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Iterative EM-Based Channel Estimation for STBC-OFDM

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***Abstract*—In this paper, an iterative EM based channel estimation algorithm is studied for STBC-OFDM systems. Compared to the time domain EM based channel estimation algorithm which needs matrix inversion, a frequency domain EM based channel estimation algorithm is proposed by estimating the channel coefficients for each subcarrier. The proposed channel estimation algorithm decreased the complexity without sacrificing the performance. The time domain and proposed frequency domain EM based channel estimation algorithms are compared in terms of bit error rate (BER), mean square error (MSE) and the number of iterations used in the EM algorithm.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1] is an attractive and powerful modulation technique for achieving the high data rates required for a wireless multimedia service. It transforms a frequency selective fading channel into multiple narrow flat fading parallel subchannels to mitigate intersymbol interference (ISI) caused due to multipath. Transmitter diversity is an effective technique to combat the fading effect in mobile wireless communications. A simple and powerful diversity technique using two transmit antennas was first proposed by Alamouti [2] and its various derivatives have been developed [3]. When, these diversity techniques are combined with OFDM, it increases the system performance and robustness against fading channels.

Channel estimation is a very crucial task for OFDM systems that accurate and robust channel estimation is necessary in order to demodulate the data coherently. There are many studies for the single-input single-output (SISO) OFDM case in the literature [4] [5]. These methods cannot be simply extended to the multiple antenna scenario, as the received signal at a receive antenna is the sum of the signals transmitted from all the transmit antennas and every tone at the receiver is associated with multiple channel parameters. Pilot-based channel estimation techniques are studied for Space Time Block Coded (STBC) OFDM in [6] [7]. Since this estimation techniques have limited system performance, Expectation-Maximization (EM) algorithm [8] is used to improve the estimation performance through iterations. For SISO-OFDM case, three different EM algorithms are compared in [9]. In [10] a time domain EM algorithm is proposed for Space-Time Trellis Coded (STTC) OFDM systems. This algorithm is modified for STBC-OFDM and Space Frequency Block Coded (SFBC) OFDM in [11]. All these EM algorithms used for space-time codes followed the decomposition of the superimposed signals which is given in [12]. While decomposing the received signal, the noise is also being decomposed with a coefficient which is chosen arbitrarily. In order to obtain good performance, this coefficient must be chosen correctly. These algorithms also require a matrix inversion while updating the channel coefficients which increase the computational complexity.

In this paper, we propose a frequency domain EM based channel estimation algorithm for STBC-OFDM systems in order to prevent the drawback of these algorithms. We estimate the channel coefficients at each subcarrier and do not use matrix inversion. Moreover, we do not need to choose a decomposition coefficient since it is not necessary to use decomposition process. Thus, the computational complexity is decreased without any performance loss.

This paper is organized as follows. First, we describe the uncoded STBC-OFDM system model in section II. Then, we give pilot-based channel estimation and time domain EM-based channel estimation algorithms for STBC-OFDM in section III and IV respectively. The proposed iterative EM-based channel estimation method is described in section V. Finally, we give the simulation results and conclusion in section VI and VII respectively.

II. STBC-OFDM SYSTEM MODEL

In this section, we examine the Alamouti STBC-OFDM which includes two-transmit antennas and one-receive antenna. A simplified block diagram of the system is shown in Figure 1. At time n, a data block $S(n,k)$, $k = 0,1,2,...,K-1$, where K is the number of subcarriers, is encoded into two different symbol blocks, $X_i(n,k)$, $i = 1,2$. These blocks can be given for two concatenated time as below:

$$\begin{aligned} X_1(n,k) &= S(n,k) \\ X_2(n,k) &= S(n+1,k) \\ X_1(n+1,k) &= -S(n+1,k)^* \\ X_2(n+1,k) &= S(n,k)^* \end{aligned} \tag{1}$$

where $n = 0,2,4,..,(N-1)$ and $(.)^*$ denotes complex conjugate.

