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On the validity and reliability of the (G'/G)-expansion method by using higher-order nonlinear equations

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ABSTRACT

In this study, we demonstrate the validity and reliability of the so-called (G'/G)-expansion method via symbolic computation. For illustrative examples, we choose the sixth-order Boussinesq equation and the ninth-order Korteweg-de-Vries equation. As a result, the power of the employed method is confirmed.

1. Introduction

During the past four decades or so, some efficient and powerful methods have been developed by a diverse group of scientists to find exact analytic solutions of physically important nonlinear evolution equations. For example, Hirota's bilinear method [1], inverse scattering method [2], the tanh method [3], Backlund transformation [4], symmetry method [5], the sine–cosine function method [6], the exp function method [7,8] and so on. All the methods mentioned above have some limitations in their applications and majority of the well known methods involve tedious computation if it is performed by hand.

The existence of a special class of explicit solutions called traveling waves is one of the most fundamental questions regarding nonlinear evolution equations. With the development of computer algebra systems (CAS) like *Mathematica*, *Matlab* or *Maple*, allowing us to perform the complicated and tedious algebraic calculations on computer, many direct and effective methods are also presented by the researchers.

Recently, Wang et al. [9] introduced an expansion technique called the (G'/G)-expansion method and they demonstrated that it is powerful technique for seeking analytic solutions of nonlinear partial differential equations. Bekir [10,11] applied this method to obtain traveling wave solutions of various equations. A generalization of the method has been given by Zhang et al. [12]. Also, Zhang et al. [13] made the further extension of the method for the evolution equations with variable coefficients. More recently, some useful studies by the authors [14–17] also appeared in the literature.

The (G'/G)-expansion method is based on the explicit linearization of nonlinear differential equations for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. The computations are performed with a computer algebra system to deduce few solutions of the nonlinear equations in an explicit form.

The success of the (G'/G)-expansion method for the equations studied in [9–11,14–16] is explained by the fact that the order of the reduced ODEs is equal to or less than 3 for which it is mostly possible to find out a solution of the resulted algebraic equations that determine unknown parameters. Otherwise, it is generally unable to guarantee the existence of a solution of the algebraic equations resulted. This important observation has been pointed out in [7]. Thus, in order the (G'/G)-expansion method to be universal it will be more important to seek solutions of higher-order nonlinear equations which

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can be reduced to ODEs of the order greater than 3. If (G'/G)-expansion method is shown to be applicable to these equations as well, then the usefulness of the method will be appealed furthermore.

2. Description of the (G'/G)-expansion method

Suppose we have a nonlinear partial differential equation for $u(x,t)$ in the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0, \tag{1}$$

where P is a polynomial in its arguments.

Step 1. By taking $u(x,t) = U(\xi)$, $\xi = x - ct$, we look for traveling wave solutions of Eq. (1), and transform it to the ordinary differential equation

$$Q(U, U', U'', \ldots) = 0, \tag{2}$$

where prime denotes the derivative with respect to ξ.

Step 2. Integrating Eq. (2), if possible, term by term one or more times yields constant(s) of integration. The integration constant(s) can be set to zero for simplicity.

Step 3. Suppose the solution $U(\xi)$ of Eq. (2) can be expressed as a finite series in the form

$$U(\xi) = \sum_{i=0}^{N} a_i \left(\frac{G'(\xi)}{G(\xi)} \right)^i, \tag{3}$$

where a_i are real constants with $a_N \neq 0$ to be determined, N is a positive integer to be determined, and the function $G(\xi)$ is the general solution of the auxiliary linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{4}$$

where λ, μ are real constants to be determined.

Step 4. By balancing the highest order nonlinear term(s) with the linear term(s) of highest order in Eq. (2), determine N.

Step 5. Get an algebraic equation involving powers of (G'/G) by substituting (3) together with (4) into Eq. (2). Next, equating the coefficients of each power of (G'/G) to zero, obtain a system of algebraic equations for a_i, λ, μ and c. Then, to determine these constants, solve the system with the aid of a computer algebra system. Since the solutions of Eq. (4) have been well known for us depending on the sign of the discriminant $\Delta = \lambda^2 - 4\mu$, the exact solutions of the given Eq. (1) can be obtained.

3. Applications

In this section, we work on two important nonlinear evolution equations using the (G'/G)-expansion method to illustrate its power.

