of the rotating rigid bodies. However, having lack of computational efficiency, their usage has dramatically decreased in the applications, where rapid calculations are needed. Nowadays, as an alternative solution, quaternions are mostly being used for their fewer addition and multiplication operation requirements. In this investigation, discussing the quaternion algebra, not only a new method but also a new representation of quaternion operator for transformation will be introduced to the rotations by using quaternions, and the results are analyzed in the kinematics of a spherical wrist manipulator.

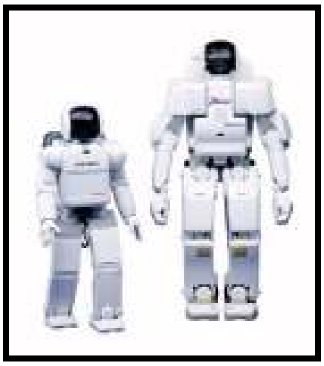


Figure 2.1. Primitive Human-like Robots
(Source: HONDA)

The parallel robot manipulators have precise positioning capability, good dynamic performance and high load carrying capacity. However, the 6 DoF parallel structures have poor workspace and the direct kinematic solution gives high coupling degree between independent loops. On the other hand, it is needed to design the given translation and rotation motion of mobile platform. Analysis of research topics mentioned in the history show that the systematic study of mobility equations of mechanical systems have been described from different points of view, but systematic study of structural synthesis is relatively weak. This investigation enunciates a new structural formula of mobility and new method of designing robot manipulators, the mobile platform that can generate general motion in space, and also generate constraint motion in subspaces. In the meantime, the structural synthesis of serial platform manipulators is identified according to the new equations for simple serial platform structural groups. General guidelines are presented with 9 new robot manipulators and tables of serial platform structural groups for designing new several serial platform manipulators.

As the developing technology gives us many possibilities, the world is near to the robots that mimic totally human motions. Due to the fact that, their mechanical designs and control system managements are not an easy task, for the current progress, most of the investigations have been done for the individual complexes; such as, human arm, leg, spine and neck. So that, future applications can combine all, and built a whole human-like system (*Fig. 2.1*). However, when compared to other areas, works on the biokinematics and robot prostheses are not sufficient. In this investigation, a new and alternative robot manipulator with variable general constraints that mimics human shoulder, elbow and wrist complex is proposed with its structural synthesis. Geometrical kinematic analysis of one of its orientation platforms is performed. By using the animation software, simulations are carried out for workspace purposes.

CHAPTER 3

QUATERNION ALGEBRA

William Rowan Hamilton searched for thirteen years for a system for the analysis of three-dimensional space. This search came to end in 1843 in four-dimensional space with his discovery of hyper-complex numbers of rank 4, named quaternions, one of the main systems of the vector analysis.

In general, quaternions are four dimensional numbers that have one scalar and one vector part. The vector part is obtained by adding the elements ***i***, ***j*** and ***k*** to the real numbers which satisfy the following relations:

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1 \tag{3.1}$$

Eq. (3.1) shows the main rule of Hamilton for dealing operations on the vector part of the quaternions. All of his concepts and ideas were developed in the light of this rule.

3.1. Preliminaries

Quaternions can be represented mainly by two alternative ways. As the name already suggests, they can be considered as the row of four real numbers that is represented by;

$$q = (q_0, q_1, q_2, q_3) \tag{3.2}$$

where, q_0, q_1, q_2 and q_3 are simply real numbers or scalars. Also, they can be denoted by scalar and vector parts as,

$$q = q_0 + \boldsymbol{q} \tag{3.3}$$

where, q_0 is some scalar and $\boldsymbol{q}$ is an ordinary vector in R^3. Eq. (3.3) can be extended to,

$$q = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3 \tag{3.4}$$

As seen in Eqs. (3.3-3.4), quaternions can be represented as the sum of scalar and vector, which is not defined in ordinary linear algebra. So that, it is important to express the operation procedures of the quaternions.

3.2. Quaternion Addition and Equality

Let us take two quaternions $q = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3$ and $p = p_0 + \boldsymbol{i}p_1 + \boldsymbol{j}p_2 + \boldsymbol{k}p_3$. These quaternions are equal if and only if they have exactly the same components, that is;

$$p = q \iff \begin{Bmatrix} p_0 = q_0 \\ p_1 = q_1 \\ p_2 = q_2 \\ p_3 = q_3 \end{Bmatrix} \tag{3.5}$$

In the addition case, the sum of two quaternions $p + q$ is described by adding the corresponding components of both quaternions, Eq. (3.6).

$$p + q = (p_0 + q_0) + \boldsymbol{i}(p_1 + q_1) + \boldsymbol{j}(p_2 + q_2) + \boldsymbol{k}(p_3 + q_3) \tag{3.6}$$

Due to the fact that there is no difference between the addition of quaternions and the row of four real numbers, quaternion addition satisfies the field properties that are applied to the addition.

The addition of two quaternions is again a new quaternion, so the set of quaternions are closed under addition, Eq. (3.7).

$$p + q = r$$
$$r = r_0 + \boldsymbol{i}r_1 + \boldsymbol{j}r_2 + \boldsymbol{k}r_3 \quad (3.7)$$

Also each quaternion has a negative or additive inverse where each component of the corresponding quaternion is negative, Eq. (3.8).

$$-r = -r_0 - \boldsymbol{i}r_1 - \boldsymbol{j}r_2 - \boldsymbol{k}r_3 \quad (3.8)$$

Moreover, there exists a zero quaternion, in which each component of the quaternion is "0", and the sum of any quaternion with the zero quaternion is again itself, Eq. (3.9).

$$p = 0 \Leftrightarrow \begin{Bmatrix} p_0 = 0 \\ p_1 = 0 \\ p_2 = 0 \\ p_3 = 0 \end{Bmatrix}$$
$$r + p = r \quad (3.9)$$

Finally, note that, the quaternion addition is commutative and associative, Eq. (3.10).

$$p + q = q + p$$
$$(p + q) + r = p + (q + r) \quad (3.10)$$

3.3. Quaternion Multiplication

When compared with the addition, quaternion multiplication is more complicated, except the multiplication by a scalar. Similar to the addition, multiplication of a quaternion by a scalar quantity is described by a quaternion, in which components of the corresponding quaternion is multiplied by the scalar Eq. (3.11).

$$Aq = p$$
$$p = Ap_0 + \boldsymbol{i}Ap_1 + \boldsymbol{j}Ap_2 + \boldsymbol{k}Ap_3 \quad (3.11)$$

On the other hand, if a quaternion is multiplied by another quaternion, more detailed procedure should be followed.

In the product of two quaternions, the fundamental rule of Hamilton, Eq. (3.1), should be satisfied. Eq. (3.1) can be opened as:

$$\begin{aligned} &\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \text{-}1 \\ &\boldsymbol{ij} = \boldsymbol{k} = \boldsymbol{\text{-}ji} \\ &\boldsymbol{jk} = \boldsymbol{i} = \boldsymbol{\text{-}kj} \\ &\boldsymbol{ki} = \boldsymbol{j} = \boldsymbol{\text{-}ik} \end{aligned} \tag{3.12}$$

and the product of two quaternions will be,

$$\begin{aligned} pq &= (p_0 + \boldsymbol{i}p_1 + \boldsymbol{j}p_2 + \boldsymbol{k}p_3)(q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3) \\ &= p_0q_0 + \boldsymbol{i}p_0q_1 + \boldsymbol{j}p_0q_2 + \boldsymbol{k}p_0q_3 + \boldsymbol{i}p_1q_0 + \boldsymbol{i}^2p_1q_1 \\ &\quad + \boldsymbol{ij}p_1q_2 + \boldsymbol{ik}p_1q_3 + \boldsymbol{j}p_2q_0 + \boldsymbol{ji}p_2q_1 + \boldsymbol{j}^2p_2q_2 \\ &\quad + \boldsymbol{jk}p_2q_3 + \boldsymbol{k}p_3q_0 + \boldsymbol{ki}p_3q_1 + \boldsymbol{kj}p_3q_2 + \boldsymbol{k}^2p_3q_3 \end{aligned} \tag{3.13}$$

When Eq. (3.12) and (3.13) are combined,

$$\begin{aligned} pq = p_0q_0 &+ \boldsymbol{i}p_0q_1 + \boldsymbol{j}p_0q_2 + \boldsymbol{k}p_0q_3 \\ &+ \boldsymbol{i}p_1q_0 - p_1q_1 + \boldsymbol{k}p_1q_2 - \boldsymbol{j}p_1q_3 \\ &+ \boldsymbol{j}p_2q_0 - \boldsymbol{k}p_2q_1 - p_2q_2 + \boldsymbol{i}p_2q_3 \\ &+ \boldsymbol{k}p_3q_0 + \boldsymbol{j}p_3q_1 - \boldsymbol{i}p_3q_2 - p_3q_3 \end{aligned} \tag{3.14}$$

and Eq. (3.14) is regrouped,

$$\begin{aligned} pq = p_0q_0 &- (p_1q_1 + p_2q_2 + p_3q_3) \\ &+ p_0(\boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3) + q_0(\boldsymbol{i}p_1 + \boldsymbol{j}p_2 + \boldsymbol{k}p_3) \\ &+ \boldsymbol{i}(p_2q_3 - p_3q_2) + \boldsymbol{j}(p_3q_1 - p_1q_3) + \boldsymbol{k}(p_1q_2 - p_2q_1) \end{aligned} \tag{3.15}$$

From this point, we should recall the cross and dot product of two vectors in three dimensional space. Let us take two vectors $\boldsymbol{a} = (a_1, a_2, a_3)$ and $\boldsymbol{b} = (b_1, b_2, b_3)$, then the dot product of two vectors will be,

$$\boldsymbol{a}.\boldsymbol{b} = (a_1b_1, a_2b_2, a_3b_3) \tag{3.16}$$

and the cross product will be,

$$\begin{aligned}\boldsymbol{a}\times\boldsymbol{b} &= \begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}\\ &= \boldsymbol{i}(a_2b_3 - a_3b_2)\\ &\quad +\boldsymbol{j}(a_3b_1 - a_1b_3)\\ &\quad +\boldsymbol{k}(a_1b_2 - a_2b_1)\end{aligned} \tag{3.17}$$

Using Eqs. (3.15), (3.16),and Eq. (3.17) the product of two quaternions becomes,

$$pq = p_0q_0 - \boldsymbol{p}.\boldsymbol{q} + p_0\boldsymbol{q} + q_0\boldsymbol{p} + \boldsymbol{p}\times\boldsymbol{q} \tag{3.18}$$

where, $\boldsymbol{p}$ and $\boldsymbol{q}$ are the vector parts of the quaternions consecutively.

As it can be easily seen from above equations, multiplication results of quaternions are still quaternions, and the fundamental rule of Hamilton violate the commutative rule. As a result, it can be said that, quaternions are closed under the multiplication and the product of quaternions are non commutative, Eq. (3.19).

$$\begin{gathered}\begin{Bmatrix} Ap = q \\ qr = s \end{Bmatrix} \Rightarrow \begin{Bmatrix} q = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3 \\ s = s_0 + \boldsymbol{i}s_1 + \boldsymbol{j}s_2 + \boldsymbol{k}s_3 \end{Bmatrix}\\ qr \neq rq\end{gathered} \tag{3.19}$$

Also quaternion product is associative and distributive over addition, Eq. (3.20).

$$\begin{aligned}&(pq)r = p(qr)\\ &p(q+r) = pq + pr\\ &(p+q)r = pr + qr\end{aligned} \tag{3.20}$$

Note that the identity for quaternion multiplication is a quaternion that has real part "1" and vector part "0", and the product of any quaternion with the identity is again itself, Eq. (3.21).

$$pq = q \Leftrightarrow \begin{Bmatrix} p_0 = 1 \\ p_1 = 0 \\ p_2 = 0 \\ p_3 = 0 \end{Bmatrix} \tag{3.21}$$

3.4. Conjugate of the Quaternion

Although it is simple, conjugate is a very important algebraic concept of the quaternions. The conjugate of quaternion q is usually denoted by $K(q)$, and it is given by,

$$\begin{aligned} K(q) &= q_0 - \boldsymbol{q} \\ &= q_0 - \boldsymbol{i}q_1 - \boldsymbol{j}q_2 - \boldsymbol{k}q_3 \end{aligned} \tag{3.22}$$

Due to the fact that, the vector parts of a quaternion and its conjugate differ only in sign, product and sum of the quaternion and its conjugate are results in scalar quantity, Eq. (3.23).

$$\begin{gathered} \begin{Bmatrix} qK(q) = K(q)q \\ \qquad = q_0^{\ 2} + q_1^{\ 2} + q_2^{\ 2} + q_3^{\ 2} \end{Bmatrix} \\ \begin{Bmatrix} q + K(q) = K(q) + q \\ \qquad = 2q_0 \end{Bmatrix} \end{gathered} \tag{3.23}$$

As additional information, conjugate of the product of two quaternions is equal to the product of the individual conjugates in reverse order Eq. (3.24).

$$K(pq) = K(q)K(p) \tag{3.24}$$

3.5. Norm of the Quaternion

As Eq. (3.23) describes, product of the quaternion and its conjugate results in the scalar quantity, which is the square of another important algebraic concept of the quaternions, called the norm of a quaternion.

The norm of a quaternion is usually denoted by $N(q)$ or $|q|$ and can be referred as the length of q. The norm is defined as,

$$N(q) = \sqrt{K(q)q} \tag{3.25}$$

Using Eq. (3.18), Eq. (3.25) can be extended to,

$$\begin{aligned} N^2(q) &= (q_0 - \boldsymbol{q})(q_0 + \boldsymbol{q}) \\ &= q_0 q_0 - (-\boldsymbol{q}).\boldsymbol{q} + q_0\boldsymbol{q} + (-\boldsymbol{q})q_0 + (-\boldsymbol{q}) \times \boldsymbol{q} \\ &= q_0{}^2 + \boldsymbol{q}.\boldsymbol{q} \\ &= q_0{}^2 + q_1{}^2 + q_2{}^2 + q_3{}^2 \\ &= |q|^2 \end{aligned} \tag{3.26}$$

As additional information, norm of the product of two quaternions is equal to the product of the individual norms, Eq. (3.27).

$$N(pq) = N(p)N(q) \tag{3.27}$$

Also note that, if the norm of a quaternion is unity, the components of the corresponding quaternions must have absolute values less than or equal to 1. Such quaternions are called as unit quaternions.

3.6. Inverse of the Quaternion

Dealing with the conjugate and the norm concepts, now we can show that every non-zero quaternion have a multiplicative inverse. The inverse of a quaternion usually denoted by q^{-1} and by the definition of inverse, product of a quaternion with its inverse should result in unity Eq. (3.28).

$$q^{-1}q = qq^{-1} = 1 \tag{3.28}$$

If we multiply them with $K(q)$ by post and pre multiplication, Eq. (3.28) becomes,

$$q^{-1}qK(q) = K(q)qq^{-1} = 1 \tag{3.29}$$

Since $qK(q) = K(q)q = N^2(q)$ we get the inverse quaternion as:

$$q^{-1} = \frac{K(q)}{N^2(q)} \tag{3.30}$$

Note that if q is a unit quaternion ($N(q) = 1$), than the inverse of the quaternion will be its conjugate as:

$$N(q) = 1 \Leftrightarrow q^{-1} = K(q) \tag{3.31}$$

CHAPTER 4

STRUCTURAL SYNTHESIS OF SERIAL PLATFORM MANIPULATORS

4.1. Structural Formula

An important class of robotic mechanical system consists of parallel platform manipulators, serial platform manipulators, multiple serial chains, and hybrid robotic mechanical systems. One or more grippers can be connected to one or several platforms. That system will describe one or more gripper robotic system. All platform robotic mechanical systems constructed from the actuators and simple structural groups consist of one or more platforms, legs, branches and hinges. Usually actuators are connected to legs. For these robotic mechanical systems loop mobility equations have been used (Freudenstein et al. 1975, Alizade 1988, Alizade et al. 2004). New method of structural synthesis of robot manipulators connects the simple structural groups to actuators and moving platform. Therefore, if the platform moves in Cartesian system coordinates, simple structural groups will be constructed in the orthogonal planes separately. For these robotic mechanical systems a new loop-legs mobility equation is used. In this section, the mobility of these systems is determined. The structural synthesis of serial platform manipulators is based on the structural synthesis of parallel platform manipulators that was described by Alizade et al. (2004).

Moving platforms that are supported by c_l legs, c_b branches, and c_h hinges, will have the total number DoF of joints of the legs as $\sum_{i=1}^{C_l} f_{li}$, branches as $\sum_{i=1}^{C_b} f_{bi}$, and the hinges as $\sum_{i=1}^{C_h} f_{hi}$, respectively. The total number of legs, branches and hinges is given as:

$$c = c_l + c_b + c_h \tag{4.1}$$

and, the total number DoF of joints of all legs, branches and hinges would be:

$$\sum_{i=1}^{C} f_{ci} = \sum_{i=1}^{C_b} f_{bi} + \sum_{i=1}^{C_l} f_{li} + \sum_{i=1}^{C_h} f_{hi} \tag{4.2}$$

All branches, legs and hinges of the manipulators create independent loops as $L_b = c_b - B + 1$, $L_l = c_l - 1$ and $L_h = c_h$, respectively, The number of independent loops in closed kinematic chains as shown by Alizade et al. (2004) can be introduced as:

$$L = L_b + L_l + L_h = c_b + c_l + c_h - B = c - B \tag{4.3}$$

Using Eqs (4.1—4.3) we can formulate the following:

- Total number of connection chains is the sum of the number of branches', legs', and hinges.
- Number of independent loops in closed branches' kinematic chains is the difference of the number of branches and platforms plus one.
- Number of independent loops in closed legs' kinematic chains is one less of the number of legs.
- Two platforms that are connected by a hinge will create independent loop.
- Number of independent loops in a closed kinematic chain is the difference of the number of the connection chains and platforms.

Rejoining the moving platforms of these branches, legs, and hinges to form separate B platforms in a space with λB DoF, is the same as removing L=c-B independent loops from the system to form kinematic chains with $\sum_{i=1}^{C} f_{ci} = \sum_{i=1}^{j} f_i$. Using structural formula (Alizade 1988) we can describe the mobility loop equation in the following form:

$$\begin{aligned} M &= \sum_{i=1}^{j} f_i - \lambda(j_B - B - c_b) + q - j_p \\ &= \sum_{i=1}^{C} f_{ci} - \lambda(c - B) + q - j_p \\ &= \lambda B + \sum_{i=1}^{C} (f_{ci} - \lambda) + q - j_p \end{aligned} \tag{4.4}$$

where, $\lambda B = M_B$ is the sum of mobilities of all platforms in the unconstrained space or subspace, and $\sum_{i=1}^{c}(f_i - \lambda) = M_c$ is the sum of constraints imposed by the legs, branches and hinges.

Each leg, branch, and hinge separately introduces an insufficient $(f_{ci} - \lambda < 0)$, sufficient $(f_{ci} - \lambda = 0)$, or a redundant $(f_{ci} - \lambda > 0)$ kinematic chain. Sum of degrees of freedom of all platforms and the degrees of constraint that is imposed by kinematic chains describe the mobility of serial platform and parallel platform manipulators.

Mobility loop equation, Eq. (4.4), for robotic systems with independent loops with variable general constraint could be described as follows:

$$M = (B - c)\lambda + \sum_{i=1}^{C} f_{ci} + q - j_p \tag{4.5}$$

where, $\lambda = 2,3,4,5,6$.

Different new platform manipulators could be designed in subspaces $\lambda = 2,3,4,5$ and in general space $\lambda = 6$.

The aim of the new method of structural synthesis is:

- Using Eq. (4.5) we can describe the simple structure groups (M = 0) for subspaces $\lambda = 2,3,4,5$ and for general space $\lambda = 6$,as shown in Alizade et al. (2004). A classification of sets of lines linearly dependent on one, two, three, four and five given lines has been introduced by McCarthy notation in McCarthy (2000).
- Simple structural groups can be connected to the general moving platform and the actuators that are positioned in the orthogonal Cartesian planes.
- Each actuators will moved (or rotated if it is possible) along the orthogonal Cartesian coordinate system.

Now, our problem is to describe a new structural formula for platform manipulators which operates in Cartesian space or subspace and its legs consist of simple structural groups and actuators operates in orthogonal planes.

Euclidian space geometry introduces that any three vectors that are not on the same plane define a space with dimension D=3, also any two non-zero independent vectors define a plane with dimension d=2, and in the end, one vector define a line passing through origin of coordinate system d=1.

Let the number of independent parameters describing the structural groups of three legs $c_l = 3$ that are placed in three orthogonal planes is λ. The general moving platform and the actuators, positioned along orthogonal axis, are connected by simple structural groups with general constraint parameters $\lambda = 3,4,5,6$. Each simple structural group creates the legs and introduces the plane or line with one or two dimensions, thus the total number of leg dimensions are $\sum_{l=1}^{C_l} d_l$, where d =1 or d =2. The dimension of constraint of the general moving platform that is imposed by dimensions of each leg can be written as $\sum_{l=1}^{C_l}(d_l - D)$. Thus, the motion of the general moving platform in Cartesian orthogonal system will be in the following form:

$$m_p = (\lambda + 3) + \sum_{l=1}^{C_l}(d_l - D) \tag{4.6}$$

The mobility of legs of the general moving platform is:

$$M_l = \sum_{l=1}^{C_l}(f_l - \lambda_l) + q - j_p \tag{4.7}$$

where λ_l is the general leg constraint, f_l is DoF of leg kinematic pairs.

As a result, the mobility of parallel Cartesian platform robot manipulators consists of the motion of the general moving platform m_p, and the mobility of legs M_l moving in orthogonal planes.

$$M = m_p + M_l \tag{4.8}$$

Combining the Eq.(4.6—4.8) we can describe the new structural formula for the mobility loop-legs equations as follows:

$$M = (\lambda + 3) + \sum_{l=1}^{C_l}(d_l - D) + \sum_{l=1}^{C_l}(f_l - \lambda_l) + q - j_p \tag{4.9}$$

Example 1: Let us design three parallel Cartesian platform manipulators, where the motion of the general moving platform has translational motions Pz, Py-Pz, and Px-Py-Pz, respectively.

A) For the first orthogonal robot manipulator we will take three simple structural groups RRR from the subspace $\lambda = 3$, one linear actuator moving along z-axis, and for symmetry two links rotating around x- and y-axes (*Table 4.1.1*). In each orthogonal plane, simple structural groups will be connected to the general moving platform, actuator and two rotation links. Using mobility loop-legs equation, Eq. (4.9), we can calculate the mobility of the type PRR-[RRR]-2RRR parallel orthogonal robot manipulator as: $q=0, j_p=0, \lambda=3, \lambda_l=(3,3,4), d_l=(1,1,2), \sum f_l=12, M=3.$ By using Eqs, (4.6) and (4.7) the motion of the general moving platform and the mobility of legs will be $m_p = 1$ and $M_l = 2$, respectively.

