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Adaptive actuator failure compensation for redundant manipulators

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SUMMARY
This paper presentsan adaptive actuator failure compensation method, which compensates for uncertainties due to unknown actuator failures for redundant manipulator systems. The method is first developed for manipulators whose joints are concurrently actuated. While physical realization of concurrently actuated manipulators and the advantages of their use have been understood before, in this paper failure modeling, controller structure, and adaptive update rules for handling uncertainties from the actuator failures are studied. The adaptive actuator failure compensation method is then expanded for a cooperating multiple manipulator system with uncertain actuator failures. Dynamic equations of such a multiple manipulator system in the task space are derived and the adaptive actuator failure compensation problem is formulated in the task space, for which a compensation controller structure is proposed with stable adaptive parameter update laws. The adaptive control scheme is able to compensate for the uncertainties of system parameters and actuator failures in a more general sense. For both cases, closed-loop system stability and asymptotic tracking are proved, despite uncertain system failures.

KEYWORDS: Actuator failure compensation, Adaptive controller design, Redundant manipulator failure compensation.

1. Introduction

This paper intends to investigate a new method for actuator failure compensation for redundant manipulators. It starts with motivation for this work: redundancy and actuator failure compensation in robotics and continues by explaining the need for a concurrently actuated manipulator, studying the physical realization aspects of concurrently actuated manipulators and proposing a new control method for post-failure control, where the number and location of the failed actuators as well as the failure values are unknown. Later, the control theory is applied into cooperating multiple manipulator system where the system redundancy is achieved by using more than one manipulator.

When high system reliability and safety are expected from a robotic manipulator, fault tolerance is employed into system design for applications where the task of themanipulator is too important to stop during the operation because of a failure, such as hazardous environments (i.e., nuclear waste handling and surgery), or it is too difficult to give service to the manipulator after it fails, such as space and underwater applications.

In order to make the manipulator fault tolerant, mostly in the literature, it is built as a kinematically redundant manipulator. The number of joints of the manipulator determines the degree of freedom (DOF) of the manipulator, considering that the joints are not coupled. This characteristic shows the reach ability of the manipulator with arbitrary orientation in its workspace.

In kinematically redundant manipulators, before a failure occurs, the redundancy can be used to optimize the motion of the manipulator. The optimization criteria can be minimization of the joint disturbance torque for independent joint controlled manipulators,[1] optimization of the manipulator motion with end-effector path constraints,[2] or multiple criteria[3] such as motion optimization, minimum time, minimum energy, and minimum distance. After the failure occurs, different algorithms are used to detect the failure and isolate the failed joint, such as observers,[4] position and velocity tracking errors,[5] full manipulator dynamics,[6] and neural networks.[7] By isolating the failed joint, new mechanical and control structures are used to drive the failed manipulator.[8]

Another way of making a manipulator redundant is by using concurrent actuators at the joints.[9,10] Redundancy is introduced and different manipulator mechanical architectures are ranked by fault tolerance measure for fault tolerance capacity in ref. [9] By using fault tolerance capacity, designers of the manipulator can categorize the manipulator mechanical structure. A parallel-coupled micro-macro actuator system has been designed by Morrell and Salisbury in ref. [10] to achieve a low impedance system and a wide range of applied force. In concurrently actuated manipulators, unknown actuator failure compensation by adaptive control without detecting the failure is an ongoing research.

In the second part of the research, the object is considered to be manipulated with multiple manipulators not only because it requires multiple manipulators to be moved but also to ensure that if actuator failures occur the remaining manipulators will be able to accomplish moving the object as

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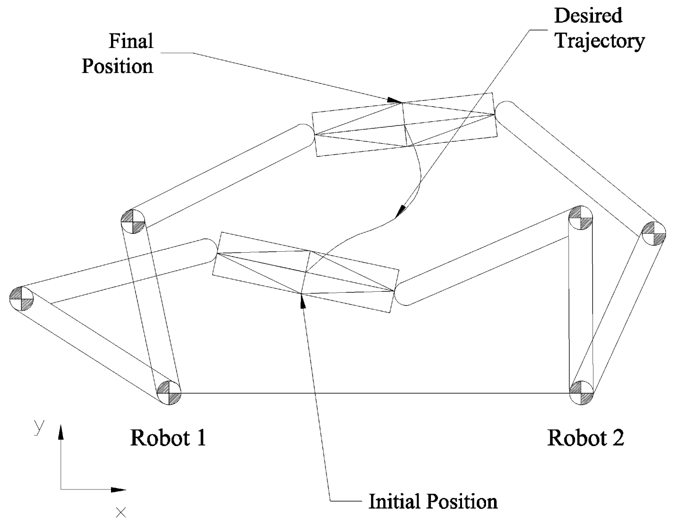

Fig. 1. Cooperating multiple manipulators.

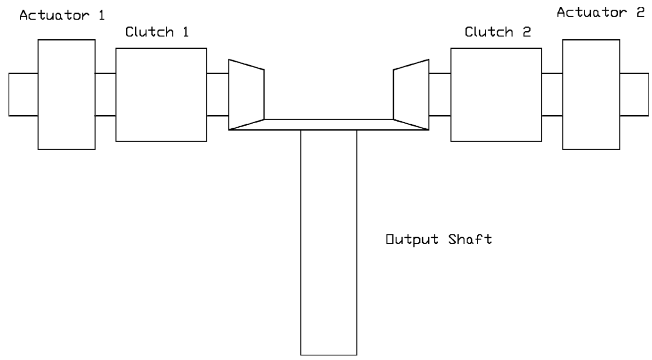

Fig. 2. Dual actuation system.

desired. In this section, an adaptive scheme for a cooperating multiple manipulator system with actuator failures in the task space is proposed, where system stability and tracking error convergence are achieved without detecting the failed actuator or having prior knowledge of failure. An adaptive actuator failure compensation controller for a platform manipulator was designed in ref. [11], but the interaction between the expandable legs and the upper platform was ignored.

In this current study, the effect of a manipulator on other manipulators in the cooperating manipulator system is also considered. Cooperating multiple manipulators are needed to handle the common object in which many robotic applications such as moving a massive object, handling flexible payload, or assembling applications are not feasible for one manipulator, because of the complexity of the application. Legged vehicles and multi-fingered hands can also be categorized as cooperating multiple manipulator system. With a set of closed kinematic chains, for instance, the manipulators are holding a common rigid object (Fig. 1), multiple manipulator system have more complexity in control design due to the dynamic interaction between the manipulators, because the system has a set of kinematic and dynamic constraints, where manipulators are controlled cooperatively to avoid internal stress on the payload. The controller should be designed to ensure the load sharing and compensate for the variation of the payload.

Manipulator task space controllers have been studied before such as adaptive control design[12,13] and PID control design.[14] In ref. [12] an adaptive control algorithm is designed for task space control of manipulators where the inverse of the Jacobian is not required and the requirement of bounded inverse of the inertia matrix is eliminated, using knowledge of the joint acceleration vector. In ref. [13] an iterative learning algorithm is used with adaptive controller design to eliminate the computation need for real-time parameter identification, but learning algorithms can not handle large modeling uncertainties and external disturbance.

This paper is organized as follow: Physical realization of the concurrent actuated joints for robotic applications is explained in Section 2. Actuator failures in concurrent actuated systems are studied in Section 3. Section 3.1. states the actuator failure problem formulation in robotics. In Section 3.2., a new adaptive algorithm for control of a redundant manipulator with actuator failures (whose location, number, and failure value are unknown) is developed. Dynamic equations of the cooperating manipulator in the task space are derived in Section 4. In Section 5 an adaptive control scheme is developed to compensate for uncertainties arising from actuator failures. Lyapunov stability analysis proves boundedness of the closed-loop signals and asymptotic tracking of a reference trajectory for the object. Section 6 gives the conclusions of this work.