3.1. Solutions of the sB equation

Let us consider the following special sixth-order Boussinesq (sB) equation in the form

$$u_{tt} - u_{xx} - (15uu_{4x} + 30u_x u_{3x} + 15(u_{2x})^2 + 45u^2 u_{2x} + 90uu_x^2 + u_{6x}) = 0, \tag{5}$$

where u_{kx} denotes the partial derivative $\partial^k u / \partial x^k$.

The famous Boussinesq equation is a nonlinear frequency dispersion equation which arises in hydrodynamics and some physical applications. Moreover, the Boussinesq type equations not only play an important role in soliton theory, but also represent very good prospect in many applied fields such as the coastal engineering where they are often used in computer models to simulate the water waves in shallow seas.

In a recent study, by generalizing the bilinear form of the Boussinesq equation, Wazwaz [18,19] has formally derived the new nonlinear dispersive Eq. (5), showed that it is not completely integrable and admits solitary wave solutions. By means of the tanh–coth method, he successfully studied multi-soliton solutions to Eq. (5).

Now, to seek for the traveling wave solutions of Eq. (5), we make the transformation $u(x,t) = U(\xi)$, $\xi = x - ct$, where c is the wave speed. Then, we get

$$(c^2 - 1)U'' - \left(15UU^{(4)} + 30U'U''' + 15(U'')^2 + 45U^2U'' + 90U(U')^2 + U^{(6)}\right) = 0, \tag{6}$$

where primes and $U^{(k)}$ denote the derivatives with respect to ξ. This reduced ODE is of the sixth-order. Now, we make the ansatz (3) for the solution of Eq. (6). Balancing the terms $U^{(6)}$ and $UU^{(4)}$ yields the leading order $N = 2$. Thus, we can write the solution of Eq. (6) in the form

$$U = a_0 + a_1\left(\frac{G'}{G}\right) + a_2\left(\frac{G'}{G}\right)^2. \tag{7}$$

Eq. (6) suggests that we calculate the derivatives $U', U'', \ldots, U^{(6)}$ from (4) and (7). Since the formulas are enormously long, we demonstrate some of them as follows:

$$U'(\xi) = -2a_2\left(\frac{G'}{G}\right)^3 - (a_1 + 2a_2\lambda)\left(\frac{G'}{G}\right)^2 - (a_1\lambda + 2a_2\mu)\left(\frac{G'}{G}\right) - a_1\mu, \tag{8}$$

$$\begin{aligned} U''(\xi) = {} & 6a_2\left(\tfrac{G'}{G}\right)^4 + (2a_1 + 10a_2\lambda)\left(\tfrac{G'}{G}\right)^3 + (8a_2\mu + 3a_1\lambda + 4a_2\lambda^2)\left(\tfrac{G'}{G}\right)^2 \\ & + (6a_2\lambda\mu + 2a_1\mu + a_1\lambda^2)\left(\tfrac{G'}{G}\right) + 2a_2\mu^2 + a_1\lambda\mu, \end{aligned} \tag{9}$$

$$\vdots$$

$$U^{(6)}(\xi) = 5040a_2\left(\frac{G'}{G}\right)^8 + (720a_1 + 19440\lambda a_2)\left(\frac{G'}{G}\right)^7 + \cdots \tag{13}$$

Substituting (7)–(9) and (13) into (6), setting the coefficients of $(G'(\xi)/G(\xi))^i (i = 0, 1, \ldots, 8)$ to zero, we obtain a system of nonlinear algebraic equations. Solving the resulting system, we have the following sets of solutions:

First solution set:

$$a_1 = -2\lambda, \quad a_2 = -2, \quad c = \mp\sqrt{1 + \lambda^4 + 22\lambda^2\mu + 76\mu^2 + 15\lambda^2 a_0 + 120\mu a_0 + 45a_0^2}. \tag{14}$$

Second solution set:

$$a_0 = \frac{1}{3}(-\lambda^2 - 8\mu), \quad a_1 = -4\lambda, \quad a_2 = -4, \quad c = \mp\sqrt{1 + \lambda^4 - 8\lambda^2\mu + 16\mu^2}. \tag{15}$$