B) For the second orthogonal robot manipulator, we will take three simple structural groups [$n = 4, p_1 = 6, M = 0, \lambda = 3$] which will be connected to the general moving platform, two linear actuators along y and z-axes, and for symmetry one link rotating, around x-axis (*Table 4.1.2*). Using the same procedure, we can calculate the structural parameters of the type 2PR(RRR)[RR-RR-RR]-(RRR)RR parallel orthogonal robot manipulator as: $M = 3,\ m_p = 2,\ M_l = 1.$ Note that the mobility of each leg will be calculated from two loops as: M_l=[(4-3)+(3-4)]+[(4-3)+(3-4)]+[(4-3)+(3-3)]=1

C) For the third orthogonal manipulator, we will follow the same steps in Example 1.B except three actuators will be used along the x, y, and z-axes (*Table 4.1.3*). The structural parameters for 3PR(RRR)[RR-RR-RR] parallel orthogonal manipulator will be $M = 3,\ m_p = 3,\ M_l = 0.$

Table 4.1. New Parallel Cartesian Robot Manipulator Types

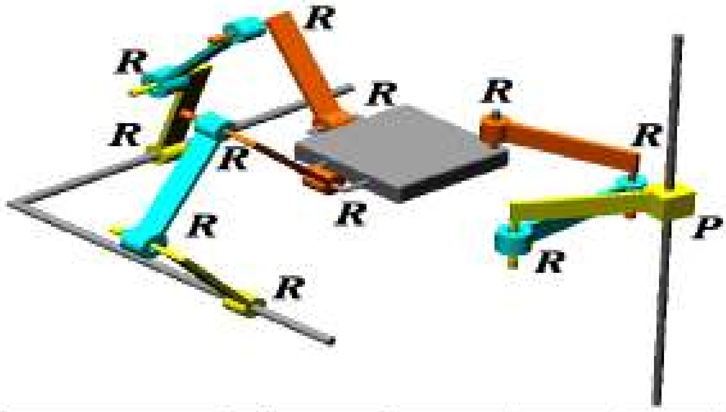
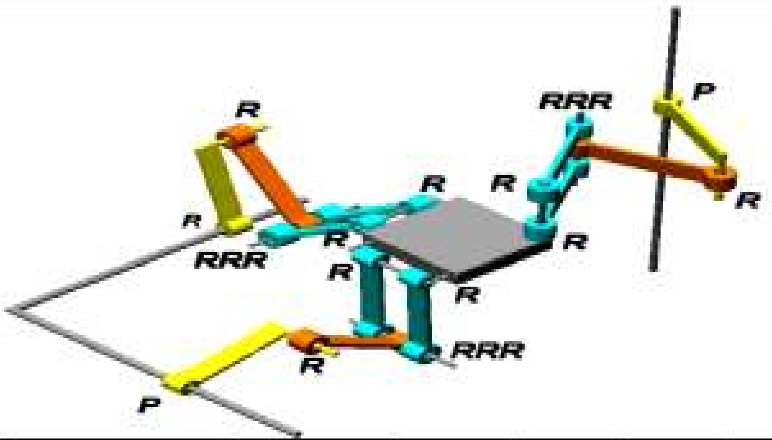
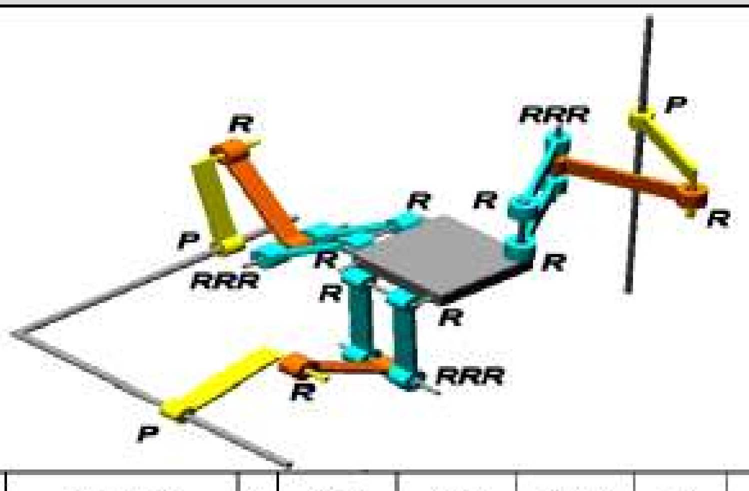
Type	*Illustration*							
	Motion	λ	λ_l	d_l	Σf_l	m_p	M_l	M
1	2	3	4	5	6	7	8	9
1								
PRR-[RRR]-2RRR	Pz	3	3,3,4	1,1,2	4,4,4	1	2	3
2								
2PR(RRR)-[RR-RR-RR] - (RRR)RR	Py,Pz	3	3,4,4	1,2,2	7,7,7	2	1	3
3								
3PR(RRR)-[RR-RR-RR]	Px,Py,Pz	3	4,4,4	2,2,2	7,7,7	3	0	3

Table 4.1 (cont.). New Parallel Cartesian Robot Manipulator Types

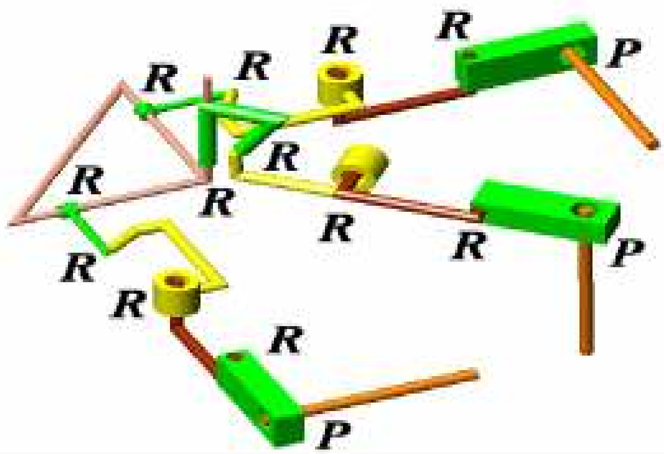
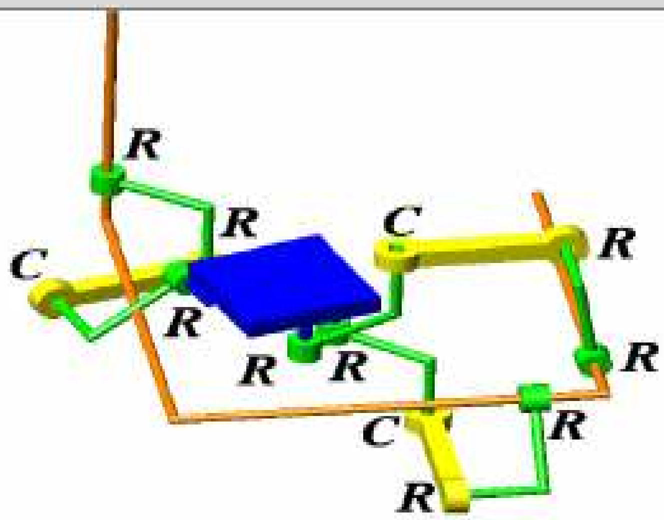
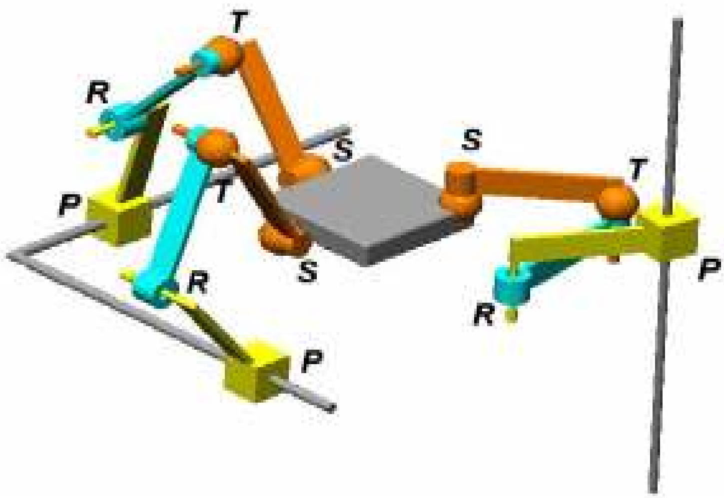
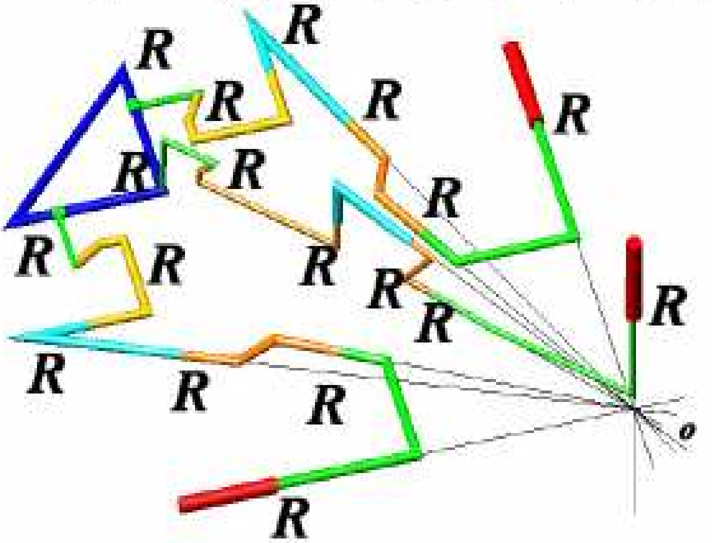
1	2	3	4	5	6	7	8	9
4								
3PRRR-[RRR]	Rx,Ry,Rz,Pxy	4	5,5,5	2,2,2	5,5,5	4	0	4
5								
3RRC-[RRR]	Px,Py,Pz,Rz	4	5,5,5	2,2,2	5,5,5	4	0	4
6								
3PRT-[SSS]	Px,Py,Pz,Rx,Ry,Rz	6	6,6,6	2,2,2	7,7,7	6	3	9

Table 4.1 (cont.). New Parallel Cartesian Robot Manipulator Types

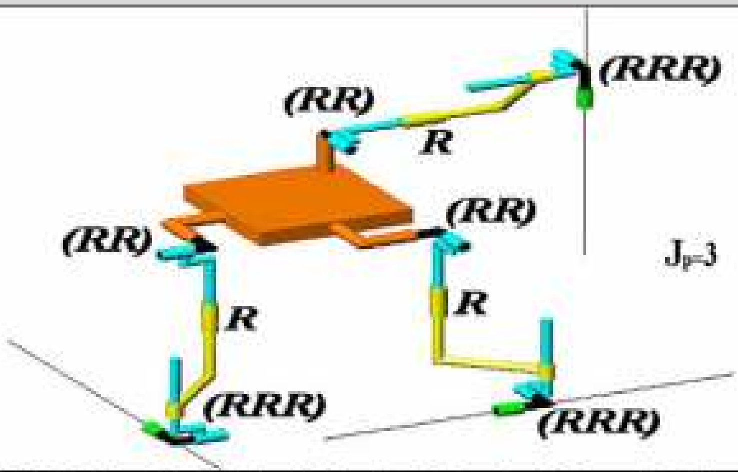
1	2	3	4	5	6	7	8	9

7



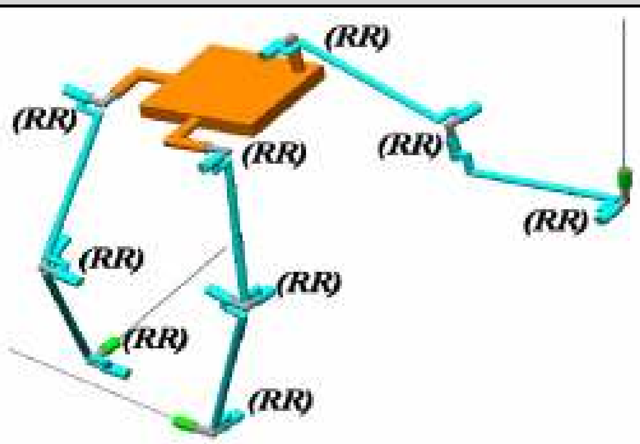
3RRRRR-[RRR]	Px,Py,Pz,Rx,Ry,Rz	6	6,6,6	2,2,2	6,6,6	6	0	6

8



3RRRRR-[RRR]	Px,Py,Pz,Rx,Ry,Rz	6	6,6,6	2,2,2	6,6,6	6	-3†	3

9



3RRRRR-[RRR]	Px,Py,Pz,Rx,Ry,Rz	6	6,6,6	2,2,2	6,6,6	6	0	6

† M_l=*-3 comes from the passive degrees of freedom* j_p=3

Example 2: Let us design two parallel Cartesian platform manipulators. In the first, the motion of the general moving platform has three rotational motions Rx, Ry, Rz and one translational motion on the line in x-y plane (Pxy). In the second, the motion of the general moving platform has one rotational motion Rz and three translational motions Px, Py and Pz.

A) For the first orthogonal robot manipulator, we will take three simple structural groups RRRR from the subspace $\lambda = 4$, and three linear actuators moving along x, y and z-axes (*Table 4.1.4*). In each orthogonal plane, simple structural groups will be connected to the general moving platform and actuators. Using mobility loop-legs equations, Eq. (4.9), Eq. (4.7) and Eq. (4.6), we can calculate the structural parameters of the type 3PRRR[RRR] parallel orthogonal robot manipulator as: $M = 4,\ m_p = 4,\ M_l = 0.$ Note that, to reach the given motion of the general moving platform we need to add one more actuator.

B) For the second orthogonal robot manipulator we will take three simple structural groups RCR from the subspace $\lambda = 4$, which will be connected to the general moving platform, and three rotational actuators in x, y and z-axes (*Table 4.1.5*). Using the same procedure, we can calculate the structural parameters of the type 3RRC[RRR] parallel orthogonal robot manipulator as: $M = 4, m_p = 4, M_l = 0.$ We need additional one actuator to reach the given motion of the general moving platform.

Example 3: Let us design a parallel Cartesian platform manipulator, where the motion of the general moving platform has three rotational motions Rx, Ry, Rz and three translational motions Px, Py and Pz. First, we will take three simple structural groups STR from the space $\lambda = 6$, and three linear actuators moving along x, y and z-axes (*Table 4.1.6*). In each orthogonal plane, simple structural groups will be connected to the general moving platform and actuators. Using mobility loop-legs equations, Eq. (4.9), Eq. (4.7) and Eq. (4.6), we can calculate the structural parameters of the type 3PRT[SSS] parallel orthogonal robot manipulator as: $M = 9,\ m_p = 6,\ M_l = 3.$ Note that, to reach the given motion of the general moving platform we need to add six more actuators. Also, to get rid of excessive mobility $(M = 6,\ m_p = 6,\ M_l = 0)$, we can connect each $\lambda = 6$ structural group directly to the x, y and z-axes (*Table 4.1.6-4.1.7-4.1.9*).

A Side Note: Due to the fact that, using the same analogy, in all our trials with $\lambda = 5$ structural groups, the legs of the parallel Cartesian platform manipulators are converted to $\lambda = 6$ structural groups, and the motion of the manipulators is transformed into Rx, Ry, Rz, Px, Py and Pz (*Table 4.1.6-4.1.7-4.1.8-4.1.9*). So that investigation for $m_p = 5$ with $\lambda = 5$ structural groups will be continued in future.

4.2. Structural Synthesis and Classification of Simple Serial Platform Structural Groups

Serial platform kinematic chains means that, at least two platforms are connected by hinge kinematic pairs (and, therefore, zero number of branches as well) and all legs are going from the mobile platforms to the frame.

The problem of creating simple structural groups for plane and spatial serial platform kinematic chains is considered by developing platforms and closed loops. Simple serial platform structural group is the one that can not be split into several other structural groups with smaller number of links. A simple serial platform structural group has the mobility equal to zero (M=0), thus the number of input parameters is zero.

The plane simple structural groups can be created by lower and higher kinematic pairs, and the spatial structural groups can be created by hinge, revolute, spheric and slotted spheric kinematic pairs. Using exchangeability of kinematic pairs we can describe different structure of simple structural group (the hinge joint between mobile platforms is not changed)

For creating simple structural platform structural groups, mobility loop equation, Eq. (4.5), could be described as follows:

$$(B - c)\lambda + \sum_{i=1}^{c} f_{ci} = 0 \tag{4.10}$$

where $c = c_l + c_h$, as $c_b = 0$.

Simple structural group, Eq. (4.9), for subspace λ=3, and for general space λ=6 can be introduced respectively as:

$$\sum_{i=1}^{c_l+c_h} f_{ci} = 3(c_l + c_h - B) \tag{4.11}$$

$$\sum_{i=1}^{c_l+c_h} f_{ci} = 6(c_l + c_h - B) \tag{4.12}$$

The additional conditions of structural synthesis of serial platform kinematic chains can be introduced as following equalities and inequalities:

$$\begin{aligned} &a) L = c_l + c_h - B \quad ; \quad b) c_l = j_B - 2c_h \quad ; \quad c) j = 3L, j = 6L \\ &d) j_l = j / c_l \quad ; \quad e) 3 \le j_B \le 6 \quad ; \quad f) c = c_l + c_h \quad ; \quad g) B \ge 2 \end{aligned} \tag{4.13}$$

Using objective functions (4.10—4.12) and additional equality and inequality constraint conditions (4.13), computer software of structural synthesis of simple serial platform structural groups has been created. Results of plane and spatial simple serial structural groups are presented in the following Tables 4.2 and 4.3.

The algorithm of structural synthesis of serial platform simple structural groups can be summarized step by step as follows:

- Take subspace λ=3, or general space λ=6.
- Select values for B and j_B (Eq. 4.13e, and 4.13g)
- Select value for hinge joints c_h and calculate the number of legs c_l (Eq. 4.13b)
- Calculate the number of independent loops L (Eq. 4.13a)
- Calculate the number of joints j with one DoF (Eq. 4.13c)
- Place the joints on legs (Eq. 4.13d) and selected hinge joints c_h between mobile platforms.
- Using the principle of exchangeability of kinematic pair, replace the joints with one DoF with higher and other kinematic pairs.
- The mobility of manipulator is equal to the number of actuators (Eq. 4.5) added to the legs of simple serial platform structural group.

The simple serial platform structural groups in subspace λ=3 and in general space λ=6 have been introduced by serial platform kinematic chains with open loops B_o, closed loops B_c, and mixed open and closed loops B_o+B_c, as shown in *Table 4.2* and *Table 4.3,* respectively.

Table 4.2. Simple Structural Groups of Serial Platform Manipulators in Subspace λ=3

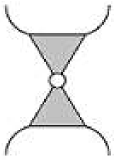
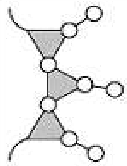
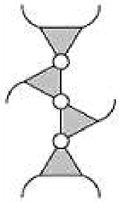
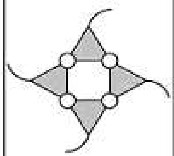
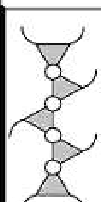
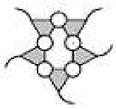
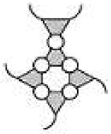
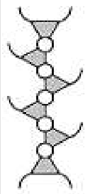
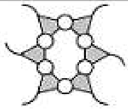
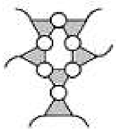
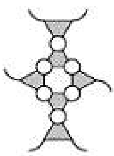
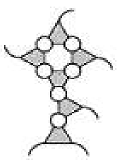
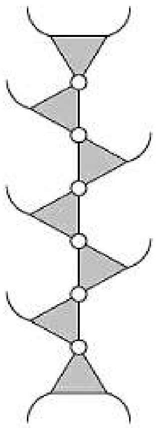
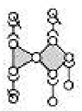
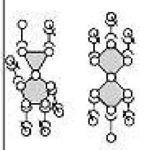
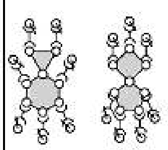
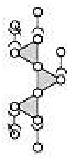
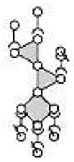
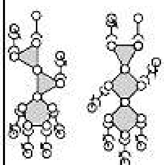
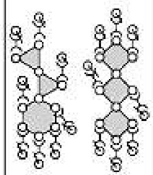
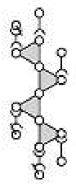
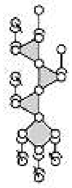
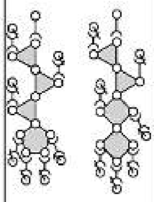
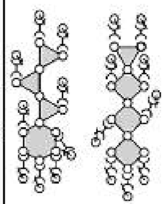
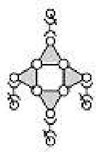
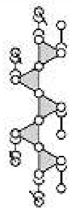
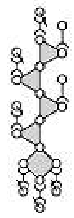
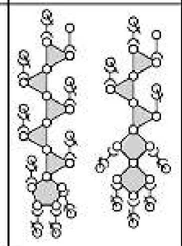
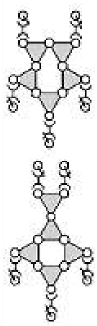
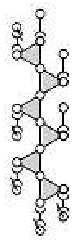
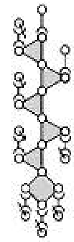
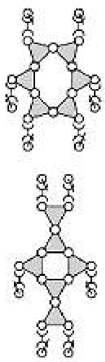
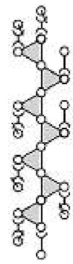
B_c+B_o	C_l / C_h	4	5	6	7	8	9	B_o
-	1							2
-	2							3
-	3							4
4	4							5
5	5		 					6
6	6			   				7

Table 4.3. Simple Structural Groups of Serial Platform Manipulators in Subspace λ=6

B_c+B_o	C_l / C_h	4	5	6	7	8	9	B_o
-	1							2
-	2							3
-	3							4
4	4							5
5	5							6
6	6							7

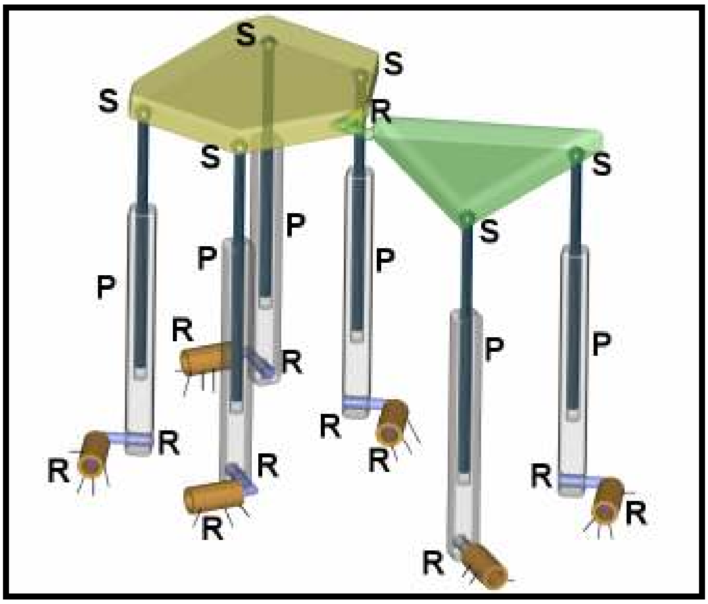


Figure 4.1. 6 DoF Spatial Serial Platform Robot Manipulator

Example: Designing a spatial robot manipulator with 6 DoF and two grippers placed on two mobile platforms. Select from *Table 4.3* simple serial platform structural group with c_l=6, and c_h=1, which consist of two mobile platforms B=2, and one hinge joint c_h=1, thus $c = c_l + c_h$=7 and joints on triangular platform j_{B1}=3, and on pentagonal platform j_{B2}=5, and $j_B = j_{B1} + j_{B2}$=8. The number of joints of simple structural group is Σ f_i = 30. Six actuators will be placed on six legs and spherical, prismatic, and revolute pairs are used in this structure. Using Eq. (5) gives, $M = (B - c)\lambda + \sum_{i=1}^{C} f_{ci}$ = (2-7)6 + 36 = 6. The structure of serial platform robot manipulator is introduced in ***Fig.4.1***.

4.3. Structural Synthesis of Parallel Cartesian Platform Robot Manipulators

Kinematic chain shown in *Table 4.4.1* is referred as a simple Cartesian structural group. Simple Cartesian structural group can be obtained by successive coupling of the three simple structural groups in the orthogonal planes to the general moving platform. Thus, simple Cartesian structural group is one of the orthogonal parallel groups that can not be split into several orthogonal parallel structural groups with smaller member of links. A simple Cartesian structural group has zero number of mobility (Eq. 4.9), that is number of input links equals to zero.

Such structural group can be reached from Eq. (4.14) if and only if the motion of the general moving platform m_p=0 and the leg mobility M_l=0:

$$(\lambda+3)+\sum_{l=1}^{C_l}(d_l-D)+\sum_{l=1}^{C_l}(f_l-\lambda_l)=0 \tag{4.14}$$

$$(\lambda+3)+\sum_{l=1}^{C_l}(d_l-D)=0 \tag{4.15}$$

$$\sum_{l=1}^{C_l}(f_l-\lambda_l)=0 \tag{4.16}$$

The kinematic chain shown in *Table 4.4.1* is the first simple Cartesian structural group that was constructed in each orthogonal plane by simple structural group RRR, with the general constraint parameter $\lambda_l = 3$, and the number of dimension d=1.

The result of the generation principle of parallel Cartesian platform robot manipulators is shown in *Table 4.4*. Every Cartesian robot manipulator was generated by the successive joining of orthogonal simple structural groups with the actuators on the orthogonal frames. *Table 4.4.2*, *4.4.4*, and *4.4.7* shows Cartesian robots with mobility of legs M_l=1,2,3 (Eq. 4.16), motion of the general moving platform m_p=0 (Eq. 4.15), and mobility of robot manipulator M =1,2,3, (Eq. 4.14), respectively.

The generation of a parallel Cartesian platform robot manipulator with one motion of the general moving platform m_p=1 can be generated by three kinematic chains as was shown in *Table 4.4.3*, *4.4.5*, and *4.4.8*. The mobilities of legs are M_l=0,1 and 2, and the mobilities of manipulators are M =1,2, and 3 respectively.

Table 4.4.6, and *4.4.9* indicate the two motions of the general moving platform m_p=2 with other parameters as M_l=0,1 and M =2,3. The generation of the Cartesian robot manipulator with three motion of the general moving platform m_p=3, mobility of legs M_l=0, and the mobility M =3 with three linear actuators moving along the orthogonal axes were shown in *Table 4.4.10*.

In the end, the generation of the variations of actuators for simple structural group RRR ($\lambda = 3$) in the orthogonal planes gives five parallel Cartesian platform robot manipulators (*Table 4.4.5, 4.4.6, 4.4.8, 4.4.9, and 4.4.10)*

Table 4.4. Variations of Actuators for Simple Structural Group RRR (λ=3) of Parallel Cartesian Platform Robot Manipulators

	Type	d_i	λ_l	f_l	m_p	M_l	M
1	RR – RRR – RR RR	1,1,1	3,3,3	3,3,3	0	0	0
2	RR – RRR – RRR RR	1,1,1	3,3,3	3,3,4	0	1	1
3	RR – RRR – RRP RR	1,1,2	3,3,4	3,3,4	1	0	1
4	RR – RRR – RRR RRR	1,1,1	3,3,3	3,4,4	0	2	2
5	RR – RRR – RRP RRR	1,1,2	3,3,4	3,4,4	1	1	2
6	RR – RRR – RRP RRP	1,2,2	3,4,4	3,4,4	2	0	2
7	RRR – RRR – RRR RRR	1,1,1	3,3,3	4,4,4	0	3	3
8	RRR – RRR – RRP RRR	1,1,2	3,3,4	4,4,4	1	2	3
9	RRR – RRR – RRP RRP	1,2,2	3,4,4	4,4,4	2	1	3
10	PRR – RRR – RRP RRP	2,2,2	4,4,4	4,4,4	3	0	3

CHAPTER 5

TYPE SYNTHESIS OF HUMAN ARM

As a definition, type synthesis is a process, where a given task to be produced by a mechanism is analysed to find the type that will best perform it, as a linkage, a cam mechanism, a gear train, or their combinations. So that, before designing the mechanism, we need to specify our task clearly. As a result, in the design of a manipulator that will mimic human arm motion, the complete structure of the arm should be investigated.