2. Realization of Concurrent Actuated Joints

It is practically possible to connect different actuators mechanically to build a concurrent actuated joint. In ref. [9], instead of having a single actuator at the link, another actuator is also attached to the same link, allowing the joint to still be controllable, in case any of the actuators fail. When the failure occurs, the failed actuator can be disengaged by a clutch mechanism, so the remaining actuator can still drive the system. An example of a dual actuator system is shown in Fig. 2. Dual actuation can also be used to eliminate the backlash effects and torque saturation at the joint. Instead of a gear box, a belt drive is used in ref. [10], where a micro-actuator is directly attached and a macro-actuator is coupled by a compliant transmission to the joint axis, which is shown in Fig. 3.

As an alternative of using separate actuators and a mechanical connection to form a concurrent actuated joint, the actuator itself can be built so that it is redundant. In ref. [15], by using separate stator winding phases which are electrically, magnetically, thermally, and physically independent of all others, a fault-tolerant actuator is achieved, shown in Fig. 4. Another way of creating a dynamically redundant actuator is to use a multiple segment/modular motor,[16] where two segments of the electric motor have separate stator winding, while sharing the same rotor, which is illustrated in Fig. 5.

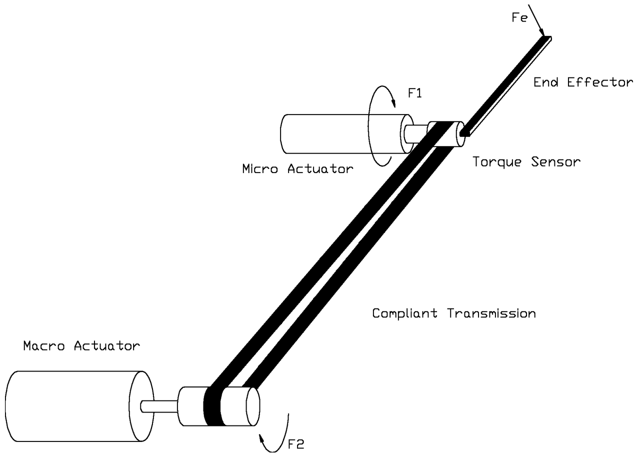

Fig. 3. Parallel-coupled micro-macro actuators.

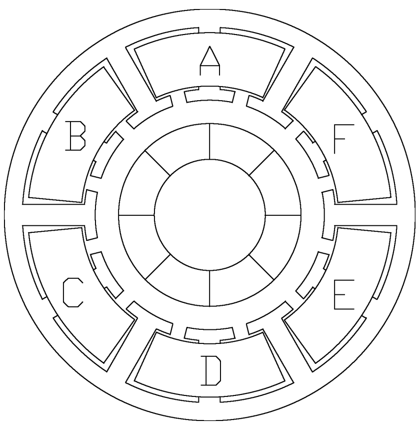

Fig. 4. Parallel phase stator windings.

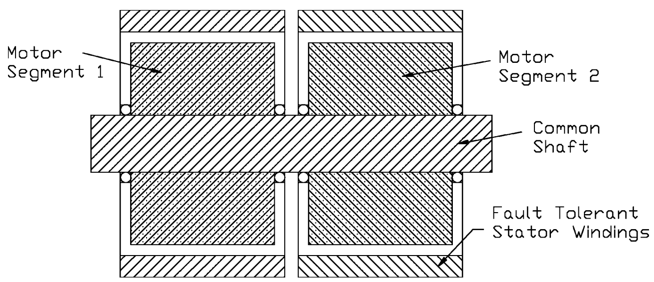

Fig. 5. Multiple segment modular motor.

3. Concurrent Actuator Failure Compensation

The problem of compensating for the actuator failure in concurrent actuated systems has been studied for flight control systems. In ref. [17], the actuator failure case is such that, m actuators are connected concurrently, up to p actuators may fail and remaining actuators are still capable of driving the system. After the unknown time of failure, the failed actuator applies constant unknown input to the system. Under these conditions, the authors designed an adaptive control law, proved the system stability and showed the desired system performance.

In this section, an adaptive compensation scheme for concurrently actuated manipulators, where at the ith joint m_i actuators are connected concurrently, is developed. After p number of actuators fail at unknown times and apply unknown constant torques, the adaptive controller stabilizes the system, the tracking error converges to zero and all system signals are bounded.

3.1. Problem formulation

The dynamic model of a concurrently actuated manipulator system is formulated as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \tag{3.1}$$

where $q, \dot{q}, \ddot{q} \in R^n$ are joint variables position, velocity and acceleration vectors; $D(q) \in R^{n\times n}$ is the inertia matrix, $C(q, \dot{q}) \in R^{n\times n}$ is the Coriolis and centrifugal term, $g(q) \in R^{n\times 1}$ is the gravity term, $\tau \in R^{n\times 1}$ is the joint torque vector and n is the degree of freedom (number of joints).

In a concurrent actuation case, at the ith joint, $i \in \{1, 2, \ldots, n\}$, m_i actuators are connected concurrently and the number of concurrent actuators m_i can be different at each joint. For the ith joint, the dynamics is written as

$$D_i(q)\ddot{q} + C_i(q, \dot{q})\dot{q} + g_i(q) = \tau_i, \tag{3.2}$$

where $D_i(q)$, $C_i(q, \dot{q})$, and $g_i(q)$ are the ith row of $D(q)$, $C(q, \dot{q})$, and $g(q)$ respectively.

To model the redundancy, the joint torque is formulated as

$$\tau_i = \tau_{i1} + \tau_{i2} + \cdots + \tau_{im_i} \tag{3.3}$$

where τ_{ij} is the torque applied to ith joint by the jth actuator.

The actuator failures are modeled as

$$\tau_{ij}(t) = \sum_{k=1}^{l_{ij}} \tau_{ijk} f_{ijk}(t), \quad t \geq t_{ij}, \tag{3.4}$$

for $j = 1, 2, \ldots, m_i$ and $i = 1, 2, \ldots, n$, with some unknown constant τ_{ijk} and known failure signal $f_{ijk}(t)$, where the failure time instant t_{ij} is unknown.[18] A commonly used failure model is

$$\tau_{ij}(t) = \bar{\tau}_{ij}, \quad t \geq t_{ij}, \tag{3.5}$$

with unknown constant failure values $\bar{\tau}_{ij}$, which is considered in this paper's design and analysis of adaptive failure compensation schemes (the general case (3.4) can be similarly handled).

The basic assumption for, the existence of an adaptive compensation scheme for unknown system and failure parameters is as follow:

> (A.1) the system (3.1) is designed such that for each joint in the presence of up to $m_i - 1$ actuator failures, the concurrently actuated manipulator system can still achieve a desired control objective by the remaining actuators, when implemented with known system and failure parameters.

The main objective of adaptive control is to adjust the remaining actuators to achieve the desired system performance, when there are up to $m_i - 1$ unknown actuator failures in the ith joint and parameter uncertainties of the system. As seen from the following design and analysis, the basic assumption (A.1) is satisfied for the system (3.1).