Substituting the solution set (14) into (7); we obtain the hyperbolic function traveling wave solutions

$$u_{1,2}(x,t) = \frac{\lambda^2}{2} + a_0 - \frac{\lambda^2 - 4\mu}{2}\left(\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct) + C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct)}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct) + C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct)}\right)^2, \tag{16}$$

where $\lambda^2 > 4\mu$, c is as in (14), C_1 and C_2 are arbitrary constants, the trigonometric function traveling wave solutions

$$u_{3,4}(x,t) = \frac{\lambda^2}{2} + a_0 + \frac{\lambda^2 - 4\mu}{2}\left(\frac{-C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct) + C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct)}{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct) + C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct)}\right)^2, \tag{17}$$

where $\lambda^2 < 4\mu$, c is as in (14), C_1 and C_2 are arbitrary constants, and the rational function solutions

$$u_{5,6}(x,t) = 2\mu + a_0 - \frac{2C_2^2}{\left(C_1 + C_2\left(x \pm \left(\sqrt{1 + 180\mu^2 + 180\mu a_0 + 45a_0^2}\right)t\right)\right)^2}, \tag{18}$$

where C_1 and C_2 are arbitrary constants.

In particular, if we take $C_2 \neq 0, C_1^2 < C_2^2$ above, then the solutions (16) give the solitary wave solutions

$$u_{7,8}(x,t) = \frac{\lambda^2}{2} + a_0 - \frac{\lambda^2 - 4\mu}{2}\tanh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(x - ct) + \xi_0\right), \quad \xi_0 = \tanh^{-1}\left(\frac{C_1}{C_2}\right), \tag{19}$$

where $\lambda^2 > 4\mu$, c is as in (14), and the solutions (17) give the periodic solutions

$$u_{9,10}(x,t) = \frac{\lambda^2}{2} + a_0 + \frac{\lambda^2 - 4\mu}{2}\cot^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(x - ct) + \xi_0\right), \quad \xi_0 = \tan^{-1}\left(\frac{C_1}{C_2}\right), \tag{20}$$

where $\lambda^2 < 4\mu$, and c is as in (14).

Also, substituting the solution set (15) into (7); we obtain the hyperbolic function traveling wave solutions

$$u_{11,12}(x,t) = \frac{2\lambda^2 - 8\mu}{3} - (\lambda^2 - 4\mu)\left(\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct) + C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct)}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct) + C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct)}\right)^2, \tag{21}$$

where $\lambda^2 > 4\mu$, $c = \mp\sqrt{1+(\lambda^2-4\mu)^2}$, C_1 and C_2 are arbitrary constants, the trigonometric function traveling wave solutions

$$u_{13,14}(x,t) = \frac{2\lambda^2-8\mu}{3} + (\lambda^2-4\mu)\left(\frac{-C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct) + C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct)}{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct) + C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct)}\right)^2, \tag{22}$$

where $\lambda^2 < 4\mu$, $c = \mp\sqrt{1+(\lambda^2-4\mu)}^2$, C_1 and C_2 are arbitrary constants, and the rational function solutions

$$u_{15,16}(x,t) = -\frac{4C_2^2}{(C_1 + C_2(x \pm t))^2}, \tag{23}$$

where C_1 and C_2 are arbitrary constants.

In particular, if we take $C_2 \neq 0, C_1^2 < C_2^2$ above, then the solutions (21) give the solitary wave solutions

$$u_{17,18}(x,t) = \frac{2\lambda^2-8\mu}{3} - (\lambda^2-4\mu)\tanh^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(x \pm \left(\sqrt{1+(\lambda^2-4\mu)^2}\right)t\right) + \xi_0\right), \tag{24}$$

where $\lambda^2 > 4\mu$, $\xi_0 = \tanh^{-1}(C_1/C_2)$, and the solutions (22) give the periodic solutions

$$u_{19,20}(x,t) = \frac{2\lambda^2-8\mu}{3} + (\lambda^2-4\mu)\cot^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(x \pm \left(\sqrt{1+(\lambda^2-4\mu)^2}\right)t\right) + \xi_0\right), \tag{25}$$

where $\lambda^2 < 4\mu$ and $\xi_0 = \tan^{-1}(C_1/C_2)$.

Meanwhile, we would like to mention that the rational solutions derived here do not appear in [18]. Furthermore, it is possible to recover some of the soliton solutions obtained in [18] by taking $\xi_0 = 0$ in Eqs. (19) and (24) and manipulation.

