Differential and Coherent Detection Schemes For Space-Time Block Codes

By

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ABSTRACT

The single most important technique providing reliable communication over wireless channels is diversity. One of the diversity techniques is space diversity. Typical examples of space diversity are multiple transmit and/or receive antenna communications. By employing multiple antennas either at the transmitter or at the receiver, multiple antenna communications introduce a space diversity to combat fading without necessarily sacrificing bandwidth resources; thus they become attractive solutions for broadband wireless applications.

Space-time coding is a new diversity method for communication over wireless channels using multiple transmit antennas. Two types of space-time coding have been studied for a few years: trellis and block coding. Space–time trellis codes perform extremely well at the cost of relatively high complexity. Owing to this, in this thesis, space-time block codes are investigated in detail. Not only these codes support an extremely simple maximum likelihood detection algorithm based only on linear processing at the receiver, but also they can be used for multiple transmit antenna differential detection.

In this thesis, coherent and differential detection schemes for space-time block coding are investigated in detail over non-dispersive Rayleigh fading channels. In coherent detection systems, Alamouti's and Tarokh's codes with two, three and four transmit antennas are simulated for BPSK and QPSK modulation. Furthermore, Tarokh's and Hughes' differential detection schemes with two transmit antennas are investigated and a unifying approach to these structures is developed with a new design criteria for optimal unitary group codes. Then, the codes chosen by Tarokh and Hughes are simulated using a generalized differential detection scheme for BPSK and QPSK modulation. Moreover, codes of Hochwald are also simulated in order to compare them to the codes of Tarokh.

ÖZ

Çeşitleme, kablosuz kanallar üzerinden güvenilir bir haberleşme sağlanabilmesi için tek ve en önemli yöntemdir. Çeşitleme tekniklerinden biri uzay çeşitlemesidir. Uzay çeşitlemesinin tipik örnekleri çoklu verici ve/veya alıcı antenli haberleşme sistemleridir. Vericide ya da alıcıda çoklu anten kullanımı ile, çoklu antenli sistemler, gerekli bant genişliğinden feda etmeksizin bayılma ile baş edebilen, bir uzay çeşitlemesi oluştururlar. Böylece uzay çeşitleme sistemleri, geniş bant kablosuz iletişim uygulamaları için cazip çözümler olurlar.

Uzay-zaman kodlama, çoklu verici anten kullanılan kablosuz kanallar üzerinden haberleşmek için kullanılan, yeni bir çeşitleme yöntemidir. Bir kaç yıldır üzerinde çalışılan, uzay-zaman kodlamanın iki şekli, kafes ve blok kodlamadır. Uzay-zaman kafes kodlamanın başarımı, oldukça yüksek karmaşıklığının yanında, son derece iyidir. Bu karmaşıklık sebebiyle, bu tezde uzay zaman blok kodlama ayrıntılı olarak incelenmiştir. Bu kodlar sadece, alıcıda yalnızca doğrusal işlemlere dayanan oldukça basit en büyük olabilirlik algıma metodunu desteklemekle kalmayıp, bunun yanında çoklu antenli ayrımsal algılama için de kullanılabilir.

Uzay-zaman blok kodlama için evreuyumlu ve ayrımsal algılama yöntemleri, bozucu olmayan Rayleigh bayılmalı kanallar için ayrıntılı olarak incelenmiştir. Uyumlu algılama sistemlerinde, BPSK ve QPSK kiplenimleri için, Alamouti ve Tarokh kodlarının 2,3 ve 4 verici antenli sistemler için benzetimleri yapılmıştır. Buna ek olarak Tarokh ve Hughes ayrımsal algılama yöntemleri 2 verici antenli sistemler için incelenip, bu yapılara ilişkin bütünleştirici bir yaklaşım ile en iyi birimcil kodlar için yeni tasarım kriterleri geliştirilmiştir. Daha sonra Tarokh ve Hughes tarafından seçilen kodlar, BPSK ve QPSK için genelleştirilmiş ayrımsal algılama yöntemleri kullanılarak benzetimleri yapılmıştır. Tarokh'un kodlarının performansları ile kıyaslanmak üzere, Hochwald'ın seçtiği kodlar için de benzetimler yapılmıştır.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABBREVIATIONS

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
CSI	Channel State Information
DPSK	Differential Phase Shift Keying
GSM	Global System for Mobile Communications
IEEE	Institute of Electrical and Electronics Engineers
IFFT	Inverse Fast Fourier Transform
ISI	Intersymbol Interference
ML	Maximum Likelihood
MMSE	Minimum Mean Squared Error
MRC	Maximal Ratio Combining
MRRC	Maximal Ratio Receive Combining
OFDM	Orthogonal Frequency Division Modulation
PDF	Probability Density Function
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
SNR	Signal to Noise Ratio
SF	Space-Frequency
ST	Space-Time
STBCing	Space-Time Block Coding
STBCs	Space-Time Block Codes
STTCing	Space-Time Trellis Coding
STTCs	Space-Time Trellis Codes
TCM	Trellis Coded Modulation
TDD	Time Division Duplex
TDMA	Time Division Multiple Access

INTRODUCTION

Future wireless communication systems are expected to provide users with a variety of multimedia services. In order to achieve this, high data rates are needed to enable reliable transmission over wireless channels. So research efforts are carried through to evolve efficient coding and modulation schemes and signal processing techniques to improve the quality and spectral efficiency of wireless communications and better techniques for sharing the limited spectrum among different high capacity users.

Incidentally, these evolutions must overcome the performance limitations. The first of the major design considerations is the physical limitations of the wireless channels. The channel is capable of receiving time varying harm such as noise, interference, and multipath. The second one is limitations on the power and size of the communications and computing devices in a mobile handset. Most personal communications and wireless services portables are carried in a briefcase and/or pocket. Therefore they are also small and lightweight. In order to using small batteries, low power requirement is needed.

Perhaps the single most important technique providing reliable communications over wireless channels is diversity techniques, which provides the receiver some less-attenuated replica of the transmitted signal.

According to the domain where the diversity is created, diversity techniques may be divided into three categories, time diversity, frequency diversity and space diversity. Time and frequency diversity introduces redundancy in time and frequency domain respectively, and therefore causes loss in bandwidth efficiency. Typical examples of space diversity are multiple transmit and/or receive antenna communications. By employing antennas at the transmitter or the receiver, multiple antenna communications come into space diversity to combat fading without necessarily sacrificing bandwidth resources; thus they become attractive solutions for broadband wireless applications.

Depending on where multiple antennas are used (transmitter or receiver), two types of space diversity can be used: receive antenna diversity and transmit antenna diversity. Receive antenna diversity has been integrated in wireless systems such as Global System for Mobile Communications (GSM) and IS-136 to improve the uplink (from mobiles to base-stations) transmissions. However, due to the cost, size, and power limitations of the remote units, receiver diversity appears impractical for downlink (from base-stations to mobiles) transmissions. The use of multiple antennas makes the remote units larger and more expensive. As a result, diversity techniques have almost exclusively been applied to base stations. A base station often serves hundreds to thousands of remote units. It is therefore more economical to add equipment to base stations rather than the remote units. There is now a large works on coding and modulations for receive diversity [4, 11-13].

Theoretical expressions of transmit diversity were covered in [5, 10]. Telatar [10] was the first to obtain expressions for capacity and error exponents for multiple transmit antenna system in the presence of Gaussian noise. Here, capacity is derived under the assumption that fading is independent from one channel use to the other. At about the same time, Foschini and Gans [5] derived the outage capacity under the assumption that fading is quasistatic; i.e., constant over a long period of time, and then changes in an independent manner. A major conclusion of these works is that the capacity of a multi-antenna system far exceeds that of a single-antenna system. In particular, the capacity grows at least linearly with the number of transmit antennas as long as the number of receive antennas is greater than or equal to the number of transmit antennas.

Among various transmit antenna diversity schemes, particularly popular recently in space-time (ST) coding that relies on multiple antenna transmissions and appropriate signal processing at the receiver to provide diversity and coding gains over uncoded single-antenna transmissions. Space Time Codes are a new approach that it improves the error rate performance or the capacity of the system by provide diversity gain without any feedback from the receiver to the transmitter, without bandwidth expansion.

More recently, space-time trellis coding (STTCing) has been proposed [1]. This technique combines signal processing at the receiver with coding techniques appropriate to multiple transmit antennas and provides significant gain over [2, 3]. Specific space-time trellis codes (STTCs) designed for 2-4 transmit antennas perform extremely well in slow fading environments and come within 2-3 dB of the outage capacity computed by Telatar [10] and independently by Foschini and Gans [5]. The bandwidth efficiency is about 3-4 times that of the previous systems. The ST codes presented in [1] provide the best possible trade-off between constellation size, data rate, diversity advantage, and trellis complexity. Space–time trellis codes perform extremely well at the cost of relatively high complexity. When the numbers of transmit antennas are fixed, the decoding complexity of STTCing (measured by the number of trellis states in the decoder) increases exponentially as a function of both the diversity level and the transmission rate.

In addressing the issue of decoding complexity, Alamouti discovered a remarkable scheme for transmissions using two transmit antennas [7]. This scheme has a simple decoding algorithm that can be generalized to an arbitrary number of receiver antennas. And also this scheme is significantly less complex than Space–time trellis coding using two transmitter antennas, although there is a loss in performance [8]. Despite the associated performance penalty, Alamouti's scheme is appealing in terms of its simplicity and performance. Introduced in [6] and [8], Space–time block coding generalizes the transmission scheme discovered by Alamouti to an arbitrary number of transmit antennas and is able to achieve the full diversity promised by the transmit and receive antennas. These codes retain the property of having a very simple maximum likelihood-decoding algorithm based only on linear processing at the receiver [6, 8]. For more details on transmit diversity for the case that the receiver knows the channel see [1, 6, 8] and the references therein.

The coherent decoding in both STTCing and STBCing requires that perfect estimates of current channel fading are available at the receiver. The channel state information can be obtained at the receiver by sending training or pilot symbols or sequences to estimate the channel from each of the transmit antennas to the receive antenna [14–21]. However, it is not always feasible or advantageous to use these schemes, especially when many antennas are used or either end of the link is moving so fast that the channel is changing very rapidly. Since the fading rate or number of transmit antennas increases, learning channel state information becomes increasingly difficult. In an effort to increase channel capacity or lower error probability, it is accepted practice to increase the number of transmitter antennas. But increasing the number of transmitter antennas increases the required training interval for learning the channel and reduces the available time in which data may be transmitted before the fading coefficients change. The number of pilot signals used to track the channel must also grow. Given a restriction on total pilot or training power, we must allocate less power per antenna with every added antenna. Finally, instability in local oscillators and phase-lock devices and inaccurate knowledge of Doppler shifts, which may be different for each antenna, may also limit channel-tracking ability at the receiver.

Motivated by these considerations, there is much interest in ST transmission schemes that do not require either the transmitter or receiver to know the channel. A natural way of dealing with unknown channels is differential modulation.

For one transmit antenna, differential detection schemes, such as differential phase shift keying (DPSK), exist that neither require the knowledge of the channel nor employ pilot symbol transmission. These differential decoding schemes are used, for instance, in IS-54 standard of the Institute of Electrical and Electronics Engineers (IEEE). It is natural to consider extensions of these schemes to multiple transmit antennas.

The basic idea behind all of these considerations is to introduce proper encoding between two consecutive code matrices so that the decoding at the receiver is independent of the underlying channels [22-28]. As expected, the price paid for is code advantage in addition to 3 dB loss in SNR compared to coherent decoding.

In this thesis, first three chapters are introduction to space-time block coding (STBCing). In fourth chapter, Alamouti's and Tarokh's schemes on coherent detection systems are researched for two, three and four transmit antennas [6-8]. Then Tarokh's and Hughes's schemes on differential detection of space-time block codes (STBCs) are investigated for two transmit antennas [22,24] and a general scheme is formed with generalized code design criteria in fifth chapter. In the last chapter, performance results of various codes for coherent and differential detection schemes are given. Furthermore, discussion on these results is also given in this chapter.

CHAPTER 1

FADING IN COMMUNICATION CHANNELS

In order to fully understand wireless communications, we need to have a basic idea on the characteristics of wireless channels. The behavior of a typical mobile wireless channel is considerably more complex than that of an additive white Gaussian noise (AWGN) channel. Besides the thermal noise at the receiver front end (which is modeled by AWGN), there are several other well-studied channel impairments [29] in a typical wireless channel:

Path Loss, which describes the loss in power as the radio signal, propagates in space

Shadowing, which is due to the presence of fixed obstacles in the propagation path of the radio signal

Fading, which accounts for the combined effect of multiple propagation paths, rapid movements of mobile units (transmitters/receivers) and reflectors.

We will give a brief introduction on these three impairments. Our focus is on fading and how spread spectrum techniques can help to combat fading in wireless channels.

1.1 Path Loss

In any real channel, signals attenuate as they propagate. For a radio wave transmitted by a point source in free space, the loss in power, known as *path loss*, is given by

$$L = \left(\frac{4\pi d}{\lambda} \right)^2 . \tag{1.1}$$

where λ is the wavelength of the signal, and d is the distance between the source and the receiver. The power of the signal decays as the square of the distance. In land mobile wireless communication environments, similar situations are observed. The mean power of a signal decays as the n^{th} power of the distance:

$$L = cd^n . \tag{1.2}$$

where c is a constant and the exponent n typically ranges from 2 to 5 [1]. The exact values of c and n depend on the particular environment. The loss in power is a factor that limits the coverage of a transmitter.

1.2 Shadowing

Shadowing is due to the presence of large-scale obstacles in the propagation path of the radio signal. Due to the relatively large obstacles, movements of the mobile units do not affect the short-term characteristics of the shadowing effect. Instead, the natures of the terrain surrounding the base station and the mobile units as well as the antenna heights determine the shadowing behavior.

Usually, shadowing is modeled as a slowly time-varying multiplicative random process. Neglecting all other channel impairments, the received signal $r(t)$ is given by:

$$r(t) = g(t)s(t), \tag{1.3}$$

where $s(t)$ is the transmitted signal and $g(t)$ is the random process which models the shadowing effect. For a given observation interval, we assume $g(t)$ is a constant g, which is usually modeled [29] as a lognormal random variable whose density function is given by

$$p(g) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma g}\exp\left(-\dfrac{(\ln g - \mu)^2}{2\sigma^2}\right) & g \geq 0 \\ 0 & g < 0. \end{cases} \tag{1.4}$$

We notice that $\ln g$ is a Gaussian random variable with mean μ and variance σ^2. This translates to the physical interpretation that μ and σ^2 are the mean and variance of the loss measured in decibels (up to a scaling constant) due to shadowing. For cellular and micro cellular environments, σ, which is a function of the terrain and antenna heights, can range from 4 to 12 dB [2].

1.3 Fading

Fading is the term used to describe the rapid fluctuations in the amplitude of the received radio signal over a short period of time. Fading is a common phenomenon in mobile communication channels caused by the interference between two or more versions of the transmitted signals which arrive at the receiver at slightly different times. The resultant received signal can vary widely in amplitude and phase, depending on various factors such as the intensity, relative propagation time of the waves, bandwidth of the transmitted signal etc. The performance of a system (in terms of probability of error) can be severely degraded by fading. Special techniques may be required to achieve satisfactory performance.

1.3.1 Parameters of Fading Channels

The following parameters are often used to characterize a fading channel:

Multipath spread T_m

Suppose that we send a very narrow pulse in a fading channel. We can measure the received power as a function of time delay as shown in Figure 1.1. The average received power $P(\tau)$ as a function of the *excess time delay* τ (Excess time delay = time delay - time delay of first path) is called the *multipath intensity profile* or the *delay power spectrum*. The range of values of τ over which $P(\tau)$ is essentially non-zero is called the *multipath spread* of the channel, and is often denoted by T_m. It essentially tells us the maximum delay between paths of significant power in the channel. For urban environments, T_m can range from 0.5μs to 5μs [29].

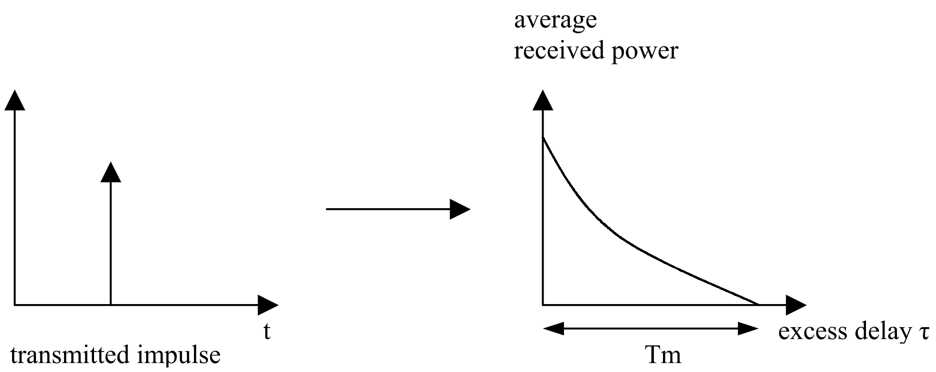

Figure 1.1 Multipath delay profile

Coherence bandwidth $(\Delta f)_c$

In a fading channel, signals with different frequency contents can undergo different degrees of fading. The *coherence bandwidth*, denoted by $(\Delta f)_c$, gives an idea of how far apart in frequency for signals to undergo different degrees of fading. Roughly speaking, if two sinusoids are separated in frequency by more than $(\Delta f)_c$, then they would undergo different degrees of (often assumed to be independent) fading. It can be shown that $(\Delta f)_c$ is related to T_m by

$$(\Delta f)_c \approx 1/T_m. \tag{1.5}$$

Coherence time $(\Delta t)_c$

In a time-varying channel, the channel impulse response varies with time. The *coherence time*, denoted by $(\Delta t)_c$, gives a measure of the time duration over which the channel impulse response is essentially invariant (or highly correlated) . Therefore, if a symbol duration is smaller than $(\Delta t)_c$, then the channel can be considered as time invariant during the reception of a symbol. Of course, due to the time-varying nature of the channel, different time-invariant channel models may still be needed in different symbol intervals.

Doppler spread B_d

Due the time-varying nature of the channel, a signal propagating in the channel may undergo Doppler shifts (frequency shifts). When a sinusoid of frequency is transmitted through the channel, the received power spectrum can be plotted against the Doppler shift as in Figure 1.2. The result is called the *Doppler power spectrum*. The *Doppler spread*, denoted by B_d, is the range of values that the Doppler power spectrum is essentially non-zero. It essentially gives the maximum range of Doppler shifts. It can be shown that $(\Delta t)_c$ and B_d are related by

$$(\Delta t)_c \approx 1/B_d \tag{1.6}$$

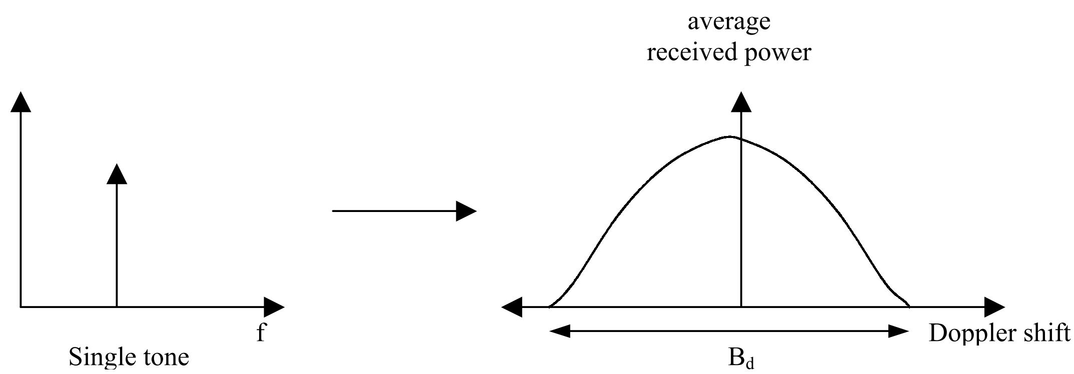

Figure 1.2 Doppler power spectrum

1.3.2 Classification of Fading Channels

There are different types of fading according to the relation between signal and channel parameters [29]. Signal fading can be either flat or frequency selective determined comparing the signal bandwidth to the coherence bandwidth of the channel which can be defined as the maximum frequency separation for which the signals are still correlated. If the coherence bandwidth is greater than the signal bandwidth, the transmitted signal undergoes flat fading. Otherwise, the transmitted signal undergoes frequency-selective fading. On the other hand, based on Doppler spread, signal fading can be either fast or slow. Relative motion between the transmitter and the receiver results in Doppler spread in the received signal. If the Doppler spread of the channel is much less than the signal bandwidth, the signal undergoes slow fading. Practically, fast fading occurs at low data rates. Figure 1.3 shows a tree of the four different types of fading.

Small-Scale Fading
(Based on multipath time delay)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Figure 1.3 Types of small-scale fading

1.3.3 A Mathematical Model for Fading Channels

Consider a transmitted signal $s(t) = A\cos(2\pi f_c t)$ through a fading channel. Ignoring the effects of noise, the received signal can be expressed as:

$$r(t) = A\sum_{i=1}^{N}\alpha_i \cos(2\pi f_c t + \theta_i) \tag{1.7}$$

where the number variables α_i and θ_i are the attenuation and the phase-shift of the i^{th} multipath component, respectively. The above expression can be rewritten as:

$$r(t) = A\left(\left(\sum_{i=1}^{N}\alpha_i \cos(\theta_i)\right)\cos(2\pi f_c t) - \left(\sum_{i=1}^{N}\alpha_i \sin(\theta_i)\right)\sin(2\pi f_c t)\right) \tag{1.8}$$

We introduce two random processes $X_1(t)$ and $X_2(t)$, such that the above equation becomes:

$$r(t) = A\big(X_1(t)\cos(2\pi f_c t) - X_2(t)\sin(2\pi f_c t)\big) \tag{1.9}$$

When there are a large number of scatterers in the channel that contribute to the signal at the receiver, $X_1(t)$ and $X_2(t)$ become Gaussian random variables with zero mean and σ^2 variance according to the central limit theorem. Eq. (1.9) can be rewritten as:

$$r(t) = AR(t)\cos(2\pi f_c t + \theta(t)) \tag{1.10}$$

where the amplitude $R(t)$ and the phase $\theta(t)$ of received waveform $r(t)$ are given by:

$$R(t) = \sqrt{X_1(t)^2 + X_2(t)^2} \tag{1.11}$$

$$\theta(t) = \tan^{-1}\left(\frac{X_2(t)}{X_1(t)}\right), \tag{1.12}$$

respectively.

Since the processes $X_1(t)$ and $X_2(t)$ are Gaussian with zero mean, the envelope of the channel response at any time instant, $R(t)$, has a Rayleigh probability density function (pdf) and the phase is uniformly distributed in the interval $[0,2\pi]$.

The Rayleigh probability density function (pdf) is defined as:

$$P_{Rayleigh}(r) = \begin{cases} \dfrac{r}{\sigma^2}\exp\left(-\dfrac{r^2}{2\sigma^2}\right) & 0 \le r \le \infty \\ 0 & r < 0, \end{cases} \tag{1.13}$$

where

$$\sigma^2 = \frac{E\left[r^2\right]}{2}. \tag{1.14}$$

When there are fixed scatterers or signal reflectors in the medium in addition to randomly moving scatterers, $R(t)$ can no longer be modeled as having zero mean. In this case, the envelope $R(t)$ has a Rician distribution and the channel is said to be a Rician fading channel. Another distribution function that has been used to model the envelope of fading signals is the Nakagami-m distribution. These fading channel models are considered in [29,30].

The distortion in the phase can be easily overcome if differential modulation is employed. Whereas the amplitude distortion $R(t)$ severely degrades the performance of digital communication systems over fading channels. It is usually reasonable to assume that the fading stays essentially constant for at least one signaling interval.

1.3.4 Rayleigh Fading Channel Model

Assuming the transmitted signal undergoes slow fading, the amplitude and phase of the received signal are constants during one bit duration. The received baseband signal at the base station is

$$r_j = \sum_{i=0}^{N-1} \sqrt{E_i/T} e^{j\theta_i} \alpha_i C_{ij} + n_j \tag{1.15}$$

where the amplitude $\sqrt{E_i/T}\alpha_i$ has a Rayleigh distribution, the phase θ_i is uniformly distributed between [0,2π], $A_i = \sqrt{E_i/T} e^{j\theta_i} \alpha_i$ is defined as the complex parameter to be estimated, and n_j is complex baseband white Gaussian noise with bandpass power spectral density $N_0/2 = \sigma^2$ for both real and imaginary parts. A_i has a Gaussian distribution. The estimated power is the square of A_i. We assume that systems are synchronized and there is no intersymbol interference (ISI).

1.3.5 Rayleigh Fading Simulator

Clarke's model [29] is used to generate Rayleigh fading coefficients. It is assumed that each arriving wave at the receiver has arbitrary phase and angle of arrival and equal average amplitude. There is no direct line-of-sight path and no excess delay for each wave. The Doppler shift is bounded by the maximum Doppler frequency f_m:

$$f_m = \frac{vf_c}{c} \tag{1.16}$$

where v is the vehicle speed, f_c is the carrier frequency and c is the speed of light. For normal highway driving, f_m is around 80 Hz. The spectral shape [29] in Clarke's model is

$$S_{E_Z} = \frac{1.5}{\pi f_m \sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}. \tag{1.17}$$

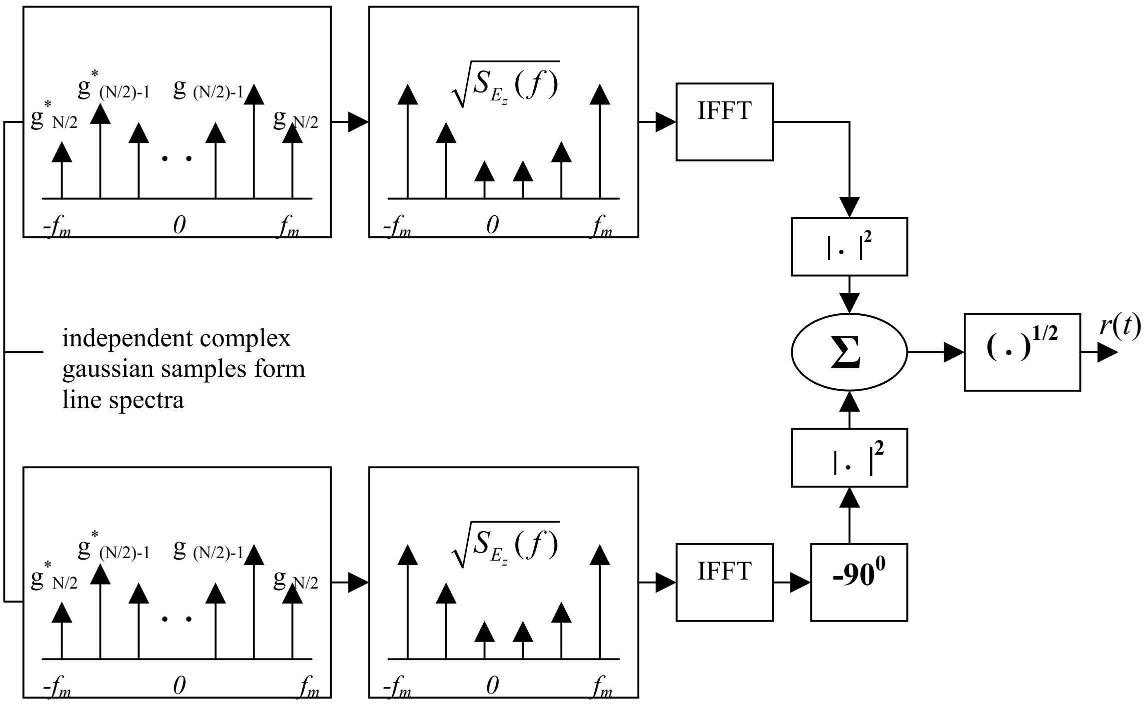

Figure 1.4 Generation of Rayleigh fading coefficients at baseband

Figure 1.4 shows the block diagram of the Rayleigh fading simulator. The first step is to generate independent complex Gaussian random variables to form baseband line spectrum for positive frequency components. Negative frequency components are obtained by conjugating positive frequency components. Then, the line spectrum is multiplied with the spectrum $\sqrt{S_{E_Z}}$. Next, inverse fast Fourier transform (IFFT) is performed on these signals obtaining the real and imaginary parts for the complex

Rayleigh fading output $x(t)e^{j\theta(t)}$, here $x(t)$ is Rayleigh distributed and $\theta(t)$ is uniformly distributed in the interval or between 0 and 2π $[0,2\pi]$.

Figure 1.5 shows the Rayleigh fading envelopes with maximum Doppler frequencies of 10Hz and 70Hz. Slow fading is observed when the vehicle speed is low and the Doppler frequency is small. Increased vehicle speed and Doppler frequency results in faster fading.