The bones of the human arm start from the shoulder to the wrist *Fig. (5.1).* In the first section, humerus creates the upper arm, and in the second section two parallel bones ulna and radius create the lower arm.

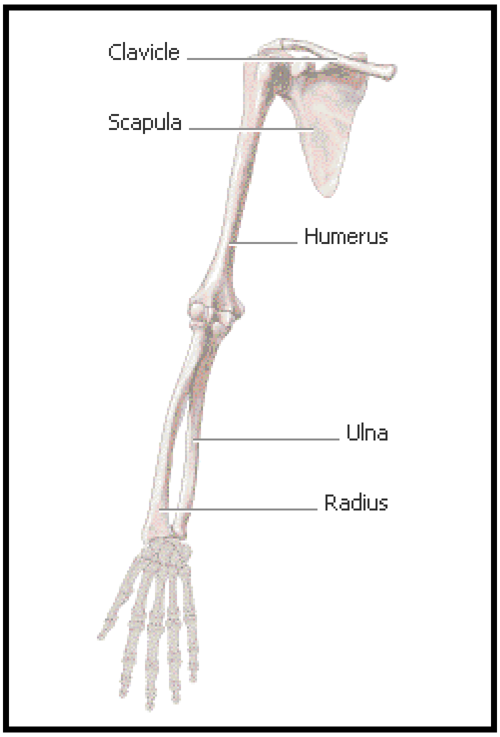


Figure 5.1. Structure of Human Arm
(Source: Encarta)

The nearly spherical head of the humerus stays in the cavity of the scapula, where it creates the shoulder joint. The shoulder joint is a ball and socket type joint that gives the full circular motion to the arm. The end of the humerus joins the bones of the lower arm at the elbow to form a revolute joint that permits the forearm to bend up and down. Also, to give the twist motion to the forearm, the radius can rotate over the ulna. Note that clavicle is a slender *f*-shaped bone that connects the upper arm to the trunk of the body and holds the shoulder joint away from the body to allow for greater freedom of movement.

5.1. The Clavicle

The clavicle forms the anterior portion of the shoulder girdle. It is a long bone, curved somewhat like the italic letter *f*, and placed nearly horizontally at the upper and anterior part of the thorax, immediately above the first rib *Fig. (5.2)*.

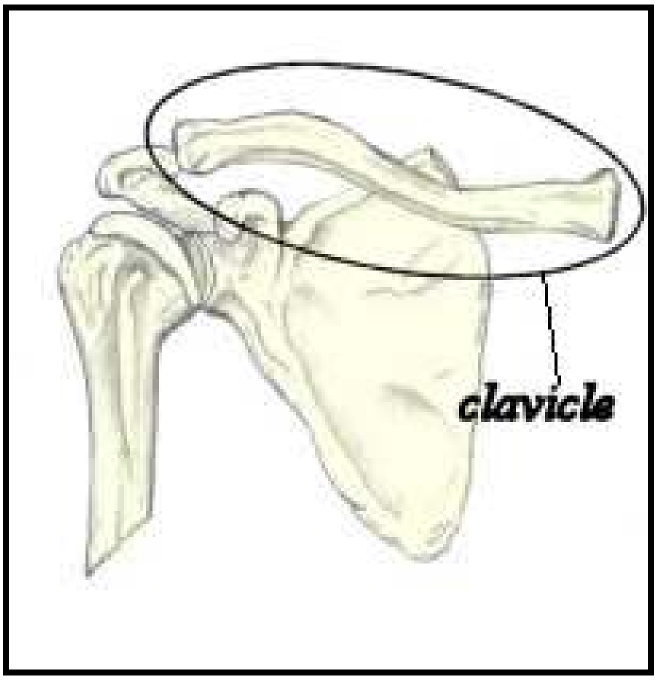


Figure 5.2. Clavicle
(Source: Sports Injuries)

The clavicle is designed to support the shoulder by the help of scapula, acting like a support that helps to align the shoulder with the rest of the chest. It has three rotational motions, as it is connected to the thorax with 3-DoF ball and socket type joint. The rotations are represented in detail in *Fig. (5.3)* and *(5.4)*.

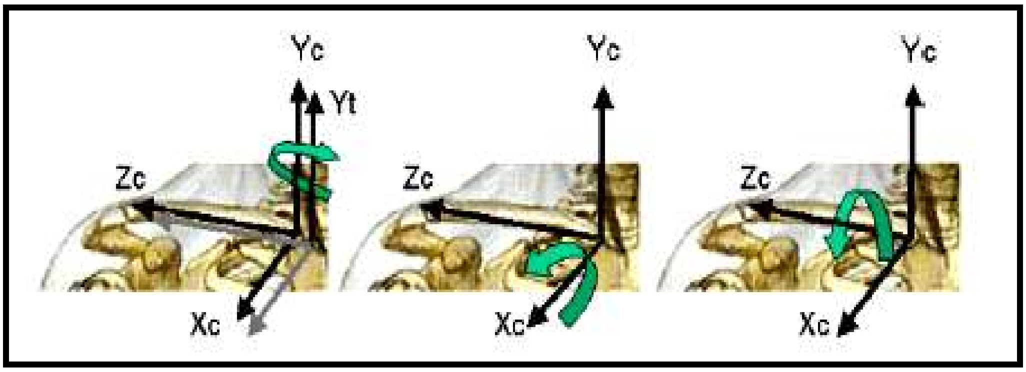


Figure 5.3. Clavicle Rotations (t: Thorax, c: Clavicle Coordinate Systems)
(Source: ISB)

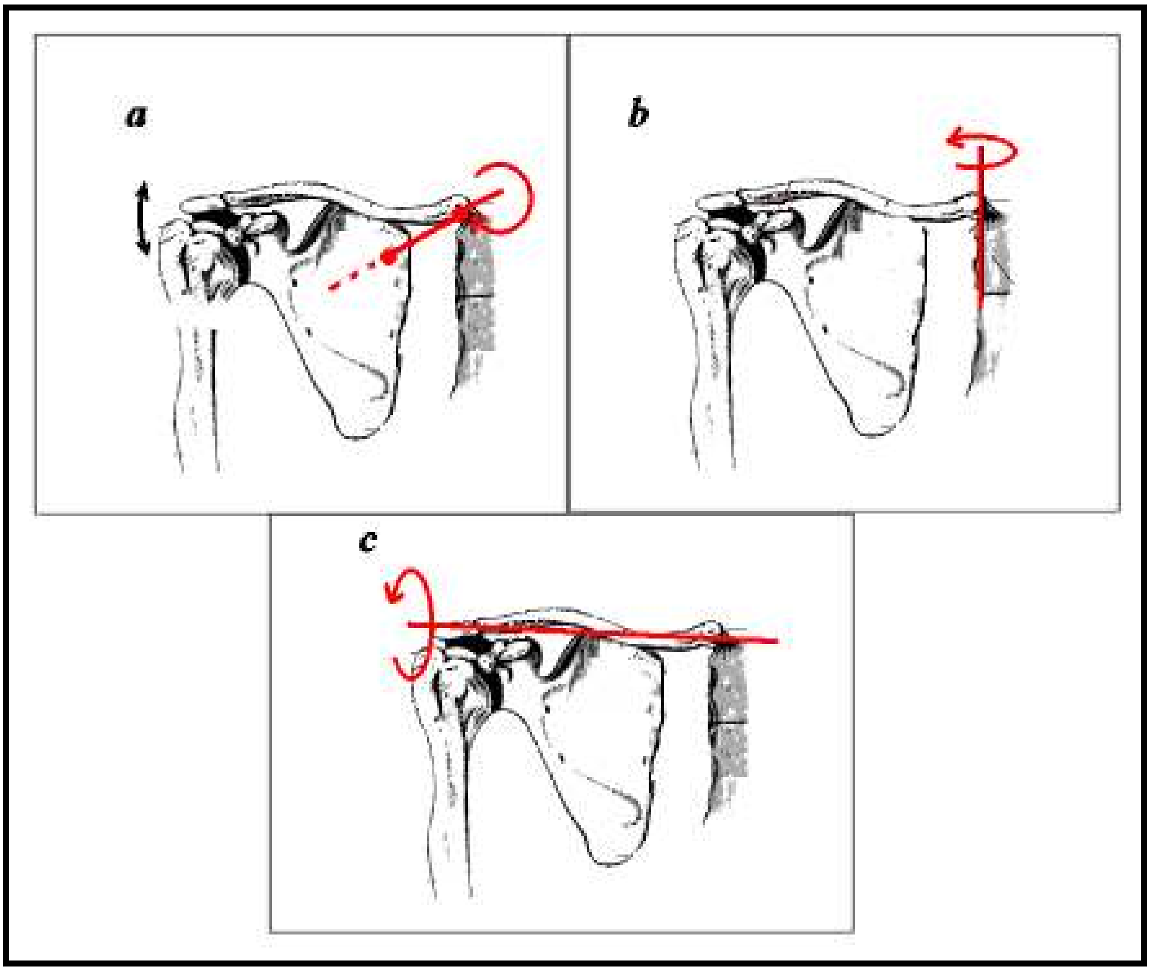


Figure 5.4. Clavicle Rotations in Detail
(a: Elevation and Depression, b: Protraction and Retraction
c: Backward and Forward Rotation)
(Source: D. M. Thompson)

After the detailed investigation of the clavicle motions, three degrees of freedom parallel orientation manipulator is selected for the mechanism that will mimic the human clavicle *Fig. (5.5)*.

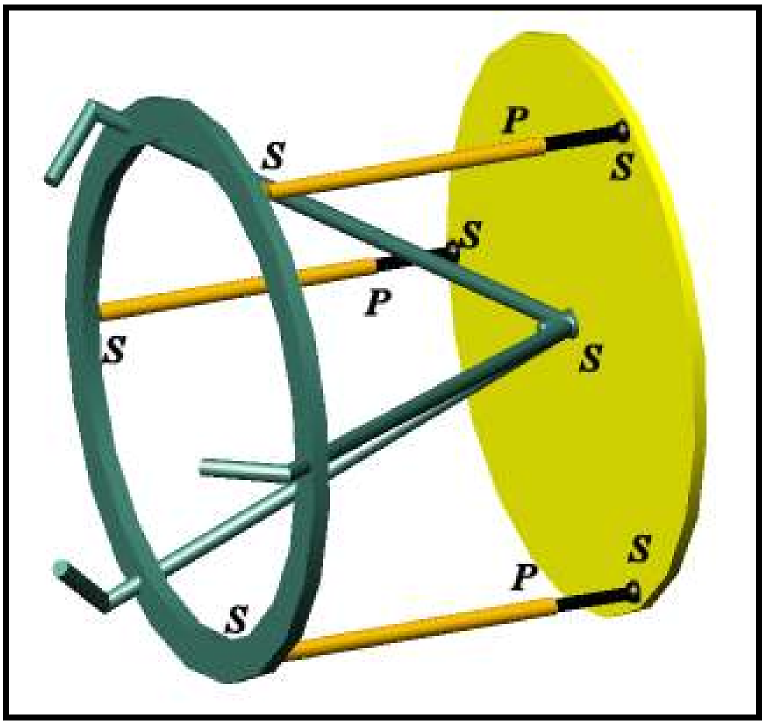


Figure 5.5. 3-DoF Orientation Platform

The parallel manipulator consists of two platforms (base and mobile), mainly connected by a spherical pair that forms the rotational center of the orientation platform. For the actuation purposes three linear actuators are connected to each platform by spherical pairs. The advantage of this configuration is the fact that, the platform actuation is so simple and the singularities in its desired workspace are overcome. Note that, although the platform works in the space λ=6, it is constrained by the main spherical joint and works as a spherical mechanism, so that it has three rotations around the Cartesian axes. The mobility calculation of the moving platform can be carried out as;

$$\begin{aligned} M &= \sum_{i=1}^{j} f_i - \lambda(j_B - B - c_b) + q - j_p \\ &= 24 - 6(4 - 1 - 0) + 0 - 3 \\ &= 3 \end{aligned} \tag{5.1}$$

5.2. The Humerus

The humerus is the longest bone in the arm that begins from the shoulder to the elbow *Fig. (5.6).* The head of the humerus articulates with the glenoid cavity of the scapula at the shoulder joint that is also called glenohumeral joint *Fig. (5.6).* It is a ball and socket type joint, which allows a wide range of circular movement.

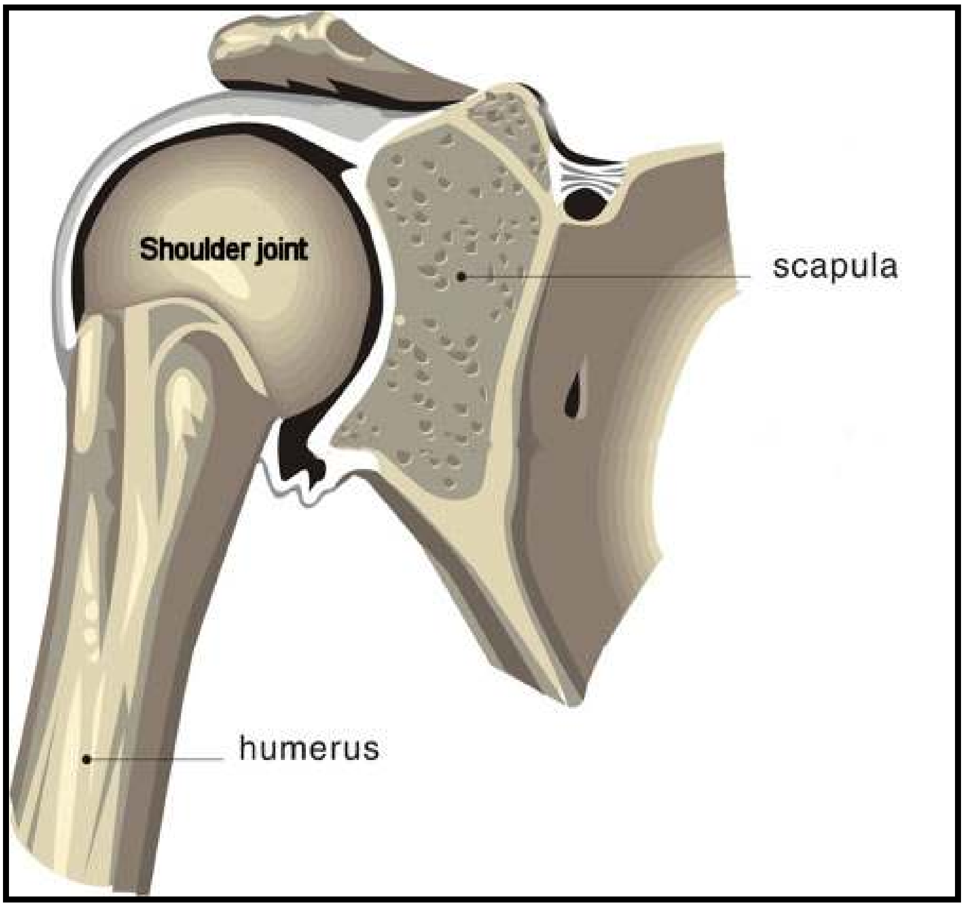


Figure 5.6. Humerus and the Shoulder Joint
(Source: Staticfiles)

Although it seems, shoulder joint is enough for the analysis of the motion of the humerus, it is not the case. Also, the motion of the scapula should be included in the analysis. Detailed representations of the both the motions of scapula and the humerus are given in *Fig. (5.7), (5.8)* and *(5.9).*

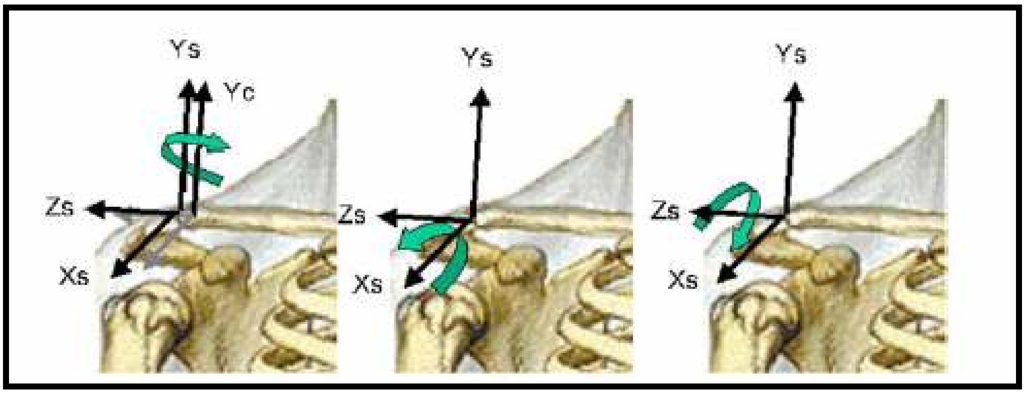


Figure 5.7. Scapula Rotations (s: Scapula, c: Clavicle Coordinate Systems)

(Source: ISB)

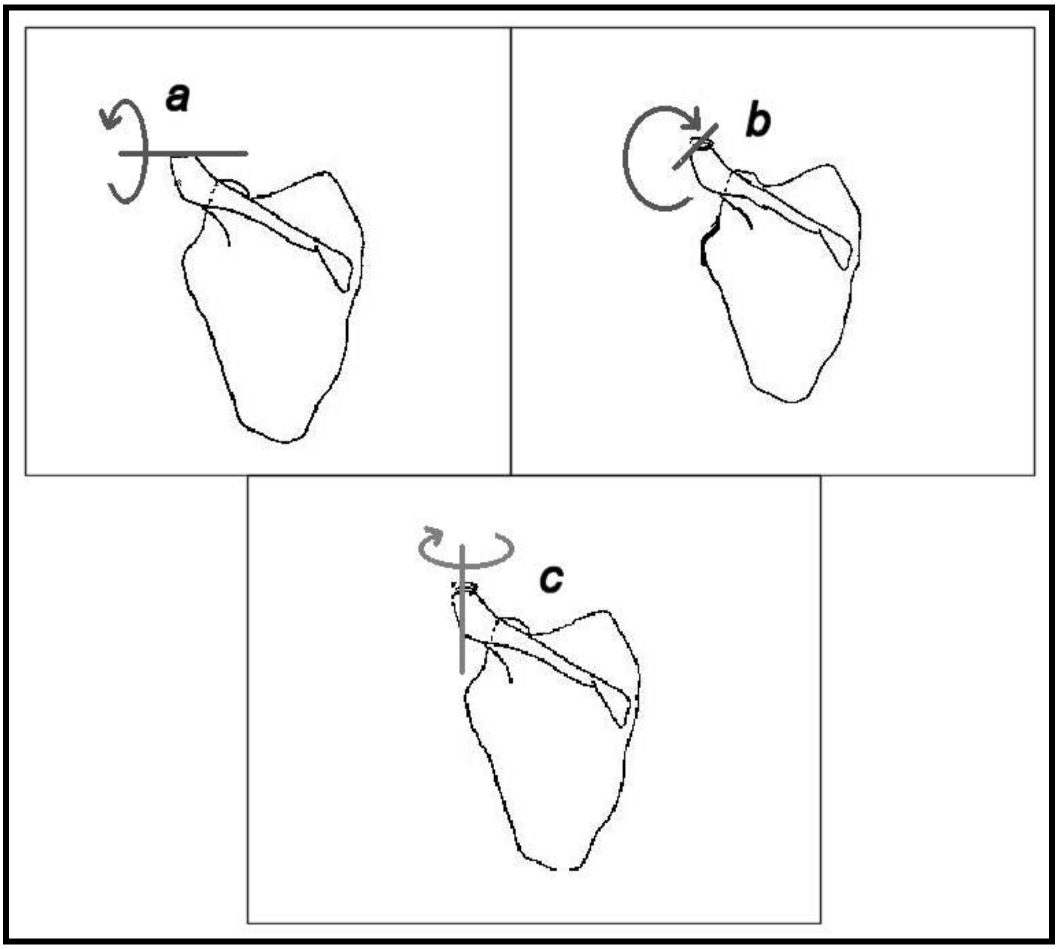


Figure 5.8. Scapula Rotations in Detail

(a:Tipping, b: Upward and Downward Rotation

c: Winging)

(Source: D. M. Thompson)

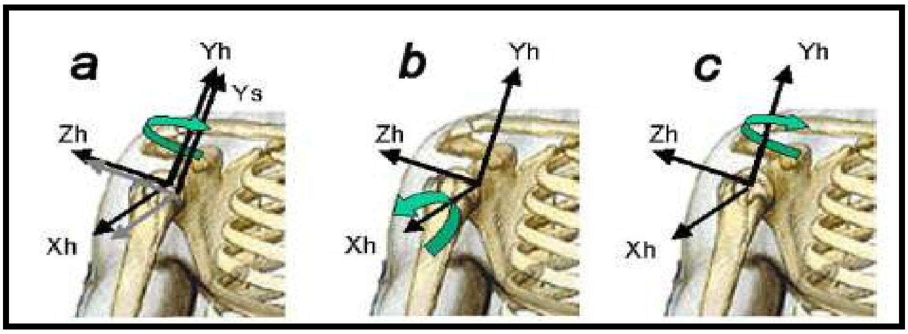


Figure 5.9. Humerus Rotations Relative to Scapula, Y-X-Y Sequence
(s: Scapula, h: Humerus Coordinate Systems)
(a: Plane of Elevation, b: Negative Elevation, c: Axial Rotation)
(Source: ISB)

The figures show that, the motion of the humerus is so complex by the addition of scapula motions. As a result, to fulfil the desired motion, a manipulator that has a spherical motion capability with large workspace without singularities should be chosen for the imitation. From this point of view, the optimized version (Gosselin 1994) of the spherical parallel manipulator that is patented by R.I. Alizade in Russian patent "1144875" in subspace λ=3 is selected, *Fig (5.10).*

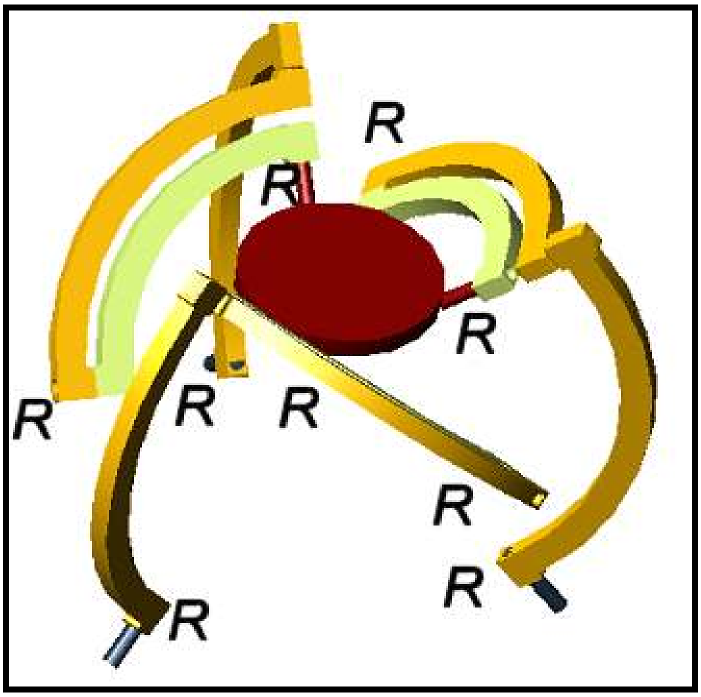


Figure 5.10. Agile Eye, Optimized3-DoF Spherical Parallel Platform

When compared with the non-optimized versions, agile eye has the largest workspace that is free from the singularities. Being a spherical parallel manipulator, it has one platform and three legs, where in each leg there exist three revolute pairs, one of each is a revolute actuator. The speciality of the manipulator is the fact that all of the revolute joint directions in each leg intersect in one point, where the rotation center is staying. In addition to its larger workspace, the agility of the manipulator is high when compared to its alternatives as orientation platforms. So that it is suitable for the humerus manipulator. The mobility calculation of the platform can be carried out as:

$$
\begin{aligned}
M &= \sum_{i=1}^{j} f_i - \lambda(j_B - B - c_b) + q - j_p \\
&= 9 - 3(3 - 1 - 0) + 0 - 0 \qquad (5.2) \\
&= 3
\end{aligned}
$$

5.3. The Radius and the Ulna

The radius and the ulna create the lower portion of the arm *Fig. (5.11).* These are the two bones placed in the forelimb in parallel to each other.

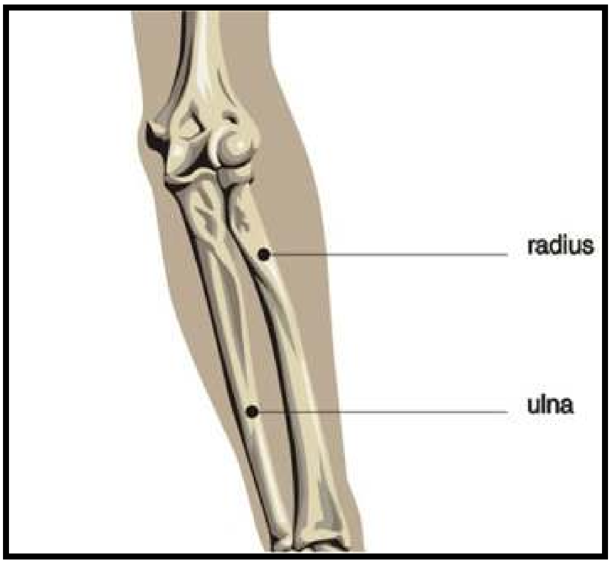


Figure 5.11. Radius and Ulna
(Source: Staticfiles)

The main function of the radius is to act as the main supporting bone of the forelimb. It articulates both with the humerus to form the elbow joint, and with the carpal bones to form the main joint of the wrist. Also, the radius can rotate over the ulna to give the twist motion to the wrist, as stated before. When compared in structure, it is shorter than the ulna, which serves as a point for muscle attachment. The motions of the bone couples include one degree of freedom motion in the elbow that is used to bend the forearm up and down, and one degree of freedom motion in the wrist that gives the hand its axial rotation, *Fig. (5.12)*.