When p_i actuators are failed at the ith joint, that is, $\tau_{ij}(t) = v_{ij}(t)$, where $v_{ij}(t)$ is the applied control input to be determined, for $j \neq j_1, \dots, j_{p_i}$ and $\tau_{ij}(t) = \bar{\tau}_{ij}$, where $\bar{\tau}_{ij}$ is an unknown constant torque produced by a failed actuator, for $j = j_1, \dots, j_{p_i}$, the dynamic Eq. (3.2) becomes.

$$D_i(q)\ddot{q} + C_i(q,\dot{q})\dot{q} + g_i(q) = \sum_{j \neq j_1,\dots,j_{p_i}} v_{ij}(t) + \sum_{j = j_1,\dots,j_{p_i}} \bar{\tau}_{ij}, \tag{3.6}$$

with $\{j_1, j_2, \dots, j_{p_i}\} \subset \{1, 2, \dots, m_i\}$ indicating a certain failure pattern.

The control objective is to design a feedback control law $v_{ij}(t)$ for the dynamic system (3.6) to ensure that all closed-loop system signals and parameter estimates are bounded and that the manipulator output $q(t)$ asymptotically tracks a given reference output $q_d(t)$.

3.2. Adaptive control design

Define the tracking error e and the filtered tracking errors r and v as

$$e = q - q_d, \quad r = \dot{e} + \lambda e, \quad v = \dot{q}_d - \lambda e, \tag{3.7}$$

where $\lambda > 0$ is a design parameter.

The closed-loop Eq. (3.6) can be expressed as

$$D_i(q)\dot{r} + C_i(q,\dot{q})r = -Y_i(q,\dot{q},v,\dot{v})\theta_i + \sum_{j \neq j_1,\dots,j_{p_i}} v_{ij}(t) + \sum_{j = j_1,\dots,j_{p_i}} \bar{\tau}_{ij} \tag{3.8}$$

where

$$Y_i(q,\dot{q},v,\dot{v})\theta_i = D_i(q)\dot{v} + C_i(q,\dot{q})v + g_i(q), \tag{3.9}$$

θ_i is the unknown parameter vector and Y_i is the known function for $i = 1, 2, \dots, n$.

In order to achieve the desired system performance, the following control structure is used:

$$v_{ij}(t) = Y_i(q,\dot{q},v,\dot{v})\hat{\theta}_{ij} + \hat{p}_{ij} - K_{ij} r_i, \tag{3.10}$$

where $K_{ij} > 0$, $j = 1, 2, \dots, m_i$, $i = 1, 2, \dots, n$, are scalar gains, $\hat{\theta}_{ij}$ and $\hat{p}_{ij}$ are parameter estimates to be determined from adaptive laws.

From the failure model (3.5) and the controller structure (3.10) when p_i actuators fail at the ith joint, that is, $\tau_{ij}(t) = \bar{\tau}_{ij}$, $j = j_1, j_2, \dots, j_{p_i}$, where the failed actuators will not apply any torque, the closed-loop Eq. (3.8) becomes

$$\begin{aligned} D_i(q)\dot{r} + C_i(q,\dot{q})r &= -Y_i(q,\dot{q},v,\dot{v})\theta_i \\ &+ \sum_{j \neq j_1,\dots,j_{p_i}} [Y_i(q,\dot{q},v,\dot{v})\hat{\theta}_{ij} + \hat{p}_{ij} - K_{ij} r_i] \\ &+ \sum_{j = j_1,\dots,j_{p_i}} \bar{\tau}_{ij}, \end{aligned} \tag{3.11}$$

for $i = 1, 2, \dots, n$. The parameters θ_{ij} and p_{ij}, the nominal values of $\hat{\theta}_{ij}$ and $\hat{p}_{ij}$, exist to satisfy the matching equations:

$$\sum_{j \neq j_1,\dots,j_{p_i}} \theta_{ij} = \theta_i \tag{3.12}$$

$$\sum_{j \neq j_1,\dots,j_{p_i}} p_{ij} + \sum_{j = j_1,\dots,j_{p_i}} \bar{\tau}_{ij} = 0 \tag{3.13}$$

where θ_{ij} and p_{ij} change their values when new failures appear.

In the closed-loop Eq. (3.11), by considering the Eqs. (3.12), (3.13) and adding and subtracting the same term, $\sum_{j \neq j_1,\dots,j_{p_i}} (Y_i\theta_{ij} + p_{ij})$, the closed-loop equation is rewritten as

$$\begin{aligned} D_i(q)\dot{r}_i + C_i(q,\dot{q})r_i &= \sum_{j \neq j_1,\dots,j_{p_i}} Y_i\tilde{\theta}_{ij} \\ &+ \sum_{j \neq j_1,\dots,j_{p_i}} \tilde{p}_{ij} - \sum_{j \neq j_1,\dots,j_{p_i}} K_{ij} r_i, \end{aligned} \tag{3.14}$$

where $\tilde{\theta}_{ij} = \hat{\theta}_{ij} - \theta_{ij}$, $\tilde{p}_{ij} = \hat{p}_{ij} - p_{ij}$.

With a slight abuse of the notation $j_1, \dots, j_n$, for the n-link manipulator, the closed-loop system can be written as

$$D(q)\dot{r} + C(q,\dot{q})r = \begin{bmatrix} \sum_{j_1 \neq j_{11},\dots,j_{1p_1}} Y_1\tilde{\theta}_{1j_1} \\ \vdots \\ \sum_{j_n \neq j_{n1},\dots,j_{np_n}} Y_n\tilde{\theta}_{nj_n} \end{bmatrix} + \begin{bmatrix} \sum_{j_1 \neq j_{11},\dots,j_{1p_1}} \tilde{p}_{1j_1} \\ \vdots \\ \sum_{j_n \neq j_{n1},\dots,j_{np_n}} \tilde{p}_{nj_n} \end{bmatrix} - \begin{bmatrix} \sum_{j_1 \neq j_{11},\dots,j_{1p_1}} K_{1j_1} r_1 \\ \vdots \\ \sum_{j_n \neq j_{n1},\dots,j_{np_n}} K_{nj_n} r_n \end{bmatrix}. \tag{3.15}$$

Suppose that failures happen at time instants t_k, $k = 1, 2, \dots, N$ and $0 < t_1 < t_2 < \cdots < t_N$ (at each time instant t_k, there may be more than one actuator failures at different joints). We consider such a Lyapunov function as

$$\begin{aligned} V = V_k &= \frac{1}{2} r^T D(q) r + \frac{1}{2}\sum_{i=1}^{n} \sum_{j_i \neq j_{i1},\dots,j_{ip_i}} \tilde{\theta}_{ij_i}^T \Gamma_{ij_i}^{-1} \tilde{\theta}_{ij_i} \\ &+ \frac{1}{2}\sum_{i=1}^{n} \sum_{j_i \neq j_{i1},\dots,j_{ip_i}} \gamma_{ij_i}^{-1} \tilde{p}_{ij_i}^2 \end{aligned} \tag{3.16}$$

for each time interval (t_k, t_{k+1}), $k = 0, 1, \dots, N$, with $t_0 = 0$ and $t_{N+1} = \infty$, corresponding to a certain failure pattern as $\{j_1, j_2, \dots, j_{p_i}\}$ for the ith joint, where $\Gamma_{ij_i} = \Gamma_{ij_i}^T > 0$ and $\gamma_{ij_i} > 0$.