3.2. Solutions of the nKdV equation

Now, let us consider the following ninth-order Korteweg-de-Vries (nKdV) equation in the form

$$\begin{aligned} &u_t + 45u_xu_{6x} + 45uu_{7x} + 210u_{3x}u_{4x} + 210u_{2x}u_{5x} + 1575u_x(u_{2x})^2 + 3150uu_{2x}u_{3x} + 1260uu_xu_{4x} + 630u^2u_{5x} \\ &\quad + 9450u^2u_xu_{2x} + 3150u^3u_{3x} + 4725u^4u_x + u_{9x} \\ &\quad = 0, \end{aligned} \tag{26}$$

where u_{kx} denotes the partial derivative $\partial^k u/\partial x^k$. In a recent study, Wazwaz [18,19] has formally derived the new nonlinear dispersive Eq. (26) by generalizing the bilinear form of the KdV equation and showed that it is not completely integrable admitting soliton solutions. He successfully studied solitary wave solutions to Eq. (26) by means of the tanh–coth method.

Now, to seek for the traveling wave solutions of Eq. (26), we make the transformation $u(x,t) = U(\xi)$, $\xi = x - ct$, where c is the wave speed. Then, we get

$$\begin{aligned} &-cU' + 45U'U^{(6)} + 45UU^{(7)} + 210U'''U^{(4)} + 210U''U^{(5)} + 1575U'(U'')^2 + 3150U'U''U''' + 1260UU'U^{(4)} + 630U^2U^{(5)} \\ &\quad + 9450U^2U'U'' + 3150U^3U''' + 4725U^4U' + U^{(9)} \\ &\quad = 0, \end{aligned} \tag{27}$$

where primes and $U^{(k)}$ denote the derivatives with respect to ξ. This reduced ODE is of the ninth-order. Now, we make the ansatz (3) for the solution of Eq. (29). Balancing the terms $U^{(9)}$ and $U'U^{(6)}$ yields the leading order $N = 2$. Thus, we can write the solution of Eq. (27) in the form

$$U = a_0 + a_1\left(\frac{G'}{G}\right) + a_2\left(\frac{G'}{G}\right)^2. \tag{28}$$

Eq. (27) suggests that we calculate the derivatives $U', U'', \ldots, U^{(9)}$ from Eqs. (4) and (28). Since the formulas are too long, for the sake of saving space, we demonstrate some of them here as follows:

$$U'(\xi) = -2a_2\left(\frac{G'}{G}\right)^3 - (a_1 + 2a_2\lambda)\left(\frac{G'}{G}\right)^2 - (a_1\lambda + 2a_2\mu)\left(\frac{G'}{G}\right) - a_1\mu, \tag{29}$$

$$\begin{aligned} U''(\xi) = {}& 6a_2\left(\tfrac{G'}{G}\right)^4 + (2a_1 + 10a_2\lambda)\left(\tfrac{G'}{G}\right)^3 + (8a_2\mu + 3a_1\lambda + 4a_2\lambda^2)\left(\tfrac{G'}{G}\right)^2 \\ & + (6a_2\lambda\mu + 2a_1\mu + a_1\lambda^2)\left(\tfrac{G'}{G}\right) + 2a_2\mu^2 + a_1\lambda\mu, \end{aligned} \tag{30}$$

$$\vdots$$

$$U^{(9)}(\xi) = -3,628,800a_2\left(\frac{G'}{G}\right)^{11} + (-362880a_1 - 19,595,520\lambda a_2)\left(\frac{G'}{G}\right)^{10} + \cdots \tag{37}$$

Substituting (28)–(30) and (37) into (27), setting the coefficients of $(G'(\xi)/G(\xi))^i (i = 0, 1, \dots, 11)$ to zero, we obtain a system of nonlinear algebraic equations. We omit to display them here. Now, solving the resulting system, we have the following sets of solutions:

First solution set:

$$\begin{aligned} &a_1 = -2\lambda, \quad a_2 = -2, \\ &c = \lambda^8 + 74\lambda^6\mu + 1536\lambda^4\mu^2 + 9104\lambda^2\mu^3 + 9616\mu^4 + 45\lambda^6 a_0 + 1980\lambda^4\mu a_0 \\ &\quad + 19800\lambda^2\mu^2 a_0 + 37440\mu^3 a_0 + 630\lambda^4 a_0^2 + 13860\lambda^2\mu a_0^2 + 47880\mu^2 a_0^2 \\ &\quad + 3150\lambda^2 a_0^3 + 25200\mu a_0^3 + 4725 a_0^4, \end{aligned} \tag{38}$$