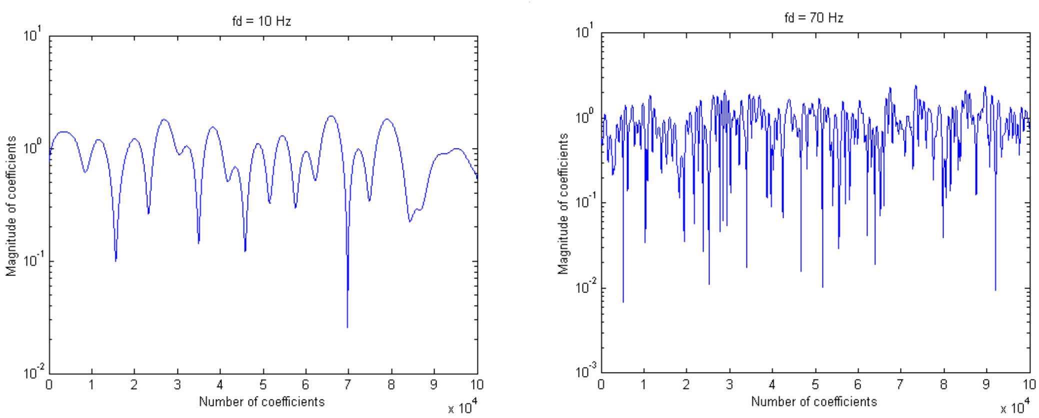

Figure 1.5 Rayleigh fading envelopes of signals with different maximum Doppler frequencies

CHAPTER 2

DIVERSITY IN WIRELESS RADIO: Principles and Methods

2.1 Objective of Diversity

Diversity is a powerful communication technique that provides wireless link improvement. The signals in terrestrial communication systems experience multipath interference and fading. This fading can become quite severe in urban environments. Mobiles in motion also experience fading due to the Doppler spreading of various multipath components. If the fading causes many channel errors, methods to combat fading must be used. One of the most efficient and simple techniques to overcome the destructive effects of fading is diversity.

2.2 Diversity Techniques

Diversity is an efficient technique to exploit the random nature of radio propagation by finding methods to generate and extract independent signal paths for communication. The concept behind diversity is relatively simple [29]: if one signal path undergoes a deep fade at a particular point of time, another independent path may have a strong signal. By having more than one path to select from, both the instantaneous and average signal to noise ratio (SNR) can be improved in the receiver by a large amount. There are many different types of diversity that have been employed in various systems. These are:

Frequency diversity, transmitting or receiving the signal at different frequencies;
Time diversity, transmitting or receiving the signal at different times;
Space diversity, transmitting or receiving the signal at different locations;
Polarization diversity, transmitting or receiving the signal with different polarizations;
Others: ***Angle diversity***, ***Path diversity*** … etc.

One method of *frequency diversity* is the transmission of the same data on two frequencies separated by more than the coherence bandwidth of the channel that frequencies will not experience the same fades. This type of frequency diversity, however, is rarely used in conventional mobile systems because the coherence bandwidth is large. Frequency diversity is inherent in spread spectrum systems, where the chipped rate is greater than the coherence bandwidth. Some multicarrier (orthogonal frequency division modulation - OFDM) and multitone systems can also achieve frequency diversity if the separation in frequency is more than the coherence bandwidth. In time division multiple access (TDMA) systems, frequency diversity is obtained by the use of equalizers when the multipath delay spread is a significant fraction of a symbol period. GSM uses frequency hopping to provide frequency diversity. A major disadvantage of frequency diversity is its ineffectiveness on flat (non-frequency selective) channels.

Time diversity repeatedly transmits information at time spacings that exceed the coherence time of the channel so that multiple repetitions of the signal will be received in independent fading conditions. A more efficient and practical method of time diversity for digital spread spectrum systems is the use of interleaving and error correcting codes, as employed in IS-95 system. Another modern implementation of time diversity involves the use of the RAKE receiver for spread spectrum code division multiple access (CDMA) where the multipath channel provides redundancy in the transmitted information. Major disadvantages of time diversity are delay and ineffectiveness over slowly fading channels.

Space diversity, also known as antenna diversity, is one of the most popular forms of diversity used in wireless systems. Space diversity can be used in many different forms at either the mobile or the base station, or both. Spatial diversity usually implies receive diversity, although transmit diversity is also a form of space diversity. Spatial diversity requires far enough separation between the antennas depending on the angular spread. For handsets the angular spread and the corresponding required spatial separation are 360^o and $\lambda/4$, respectively whereas these values are a few degrees and 10 to 20 λ for outdoors base stations. Currently, multiple antennas at base stations are used for receive diversity at the base. However, it is difficult to have more than one or two antennas at the portable unit owing to the size limitations and cost of multiple chains of RF down conversion.

At the base station, space diversity is considerably less practical than at the mobile because the narrow angle of incident fields requires large antenna spacing [31]. The comparatively high cost of using space diversity at the base station prompts the considerations of using orthogonal polarization to exploit *polarization diversity*. In polarization diversity, two antennas with different polarization are used to receive (or transmit) the signal. Different polarization will ensure that the fading channel corresponding to each of the two antennas will be independent without having to place the two antennas far apart. The use of polarization diversity, though not currently in use in IS-95, is certainly a future possibility to be considered for signal reception enhancement to combat fading.

2.3 Space Diversity Techniques

Depending on whether multiple antennas are used for transmission or reception, two types of spatial diversity can be used: *receive-antenna diversity* and *transmit-antenna diversity*.

2.3.1 Receive Diversity Techniques

In receive-antenna diversity schemes, multiple antennas are deployed at the receiver to acquire separate copies of the transmitted signals these antennas are well separated to ensure independent fading channels and their outputs are properly combined to mitigate channel fading. In fact, receive-antenna diversity has been incorporated in wireless systems such as GSM and IS-136 to improve the up-link (from mobiles to base station) transmissions (see Reference [32] and references cited therein).

There are several possible diversity reception methods employed in communication receivers. The most common techniques are:

2.3.1.1 Selection Diversity:

Selection Diversity is one of the most simplest diversity techniques. The best received signal is used, and this signal can be chosen based on several quality metrics, including total received power, SNR, etc.

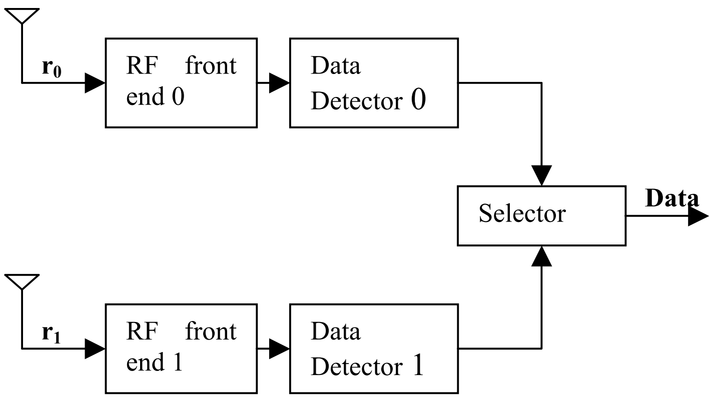

Figure 2.1 Selection Diversity

2.3.1.2 Switched Diversity:

This is very similar to selection diversity. The received signals are scanned in a fixed sequence until one is found to be above the predetermined threshold. This signal is then received until its level falls below the threshold and the scanning process is again initiated.

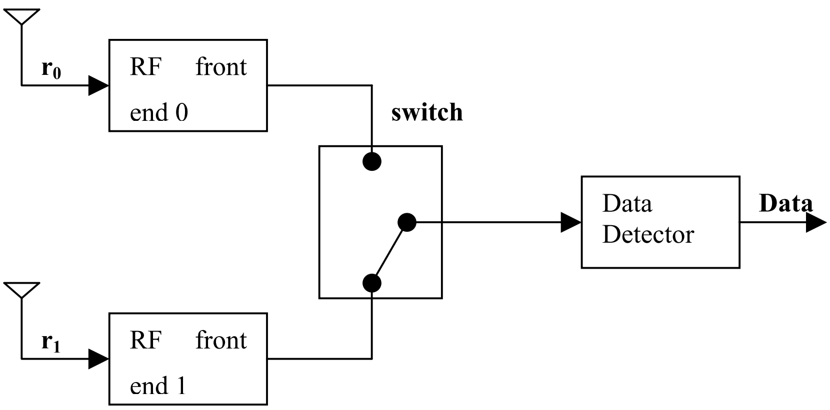

Figure 2.2 Switched Diversity

2.3.1.3 Linear Combining:

As the name implies, the signal used for detection in linear combining techniques is a linear combination of a weighted replica of all received signals.

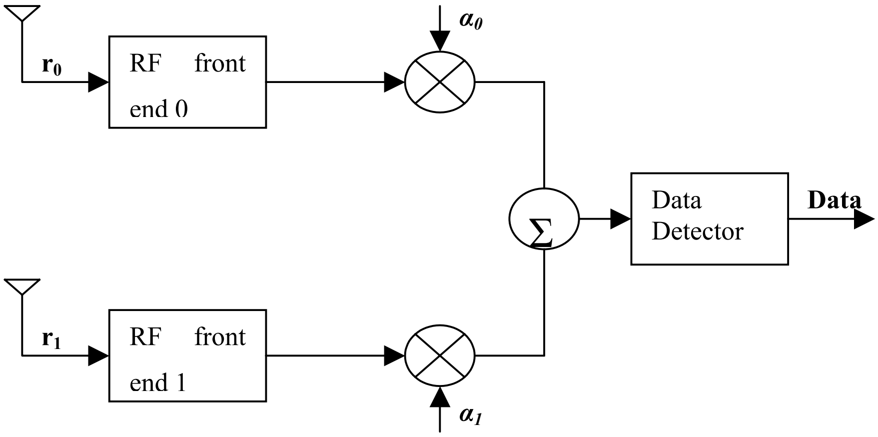

Figure 2.3 Linear combining diversity

In Figure 2.3, let $\mathbf{r}_0$ and $\mathbf{r}_1$ be the received signals at antennas 0 and 1, respectively,

$$\begin{aligned} \mathbf{r_0} &= A_0 e^{j\theta_0}\mathbf{s} + \mathbf{n_0} \\ \mathbf{r_1} &= A_1 e^{j\theta_1}\mathbf{s} + \mathbf{n_1} \end{aligned} \tag{2.1}$$

where $\mathbf{s}$ is the information symbol, $\mathbf{n}_0$ and $\mathbf{n}_1$ are the additive white Gaussian noise at antenna 0 and 1 respectively, and A_i and θ_i (i = 0, 1) are the corresponding amplitude and phase of the fading channel, respectively. The receiver uses the linear combination $\tilde{\mathbf{r}} = \boldsymbol{\alpha}_0\mathbf{r_0} + \boldsymbol{\alpha}_1\mathbf{r_1}$. The weighting coefficients $\boldsymbol{\alpha}_0$ *and* $\boldsymbol{\alpha}_1$ can be chosen in several ways.

In *equal gain combining,* all the received signals are co-phased at the receiver and added together without any weighting. Therefore the weights are chosen as $\boldsymbol{\alpha}_0 = e^{-j\theta_0}$ and $\boldsymbol{\alpha}_1 = e^{-j\theta_1}$.

A second approach is *maximal ratio combining* (MRC), where the two signals are co-phased and also weighted with their corresponding amplitudes A_0 and A_1 to provide the optimal SNR at the output. In this case $\boldsymbol{\alpha}_0 = A_0 e^{-j\theta_0}$ and $\boldsymbol{\alpha}_1 = A_1 e^{-j\theta_1}$.

A third approach is *minimum mean squared error* (MMSE) *combining*, where the weighting coefficients are chosen during a training phase such that

$$(\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1) = \arg\min_{\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1} \left|\boldsymbol{\alpha}_0\mathbf{r_0} + \boldsymbol{\alpha}_1\mathbf{r_1} - \mathbf{s}\right|^2. \tag{2.2}$$

2.3.2 Receive Diversity Performance

The performance of MMSE linear combining and MRC are essentially the same. In general, there will be a dramatic improvement in the average SNR, even with two-branch selection diversity. For all the above approaches to receive diversity, the average SNR will increase with the number of receive antennas. However, for the selection diversity, the SNR increases very slowly with the number of receive antennas. For MRC, the average SNR will increase linearly with the number of receive antennas. For equal gain combining the rate of SNR increase will be slightly less than that of MRC. In fact, the difference between the two is only 1.05 dB in the limit of an infinite number of receive antennas [33].

2.3.3 Transmit Diversity Techniques

Owing to size/power limitations at the mobile units, receive antenna diversity appears less practical for the downlink (from base stations to mobiles) transmissions. As a result, the downlink becomes the capacity bottleneck in current wireless systems [34,35], which has motivated rapidly growing research work on transmit antenna diversity. Transmit-antenna diversity relies on multiple antennas at the transmitter and is suitable for downlink transmissions because having multiple antennas at the base station is certainly feasible. The information theoretic aspects of transmit diversity were addressed in [5,10,36,37]. Previous work on transmit diversity can be classified into three broad categories: schemes using feedback, schemes with feedforward or training information but no feedback, and blind schemes.

2.3.3.1 Transmit Diversity with Feedback from Receiver

The first category uses feedback, either explicitly or implicitly, from the receiver to the transmitter to train the transmitter. Figure 2.4 shows a conceptual block diagram for transmit diversity with feedback. A signal is weighted differently and transmitted from two different antennas. The weights $\boldsymbol{\alpha}_0$ and $\boldsymbol{\alpha}_1$ are varied such that the received signal power $|r(t)|^2$ is maximized. The weights are adapted with feedback information from the receiver.

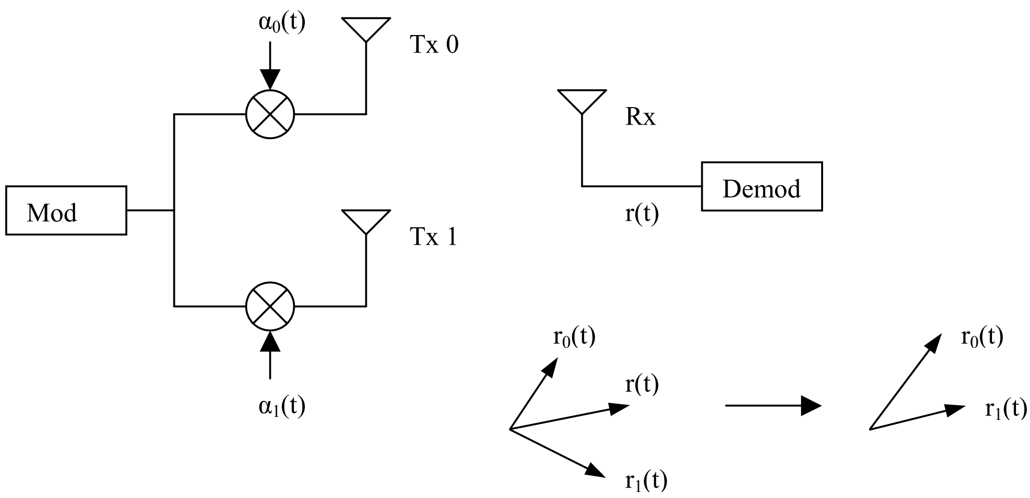

Figure 2.4 Transmit diversity with feedback from receiver

For instance, in time division duplex (TDD) systems [38], the same antenna weights are used for reception and transmission, so feedback is implicit in the exploitation of channel symmetry. These weights are chosen during reception to maximize the receive SNR, and during transmission to weight the amplitudes of the transmitted signals, and, therefore, will also maximize SNR at the receiver. Explicit feedback includes switched diversity systems with feedback [39]. However, in practice, movement by either the transmitter or the receiver or the surroundings such as cars, and interference dynamics causes a mismatch between the channel perceived by the transmitter and that perceived by the receiver. Thus, usage of transmit-antenna diversity schemes with no feedback is well motivated for future broadband wireless systems that are characterized by high mobility.

2.3.3.2 Transmit Diversity via Delay Diversity

Transmit diversity schemes mentioned in the second category use linear processing at the transmitter to spread the information across antennas. At the receiver, an optimal receiver recovers information. Feedforward information is required to estimate the channel from the transmitter to the receiver. These estimates are used to compensate for the channel response at the receiver. The first scheme of this type is the delay diversity scheme (see Figure 2.5) proposed by Wittneben [2] and it includes the delay diversity scheme of [3] as a special case. The linear processing techniques were also studied in [40, 41]. Delay diversity schemes are indeed optimal in providing diversity, in the sense that the diversity gain experienced at the receiver is equal to the optimal diversity gain obtained with receive diversity [42, 43]. The delay diversity scheme provides diversity benefit by introducing intentional multipath which can be exploited at the receiver by using an equalizer. The linear filtering used at the transmitter to create delay diversity can also be viewed as a channel code that takes binary or integer input and creates real valued output. The advantage of delay diversity over other transmit diversity schemes is that it will achieve the maximum possible diversity order *N*, i.e. number of transmit antennas, without any sacrifice in the bandwidth.

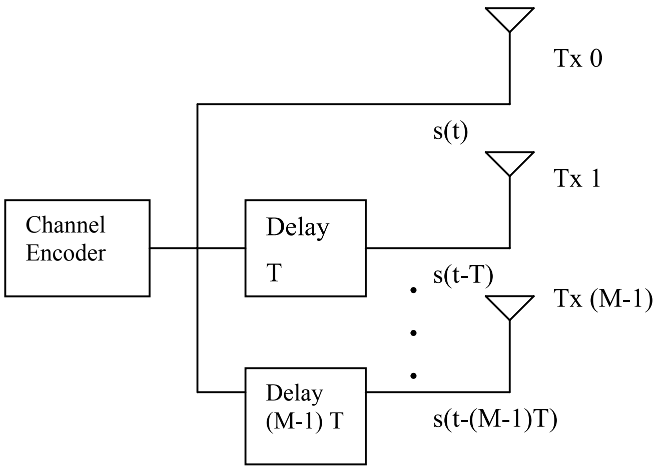

Figure 2.5 Transmit delay diversity

2.3.3.3 Transmit Diversity with Frequency Weighting

The third category does not require feedback or feedforward information. Instead, it uses multiple transmit antennas combined with channel coding to provide diversity. An example of this approach is the use of channel coding along with phase sweeping [44] or of frequency offset [45] with multiple transmit antennas, to mitigate the harm of scenario B, as shown in Figure 2.6. An appropriately designed channel code/interleaver pair is used to simulate fast fading.

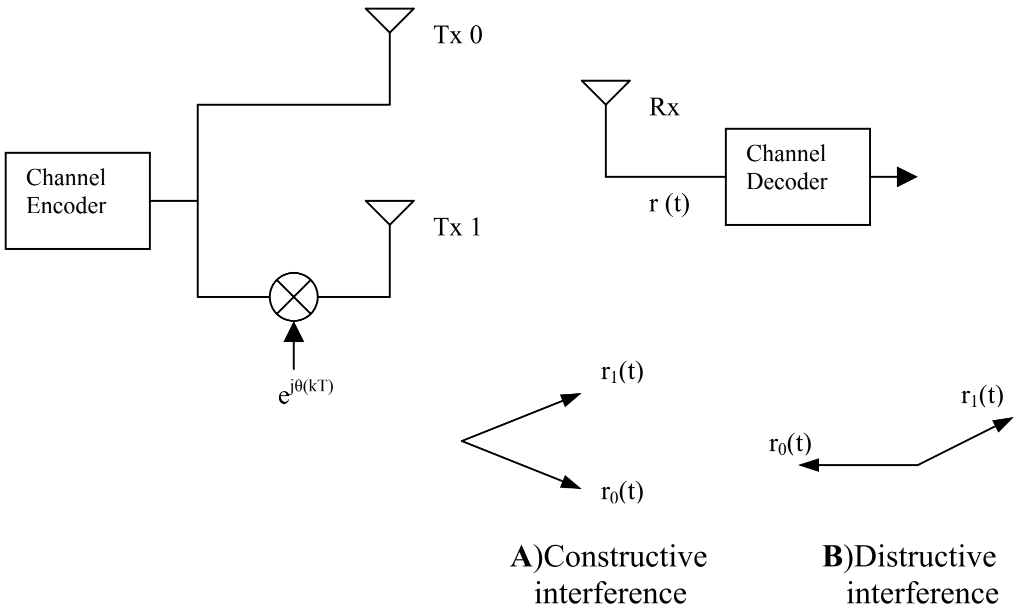

Figure 2.6 Transmit diversity with frequency weighting

2.3.3.4 Transmit Diversity with Channel Coding

Another approach in the third category is to encode information by a channel code which has a minimum Hamming distance $d_{min} \leq N$ (Figure 2.7) and to transmit the code symbols using different antennas in an orthogonal manner. This can be done by either time multiplexing [44], or by using orthogonal spreading sequences for different antennas [46]. Channel coding schemes can achieve a diversity of order N using maximum likelihood (ML) detection or MRC at the receiver. After receiving the N symbols, the decoder performs ML decoding to decode the received codeword. The disadvantage of these schemes as compared to the previous two categories is the loss in bandwidth efficiency by a factor of $1/N$ owing to the use of the channel code. Using appropriate coding, it is possible to relax the orthogonality requirement needed in these schemes and to obtain a diversity as well as a coding gain without sacrificing bandwidth. This will be possible if one views the whole system as a multiple input/multiple output system and uses channel codes that are designed with that view in mind.

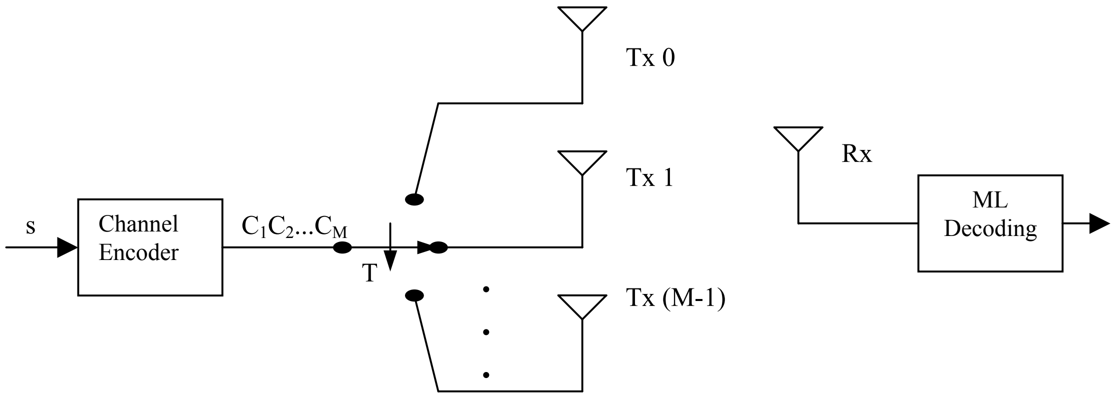

Figure 2.7 Transmit diversity with channel coding

2.3.4 General Structure of Transmit Diversity Schemes

In general, all transmit diversity schemes described earlier can be represented by a single transmitter structure as shown in Figure 2.8.

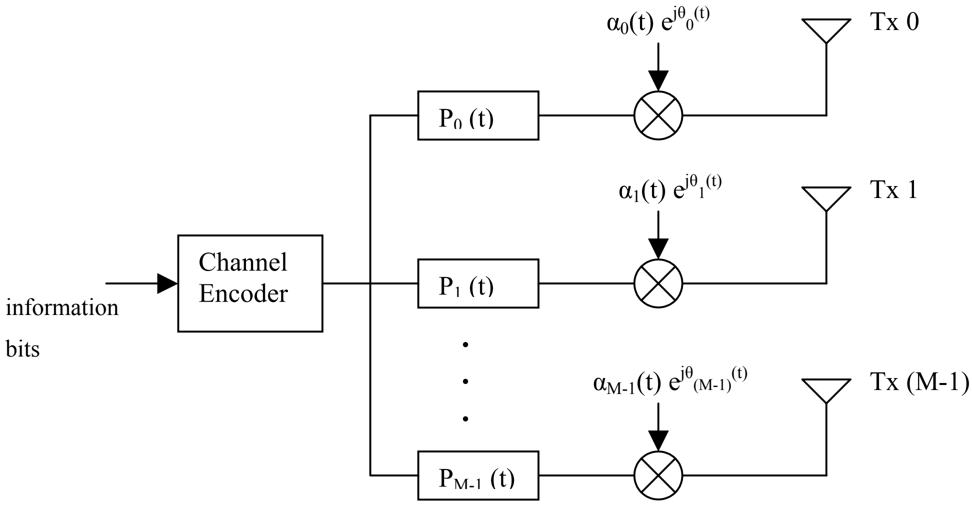

Figure 2.8. General structure of transmit diversity schemes

By appropriately selecting the pulse shaping function $p_i(t)$ and the weight $\alpha_i(t)e^{j\theta_i(t)}$, we can obtain any of the transmit diversity schemes. For example, the delay diversity scheme can be obtained from the above structure by setting all the weights to 1 and the pulse shaping functions to simple time shifts. Note that for all the transmit diversity schemes in the third category any channel code could be used. As pointed out, the use of a channel code in combination with multiple transmit antennas would achieve diversity, but will suffer a loss in bandwidth owing to channel coding. However, by using channel codes that are specifically designed for multiple transmit antennas, one can achieve the needed diversity gain without any sacrifice in bandwidth. These codes are called *ST codes*. ST coding [1,6-8,22,23,47–51] is a coding technique that is designed for use with multiple transmit antennas. ST codes introduce temporal and spatial correlation into signals transmitted from different antennas, so as to provide diversity at the receiver, and coding gain over an uncoded system without sacrificing the bandwidth. The spatio–temporal structure of these codes can be exploited to further increase the capacity of wireless systems with a relatively simple receiver structure [52]. In the next section we will review ST coding and its associated signal-processing framework.

CHAPTER 3

SPACE-TIME CODING

3.1 Basic System Model

In this section, we will describe a basic model for a mobile communication system that employs space–time coding with M transmit antennas and N receive antennas as shown in Figure 3.1.

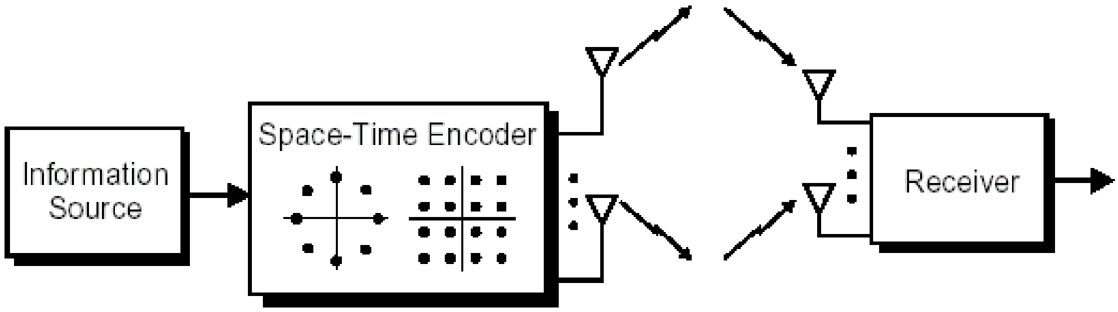

Figure 3.1 A basic system model

At the transmitter, information data symbols $s(l)$ belonging to constellation set A are analyzed with a vector of size τ_sx1. The space-time encoder maps this information symbol vector $s(l)$ at frame l to one of the M code vectors, $c_1(l), c_2(l)...c_M(l)$. At each frame slot l, τ_cx1 code vector $c_j(l)$, $j = 1,2\ldots M$ are transmitted simultaneously from the M transmit antennas. It is emphasized that the M vectors are transmitted simultaneously each from a different transmitter antenna and that all these vectors have the same transmission period $\tau_c T$. The encoder chooses the coded M vectors to transmit in a way to maximize both the coding gain and diversity gain.

We assume that the channels' delay spread is small compared to T but their coherence time is larger than $\tau_c T$. Under these assumptions, the channels between pairs of transmit- and receive-antennas are flat faded and can be modeled as complex constants within one block. When the channels are flat, no ISI occurs in the time domain. The path gain from transmit antenna j to receive antenna i is defined to be $h_{i,j}$ and this gains are modeled as samples of independent complex Gaussian variables with variance 0.5 per real dimension. The wireless channel is assumed to be a quasi-static fading channel for which the path gains are constant over a frame of length L and vary from one frame to another. Also, let us assume that Es is the total energy transmitted from all antennas per input symbol. Therefore, the energy per input symbol transmitted from each transmit antenna is E_s/M. The constellation points are scaled by a factor of $\sqrt{E_s / M}$ such that the average energy of the constellation points is 1.

Signals arriving at different receive antennas undergo independent fading. The signal at each receive antenna is a noisy superposition of the faded versions of the M transmitted signals. Let $r_i(l)$, i=1...N be the received signal at antenna after matched filtering. Assuming ideal timing and frequency information, we have

$$r_i(l) = \sqrt{E_S / M} \cdot \sum_{j=1}^{M} h_{ij}(l) c_j(l) + \eta_i(l), \quad i = 1, \ldots, N \tag{3.1}$$

where $\eta_i(l)$ are independent samples of a zero-mean complex white Gaussian process with two-sided power spectral density $N_0/2$ per dimension. It is also assumed that $\eta_i(l)$ and $\eta_k(l)$ are independent for $i \neq k,\ 1 \leq i,\ k \leq N$. The gain $h_{ij}(l)$ models the complex fading channel gain from transmit antenna j to receive antenna i. It is assumed that $h_{ij}(l)$ and $h_{qk}(l)$ are independent for $i \neq q$ or $j \neq k$ where $1 \leq i$, $q \leq N$, $1 \leq j$ and $k \leq M$. This condition is satisfied if the transmit antennas are well separated corresponding to a distance more than λ/2, or by using antennas with different polarization.