After a K-length Inverse Fast Fourier Transform (IFFT)

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operation, it is expressed as below:

$$x_i(n,m) = \frac{1}{K}\sum_{k=0}^{K-1} X_i(n,k)e^{\frac{j2\pi mk}{K}},\ i = 1,2. \qquad (2)$$

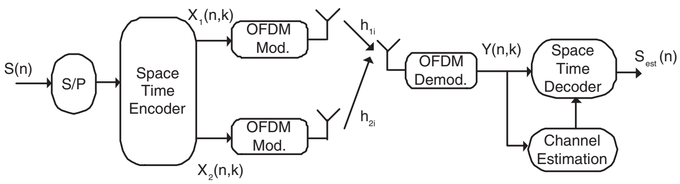

Fig. 1. Alamouti STBC-OFDM System Model

At the transmitter side, the cyclic-prefix (CP) is added to mitigate ISI after IFFT operation because of the property of the OFDM system. Each block is transmitted through different antennas over the same bandwidth using K OFDM subcarriers. At time n, $x_1(n,m)$ and $x_2(n,m)$, at time $n+1$, $-x_2^*(n,m)$ and $x_1^*(n,m)$ are transmitted from the first and second antennas, respectively. At the receiver side, firstly CP is removed and then Fast Fourier Transform (FFT) operation is performed. The received signal is the superposition of the transmitted signals and can be expressed as follows:

$$Y(n,k) = \sum_{i=1}^{2} X_i(n,k)H_i(n,k) + N(n,k), \qquad (3)$$

where $N(n,k)$ is the AWGN with zero mean and σ_N^2 variance for both real and imaginary components and $H_i(n,k)$ denotes the channel frequency response of the multipath channel and the kth subchannel between the ith transmit and receive antenna.

Assuming that the channel is quasi-static and satisfies $H_i(n,k) = H_i(n+1,k) = H_i(k), (n = 0,2,4,...)$, the demodulated signals $Y(n,k)$ and $Y(n+1,k)$ are then decoded by the linear maximum-likelihood space-time decoder:

$$\begin{aligned}\tilde{S}(n,k) &= H_1^*(k)Y(n,k) + H_2(k)Y^*(n+1,k)\\ \tilde{S}(n+1,k) &= H_2^*(k)Y(n,k) - H_1(k)Y^*(n+1,k). \qquad (4)\end{aligned}$$

Finally, hard decision symbols are found as follows:

$$\begin{aligned}\hat{S}(n,k) &= \min_{S\epsilon\Omega} d^2(\tilde{S}(n,k),S)\\ \hat{S}(n+1,k) &= \min_{S\epsilon\Omega} d^2(\tilde{S}(n+1,k),S) \qquad (5)\end{aligned}$$

where Ω is a set of all probable transmitted symbols.

Thus, accurate estimation of channel parameters is the key for decoding of the space-time codes. The remaining part of this paper will focus on the channel estimation issue.

III. Pilot-based STBC-OFDM channel estimation

Pilot-based channel estimation is a widely used estimation technique due to its low computational complexity. The aim of this technique is to use distributed pilot symbols at certain locations in the OFDM time-frequency lattices to estimate the channel coefficients. The channel estimation for STBC-OFDM can be done by transmitting pilot symbols from different antennas at the same frequency bin simultaneously. Therefore, the received signal at the pilot positions will be the superposition of the signals coming from different transmit antennas. Thus, estimation issue becomes more complicated when compared to SISO-OFDM systems.