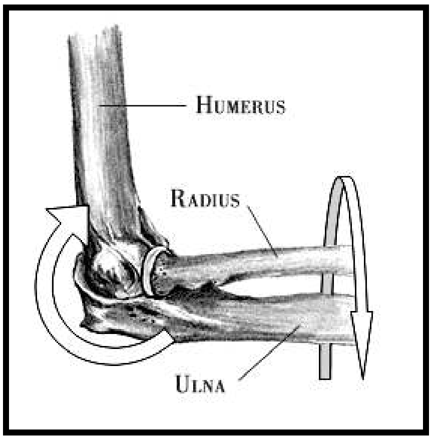


Figure 5.12. Radius and Ulna Rotations
(Source: Orthopedic Center)

Due to the simplicity of the motions, and the workspace, the manipulator that will mimic the forearm is selected to be an orientation platform with two degrees of freedom, *Fig. (5.13)*.

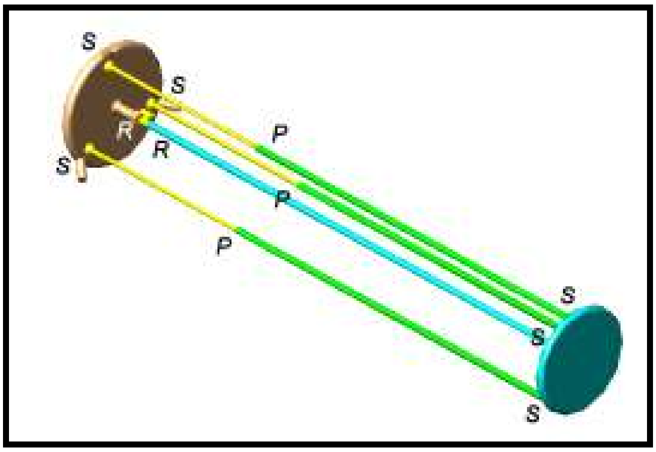


Figure 5.13. 2-DoF Orientation Platform

Similar to the previous example, the manipulator in λ=6 has two platforms (base and moving), but this time connected by two revolute actuators. The motion is carried out by these actuators, where one of the actuators controls the elbow motion, while the other controls the wrist twist. Other legs around the platforms are used for both rigidity of the manipulator, and the constraint for the wrist twist, as human wrist cannot rotate in one full circle. The mobility calculation of the manipulator can be carried out as:

$$\begin{aligned} M &= \sum_{i=1}^{j} f_i - \lambda(j_B - B - c_b) + q - j_p \\ &= 23 - 6(4 - 1 - 0) + 0 - 3 \\ &= 2 \end{aligned} \tag{5.3}$$

5.4. Combined Manipulator for Human Arm

By the combination of the proposed mechanisms, we end up with a new manipulator with variable general constraints that mimics human shoulder, elbow and wrist* complex, *Fig. (5.14).* Variability comes from the different subspace and spaces of the individual manipulators, as λ=6, and λ=3.

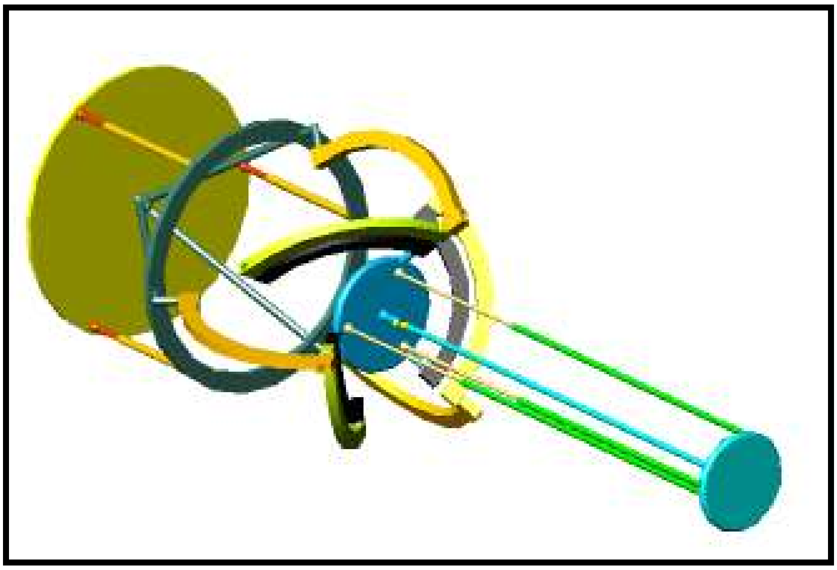


Figure 5.14. New Manipulator with Variable General Constraints that Mimics Human Shoulder, Elbow and Wrist Complex

* *Note that the proposed human arm manipulator just gives the axial motion of the wrist. Individual serial mechanisms for the wrist will be investigated in the following chapter.*

The mobility calculation of the new combined manipulator can be carried out as:

$$
\begin{aligned}
M &= \sum_{i=1}^{j} f_i - \sum_{K=1}^{L} \lambda_K + q - j_p \\
&= 56 - (6 \times 3 + 3 \times 2 + 6 \times 3) + 0 - 6 \\
&= 8
\end{aligned}
\tag{5.4}
$$

CHAPTER 6

KINEMATICS OF HUMAN WRIST MANIPULATOR

6.1. Quaternions as a Product of Two Lines

As it is stated before, the fundamental ***i, j, k*** of quaternions furnishs a set of six quantities, three distinct space units and their combination by multiplication.

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1 \tag{6.1}$$

Using Eq. (6.1), a directed line in space comes to be represented as $\boldsymbol{i}x+\boldsymbol{j}y+\boldsymbol{k}z$, while the product of two lines is the quaternion:

$$\begin{aligned}\tilde{q} &= (\boldsymbol{i}x_1 + \boldsymbol{j}y_1 + \boldsymbol{k}z_1)(\boldsymbol{i}x_2 + \boldsymbol{j}y_2 + \boldsymbol{k}z_2) \\ &= -(x_1x_2 + y_1y_2 + z_1z_2) + \boldsymbol{i}(y_1z_2 - z_1y_2) \\ &\quad + \boldsymbol{j}(z_1x_2 - x_1z_2) + \boldsymbol{k}(x_1y_2 - y_1x_2)\end{aligned} \tag{6.2}$$

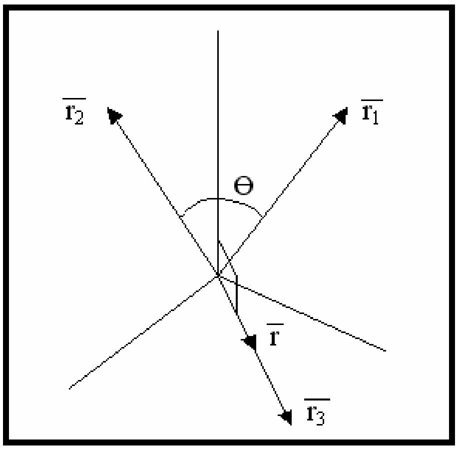


Figure 6.1. Multiplication of Two Lines

Suppose the lines $\bar{r}$, $\bar{r}_1$ and $\bar{r}_2$ to be each at unit length. If θ be the angle between the two unit vectors $\bar{r}_1$ and $\bar{r}_2$, and $\bar{r}_3$ is the line that is perpendicular to each of them, we have,

$$\bar{r}_1 \cdot \bar{r}_2 = x_1x_2 + y_1y_2 + z_1z_2 = |\bar{r}_1||\bar{r}_2|Cos\theta \tag{6.3}$$

where,

$$|\bar{r}_1| = |\bar{r}_2| = 1 \text{ and } x_1x_2 + y_1y_2 + z_1z_2 = Cos\theta \tag{6.4}$$

Also,

$$\bar{r}_3 = \bar{r}_1 \times \bar{r}_2 = \boldsymbol{i}(y_1z_2 - z_1y_2) + \boldsymbol{j}(z_1x_2 - x_1z_2) + \boldsymbol{k}(x_1y_2 - y_1x_2) \tag{6.5}$$

$$|\bar{r}_1 \times \bar{r}_2| = |\bar{r}_1||\bar{r}_2|Sin\theta = 1.Sin\theta \tag{6.6}$$

If we write 1 as a length of unit vector *r(x, y, z)* Eq. (6.6) becomes,

$$|\bar{r}_1 \times \bar{r}_2| = \sqrt{x^2 + y^2 + z^2}.Sin\theta \tag{6.7}$$

If we use Eq. (6.5) and (6.6), we get,

$$\begin{aligned} |\bar{r}_1 \times \bar{r}_2| &= \sqrt{(y_1z_2 - z_1y_2)^2 + (z_1x_2 - x_1z_2)^2 + (x_1y_2 - y_1x_2)^2} \\ &= \sqrt{(Sin^2\theta)(x^2 + y^2 + z^2)} \end{aligned} \tag{6.8}$$

The components of Eq.(6.8) are,

$$\begin{aligned} (y_1z_2 - z_1y_2) &= xSin\theta \\ (z_1x_2 - x_1z_2) &= ySin\theta \\ (x_1y_2 - y_1x_2) &= zSin\theta \end{aligned} \tag{6.9}$$

By using Eq. (6.2), (6.3) and (6.9), product of two unit lines can be expressed as;

$$\tilde{q} = -Cos\theta + \boldsymbol{i}xSin\theta + \boldsymbol{j}ySin\theta + \boldsymbol{k}zSin\theta \tag{6.10}$$

Thus, the quaternion (6.10) as a product of two lines becomes

$$\tilde{q} = -Cos\theta + (\boldsymbol{i}x + \boldsymbol{j}y + \boldsymbol{k}z)Sin\theta \tag{6.11}$$

or,

$$\tilde{q} = (a + ib + jc + kd) \tag{6.12}$$

where,

$$\begin{aligned} a &= -Cos\theta \\ b &= xSin\theta \\ c &= ySin\theta \\ d &= zSin\theta \end{aligned} \tag{6.13}$$

The conjugate of $\tilde{q}$ may be written as,

$$\tilde{q}^{-1} = -Cos\theta - (\boldsymbol{i}x + \boldsymbol{j}y + \boldsymbol{k}z)Sin\theta \tag{6.14}$$

or,

$$\tilde{q}^{-1} = (a - \boldsymbol{i}b - \boldsymbol{j}c - \boldsymbol{k}d) \tag{6.15}$$

Now the product of two quaternions can be at once expressed as a third quaternion;

$$\begin{aligned} \tilde{q} &= \tilde{q}_1\tilde{q}_2 \\ &= (a + \boldsymbol{i}b + \boldsymbol{j}c + \boldsymbol{k}d)(a' + \boldsymbol{i}b' + \boldsymbol{j}c' + \boldsymbol{k}d') \\ &= A + \boldsymbol{i}B + \boldsymbol{j}C + \boldsymbol{k}D \end{aligned} \tag{6.16}$$

where the components of (6.16) are

$$\begin{aligned} A &= aa' - bb' - cc' - dd'; \quad & B &= ab' + ba' + cd' - dc'; \\ C &= ac' + ca' + db' - bd'; \quad & D &= ad' + da' + bc' - cb'; \end{aligned} \tag{6.17}$$

It should be noted that for the later use, a new quaternion operator can be introduced for rotations according to Eq. (6.11) and (6.14) as,

$$\tilde{q}^{-1}(r)\tilde{q} \tag{6.18}$$

where, $\tilde{q} = -Cos\theta + \hat{m}Sin\theta$, $\hat{m}$ is the unit normed vector that passes through origin and $\tilde{q}^{-1} = -Cos\theta - \hat{m}Sin\theta$ is the conjugate of unit quaternion $\tilde{q}$.

The quaternion operator Eq. (6.18) rotates any vector $\bar{r}$ that passes through origin, by 2θ around $\hat{m}$ axis and can be reformed as,

$$q(r)q^{-1} \tag{6.19}$$

If and only if, $q = Cos\theta + \hat{m}Sin\theta$ and $q^{-1} = Cos\theta - \hat{m}Sin\theta$.

Let us now proof transformation from the view of the quaternion operator in Eq. (6.18) to the view of the quaternion operator in Eq. (6.19).

Multiplying the quaternion $\tilde{q}$ and the conjugate quaternion $\tilde{q}^{-1}$ by (-1), the result will be,

$$\begin{aligned}
(-1)\tilde{q} &= (-a - \boldsymbol{i}b - \boldsymbol{j}c - \boldsymbol{k}d) \\
&= Cos\theta - (\boldsymbol{i}x + \boldsymbol{j}y + \boldsymbol{k}z)Sin\theta = Cos\theta - \hat{m}Sin\theta \\
&= q^{-1} \\
(-1)\tilde{q}^{-1} &= (-a + \boldsymbol{i}b + \boldsymbol{j}c + \boldsymbol{k}d) \\
&= Cos\theta + (\boldsymbol{i}x + \boldsymbol{j}y + \boldsymbol{k}z)Sin\theta = Cos\theta + \hat{m}Sin\theta \\
&= q
\end{aligned} \tag{6.20}$$

Multiplying the quaternion operator in Eq. (6.18) by (-1) and using Eq. (6.20), the result becomes,

$$(-1)[\tilde{q}^{-1}(...)\tilde{q}] = (-1)\tilde{q}^{-1}(...)(-1)\tilde{q} = q(...)q^{-1} \tag{6.21}$$

Similarly, multiplying the quaternion operator in Eq. (6.19) by (-1) makes the equation,

$$(-1)[q(...)q^{-1}] = (-1)q(...)(-1)q^{-1} = \tilde{q}^{-1}(...)\tilde{q} \tag{6.22}$$

Two quaternion operators in Eq. (6.21) and (6.22) are equal, if it satisfies the following conditions:

$$\begin{aligned} \tilde{q}^{-1} &= -q = -Cos\theta - \hat{m}Sin\theta \\ \tilde{q} &= -q^{-1} = -Cos\theta + \hat{m}Sin\theta \end{aligned} \tag{6.23}$$

Eq. (6.23) describes an important new definition in quaternion algebra.

Definition:

Two quaternion operators will be equal, if quaternion and its conjugate in former quaternion operator are equal to the negative conjugate and its negative quaternion in later quaternion operator respectively.

6.2. Rigid Body Rotations by Using Sequential Method by Quaternion Operators

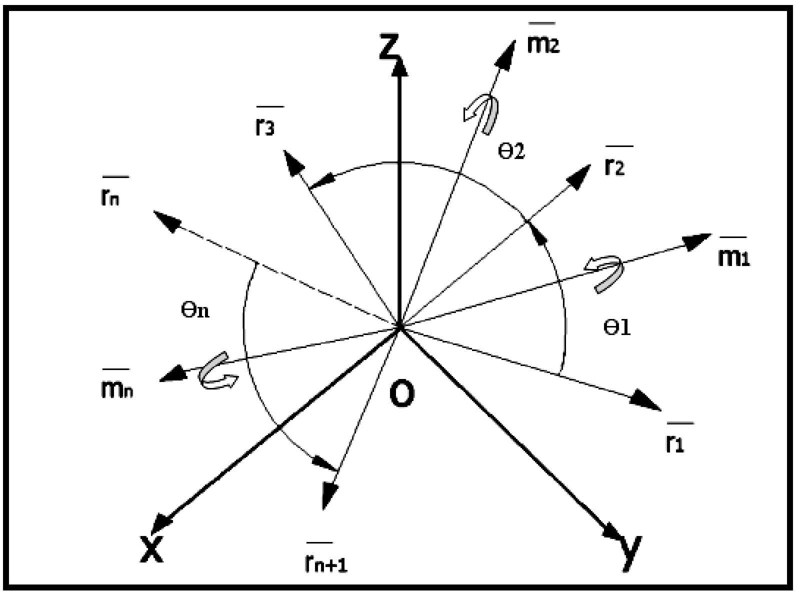


Figure 6.2. Sequential Rotations of $\bar{r}_1$

Fig. (6.2) shows a vector $\bar{r}_i$ that passes through successive rotations about $\bar{m}_i$ by θ_i (i=1, 2…n). These rotations can be carried out by a quaternion operator in Eq. (6.18) or (6.19).

Now, let $q_i = Cos\frac{\theta_i}{2} + \hat{m}_i Sin\frac{\theta_i}{2}$ a unit quaternion and thus the quaternion operator in Eq. (6.19) is chosen.

Any $\bar{r}_{i+1}$ can be found by using the equation,

$$r_{i+1} = q_i (r_i) q_i^{-1} \tag{6.24}$$

and generalizing the case in Eq. (6.24),

$$r_{n+1} = q_n q_{n-1} \ldots\ldots. q_2 q_1 (r_1) q_1^{-1} q_2^{-1} \ldots\ldots. q_{n-1}^{-1} q_n^{-1} \tag{6.25}$$

It has been shown that rigid body rotations can be modeled in sequential method by quaternion operators, Eq. (6.25), and it should be noted that sequential model is closer to the traditional motion of robot manipulators.

6.3. Rigid Body Rotations by Using New Modular Method by Quaternion Operators

Fig. (*6.3)* interprets a new method for rotations of a rigid body by using quaternion operators. In this new method, $\bar{r}_1$ and all $\bar{m}_i$'s except $\bar{m}_n$ is rotated around the $\bar{m}_n$ axis by θ_n and new vectors are reached as, $\bar{r}_1^1$ and $\bar{m}_i^{\,1}$ (i=1,2,……n-1) and this procedure is continued until $\bar{r}_1^n$ is being reached.

Now, let us introduce a unit quaternion $q_i^* = Cos\frac{\theta_k}{2} + \hat{m}_k^{i-1} Sin\frac{\theta_k}{2}$ where $k = p + 1 - i$, (i=1,2,…..n), where p = number of total rotations and a quaternion operator $q^*(r)q^{*-1}$.

Any $\bar{r}_1^i$ can be found by using the equation,

$$r_1^i = q_i^*(r)q_i^{*-1} \tag{6.26}$$

and any m_j^i can be found by using the equation,

$$m_j^i = q_i^*(m_j^{i-1})q_i^{*-1} \tag{6.27}$$

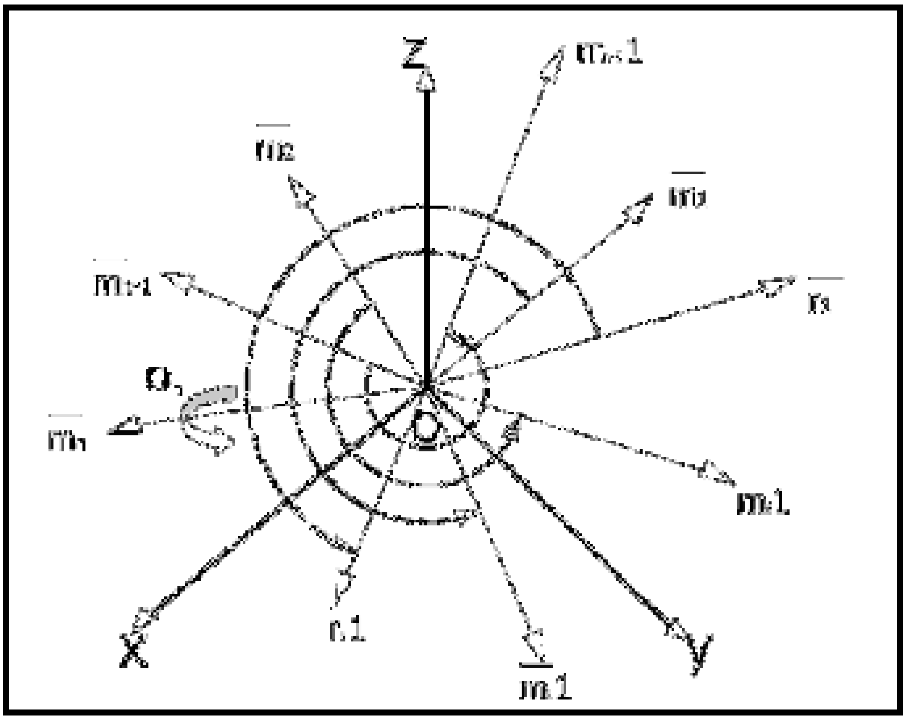


Figure 6.3. Modular Method Rotations

and generalizing the case in Eq. (6.26) and (6.27),

$$r_1^n = q_n^* q_{n-1}^* ....q_2^* q_1^* (r_1) q_1^{*-1} q_2^{*-1} ....q_{n-1}^{*-1} q_n^{*-1} \tag{6.28}$$

The sequential Eq. (6.25) and modular Eq. (6.28) methods of rigid body rotations by quaternion operators are identical; however, modular method is closer to the human motion of robot manipulators.

Theorem:

If $\bar{m}_i$ (i=1, 2…n) be unit vectors which pass through origin of fixed coordinate system and $R_{m_i}(\theta_i)$, (i=1, 2…n), (n≥2) be rotation operators about $\bar{m}_i$ axes by θ_i

angles, and these rotations are applied to an $\bar{r}_1$ vector in $R_{m_i}(\theta_1)$ to $R_{m_i}(\theta_n)$ order, solution vector is assumed to be $\bar{r}_{n+1}$ vector, where subscript shows the operation numbers. When these rotation operations are applied to an $\bar{r}_2$ vector and also to $\bar{m}_i$ (i= 1,….,n-1) rotation axes at the same time, but in reverse order from θ_n around the $\bar{m}_n$, we get $\bar{r}_2^1$ and $\bar{m}_i^1$ (i=1,….,n-1). If we apply the same procedure to the resulting vectors, finally we end up with $\bar{r}_2^n$. Solutions of these two different set of operations, $\bar{r}_{n+1}$ and $\bar{r}_2^n$, are exactly the same if and only if $\bar{r}_1$ and $\bar{r}_2$ are equal.

Proof:

In the first case, rotation operators $R_{m_i}(\theta_i)$ are applied to vector $\bar{r}_1$ respectively as fallows,

$$\bar{r}_2 = R_{m_1}(\theta_1)r_1,\ \bar{r}_3 = R_{m_2}(\theta_2)r_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,\ \bar{r}_{n+1} = R_{m_n}(\theta_n)r_n \qquad (6.29)$$

Using quaternion operators, Eq. (6.29) can be shown as,

$$r_{n+1} = q_n q_{n-1} \ldots\ldots. q_2 q_1 (r_1) q_1^{-1} q_2^{-1} \ldots\ldots. q_{n-1}^{-1} q_n^{-1} \qquad (6.30)$$

If n=2, Eq. (6.30) is reduced to,

$$r_3 = q_2 q_1 (r_1) q_1^{-1} q_2^{-1} \qquad (6.31)$$

Using Eq. (6.31) where we set $\bar{m}_1 = m_1 i + m_2 j + m_3 k$ *and* $\bar{m}_2 = n_1 i + n_2 j + n_3 k$, we reached by sequential method analytical expression of rigid body rotations with respect to two axes as equations shown in Appendix A.

In the second case, first m_i (i=1,2….,n-1) axes and $\bar{r}_2$ are rotated around $\bar{m}_n$ by the angle θ_n and we get $\bar{r}_2^1$ and $\bar{m}_i^1$ (i=n-1,….,1). We continued this process until we reach $\bar{r}_2^n$. These operations are summarized by using quaternion operators as,

$$r_2^n = q_n^* q_{n-1}^* \ldots\ldots. q_2^* q_1^* (r_2) q_1^{*-1} q_2^{*-1} \ldots\ldots. q_{n-1}^{*-1} q_n^{*-1} \qquad (6.32)$$

If n=2, Eq. (6.32) is reduced to,

$$r_2^2 = q_2^* q_1^* (r_1) q_1^{*-1} q_2^{*-1} \tag{6.33}$$

Using Eq. (6.33) where we set $\overline{m}_1 = m_1 i + m_2 j + m_3 k$, *and* $\overline{m}_2 = n_1 i + n_2 j + n_3 k$, we reached by modular method analytical expression of rigid body rotations with respect to two axes as *Fig. (6.4).*

As it was shown in Appendix A, equations, $r_3 = q_2 q_1 (r_1) q_1^{-1} q_2^{-1}$ and $r_2^2 = q_2^* q_1^* (r_2) q_1^{*-1} q_2^{*-1}$ are identical.

As a result, the sequential and modular methods of rigid body rotations by quaternion operators are identical.

6.4. Spherical Wrist Motion through Quaternions

Suppose 2-DoF spherical manipulator is given as *Fig. (6.4),* and it is desired to find the position of the gripper, $\overline{z}_2$, after its rotation, first around $\overline{z}_1$ by θ_1, and then around $\overline{z}_0$ by θ_2.