Let $c_l = [c_1(l), c_2(l) \ldots c_M(l)]^T$ be the Mxτ_c code matrix transmitted from the M antennas at frame l, $h_i(l) = [h_{i1}(l), h_{i2}(l) \ldots h_{iM}(l)]^T$ be the corresponding M x 1 channel vector from the M transmit antennas to the i^{th} receive antenna Because of quasi-static flat fading channel, and $r(l) = [r_1(l), r_2(l) \ldots r_N(l)]^T$ be Nxτ_c the received signal vector. Also, let $\eta(l) = [\eta_1(l), \eta_2(l) \ldots \eta_M(l)]^T$ be the Nxτ_c noise vector at the receive antennas. Let us define the NxM channel matrix $H(l) = [h_1(l), h_2(l) \ldots h_N(l)]^T$ from the M transmit to the N receive antennas. Eq. (3.1) can be rewritten in a matrix form as

$$r(l) = \sqrt{E_S} \cdot \mathrm{H}(l) \cdot c_l + \eta(l) \tag{3.2}$$

We can easily see that the SNR *per receive antenna* is given by $\rho = \dfrac{E_S}{N_0}$. (3.3)

3.2 Performance Criterion

We assume that the elements of the signal constellation are contracted by a factor of $\sqrt{E_s / M}$ so that the average energy of the constellation is 1. In this light, the design criterion to be established here is not constellation dependent and applies equally well to 4-PSK, 8-PSK and 16-QAM.

Suppose that the *code vector* sequence $C = c_1, c_2, \ldots, c_L$ was transmitted. We consider the probability that the decoder decides erroneously in favor of the legitimate code vector sequence $\widetilde{C} = \widetilde{c}_1, \widetilde{c}_2, \ldots, \widetilde{c}_L$.

We define the pairwise block error event $\{c \to \widetilde{c}\}$ as the event that the receiver decodes the block $\widetilde{c}$ erroneously when the block c is actually sent. Let $\Pr\{c \to \widetilde{c}\}$ be the pairwise block error probability averaged over the fading channels. The goal is to pursue optimal ST codes so that $\Pr\{c \to \widetilde{c}\}$ is minimized.

Assuming that for each frame or block of data of *L* length the ideal channel state information (CSI) $H(l), l = 1, \ldots, L$ are available at the receiver, the probability of transmitting *C* and deciding in favor of $\widetilde{C}$ is well upper bounded by [30]

$$P(C \to \widetilde{C} \mid H(l), l = 1, \ldots, L) = Q\left(\sqrt{\frac{D^2(C, \widetilde{C}) E_S}{2N_o}}\right) \leq \exp(-D^2(C, \widetilde{C}) \cdot E_S / 4N_o) \qquad (3.4)$$

where $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^\infty \exp(-x^2/2)dx$ and $D^2\left(C, \widetilde{C}\right) = \sum_{l=1}^{L} \left\| H(l)(c_l - \widetilde{c}_l) \right\|^2$

It is clear that in order to minimize the pairwise error probability we need to maximize $D^2\left(C, \widetilde{C}\right)$. However, $D^2\left(C, \widetilde{C}\right)$ is a function of the maximum Doppler frequency. Therefore, we will derive the performance criterion for designing the ST code, assuming that the fading is constant during a frame and vary from one frame to another. In this case H *(l) = [h $_1$ (l), h $_2$ (l)... h $_N$ (l)]* ^{T}*, l* = 1...*L* and we can easily verify that

$$D^2\left(C, \widetilde{C}\right) = \sum_{l=1}^{L} \left\| H(l)(c_l - \widetilde{c}_l) \right\|^2 = \sum_{l=1}^{L} \sum_{i=1}^{N} h_i^* (c_l - \widetilde{c}_l)(c_l - \widetilde{c}_l)^* h_i = \sum_{i=1}^{N} h_i^* A(C, \widetilde{C}) h_i \qquad (3.5)$$

$$A(C, \widetilde{C}) = \sum_{l=1}^{L} (c_l - \widetilde{c}_l)(c_l - \widetilde{c}_l)^* \qquad (3.6)$$

where $(.)^*$ denotes the conjugate operation for scalars and the conjugate transpose for matrices and vectors.

To this end, the required mathematical background on linear algebra is first reviewed [53]:

1. Let $x = (x_1, x_2 \dots x_k)$ and $y = (y_1, y_2 \dots y_k)$ be complex vectors in C^k. The inner product of x and y is given by $x \cdot y = \sum_{i=1}^{k} x_i \bar{y}_i$ where $\bar{y}_i$ denotes the complex conjugate of y_i.
2. For any matrix A ($\forall A \in C^N x C^M$), let A^* denote the Hermitian, i.e. the transpose conjugate of A.
3. An nxn matrix A is Hermitian if and only if $A = A^*A$ is non-negative definite if $xAx^* \geq 0$ for any 1xn complex vector x.
4. An nxn matrix U is unitary if and only if $UU^* = I$ where I is identity matrix.
5. An n x l matrix B is a square root of an nxn matrix A if $BB^* = A$.
6. An eigenvector v of an n x n matrix A corresponding to eigenvalue λ is 1xn vector of unit Euclidean length such that $vA = \lambda v$ for some complex number λ. The number of eigenvectors of A corresponding to eigenvalue zero is $n - r$, where r is the rank of A.
7. Any matrix A with square root B is nonnegative definite.
8. Given a Hermitian matrix A with the eigenvectors of span C^n, the complex space of n dimensions, it is easy to construct an orthonormal basis of C^n consisting of eigenvectors A.
9. There exists a unitary matrix U and a real diagonal matrix D such that $UAU^*=D$. The rows of U are an orthonormal basis C^n consisting of eigenvectors of A.

And so, under these conditions we can say that the MxM matrix $A\left(C,\tilde{C}\right)$ is Hermitian and is equal to $B\left(C,\tilde{C}\right)B^*\left(C,\tilde{C}\right)$ where $B\left(C,\tilde{C}\right)$ is MxL and represents the error sequence $C-\tilde{C}$. The matrix B is a square root of A. Since A is Hermitian we can write A as $U\Lambda U^*$ where U is unitary and Λ is a diagonal matrix where the diagonal elements $\lambda_n, n = 1,\dots,M$ are the nonnegative eigenvalues of A. Therefore, we can write $D^2\left(C,\tilde{C}\right)$ as

$$D^2\left(C,\tilde{C}\right) = \sum_{i=1}^{N} h_i^* U\Lambda U^* h_i = \sum_{i=1}^{N} \beta_i^* \Lambda \beta_i \qquad (3.7)$$

where $\beta_i = U^* h_i$ and $\beta_i = \left(\beta_{1,i},\dots,\beta_{M,i}\right)$. Since U is unitary and h_i is a complex Gaussian random vector with zero mean and covariance I, then β_i will be also a complex Gaussian random vector with zero mean and covariance I. Hence, we will have

$$D^2\left(C,\widetilde{C}\right)=\sum_{j=1}^{M}\sum_{i=1}^{N}\lambda_j\left|\beta_{j,i}\right|^2 . \tag{3.8}$$

The random variable $v_{ij}=\left|\beta_{j,i}\right|^2$ has a χ^2 distribution with two degrees of freedom, that is

$$v_{ij} \sim f_v(v)=e^{-v} \text{ for } v>0 \text{ and 0 otherwise.} \tag{3.9}$$

Thus, to compute an upper bound on the average pairwise error probability [49] we simply average the right-hand side of Eq. (3.4) (see Appendix 1) to arrive at

$$P(C\to\widetilde{C})\le\left(\prod_{j=1}^{M}\frac{1}{1+\lambda_j\cdot(E_S/4N_0)}\right)^N . \tag{3.10}$$

Let r denote the rank of the matrix A which is also equal to the rank of B. Then, A has exactly $M - r$ zero eigenvalues. Without loss of generality, let us assume that $\lambda_1,\lambda_2,\ldots,\lambda_r$ are the nonzero eigenvalues, then it follows from Eq. (3.10) that

$$\left(\prod_{j=1}^{M}\frac{1}{1+\lambda_j\cdot(E_S/4N_0)}\right)^N=\left(\prod_{j=1}^{r}\frac{1}{1+\lambda_j\cdot(E_S/4N_0)}\right)^N\le\left(\prod_{j=1}^{r}\frac{1}{\lambda_j\cdot(E_S/4N_0)}\right)^N$$

$$P(C\to\widetilde{C})\le\left(\prod_{j=1}^{r}\lambda_j\right)^{-N}\cdot\left(E_S/4N_0\right)^{-rN} \tag{3.11}$$

We can easily see that the probability of error bound in Eq. (3.11) is similar to the probability of error bound for trellis coded modulation (TCM) for fading channels and, thus, a diversity gain of rN and a coding gain of $g_r=(\lambda_1,\lambda_2,\ldots,\lambda_r)^{1/r}$ are achieved [4]. From the above analysis, we arrive at the following design criteria.

The Rank Criterion: In order to achieve the maximum diversity *MN,* the matrix $B\left(C,\widetilde{C}\right)$ has to be full rank for any two code vector sequences C and $\widetilde{C}$. If $B\left(C,\widetilde{C}\right)$ has a minimum rank r over the set of two-tuples of distinct code vector sequences, then a diversity of rN is achieved.

The Determinant Criterion: Suppose that a diversity benefit of rN is our target. The minimum of $g_r=(\lambda_1,\lambda_2,\ldots,\lambda_r)^{1/r}$ taken over all pairs of distinct code vector sequences C and $\widetilde{C}$ is the coding gain. The design target is to maximize g_r.

Some remarks are now in order:

Remark 1*:* The diversity advantage rN and the coding advantage g_r affect the bound in Eq. (3.11) in different ways. At high SNR, it is clear that rN plays a more important role to reduce the upper bound in Eq. (3.11) than g_r. Thus, ST coding designs should first satisfy the rank criterion and then the determinant criterion. If a tradeoff has to be made, the coding advantage should be the candidate.

Remark 2: The aforementioned ST coding design criteria are based on ML decoding and provide guidelines to design ST coding schemes.

Remark 3: In order to achieve the maximum diversity advantage, we infer from the dimensionality of B that one should select $\tau_c \geq M$.

Remark 4: Although multiple receive antennas (N > 1) are helpful to increase the diversity advantage, they are optional in ST coding designs. All design criteria are meaningful even for a single receive antenna provided that the SNR is high.

3.3 Code Construction

3.3.1 Space-Time Trellis Codes

We advance to use the criteria derived in the previous section to design trellis codes for a wireless communication system that employs transmit antennas and optional receive antenna diversity where the channel is a quasi-static flat fading channel.

STTCs are defined by a trellis and a signaling constellation such as BPSK, QPSK and 16QAM. In our example, suppose that the transmitter has a two antenna (M=2), and the signal constellation is QPSK.

A TCM encoder for ST coding is shown in Figure 3.2. At each symbol time, k information bits are fed into the convolutional encoder. The encoder has v memory elements, hence it has V=2^v states. Every symbol time, the encoder outputs m coded bits which are mapped to a signal constellation with M=2^m possible symbols. At each symbol time, the output symbol vector C = [C_1 C_2 ... C_M] has M symbols for the M transmit antennas.

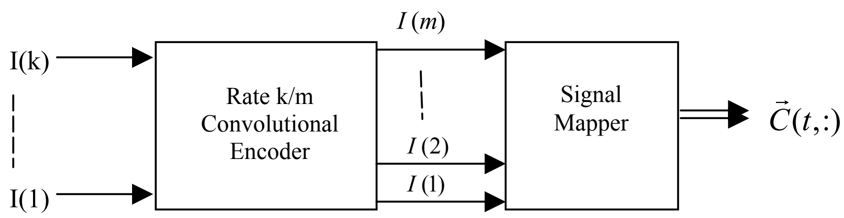

Figure 3.2 TCM encoder block diagram

The trellis is a graph that shows which symbols the convolutional encoder is allowed to transmit given the current state, and also which future state the encoder will enter when a particular symbol is transmitted. There are V=2^v states and $K=2^k$ branches from each state. Each branch has M symbols, one for each transmitter antenna.

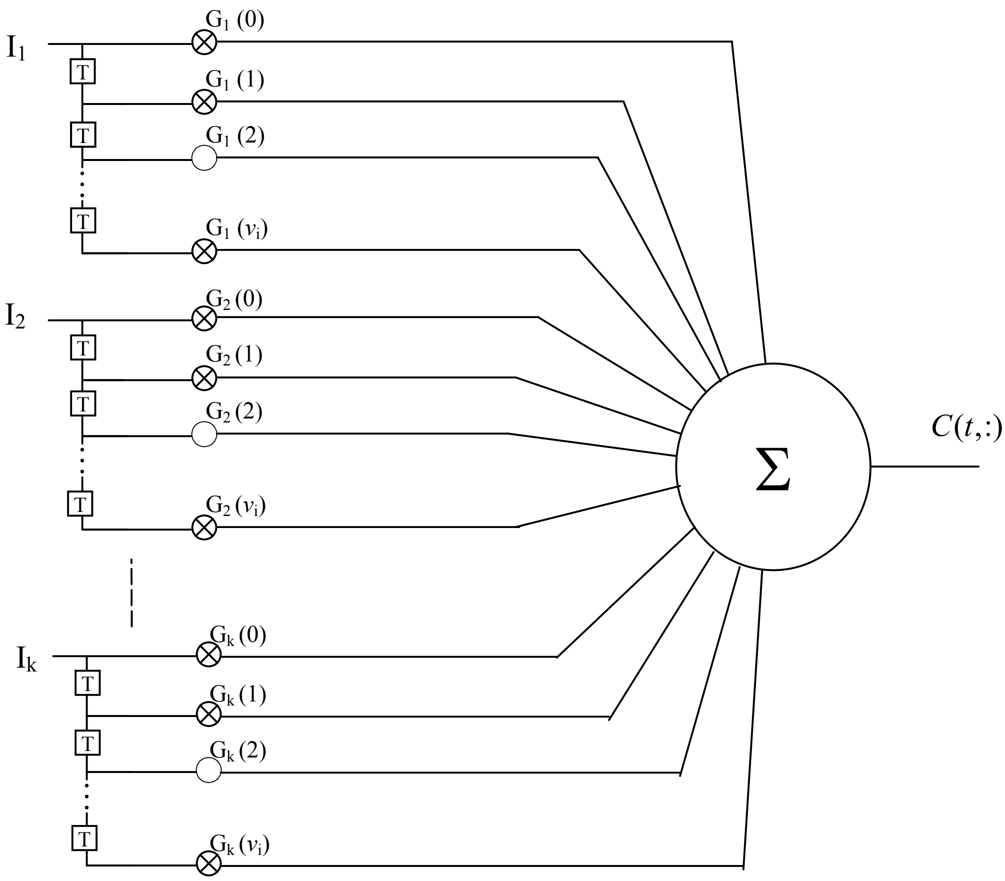

Figure 3.3 STTC feed forward encoder with k input bits.

Figure 3.3 depicts that an encoder is implemented as a feedforward shift register with a memory order v. The input to the encoder is a binary sequence (I_1, I_2… I_k). The encoder memory for each input may be written $I_i(t) = [I_i(t) \quad I_i(t-1) \quad \cdots \quad I_i(t-v_i)]^T$. The encoder for each input bit may have different memory length v_i, and the total memory of the encoder is $v = \sum_{i=1}^{k} v_i$. The encoder may be represented by k generator matrices as i=1:k, $G_i = [G_i(0) \quad G_i(1) \quad \cdots \quad G_i(v_i)]$. The output is obtained as a modulo M sum of the linear combinations of the current and delayed binary inputs. The linear combination vectors are $G_i(l) = [g_i(1,l) \quad g_i(2,l) \quad \cdots \quad g_i(M,l)]^T$, i=1:k, l=1:v_i. The output can be expressed as,

$$s(t,:) = \mathrm{mod}\left(\left(\sum_{i=1}^{k} G_i \cdot I_i(t)\right)^T, M\right),\ C(t,:) = map(s(t,:)) \tag{3.15}$$

$$C(t,:) = [C(t,1) \quad C(t,2) \quad \cdots \quad C(t,M)] \tag{3.16}$$

Given this encoder structure, we would like to find a set of coefficients, i.e. generator matrices that will satisfy the design criteria presented in previous section. It is important to note that this structure does not guarantee geometrically uniformity of the code [54].

STTCing employs ML decoding and its encoding is optimal in terms of maximizing both diversity and coding advantages [1].

3.3.2 STC: Q-PSK Example

For example, consider Q-PSK, 4 states STTC with two transmit-antennas (M=2). Figure 3.4 depicts the corresponding 4-PSK constellation labeling.

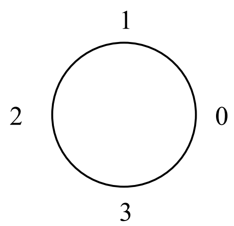

Figure 3.4 QPSK Constellation

Figure 3.5 depicts the corresponding Q-PSK encoding trellis (see Reference [1] for further details).

Generator matrices constructed by Tarokh in [1] are

$$G_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\ G_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ v\text{=2, m=2, k=2, M=2}.$$

With necessary computations as mentioned before, we found trellis diagram such as in Figure 3.5.

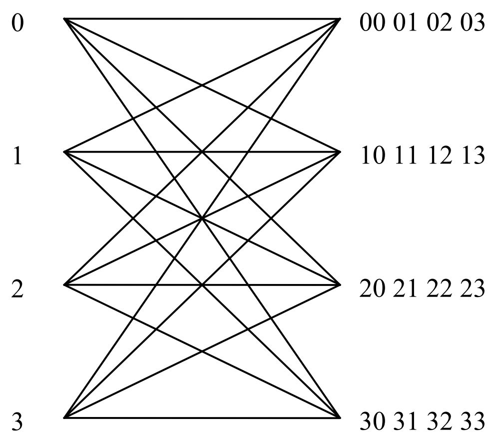

Figure 3.5 Q-PSK 4-State Space-Time Code with 2 Tx Antennas

Using this encoding trellis method, the ST encoder maps the data frame $s(l)$ to the code matrix $C(l)$. As an example, when $s(l) = \begin{bmatrix}1, & 3, & 2, & 3, & 0, & 1, & \cdots\end{bmatrix}^T$, the code matrix is given by

$$\mathrm{c}(l) = \begin{bmatrix} 0 & 1 & 3 & 2 & 3 & 0 & 1 & \cdots \\ 1 & 3 & 2 & 3 & 0 & 1 & \cdots & \end{bmatrix} \tag{3.17}$$

where we observe that this example is exactly the delay diversity scheme in Reference [3].

STTCing achieves the maximum diversity advantage MN and the maximum coding advantage. However, for a fixed number of transmit antennas, its decoding complexity increases exponentially with the transmission rate [1]. As discussed in section 3.2, when a tradeoff has to be made between coding advantage and diversity advantage, it is appropriate to sacrifice the coding advantage. We show next how STBCing trades decoding simplicity with possibly reduced coding advantage.

CHAPTER 4

SPACE–TIME BLOCK CODING

4.1 Introduction

STBCing is a new method for communication over wireless channels using multiple transmit antennas. It is now adapted for use in third generation wireless systems by WCDMA and CDMA 2000 standardization bodies. Not only these codes support an extremely simple maximum likelihood detection algorithm based only on linear processing at the receiver, but also they can be used for multiple transmit antenna differential detection.

The STTCs presented in [1] provide the best possible tradeoff between constellation size, data rate, diversity advantage, and trellis complexity. STTCs perform extremely well at the cost of relatively high complexity. When the numbers of transmit antennas is fixed, the decoding complexity of STTCing (measured by the number of trellis states in the decoder) increases exponentially as a function of both the diversity level and the transmission rate.

In addressing the issue of decoding complexity, Alamouti discovered a remarkable scheme for transmissions using two transmit antennas [7]. This scheme has a simple decoding algorithm that can be generalized to an arbitrary number of receiver antennas. And also this scheme is significantly less complex than STTCing using two transmitter antennas, although there is a loss in performance [8]. Despite the associated performance penalty, Alamouti's scheme is appealing in terms of its simplicity and performance. Introduced in [6,8], STBCing generalizes the transmission scheme discovered by Alamouti to an arbitrary number of transmit antennas and is able to achieve the full diversity promised by the transmit and receive antennas. These codes retain the property of having a very simple maximum likelihood-decoding algorithm based only on linear processing at the receiver [6,8].

4.2 Maximal-Ratio Receive Combining (MRRC) Scheme

In conventional transmission systems, we have a single transmitter, which transmits information to a single receiver. In Rayleigh fading channels, the transmitted symbols experience severe magnitude fluctuation and phase rotation. In order to mitigate this problem, we can employ several receivers that receive replicas of the same transmitted symbol through independent fading paths. Even if a particular path is severely faded, we may still be able to recover a reliable estimate of the transmitted symbols through other propagation paths. However, at the station, we have to combine the received symbols of the different propagation paths, which involves additional complexity. An optimal combining method often used in practice is referred to as the MRRC technique.

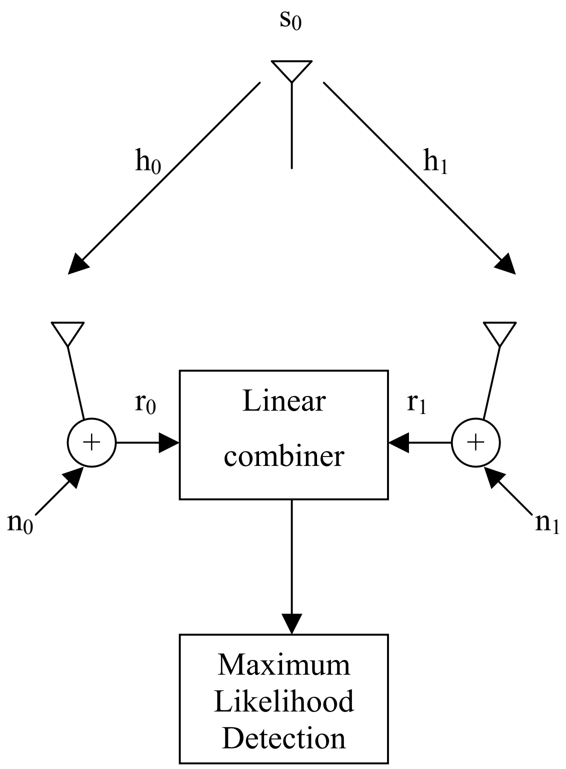

Figure 4.1 Configuration of MRRC

Figure 4.1 shows the baseband representation of the classical two-branch MRRC. At a given time, a signal s_0 is sent from the transmitter. The channel may be modeled by a complex multiplicative distortion composed of a magnitude response and a phase response. The channel between the transmit antenna and receive antenna zero is denoted by h_0 and between the transmit antenna and receive antenna one is denoted by h_1 where

$$h_0 = \alpha_0 e^{j\theta_0}, \quad h_1 = \alpha_1 e^{j\theta_1} \tag{4.1}$$

In this formulas, α_0, α_1 are the fading magnitudes and θ_0, θ_1 are the phase values. For simplicity, all channels are assumed to be constituted of a single nondispersive or flat-fading propagation path.

Noise and interference are added at the two receivers, as shown in Figure 4.1. Therefore the resulting received baseband signals are

$$r_0 = h_0 s_0 + n_0 \ , \ \ r_1 = h_1 s_0 + n_1 \tag{4.2}$$

where n_0 and n_1 represent complex noise and interference. In matrix form, this can be written as

$$\begin{pmatrix} r_0 \\ r_1 \end{pmatrix} = \begin{pmatrix} h_0 \\ h_1 \end{pmatrix} s_0 + \begin{pmatrix} n_0 \\ n_1 \end{pmatrix} \tag{4.3}$$

Assuming that perfect channel information is available, the received signals r_0 and r_1 can be multiplied by the conjugate of the complex channel transfer functions h_0 and h_1, respectively, in order to remove the channel's effects.

Maximal-ratio receive combining scheme:

$$S = ar_0 + br_1 , \ \ a,b = coefficients$$

$$= (ah_0 + bh_1)s_m + an_0 + bn_1$$

$$SNR \propto \frac{|ah_0 + bh_1|^2}{E\{|an_0 + bn_1|^2\}} = \frac{|ah_0 + bh_1|^2}{(|a|^2 + |b|^2)\sigma^2}$$

$SNR = max$ if $a = c.h_0^*, b = c.h_1^*$ where c is an arbitrary constant.

Assuming c=1, output of the combination is

$$\tilde{s}_0 = h_0^* r_0 + h_1^* r_1 = h_0^*(h_0 s_m + n_0) + h_1^*(h_1 s_m + n_1) = (\alpha_0^2 + \alpha_1^2)s_m + h_0^* n_0 + h_1^* n_1 \tag{4.4}$$

The combined signal $\tilde{s}_0$ is then passed to the ML detector, as shown in Figure 4.1.

The conditional pdf $p(r \mid s_m)$ or any monotonic function of it is usually called the *likelihood function*. The decision criterion based on the maximum of $p(r \mid s_m)$ over M signals is called the *maximum-likelihood (ML) criterion* [30].

Assuming n_0 and n_1 are Gaussian distributed $N(0, \sigma^2)$, the monotonic function of conditional pdf $p(r_0,r_1 \mid s_m)$ is

$$\ln p(r_0, r_1 \mid s_m) = -(r_0 - h_0 s_m, r_1 - h_1 s_m)^* \begin{pmatrix} r_0 - h_0 s_m \\ r_1 - h_1 s_m \end{pmatrix} + \text{constant} \tag{4.5}$$

The criterion for deciding on the transmitted symbol s_k is

$$\tilde{s}_m = \arg\min_{s_k}\left\{d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k)\right\} \tag{4.6}$$

where $d^2(x,y)$ is the squared Euclidean distance between signals x and y calculated by the following expression: $d^2(x,y) = (x-y)(x^* - y^*)$

Thus, the ML decision rule at the receiver for the received signals is to choose signal $\tilde{s}_m = s_m$ if and only if (iff)

$$d^2(r_0, h_0 s_m) + d^2(r_1, h_1 s_m) \le d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k)\,,\ \forall m \ne k \tag{4.7}$$

$$\begin{aligned} d^2(r_0, h_0 s_k) &= (h_0(s_m - s_k) + n_0)\ (h_0^*(s_m^* - s_k^*) + n_0^*) \\ &= \alpha_0^2\left(|s_m|^2 + |s_k|^2 - s_m s_k^* - s_k s_m^*\right) + |n_0|^2 + h_0 s_m n_0^* - h_0 s_k n_0^* + h_0^* s_m^* n_0 - h_0^* s_k^* n_0 \end{aligned} \tag{4.8}$$

$$\begin{aligned} d^2(r_1, h_1 s_k) &= (h_1(s_m - s_k) + n_1)\ (h_1^*(s_m^* - s_k^*) + n_1^*) \\ &= \alpha_1^2\left(|s_m|^2 + |s_k|^2 - s_m s_k^* - s_k s_m^*\right) + |n_1|^2 + h_1 s_m n_1^* - h_1 s_k n_1^* + h_1^* s_m^* n_1 - h_1^* s_k^* n_1 \end{aligned} \tag{4.9}$$

summing Eq. (4.8) and Eq. (4.9) with $-s_m s_k^* - s_k s_m^* = -2\,\mathrm{Re}\{s_m s_k^*\}$ we get;

$$\begin{aligned} d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k) = {} & (\alpha_0^2 + \alpha_1^2)|s_m|^2 + (\alpha_0^2 + \alpha_1^2)|s_k|^2 + |n_0|^2 + |n_1|^2 - 2(\alpha_0^2 + \alpha_1^2)\mathrm{Re}\{s_m s_k^*\} \\ & + (h_0 n_0^* + h_1 n_1^*)s_m - (h_0 n_0^* + h_1 n_1^*)s_k + (h_0^* n_0 + h_1^* n_1)s_m^* - (h_0^* n_0 + h_1^* n_1)s_k^* \end{aligned}$$

$$d^2(r_0, h_0 s_m) + d^2(r_1, h_1 s_m) = |n_0|^2 + |n_1|^2$$

choose $\tilde{s}_m = s_m$ iff,

$$\begin{aligned} 0 \le {} & (\alpha_0^2 + \alpha_1^2)|s_m|^2 + (\alpha_0^2 + \alpha_1^2)|s_k|^2 - 2(\alpha_0^2 + \alpha_1^2)\mathrm{Re}\{s_m s_k^*\} \\ & + 2\,\mathrm{Re}\{s_m(h_0 n_0^* + h_1 n_1^*)\} - 2\,\mathrm{Re}\{s_k(h_0 n_0^* + h_1 n_1^*)\} \end{aligned} \tag{4.10}$$

$$\begin{aligned} \Rightarrow \quad & -(\alpha_0^2 + \alpha_1^2)|s_m|^2 - 2\,\mathrm{Re}\{s_m(h_0 n_0^* + h_1 n_1^*)\} \\ & \le (\alpha_0^2 + \alpha_1^2)|s_k|^2 - 2(\alpha_0^2 + \alpha_1^2)\mathrm{Re}\{s_m s_k^*\} - 2\,\mathrm{Re}\{s_k(h_0 n_0^* + h_1 n_1^*)\} \end{aligned} \tag{4.11}$$

The maximal-ratio combiner may then construct the signal $\tilde{s}_0$, as shown in Figure 4.1. So, we have

$$\tilde{s}_0 s_m^* = (\alpha_0^2 + \alpha_1^2)|s_m|^2 + (h_0^* n_0 + h_1^* n_1)s_m^* \tag{4.12}$$

$$\tilde{s}_0 s_k^* = (\alpha_0^2 + \alpha_1^2)s_m s_k^* + (h_0^* n_0 + h_1^* n_1)s_k^*. \tag{4.13}$$

Using Eqs. (4.12, 4.13) with Eq. (4.11), we have

$$\left(\alpha_0^2+\alpha_1^2\right)|s_m|^2-2\operatorname{Re}\left\{\tilde{s}_0 s_m^*\right\}\le\left(\alpha_0^2+\alpha_1^2\right)|s_k|^2-2\operatorname{Re}\left\{\tilde{s}_0 s_k^*\right\} \quad\Rightarrow$$

$$\left(\alpha_0^2+\alpha_1^2-1\right)|s_m|^2+|s_m|^2-2\operatorname{Re}\left\{\tilde{s}_0 s_m^*\right\}+|s_0|^2\le\left(\alpha_0^2+\alpha_1^2-1\right)|s_k|^2+|s_k|^2-2\operatorname{Re}\left\{\tilde{s}_0 s_k^*\right\}+|s_0|^2$$

$$\Rightarrow\left(\alpha_0^2+\alpha_1^2-1\right)|s_m|^2+d^2\left(\tilde{s}_0,s_m\right)\le\left(\alpha_0^2+\alpha_1^2-1\right)|s_k|^2+d^2\left(\tilde{s}_0,s_k\right)\quad\forall m\ne k \tag{4.14}$$

For PSK signals (equal energy constellations)

$$|s_m|^2=|s_k|^2=E_s \qquad \forall m,k \tag{4.15}$$

where E_s is the energy of the signal. Therefore, for PSK signals, the decision rule in Eq. (4.14) may be simplified to choose s_m iff

$$d^2\left(\tilde{s}_0,s_m\right)\le d^2\left(\tilde{s}_0,s_k\right) \qquad \forall m\ne k \tag{4.16}$$

We can see that ML transmitted symbol is the one having the minimum Euclidean distance from the combined signal $\tilde{s}_0$. The ML detector may produce s_m, which is a ML estimate of s_0.