Second solution set:

$$a_0 = -2\lambda^2, \quad a_1 = -8\lambda, \quad a_2 = -8, \quad \mu = \frac{\lambda^2}{4}, \quad c = 0. \tag{39}$$

Third solution set:

$$a_0 = \frac{-3\lambda^2}{2}, \quad a_1 = -6\lambda, \quad a_2 = -6, \quad \mu = \frac{\lambda^2}{4}, \quad c = 0. \tag{40}$$

Substituting the solution set (38) into (28); we obtain the hyperbolic function traveling wave solution

$$u_1(x,t) = \frac{\lambda^2}{2} + a_0 - \frac{\lambda^2 - 4\mu}{2}\left(\frac{C_1 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct) + C_2 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct)}{C_1 \sinh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct) + C_2 \cosh\frac{\sqrt{\lambda^2-4\mu}}{2}(x-ct)}\right)^2, \tag{41}$$

where $\lambda^2 > 4\mu$, c is as in (38), C_1 and C_2 are arbitrary constants, the trigonometric function traveling wave solution

$$u_2(x,t) = \frac{\lambda^2}{2} + a_0 + \frac{\lambda^2 - 4\mu}{2}\left(\frac{-C_1 \sin\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct) + C_2 \cos\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct)}{C_1 \cos\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct) + C_2 \sin\frac{\sqrt{4\mu-\lambda^2}}{2}(x-ct)}\right)^2, \tag{42}$$

where $\lambda^2 < 4\mu$, c is as in (38), C_1 and C_2 are arbitrary constants, and the rational function solution

$$u_3(x,t) = \frac{\lambda^2}{2} + a_0 - \frac{2C_2^2}{\left(C_1 + C_2\left(x - \left(\frac{4725}{16}\lambda^8 + \frac{4725}{2}\lambda^6 a_0 + \frac{14175}{2}\lambda^4 a_0^2 + 9450\lambda^2 a_0^3 + 4725 a_0^4\right)t\right)\right)^2}, \tag{43}$$

where C_1 and C_2 are arbitrary constants.

In particular, if we take $C_2 \neq 0, C_1^2 < C_2^2$ above, then the solution (41) gives the solitary wave solution

$$u_4(x,t) = \frac{\lambda^2}{2} + a_0 - \frac{\lambda^2 - 4\mu}{2}\tanh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(x-ct) + \xi_0\right), \quad \xi_0 = \tanh^{-1}\left(\frac{C_1}{C_2}\right), \tag{44}$$

where $\lambda^2 > 4\mu$, c is as in (38), and the solution (42) gives the periodic solution as

$$u_5(x,t) = \frac{\lambda^2}{2} + a_0 + \frac{\lambda^2 - 4\mu}{2}\cot^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(x-ct) + \xi_0\right), \quad \xi_0 = \tan^{-1}\left(\frac{C_1}{C_2}\right). \tag{45}$$

where $\lambda^2 < 4\mu$ and c is as in (38). Furthermore, it is possible to recover the soliton solutions obtained in [18] by taking $\xi_0 = 0$ in (44) and manipulation.

Also, substituting the solution sets Eqs. (39) and (40) into (28); we obtain the rational function solutions

$$u_6(x,t) = \frac{-8C_2^2}{(C_1 + C_2 x)^2}, \tag{46}$$

$$u_7(x,t) = \frac{-6C_2^2}{(C_1 + C_2 x)^2}, \tag{47}$$

where C_1 and C_2 are arbitrary constants. We note that Eqs. (46) and (47) do not appear in [18].

4. Conclusion

We successfully obtained exact and explicit analytic solutions (including new ones) with arbitrary parameters to the sB equation and the nKdV equation via the (G'/G)-expansion method. The extracted solutions in this work are wider if compared to the earlier works of others. In general, it is too difficult to solve these higher-order equations by traditional methods.

The procedure is simple, direct and constructive with the help of a computer algebra system. We foresee that our results can be found potentially useful for applications in mathematical physics and applied mathematics including numerical simulation.

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