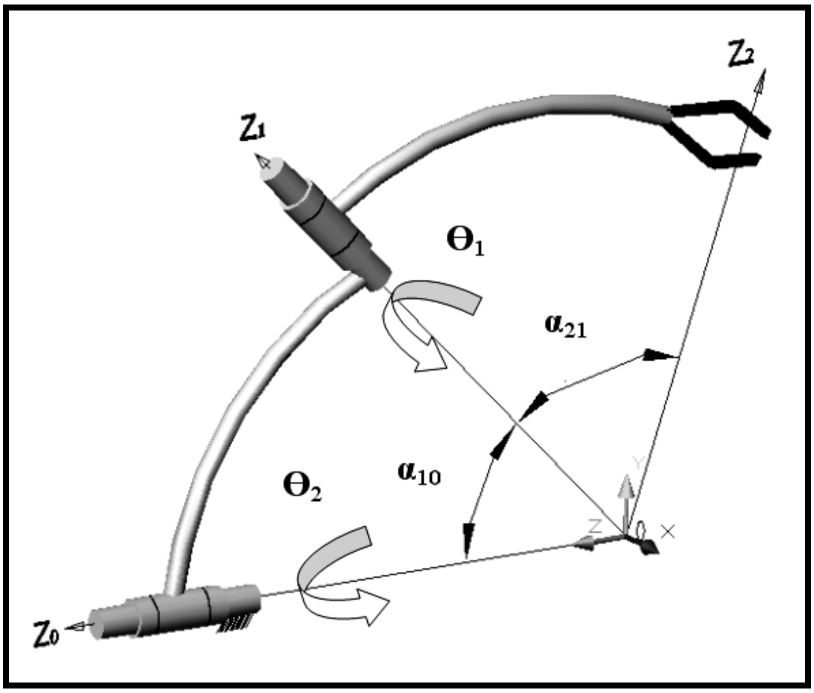


Figure 6.4. Position Vectors of a Spherical Serial Wrist with 2-DoF

Applying the first sequential method, the position of the gripper after the first rotation can be calculated as,

$$z_2^1 = q_1(z_2)q_1^{-1} \tag{6.34}$$

where,

$$q_1 = Cos\frac{\theta_1}{2} + \hat{z}_1 Sin\frac{\theta_1}{2} \tag{6.35}$$

and the final position will be,

$$z_2^2 = q_2(z_2^1)q_2^{-1} \tag{6.36}$$

where,

$$q_2 = Cos\frac{\theta_2}{2} + \hat{z}_0 Sin\frac{\theta_2}{2} \tag{6.37}$$

Combining Eq. (6.34÷6.37) we get,

$$z_2^2 = q_2 q_1(z_2)q_1^{-1}q_2^{-1} \tag{6.38}$$

If the desired method will be the modular method, to find the final position of the gripper, both $\bar{z}_1$ and $\bar{z}_2$ will be rotated first around $\bar{z}_0$ by θ_2 to find $\bar{z}_2^{1*}$ and $\bar{z}_1^{1*}$ as,

$$z_1^{1*} = q_1^{\,*}(z_1)q_1^{*-1},\ z_2^{1*} = q_1^{\,*}(z_2)q_1^{*-1} \tag{6.39}$$

where,

$$q_1^{\,*} = Cos\frac{\theta_2}{2} + \hat{z}_0 Sin\frac{\theta_2}{2} \tag{6.40}$$

To reach the final position, $\bar{z}_2^{1*}$ should be rotated around $\bar{z}_1^{1*}$ by θ_1 and $\bar{z}_2^2$ will be,

$$z_2^2 = z_2^{2*} = q_2^{\ *}(z_2^{1*})q_2^{*-1} \tag{6.41}$$

where,

$$q_2^{\ *} = Cos\frac{\theta_1}{2} + \hat{z}_1^{1*}Sin\frac{\theta_1}{2} \tag{6.42}$$

Note that, both Eq. (6.38) and (6.42) will give the same result for the final position of the gripper.

The same procedure can be applied to more complex systems as 3-DoF spherical manipulator in *Fig. (6.5).*

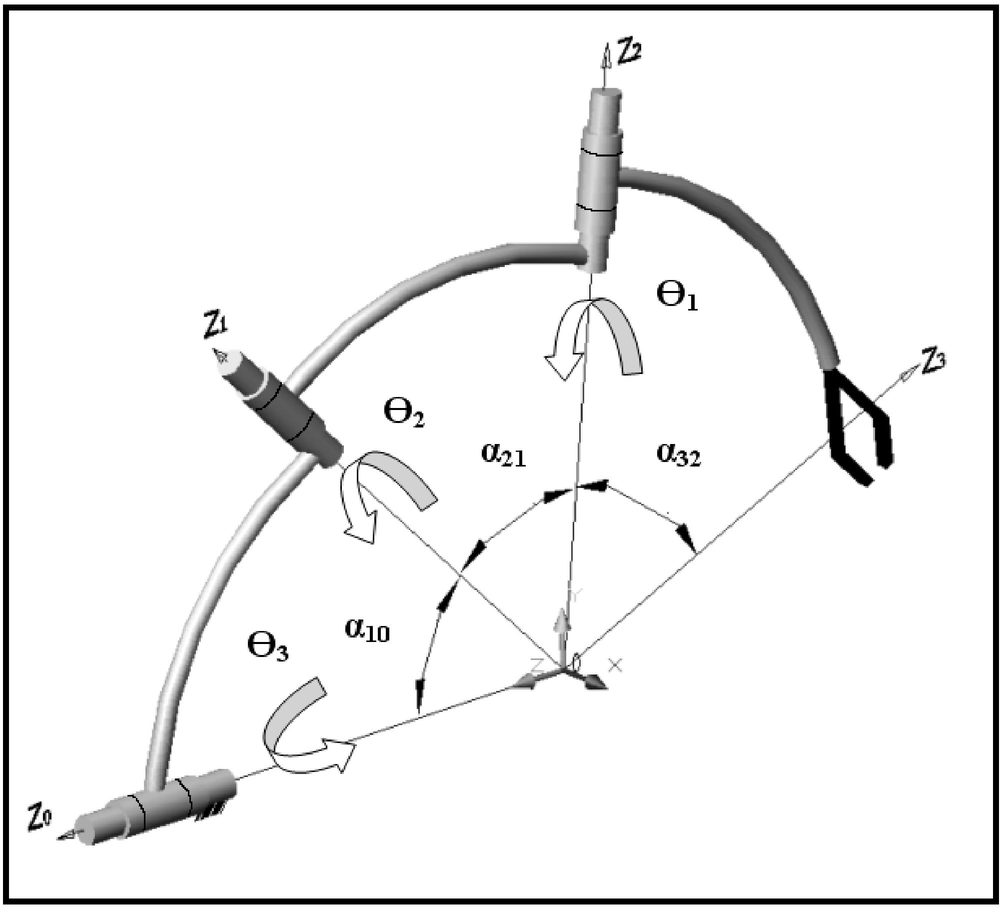


Figure 6.5. Position Vectors of a Spherical Serial Wrist with 3-DoF

By sequential method the final position of the gripper $\bar{z}_3$ will be,

$$z_3^3 = q_3 q_2 q_1 (z_3) q_1^{-1} q_2^{-1} q_3^{-1} \tag{6.43}$$

where,

$$\begin{aligned} q_1 &= Cos\frac{\theta_1}{2} + \hat{z}_2 Sin\frac{\theta_1}{2} \\ q_2 &= Cos\frac{\theta_2}{2} + \hat{z}_1 Sin\frac{\theta_2}{2} \\ q_3 &= Cos\frac{\theta_3}{2} + \hat{z}_0 Sin\frac{\theta_3}{2} \end{aligned} \tag{6.44}$$

If the modular method is the case to reach the final position of the gripper, following equations should be used:

$$\begin{gathered} z_1^{1*} = q_1^*(z_1) q_1^{*-1}, z_2^{1*} = q_1^*(z_2) q_1^{*-1}, z_3^{1*} = q_1^*(z_3) q_1^{*-1} \\ q_1^* = Cos\frac{\theta_3}{2} + \hat{z}_0 Sin\frac{\theta_3}{2} \\ z_2^{2*} = q_2^*(z_2^{1*}) q_2^{*-1}, z_3^{2*} = q_2^*(z_3^{1*}) q_2^{*-1} \\ q_2^* = Cos\frac{\theta_2}{2} + \hat{z}_1^{1*} Sin\frac{\theta_2}{2} \\ z_3^3 = z_3^{3*} = q_3^*(z_3^{2*}) q_3^{*-1} \\ q_3^* = Cos\frac{\theta_1}{2} + \hat{z}_2^{2*} Sin\frac{\theta_1}{2} \end{gathered} \tag{6.45}$$

Note that, both Eq. (6.43) and (6.45) will give the same result for the final position of the gripper.

Further analysis can be applied to the systems with higher degrees of freedom as in *Fig. (6.6)*.

By sequential method the final position of the gripper $\bar{z}_4$ will be,

$$z_4^4 = q_4 q_3 q_2 q_1 (z_4) q_1^{-1} q_2^{-1} q_3^{-1} q_4^{-1} \tag{6.46}$$

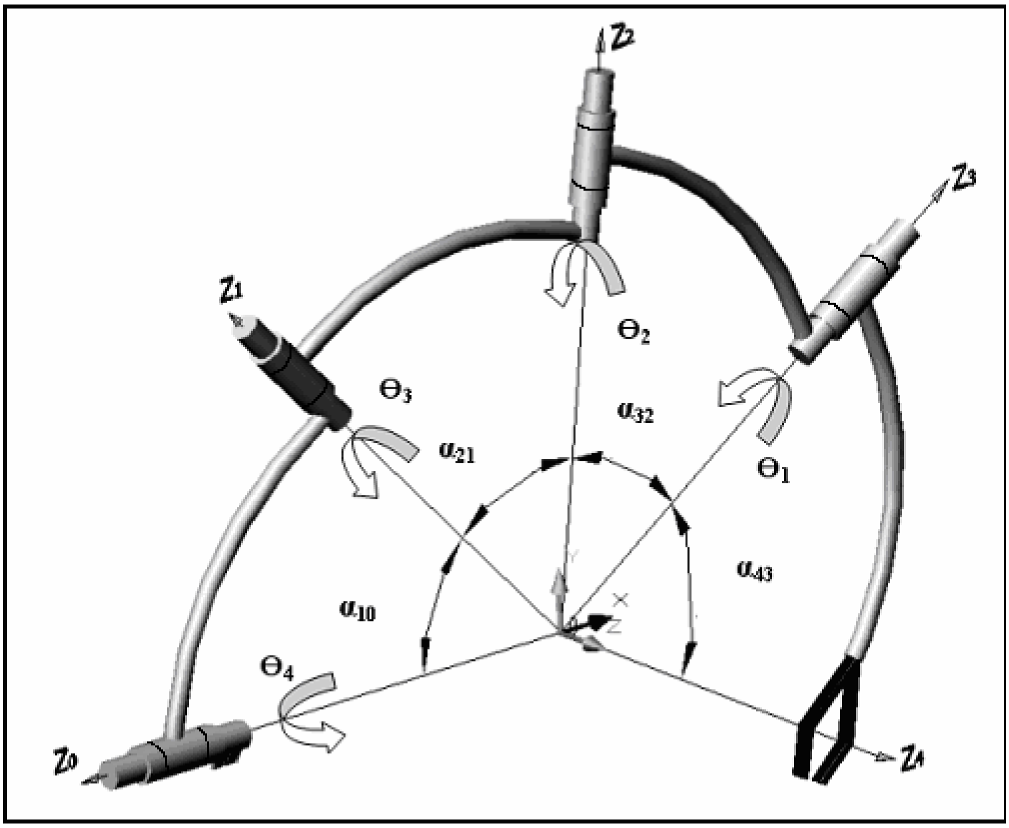


Figure 6.6. Position Vectors of a Spherical Serial Wrist with 4-DoF

where,

$$
\begin{aligned}
q_1 &= Cos\frac{\theta_1}{2} + \hat{z}_3 Sin\frac{\theta_1}{2} \\
q_2 &= Cos\frac{\theta_2}{2} + \hat{z}_2 Sin\frac{\theta_2}{2} \\
q_3 &= Cos\frac{\theta_3}{2} + \hat{z}_1 Sin\frac{\theta_3}{2} \\
q_4 &= Cos\frac{\theta_4}{2} + \hat{z}_0 Sin\frac{\theta_4}{2}
\end{aligned} \tag{6.47}
$$

If the modular method is the case to reach the final position of the gripper, following equations should be used:

$$
\begin{gathered}
z_1^{1*} = q_1^{\ *}(z_1)q_1^{*-1}, z_2^{1*} = q_1^{\ *}(z_2)q_1^{*-1} \\
z_3^{1*} = q_1^{\ *}(z_3)q_1^{*-1}, z_4^{1*} = q_1^{\ *}(z_4)q_1^{*-1}, q_1^{\ *} = Cos\frac{\theta_4}{2} + \hat{z}_0 Sin\frac{\theta_4}{2}
\end{gathered} \tag{6.48a}
$$

$$
\begin{gathered}
z_2^{2*} = q_2^{\,*}(z_2^{1*})q_2^{*-1}, z_3^{2*} = q_2^{\,*}(z_3^{1*})q_2^{*-1}, z_4^{2*} = q_2^{\,*}(z_4^{1*})q_2^{*-1} \\
q_2^{\,*} = Cos\frac{\theta_3}{2} + \hat{z}_1^{1*} Sin\frac{\theta_3}{2} \\
z_3^{3*} = q_3^{\,*}(z_3^{2*})q_3^{*-1}, z_4^{3*} = q_3^{\,*}(z_4^{2*})q_3^{*-1} \\
q_3^{\,*} = Cos\frac{\theta_2}{2} + \hat{z}_2^{2*} Sin\frac{\theta_2}{2} \\
z_4^{4} = z_4^{4*} = q_3^{\,*}(z_4^{3*})q_3^{*-1} \\
q_4^{\,*} = Cos\frac{\theta_3}{2} + \hat{z}_3^{3*} Sin\frac{\theta_3}{2}
\end{gathered}
\tag{6.48b}
$$

Again, note that, both Eq. (6.46) and (6.48) will give the same result for the final position of the gripper.

6.5. Workspaces of the Spherical Wrists

By using the computed equations of the 2-DoF, 3-DoF and 4-DoF spherical wrists, the workspaces of the grippers are analysed *Fig. (6.7), (6.8),* and *(6.9).*

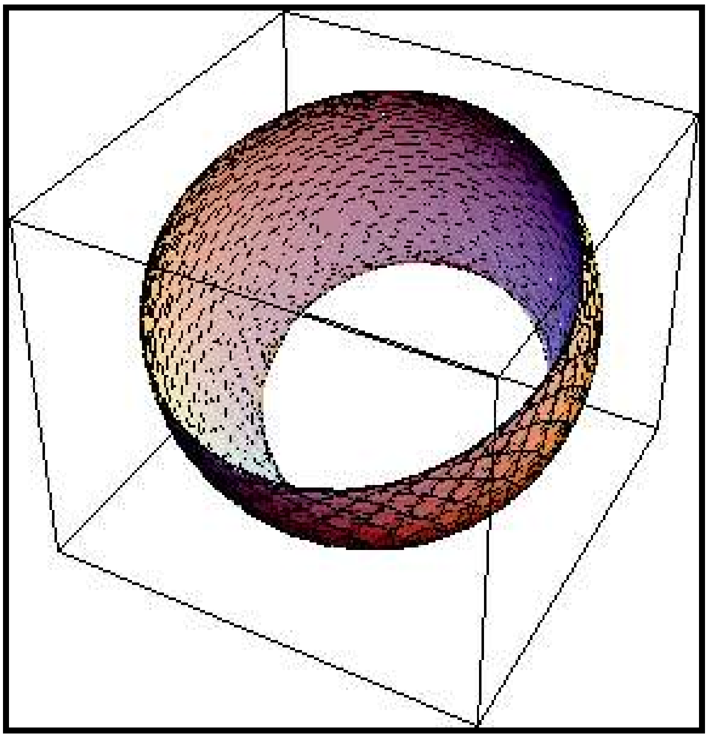


Figure 6.7. Workspace of a Spherical Serial Wrist with 2-DoF

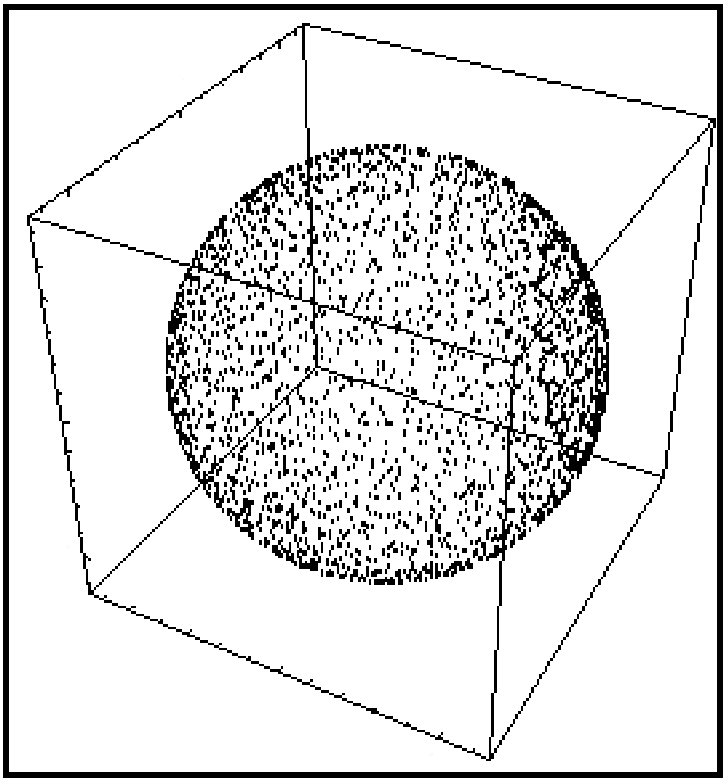


Figure 6.8. Workspace of a Spherical Serial Wrist with 3-DoF

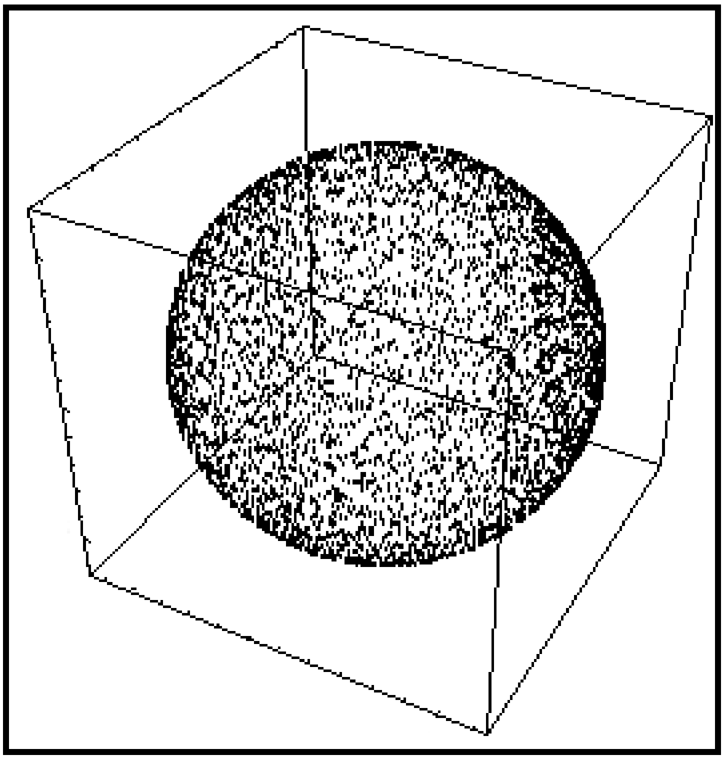


Figure 6.9. Workspace of a Spherical Serial Wrist with 4-DoF

CHAPTER 7

GEOMETRICAL ANALYSIS OF THE HUMAN CLAVICLE AND ELBOW MANIPULATOR

7.1. Geometrical Analysis of Spatial 3-DoF Orientation Mechanism in λ=6

Now, the first part, proposed for the clavicle, of the new manipulator design with variable general constraints that mimics human shoulder, elbow and wrist complex will be analysed with respect to the geometrical approach *Fig. (7.1-7.2)*.

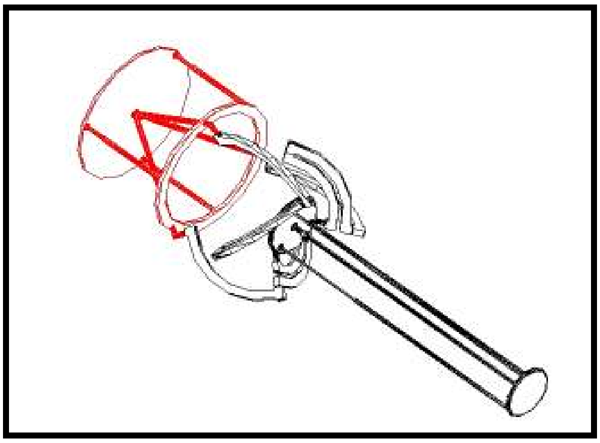


Figure 7.1. Orientation Platform (Red) of the New Manipulator Design

Figure 7.2. Orientation Platform (Closed View)

Eq.(7.1) shows the simple equation of a sphere with radius "r" whose center is fixed at the origin *Fig.(7.3)* and the Eq. *(7.2)* shows the case when the center is away from the origin (x_1, y_1, z_1) *Fig. (7.3).*

$$x^2 + y^2 + z^2 = r^2 \tag{7.1}$$

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r^2 \tag{7.2}$$

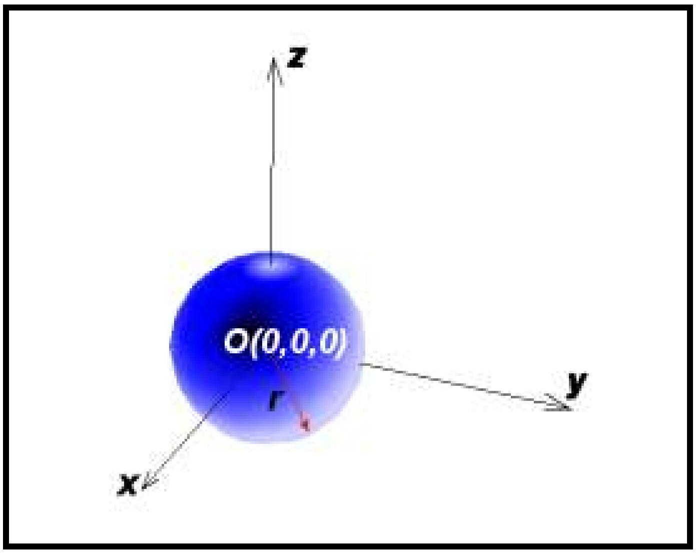


Figure 7.3. Sphere with Radius "r" whose Center is Fixed at the Origin

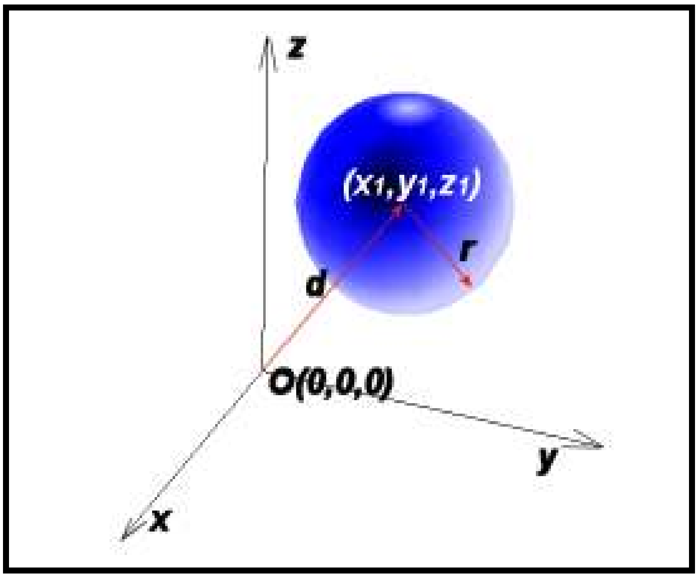


Figure 7.4. Sphere with Radius "r" whose Center is Away from the Origin

Now let us consider the generalized mechanism in *Fig. (7.5)*. We will set the origin point of the Cartesian coordinate system as the rotation center of the mechanism where z axis is perpendicular to the upper platform in the initial configuration.