4.3 Space-Time Block Codes

In analogy to the MRRC matrix formula of Eq. (4.3), a STBC describing the relationship between the original transmitted signal s and the signal replicas artificially created at the transmitter for transmission over various diversity is defined an τxM dimensional transmission matrix C. Hence, a general form of the transmission matrix of a STBC is written as

$$C=\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1M} \\ c_{21} & c_{22} & \cdots & c_{2M} \\ . & . & . & . \\ . & . & . & . \\ c_{\tau 1} & c_{\tau 2} & \cdots & c_{\tau M} \end{bmatrix} \tag{4.17}$$

The entries of the matrix c_{tj} are linear combinations of k-ary input symbols $s_1,s_2,\ldots,s_k$ and their conjugates. The k-ary input symbols are used to represent the information-bearing binary bits to be channels. In a signal constellation having 2^b constellation points, a number b of binary bits are used to represent a symbol s_i. Hence, a block of kxb binary bits is entered into the ST block encoder at a time and it is, therefore, referred to as a STBC. The number of transmission antennas is M, and it is used to separate different codes from each other. If c_{tj} represents the element in the t th row and the j th column of C, the entries c_{tj}, $j = 1,2,\ldots,M$ are transmitted simultaneously from transmit antennas $1,2,\ldots,M$ at each time slot $t = 1,2,\ldots,\tau$. So, the j th column of represents the transmitted symbols from the j th antenna and the t th row of C represents the transmitted symbols at time slot t. Since τ time slots are used to transmit k symbols, we define the code rate R of the code to be $R = k/\tau$.

4.3.1 Two-Branch Transmitter Diversity Based STBCs

The technique proposed in Alamouti's paper [7] is a simple transmit diversity scheme which improves the signal quality at the receiver on one side of the link by simple processing across two transmit antennas on the opposite side. The obtained diversity order is equal to applying MRRC with two antennas at the receiver. The scheme may easily be generalized to two transmit antennas and N receive antennas to provide a diversity order of 2N. This is done without any feedback from the receiver to the transmitter and with small computation complexity. The scheme requires no bandwidth expansion, as redundancy is applied in space across multiple antennas, not in time or frequency.

Steps of Alamouti's scheme

(a) Encode (ST code) at the transmitter

(b) Combine at the receiver(s)

(c) Detect using ML.

At a given symbol period, two signals are simultaneously transmitted from the two antennas. The signal transmitted is shown in Table 4.1 and also Figure 4.2. In Table.4.1, the encoding is done in space and time (ST coding). The encoding, however, may also be done in space and frequency. Instead of two adjacent symbol periods, two adjacent carriers may be used (SF coding).

Table 4.1 Transmitted signal sequence in space and time

	Antenna 0	*Antenna 1*
Time t	s_0	s_1
Time t+T	$-s_1^*$	s_0^*

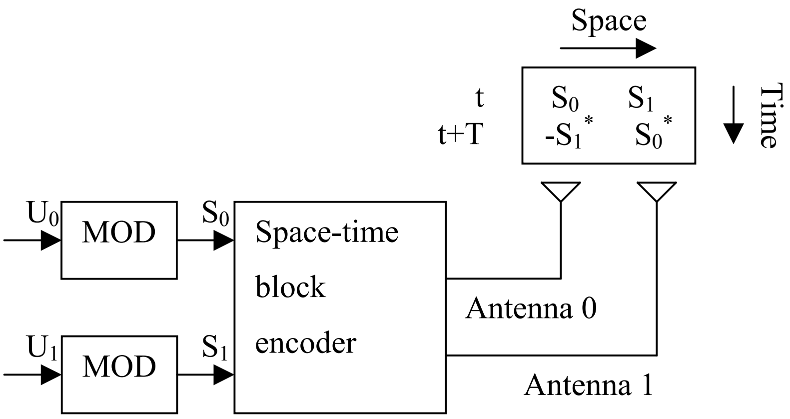

Figure 4.2 Transmitter of STBCing

The signal represented in table can be written in the matrix form:

$$C = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix}. \tag{4.18}$$

The channel at time t is modeled by a complex multiplicative distortion $h_j(t)$ for transmit antenna j. Assuming that fading is constant during two symbols time

$$h_j(t) = h_j(t+T) = h_j = \alpha_j e^{j\theta_{ji}}, \quad \text{j} = 0,1 \tag{4.19}$$

where T is the symbol duration.

4.3.1.1 Two-Branch Transmit Diversity with One Receiver

Let us now consider that the C code in Eq. (4.18) is sent from two transmitter. The channel vector may be defined as $H = (h_0, h_1)^T$.

The receiver in each time slot adds independent noise samples and so the received signals can be expressed as

$$\begin{aligned} r_0 &= r(t) = h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \end{aligned} \tag{4.20}$$

where r_0 is the first received signal and r_1 is the second. Note that the received signal r_0 consists of the transmitted signals s_0 and s_1, while r_1 consists of their conjugates.

$$\text{in vector form } Y = \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} = C \cdot H + n \qquad \left(\eta = \begin{bmatrix} n_0 \\ n_1 \end{bmatrix} \right) \tag{4.21}$$

where n_0 and n_1 are complex random variables representing receiver noise and interference.

The scheme showed in Figure 4.3 uses two transmit antennas and one receive antenna.

In order to determine the transmitted symbols, we have to extract the signals s_0 and s_1 from received signals r_0 and r_1. Therefore, both signals r_0 and r_1 are passed to the combiner.

The outputs of the combiner shown in Figure 4.3 are

$$\begin{aligned} \tilde{s}_0 &= h_0^* r_0 + h_1 r_1^* \\ \tilde{s}_1 &= h_1^* r_0 - h_0 r_1^* \end{aligned} \qquad \text{in vector form } \tilde{C} = \begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \begin{bmatrix} h_0^* & h_1 \\ h_1^* & -h_0 \end{bmatrix} \cdot \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix} \tag{4.22}$$

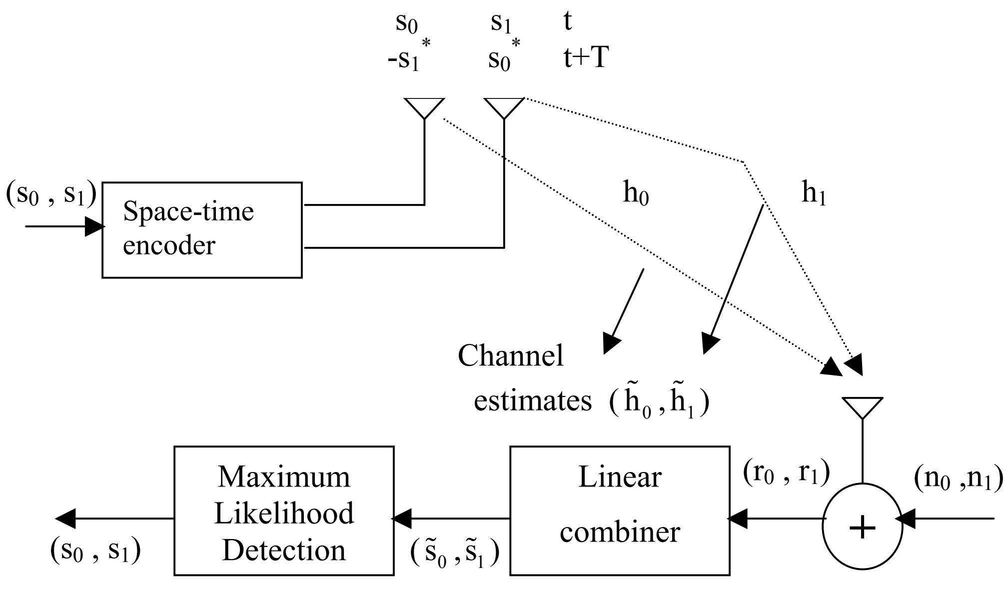

Figure 4.3 Two-Branch Transmit Diversity Scheme with One Receiver

Then we obtain the estimated code vectors as:

$$\begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = (|h_0|^2 + |h_1|^2) \cdot \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} h_0^* & h_1 \\ h_1^* & -h_0 \end{bmatrix} \cdot \begin{bmatrix} n_0 \\ n_1^* \end{bmatrix} \tag{4.23}$$

By using the channel coefficients under assumption that perfect CSI,

$$\begin{aligned}\tilde{s}_0 &= \left(\alpha_0^2 + \alpha_1^2\right) \cdot s_0 + h_0^* n_0 + h_1\, n_1 \\ \tilde{s}_1 &= \left(\alpha_0^2 + \alpha_1^2\right) \cdot s_1 - h_0 n_1^* + h_1^* n_1^*\end{aligned} \tag{4.24}$$

Both signals $\tilde{s}_0$ and $\tilde{s}_1$ are then passed to the maximum likelihood detector of Figure 4.3, which applies Eq. (4.16) to determine the most likely transmitted symbols. The maximum diversity advantage = 2 is achieved.

4.3.1.2 Two-Branch Transmit Diversity with Two Receiver

STBC with one receiver introduced in previous section can be extended to an arbitrary number of receivers. The encoding and transmission sequence will be identical to the case of a single receiver.

For illustration, we discuss the special case of two transmit and two receive antennas in detail. The generalization to N receive antennas will be given in the next section.

The scheme showed in Figure 4.4 uses two transmit antennas and two receive antennas.

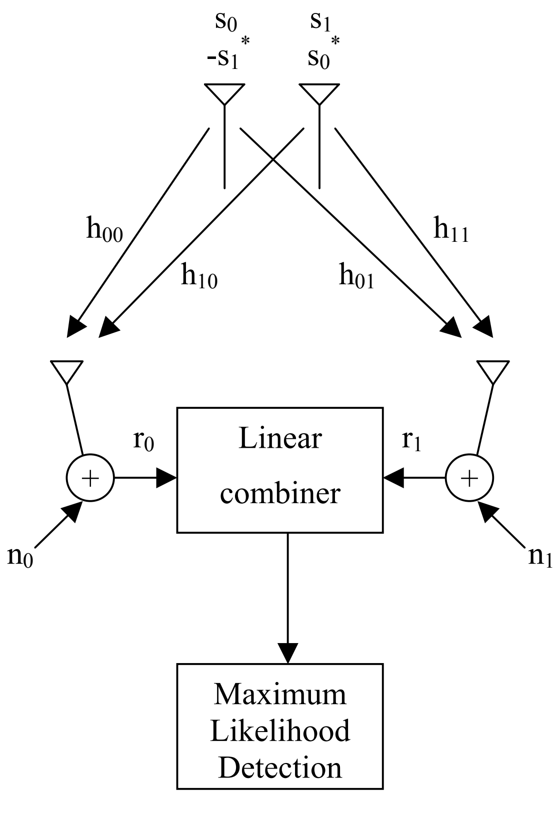

Figure 4.4 Two-Branch Transmit Diversity Scheme with Two Receivers

The encoding and transmission sequence of the information symbols and notation for the received signals at the two receive antennas are shown in Table 4.2.

Table 4.2 Transmitted and received signal sequence in space and time

	Tx Antenna 0	*Tx Antenna 1*	*Rx Antenna 0*	*Rx Antenna 1*
Time t	s_0	s_1	r_{00}	r_{10}
Time t+T	$-s_1^*$	s_0^*	r_{01}	r_{11}

Table 4.3 defines the channels between transmit and receive antennas.

Table 4.3 The channels between transmit and receive antennas

	Rx Antenna 0	*Rx Antenna 1*
Tx Antenna 0	h_{00}	h_{10}
Tx Antenna 1	h_{01}	h_{11}

The subscript i in the notation h_{ij}, n_{it}, r_{it} represents the receiver index, i=0,1...N-1. The subscript j denotes the transmitter index in h_{ij} and the subscript t denotes time slots in n_{it}, r_{it}.

Therefore, at the receiver side, we have

$$\begin{aligned} r_{00} &= h_{00}s_0 + h_{01}s_1 + n_{00} \qquad & r_{01} &= -h_{00}s_1^* + h_{01}s_0^* + n_{01} \\ r_{10} &= h_{10}s_0 + h_{11}s_1 + n_{10} \qquad & r_{11} &= -h_{10}s_1^* + h_{11}s_0^* + n_{11} \end{aligned} \tag{4.25}$$

These equations can be also presented in matrix form:

$$C = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} \left[r_{00} \quad r_{01}\right] &= \left[h_{00} \quad h_{01}\right]C^T + \left[n_{00} \quad n_{01}\right] \\ \left[r_{10} \quad r_{11}\right] &= \left[h_{10} \quad h_{11}\right]C^T + \left[n_{10} \quad n_{11}\right] \end{aligned} \tag{4.26}$$

For convenience, we have conjugated the equation for $r_{01}$and r_{11} so that the signal s_0 and s_1 don't need to be conjugated.

$$\begin{bmatrix} r_{00} \\ r_{01}^* \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} \\ h_{01}^* & -h_{00}^* \end{bmatrix} . \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_{00} \\ n_{01}^* \end{bmatrix}, \qquad \begin{bmatrix} r_{10} \\ r_{11}^* \end{bmatrix} = \begin{bmatrix} h_{10} & h_{11} \\ h_{11}^* & -h_{10}^* \end{bmatrix} . \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_{10} \\ n_{11}^* \end{bmatrix} \tag{4.27}$$

At the combiner of Figure 4.4, the received signals are combined to extract the transmitted signals s_0 and s_1 from the received signals r_{00}, r_{01}, r_{10}, and r_{11} according to

$$\begin{aligned} \widetilde{s}_0 &= h_{00}^* r_{00} + h_{01} r_{01}^* + h_{10}^* r_{10} + h_{11} r_{11}^* \\ \widetilde{s}_1 &= h_{01}^* r_{00} - h_{11} r_{01}^* + h_{11}^* r_{10} - h_{10} r_{11}^* \end{aligned} \tag{4.28}$$

Again we can show the above expressions in matrix form and we can simplify:

$$\begin{bmatrix}\tilde{s}_0\\ \tilde{s}_1\end{bmatrix}=\begin{bmatrix}h_{00}^* & h_{01}\\ h_{01}^* & -h_{00}\end{bmatrix}.\begin{bmatrix}r_{00}\\ r_{01}^*\end{bmatrix}+\begin{bmatrix}h_{10}^* & h_{11}\\ h_{11}^* & -h_{10}\end{bmatrix}.\begin{bmatrix}r_{10}\\ r_{11}^*\end{bmatrix} \tag{4.29}$$

$$=\underbrace{\begin{bmatrix}h_{00}^* & h_{10}\\ h_{01}^* & -h_{00}\end{bmatrix}.\begin{bmatrix}h_{00} & h_{01}\\ h_{01}^* & -h_{00}^*\end{bmatrix}.\begin{bmatrix}s_0\\ s_1\end{bmatrix}+\begin{bmatrix}h_{10}^* & h_{11}\\ h_{11}^* & -h_{10}\end{bmatrix}.\begin{bmatrix}h_{10} & h_{11}\\ h_{11}^* & -h_{10}^*\end{bmatrix}.\begin{bmatrix}s_0\\ s_1\end{bmatrix}}_{I}$$

$$+\begin{bmatrix}h_{00}^* & h_{01}\\ h_{01}^* & -h_{00}\end{bmatrix}.\begin{bmatrix}n_{00}\\ n_{01}^*\end{bmatrix}+\begin{bmatrix}h_{10}^* & h_{11}\\ h_{11}^* & -h_{10}\end{bmatrix}.\begin{bmatrix}n_{10}\\ n_{11}^*\end{bmatrix}$$

$$I=\left(\begin{bmatrix}\left(h_{00}^*.h_{00}+h_{01}.h_{01}^*\right) & \left(h_{00}^*.h_{01}-h_{01}.h_{00}^*\right)\\ \left(h_{01}^*.h_{00}-h_{00}.h_{01}^*\right) & \left(h_{01}^*.h_{01}+h_{00}.h_{00}^*\right)\end{bmatrix}+\begin{bmatrix}\left(h_{10}^*.h_{10}+h_{11}.h_{11}^*\right) & \left(h_{10}^*.h_{11}-h_{11}.h_{10}^*\right)\\ \left(h_{11}^*.h_{10}-h_{10}.h_{11}^*\right) & \left(h_{11}^*.h_{11}+h_{10}.h_{10}^*\right)\end{bmatrix}\right).\begin{bmatrix}s_0\\ s_1\end{bmatrix}$$

$$I=\begin{bmatrix}\left(|h_{00}|^2+|h_{01}|^2+|h_{10}|^2+|h_{11}|^2\right) & 0\\ 0 & \left(|h_{00}|^2+|h_{01}|^2+|h_{10}|^2+|h_{11}|^2\right)\end{bmatrix}.\begin{bmatrix}s_0\\ s_1\end{bmatrix}$$

Thus, we have

$$\begin{aligned}\tilde{s}_0&=\left(\alpha_{00}^2+\alpha_{01}^2+\alpha_{10}^2+\alpha_{11}^2\right)s_0+h_{00}^*n_{00}+h_{01}n_{01}^*+h_{10}^*n_{10}+h_{11}n_{11}^*\\ \tilde{s}_1&=\left(\alpha_{00}^2+\alpha_{01}^2+\alpha_{10}^2+\alpha_{11}^2\right)s_1-h_{00}n_{01}^*+h_{01}^*n_{00}-h_{10}n_{11}^*+h_{11}^*n_{10}\end{aligned} \tag{4.30}$$

These combined signals are then sent to the ML detector, which for signals s_0 and s_1 uses the decision criteria expressed in Eq. (4.16). Hence the maximum diversity advantage of 4 is achieved.

4.3.1.3 Two-Branch Transmit Diversity with N Receivers

There may be applications where a higher order of diversity is needed and multiple receive antennas at the remote units are feasible. In such cases, it is possible to provide diversity orders of 2N with two transmit and N receive antennas.

Using the notation h_{ij}, n_{it}, r_{it} in previous section (4.3.1.2), we can generalize equations to N receive antennas.

Generalization of the received signals given in Eq. (4.25) is

$$\begin{aligned}r_{i0}&=h_{i0}s_0+h_{i1}s_1+n_{i0}\\ r_{i1}&=-h_{i0}s_1^*+h_{i1}s_0^*+n_{i1}\end{aligned} \tag{4.31}$$

Where $i=0,1\ldots(N-1)$ and N is the number of receivers.

At the combiner, generalized receiver signals in Eq. (4.31) are combined

$$\begin{aligned}\tilde{s}_0 &= \sum_{i=0}^{N-1} h_{i0}^* r_{i0} + h_{i1} r_{i1}^* \\ \tilde{s}_1 &= \sum_{i=0}^{N-1} h_{i1}^* r_{i0} - h_{i0} r_{i1}^*\end{aligned} \tag{4.32}$$

We can also simplify Eq. (4.32) to

$$\begin{aligned}\tilde{s}_0 &= \sum_{i=0}^{N-1} \left(\left|h_{i0}\right|^2 + \left|h_{i1}\right|^2 \right) s_0 + h_{i0}^* n_{i0} + h_{i1} n_{i1}^* \\ \tilde{s}_1 &= \sum_{i=0}^{N-1} \left(\left|h_{i0}\right|^2 + \left|h_{i1}\right|^2 \right) s_1 + h_{i1}^* n_{i0} - h_{i0} n_{i1}^*\end{aligned} \tag{4.33}$$

These combined signals are then sent to the maximum likelihood detector. So we have 2N different (independent) channels, so the degree of spatial diversity is 2N.

All of these equations given above can be represented in matrix form:

If the channel matrix is

$$H = \begin{bmatrix} h_{10} & h_{1,1} \\ h_{20} & h_{2,1} \\ \vdots & \vdots \\ h_{(N-1)0} & h_{(N-1)1} \end{bmatrix} \tag{4.34}$$

where $h_{ij} = \alpha_{ij} e^{j\theta_{ij}}$ $i = 0,1\ldots(N-1)$, $j = 0,1$, Received signal matrix whose elements are given in Eq. (4.31) is

$$Y = \begin{bmatrix} r_{10} & r_{11} \\ r_{20} & r_{21} \\ \vdots & \vdots \\ r_{(N-1)0} & r_{(N-1)1} \end{bmatrix}. \tag{4.35}$$

Matrix Y is also derived with matrix computations

$$Y = CH + \eta \tag{4.36}$$

Where C is given in Eq. (4.18) and noise matrix η,

$$\eta = \begin{bmatrix} n_{10} & n_{11} \\ n_{20} & n_{21} \\ \vdots & \vdots \\ n_{(N-1)0} & n_{(N-1)1} \end{bmatrix}. \tag{4.37}$$

Combination process in matrix form is

$$\widetilde{S}=\begin{bmatrix}\widetilde{s}_0\\ \widetilde{s}_1\end{bmatrix}=\sum_{i=0}^{N-1}\begin{bmatrix}h_{i0}^* & h_{i1}\\ h_{i1}^* & -h_{i0}\end{bmatrix}\cdot\begin{bmatrix}r_{i0}\\ r_{i1}^*\end{bmatrix}. \tag{4.38}$$

4.3.2 Generalized Orthogonal Designs as STBCs

In previous sections, we have detailed Alamouti's simple two-transmitter complex orthogonal STBC of Eq. (4.18). It is shown that using two transmit antennas and one receive antenna, this scheme provide the same diversity order as MRRC with one transmit antenna and two receive antennas. It also shown that it may be generalized to two transmit and N receive antennas.

It is natural to ask for extensions of these schemes to an arbitrary number of transmit antennas with real or complex orthogonal designs.

4.3.2.1 Real Orthogonal Designs

A real orthogonal design of size M is an M x M orthogonal matrix whose rows are permutations of real input symbols $\pm s_1, \pm s_2, \ldots, \pm s_M$. Without loss of generality, the first row can be assigned as $(s_1, s_2, \ldots, s_M)$.

The existence of real orthogonal designs for different values of M is known as the Hurwitz-Radon problem in mathematics [55]. It was shown that real orthogonal designs exist if and only if M = 2, 4, or 8.

For example, real orthogonal designs for M = 2, M = 4 and M = 8 are

$$\begin{bmatrix}\mathbf{s}_1 & \mathbf{s}_2\\ -\mathbf{s}_2 & \mathbf{s}_1\end{bmatrix},$$

$$\begin{bmatrix}\mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4\\ -\mathbf{s}_2 & \mathbf{s}_1 & -\mathbf{s}_4 & \mathbf{s}_3\\ -\mathbf{s}_3 & \mathbf{s}_4 & \mathbf{s}_1 & -\mathbf{s}_2\\ -\mathbf{s}_4 & -\mathbf{s}_3 & \mathbf{s}_2 & \mathbf{s}_1\end{bmatrix},$$

$$\begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4 & \mathbf{s}_5 & \mathbf{s}_6 & \mathbf{s}_7 & \mathbf{s}_8 \\ -\mathbf{s}_2 & \mathbf{s}_1 & \mathbf{s}_4 & -\mathbf{s}_3 & \mathbf{s}_6 & -\mathbf{s}_5 & -\mathbf{s}_8 & \mathbf{s}_7 \\ -\mathbf{s}_3 & -\mathbf{s}_4 & \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_7 & \mathbf{s}_8 & -\mathbf{s}_5 & -\mathbf{s}_6 \\ -\mathbf{s}_4 & \mathbf{s}_3 & -\mathbf{s}_2 & \mathbf{s}_1 & \mathbf{s}_8 & -\mathbf{s}_7 & \mathbf{s}_6 & -\mathbf{s}_5 \\ -\mathbf{s}_5 & -\mathbf{s}_6 & -\mathbf{s}_7 & -\mathbf{s}_8 & \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4 \\ -\mathbf{s}_6 & \mathbf{s}_5 & -\mathbf{s}_8 & \mathbf{s}_7 & -\mathbf{s}_2 & \mathbf{s}_1 & -\mathbf{s}_4 & \mathbf{s}_3 \\ -\mathbf{s}_7 & \mathbf{s}_8 & \mathbf{s}_5 & -\mathbf{s}_6 & -\mathbf{s}_3 & \mathbf{s}_4 & \mathbf{s}_1 & -\mathbf{s}_2 \\ -\mathbf{s}_8 & -\mathbf{s}_7 & \mathbf{s}_6 & \mathbf{s}_5 & -\mathbf{s}_4 & -\mathbf{s}_3 & \mathbf{s}_2 & \mathbf{s}_1 \end{bmatrix}.$$

A STBC based on real orthogonal designs of size n (n = 2, 4, or 8) can be constructed as follow. The encoder takes in a block of Mb bits. For each i, $1 \le i \le M$, the encoder select a symbol s_i from a real constellation A of size 2^b. The encoder then uses s_i to build an orthogonal matrix $C(s_1, s_2, \ldots, s_M)$ based on real orthogonal designs of size M. At time t, the M antennas transmit the t^th^ row of $C(s_1, s_2, \ldots, s_M)$.

Theorem: The diversity order of the above coding schemes is MN.

Proof: The rank criterion requires that the matrix $C(\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_M)$- $C(s_1, s_2, \ldots, s_M)$ be nonsingular for any two distinct code sequences $(\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_M) \neq (s_1, s_2, \ldots, s_M)$. Clearly,

$$\tilde{C} = C(\tilde{s}_1 - s_1, \tilde{s}_2 - s_2, \ldots, \tilde{s}_M - s_M) = C(\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_M) - C(s_1, s_2, \ldots, s_M) \quad (4.39)$$

where $\tilde{C}$ is the matrix constructed from C by replacing s_i with $\tilde{s}_i - s_i$ for all i = 1,2,...,M. The determinant of the orthogonal matrix C is easily seen to be

$$\det\left(CC^T\right)^{1/2} = \left[\sum_i x_i^2\right]^{n/2} \quad (4.40)$$

where C^T is the transpose of C. Hence

$$\det\left(\tilde{C}\tilde{C}^T\right)^{1/2} = \left[\sum_i (\tilde{s}_i - s_i)(\tilde{s}_i - s_i)^*\right]^{n/2} = \left[\sum_i |(\tilde{s}_i - s_i)|^2\right]^{n/2} \quad (4.41)$$

which is nonzero. It follows that $C(\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_M) - C(s_1, s_2, \ldots, s_M)$ is nonsingular and maximum diversity order MN is achieved.

Clearly, since the maximum rate is b bits/s/Hz and the block length is τ = M, is p, the rate of this code is defined as ratio between the actual transmitted bits and the maximum transmitted bits, which is $R = Mb/\tau b = M/M = 1$, full rate. It is shown to be optimal under the diversity order MN for the constellation size 2^b.

Not only does this create transmission schemes for any number of transmit antennas, but also generalizes the Hurwitz-Radon theory to non-square matrices, which have not full rate [8]. In this paper, generalized real orthogonal designs of size 3, 5, 6, and 7 are explicted constructed. Therefore, block codes for any number of antennae between 2 and 8 are available.