One of the techniques to solve this problem is sending the pilot symbols using Space-Time Block Codes [6]. The pilot arrangement structure can be seen in Figure 2. It is seen from the figure that at time n $pilot1$ and $pilot2$, at time $n+1$ $-pilot2^*$ and $pilot1^*$ are sent from the first and second antennas, respectively. The received signal model for this system can be expressed in matrix form as:

$$\underline{Y}_{pilot} = \mathbf{X}_{pilot}\underline{H}_{pilot} + \underline{N}_{pilot} \qquad (6)$$

where

$$\begin{aligned}\underline{Y}_{pilot} &= \begin{bmatrix} Y(n,k_p)\\ Y(n+1,k_p)\end{bmatrix}\\ \mathbf{X}_{pilot} &= \begin{bmatrix} X_1(n,k_p) & X_2(n,k_p)\\ -X_2(n,k_p)^* & X_1(n,k_p)^*\end{bmatrix}\\ \underline{H}_{pilot} &= \begin{bmatrix} H_1(k_p)\\ H_2(k_p)\end{bmatrix}\\ \underline{N}_{pilot} &= \begin{bmatrix} N_1(n,k_p)\\ N_2(n+1,k_p)\end{bmatrix}.\end{aligned}$$

In Equation (6), k_p is the subcarrier index represents the pilot positions, $Y(n,k_p)$ and $Y(n+1,k_p)$ are the received signals at time n and $n+1$, respectively. The STBC-OFDM system assumes that $H_i(n,k_p) = H_i(n+1,k_p) = H_i(k_p), n = 0,2,4,...$ and for $i = 1,2$. We can also simplify the equations belongs to pilot positions for time n and $n+1$ as:

Time n:

$$Y(k_p) = X_1(k_p)H_1(k_p) + X_2(k_p)H_2(k_p) + N(k_p) \qquad (7)$$

Time $n+1$:

$$Y(k_p) = -X_2(k_p)^*H_1(k_p) + X_1(k_p)^*H_2(k_p) + N(k_p). \qquad (8)$$

The estimated channel coefficients at pilot positions can be found by using the Equations (7) and (8) as below:

$$\begin{aligned}\hat{H}_1(k_p) &= \frac{1}{2}(Y(n,k_p)X_1^*(n,k_p) - Y(n+1,k_p)X_2(n,k_p))\\ \hat{H}_2(k_p) &= \frac{1}{2}(Y(n,k_p)X_2^*(n,k_p) + Y(n+1,k_p)X_1(n,k_p)) \qquad (9)\end{aligned}$$

where the signal power is normalized to 1.

The estimated channel vectors can be reconstructed for each OFDM symbol

$$\begin{aligned}\underline{\hat{H}}_1 &= [\hat{H}_1(1), \hat{H}_1(D_f+1), ..., \hat{H}_1((N_f-1)D_f+1)]^T\\ \underline{\hat{H}}_2 &= [\hat{H}_2(1), \hat{H}_2(D_f+1), ..., \hat{H}_2((N_f-1)D_f+1)]^T \qquad (10)\end{aligned}$$

where D_f is the distance between the subcarriers and N_f is the number of pilot symbols in the frequency axes. We use Discrete Fourier Transform (DFT)-based interpolation technique in the frequency axes in order to estimate all channel

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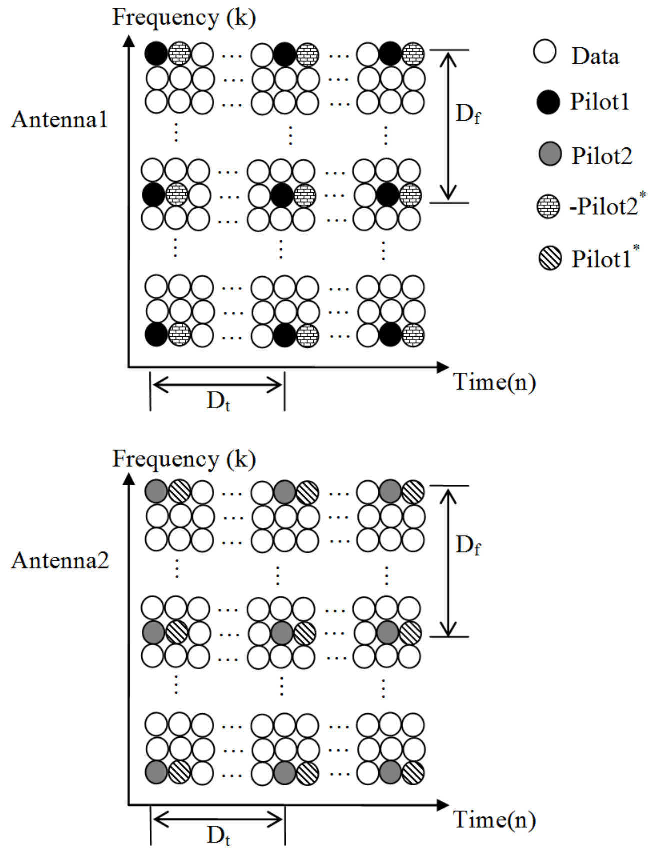