To start to analyse the workspace of the first point D_1 we will draw the vector from the origin to the point D_1. From this point we can describe the workspace of r_1 as a sphere "A" whose equation is,

$$x_{D_1}{}^2 + y_{D_1}{}^2 + z_{D_1}{}^2 = r_1^2 \tag{7.3}$$

In second operation, we will pass to the leg d_1. By using the constructional parameters we can easily find the coordinates of $O_1\ (x_1, y_1, z_1)$. Due to the limitation of the leg d_1 the work space of the vector will be the volume between the spheres B and C. We can show this in equation as,

$$\begin{gathered}(x_{D_1} - x_1)^2 + (y_{D_1} - y_1)^2 + (z_{D_1} - z_1)^2 = d_1{}^2 \\ d_1 = d_{1\min} \to d_{1\max}\end{gathered} \tag{7.4}$$

By the same analogy, we can write other 4 equations for the other legs,

$$x_{D_2}{}^2 + y_{D_2}{}^2 + z_{D_2}{}^2 = r_2^2 \tag{7.5}$$

$$\begin{gathered}(x_{D_2} - x_2)^2 + (y_{D_2} - y_2)^2 + (z_{D_2} - z_2)^2 = d_2{}^2 \\ d_2 = d_{2\min} \to d_{2\max}\end{gathered} \tag{7.6}$$

$$x_{D_3}{}^2 + y_{D_3}{}^2 + z_{D_3}{}^2 = r_3^2 \tag{7.7}$$

$$\begin{gathered}(x_{D_3} - x_3)^2 + (y_{D_3} - y_3)^2 + (z_{D_3} - z_3)^2 = d_3{}^2 \\ d_3 = d_{3\min} \to d_{3\max}\end{gathered} \tag{7.8}$$

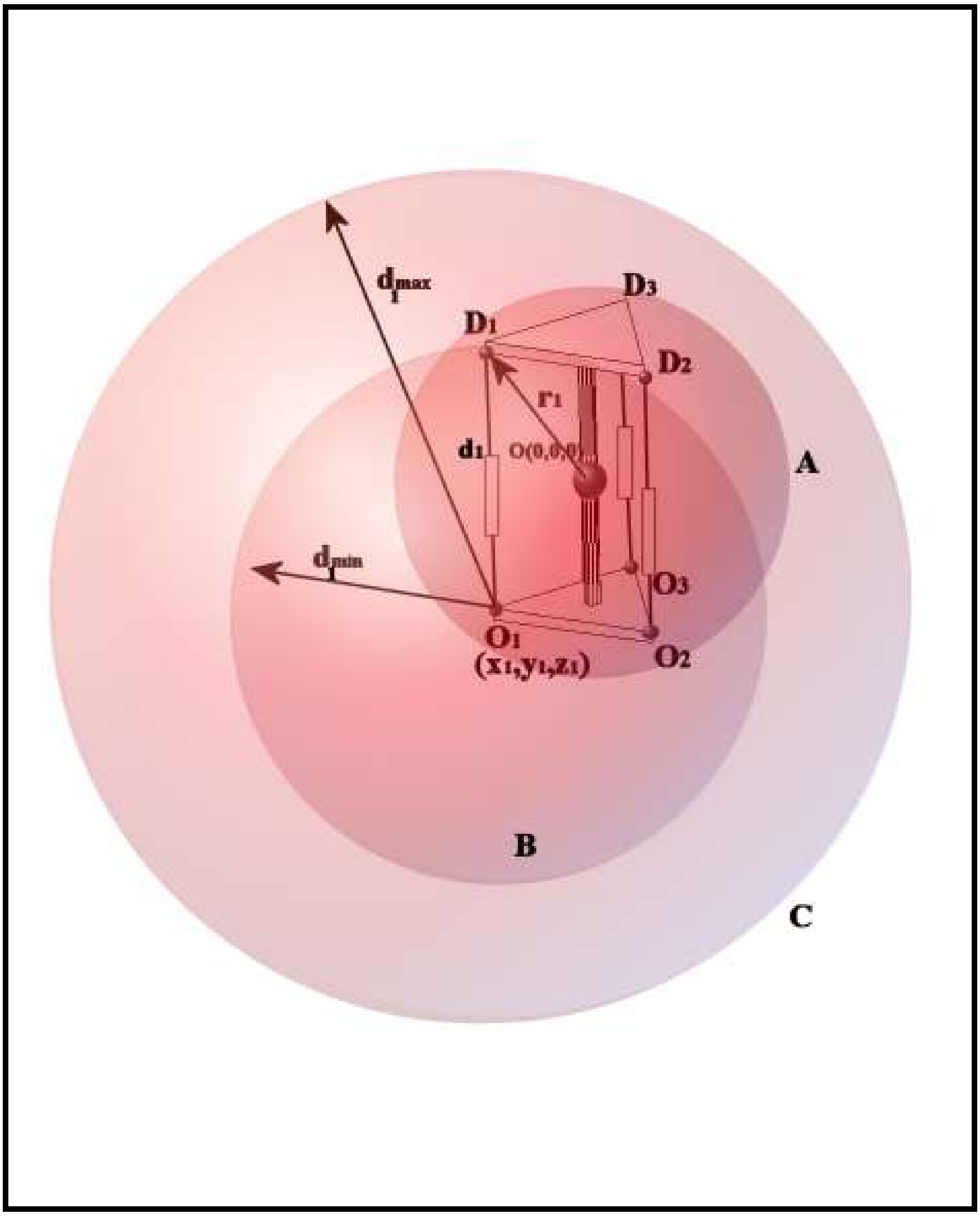


Figure 7.5. Generalized Orientation Mechanism

At this point we have 6 equations with 9 unknowns, of which 6 of them are independent and 3 of them are dependent with respect to the Eq. (7.3), (7.5) and (7.7), remaining 3 equations comes from the construction parameters of the upper platform *Fig. (7.6).*

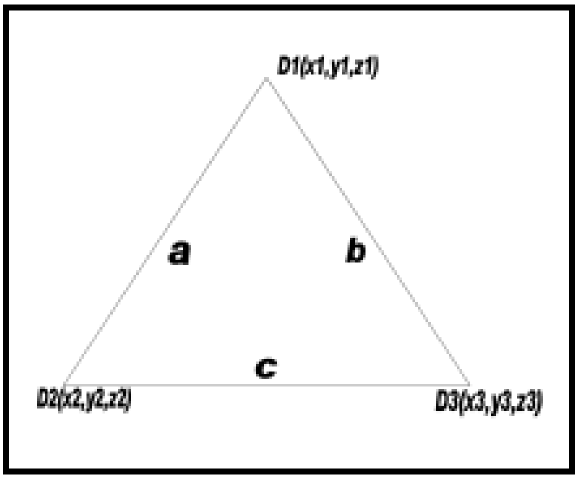


Figure 7.6. Construction Parameters of Upper Platform (a, b, c)

By using the fixed lengths, a, b, and c,

$$(x_{D_1}-x_{D_2})^2+(y_{D_1}-y_{D_2})^2+(z_{D_1}-z_{D_2})^2=a^2 \tag{7.9}$$

$$(x_{D_2}-x_{D_3})^2+(y_{D_2}-y_{D_3})^2+(z_{D_2}-z_{D_3})^2=c^2 \tag{7.10}$$

$$(x_{D_1}-x_{D_3})^2+(y_{D_1}-y_{D_3})^2+(z_{D_1}-z_{D_3})^2=b^2 \tag{7.11}$$

Now from the 9 equations Eq. (7.3÷7.11), we can easily start to compute the unknowns $x_{D_i}, y_{D_i}, z_{D_i}$ (i=1, 2, 3).

From the Eq. (7.3), (7.5) and (7.7), we will get,

$$\begin{aligned} z_{D_1} &= \pm\sqrt{r_1^2 - x_{D_1} - y_{D_1}} \\ z_{D_2} &= \pm\sqrt{r_2^2 - x_{D_2} - y_{D_2}} \\ z_{D_3} &= \pm\sqrt{r_3^2 - x_{D_3} - y_{D_3}} \end{aligned} \tag{7.12}$$

by using the transformations in Eq. (7.13), equation sets can be rewritten as,

$$\begin{aligned} x_{D_1} &= \tilde{x}_1 \\ y_{D_1} &= \tilde{y}_1 \\ z_{D_1} &= \tilde{z}_1 \end{aligned} \tag{7.13}$$

$$
\begin{aligned}
(\tilde{x}_1 - x_1)^2 + (\tilde{y}_1 - y_1)^2 + (\tilde{z}_1 - z_1)^2 &= d_1^{\,2} \\
(\tilde{x}_2 - x_2)^2 + (\tilde{y}_2 - y_2)^2 + (\tilde{z}_2 - z_2)^2 &= d_2^{\,2} \\
(\tilde{x}_3 - x_3)^2 + (\tilde{y}_3 - y_3)^2 + (\tilde{z}_3 - z_3)^2 &= d_3^{\,2} \\
(\tilde{x}_1 - \tilde{x}_2)^2 + (\tilde{y}_1 - \tilde{y}_2)^2 + (\tilde{z}_1 - \tilde{z}_2)^2 &= a^2 \\
(\tilde{x}_1 - \tilde{x}_3)^2 + (\tilde{y}_1 - \tilde{y}_3)^2 + (\tilde{z}_1 - \tilde{z}_3)^2 &= b^2 \\
(\tilde{x}_3 - \tilde{x}_2)^2 + (\tilde{y}_3 - \tilde{y}_2)^2 + (\tilde{z}_3 - \tilde{z}_2)^2 &= c^2
\end{aligned}
\tag{7.14}
$$

expanding the equations,

$$
\begin{aligned}
\tilde{x}_1^{\,2} + \tilde{y}_1^{\,2} + \tilde{z}_1^{\,2} + x_1^{\,2} + y_1^{\,2} + z_1^{\,2} - 2\tilde{x}_1 x_1 - 2\tilde{y}_1 y_1 - 2\tilde{z}_1 z_1 &= d_1^{\,2} \\
\tilde{x}_2^{\,2} + \tilde{y}_2^{\,2} + \tilde{z}_2^{\,2} + x_2^{\,2} + y_2^{\,2} + z_2^{\,2} - 2\tilde{x}_2 x_2 - 2\tilde{y}_2 y_2 - 2\tilde{z}_2 z_2 &= d_2^{\,2} \\
\tilde{x}_3^{\,2} + \tilde{y}_3^{\,2} + \tilde{z}_3^{\,2} + x_3^{\,2} + y_3^{\,2} + z_3^{\,2} - 2\tilde{x}_1 x_1 - 2\tilde{y}_1 y_1 - 2\tilde{z}_1 z_1 &= d_3^{\,2} \\
\tilde{x}_1^{\,2} + \tilde{y}_1^{\,2} + \tilde{z}_1^{\,2} + \tilde{x}_2^{\,2} + \tilde{y}_2^{\,2} + \tilde{z}_2^{\,2} - 2\tilde{x}_1 \tilde{x}_2 - 2\tilde{y}_1 \tilde{y}_2 - 2\tilde{z}_1 \tilde{z}_2 &= a^2 \\
\tilde{x}_1^{\,2} + \tilde{y}_1^{\,2} + \tilde{z}_1^{\,2} + \tilde{x}_3^{\,2} + \tilde{y}_3^{\,2} + \tilde{z}_3^{\,2} - 2\tilde{x}_1 \tilde{x}_3 - 2\tilde{y}_1 \tilde{y}_3 - 2\tilde{z}_1 \tilde{z}_3 &= b^2 \\
\tilde{x}_2^{\,2} + \tilde{y}_2^{\,2} + \tilde{z}_2^{\,2} + \tilde{x}_3^{\,2} + \tilde{y}_3^{\,2} + \tilde{z}_3^{\,2} - 2\tilde{x}_2 \tilde{x}_3 - 2\tilde{y}_2 \tilde{y}_3 - 2\tilde{z}_2 \tilde{z}_3 &= c^2
\end{aligned}
\tag{7.15}
$$

using Eq. (7.3), (7.5) and (7.7), and arranging the terms,

$$
\begin{aligned}
\tilde{x}_1 x_1 + \tilde{y}_1 y_1 + \tilde{z}_1 z_1 &= \frac{1}{2}(-d_1^{\,2} + r_1^{\,2} + x_1^{\,2} + y_1^{\,2} + z_1^{\,2}) \\
\tilde{x}_2 x_2 + \tilde{y}_2 y_2 + \tilde{z}_2 z_2 &= \frac{1}{2}(-d_2^{\,2} + r_2^{\,2} + x_2^{\,2} + y_2^{\,2} + z_2^{\,2}) \\
\tilde{x}_3 x_3 + \tilde{y}_3 y_3 + \tilde{z}_3 z_3 &= \frac{1}{2}(-d_3^{\,2} + r_3^{\,2} + x_3^{\,2} + y_3^{\,2} + z_3^{\,2}) \\
\tilde{x}_1 \tilde{x}_2 + \tilde{y}_1 \tilde{y}_2 + \tilde{z}_1 \tilde{z}_2 &= \frac{1}{2}(-a^2 + r_1^{\,2} + r_2^{\,2}) \\
\tilde{x}_1 \tilde{x}_3 + \tilde{y}_1 \tilde{y}_3 + \tilde{z}_1 \tilde{z}_3 &= \frac{1}{2}(-b^2 + r_1^{\,2} + r_3^{\,2}) \\
\tilde{x}_2 \tilde{x}_3 + \tilde{y}_2 \tilde{y}_3 + \tilde{z}_2 \tilde{z}_3 &= \frac{1}{2}(-c^2 + r_2^{\,2} + r_3^{\,2})
\end{aligned}
\tag{7.16}
$$

For simplifying purposes, new parameters can be used as,

$$
\begin{aligned}
p &= \frac{1}{2}(-d_1{}^2 + r_1{}^2 + x_1{}^2 + y_1{}^2 + z_1{}^2) \\
q &= \frac{1}{2}(-d_2{}^2 + r_2{}^2 + x_2{}^2 + y_2{}^2 + z_2{}^2) \\
t &= \frac{1}{2}(-d_3{}^2 + r_3{}^2 + x_3{}^2 + y_3{}^2 + z_3{}^2) \\
s_1 &= \frac{1}{2}(-a^2 + r_1{}^2 + r_2{}^2) \\
s_2 &= \frac{1}{2}(-b^2 + r_1{}^2 + r_3{}^2) \\
s_3 &= \frac{1}{2}(-c^2 + r_2{}^2 + r_3{}^2)
\end{aligned} \tag{7.17}
$$

and using equation set (7.12),

$$
\begin{aligned}
(\pm\sqrt{r_1{}^2 - \tilde{x}_1 - \tilde{y}_1})z_1 &= p - \tilde{x}_1 x_1 - \tilde{y}_1 y_1 \\
(\pm\sqrt{r_2{}^2 - \tilde{x}_2 - \tilde{y}_2})z_2 &= q - \tilde{x}_2 x_2 - \tilde{y}_2 y_2 \\
(\pm\sqrt{r_3{}^2 - \tilde{x}_3 - \tilde{y}_3})z_3 &= t - \tilde{x}_3 x_3 - \tilde{y}_3 y_3 \\
(\pm\sqrt{r_1{}^2 - \tilde{x}_1 - \tilde{y}_1})(\pm\sqrt{r_2{}^2 - \tilde{x}_2 - \tilde{y}_2}) &= s_1 - \tilde{x}_1\tilde{x}_2 - \tilde{y}_1\tilde{y}_2 \\
(\pm\sqrt{r_1{}^2 - \tilde{x}_1 - \tilde{y}_1})(\pm\sqrt{r_3{}^2 - \tilde{x}_3 - \tilde{y}_3}) &= s_2 - \tilde{x}_1\tilde{x}_3 - \tilde{y}_1\tilde{y}_3 \\
(\pm\sqrt{r_2{}^2 - \tilde{x}_2 - \tilde{y}_2})(\pm\sqrt{r_3{}^2 - \tilde{x}_3 - \tilde{y}_3}) &= s_3 - \tilde{x}_2\tilde{x}_3 - \tilde{y}_2\tilde{y}_3
\end{aligned} \tag{7.18}
$$

Squaring both sides and arranging,

$$
\begin{gathered}
\tilde{x}_1^2(-z_1{}^2 - x_1^2) + \tilde{y}_1^2(-z_1{}^2 - y_1^2) + \tilde{x}_1(2px_1) + \tilde{y}_1(2py_1) + \tilde{x}_1\tilde{y}_1(-2x_1y_1) = p^2 - z_1^2 r_1^2 \\
\tilde{x}_2^2(-z_2{}^2 - x_2^2) + \tilde{y}_2^2(-z_2{}^2 - y_2^2) + \tilde{x}_2(2qx_2) + \tilde{y}_2(2qy_2) + \tilde{x}_2\tilde{y}_2(-2x_2y_2) = q^2 - z_2{}^2 r_2^2 \\
\tilde{x}_3^2(-z_3{}^2 - x_3^2) + \tilde{y}_3^2(-z_3{}^2 - y_3^2) + \tilde{x}_3(2tx_3) + \tilde{y}_3(2ty_3) + \tilde{x}_3\tilde{y}_3(-2x_3y_3) = t^2 - z_3{}^2 r_3^2 \\
\tilde{x}_1^2(-r_2^2) + \tilde{y}_1^2(-r_2^2) + \tilde{x}_2{}^2(-r_1^2) + \tilde{y}_2{}^2(-r_1^2) + \tilde{x}_1^2\tilde{y}_2{}^2 + \tilde{y}_1^2\tilde{x}_2{}^2 + \tilde{x}_1\tilde{x}_2(2s_1) + \tilde{y}_1\tilde{y}_2(2s_1) \\
+ \tilde{x}_1\tilde{x}_2\tilde{y}_1\tilde{y}_2(-2) = s_1{}^2 - r_1^2 r_2^2 \\
\tilde{x}_1^2(-r_3^2) + \tilde{y}_1^2(-r_3^2) + \tilde{x}_3{}^2(-r_1^2) + \tilde{y}_3{}^2(-r_1^2) + \tilde{x}_1^2\tilde{y}_3{}^2 + \tilde{y}_1^2\tilde{x}_3{}^2 + \tilde{x}_1\tilde{x}_3(2s_1) + \tilde{y}_1\tilde{y}_3(2s_1) \\
+ \tilde{x}_1\tilde{x}_3\tilde{y}_1\tilde{y}_3(-2) = s_2{}^2 - r_1^2 r_3^2 \\
\tilde{x}_2^2(-r_3^2) + \tilde{y}_2^2(-r_3^2) + \tilde{x}_3^2(-r_2^2) + \tilde{y}_3{}^2(-r_2^2) + \tilde{x}_2{}^2\tilde{y}_3{}^2 + \tilde{y}_2{}^2\tilde{x}_3{}^2 + \tilde{x}_2\tilde{x}_3(2s_1) + \tilde{y}_2\tilde{y}_3(2s_2) \\
+ \tilde{x}_2\tilde{x}_3\tilde{y}_2\tilde{y}_3(-2) = s_2{}^2 - r_2^2 r_3^2
\end{gathered} \tag{7.19}
$$

and, in the end, after the combinations of the known values, our final equation sets, that has 6 independent variables will be,

$$
\begin{gathered}
A_1\tilde{x}_1^{\,2} + A_2\tilde{y}_1^{\,2} + A_3\tilde{x}_1 + A_4\tilde{y}_1 + A_5\tilde{x}_1\tilde{y}_1 = A \\
B_1\tilde{x}_2^{\,2} + B_2\tilde{y}_2^{\,2} + B_3\tilde{x}_2 + B_4\tilde{y}_2 + B_5\tilde{x}_2\tilde{y}_2 = B \\
C_1\tilde{x}_3^{\,2} + C_2\tilde{y}_3^{\,2} + C_3\tilde{x}_3 + C_4\tilde{y}_3 + C_5\tilde{x}_3\tilde{y}_3 = C \\
D_1\tilde{x}_1^{\,2} + D_2\tilde{y}_1^{\,2} + D_3\tilde{x}_2^{\,2} + D_4\tilde{y}_2^{\,2} + D_5\tilde{x}_1^{\,2}\tilde{y}_2^{\,2} + D_6\tilde{y}_1^{\,2}\tilde{x}_2^{\,2} + D_7\tilde{x}_1\tilde{x}_2 + D_8\tilde{y}_1\tilde{y}_2 \\
+D_9\tilde{x}_1\tilde{x}_2\tilde{y}_1\tilde{y}_2 = D \\
E_1\tilde{x}_1^{\,2} + E_2\tilde{y}_1^{\,2} + E_3\tilde{x}_3^{\,2} + E_4\tilde{y}_3^{\,2} + E_5\tilde{x}_1^{\,2}\tilde{y}_3^{\,2} + E_6\tilde{y}_1^{\,2}\tilde{x}_3^{\,2} + E_7\tilde{x}_1\tilde{x}_3 + E_8\tilde{y}_1\tilde{y}_3 \\
+E_9\tilde{x}_1\tilde{x}_3\tilde{y}_1\tilde{y}_3 = E \\
F_1\tilde{x}_2^{\,2} + F_2\tilde{y}_2^{\,2} + F_3\tilde{x}_3^{\,2} + F_4\tilde{y}_3^{\,2} + F_5\tilde{x}_2^{\,2}\tilde{y}_3^{\,2} + F_6\tilde{y}_2^{\,2}\tilde{x}_3^{\,2} + F_7\tilde{x}_2\tilde{x}_3 + F_8\tilde{y}_2\tilde{y}_3 \\
+F_9\tilde{x}_2\tilde{x}_3\tilde{y}_2\tilde{y}_3 = F
\end{gathered}
\tag{7.20}
$$

As we have six independent equations and six unknowns, there exists a unique solution for the position of the platform with respect to the given parameters. So that, solving the Eq. (7.20) by numerical methods and get the positions of the corners, any position and orientation of the center of the orientation platform can be found by using the input parameters.

7.2. Geometrical Analysis of Spatial 2-DoF Orientation Mechanism in λ=6

Unlike the 3-DoF orientation mechanism, geometric analysis of the manipulator proposed for the ankle joint is relatively simple Fig (7.7) and (7.8), as the control of the platform is carried out by two actuators separately.

In *Fig. (7.9),* on the closed view of the platform base, two revolute actuators A, and B can easily be seen. As it is mentioned, revolute actuator A is responsible for the position of the platform and the other revolute actuator is responsible for the orientation of the platform.

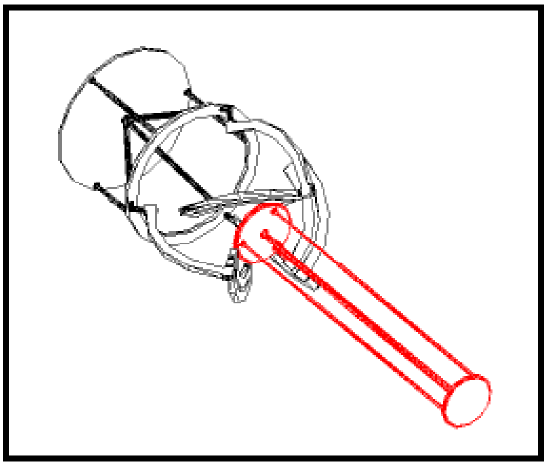


Figure 7.7. 2-DoF Orientation Platform (Red) of the New Manipulator Design

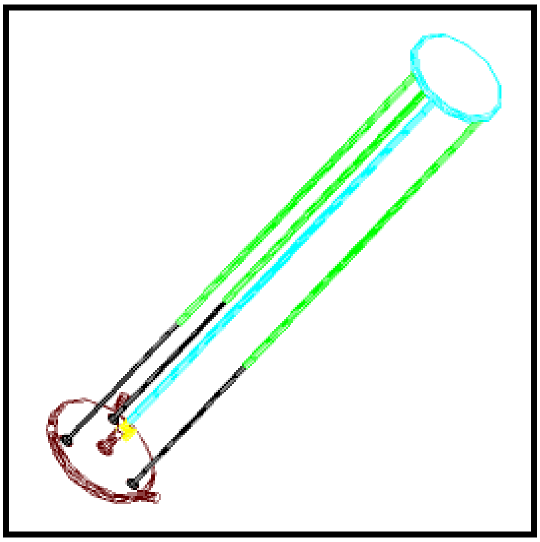


Figure 7.8. 2-DoF Orientation Platform (Closed View)

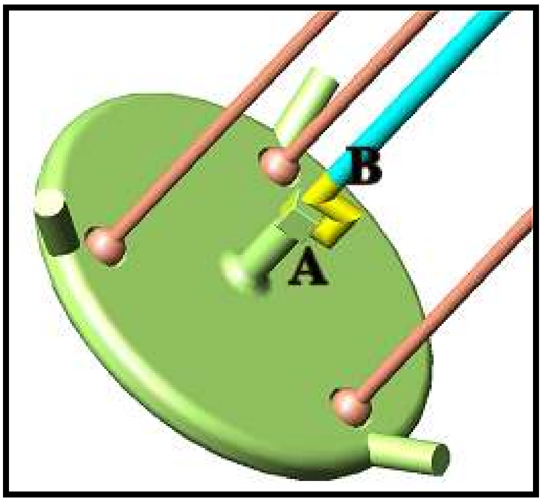


Figure 7.9. 2-DoF Orientation Platform (Base)

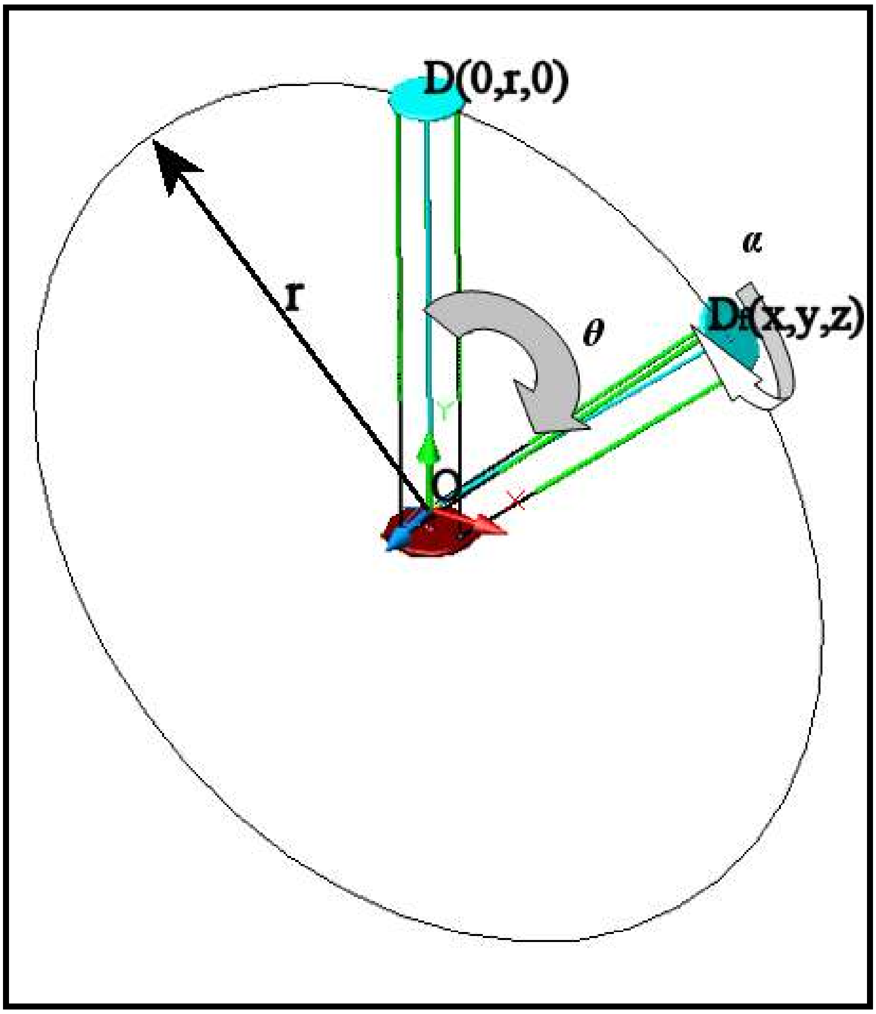


Figure 7.10. Generalized 2-DoF Orientation Platform

Now, consider the manipulator in Fig (7.10) will be rotated by the angle $\boldsymbol{\theta}$ around z axis by using revolute actuator A in clock wise direction, and rotated by the angle $\boldsymbol{\alpha}$ around the floating y axis. Note that in this operation, the platform of the shoulder is taken as a fixed frame. Due to the fact that, the center of the upper platform (D), will always remain on the circle with radius r, the last position of the platform will be,

$$
\begin{aligned}
x &= rSin(\theta) \\
y &= rCos(\theta) \\
z &= 0
\end{aligned}
\tag{7.21}
$$

and the final orientation will be the angle $\boldsymbol{\alpha}$.