4.3.2.2 Complex Orthogonal Designs

A complex orthogonal design of size M is an M x M orthogonal matrix whose rows are permutations of complex input symbols $\pm s_1, \pm s_2, \ldots, \pm s_M$, their conjugates, or multiples of these indeterminates by $\pm\sqrt{-1}$. Without loss of generality, the first row can be assigned as $(s_1, s_2, \ldots, s_M)$.

It is shown that full rate complex orthogonal designs exist if and only if M = 2 [8]. Again, allowing linear processing of transmit signals does not increase the set of M for which the designs exist. To extend the set of M for complex orthogonal designs, generalized complex orthogonal designs are defined in an analogous fashion as generalized real orthogonal designs. However, block codes are guaranteed to exist for any value of M only for rate $R \le 1/2$. For rate $R > 1/2$, by allowing linear processing of transmit signals, block codes of rate 3/4 for $M = 3$ and 4 are shown to exist by explicit construction. The problem of complex orthogonal designs of rate $R > 1/2$ is still not well understood.

For instance, rate 1/2 codes for transmission using three and four transmit antennas are given by

$$C_3 = \begin{bmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & -s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* \\ -s_2^* & s_1^* & -s_4^* \\ -s_3^* & s_4^* & s_1^* \\ -s_4^* & -s_3^* & s_2^* \end{bmatrix} \quad \text{and} \quad C_4 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}. \qquad (4.42)$$

Note that matrices obtained first four rows of C_3 and C_4 are the same in real orthogonal designs. These transmission schemes and their analogs for higher M give full diversity but lose half of the theoretical bandwidth efficiency. Furthermore, the number of transmission slots across that the channels is required to have a constant fading envelope is eight, namely, four times higher than that of the space-time code at Eq. (4.18).

In order to increase the associated bandwidth efficiency, for $M = 3$ and 4, Tarokh [8] construct rate 3/4 generalized complex linear processing orthogonal designs given by

$$C_3^{'} = \begin{bmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{\left(-s_1 - s_1^* + s_2 - s_2^*\right)}{2} \\ \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{\left(s_2 + s_2^* + s_1 - s_1^*\right)}{2} \end{bmatrix}$$

$$C_4^{'} = \begin{bmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{\left(-s_1 - s_1^* + s_2 - s_2^*\right)}{2} & \frac{\left(-s_2 - s_2^* + s_1 - s_1^*\right)}{2} \\ \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{\left(s_2 + s_2^* + s_1 - s_1^*\right)}{2} & -\frac{\left(s_1 + s_1^* + s_2 - s_2^*\right)}{2} \end{bmatrix}. \qquad (4.43)$$

These codes are designed using the theory of amicable designs [55].

Because of orthogonality, the same procedure used for decoding the simple 2x2 STBCs can be used for this code too.

CHAPTER 5

DIFFERENTIAL SPACE-TIME BLOCK CODING

5.1 Introduction

The coherent decoding in both STTCing and STBCing requires that perfect estimates of current channel fading are available at the receiver. The channel state information (CSI) can be obtained at the receiver by sending training or pilot symbols or sequences to estimate the channel from each of the transmit antennas to the receive antenna [14–21]. However, it is not always feasible or advantageous to use these schemes, especially when many antennas are used or either end of the link is moving so fast that the channel is changing very rapidly. Since the fading rate or number of transmit antennas increases, learning CSI becomes increasingly difficult. In an effort to increase channel capacity or lower error probability, it is accepted practice to increase the number of transmitter antennas. But increasing the number of transmitter antennas increases the required training interval for learning the channel and reduces the available time in which data may be transmitted before the fading coefficients change. The number of pilot signals used to track the channel must also grow. Given a restriction on total pilot or training power, we must allocate less power per antenna with every added antenna. Finally, instability in local oscillators and phase-lock devices and inaccurate knowledge of Doppler shifts, which may be different for each antenna, may also limit channel-tracking ability at the receiver.

Motivated by these considerations, there is much interest in ST transmission schemes that do not require either the transmitter or receiver to know the channel. A natural way of dealing with unknown channels is differential modulation.

For one transmit antenna, differential detection schemes, such as DPSK, exist that neither require the knowledge of the channel nor employ pilot symbol transmission. These differential decoding schemes are used, for instance, in the IEEE IS-54 standard. It is natural to consider extensions of these schemes to multiple transmit antennas.

A partial solution to this problem was proposed in [23] for the 2x2 code, where it was assumed that the channel is not known at the receiver. In this scheme, the detected pair of symbols at time $t-T$ is used to estimate the channel at the receiver and these channel estimates are used for detecting the pair of symbols at time t. However, the scheme in [23] requires the transmission of known pilot symbols at the beginning and hence is not fully differential. The scheme in [23] can be thought of as a joint data channel estimation approach, which can lead to error propagation. In [22], a true differential detection scheme for the 2x2 code was constructed. This scheme shares many of the desirable properties of DPSK: it can be demodulated with or without CSI at the receiver, achieves full diversity gain in both cases, and there is a simple noncoherent receiver that performs within 3 dB of the coherent receiver. However, this scheme has some limitations. First, the encoding scheme expands the signal constellation for non-binary signals. Second, it is limited only to the $M = 2$ STBC for complex constellations and to the case $M \leq 8$ for real constellations. These observations are based on the results in [8] that the 2x2 STBC is an orthogonal design and complex orthogonal designs do not exist for M > 2.

In [24], another approach for differential modulation with transmit diversity based on group codes was proposed. This approach can be applied to any number of antennas and to any constellation. The group structure of these codes greatly simplifies the analysis of these schemes, and may also yield simpler and more transparent modulation and demodulation procedures.

A different differential approach to transmit diversity when the channel is not known at the receiver is reported in [25, 26] but this approach requires exponential encoding and decoding complexities.

Currently, there are many researches on differential modulation schemes such as unitary codes based on amicable orthogonal designs in [27] and double differential ST coding schemes in [28].

The basic idea behind all of these schemes is to introduce proper encoding between two consecutive code matrices so that the decoding at the receiver is independent of the underlying channels. As expected, the price paid for is code advantage in addition to 3 dB loss in SNR compared to coherent decoding.

5.2 A Simple Differential Detection Scheme for Transmit Diversity

Let us assume that we use a constellation A with 2^b-PSK for some $b = 1,2,3,\dots$, but in reality only BPSK, QPSK, and 8-PSK are of interest [30]. Thus,

$$A=\left\{\frac{e^{j2\pi k/2^b}}{\sqrt{2}}\middle|\, k=0,1,\ldots,2^b-1\right\} \tag{5.1}$$

where $j=\sqrt{-1}$.

Given a pair of 2^b-PSK constellation symbols x_1 and x_2, we first observe that the complex vectors $(x_1 \quad x_2)$ and $(-x_2^* \quad x_1^*)$ are orthogonal to each other and have unit lengths. Any two-dimensional vector $X_{34}=(x_3 \quad x_4)$ can be uniquely represented in the orthonormal basis given by $(x_1 \quad x_2)$ and $(-x_2^* \quad x_1^*)$ as

$$(x_3 \quad x_4)=A(x_1 \quad x_2)+B(-x_2^* \quad x_1^*). \tag{5.2}$$

The coefficients A and B can be given by

$$\left.\begin{array}{l} x_3=Ax_1-Bx_2^* \\ x_4=Ax_2+Bx_1^* \end{array}\right\} \Rightarrow \begin{array}{l} A=\dfrac{x_3x_1^*+x_4x_2^*}{|x_1|^2+|x_2|^2} \\ B=\dfrac{-x_3x_2+x_4x_1}{|x_1|^2+|x_2|^2} \end{array} \overset{|x_1|^2+|x_2|^2=1}{\Rightarrow} \begin{array}{l} A=x_3x_1^*+x_4x_2^* \\ B=-x_3x_2+x_4x_1 \end{array}. \tag{5.3}$$

The pair of A and B, which create X_{qk}, can be represented with a unique complex vector $P_{qk}=(A \quad B)$. All the vectors P_{qk} consist of the set v_{qk}. Since $X_{qk}\in A\times A$, the set v_{qk} has the following properties:

• *Property A:* It consists of 2^{2b} unit-length distinct vectors with P elements corresponding to the pairs of constellation symbols.

• *Property B:* The minimum distance between any two distinct elements of v_{qk} is equal to the minimum distance of the 2^b-PSK constellation A.

We define an arbitrary one-to-one mapping β shown in Eq. (5.3), which maps $2b$ bits onto v_{qk}. Note that the choice of the set v_{qk} and the mapping β is completely arbitrary as long as vectors P_{qk} are unit-length and the mapping β is one-to-one. These properties hold because the mapping β is just a change of basis from standard basis given by the set of vectors $\{(1 \quad 0),(0 \quad 1)\}$ to the orthonormal basis given by $\{(x_1 \quad x_2),(-x_2^* \quad x_1^*)\}$ which preserves the distances between the points of the two-dimensional complex space.

We start with an arbitrary vector $X_{12}\in A\times A$ and let $v=\{v_{qk}, \quad \forall v_{qk}\in A\times A\}$. $X_{12}=(1/\sqrt{2} \quad 1/\sqrt{2})$ can be chosen because the 2^b-PSK constellation A always contains the signal point $1/\sqrt{2}$. Given a block of $2b$ bits, the first b bits are mapped onto a constellation symbol x_q and the second b bits are mapped onto a constellation symbol x_k using Gray mapping. Then, P_{qk} is defined by Eq. (5.3) with $x_3 = x_q$ and $x_4 = x_k$. Conversely, given $P_{qk} = \begin{pmatrix} A_{qk} & B_{qk} \end{pmatrix}$, the pair X_{qk} can be recovered by Eq. (5.2).

5.2.1 The Encoding Algorithm for Alamoutis' Code (4.18)

The transmitter begins the transmission by sending arbitrary symbols s_1 and s_2 at time 1 and symbols $-s_2^*$ and s_1^* at time 2 which are unknown to the receiver. These two transmissions do not convey any information. The transmitter subsequently encodes the rest of data by Eq. (5.2) and Eq. (5.3).

The block diagram of the encoder is given in Figure 5.1.

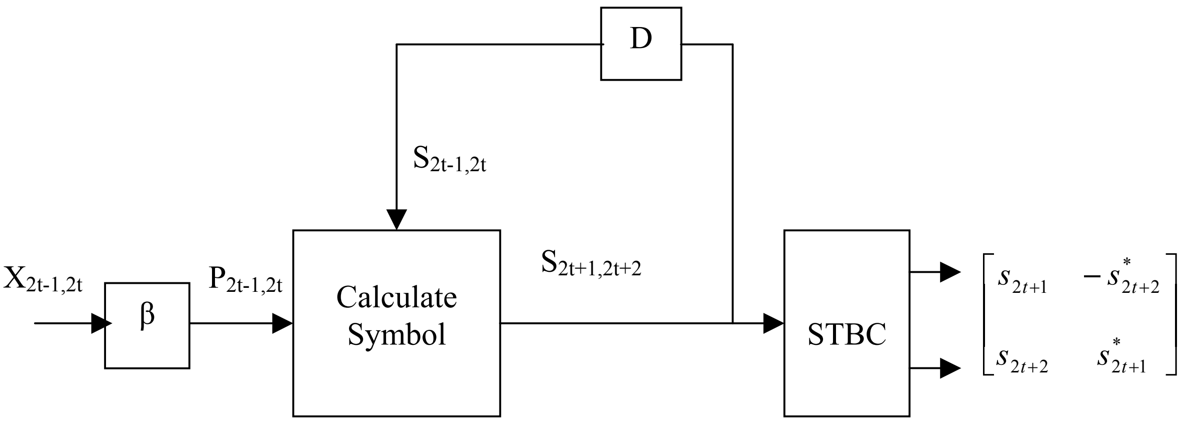

Figure 5.1 Block diagram of the encoder

Suppose that s_{2t-1} and s_{2t} are sent, respectively, from transmit antennas one and two at time $2t-1$, and that $-s_{2t}^*, -s_{2t-1}^*$ are sent, respectively, from antennas one and two at time $2t$. At time $2t+1$, a block of $2b$ bits arrives at the encoder. The transmitter uses the mapping β and computes $\beta(X_{2t-1,2t}) = P_{2t-1,2t}$ as shown in Eq. (5.4). Then the pair $S_{2t+1,2t+2}$ is recovered and transmitted like s_{2t-1} and s_{2t}. This process is inductively repeated until the end of the frame or end of the transmission represented by the set of equations:

$$\begin{aligned} A_{2t-1,2t} &= x_{2t-1}x_1^* + x_{2t}x_2^* \\ B_{2t-1,2t} &= -x_{2t-1}x_2 + x_{2t}x_1 \end{aligned} \tag{5.4}$$

$$S_{2t+1,2t+2} = \begin{pmatrix} s_{2t+1} & s_{2t+2} \end{pmatrix} = A_{2t-1,2t} \begin{pmatrix} s_{2t-1} & s_{2t} \end{pmatrix} + B_{2t-1,2t} \begin{pmatrix} -s_{2t}^{*} & s_{2t-1}^{*} \end{pmatrix} \tag{5.5}$$

5.2.2 The Differential Decoding Algorithm for Code (4.18)

We will first present the results for one receive antenna and then the ones generalized for more than one antenna. Channel coefficients are assumed to be the same for four-time interval

$$\begin{aligned} h_{11}(2t-1) = h_{11}(2t) = h_{11}(2t+1) = h_{11}(2t+2) = h_{11} \\ h_{12}(2t-1) = h_{12}(2t) = h_{12}(2t+1) = h_{12}(2t+2) = h_{12}. \end{aligned} \tag{5.6}$$

Let us assume that signals r_{2t-1}, r_{2t}, r_{2t+1} and r_{2t+2}, given as:

$$\begin{aligned} r_{2t-1} &= s_{2t-1}h_{11} + s_{2t}h_{12} + \eta_{2t-1} \\ r_{2t}^{*} &= s_{2t-1}h_{12}^{*} - s_{2t}h_{11}^{*} + \eta_{2t} \\ r_{2t+1} &= s_{2t+1}h_{11} + s_{2t+2}h_{12} + \eta_{2t+1} \\ r_{2t+2}^{*} &= s_{2t+1}h_{12}^{*} - s_{2t+2}h_{11}^{*} + \eta_{2t+2} \end{aligned} \tag{5.7}$$

are received. In order to detect which information bits $X_{2t-1,2t}$ are sent at times *2t+1* and *2t+2*, we must estimate the pair of $P_{2t-1,2t}$ using the rule:

$$\begin{aligned} A_{2t-1,2t} &= s_{2t+1}s_{2t-1}^{*} + s_{2t+2}s_{2t}^{*}, \\ B_{2t-1,2t} &= -s_{2t+1}s_{2t} + s_{2t+2}s_{2t-1}. \end{aligned} \tag{5.8}$$

This can be attained by

$$\begin{aligned} R_1 &= r_{2t+1}r_{2t-1}^{*} + r_{2t+2}^{*}r_{2t} = \left(|h_{11}|^2 + |h_{12}|^2\right) A_{2t-1,2t} + N_1, \\ R_2 &= -r_{2t-1}r_{2t+2}^{*} + r_{2t+1}r_{2t}^{*} = \left(|h_{11}|^2 + |h_{12}|^2\right) B_{2t-1,2t} + N_2. \end{aligned} \tag{5.9}$$

Thus, we can write

$$\begin{pmatrix} R_1 & R_2 \end{pmatrix} = \left(|h_{11}|^2 + |h_{12}|^2\right) P_{2t-1,2t} + \begin{pmatrix} N_1 & N_2 \end{pmatrix}. \tag{5.10}$$

The receiver now computes the closest vector of v to $\begin{pmatrix} R_1 & R_2 \end{pmatrix}$ as the solution of

$$ML = \arg\min_{v} \left\| \begin{pmatrix} R_1 & R_2 \end{pmatrix} - P_{2t-1,2t} \right\|. \tag{5.11}$$

Once this vector is computed, the inverse mapping of β is applied and the transmitted bits are recovered. The block diagram of the receiver is given in Figure 5.2.

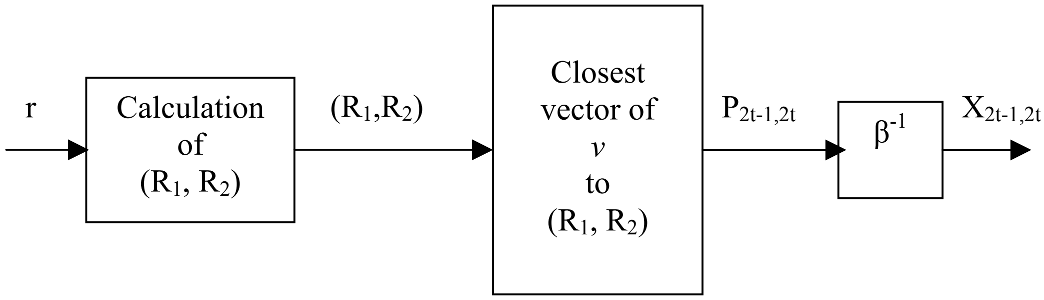

Figure 5.2 The block diagram of the differential receiver

The same procedure can be used for more than one receive antenna. For each receive antenna i, using the same method for $\left(R_1 \quad R_2\right)$ given above, $\left(R_1^i \quad R_2^i\right)$ can be computed. Then, the closest vector in v to

$$\left(\sum_{i=1}^{N} R_1^i \quad \sum_{i=1}^{N} R_2^i\right) \tag{5.12}$$

is computed. Subsequently, the transmitted bits are computed by applying the inverse mapping of β.

This technique also can be generalized to more than 2 transmit antennas employing the theory of STBCing introduced in section ST block code M transmit antennas [56].

5.3 A General Approach to Differential Space-Time Schemes

In this section, a general approach to differential modulation for multiple transmit antennas based on group codes are proposed. This approach can be applied to any number of transmit and receive antennas, and any signal constellation.

In the previous section, a differential detection scheme for M=2 transmit antennas based on Alamouti's code (4.18) is introduced. This scheme shares many of the desirable properties of DPSK. However, the scheme also has a limitation. The encoding procedure significantly expands the signal constellation for nonbinary signaling (e.g., from QPSK to 9QAM) as shown in Figure 5.3.

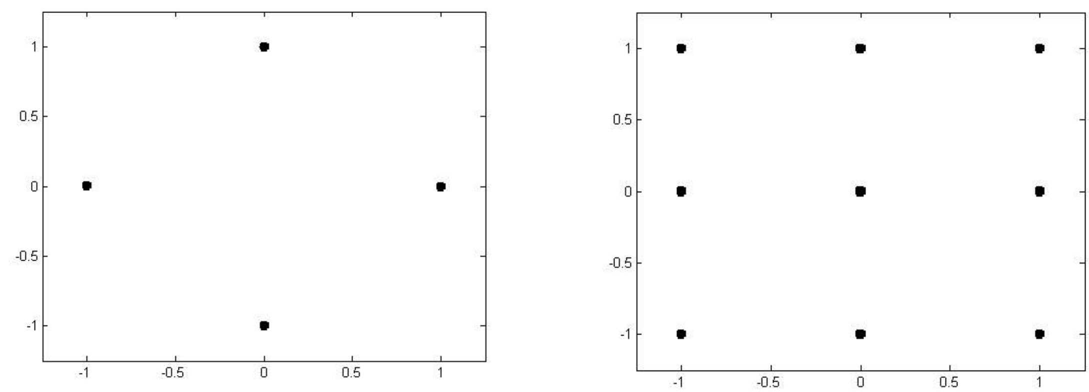

Figure 5.3 Expansion of constellation set from QPSK to 9QAM

In order to cope with this problem, the group structure, which greatly simplifies the analysis of these schemes, is proposed [14].

5.3.1 Differential Encoder

Transmitter sends $X_0 = D$ to initialize transmission where D is a M x τ matrix suppose that D is chosen such that it satisfies $DD^* = \tau I$. Then, as shown in Figure 5.4 messages are differentially encoded as

$$X_l = X_{l-1}G_l, \qquad l = 1,\ldots,L. \tag{5.13}$$

where G_l are any τ x τ unitary matrices such that $G_l G_l^* = G_l^* G_l = I$ and X_l refers to code matrices C given in previous sections with constellation *A*.

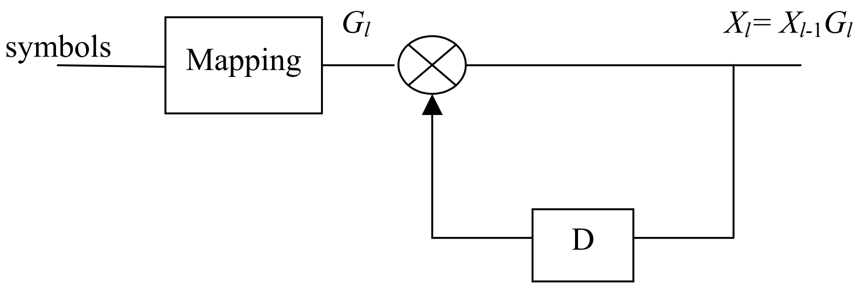

Figure 5.4 General Differential Encoder

Hughes calls the collection of matrices $DG = \{DG_l \ : \ \forall G_l\}$ a group code of length τ over constellation A [24].

The group structure ensures that $X_l \in DG$ whenever $X_{l-1} \in DG$. Moreover, the rate of the code in units of b/s/Hz is essentially

$$R = (1/\tau)\log_2|DG| \tag{5.14}$$

for large L, where $|DG|$ denotes the cardinality of DG.

We have two additional restrictions for considering the structure and performance of differentially encoded group codes. First, we assume that DG is a unitary code obeying $DG(DG)^* = \tau I$. Clearly, DG is unitary if and only if $DD^* = \tau I$ and $G_l G_l^* = G_l^* G_l = I$. Second, we assume for simplicity that $M = \tau$.

5.3.2 Differential Decoder

We now derive a receiver for differentially encoded unitary group codes Y denotes the entire received sequence given by $Y = [Y_0 \quad \cdots \quad Y_L]$, where

$$Y_l = \sqrt{\frac{E_s}{M}} H X_l + N_l \qquad l = 0,1,\ldots,L. \tag{5.15}$$

When receiver estimates G_k, it is natural to look only for the last two received blocks

$$\overline{Y}_l = [Y_{l-1} \quad : \quad Y_l]. \tag{5.16}$$

$\overline{Y}_l$ are affected by the code matrices $\overline{C}_{G_l} = [X_{l-1} \quad : \quad X_{l-1}G_l]$. Note that $DD^* = \tau I$, $D^*D = MI$ and $M = \tau$ imply that $D^*D = D\ D^* = MI$. From this, we can easily show that

$$G_l G_l^* = G_l^* G_l = I \tag{5.17}$$

which in turn leads to $X_l X_l^* = X_l^* X_l = MI$ where $X_l = DG_l$. It follows that the Mx2M matrices $\overline{C}_{G_l}$ satisfy

$$\overline{C}_{G_l}\overline{C}_{G_l}^* = [X_{l-1} \quad : \quad X_{l-1}G_l]\begin{bmatrix} X_{l-1}^* \\ (X_{l-1}G_l)^* \end{bmatrix} = 2MI \ \text{ for all } X_{l-1}. \tag{5.18}$$

If X_{l-1} were known at the receiver, the optimal decoder for this block code would be the quadratic receiver [1] characterized by the detection rule:

$$\tilde{m} = \arg\max_{m} \; p(Y|C_m) = \arg\max_{m} \; Tr\{YC_m^* C_m Y^*\} \; . \tag{5.19}$$

This equation depends only on the cross-product matrices

$$\overline{C}_{G_l}^* \overline{C}_{G_l} = \begin{bmatrix} X_{l-1}^* \\ (X_{l-1}G_l)^* \end{bmatrix} \begin{bmatrix} X_{l-1} & : & X_{l-1}G_l \end{bmatrix} = \begin{bmatrix} MI & MG_l \\ MG_l^* & MI \end{bmatrix}. \tag{5.20}$$

As these matrices do not depend on X_{l-1}, the receiver does not require knowledge of the past in order to decode the current message. The quadratic receiver in Eq. (5.21) can reduce to a simple form as shown in Eq. (5.23) following the steps

$$\tilde{G}_l = \arg \max_{D\tilde{G}_l \in DG} \; Tr\{\overline{Y}_l \overline{C}_{G_l}^* \overline{C}_{G_l} \overline{Y}_l^*\} \tag{5.21}$$

$$\begin{aligned} \overline{Y}_l \overline{C}_{G_l}^* \overline{C}_{G_l} \overline{Y}_l^* &= \begin{bmatrix} Y_{l-1} & : & Y_l \end{bmatrix} \begin{bmatrix} MI & MG_l \\ MG_l^* & MI \end{bmatrix} \begin{bmatrix} Y_{l-1} & : & Y_l \end{bmatrix}^* \\ &= M\left(Y_{l-1}Y_{l-1}^* + Y_l Y_l^* + Y_l G_l^* Y_{l-1}^* + Y_{l-1} G_l Y_l^*\right) \end{aligned} \tag{5.22}$$

Because the term of $Y_{l-1}Y_{l-1}^* + Y_l Y_l^*$ is independent of choosing G_l, $\tilde{G}_l$ can be written as

$$\tilde{G}_l = \arg \max_{D\tilde{G}_l \in DG} \; Tr\{Y_l G_l^* Y_{l-1}^* + Y_{l-1} G_l Y_l^*\} = \arg \max_{D\tilde{G}_l \in DG} \; Tr\left\{\left(Y_{l-1} G_l Y_l^*\right)^* + Y_{l-1} G_l Y_l^*\right\}$$

leading to

$$\left(Y_{l-1} G_l Y_l^*\right)^* + Y_{l-1} G_l Y_l^* = 2\,\mathrm{Re}\left(Y_{l-1} G_l Y_l^*\right) \Rightarrow \tilde{G}_l = \arg \max_{D\tilde{G}_l \in DG} \; Tr\{\mathrm{Re}\left(Y_{l-1} G_l Y_l^*\right)\} \; ,$$

and finally the simple form of the receiver is reached as

$$\tilde{G}_l = \arg \max_{D\tilde{G}_l \in DG} \; \mathrm{Re}\,Tr\left\{Y_{l-1} G_l Y_l^*\right\} = \arg \max_{D\tilde{G}_l \in DG} \; \mathrm{Re}\,Tr\left\{G_l Y_l^* Y_{l-1}\right\} \tag{5.23}$$

where last step in Eq 5.23 follows from the identity $Tr(AB) = Tr(BA)$. It is illustrated in Figure 5.5. Here, “arg” denotes the argument of the maximum and “ReTr” is the real part of the trace.

This receiver has a one-block delay. If the receiver knows both X_{l-1} and fading matrix **H**, then receiver in Eq 5.23 reduces to

$$\tilde{G}_l = \arg \max_{D\tilde{G}_l \in DG} \; \mathrm{Re}\,Tr\left\{H X_{l-1} G_l Y_l^*\right\}. \tag{5.24}$$

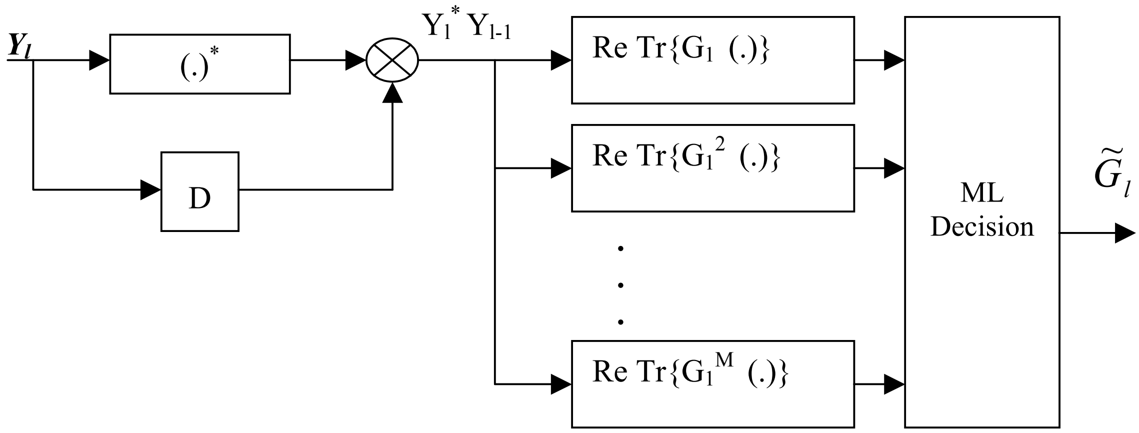

Figure 5.5 General Differential Decoder

Note that **H** can be estimated by the previous received block as

$$Y_{l-1} = \sqrt{\frac{E_s}{M}} H X_{l-1} + N_{l-1} \tag{5.25}$$

$$\tilde{H} = Y_{l-1} X_{l-1}^* = H + N_{l-1} X_{l-1}^* . \tag{5.26}$$

Thus this differential receiver has the same form as the receiver for perfect CSI and the difference are only in the quality of its channel estimates.

Generalization of differential ST schemes mentioned in the previous sections is only considered in dimension. In other words, this representation is the matrix form of the simple differential detection scheme mentioned in section 5.2.

Messages, X_l, defining in Eq. (5.13) can be shown in a similar code form to Eq. (4.18):

$$X_l = \begin{bmatrix} s_{2t+1} & -s_{2t+2}^* \\ s_{2t+2} & s_{2t+1}^* \end{bmatrix}. \tag{5.27}$$

Note that messages, X_l, are the transpose of C in Eq 4.18 introduced to simplify the notation in the subsequent derivations.