Differentiating V in the interval (t_k, t_{k+1}) yields

$$\dot{V} = r^T D(q)\dot{r} + \frac{1}{2} r^T \dot{D}(q) r + \sum_{i=1}^{n} \sum_{j_i \neq j_{i1},\ldots,j_{ip_i}} \dot{\tilde{\theta}}_{ij_i}^T \Gamma_{ij_i}^{-1} \tilde{\theta}_{ij_i} + \sum_{i=1}^{n} \sum_{j_i \neq j_{i1},\ldots,j_{ip_i}} \gamma_{ij_i}^{-1} \dot{\tilde{p}}_{ij_i} \tilde{p}_{ij_i}. \quad (3.17)$$

Substituting the (3.15) into (3.17) results

$$\dot{V} = \frac{1}{2} r^T [\dot{D}(q) - 2C(q,\dot{q})] r + r^T \begin{bmatrix} \sum_{j_1 \neq j_{11},\ldots,j_{1p_1}} Y_1 \tilde{\theta}_{1j_1} \\ \vdots \\ \sum_{j_n \neq j_{n1},\ldots,j_{np_n}} Y_n \tilde{\theta}_{nj_n} \end{bmatrix} + r^T \begin{bmatrix} \sum_{j_1 \neq j_{11},\ldots,j_{1p_1}} \tilde{p}_{1j_1} \\ \vdots \\ \sum_{j_n \neq j_{n1},\ldots,j_{np_n}} \tilde{p}_{nj_n} \end{bmatrix} - r^T \begin{bmatrix} \sum_{j_1 \neq j_{11},\ldots,j_{1p_1}} K_{1j_1} r_1 \\ \vdots \\ \sum_{j_n \neq j_{n1},\ldots,j_{np_n}} K_{nj_n} r_n \end{bmatrix} + \sum_{i=1}^{n} \sum_{j_i \neq j_{i1},\ldots,j_{ip_i}} \dot{\tilde{\theta}}_{ij_i}^T \Gamma_{ij_i}^{-1} \tilde{\theta}_{ij_i} + \sum_{i=1}^{n} \sum_{j_i \neq j_{i1},\ldots,j_{ip_i}} \gamma_{ij_i}^{-1} \dot{\tilde{p}}_{ij_i} \tilde{p}_{ij_i} \quad (3.18)$$

where the first term results in zero from the skew-symmetric property of $\dot{D}(q) - 2C(q,\dot{q})$.

The parameter update laws are chosen as

$$\dot{\hat{\theta}}_{ij} = \dot{\tilde{\theta}}_{ij} = -\Gamma_{ij} Y_i^T(q,\dot{q},v,\dot{v}) r_i, \quad \Gamma_{ij} = \Gamma_{ij}^T > 0 \quad (3.19)$$

$$\dot{\hat{p}}_{ij} = \dot{\tilde{p}}_{ij} = -\gamma_{ij} r_i, \ -\gamma_{ij} > 0, \quad (3.20)$$

with $\hat{\theta}_{ij}(0) = \hat{\theta}_{i0}$ for some $\hat{\theta}_{i0}$ such that $\sum_{j=1}^{m_i} \hat{\theta}_{i0} = m_i \hat{\theta}_{i0}$ is an initial estimate of θ_i, and $\hat{p}_{ij}(0) = 0$, where $i = 1, \ldots, n$, $j = 1, \ldots, m_i$. The derivative of the Lyapunov function is then found as

$$\dot{V} = -\sum_{i=1}^{n} \sum_{j_i \neq j_{i1},\ldots,j_{ip_i}} K_{ij_i} r_i^2 \leq 0, \quad (3.21)$$

for each time interval (t_k, t_{k+1}).

Whenever new failures occur, the Lyapunov function $V = V_k$ changes with actuator failures into V_{k+1} such that V is not continuous at the time instants t_k, $k = 0, 1, \ldots, N$. Except for a finite number (N as indicated here) of discontinuous points, V is differentiable with a negative time derivative, which means V decreases with time in each time interval (t_k, t_{k+1}) when there is no actuator failures during this time span. Starting from the first time interval $[t_0, t_1)$, we see that $V(t) \leq V(t_0)$ from $\dot{V} \leq 0$ for $\forall t \in [t_0, t_1)$. It is concluded that all closed-loop signals are bounded for $t \in [t_0, t_1)$, including $\hat{\theta}_{ij}(t)$ and $\hat{p}_{ij}(t)$. At time $t = t_1$, some actuators at some joints fail, which results in the abrupt change of V from V_0 to V_1 with a set of new finite constants θ_{ij} and p_{ij}. In addition to the new constants θ_{ij} and p_{ij} satisfying the matching conditions (3.12)–(3.13), some of the parameter estimates $\hat{\theta}_{ij}(t)$ and $\hat{p}_{ij}(t)$ are removed from the Lyapunov function V because their corresponding actuators are not working anymore. Since $\hat{\theta}_{ij}(t)$ and $\hat{p}_{ij}(t)$ are continuous and are finite at time t_1, the change of V, however, is a jumping with a finite value, that is, $V(t_1^+) = V_1(t_1)$ is bounded. Repeating the argument above, we establish the boundedness of $r(t)$, $\hat{\theta}_{ij}(t)$, and $\hat{p}_{ij}(t)$ for some j corresponding to the remaining actuators in the time interval (t_1, t_2) and prove that $V(t_2^+) = V_2(t_2)$ is bounded. Continuing in the same way, we have that $V(t) \leq V(t_k^+)$ for $\forall t \in (t_k, t_{k+1})$ with a finite $V(t_k^+)$, $k = 0, 1, \ldots, N$. Therefore, we conclude that $V(t)$ is piecewise continuous and bounded.

Recall that at each joint, there remains at least one actuator for achieving the control objective. Hence at least one pair of $\hat{\theta}_{ij}(t)$ and $\hat{p}_{ij}(t)$ with some j for each i remains in the Lyapunov function V, which implies that $\hat{\theta}_{ij}(t)$ and $\hat{p}_{ij}(t)$ with some $j \in \{1, 2, \ldots, m_i\}$ for each $i = 1, 2, \ldots, n$ are bounded for $\forall t \in [0, \infty)$. Form the adaptive update laws (3.19) and (3.20), we note that for the ith joint, $\hat{\theta}_{ij}(t)$ and $\hat{p}_{ij}(t)$ are parallel to each other with different adaptive gains for different j. Since at least one pair of them with some j is bounded for $\forall t \in [0, \infty)$, the others are also bounded in the sense that the adaptive laws for them are calculated in computing chips virtually even if the signals may not exist due to the failures in the corresponding actuators. It follows that all closed-loop signals are bounded for both the real signals applied to the manipulator system and virtual signals calculated in computing chips.

Considering the last time interval (t_N, ∞) with a finite $V(t_N^+)$, we see that it follows from (3.21) that $r(t) \in L^2$. On the other hand, from the boundedness of the closed-loop signals, it can be shown that $\dot{r}(t) \in L^\infty$ so that $\lim_{t\to\infty} r(t) = 0$, from which it can be shown that $\lim_{t\to\infty} e(t) = 0$. Thus, stability in the Lyapunov sense and asymptotic tracking: $\lim_{t\to\infty} e(t) = 0$ are established.

Remark 3.1. For time-varying actuator failures modeled as (3.4), a complete parameterization of the actuator failures can be obtained as shown in ref. [18]. With the parameterization of actuator failures and system uncertainties, the proposed adaptive compensation design in this section can be extended to achieve asymptotic tacking of reference signals for concurrently actuated manipulator system in the presence of the time-varying actuator failures. In case that the failure signal $f_{ijk}(t)$ in the failure model (3.4) is unknown, while $f_{ijk}(t)$ is bounded by some function of time, a modified bounding design of adaptive compensation can be applied to the concurrently actuated manipulator for achieving stability and desired tracking performance in the sense that the tracking error can be made as small as expected by choosing larger design parameters.

4. Dynamic Equations for Cooperating Manipulators

In the design of adaptive actuator failure compensation scheme for a cooperating multiple manipulator system, the dynamic equations of the system in the task space are derived and by modifying the control algorithm designed for concurrently actuated manipulator system the control algorithm for a cooperating multiple manipulator system is derived.