Let us now prove the positions by the help of the rotation matrices. As seen in *Fig. (7.10)* the first position of the center point of the upper platform is D(0,r,0) and the final position is D_f(0,r,0). By using the rotational sequences,

$$Df = R(-\theta, z)R(\alpha, y)D \tag{7.22}$$

Expanding and evaluating the Eq. (7.22),

$$\begin{aligned}
\begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} C(-\theta) & -S(-\theta) & 0 \\ S(-\theta) & C(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\alpha & 0 & S\alpha \\ 0 & 1 & 0 \\ -S\alpha & 0 & C\alpha \end{bmatrix} \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} rS\theta \\ rC\theta \\ 0 \end{bmatrix}
\end{aligned} \tag{7.23}$$

So, the results of Eq. (7.23) prove the geometrical interpretation for the final position of the platform Eq. (7.21). By using these equations any of the positions of the platform can be computed with respect to the given parameters.

CHAPTER 8

CONCLUSION

It has been shown in this study that, rigid body rotations can be carried out by using two different techniques through quaternion algebra with different quaternion operators. The advantage is the fact that, it is very simple procedure to use quaternions in rotation sequences when compared with the rotation matrices or other alternative tools; thus, this means fewer operations and less computational time. Analysis of the results through kinematics of spherical human wrist manipulators has proved that the theory is applicable practically as well as theoretically. Also it has been shown that sequential model is closer to the traditional motion of robot manipulators, while the new proposed modular method is closer to the human motion of robot manipulators.

Moreover, in this study, a new structural formula for spatial parallel manipulators having one general moving platform, working in Cartesian space with three legs that are placed in orthogonal planes introducing simple structural groups in subspaces λ=3, 4 and general space λ=6, and connected to actuators and to the general moving platform is introduced. History of formulas DoF is presented as 38 equations with the unique key controlling parameters. 9 new parallel Cartesian platform robot manipulators are introduced by applying new mobility loop-legs equation. Structural synthesis of serial platform manipulators and parallel Cartesian platform manipulators are considered. Simple serial platform structural groups in subspace λ=3, and general space λ=6 are presented along with examples.

Finally, after the type synthesis of human arm is completed with the illustrations of the human arm, a new manipulator with variable general constraints that mimics human shoulder, elbow and wrist complex is proposed with its structural synthesis. The manipulator has two orientation platforms in space $\lambda = 6$ and one spherical platform in subspace $\lambda = 3$. At the end of the study, geometrical kinematic analysis of the orientation platforms of the new manipulator design for the human arm is accomplished.

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APPENDIX A

RESULT OF THE EQUATIONS

$$r_3 = q_2 q_1 (r_1) q_1^{-1} q_2^{-1} \textbf{ AND } r_2^2 = q_2^{\bullet} q_1^{\bullet} (r_2) q_1^{\bullet -1} q_2^{\bullet -1}$$

$Quaternion\left[a,b,c,d\right]$

$a = \left[0\right]$

$$b = \begin{bmatrix}
\frac{1}{2} Cos^2\left[\frac{\theta_2}{2}\right](x + m_1^2 x - m_2^2 x - m_3^2 x + 2m_1 m_2 y + 2m_1 m_3 z + ((1 - m_1^2 + m_2^2 + m_3^2)x - 2m_1(m_2 y + m_3 z)) \\
Cos\left[\theta_1\right] + (-2m_3 y + 2m_2 z) Sin\left[\theta_1\right]) + Sin\left[\theta_2\right](2m_1 m_3 n_2 x - 2m_1 m_2 n_3 x + 2m_2 m_3 n_2 y - n_3 y + m_1^2 n_3 y - \\
m_2^2 n_3 y + m_3^2 n_3 y + n_2 z - m_1^2 n_2 z - m_2^2 n_2 z + m_3^2 n_2 z - 2m_2 m_3 n_3 z + (m_1(-2m_3 n_2 x + 2m_2 n_3 x) - n_3 y - m_3^2 n_3 y \\
+n_2 z - m_3^2 n_2 z + m_1^2(-n_3 y + n_2 z) + m_2^2(n_3 y + n_2 z) + m_2(-2m_3 n_2 y + 2m_3 n_2 y + 2m_3 n_3 z)) Cos\left[\theta_1\right] - 2(m_2 n_2 x \\
+m_3 n_3 x - m_1(n_2 y + n_3 z)) Sin\left[\theta_1\right]) + Sin^2\left[\frac{\theta_2}{2}\right](n_1^2 x + m_1^2 n_1^2 x - m_2^2 n_1^2 x - m_3^2 n_1^2 x + 4m_1 m_2 n_1 n_2 x - n_2^2 x - \\
m_1^2 n_2^2 x + m_2^2 n_2^2 x + 4m_1 m_3 n_1 n_3 x - n_3^2 x - m_1^2 n_3^2 x + m_2^2 n_3^2 x + m_3^2 n_3^2 x + 2m_1 m_2 n_1^2 y + 2n_1 n_2 y - 2m_1^2 n_1 n_2 y + \\
2m_2^2 n_1 n_2 y - 2m_3^2 n_1 n_2 y - 2m_1 m_2 n_2^2 y + 4m_2 m_3 n_1 n_3 y - 2m_1 m_2 n_3^2 y + 2m_1 m_3 n_1^2 z + 4m_2 m_3 n_1 n_2 z - 2m_1 m_3 n_2^2 z + \\
2n_1 n_3 z - 2m_1^2 n_1 n_3 z - 2m_2^2 n_1 n_3 z + 2m_3^2 n_1 n_3 z - 2m_1 m_3 n_3^2 z - (n_1^2((-1 + m_1^2 - m_2^2 - m_3^2)x + 2m_1(m_2 y + m_3 z)) \\
-(n_2^2 + n_3^2)((-1 + m_1^2 - m_2^2 - m_3^2)x + 2m_1(m_2 y + m_3 z)) - 2n_1(-2m_1(m_2 n_2 + m_3 n_3)x + n_2(y - m_2^2 y + m_3^2 y \\
-2m_2 m_3 z) + n_3(-2m_2 m_3 y + z + m_2^2 z - m_3^2 z) + m_1^2(n_2 y + n_3 z))) Cos\left[\theta_1\right] - 2(m_3(-2n_1 n_2 x + n_1^2 y - (n_2^2 + n_3^2)y) \\
+2m_1 n_1(-n_3 y + n_2 z) + m_2(2n_1 n_3 x - n_1^2 z + (n_2^2 + n_3^2)z)) Sin\left[\theta_1\right]))
\end{bmatrix}$$

$$
c = \begin{bmatrix}
\frac{1}{2}(-Sin\left[\theta_2\right](2m_1m_3n_1x - n_3x - m_1^2n_3x + m_2^2n_3x + m_3^2n_3x + 2m_2m_3n_1y - 2m_1m_2n_3y + n_1z - m_1^2n_1z - \\
m_2^2n_1z + m_3^2n_1z - 2m_1m_3n_3z + (-(1+m_2^2+m_3^2)n_3x + n_1(-2m_2m_3y + z + m_2^2z - m_3^2z) + m_1^2(n_3x + n_1z) \\
+2m_1(-m_3n_1x + m_2n_3y + m_3n_3z))Cos\left[\theta_1\right] - 2m_2n_1xSin\left[\theta_1\right] + 2m_1n_1ySin\left[\theta_1\right] + 2m_3n_3ySin\left[\theta_1\right] - \\
2m_2n_3zSin\left[\theta_1\right]) + Cos^2\left[\frac{\theta_2}{2}\right](2m_1m_2x + y - m_1^2y + m_2^2y - m_3^2y + 2m_2m_3z + (-2m_1m_2x + y + m_1^2y - \\
m_2^2y + m_3^2y - 2m_2m_3z)Cos\left[\theta_1\right] + 2(m_3x - m_1z)Sin\left[\theta_1\right]) + Sin^2\left[\frac{\theta_2}{2}\right](-2m_1m_2n_1^2x + 2n_1n_2x + \\
2m_1^2n_1n_2x - 2m_2^2n_1n_2x - 2m_3^2n_1n_2x + 2m_1n_2n_2^2x + 4m_1m_3n_2n_3x - 2m_1n_2n_3^2x - n_1^2y + m_1^2n_1^2y - m_2^2n_1^2y \\
+m_3^2n_1^2y + 4m_1m_2n_1n_2y + n_2^2y - m_1^2n_2^2y + m_2^2n_2^2y - m_3^2n_2^2y + 4m_2m_3n_2n_3y - n_3^2y + m_1^2n_3^2y - m_2^2n_3^2y + \\
m_3^2n_3^2y - 2m_2m_3n_1^2z + 4m_1m_3n_1n_2z + 2m_2m_3n_2^2z + 2n_2n_3z - 2m_1^2n_2n_3z - 2m_2^2n_2n_3z + 2m_3^2n_2n_3z - \\
2m_2m_3n_3^2z - (-2(1+m_2^2+m_3^2)n_1n_2x - n_2^2y + m_2^2n_2^2y - m_3^2n_2^2y + 4m_2m_3n_2n_3y + n_3^2y - m_2^2n_3^2y + m_3^2n_3^2y \\
+2m_2m_3n_2^2z - 2n_2n_3z - 2m_2^2n_2n_3z + 2m_3^2n_2n_3z - 2m_2m_3n_3^2z + n_1^2(y - m_2^2y + m_3^2y - 2m_2m_3z) + \\
m_1^2(2n_1n_2x + n_1^2y - n_2^2y + n_3^2y - 2n_2n_3z) + m_1(-2m_2(n_1^2x - n_2^2x + n_3^2x - 2n_1n_2y) + 4m_3n_2(n_3x + n_1z))) \\
Cos\left[\theta_1\right] - 2(m_3(n_1^2x + (-n_2^2 + n_3^2)x + 2n_1n_2y) + 2m_2n_2(n_3x - n_1z) + m_1(-2n_2n_3y + n_2^2z - (n_1^2 + n_3^2)z)) \\
Sin\left[\theta_1\right]))
\end{bmatrix}
$$

$$
d = \begin{bmatrix}
\frac{1}{2}(Cos^2\left[\frac{\theta_2}{2}\right](2m_1m_3x + 2m_2m_3y + z - m_1^2z - m_2^2z + m_3^2z + (-2m_1m_3x - 2m_2m_3y + z + m_1^2z + m_2^2z - m_3^2z) \\
Cos\left[\theta_1\right] + (-2m_2x + 2m_1y)Sin\left[\theta_1\right]) + Sin\left[\theta_2\right](2m_1m_2n_1x - n_2x - m_1^2n_2x + m_2^2n_2x + m_3^2n_2x + n_1y - m_1^2n_1y \\
+m_2^2n_1y - m_3^2n_1y - 2m_1m_2n_2y + 2m_2m_3n_1z - 2m_1m_3n_2z + (-(1+m_2^2+m_3^2)n_2x + m_1^2(n_2x + n_1y) + n_1(y - \\
m_2^2y + m_3^2y - 2m_2m_3z) + m_1(-2m_2n_1x + 2m_2n_2y + 2m_3n_2z))Cos\left[\theta_1\right] + 2m_3n_1xSin[\theta_1] + 2m_3n_2ySin[\theta_1] - \\
2m_1n_1zSin[\theta_1] - 2m_2n_1zSin[\theta_1]) + Sin^2\left[\frac{\theta_2}{2}\right](-2m_1m_3n_1^2x - 2m_1m_3n_2^2x + 2n_1n_3x + 2m_1^2n_1n_3x - 2m_2^2n_1n_3x \\
-2m_3^2n_1n_3x + 4m_1m_2n_2n_3x + 2m_1m_3n_3^2x - 2m_2m_3n_1^2y - 2m_2m_3n_2^2y + 4m_1m_2n_1n_3y + 2n_2n_3y - 2m_1^2n_2n_3y + \\
2m_2^2n_2n_3y - 2m_3^2n_2n_3y + 2m_2m_3n_3^2y - n_1^2z + m_1^2n_1^2z + m_2^2n_1^2z - m_3^2n_1^2z - n_2^2z + m_1^2n_2^2z + m_2^2n_2^2z - m_3^2n_2^2z + \\
4m_1m_3n_1n_3z + 4m_2m_3n_2n_3z + n_3^2z - m_1^2n_3^2z - m_2^2n_3^2z + m_3^2n_3^2z - (-2(1+m_2^2+m_3^2)n_1n_3x - 2m_2m_3n_2^2y - \\
2n_2n_3y + 2m_2^2n_2n_3y - 2m_3^2n_2n_3y + 2m_2m_3n_3^2y + n_2^2z + m_2^2n_2^2z - m_3^2n_3^2z + 4m_2m_3n_2n_3z - n_3^2z - m_2^2n_3^2z + \\
m_3^2n_3^2z + n_1^2(-2m_2m_3y + z + m_2^2z - m_3^2z) + m_1^2(2n_1n_3x - 2n_2n_3y + n_1^2z + n_2^2z - n_3^2z) + m_1(4m_2n_3(n_2x + n_1y) \\
-2m_3(n_1^2x + n_2^2x - n_3^2x - 2n_1n_3z)))Cos\left[\theta_1\right] + 2(2m_3n_3(n_2x - n_1y) + m_2(n_1^2x + (n_2^2 - n_3^2)x + 2n_1n_3z) - \\
m_1(n_1^2y + n_2^2y - n_3^2y + 2n_2n_3z))Sin\left[\theta_1\right]))
\end{bmatrix}
$$

APPENDIX B

Q-BASIC CODE FOR THE CREATION OF THE STRUCTURAL GROUPS OF SERIAL PLATFORM MANIPULATORS

```
DO
CLS
INPUT "Select the subspace 3 or 6", s
REDIM s(B)
REDIM c(2)
REDIM L(2)
REDIM j(2)
REDIM a(2)
REDIM B(2)
c(1)=B-1
c(2)=B
IF s=3 THEN
        FOR i=1 to B
                s(i)=3
        NEXT i
ELSE
        PRINT "Select the platform structures 3,4,5,6:"
        FOR i=1 to B
                PRINT "B(";i;"):";
                INPUT " ", s(i)
        NEXT i
END IF
Jb=0
FOR i=1 to B
        Jb=jb+s(i)
NEXT i
cl(1)=jb-(2*c(1))
```

```
cl(2)=jb-(2*c(2))
L(1)=cl(1)+c(1)-B
L(2)=cl(2)+c(2)-B
j(1)=s*L(1)
j(2)=s*L(2)
a(1)=INT((j(1)-c(1))(cl(1))
B(1)=j(1)-c(1)-(a(1)*cl(1))
a(2)=INT((j(2)-c(2))(cl(2))
B(2)=j(2)-c(2)-(a(2)*cl(2))
CLS
PRINT "Subspace or space=";s
PRINT "Number of platforms=";B
PRINT "Structure of platforms";
FOR i=1 to B
        PRINT s(i);
NEXT i
PRINT "*****"
PRINT "by using" ;c(1); "hinges…"
PRINT "total number of legs=";cl(1)
PRINT "total number of joints=";j(1)
PRINT "in each leg there should be ";a(1);"joints"
PRINT "remaining" ;B(1) ; " joints can be placed to any leg"
IF B>=4 THEN
        PRINT "*****"
        PRINT "by using" ;c(2); "hinges…"
        PRINT "total number of legs=";cl(2)
        PRINT "total number of joints=";j(2)
        PRINT "in each leg there should be ";a(2);"joints"
        PRINT "remaining" ;B(2) ; " joints can be placed to any leg"
IF (a(1)-INT((-B(2))/(B(2)+1)))>=s THEN PRINT "warning: impossible configuration"
END IF
INPUT "Do you want to exit (y/n)",exit$
LOOP until exit$="y"
```