X_l can be represented using Eq 5.5 in terms of X_{l-1}:

$$\begin{bmatrix} s_{2t+1} & -s_{2t+2}^* \\ s_{2t+2} & s_{2t+1}^* \end{bmatrix} = \begin{bmatrix} s_{2t-1} & -s_{2t}^* \\ s_{2t} & s_{2t-1}^* \end{bmatrix} \begin{bmatrix} A & -B^* \\ B & A^* \end{bmatrix} \tag{5.28}$$

where **A**, **B** are defined in Eq. (5.4). In fact equation above is equal to Eq. (5.13). Thus G is obtained in terms of (**A**, **B**):

$$G = \begin{bmatrix} A & -B^* \\ B & A^* \end{bmatrix}. \tag{5.29}$$

Initial matrix can be also chosen as in section 5.2:

$$D = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \tag{5.30}$$

The general approach to differential ST schemes developed in this section shows that Tarokh's and Hughes's encoding algorithms for differential ST coding are essentially the same algorithm.

Furthermore, we can show that their decoding algorithms are also equal. But because of very complex mathematical implementations, we provide the details of this proof in Appendix 2. Briefly giving the results of Appendix 2, both approaches have the same receiver models given by:

$$ML_{TAROKH} = \arg\min_{v} \left\| \begin{pmatrix} R_1 & R_2 \end{pmatrix} - P_{2t-1,2t} \right\| = \arg\min_{(A\,B)} \left\| \begin{pmatrix} R_1 & R_2 \end{pmatrix} - \begin{pmatrix} A & B \end{pmatrix} \right\|$$

and

$$ML_{HUGHES} = \widetilde{G}_l = \arg\max_{D\widetilde{G}_l \in DG} \operatorname{Re} Tr\left\{ G_l\, Y_l^* Y_{l-1} \right\}\ . \tag{5.31}$$

5.4 Design Criteria

For large SNR, the performance of differential ST modulation is approximately 3dB worse than the performance of the block code **DG** with coherent detection. Thus, the design criterion for differential modulation is the same as the one for coherent modulation: the rank and determinant criteria of [41,1]. In [1], the pairwise error probability of this receiver, $\Pr\{C_0 \to C_1\}$ was shown to be bounded by

$$\left| I + \left(\rho/4 \right) (C_0 - C_1)(C_0 - C_1)^* \right|^{-r} \tag{5.32}$$

where I is the identity matrix and $|\,.\,|$ denotes the determinant. For large SNR (ρ), this bound behaves like $\left(\Lambda\rho/4 \right)^{-rv}$, where v is the rank of $C_0 - C_1$ and Λ is the geometric mean of the non-zero singular values of this matrix [41,1]. Thus, v can be interpreted as the diversity advantage of the code pair [41], and Λ can be interpreted as the coding advantage [1]. For codes with maximum diversity $v = t$, the coding advantage (or product distance) reduces to a simple form:

$$\Lambda_p\left(C_0, C_1 \right) = \| C_0 - C_1 \|^{1/v} \tag{5.33}$$

where $\| A \| = \left| AA^* \right|$. Since the error probability is determined mainly by determinant criterion, v, however, it is natural to focus exclusively on codes with full rank, $v = t$ (equal to the number of transmit antennas). In this case, using $DD^* = D^*D = tI$, we can express this in terms of the distance between the messages

$$\Lambda_p\left(C_{G_l}, C_{\widetilde{G}_l} \right) = \left\| C_{G_l} - C_{\widetilde{G}_l} \right\|^{1/t} = \left\| DG_l - D\widetilde{G}_l \right\|^{1/t} = t \left\| G_l - \widetilde{G}_l \right\|^{1/t},$$

$$\Lambda_p\left(C_{G_l}, C_{\widetilde{G}_l} \right) = t\Lambda_p\left(G_l, \widetilde{G}_l \right). \tag{5.34}$$

These results have important implications for the theory and design of differential ST modulation. The most important one is that the design criteria for differential ST modulation are the same as in perfect CSI at receiver: choose DG so that $v = t$ and such that Λ_p is as large as possible. From Eq. (5.34), we can clearly choose D to be any matrix that satisfies $DD^* = \tau I$. Note that Λ_p does not depend on D. Thus, the initial matrix controls only the scalar constellation of DG but the choice of D does not affect the performance.

5.5 Group Design Preliminaries

We assume familiarity with linear algebra at the level of [53] and group theory at the level of [57]. A short review on group theory can also be found in Appendix 3.

In this section, we characterize all unitary group codes with $M = \tau$, full rank, and we identify those with maximum product distance, Λ_p.

We begin some useful results on 2x2 unitary matrices. Consider the matrix

$$G = \begin{bmatrix} a & c \\ b & d \end{bmatrix}. \tag{5.35}$$

If matrix G is unitary, G must satisfy $\det(G)=\exp(j\theta)$ and $G^{*}G = I$, respectively. Under these conditions, matrix G takes the form

$$G=\begin{bmatrix} a & c \\ b & d \end{bmatrix} \Rightarrow G^{*} = G^{-1} = e^{-j\theta}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix} \xrightarrow[c=-b^{*}e^{j\theta}]{d=a^{*}e^{j\theta}} G=\begin{bmatrix} a & -b^{*}e^{j\theta} \\ b & a^{*}e^{j\theta} \end{bmatrix} \quad (5.36)$$

where coefficients a and b must also satisfy $|a|^2+|b|^2=1$ for the unitary. We say that G is a diagonal matrix if $b=0$ in Eq. (5.36), which we write as $G=diag\{a,a^{*}e^{j\theta}\}$. We say G is a off-diagonal matrix if $a=0$, and we write $G=offdiag\{b,-b^{*}e^{j\theta}\}$. Additionally, the nonzero entries in diagonal and off-diagonal unitary matrices always have unit magnitude.

As mentioned before, Tarokh's differential scheme has a limitation in the dimension issue. The encoding procedure significantly expands the signal constellation for nonbinary signaling. In Figure 5.3, we can see that constellation growing for QPSK signals where the constellation is expanded from QPSK to 9QAM.

This expansion is always obtained by transmitting some symbol couples in two consecutive time slots. Therefore, we should guarantee that the codes in DG^2 have unit energy in order to find optimal codes without constellation expansion. Because of Eq. (5.36), DG and DG^2 must be

$$G=\begin{bmatrix} a & -b^{*}e^{j\theta} \\ b & a^{*}e^{j\theta} \end{bmatrix} \Rightarrow DG=\begin{bmatrix} (a-b) & -(b^{*}+a^{*})e^{j\theta} \\ (b+a) & (a^{*}-b^{*})e^{j\theta} \end{bmatrix}$$

$$\Rightarrow DG^2=\begin{bmatrix} (a-b)a-b(b^{*}+a^{*})e^{j\theta} & -(a-b)b^{*}e^{j\theta}-a^{*}(b^{*}+a^{*})e^{j2\theta} \\ (b+a)a+b(a^{*}-b^{*})e^{j\theta} & -(b+a)b^{*}e^{j\theta}+a^{*}(a^{*}-b^{*})e^{j2\theta} \end{bmatrix}, \quad (5.37)$$

where D is chosen as Eq. (5.30).

We have five equations to obtain unit energy codes DG and DG^2:

$$|a|^2+|b|^2=1, \quad |a-b|^2=1, \quad |a+b|^2=1$$

$$\left|(a-b)a-b(b^{*}+a^{*})e^{j\theta}\right|^2=1, \qquad \left|(b+a)a+b(a^{*}-b^{*})e^{j\theta}\right|^2=1, \qquad (5.38)$$

where unknown variables are *a, b* and $\exp(j\theta)$.

Let us get $a = x + yj$, $b = z + vj$. Thus, from the first three equality in Eq. (5.38), we can find

$$x^2 + y^2 + z^2 + v^2 = 1 \text{ and } xz = -yv. \tag{5.39}$$

If $\exp(j\theta)$ is not a complex vector, it will be 1 or -1. Under this condition, group G is a special unitary group. Thus we can get

$$xz = 0 \text{ and } yv = 0 \qquad \text{for } \exp(j\theta) = 1 \; or \; -1. \tag{5.40}$$

Finally, we have three equations with four unknown variables:

$$x^2 + y^2 + z^2 + v^2 = 1,\ xz = 0 \text{ and } yv = 0 \tag{5.41}$$

Now, we analyze unitary groups in the perspective of capacity. The minimum product distance in Eq. (5.34) is

$$\Lambda_p\left(C_{G_l}, C_{\widetilde{G}_l} \right) = t \min_{G_l \neq \widetilde{G}_l} \Lambda_p\left(G_l, \widetilde{G}_l \right).$$

Every unitary ST group code is equivalent to one with initial matrix D. For this D, We can chose the minimum product distance criterion to design group codes such that

$$\Lambda_p = \min_{\widetilde{G}_l \in G, \widetilde{G}_l \neq I} \Lambda_p\left(D, D\widetilde{G}_l \right) \tag{5.42}$$

is as large as possible. Once again, $\Lambda_p > 0$ if and only if the code achieves the maximum diversity advantage with full rank. Eq. (5.42) can be reduced to a simple form:

$$\Lambda_p = \min_{\widetilde{G}_l \in G, \widetilde{G}_l \neq I} \left| D(I - G)(I - G)^* D^* \right|^{1/t} = t \min_{\widetilde{G}_l \in G, \widetilde{G}_l \neq I} \left\| I - G \right\|^{1/t}. \tag{5.43}$$

From Eq. (5.37), we have the unitary matrix G

$$G = \begin{bmatrix} x + yj & (-z + vj)e^{j\theta} \\ z + vj & (x - yj)e^{j\theta} \end{bmatrix} \Rightarrow \left\| I - G \right\| = \begin{Vmatrix} 1 - x - yj & (z - vj)e^{j\theta} \\ -z - vj & 1 - (x - yj)e^{j\theta} \end{Vmatrix} \tag{5.44}$$

For special unitary group codes, $e^{j\theta} = 1$. Hence after necessary calculations, we obtain Λ_p as:

$$\Lambda_p = 4 \max_x \left| 1 - x \right| \text{ for } M = \tau, \text{(special unitary group codes)}. \tag{5.45}$$

And finally, we can design optimum group codes which supply Eq. (5.41) and maximize Eq. (5.45).

5.6 Optimal Unitary Group Codes

For $M = \tau$, Hughes showed in [24] that all full rank unitary group codes are equivalent to either a *cyclic group code* or a *dicyclic group code* (also called dihedral). For example, if $u_m = exp(2\pi j/m)$, the matrix

$$X = \begin{bmatrix} u_m & 0 \\ 0 & u_m^k \end{bmatrix} \tag{5.46}$$

generates a cyclic group of order m, which we call the (m,k) *cyclic group*. This group supplies the design criteria given in the previous section: $a = u_m$, $b = 0$, $\theta = 2\pi(k+1)/m$. It is easily shown that the minimum distance of this group is

$$\Lambda_p = \min_{1 \le l \le m-1} 8\left|\sin(l\pi/m)\sin(lk\pi/m)\right| \tag{5.47}$$

which is positive for all odd *k*. For a given *m*, the smallest distance is $\Lambda_p = 8\sin^2(\pi/m)$, which is achieved by $k = 1$ and $k = m-1$.

These groups can often be represented in smaller constellations by using an *offdiagonal* generator. For example, the group generated by

$$X = \begin{bmatrix} 0 & u_{m/2} \\ 1 & 0 \end{bmatrix} \tag{5.48}$$

is equivalent to an $(m, m/2+1)$ cyclic group since the eigenvalues of X are $\pm u_m$. This group code takes values in the *m/*2-PSK constellation if it is used with the initial matrix in Eq. (5.30). This group is also unitary: $a = 0$, $b = 1$, $\theta = \pi(m+1)/m$.

We can also consider dicyclic (dihedral) group codes for differential ST coding schemes. These groups can always be generated by two elements. For example,

$$G = \langle X, Y \rangle = \left\langle \begin{bmatrix} u_{m/2} & 0 \\ 0 & u_{m/2}^* \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\rangle \tag{5.49}$$

is also unitary: $\theta = 2\pi$, $b = u_{m/2}$, $a = 0$.

5.7 Selection of The Matrix $\mathbf{G}_l$

In section 5.2, Tarokh uses the orthogonality property to choose the vector couple of $(A \quad B)$. The information bits are mapped to this vector couple by one-to-one mapping. Group code matrices $\mathbf{G}_l$ given in previous sections are also chosen in the same way if we use orthogonal codes as Alamouti's codes. Note that this mapping algorithm is used only in the case that the determinant of all $\mathbf{G}_l$ is equal to 1 or –1. Otherwise, another mapping algorithm can be used, but it must be more complex than orthogonal mapping.

Another approach to mapping algorithms is introduced in [27]. In that paper, in order to get G_l, the authors mapped information bits to G_l / G_{l-1} instead of G_l. Using this algorithm, they could find a new double differential receiver structure.

CHAPTER 6

PERFORMANCE RESULTS

In this section, unless otherwise stated, all simulation results are obtained over uncorrelated narrowband or nondispersive Rayleigh fading channels as mentioned in Chapter 1.

Our main assumptions are given as follows:

* The average signal power received by the each receiver antenna is the same.
* The total transmit power is always 1 for orthogonal codes.
* The receiver interference and noise are assumed to have Gaussian distribution with zero mean and variance $\sigma^2 = 0.5/\text{SNR}$ per complex dimension.
* We use independent complex Gaussian random variables with variance 0.5 per real dimension to model the path gains.
* Speed of remote unit v is assumed to be 6 km/hour.
* Operating carrier frequency f_c is assumed to be 1900 MHz.
* The fade coefficients are assumed constant over a block of transmitted code in theoretic aspects. This is a reasonable assumption given that the symbol duration T is small when compared to the speed of change in a wireless channel described by the maximum Doppler frequency f_m (As mentioned in Chapter 1, Eq. (1.10) can describe maximum Doppler frequency). Choosing 10 Hz for f_m, this assumption is almost obtained unless using the same coefficients in one block.
* We transmit 410^6 bits in order to reach a sensitivity level of 10^{-5} for bit error rate.

In the following sections, we compare the performance of various combinations of STBCs in coherent and differential systems.

In coherent systems, the information source is encoded using a space–time block code, and the constellation symbols are transmitted from different antennas. The receiver estimates the transmitted bits by using the signals of the received antennas. The theory of coherent STBCing was examined in Chapter 4. In the simulations, our main assumption is that the receiver has the perfect knowledge of the channels' fading amplitudes.

In differential systems, the information sequence is mapped onto a matrix G and then G is differentially encoded and transmitted. The receiver estimates the matrix G by using the signals of the received antennas and then the transmitted bits are obtained with the aid of an inverse mapping algorithm as described in detail in Chapter 5.

In this chapter, we present the simulation results for comparing various coherent and differential STBCs. All of the simulation results including the ones in this chapter are given in Appendix 4.

6.1 Performance Comparison of Various Coherent STBCing Systems

In this section, the performance of various STBCs with perfect CSI are investigated and compared with each other and MRRC technique. All of the investigated STBCs, namely, the C_2, C_3, C_4 codes are

$$C_2 = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix},\ C_3 = \begin{bmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & -s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* \\ -s_2^* & s_1^* & -s_4^* \\ -s_3^* & s_4^* & s_1^* \\ -s_4^* & -s_3^* & s_2^* \end{bmatrix} \text{ and } C_4 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}.$$

6.1.1 MRRC and STBC C_2

Figure 6.1 and Figure 6.2 show the bit error rate (BER) performance of MRRC and the STBC C_2 for uncoded coherent BPSK and QPSK. As mentioned before, total power received from both transmit antennas in the ST coded system using C_2 is the same as the received power in transmit of the single transmit antenna system with MRRC. It can be seen in Figure 6.1 and Figure 6.2 that the performance of the STBC C_2 with two transmitter antennas and a single receiver antenna is about 3 dB worse than two branch MRRC, even though both systems have the same diversity order of two. The 3 dB penalty is incurred because the transmit power of each antenna in ST coded arrangement using C_2 is only half of the transmit power in the MRRC assisted system in order to ensure the same total transmitted power. If each transmit antenna in STBCing with C_2 was to radiate the same energy as the single transmit antenna for MRRC, however, the performance would be identical. In other words, if the BER was drawn

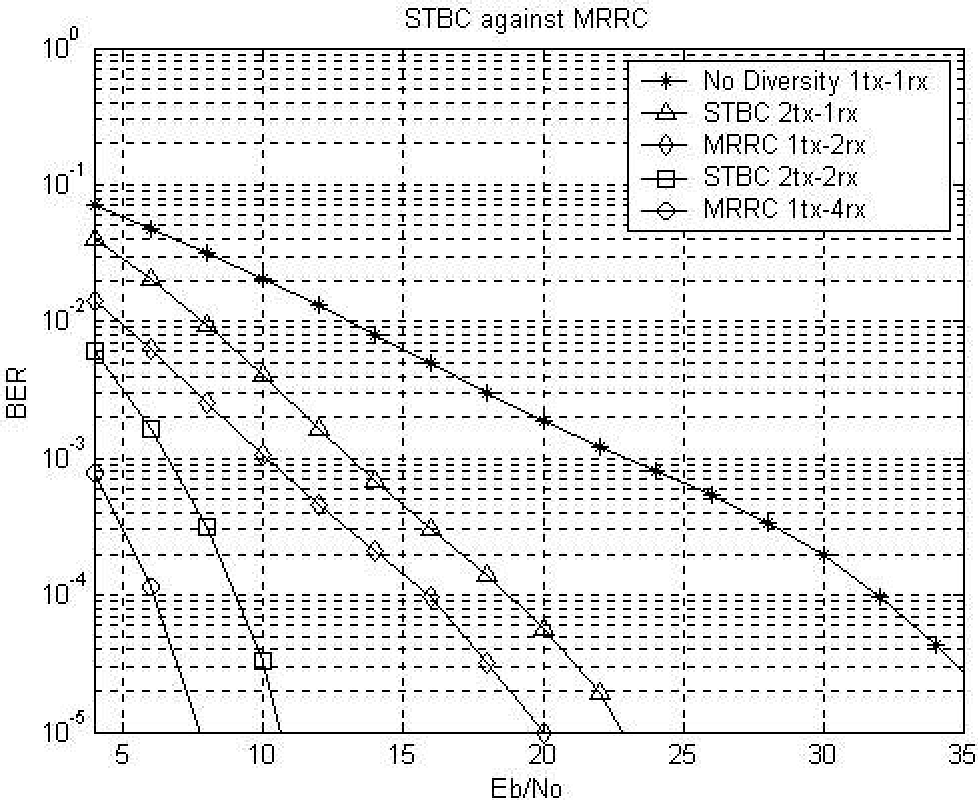

Figure 6.1 Performance comparison of the MRRC technique and STBC C_2 using BPSK over uncorrelated Rayleigh channel

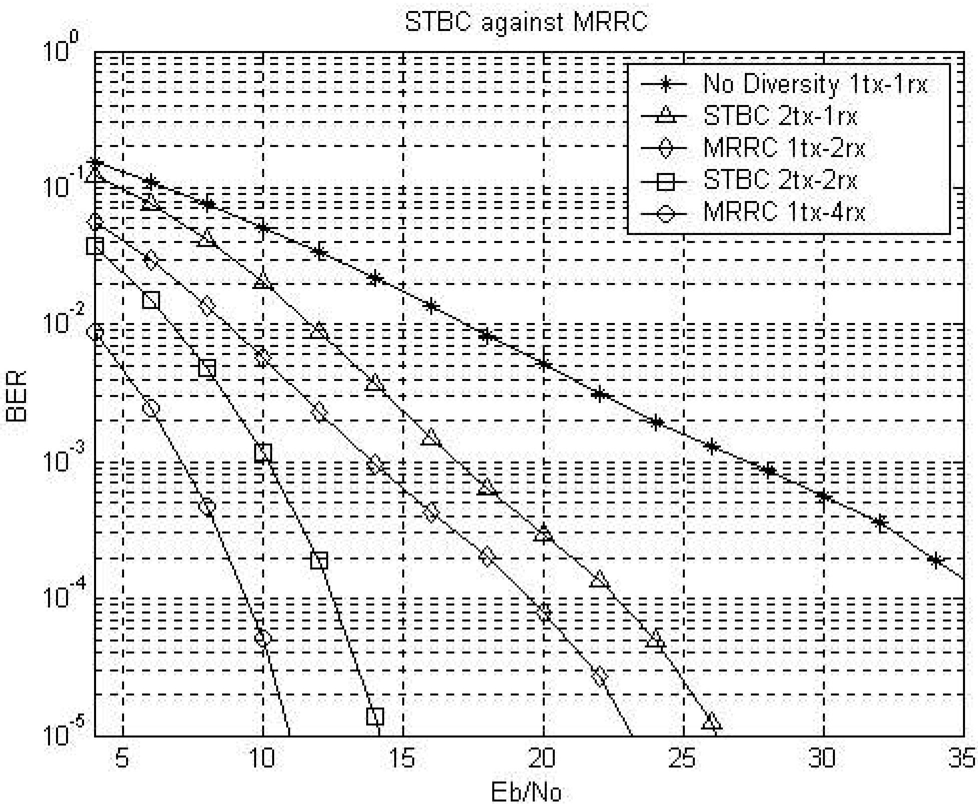

Figure 6.2 Performance comparison of the MRRC technique and STBC C_2 using QPSK over uncorrelated Rayleigh channel

against the average SNR per transmit antenna, then the performance curves for ST coded system using C_2 would shift 3 dB to the left and overlap with the MRRC curves. Nevertheless, even with the equal total received power assumption, the diversity gain for STBCing system using C_2 with one and two receive antennas at a BER of 10^{-4} is about 13 dB and 23 dB, respectively, which are 3 dB worse than MRRC with one transmit antenna and four receive antennas. The advantage of ST coded scheme is nonetheless that the increased complexity of the ST coded transmitter is more affordable at the BS than at the MS, where the MRRC system would have to be located in the receive diversity case.

6.1.2 STBCs (C_2, C_3 and C_4) with Rate One (1-bps)

Figure 6.3 compares the performances of the STBCs C_2, C_3 and C_4 having an effective throughput of 1 bps over uncorrelated Rayleigh fading channels using one receiver antenna. BPSK modulation was employed in conjunction with the STBC C_2. As mentioned before, STBCs C_3 and C_4 are half-rate codes. Therefore, QPSK modulation was used in the context of C_3 and C_4 in order to retain a throughput of 1 bps. It is seen that at the bit error rate of 10^{-4}, the rate 1/2 QPSK codes C_3 and C_4 give about 6 dB and 6.5 dB gain over the BPSK system with STBC C_2, respectively.

6.2 Performance Comparison of Various Differential STBCs

In this section, the performance of various differential STBCs are investigated and compared with each other and coherent detection techniques. All the investigated differential codes are given in Table 6.1.

Table 6.1 Cyclic codes for differential schemes

TAROKH	HOCHWALD	HUGHES
for BPSK	for BPSK	for BPSK
$\left\langle \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\rangle$, (R = 1)	$\left\langle \begin{bmatrix} j & 0 \\ 0 & j \end{bmatrix} \right\rangle$, (R = 1)	$\left\langle \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\rangle$, (R = 0.5)
for QPSK	for QPSK	for QPSK
............(*)	$\left\langle \begin{bmatrix} e^{(j\pi/8)} & 0 \\ 0 & e^{(j7\pi/8)} \end{bmatrix} \right\rangle$, (R = 2)	$\left\langle \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\rangle$, (R = 1.5)

(*) Tarokh's code for QPSK is not a group code.

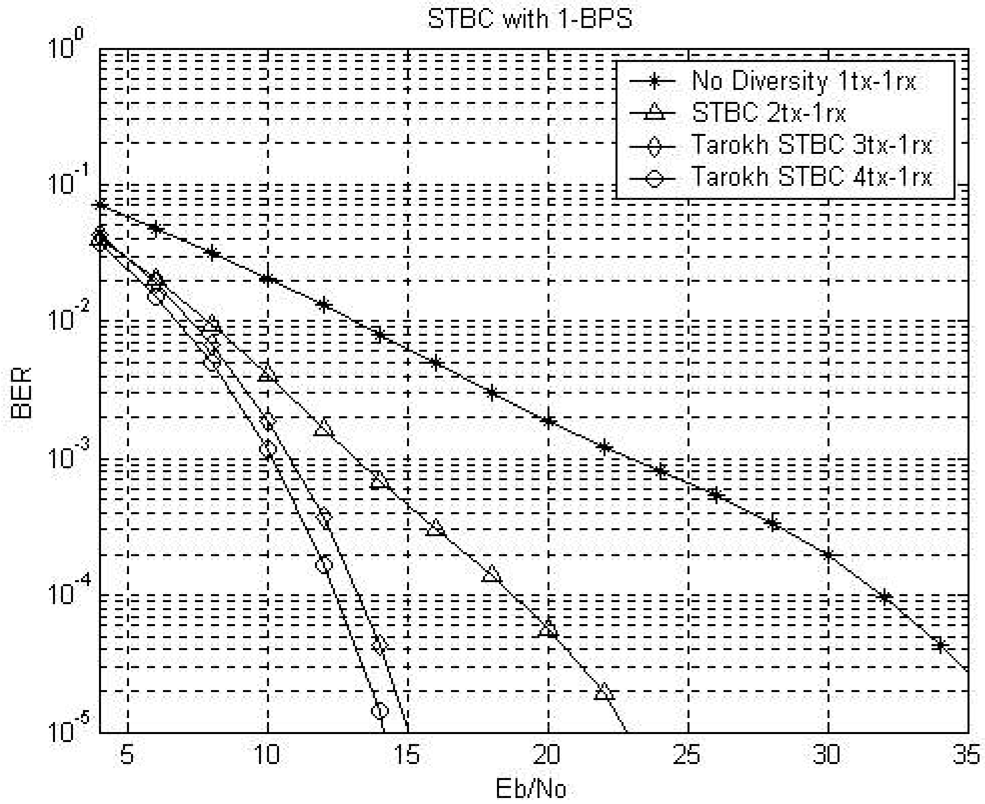

Figure 6.3 Performance comparison of the STBCs C_2, C_3 and C_4 having an effective throughput of 1 bps over uncorrelated Rayleigh fading channels

6.2.1 Differential and Coherent Detection for STBC C_2

The performance of the proposed differential detection scheme in Section 5.2 is 3 dB worse than that of the transmit diversity scheme of Alamouti (which employs coherent detection) at high signal-to-noise power ratios. In order to clarify the reason for this 3 dB loss let us expand the noise terms N_1 and N_2 in Eq. (5.9) as follows:

$$\begin{aligned} N_1 &= \begin{pmatrix} s_{2t+1} & s_{2t+2} \end{pmatrix} H\, N_{2t-1}^* + N_{2t+1}\, H^* \begin{pmatrix} s_{2t-1} & s_{2t} \end{pmatrix}^* + N_{2t+1} N_{2t-1}^* \\ N_2 &= \begin{pmatrix} s_{2t+1} & s_{2t+2} \end{pmatrix} H\, N_{2t}^* + N_{2t+1}\, H^* \begin{pmatrix} -s_{2t}^* & s_{2t}^* \end{pmatrix}^* + N_{2t+1} N_{2t}^* \end{aligned} \tag{6.1}$$

where $H = \begin{pmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{pmatrix}$ and $N_k = \begin{pmatrix} \eta_k & \eta_{k+1}^* \end{pmatrix}$.

Noting that the multiplicative term in N_1 can be ignored at high signal-to-noise power ratios, as it is much smaller compared to the other terms. Similarly, the term in N_2 can be ignored at high SNR. The two remaining terms in N_1 and N_2 double the power of noise in Eq. (5.9) as compared to the coherent detection. This doubling of the noise power is equivalent to the aforementioned 3 dB loss.

In Figure 6.4, we present the BER performance as a function of the bit energy per noise power spectral density E_b/N_0 for both of the coherent and differential detection methods with the two transmit diversity systems. Only the results for BPSK modulation are presented because the constellation set expands for other large M-ary PSK. The 3 dB loss due to differential detection can also be observed from these simulation results.