When cooperating multiple manipulators are moving an object, they form a closed kinematic chain, where the system is constrained by holonomic and nonholonomic constrains between the manipulators themselves and the object. The dynamic modeling of the cooperating multiple manipulator system in task space is derived in this section. It is assumed that each manipulator, which is non-redundant with the same n degree of freedom, does not enter any singular configuration and there is no relative motion between the object and the manipulator end-effectors. By formulating the control problem in task space, the need for solving the inverse kinematics problem is eliminated, but these control schemes still require the Jacobian matrix to be known. In ref. [14] a PID control algorithm is designed to compensate for the uncertainties in the Jacobian matrix, where an estimator is used to obtain an approximation of the Jacobian matrix.

The dynamic equation of the ith manipulator with n DOF in a multiple manipulator system is given as shown in ref.[20]:

$$D_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i - J_i^T F_i \tag{4.1}$$

for $i = 1, 2, \ldots, m$, where m is the number of manipulators in the multiple manipulator system, q_i, $\dot{q}_i$ and $\ddot{q}_i \in R^n$ are joint variable position, velocity and acceleration vectors, $D_i(q_i) \in R^{n\times n}$ is the inertia matrix, $C_i(q_i, \dot{q}_i) \in R^{n\times n}$ is the Coriolis and centrifugal terms, $g_i(q_i) \in R^{n\times 1}$ is the gravity term, $\tau_i \in R^{n\times 1}$ is the generalized torque vector, J_i^T is the Jacobian matrix defined as $J_i = \partial h_i(q_i)/\partial q_i$ with h_i being the forward kinematic output function for the ith manipulator to be defined in (4.4) and F_i is the applied generalized end-effector force vector for the ith manipulator.

Equation of motion for the common object is formulated as

$$D_o(x_c)\ddot{x}_c + C_o(x_c, \dot{x}_c)\dot{x}_c + g_o(x_c) = A^T F, \tag{4.2}$$

where $x_c \in R^{n_c\times 1}$, $\dot{x}_c$ and $\ddot{x}_c$ are position, velocity and acceleration vectors of the center of the mass of the object, n_c is the dimension of the position vector of the mass of the object, $D_o(x_c) \in R^{n_c\times n_c}$ is the inertia matrix of the object, $C_o(x_c, \dot{x}_c) \in R^{n_c\times n_c}$ is the Coriolis and centrifugal terms of the object, $g_o(x_c) \in R^{n_c\times 1}$ is the gravity term, $F = [F_1^T, \ldots, F_m^T]^T$ is the generalized end-effector force vector and $A \in R^{nm\times n_c}$ is the Jacobian matrix defined by

$$A = \left[A_1^T, \ldots, A_m^T\right]^T, \quad A_i = \frac{\partial \pi_i(x_c)}{\partial x_c}, \tag{4.3}$$

in which the constraint equations are defined as

$$\begin{aligned} x_i &= h_i(q_i) = \pi_i(x_c), \\ \dot{x}_i &= J_i\dot{q}_i = A_i\dot{x}_c, \quad i = 1, 2, \ldots, m, \end{aligned} \tag{4.4}$$

where $x_i \in R^n$ is the position of the ith manipulator in the Cartesian coordinates, h_i is the forward kinematic output function of the ith manipulator and π_i is the transformation matrix from object frame to the ith manipulator end-effector frame.

When the dynamic equation of motion of the ith manipulator is written in the object coordinate, it is formulated as ref. [21]

$$\bar{D}_i(x_c)\ddot{x}_c + \bar{C}_i(x_c, \dot{x}_c)\dot{x}_c + \bar{g}_i(x_c) = E_i^T(x_c)\tau_i - A_i^T F_i, \tag{4.5}$$

where $E_i(x_c) = J_i^{-1}A_i$, $\bar{D}_i = E_i^T D_i E_i$, $\bar{C}_i = E_i^T C_i E_i + E_i^T D_i \dot{E}_i$ and $\bar{g}_i = E_i^T g_i$.

By summing the manipulator dynamic Eq. (4.5) with the object dynamic Eq. (4.2), the dynamic equation of the system in the task space is formulated as

$$D(x_c)\ddot{x}_c + C(x_c, \dot{x}_c)\dot{x}_c + g(x_c) = E^T(x_c)\tau, \tag{4.6}$$

where $D = \sum_{i=1}^m \bar{D}_i + D_o$, $C = \sum_{i=1}^m \bar{C}_i + C_o$, $g = \sum_{i=1}^m \bar{g}_i + g_o$, $E = [E_1^T, E_2^T, \ldots, E_m^T]^T \in R^{nm\times n_c}$ and $\tau = [\tau_1^T, \tau_2^T, \ldots, \tau_m^T]^T \in R^{nm\times 1}$.

5. Cooperating Actuator Failure Compensation

In this section, the failure compensation problem for a cooperating manipulator system is formulated and an adaptive control scheme is designed to ensure system stability and asymptotic tracking of a reference for the object in the task space. By using a direct adaptive design, it is expected that there is no need for fault detection and isolation algorithms in order to ensure the desired system performance in the presence of failures.

5.1. Problem formulation

In a multiple manipulator system, the actuator failure problem is formulated as follows: at an unknown time instant, some actuators at the joints of some manipulators may fail during operation. There can be up to $nm - n_c$ failures in the multiple manipulator system (4.6), that is, at least n_c independent actuators at the joints of some manipulators are left for guaranteeing the control objective, because the manipulators with n DOF are independent of each other.

The actuator failure is modeled as

$$\tau_{ij}(t) = \bar{\tau}_{ij}, \quad t \geq t_{ij}, \tag{5.1}$$

for the ith manipulator jth joint, $i \in \{1, 2, \ldots, m\}$, $j \in \{1, 2, \ldots, n\}$, where the failure time instant t_{ij} and the constant value of $\bar{\tau}_{ij}$ are unknown.

In case of actuator failures, the actual input τ can be expressed as

$$\tau(t) = v(t) + \sigma(\bar{\tau} - v(t)) \tag{5.2}$$

where $v(t) \in R^{nm}$ is the applied control input to be determined, $\bar{\tau} = [\bar{\tau}_{11}, \ldots, \bar{\tau}_{1n}, \ldots, \bar{\tau}_{ij}, \ldots, \bar{\tau}_{mn}]^T$ is the vector of torques from the failed actuators, $\sigma = \mathrm{diag}\{\sigma_{11}, \ldots, \sigma_{1n}, \ldots, \sigma_{ij}, \ldots, \sigma_{mn}\}$ is an $nm \times nm$ diagonal matrix identifying the unknown failure pattern with $\sigma_{ij} = 1$, if the actuator at the jth joint of the ith manipulator fails, otherwise, $\sigma_{ij} = 0$.

When some actuators are failed in a certain failure pattern σ, the manipulator dynamic Eq. (4.6) becomes

$$D(x_c)\ddot{x}_c + C(x_c, \dot{x}_c)\dot{x}_c + g(x_c) = E^T(x_c)\sigma\bar{\tau} + E^T(x_c)(I - \sigma)v(t). \quad (5.3)$$

The objective of adaptive compensation control is to adjust the remaining control inputs to achieve the desired system performance, when there are up to $nm - n_c$ actuator failures at manipulator joints with unknown failure time instants t_{ij}, failure parameters $\bar{\tau}_{ij,}$ and failure locations σ_{ij}, in addition to system parameter uncertainties. More specifically, the control objective is to design a feedback control law $v(t)$ for the dynamic systems (5.3) to ensure that all closed-loop system signals and parameter estimates are bounded and that in the task space, the object position $x_c(t)$ asymptotically tracks a given reference $x_{cd}(t)$.