APPENDIX C

MOTION ANALYSIS OF NEW CARTESIAN ROBOT MANIPULATORS

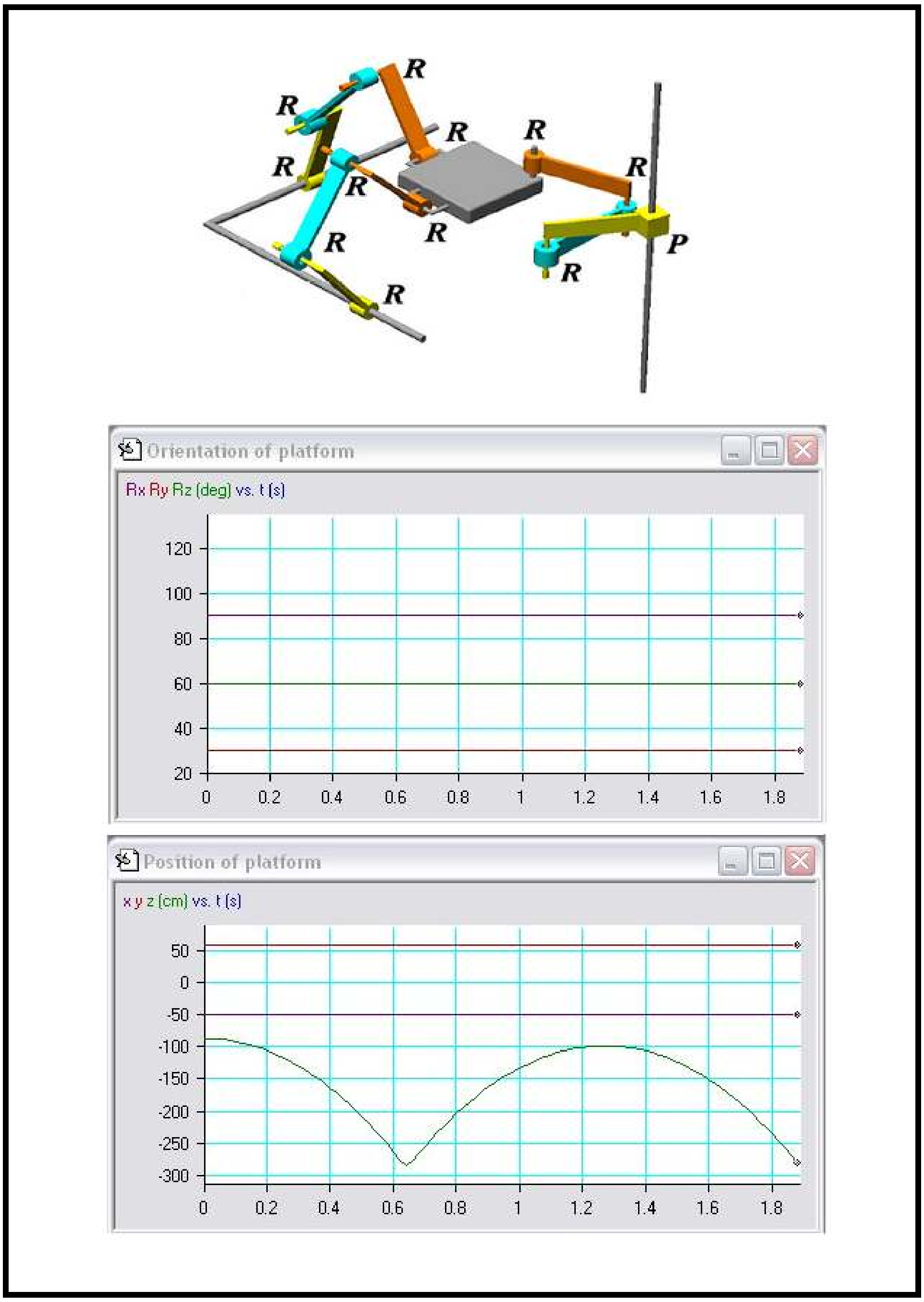

Figure C.1. Raw Motion Analysis

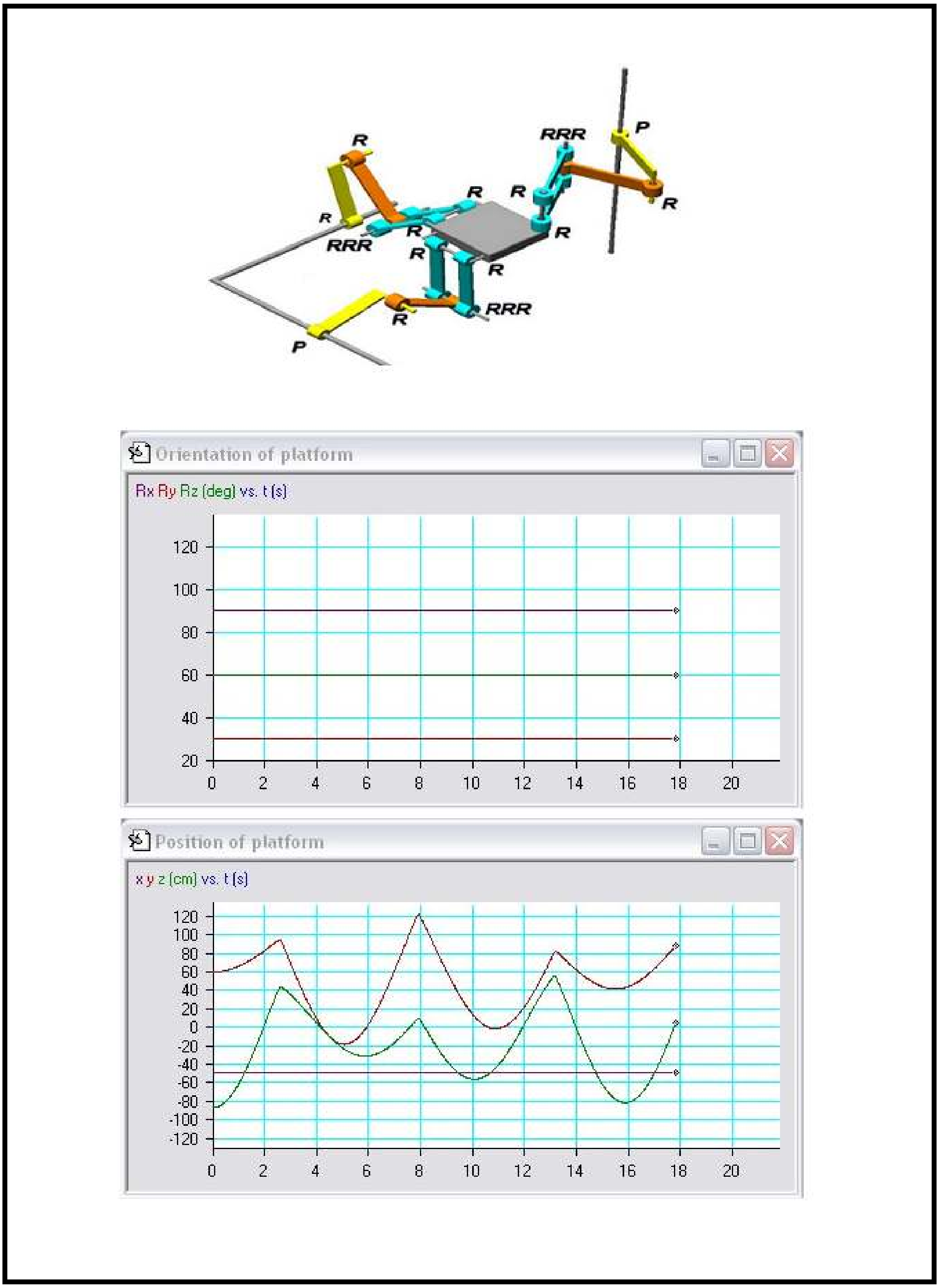

Figure C.2. Raw Motion Analysis

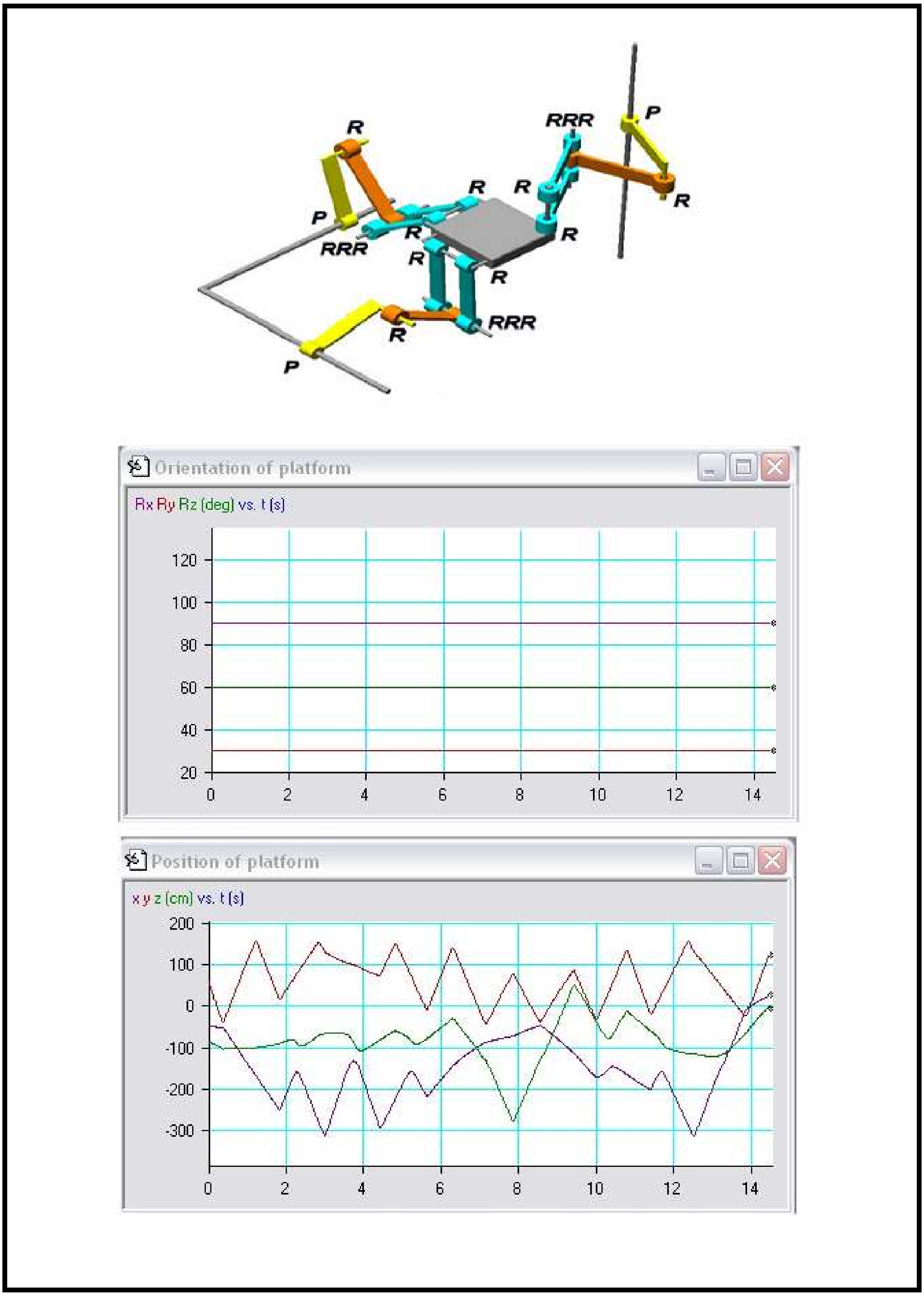

Figure C.3. Raw Motion Analysis

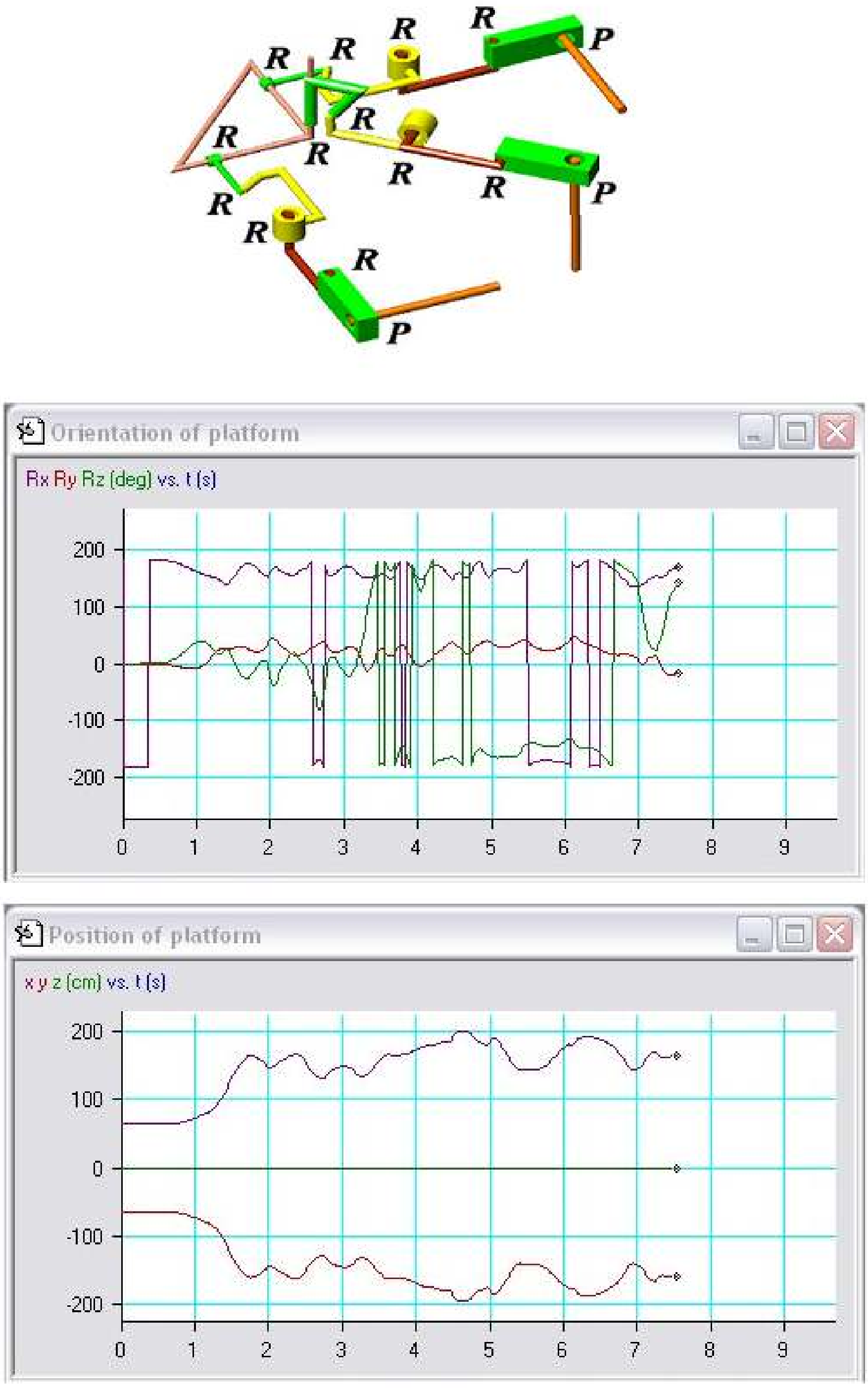

Figure C.4. Raw Motion Analysis

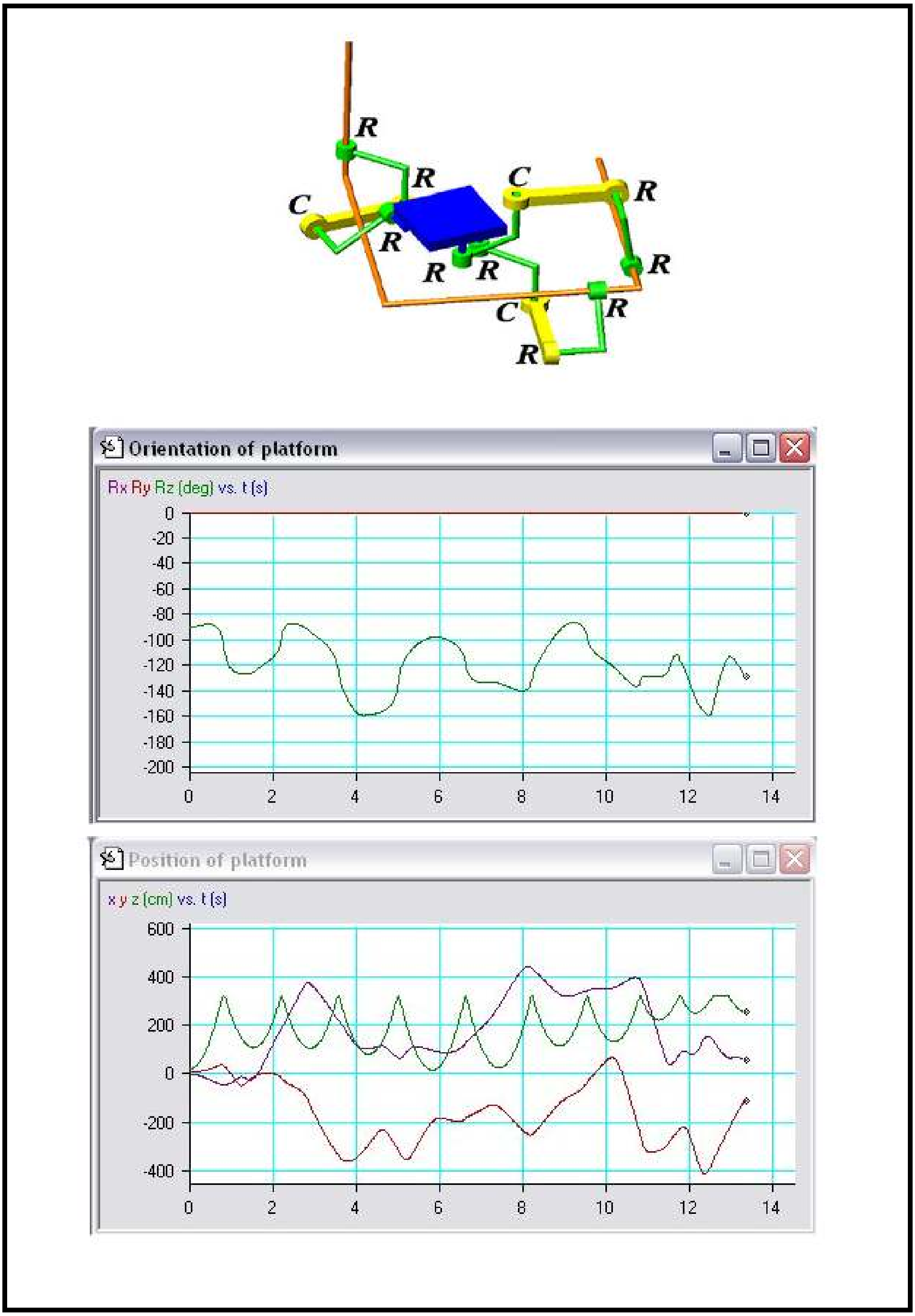

Figure C.5. Raw Motion Analysis

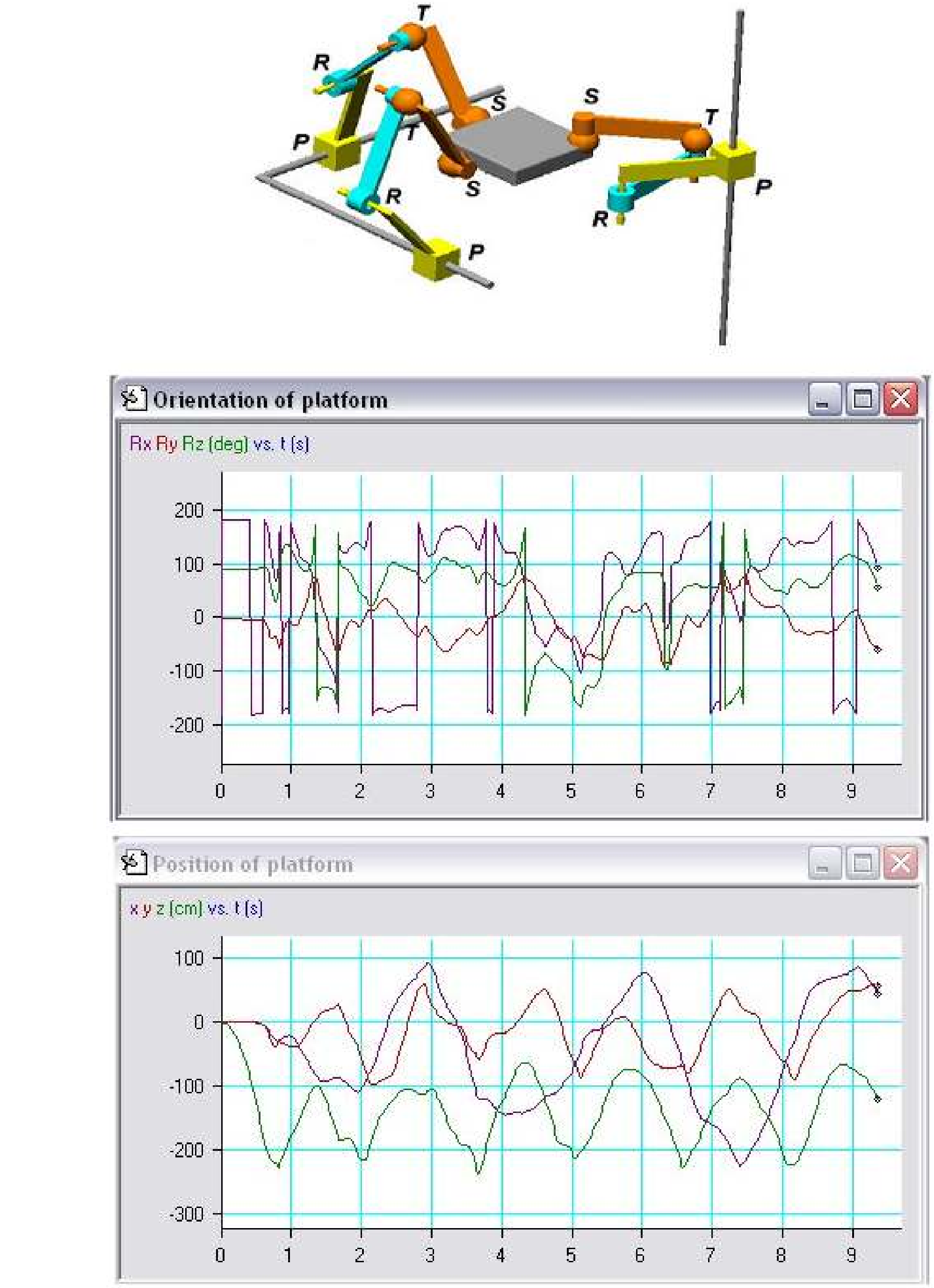

Figure C.6. Raw Motion Analysis

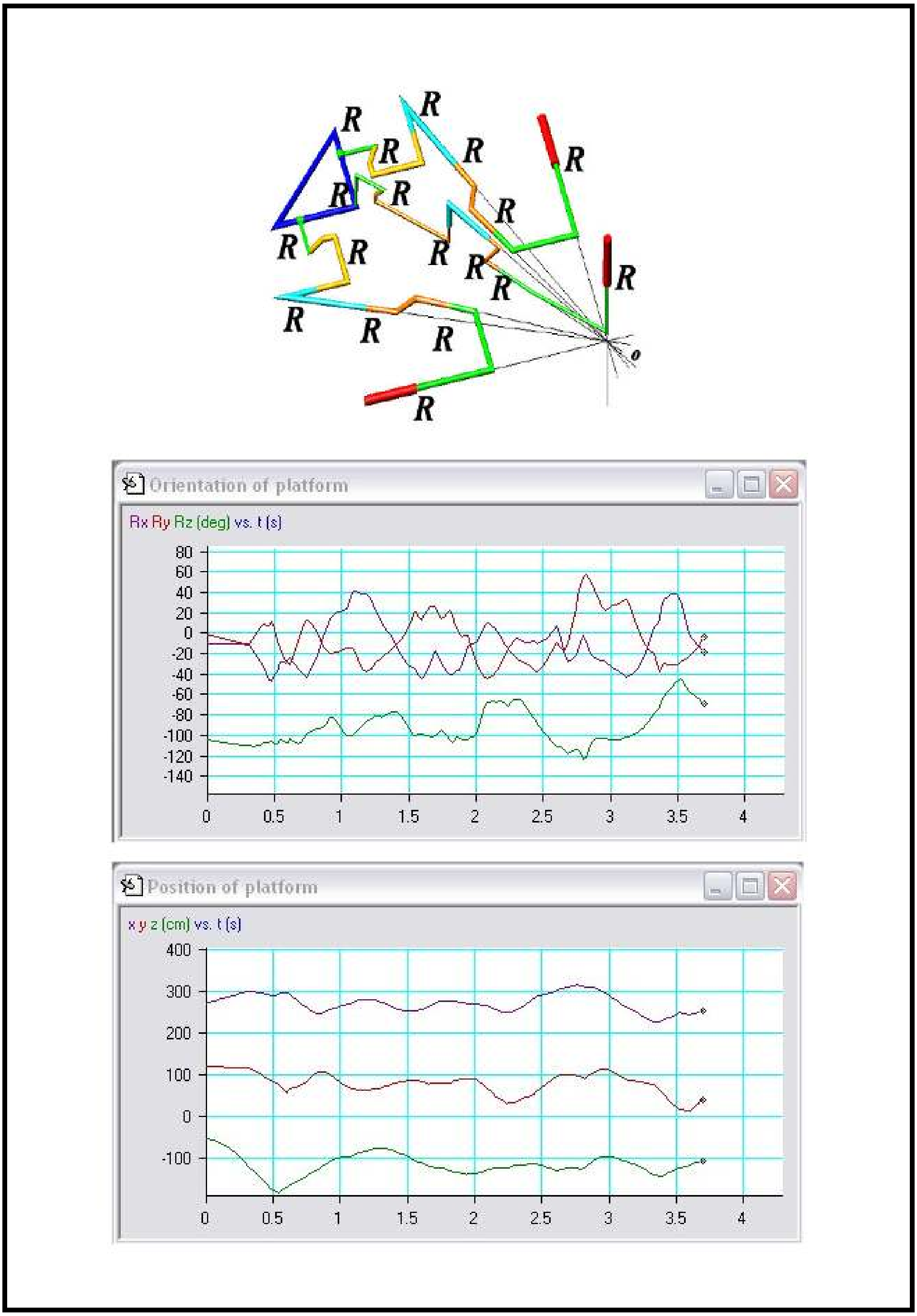

Figure C.7. Raw Motion Analysis

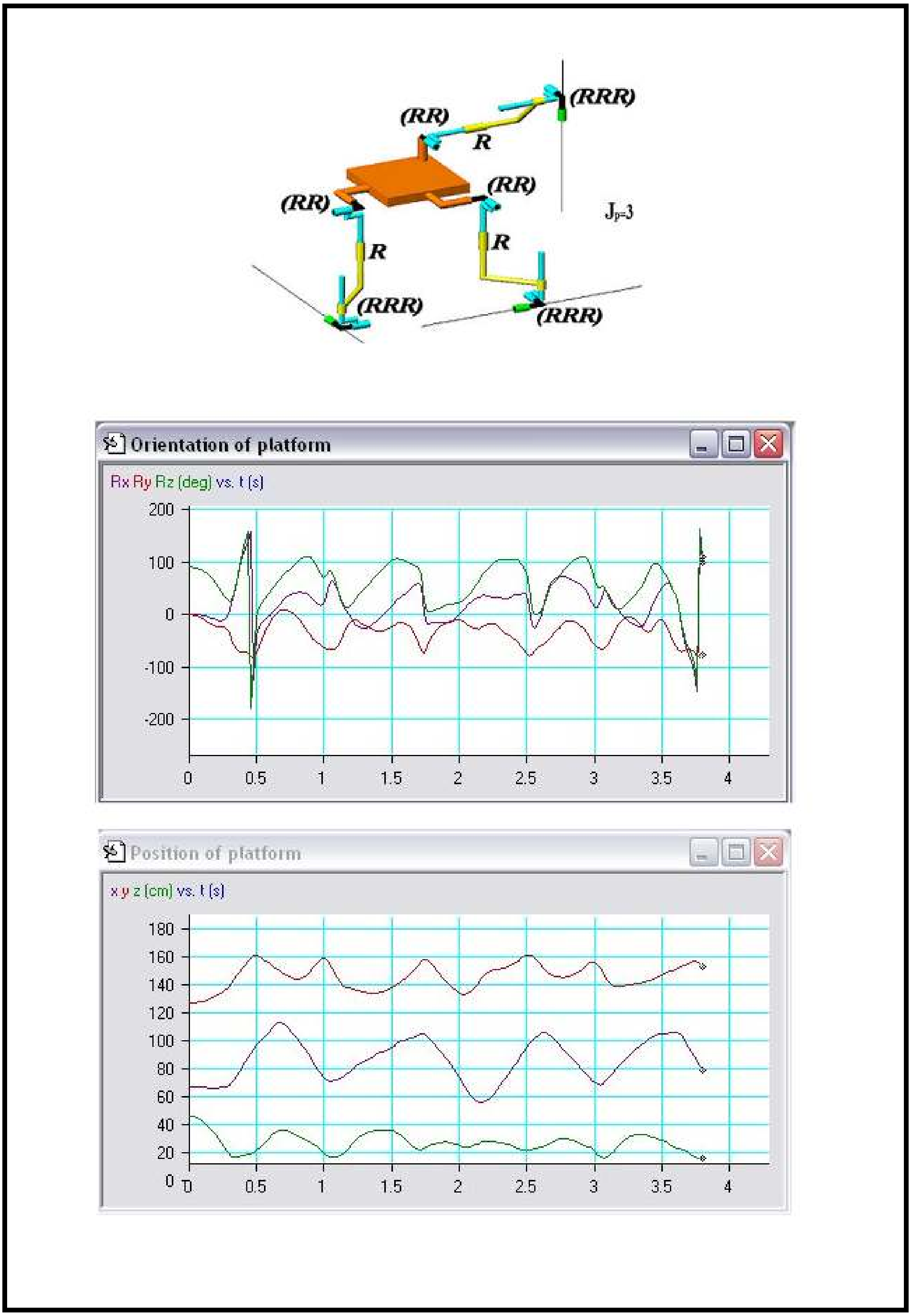

Figure C.8. Raw Motion Analysis

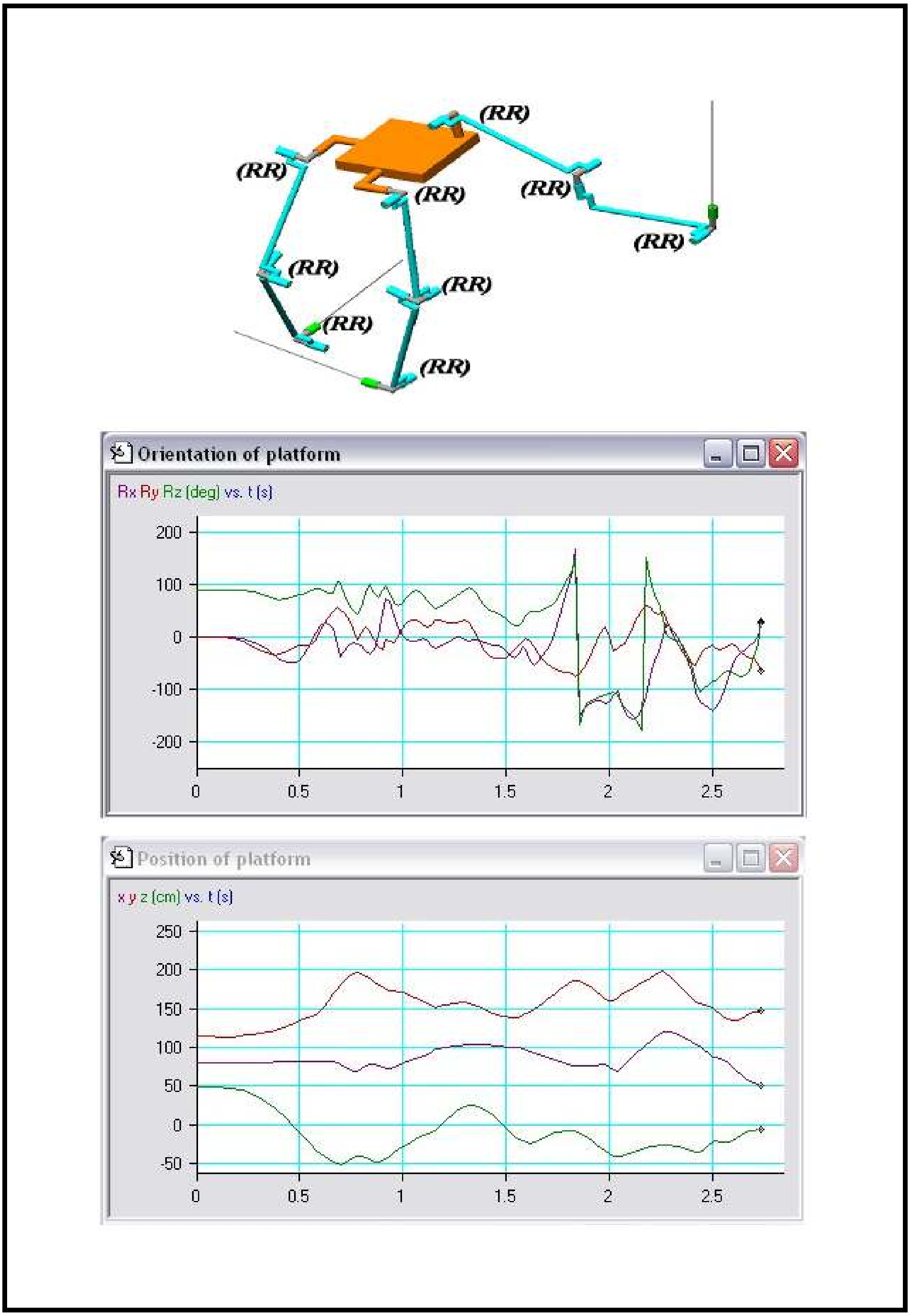

Figure C.9. Raw Motion Analysis