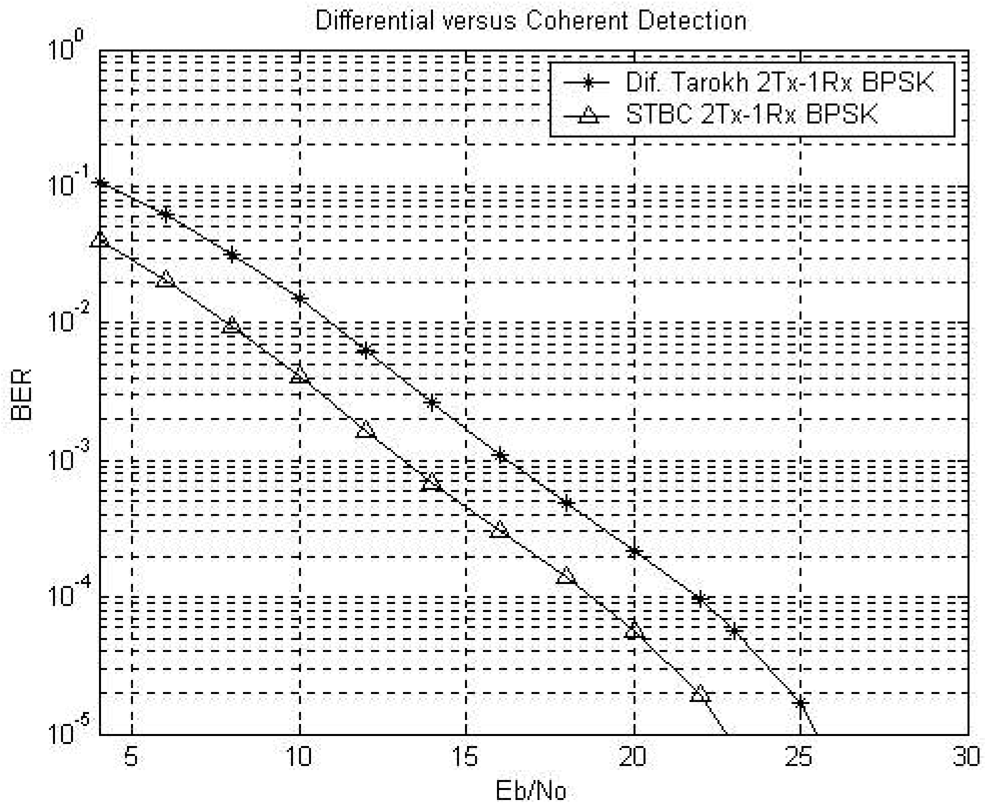

Figure 6.4 Performance comparison of differential detection and coherent detection for BPSK constellation in the system with two transmit and one receive antennas.

6.2.2 Comparison of Chosen STBCs by Hochwald, Hughes & Tarokh

Fig. 6.5 shows the performance of the rate-1 space–time block code in differential detection scheme for BPSK. As Figure 6.4 clearly depicts, noncoherent detection performs about 3 dB worse than coherent detection as expected. We have also included the results for the unitary space–time modulation scheme of Hochwald and Sweldens from [9] in order to compare rate-1 differential codes. As it can be seen in the figure, Tarokh's differential detection scheme performs more than 1 dB better than the code in table 6.1 at a bit error rate equal to 10^{-4}. The 1 dB loss is due to the codes used in [9] which have not full rank as the ones in [22].

As mentioned before, Hughes shows that at rates larger than 1, Tarokh's scheme has a limitation. Thus we provide Hughes' some other optimal unitary group codes at different rates with the corresponding simulation results. Since these codes have different rates, they are not comparable with other full rate codes.

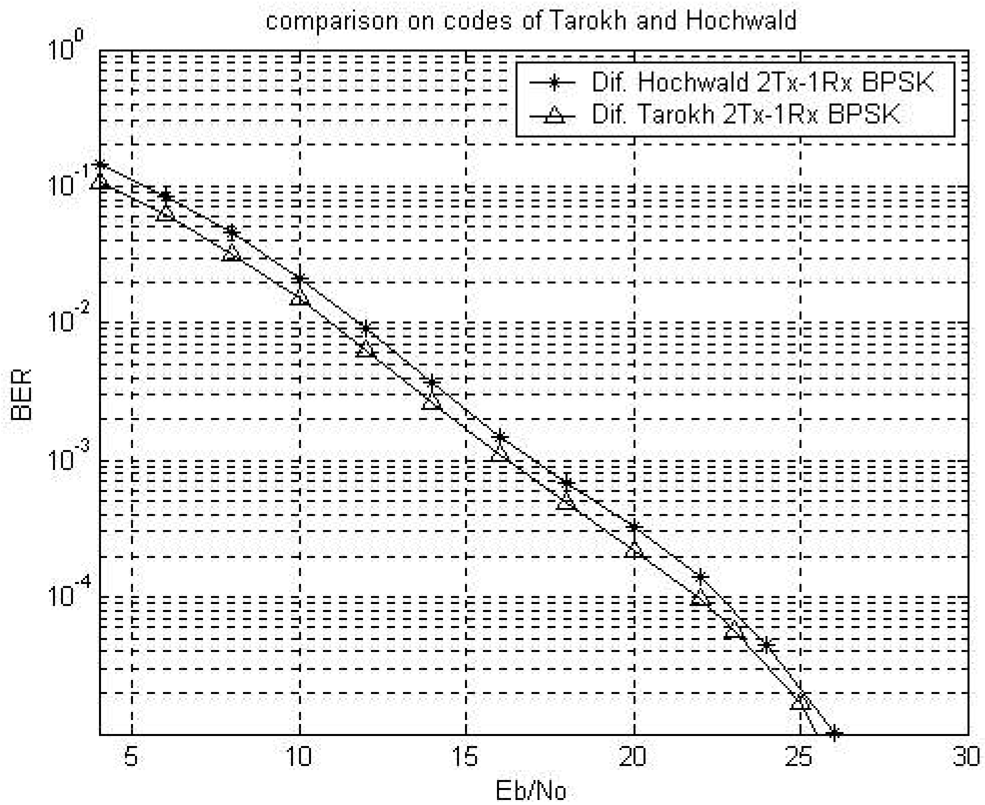

Figure 6.5 Performance comparison of different codes over general differential detection scheme using BPSK with full rate

6.3 Discussion on Coherent STBCing

The STBCing scheme requires the simultaneous transmission of M different symbols out of M antennas. If the system is radiation power limited, in order to have the same total radiated power from M transmit antennas the energy allocated to each symbol should be divided by M. This results in some dB penalty in the error performance. However, the corresponding reduction of power in each transmit chain translates to cheaper, smaller, or smaller number of linear power amplifiers. Moreover, if the limitation is only due to RF power handling (amplifier sizing, linearity, etc.), then the average radiated power from each antenna may be the same as it is in the one transmit antenna case. Thus no performance penalty is incurred.

In coherent studies, it is assumed that the receiver has perfect knowledge of the channel. The CSI may be derived by pilot symbol insertion and extraction. Known symbols are transmitted periodically from the transmitter to the receiver. The receiver extracts the samples and interpolates them to construct an estimate of the channel for every data symbol transmitted. In the coherent transmit diversity schemes discussed in this thesis, M times as many pilots as in the M.N-branch receiver combining scheme are needed.

In receive diversity combining schemes, if one of the receive chains fail, and the other receive chain is operational, then the performance loss is on the order of the diversity gain. In other words, the signal may still be detected, but with inferior quality. This is commonly referred to as soft failure. Fortunately, the new transmit diversity scheme provides the same soft failure [7].

6.4 Discussion on Differential STBCing

As mentioned in the previous section, in coherent studies, it is assumed that the receiver has perfect knowledge of the channel. The CSI may be derived by pilot symbol insertion and extraction in slowly changing channels. But measurements show that the statistics of wireless channels are highly variant and finding a general model that holds in all scenarios seems to be a very difficult if not an impossible task. Therefore, there may be scenarios or applications where coherent detection is not probable.

This motivates researchers to consider transmit diversity schemes where neither the transmitter nor the receiver requires channel state information. Differential (noncoherent) detection is a scheme where channel estimation is not needed.

Last three years, many papers are published on differential detection schemes. But, so far, in differential detection, full rate codes have not been found except for the BPSK constellation set using unitary designs although they exist in coherent detection.

CHAPTER 7

CONCLUSIONS AND SUGGESTIONS FOR THE FUTURE RESEARCH

ST coding is a new coding/signal-processing framework for wireless communication systems with multiple transmit and multiple receive antennas. This framework has the potential of dramatically improving the capacity and data rates.

ST codes designed so far come in two different types. STTCs offer the maximum possible diversity gain without any sacrifice in the transmission bandwidth. When the numbers of transmit antennas is fixed, the decoding complexity of STTCing (measured by the number of trellis states in the decoder) increases exponentially as a function of both the diversity level and the transmission rate. STBCs offer a much simpler way of obtaining transmit diversity without any sacrifice in bandwidth and without requiring huge decoding complexity. In fact, the structure of STBCing is such that it allows for very simple signal processing (linear combining) for encoding/decoding and differential encoding/detection.

In this thesis, we provided examples of STBCs for transmission using multiple transmit antennas. We described both their encoding and decoding algorithms. The encoding and decoding of these codes have very little complexity. Simulation results were provided to demonstrate that significant gains can be achieved by increasing the number of transmit chains with very little decoding complexity. Because of superior performance and low complexity at the receiver-end, STBCing is an attractive solution especially for downlink applications.

In coherent systems, it is shown that using two transmit antennas and one receive antenna, STBCs provide the same diversity order as MRRC with one transmit and two receive antennas. It is further shown that the scheme may easily be generalized to two transmit antennas and N receive antennas to provide a diversity order of $2N$. An obvious application of the scheme is to provide diversity improvement at all the remote units in a wireless system, using two transmit antennas at the base stations instead of two receive antennas at all the remote terminals. The scheme does not require any feedback from the receiver to the transmitter and its computation complexity is similar to MRRC. When compared with MRRC, if the total radiated power is to remain the same, the transmit diversity scheme has a 3-dB disadvantage because of the simultaneous transmission of two distinct symbols from two antennas. Otherwise, if the total radiated power is doubled, then its performance is identical to MRRC. Moreover, assuming equal radiated power, the scheme requires two half-power amplifiers compared to one full power amplifier for MRRC, which may be advantageous for system implementation. STBCing also requires twice the number of pilot symbols for channel estimation when pilot insertion and extraction is used.

For more than two antennas, it is shown that as we increased the effective throughput of the system, the performance of the half-rate ST codes C_3 and C_4 degraded in comparison to that of the unity rate ST code C_2. This was because higher modulation schemes had to be employed in conjunction with the half-rate ST codes C_3 and C_4 in order to maintain the same effective throughput; these codes are weaker to errors and hence the performance of the system degrades.

We have also presented a differential detection transmit diversity method. The transmission scheme exploits the diversity given by multiple transmit antennas when neither the transmitter nor the receiver knows the channel. We summarize the existing results for this scenario: Transmission schemes that approach the problem using differential detection were first proposed in [22] and independently using another approach in [24]. These were then extended in [25, 56].

This motivates us to consider the generalization of differential unitary STBCing in this thesis. All of constructions with the two transmit antenna differential detection schemes are generalized for unitary group codes.

The following is a brief discussion of possible avenues for future research. In this thesis, the signal constellations used to test the robustness of STBCs against non-dispersive channels were BPSK and QPSK. M-ary Phase Shift Keying using values of M other than two and four should be tested but they will most likely confirm to the results already generated to be fairly consistent for PSK with any value of M. It would be more beneficial to test the codes with several different types of modulation, such as Quadrature Amplitude Modulation (QAM), Frequency Shift Keying (FSK), or Minimum Shift Keying.

In this thesis, it was assumed that the fading that occurred over each transmitter-receiver pair was independent. In a physical system it may not be possible to place the antennas at the minimum required distance apart, thus leading to correlated fading between different antennas. For this reason, the performance of the systems should be shown under the condition that correlation exists between the different fading coefficients.

Future research should also focus on iterative decoding and estimation. This is a process where pilot symbols are used to estimate the channel, and then decoding is done using the channel estimates. The decoded data is then fed back into the estimator and the data is used as pilot symbols to refine the estimate of the channel. The refined estimate is then used to decode the data a second time. This process can be repeated for any number of iterations until the performance reaches a desirable level. This idea leads naturally to the concept of turbo space-time codes.

As mentioned before, in differential detection, full rate codes have not been found except for the BPSK constellation set using unitary designs although they exist in coherent detection. For this reason, future research should also focus on full rate codes realizations of order modulation schemes.

APPENDIX 1

Derivation of an Upper Bound on the Average Pairwise Error Probability

The probability of transmitting C and deciding in favor of $\tilde{C}$ is well upper bounded by [30]

$$P(C \to \tilde{C} \mid H(l), l = 1,\ldots,L) = Q\left(\sqrt{\frac{D^2(C,\tilde{C})E_S}{2N_o}}\right) \le \exp(-D^2(C,\tilde{C}) \cdot E_S / 4N_o)$$

From Eq. (3.8), this probability function is simplified as follows:

$$D^2\left(C,\tilde{C}\right) = \sum_{j=1}^{M}\sum_{i=1}^{N} \lambda_j \left|\beta_{ji}\right|^2 \Rightarrow P(C \to \tilde{C} \mid \beta_{ji}) \le \exp\left(-\left(E_S / 4N_o\right) \cdot \sum_{j=1}^{M}\sum_{i=1}^{N} \lambda_j \left|\beta_{ji}\right|^2\right)$$

$$P(C \to \tilde{C} \mid \beta_{ji}) \le \prod_{j=1}^{M}\prod_{i=1}^{N} \exp\left(-\left(E_S / 4N_o\right) \cdot \lambda_j \left|\beta_{ji}\right|^2\right). \tag{A1.1}$$

β_{ji} are independent complex Gaussian random variables with zero mean and variance *0.5* per dimensions. Thus $\left|\beta_{ji}\right|$ are independent random variables with Rayleigh distribution

$$p\left(\left|\beta_{ji}\right|\right) = \frac{\left|\beta_{ji}\right|}{\sigma^2} e^{-\left|\beta_{ji}\right|^2 / 2\sigma^2}, \quad \left|\beta_{ji}\right| \ge 0 \quad [30]$$

and $\sigma^2 = 1/2 \Rightarrow p\left(\left|\beta_{ji}\right|\right) = 2 \cdot \left|\beta_{ji}\right| \cdot e^{-\left|\beta_{ji}\right|^2}$ (A1.2)

or the random variable $\nu_{ji} = \left|\beta_{ji}\right|^2$ has a χ^2 distribution with two degrees of freedom, that is

$$p(\nu_{ji}) = \frac{1}{\sigma^n 2^{n/2} \Gamma(\frac{1}{2}n)} \nu_{ji}^{\,n/2-1} e^{-\nu_{ji}/2\sigma^2}, \quad \nu_{ji} \ge 0 \quad [30],$$

$$n = 2,\ \sigma^2 = 1/2 \text{ and } \Gamma(1) = 1 \quad \Rightarrow \quad p(\nu_{ji}) = e^{-\nu_{ji}} \tag{A1.3}$$

Thus to compute a bound on the probability of error, we average the right side of Eq. (A1.1) with respect to Eq. (A1.2) or Eq. (A1.3) (We can choose each one to arrive at Eq. (3.10), in here we are using Eq. (A1.2).

For simplify, let x = $|\beta_{ji}|$ and A_j = $(E_S/4N_o)\cdot\lambda_j$

In the general case, suppose that x_i are the real roots of the equation y = g (x). Then the pdf of the random variable $Y = g\,(X)$ may be expressed as $p(y) = \sum_{i=1}^{n} \frac{p(x_i)}{|g'(x_i)|}$

From Eq. (A1.1), y = exp ($-A_j$. x^2) $\Rightarrow \quad x_1 = \sqrt{-\frac{1}{A_j}\ln(y)}\,,\quad x_2 = -\sqrt{-\frac{1}{A_j}\ln(y)}$

But $|\beta_{ji}| \geq 0 \quad \Rightarrow \quad x \geq 0$ and x_2 is not a root of y. Thus from Eq. (A1.2)

$$p(x) = 2xe^{-x^2} \text{ and } -x^2 = \frac{1}{A_j}\ln(y)\,,$$

$$g'(x\ ) = -2A_j xy = -2A_j y\sqrt{-\frac{1}{A_j}\ln(y)}\,,$$

$$p(y) = \frac{2\left(\sqrt{-\frac{1}{A_j}\ln(y)}\right)\cdot e^{\frac{1}{A_j}\ln(y)}}{-2A_j y\sqrt{-\frac{1}{A_j}\ln(y)}} = \frac{-1}{A_j y}y^{1/A_j}\,. \tag{A1.4}$$

The *average of y* is defined by [30] as

$$E(Y) = \int_{-\infty}^{\infty} y\cdot p(y)\cdot dy \tag{A1.5}$$

From Eq. (A1.5) with Eq. (A1.4) and $x \in [0,\infty) \Rightarrow y \in [1,0)$

$$E(Y) = \int_{1}^{0}\frac{-1}{A_j}y^{1/A_j}dy = \left(\frac{1}{A_j}\cdot y^{1/A_j+1}\cdot\left(\frac{1}{A_j}+1\right)^{-1}\right)\Bigg|_0^1 = 1/\left(1+A_j\right) \tag{A1.6}$$

And finally an upper bound on the average pairwise error probability is found as:

$$P(C\to\widetilde{C}) \leq \prod_{j=1}^{M}\prod_{i=1}^{N}\frac{1}{1+\lambda_j\cdot(E_S/4N_0)} = \left(\prod_{i=1}^{N}\frac{1}{1+\lambda_j\cdot(E_S/4N_0)}\right)^M.$$

APPENDIX 2

Proof Of Equality For Two Receiver Models In Eq. (5.31)

Tarokh's receiver model:

From Section 5.2, we have

$$\begin{aligned} R_1 &= r_{2t+1} r_{2t-1}^* + r_{2t+2}^* r_{2t} = \left(|h_{11}|^2 + |h_{12}|^2\right) A_{2t-1,2t} + N_1 \\ R_2 &= -r_{2t-1} r_{2t+2}^* + r_{2t+1} r_{2t}^* = \left(|h_{11}|^2 + |h_{12}|^2\right) B_{2t-1,2t} + N_2 \end{aligned} \qquad \text{(A2.1)}$$

$$ML_{TAROKH} = \arg\min_{(A\ B)} \left\| \begin{pmatrix} R_1 & R_2 \end{pmatrix} - \begin{pmatrix} A & B \end{pmatrix} \right\|.$$

$$ML = \arg\min_{(A\ B)} \sqrt{\left(\mathrm{Re}(R_1 - A)\right)^2 + \left(\mathrm{Im}(R_1 - A)\right)^2 + \left(\mathrm{Re}(R_2 - B)\right)^2 + \left(\mathrm{Im}(R_2 - B)\right)^2} \qquad \text{(A2.2)}$$

We can easily see that some terms in Eq. (A2.2) are taken out of the expression for ML criterion.

First term is $\left(\mathrm{Re}(R_1)\right)^2 + \left(\mathrm{Re}(R_2)\right)^2 + \left(\mathrm{Im}(R_1)\right)^2 + \left(\mathrm{Im}(R_2)\right)^2$. This term can be omitted because it has always a positive value or it is zero and it is also independent from $\begin{pmatrix} A & B \end{pmatrix}$.

Second term is $\left(\mathrm{Re}(A)\right)^2 + \left(\mathrm{Re}(B)\right)^2 + \left(\mathrm{Im}(A)\right)^2 + \left(\mathrm{Im}(B)\right)^2$. This term can be also omitted because its value is equal to 1 from the unity property of $\begin{pmatrix} A & B \end{pmatrix}$.

Thus we have a simple form of ML_{TAROKH}:

$$ML = \arg\min_{(A\ B)} \left[-2\left(\mathrm{Re}(R_1)\mathrm{Re}(A) + \mathrm{Im}(R_1)\mathrm{Im}(A) + \mathrm{Re}(R_2)\mathrm{Re}(B) + \mathrm{Im}(R_2)\mathrm{Im}(B)\right)\right]$$

$$ML = \arg\max_{(A\ B)} \left(\mathrm{Re}(R_1)\mathrm{Re}(A) + \mathrm{Im}(R_1)\mathrm{Im}(A) + \mathrm{Re}(R_2)\mathrm{Re}(B) + \mathrm{Im}(R_2)\mathrm{Im}(B)\right). \qquad \text{(A2.3)}$$

Hughes's receiver model:

From Section 5.2, we have

$$ML_{HUGHES} = \widetilde{G}_l = \arg \max_{D\widetilde{G}_l \in DG} \ \mathrm{Re}\, Tr\left\{G_l\, Y_l^* Y_{l-1}\right\}.$$

As mentioned in Section 5.3, Tarokh's and Hughes's encoding algorithms for differential space-time coding are equal.

Let $Y_l = \begin{pmatrix} r_{2t+1} & r_{2t+2} \end{pmatrix}$ and $Y_{l-1} = \begin{pmatrix} r_{2t-1} & r_{2t} \end{pmatrix}$. Thus we can give ML_{HUGHES} estimator in terms of Tarokh's representation:

$$Y_l^* Y_{l-1} = \begin{bmatrix} r_3^* r_1 & r_3^* r_2 \\ r_4^* r_1 & r_4^* r_2 \end{bmatrix} \quad \Rightarrow \quad G_l\, Y_l^* Y_{l-1} = \begin{bmatrix} A & -B^* \\ B & A^* \end{bmatrix} \begin{bmatrix} r_3^* r_1 & r_3^* r_2 \\ r_4^* r_1 & r_4^* r_2 \end{bmatrix},$$

$$\operatorname{Re} Tr\left\{G_l\, Y_l^* Y_{l-1}\right\} = \operatorname{Re}\left(r_3^* r_1 A + r_4^* r_2 A^* + r_3^* r_2 B - r_4^* r_1 B^*\right). \tag{A2.4}$$

From Eq. (A2.4), we can easily show that

$$\operatorname{Re}\left(r_3^* r_1 A + r_4^* r_2 A^*\right) = \operatorname{Re}\left(r_3^* r_1 A\right) + \operatorname{Re}\left(r_4^* r_2 A^*\right) = \operatorname{Re}\left(A^* r_1^* r_3\right) + \operatorname{Re}\left(r_4^* r_2 A^*\right),$$

$$\operatorname{Re}\left(r_3^* r_2 B - r_4^* r_1 B^*\right) = \operatorname{Re}\left(r_3^* r_2 B\right) - \operatorname{Re}\left(r_4^* r_1 B^*\right) = \operatorname{Re}\left(B^* r_3 r_2^*\right) - \operatorname{Re}\left(r_4^* r_1 B^*\right). \tag{A2.5}$$

Using Eq. (A2.1), we can simplify the equations in Eq. (A2.5) as follows:

$$\operatorname{Re}\left(A^* r_1^* r_3\right) + \operatorname{Re}\left(r_4^* r_2 A^*\right) = \operatorname{Re}\left(A^*\left(r_1^* r_3 + r_4^* r_2\right)\right) = \operatorname{Re}\left(A^* R_1\right),$$

$$\operatorname{Re}\left(B^* r_3 r_2^*\right) - \operatorname{Re}\left(r_4^* r_1 B^*\right) = \operatorname{Re}\left(B^*\left(r_3 r_2^* - r_4^* r_1\right)\right) = \operatorname{Re}\left(B^* R_2\right). \tag{A2.6}$$

And finally we put Eq. (A2.6) into Eq. (A2.4):

$$\operatorname{Re}\left(A^* R_1\right) + \operatorname{Re}\left(B^* R_2\right) = \operatorname{Re}(R_1)\operatorname{Re}(A) + \operatorname{Im}(R_1)\operatorname{Im}(A) + \operatorname{Re}(R_2)\operatorname{Re}(B) + \operatorname{Im}(R_2)\operatorname{Im}(B),$$

$$\operatorname{Re} Tr\left\{G_l\, Y_l^* Y_{l-1}\right\} = \operatorname{Re}(R_1)\operatorname{Re}(A) + \operatorname{Im}(R_1)\operatorname{Im}(A) + \operatorname{Re}(R_2)\operatorname{Re}(B) + \operatorname{Im}(R_2)\operatorname{Im}(B).$$

Thus the same receiver model is obtained like Eq. (A2.3):

$$ML = \arg\max_{(A\,B)}\left(\operatorname{Re}(R_1)\operatorname{Re}(A) + \operatorname{Im}(R_1)\operatorname{Im}(A) + \operatorname{Re}(R_2)\operatorname{Re}(B) + \operatorname{Im}(R_2)\operatorname{Im}(B)\right). \tag{A2.7}$$

APPENDIX 3

A Short Review on Group Theory

A **group** is a set G together with a multiplication on G which satisfies three axioms:

(a) The multiplication is associative that is to say $(xy)z = x(yz)$ for any three (not necessarily distinct) elements of G.

(b) There is an element e in G, called an identity element, such that $xe = x = ex$ for every x in G.

(c) Each element x of G has an inverse x^{-1} which belongs to the set G and satisfies $x^{-1}x = e = xx^{-1}$.

A group is commutative, or **Abelian**, if $xy = yx$ for any two of its elements.

The **order** *(cardinality)* of a finite group is the number of elements in the group. A group that contains infinitely many elements is said to have an *infinite order*. Mathematicians usually write $|G|$ for the order of group G. If x is an element of a group, and if $x^n = e$ for some positive integer n, then we say that x has a finite order, and the smallest positive integer m such that $x^m = e$ is called the order of x. Otherwise x has an infinite order.

A **subgroup** of a group G is a subset of G which itself forms a group under the multiplication of G.

An nxn matrix A is **orthogonal** if $A^T A$ is the identity matrix. The product of two orthogonal matrices is another orthogonal matrix. In addition, the inverse of an orthogonal matrix is an orthogonal matrix, as is the identity matrix. Hence the set of orthogonal matrices form a group, called the **orthogonal group**. An orthogonal group that has a determinant equal to +1 or –1 is called the **special orthogonal group**.

A square matrix A is a **unitary matrix** if $A^{*} = A^{-1}$, where A^{*} denotes the adjoint matrix and A^{-1} is the matrix inverse. The definition of a unitary matrix guarantees that $A^{*}A = I$, where I is the identity matrix. Note that transpose is a much simpler computation than inverse. For real matrices, unitary is the same as orthogonal. Also, the norm of the determinant of A is $|\det A| = 1$. If $\det A = 1$ then A is a **special unitary matrix**. The product of two unitary matrices is another unitary matrix. The inverse of a unitary matrix is another unitary matrix, and identity matrices are unitary. Hence the set of unitary matrices form a group, called the **unitary group**.

The **adjoint matrix**, sometimes also called the adjugate matrix or conjugate transpose, of an mxn matrix A is the nxm matrix defined by $A^{*} = \overline{A}^{T}$, where the adjoint operator is denoted with a star, $(.)^{T}$ denotes the transpose, and $\overline{A}$ denotes the conjugate operation.

One very common type of group is the cyclic group. If there is an element x in G that generates all of G (in other words, for which $\langle x \rangle = G$), we say that G is a **cyclic group**. *n*th powers of *x* must be the identity matrix and *n* is the order of G. Cyclic groups are also Abelian.

Let m be an integer which is greater than or equal to 2. $4m$ elements form a group $G = \{e, x, x^{2}, \ldots, x^{2m-1}, y, xy, x^{2}y, \ldots, x^{2m-1}y\}$ where $x^{2m} = e$, $y^{2} = e$ and $yx = x^{2m-1}y$. Mathematicians call G the **dicyclic group** *(dihedral group)* of order 4*m*. Dicyclic groups are not commutative.

The quaternion can be represented using complex $2x2$ matrices like Eq. (5.36). While the quaternions are not commutative, they are associative, and they form a group known as the **quaternion group**.