5.2. Adaptive controller design

Because of the uncertainties in actuator failures and manipulator system parameters, an adaptive design is used to generate $v(t) = [v_1(t), v_2(t), \ldots, v_{nm-1}(t), v_{nm}(t)]^T \in R^{nm}$, to compensate for actuator failures and system uncertainties.

The advantage of designing the controller in the task space is that there is no need to measure the forces acting on the end-effectors of the manipulators, since these internal forces are eliminated in the dynamical modeling. However, the task space control of a manipulator system requires knowledge of the common object position and velocity vectors, x_c and $\dot{x}_c$, which can be measured by using vision systems.[19]

Define the tracking error $e_x \in R^{n_c}$ and the filtered tracking errors $r_x \in R^{n_c}$ and $v_x \in R^{n_c}$ as

$$e_x = x_c - x_{cd}, \quad r_x = \dot{e}_x + \lambda e_x, \quad v_x = \dot{x}_{cd} - \lambda e_x, \quad (5.4)$$

where x_{cd} is the reference for x_c and $\lambda \in R^{n_c \times n_c} > 0$ is a gain matrix.

The closed-loop Eq. (5.3) can be expressed as

$$D(x_c)\dot{r}_x + C(x_c, \dot{x}_c)r_x = -Y_x(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_x + E^T(x_c)\sigma\bar{\tau} + E^T(x_c)(I - \sigma)v(t), \quad (5.5)$$

where

$$Y_x(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_x = D(x_c)\dot{v}_x + C(x_c, \dot{x}_c)v_x + g(x_c), \quad (5.6)$$

$\theta_x \in R^{n_\theta}$ is an unknown system parameter vector and $Y_x \in R^{n_c \times n_\theta}$ is a matrix of known functions.

Failure parameterization. Suppose that for any up to $nm - n_c$ actuator failures, there is no singular configuration in the multiple manipulator system, which implies that the matrix $E^T(x_c)(I - \sigma)$ has full row rank for $\forall\sigma \in \Sigma$, where $\Sigma = \{\sigma_i |\ i = 1, 2, \ldots, N\}$ is the set of all failure patterns with $N = \sum_{k=0}^{nm-n_c} \binom{nm}{k}$, that is, σ_i is a particular expression of σ defined in (5.2) and there are totally N of them. Since actuator failures can be identified by their failure patterns, we use $\sigma_{\bar{i}}$ with some $\bar{i} \in \{1, 2, \ldots, N\}$ to represent one specific failure pattern among all N actuator failure patterns. It is also noted that only one actuator pattern in Σ will happen at a time. Introducing $\bar{E}_i(x_c) = (I - \sigma_i)E(x_c)$ for $i = 1, 2, \ldots, N$, we define

$$\begin{aligned} Y &= \left[\bar{E}_1\left(\bar{E}_1^T\bar{E}_1\right)^{-1}Y_x, \bar{E}_2\left(\bar{E}_2^T\bar{E}_2\right)^{-1}Y_x, \ldots, \bar{E}_N\left(\bar{E}_N^T\bar{E}_N\right)^{-1}Y_x\right] \in R^{nm\times Nn_\theta}, \\ H &= \left[\bar{E}_1\left(\bar{E}_1^T\bar{E}_1\right)^{-1}E^T, \bar{E}_2\left(\bar{E}_2^T\bar{E}_2\right)^{-1}E^T, \ldots, \bar{E}_N\left(\bar{E}_N^T\bar{E}_N\right)^{-1}E^T\right] \in R^{nm\times Nnm}, \\ G &= \left[\bar{E}_1\left(\bar{E}_1^T\bar{E}_1\right)^{-1}, \bar{E}_2\left(\bar{E}_2^T\bar{E}_2\right)^{-1}, \ldots, \bar{E}_N\left(\bar{E}_N^T\bar{E}_N\right)^{-1}\right] \in R^{nm\times Nn_c}. \end{aligned} \quad (5.7)$$

For convenience, we denote σ_1 as the pattern for the no failure case, that is, $\sigma_1 = 0$, a zero matrix and $\bar{E}_1(x_c) = E(x_c)$, corresponding to $\binom{nm}{0}$ in $N = \sum_{k=0}^{nm-n_c}\binom{nm}{k}$.

With Y, H, and G defined in (5.7), we propose the following nominal controller

$$v^*(t) = Y(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_0 - H(x_c)\rho_0 - G(x_c)K_0 r_x, \quad (5.8)$$

where $\theta_0 = [\theta_{01}^T, \theta_{02}^T, \ldots, \theta_{0N}^T]^T \in R^{Nn_\theta}$ and $\rho_0 = [\rho_{01}^T, \rho_{02}^T, \ldots, \rho_{0N}^T]^T \in R^{Nnm}$ are parameter vectors with $\theta_{0i} \in R^{n_\theta}$ and $\rho_{0i} \in R^{nm}$, $i = 1, 2, \ldots, N$ and $K_0 = [K_{01}, K_{02}, \ldots, K_{0N}]^T \in R^{Nn_c\times n_c}$ is a matrix with N diagonal matrices $K_{0i} \in R^{n_c\times n_c}$, $i = 1, 2, \ldots, N$.

Substituting (5.8) into (5.5), we obtain the closed-loop system

$$\begin{aligned} D(x_c)\dot{r}_x + C(x_c, \dot{x}_c)r_x &= -Y_x(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_x + E^T(x_c)\sigma\bar{\tau} \\ &\quad + E^T(x_c)(I - \sigma)\big(Y(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_0 \\ &\quad - H(x_c)\rho_0 - G(x_c)K_0 r_x\big). \end{aligned} \quad (5.9)$$

To ensure asymptotic tracking, matching conditions are derived for the nominal controller as

$$\begin{aligned} E^T(x_c)(I - \sigma)Y(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_0 &= Y_x(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_x \\ E^T(x_c)(I - \sigma)H(x_c)\rho_0 &= E^T(x_c)\sigma\bar{\tau} \\ E^T(x_c)(I - \sigma)G(x_c)K_0 r_x &= K_D r_x, \end{aligned} \quad (5.10)$$

where $K_D \in R^{n_c\times n_c}$ is a diagonal matrix whose diagonal elements are defined to be positive constants. With the matching conditions (5.10) satisfied, the closed-loop system becomes

$$D(x_c)\dot{r}_x + C(x_c, \dot{x}_c)r_x = -K_D r_x. \quad (5.11)$$

The stability and asymptotic tracking property of the closed-loop system (5.11) can be shown by the Lyapunov function $V = \frac{1}{2}r_x^T D(x_c)r_x$ whose time derivative $\dot{V} = -r_x^T K_D r_x$.

The matching conditions given by (5.10) are satisfied because the Eq. (5.10) is solvable for $\forall\sigma \in \Sigma$. One solution to (5.10) for a specific failure pattern $\sigma_{\bar{i}}$ is

$$\begin{cases} \theta_{0\bar{i}} = \theta_x,\ \rho_{0\bar{i}} = \sigma\bar{\tau},\ K_{0\bar{i}} = K_D \\ \quad \text{if } \sigma = \sigma_{\bar{i}}, \bar{i} \in \{1, 2, \ldots, N\}, \\ \theta_{0i} = 0,\ \rho_{0i} = 0,\ K_{0\bar{i}} = 0 \\ \quad \text{for } i = 1, 2, \ldots, N \text{ and } i \neq \bar{i}. \end{cases} \tag{5.12}$$

Since we do not know which actuator will fail during system operation, we choose a controller structure for each $v_j^*(t)$ instead of $v^*(t)$ as

$$v_j^*(t) = Y_j(x_c, \dot{x}_c, v_x, \dot{v}_x)\theta_j - H_j(x_c)\rho_j - G_j(x_c)K_j r_x, \quad j = 1, 2, \ldots, nm, \tag{5.13}$$

where Y_j is the jth row of Y, H_j is the jth row of H, and G_j is the jth row of G, $\theta_j = [\theta_{j1}^T, \theta_{j2}^T, \ldots, \theta_{jN}^T]^T \in R^{Nn_\theta}$ and $\rho_j = [\rho_{j1}^T, \rho_{j2}^T, \ldots, \rho_{jN}^T]^T \in R^{Nnm}$ and $K_j = [K_{j1}, K_{j2}, \ldots, K_{jN}]^T \in R^{Nn_c \times n_c}$. Based on the matching conditions (5.10), the nominal values of θ_j, ρ_j and K_j, $j = 1, 2, \ldots, nm$, can be

$$\theta_j = \theta_0, \quad \rho_j = \rho_0, \quad K_j = K_0, \quad j = 1, 2, \ldots, nm. \tag{5.14}$$

Control law. Suppose that at time t, there are p failed actuators with a failure pattern $\sigma = \sigma_{\bar{i}}$, that is, for $u(t) = \tau(t) = [u_1(t), u_2(t), \ldots, u_{nm-1}(t), u_{nm}(t)]^T \in R^{nm}$ and $\bar{u} = \bar{\tau} = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_{nm-1}, \bar{u}_{nm}]^T \in R^{nm}$, it holds that $u_j = \bar{u}_j$, $j = j_1, j_2, \ldots, j_p$, for some subset $\{j_1, j_2, \ldots, j_p\} \subset \{1, 2, \ldots, nm\}$ and that $u_j(t) = v_j(t)$, all $j \in \{1, 2, \ldots, nm\}$ but $j \neq j_1, j_2, \ldots, j_p$. For $j_1, j_2, \ldots, j_p$ unknown, the adaptive control law is designed for all $v_j(t)$ as

$$v_j(t) = Y_j(x_c, \dot{x}_c, v_x, \dot{v}_x)\hat{\theta}_j - H_j(x_c)\hat{\rho}_j - G_j(x_c)\hat{K}_j r_x, \quad j = 1, 2, \ldots, nm, \tag{5.15}$$

where $\hat{\theta}_j \in R^{Nn_\theta}$ and $\hat{\rho}_j \in R^{Nnm}$ are the estimates of θ_j and ρ_j given by (5.14). With the adaptive controller (5.15), the closed-loop system for $\sigma = \sigma_{\bar{i}}$ is given by

$$\begin{aligned} D(x_c)\dot{r}_x + C(x_c, \dot{x}_c)r_x &= -Y_x\theta_x + E^T\sigma_{\bar{i}}\bar{\tau} + \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T Y_j\hat{\theta}_j \\ &\quad - \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T H_j\hat{\rho}_j - \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T G_j\hat{K}_j r_x \\ &= -Y_x\theta_x + E^T\sigma_{\bar{i}}\bar{\tau} + \bar{E}_{\bar{i}}^T Y\theta_0 - \bar{E}_{\bar{i}}^T H\rho_0 \\ &\quad + \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T Y_j\tilde{\theta}_j - \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T H_j\tilde{\rho}_j \\ &\quad - \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T G_j\tilde{K}_j r_x - K_D r_x \\ &= \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T(x_c)Y_j(x_c, \dot{x}_c, v_x, \dot{v}_x)\tilde{\theta}_j \\ &\quad - \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T(x_c)H_j(x_c)\tilde{\rho}_j \\ &\quad - \sum_{j\neq j_1,\ldots j_p} E_{(j)}^T(x_c)G_j(x_c)\tilde{K}_j(x_c)r_x - K_D r_x, \end{aligned} \tag{5.16}$$

where $E_{(j)}$ is the jth row of E and $\tilde{\theta}_j = \hat{\theta}_j - \theta_j$, $\tilde{\rho}_j = \hat{\rho}_j - \rho_j$ and $\tilde{K}_j = \hat{K}_j - K_j$, are the parameter errors (for K_j defined to be with a positive K_D in (5.10), (5.12) and (5.14)).

Adaptive scheme. Let $\hat{K}_{jil}$ denote the lth diagonal elements of the diagonal matrix $\hat{K}_{ji}$, $j = 1, 2, \ldots, nm$, $i = 1, 2, \ldots, N$, $l = 1, 2, \ldots, n_c$. The parameter update laws of $\hat{\theta}_j(t)$, $\hat{\rho}_j(t)$ and $\hat{K}_{jil}(t)$ are designed as

$$\dot{\hat{\theta}}_j = -\Gamma_{\theta j}Y_j^T(x_c, \dot{x}_c, v_x, \dot{v}_x)E_{(j)}(x_c)r_x, \quad \Gamma_{\theta j} = \Gamma_{\theta j}^T > 0 \tag{5.17}$$

$$\dot{\hat{\rho}}_j = \Gamma_{\rho j}H_j^T(x_c)E_{(j)}(x_c)r_x, \quad \Gamma_{\rho j} = \Gamma_{\rho j}^T > 0 \tag{5.18}$$

$$\dot{\hat{K}}_{jil} = \gamma_{jil}G_{jil}(x_c)r_{xl}E_{(j)}(x_c)r_x,\ \gamma_{jil} > 0, \tag{5.19}$$

for $j = 1, 2, \ldots, nm$, $i = 1, 2, \ldots, N$ and $l = 1, 2, \ldots, n_c$, with $\hat{\theta}_j(0) = \hat{\theta}_0$, $\hat{\rho}_j(0) = 0$ and $\hat{K}_{jil}(0) = \hat{K}_{jil0}$ as initial estimates of θ_0, $\rho_0 = 0$ and K_0 defined by (5.12) with $i = 1$ (the no failure case, such that $\bar{\tau} = 0$ holds), where $G_{jil}(x_c)$ is the $[(i-1)n_c + l]$th component of $G_j(x_c)$, r_{xl} is the lth component of r_x, $\Gamma_{\theta j} = \Gamma_{\theta j}^T > 0$, $\Gamma_{\rho j} = \Gamma_{\rho j}^T > 0$ and $\gamma_{jil} > 0$ are the adaptive gains.

5.3. Stability analysis

Suppose that failures happen at time instants t_k, $k = 1, 2, \ldots, M$ and $0 < t_1 < t_2 < \cdots < t_M$ (at each time instant t_k, there may be more than one actuator failures at some joints of some manipulators). We consider such a Lyapunov function as

$$\begin{aligned} V = V_k &= \frac{1}{2}r_x^T D(x_c)r_x \\ &\quad + \frac{1}{2}\sum_{j\neq j_1,\ldots,j_p} \tilde{\theta}_j^T\Gamma_{\theta j}^{-1}\tilde{\theta}_j + \frac{1}{2}\sum_{j\neq j_1,\ldots,j_p}^{n} \tilde{\rho}_j^T\Gamma_{\rho j}^{-1}\tilde{\rho}_j \\ &\quad + \frac{1}{2}\sum_{j\neq j_1,\ldots,j_p}^{n}\sum_{i=1}^{N}\sum_{l=1}^{n_c}\gamma_{jil}^{-1}\tilde{K}_{jil}^2 \end{aligned} \tag{5.20}$$

for each time interval (t_k, t_{k+1}), $k = 0, 1, \ldots, M$, with $t_0 = 0$ and $t_{M+1} = \infty$, corresponding to a certain failure pattern as $\{j_1, j_2, \ldots, j_p\}$, where $\Gamma_{\theta j} = \Gamma_{\theta j}^T > 0$, $\Gamma_{\rho j} = \Gamma_{\rho j}^T > 0$ and $\gamma_{jil} > 0$.