APPENDIX 4

All of The Simulation Results

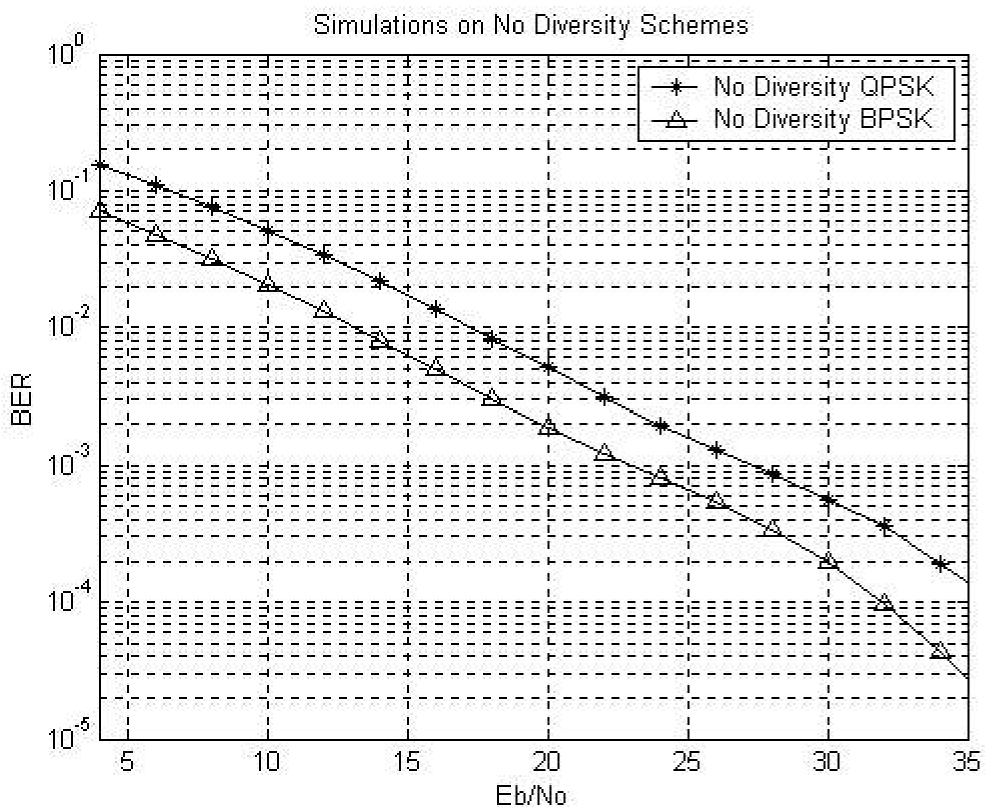

Figure A4.1 Performance curves of no diversity scheme using BPSK and QPSK over uncorrelated Rayleigh channel

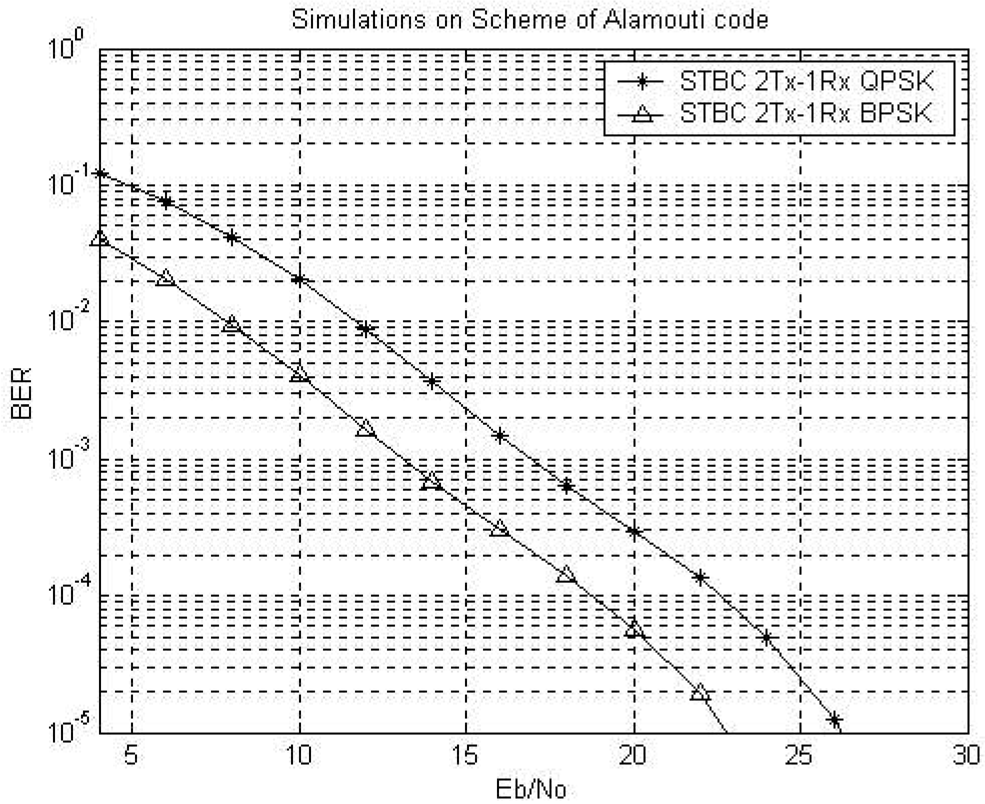

Figure A4.2 Performance curves of Alamouti's scheme with two transmit and one receive antennas using BPSK and QPSK over uncorrelated Rayleigh channel

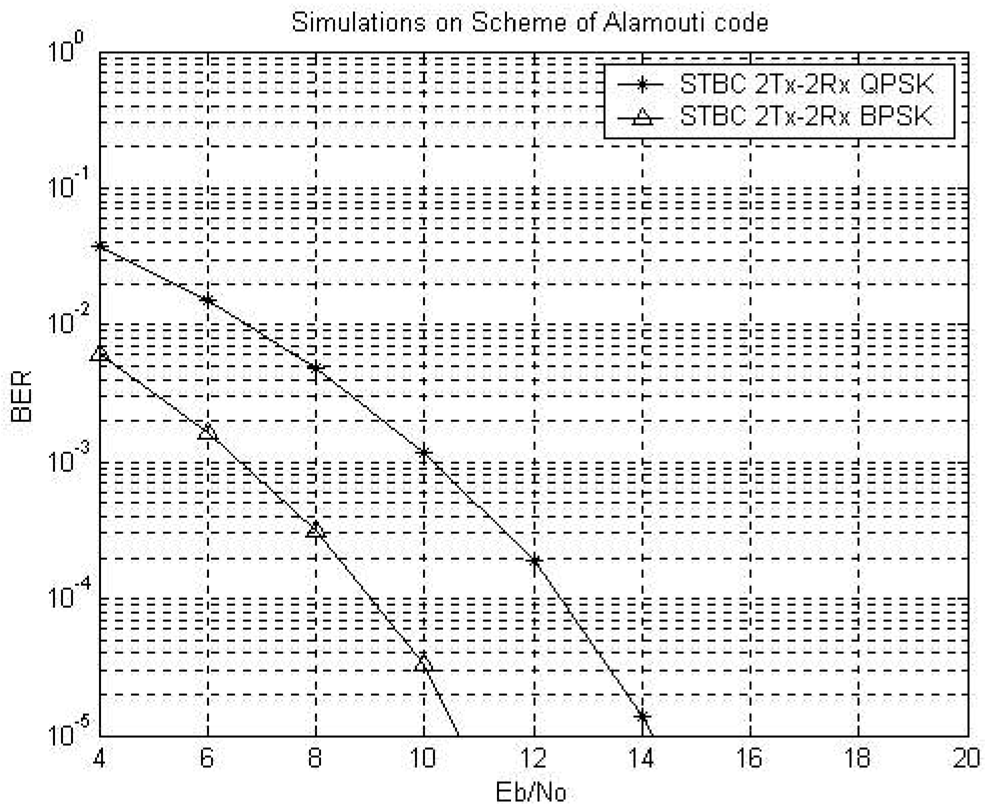

Figure A4.3 Performance curves of Alamouti's scheme with two transmit and two receiver antennas using BPSK and QPSK over uncorrelated Rayleigh channel

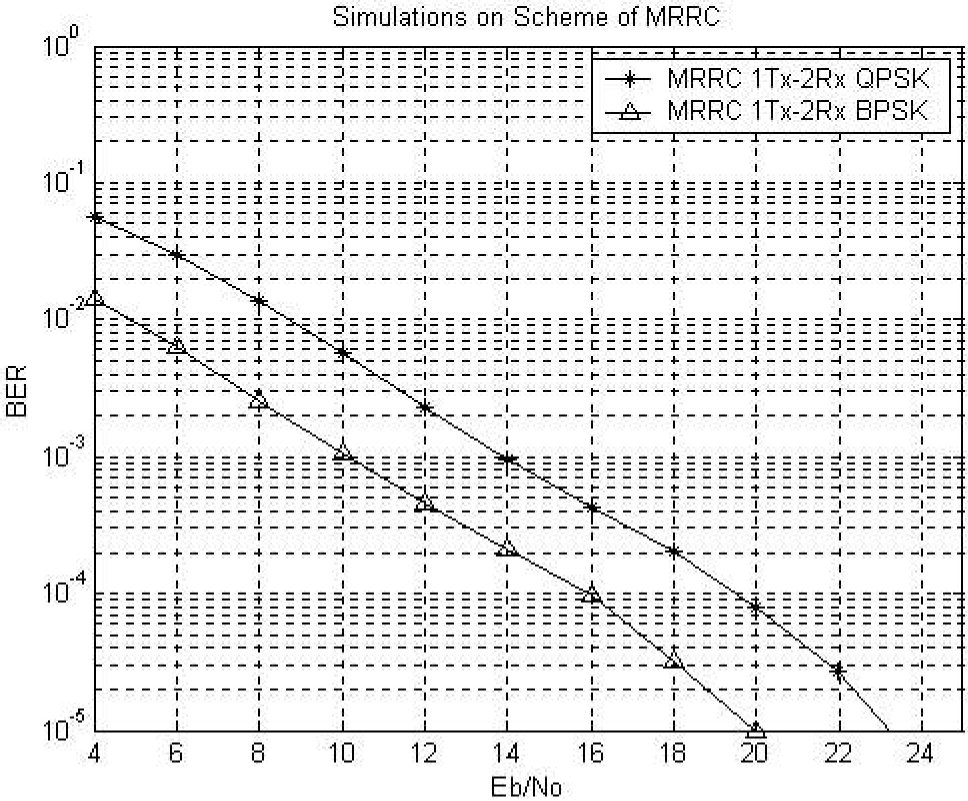

Figure A4.4 Performance curves of MRRC scheme with one transmit and two receive antennas using BPSK and QPSK over uncorrelated Rayleigh channel

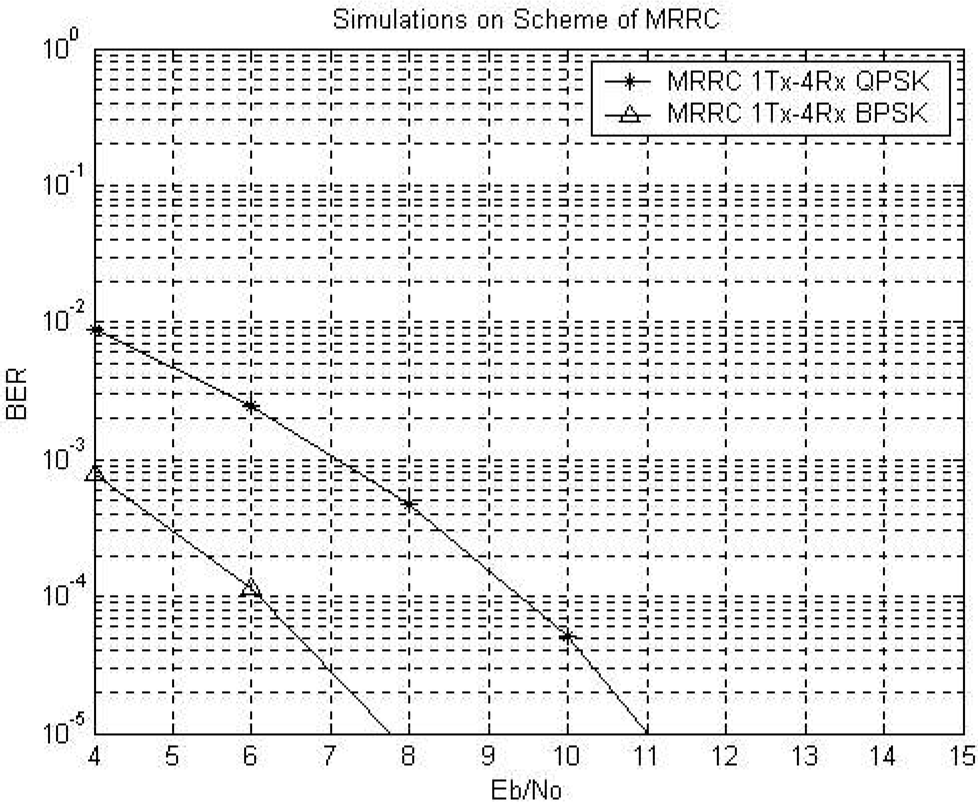

Figure A4.5 Performance curves of MRRC scheme with one transmit and four receive antennas using BPSK and QPSK over uncorrelated Rayleigh channel

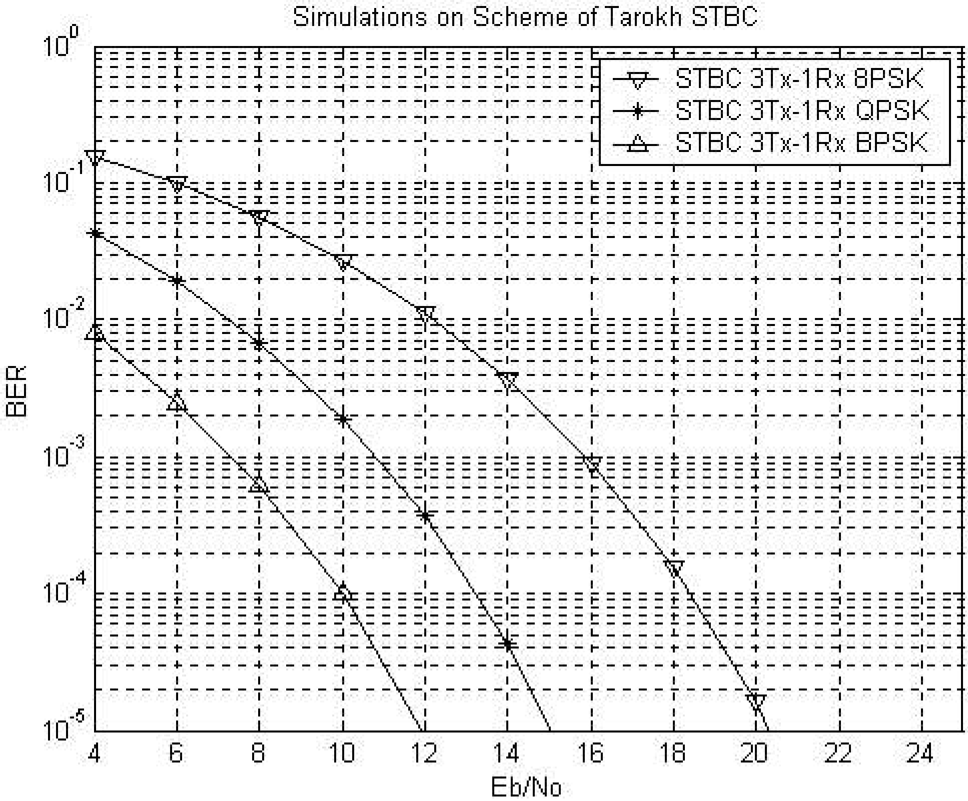

Figure A4.6 Performance curves of Tarokh's scheme in Eq. (4.42) with three transmit and one receive antennas using BPSK and QPSK over uncorrelated Rayleigh channel

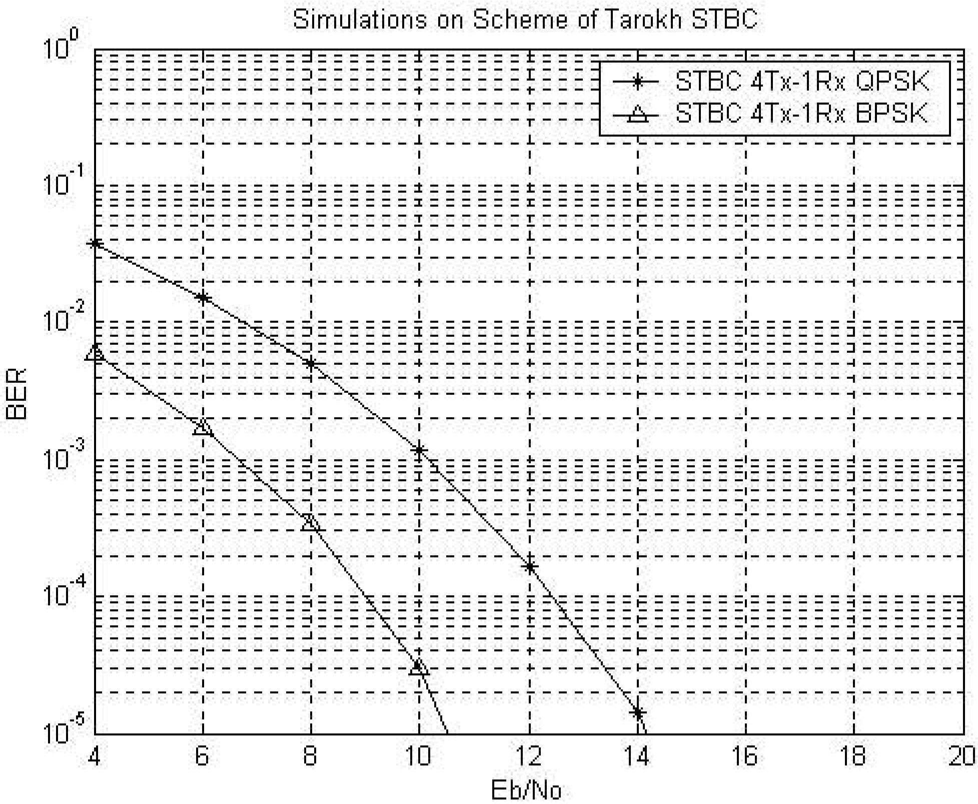

Figure A4.7 Performance curves of Tarokh's scheme in Eq. (4.42) with four transmit and one receive antennas using BPSK and QPSK over uncorrelated Rayleigh channel

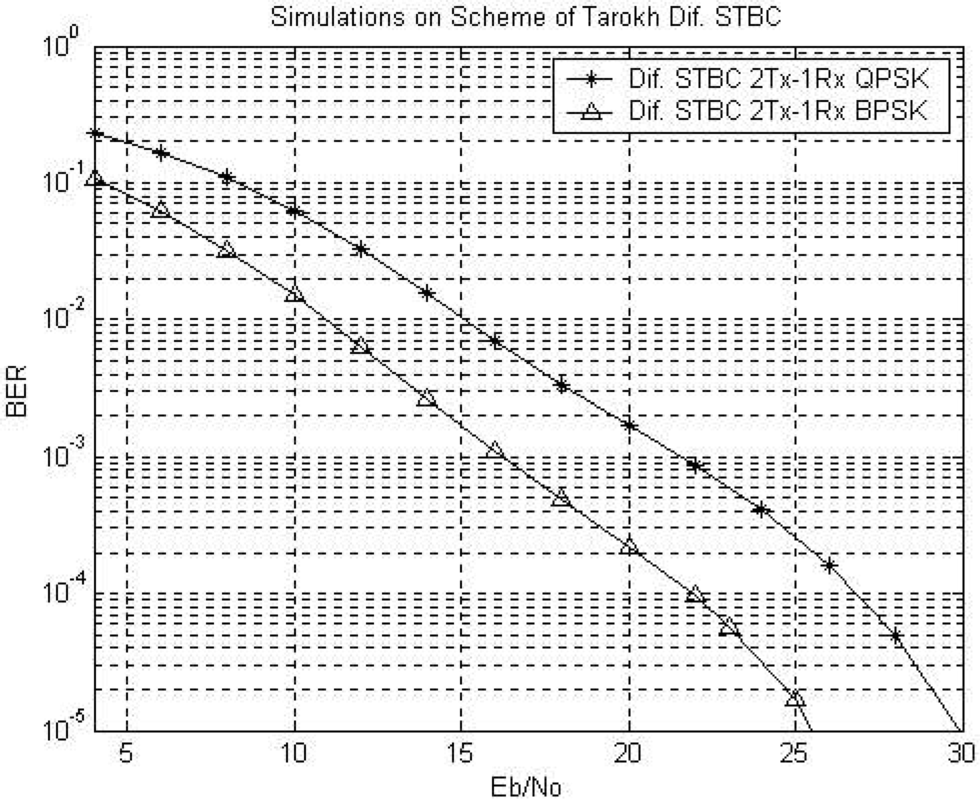

Figure A4.8 Performance curves of Tarokh's differential scheme with two transmit and one receive antennas using BPSK and QPSK over uncorrelated Rayleigh channel

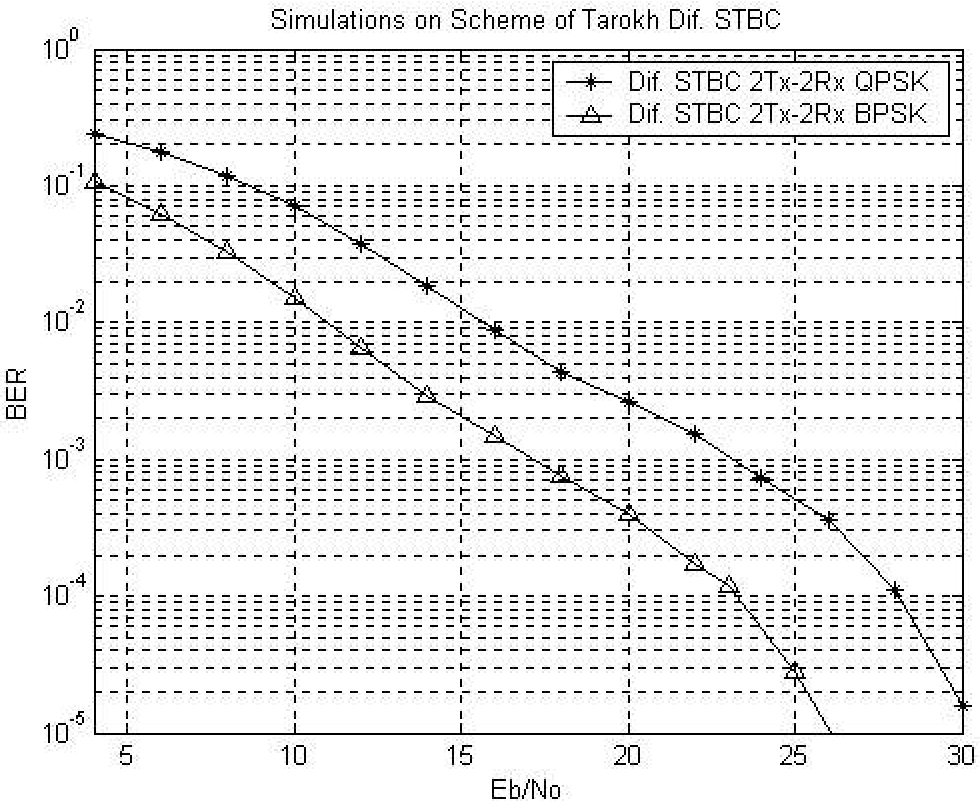

Figure A4.9 Performance curves of Tarokh's differential scheme with two transmit and two receive antennas using BPSK and QPSK over uncorrelated Rayleigh channel

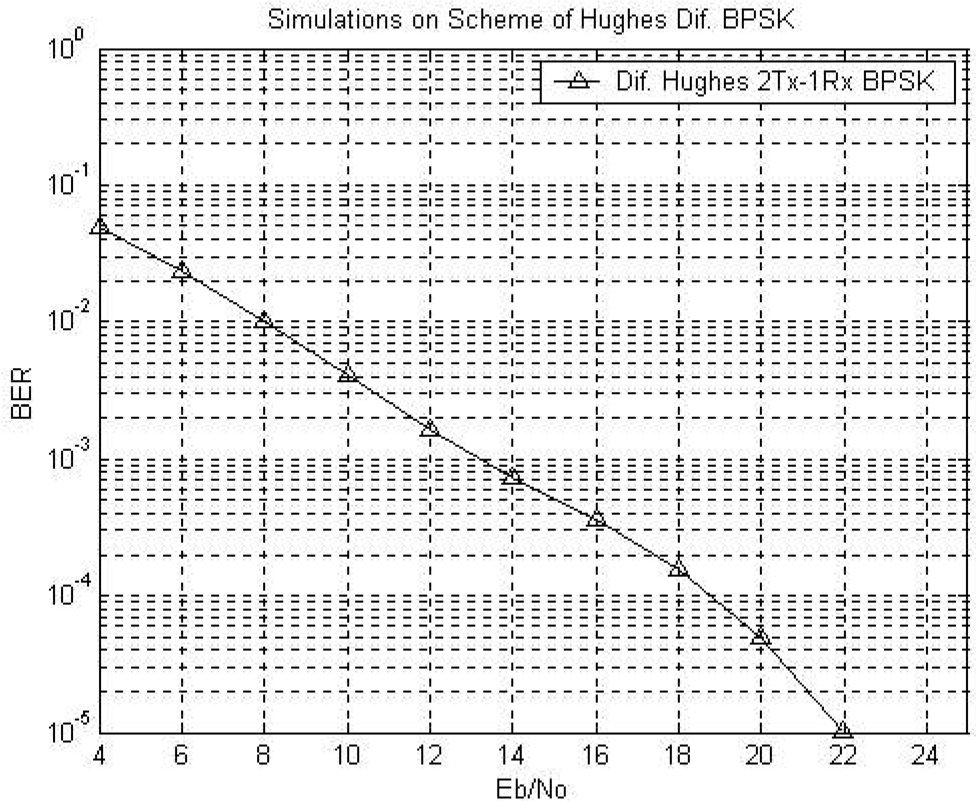

Figure A4.10 Performance curve of Hughes' differential scheme with two transmit and one receive antennas using half rate BPSK over uncorrelated Rayleigh channel

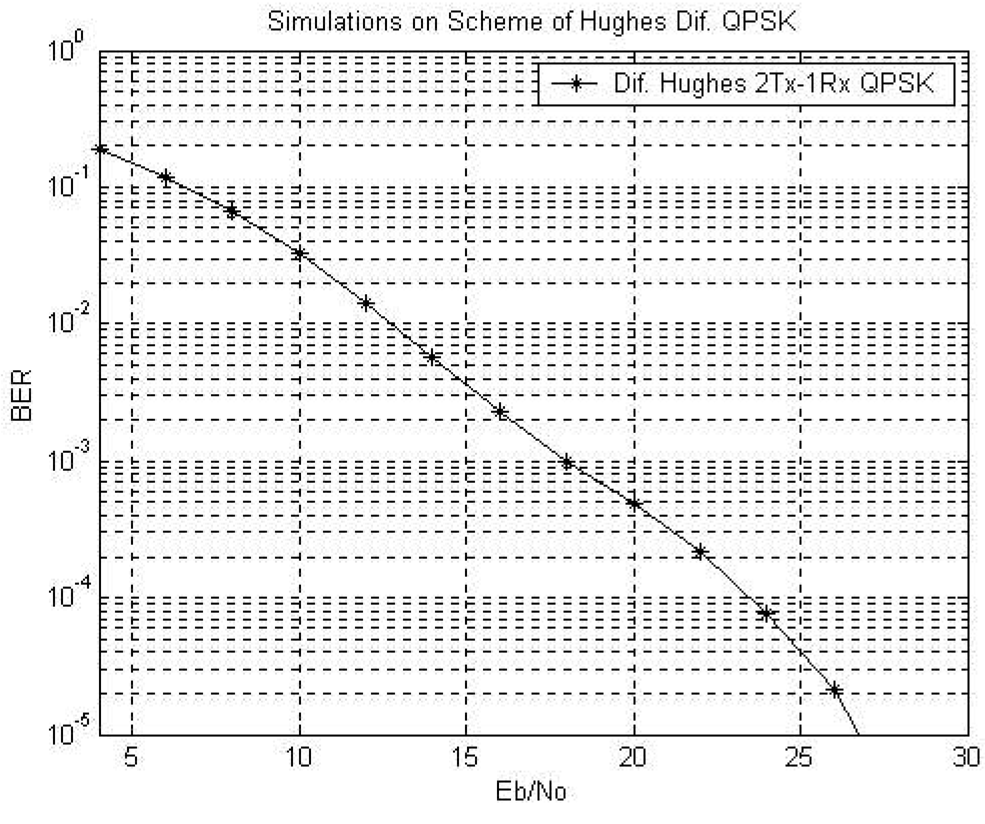

Figure A4.11 Performance curve of Hughes' differential scheme with two transmit and one receive antennas using 3/2 rate QPSK over uncorrelated Rayleigh channel

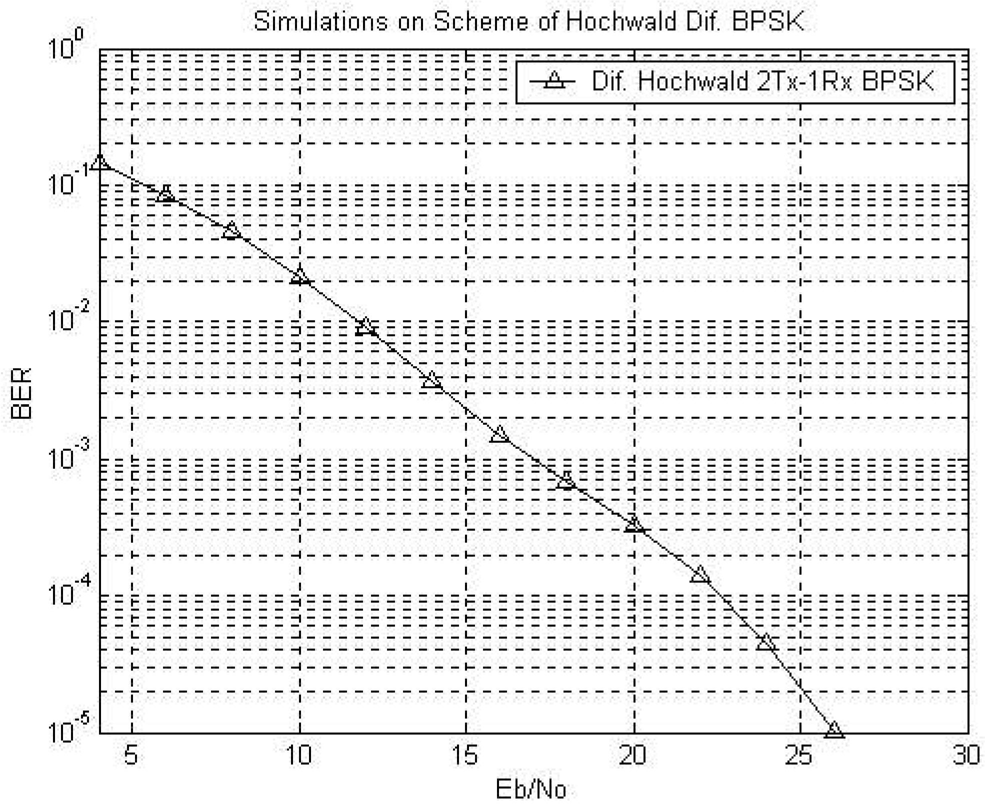

Figure A4.12 Performance curve of Hochwald's differential scheme with two transmit and one receive antennas using BPSK over uncorrelated Rayleigh channel

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