Differentiating V with respect to time in the interval (t_k, t_{k+1}) along (5.16) yields

$$\begin{aligned} \dot{V} &= \frac{1}{2}r_x^T[\dot{D}(x_c) - 2C(x_c, \dot{x}_c)]r_x \\ &\quad + \sum_{j\neq j_1,\ldots j_p} r_x^T E_{(j)}^T(x_c)Y_j(x_c, \dot{x}_c, v_x, \dot{v}_x)\tilde{\theta}_j \end{aligned}$$

$$
\begin{aligned}
&- \sum_{j \neq j_1,\ldots j_p} r_x^T E_{(j)}^T(x_c) H_j(x_c) \tilde{\rho}_j \\
&- \sum_{j \neq j_1,\ldots j_p} r_x^T E_{(j)}^T(x_c) G_j(x_c) \tilde{K}_j r_x - r_x^T K_D r_x \\
&+ \sum_{j \neq j_1,\ldots,j_p} \dot{\tilde{\theta}}_j^T \Gamma_{\theta j}^{-1} \tilde{\theta}_j + \sum_{j \neq j_1,\ldots,j_p}^{n} \dot{\tilde{\rho}}_j^T \Gamma_{\rho j}^{-1} \tilde{\rho}_j \\
&+ \sum_{j \neq j_1,\ldots,j_p}^{n} \sum_{i=1}^{N} \sum_{l=1}^{n_c} \gamma_{jil}^{-1} \dot{\tilde{K}}_{jil} \tilde{K}_{jil},
\end{aligned} \tag{5.21}
$$

where the first term results in zero from the skew-symmetric property of $\dot{D}(q) - 2C(q, \dot{q})$. With the adaptive update laws (5.17)–(5.19), the time-derivative of the Lyapunov function V becomes

$$
\dot{V} = -r_x^T K_D r_x \leq 0. \tag{5.22}
$$

When new actuator failures occur, the Lyapunov function $V = V_k$ changes with failures into V_{k+1}. Hence V is not continuous at the time instants t_k, $k = 0, 1, \ldots, M$. Except for a finite number (M as indicated here) of discontinuous points, V is differentiable with a negative time derivative, that is, V decreases with time in each time interval (t_k, t_{k+1}) when there is no actuator failures during this time span.

Starting from the first time interval $[t_0, t_1)$, we see that $V(t) \leq V(t_0)$ from $\dot{V} \leq 0$ for $\forall t \in [t_0, t_1)$. It is concluded that all signals are bounded for $t \in [t_0, t_1)$, including the estimates $\hat{\theta}_j(t)$, $\hat{\rho}_j(t)$ and $\hat{K}_{jil}(t)$ for $j = 1, 2, \ldots, nm$, $i = 1, 2, \ldots, N$, $l = 1, 2, \ldots, n_c$.

At time $t = t_1$, when some actuators fail, that is, the control signals to some joints are stopped by some constant torques with unknown values, V changes abruptly from V_0 to V_1. First of all, based on a new failure pattern, the unknown parameters θ_j, ρ_j, and K_{jil} change into a set of new θ_j, ρ_j and K_{jil} with finite values. It is also noted that some of the parameter estimates $\hat{\theta}_j(t)$, $\hat{\rho}_j(t)$, and $\hat{K}_{jil}(t)$ with some $j \in \{1, 2, \ldots, nm\}$ are removed from the Lyapunov function V because their corresponding actuators are not working anymore. Since $\hat{\theta}_j(t)$, $\hat{\rho}_j(t)$, and $\hat{K}_{jil}(t)$ are continuous and are finite at time t_1 and the unknown parameters θ_j, ρ_j, K_{jil} have finite values to satisfy the matching conditions under the current failure pattern, the change of V is a finite value jumping, which means that $V(t_1^+) = V_1(t_1)$ is bounded.

Repeating the argument above, we establish the boundedness of $r(t)$, $\hat{\theta}_j(t)$, $\hat{\rho}_j(t)$, and $\hat{K}_{jil}(t)$ for some j corresponding to the remaining actuators in the time interval (t_1, t_2) and prove that $V(t_2^+) = V_2(t_2)$ is bounded. Continuing in the same way, we have that $V(t) \leq V(t_k^+)$ for $\forall t \in (t_k, t_{k+1})$ with a finite $V(t_k^+)$, $k = 0, 1, \ldots, M$. Therefore, we conclude that $V(t)$ is piecewise continuous and bounded.

Recall that any n_c actuators of the nk actuators are independent and assumed to guarantee the nonsingular property, that is, each row $E_{(j)}(x_c)$ of $E(x_c)$ can be represented by a linear combination of any other n_c rows of $E(x_c)$. Form the adaptive update laws (5.8)–(5.10), we thus know that for the jth designed control input $v_j(t)$, the adaptive laws of its estimates $\hat{\theta}_j(t)$, $\hat{\theta}_j(t)$, and $\hat{K}_{jil}(t)$ are a linear combination of the adaptive laws for the estimates of any other n_c control inputs. On the other hand, at any time there remain at least n_c actuators for achieving the control objective. Hence at least n_c sets of $\hat{\theta}_j(t)$, $\hat{\rho}_j(t)$, and $\hat{K}_{jil}(t)$ with some j remain in the Lyapunov function V, which implies that $\hat{\theta}_j(t)$, $\hat{\rho}_j(t)$, and $\hat{K}_{jil}(t)$ some $j \in \{1, 2, \ldots, nm\}$ (no less than n_c different j) for each $i = 1, 2, \ldots, N$ and $l = 1, 2, \ldots, n_c$ are bounded for $\forall t \in [0, \infty)$. Since at least n_c sets of them with some j are bounded for $\forall t \in [0, \infty)$, the others are also bounded in the sense that the estimates obey their adaptive laws, which are linear combinations of the remaining n_c sets, with different initial values. Notice that even if those estimate signals may not be applied to the system if the corresponding actuators are failed, the adaptive laws of them are still calculated in computing chips virtually. It follows that all closed-loop signals are bounded for both the real signals applied to the manipulator system and virtual signals calculated in computing chips.

Considering the last time interval (t_M, ∞) with a finite $V(t_M^+)$, we see that it follows from (5.22) that $r_x(t) \in L^2$. On the other hand, from the boundedness of the closed-loop signals, it can be shown that $\dot{r}_x(t) \in L^\infty$ so that $\lim_{t\to\infty} r_x(t) = 0$, from which it follows that $\lim_{t\to\infty} e_x(t) = 0$ and $\lim_{t\to\infty} \dot{e}_x(t) = 0$.

Thus, stability in the Lyapunov sense and asymptotic tracking: $\lim_{t\to\infty} e(t) = 0$ are established for the adaptive actuator failure compensation design of the cooperating multiple manipulator system in the task space, despite actuator failures.

6. Conclusions

In this paper, actuator failure compensation for redundant manipulators is addressed, using a new adaptive control method to compensate for uncertainties from actuator failures. The developed adaptive compensation scheme consists of a parametrized controller structure to handle all possible actuator failure patterns and an adaptive law to update the controller parameters when the failure parameters are unknown. Such an adaptive failure compensation scheme is applicable to concurrently actuated manipulator systems whose actuators may fail as well as to cooperating multiple manipulator system whose actuators and joints may fail during the system operation. With a modified Lyapunov stability analysis to deal with discontinuous parameters due to failures, the stability and asymptotic output tracking of the closed-loop systems are proved, in the presence of uncertainties of actuator failures.

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