Fig. 2. Pilot Symbol Pattern for one STBC-OFDM frame

coefficients belonging to all subcarriers. First, we transform the frequency channel estimates $\underline{\hat{H}}_1$ and $\underline{\hat{H}}_2$ into time domain as follows:

$$\begin{aligned}\underline{\hat{h}}_1 &= \mathbf{F}^{-1}\underline{\hat{H}}_1 \\ \underline{\hat{h}}_2 &= \mathbf{F}^{-1}\underline{\hat{H}}_2\end{aligned} \tag{11}$$

where $\mathbf{F}$ is the $N_f \times N_f$ DFT matrix. Then we apply a filtering matrix to $\underline{\hat{h}}_1$ and $\underline{\hat{h}}_2$ assuming that the channel response is limited to L_f and obtain the filtered channel response:

$$\begin{aligned}\underline{\hat{h}}_1^{(0)} &= \mathbf{W}\underline{\hat{h}}_1 \\ \underline{\hat{h}}_2^{(0)} &= \mathbf{W}\underline{\hat{h}}_2.\end{aligned} \tag{12}$$

For the above equations $\mathbf{W}$, is the $L_f \times N_f$ filtering matrix. Then we apply DFT in order to obtain the initial estimates for the nth OFDM symbol as follows:

$$\begin{aligned}\underline{\hat{H}}_1^{(0)} &= \mathbf{V}\underline{\hat{h}}_1^{(0)} \\ \underline{\hat{H}}_2^{(0)} &= \mathbf{V}\underline{\hat{h}}_2^{(0)},\end{aligned} \tag{13}$$

where $\mathbf{V}$ is the $K \times L_f$ matrix obtained from the first L_f columns of the $K \times K$ DFT matrix.

In order to estimate the other channel coefficients in the OFDM frame, we simply apply linear interpolation in the time domain using the estimated channel coefficients for pilot OFDM symbol.

IV. Time Domain EM algorithm for STBC-OFDM

The channel estimation performance using pilot symbols can be improved by using Expectation-Maximization algorithm iteratively. The output of the STBC-OFDM system is the superposition of the transmitted symbols from different transmit antennas. Thus, the output data must be decomposed in order to estimate the channel coefficients. An EM algorithm is proposed for the general estimation problem of superimposed signals in [12]. Then this method is used to solve the channel estimation problem in time domain for STBC-OFDM in [11]. The observed data $\underline{Y}$ can be decomposed into two components following [11] as:

$$\underline{Z}_i = \mathbf{X}_i\mathbf{V}\underline{h}_i + \underline{N}_i, \qquad i = 1, 2 \tag{14}$$

where $\underline{N}_i$ are obtained by arbitrarily decomposing the total noise $\underline{N}$ into 2 components such that $\underline{N}_1 + \underline{N}_2 = \underline{N}$. Thus, the relation between the complete data $(\underline{Z}_1, \underline{Z}_2)$ and incomplete data $\underline{Y}$ is given by $\underline{Y} = \underline{Z}_1 + \underline{Z}_2$.

The above described EM based STBC-OFDM channel estimation algorithm can take the following form for each antenna $i = 1, 2$:

E-Step:

$$\underline{Z}_i^{(p)} = \mathbf{X}_i\mathbf{V}\underline{\hat{h}}_i^{(p)}. \tag{15}$$

$$\underline{\hat{Z}}_i^{(p)} = \underline{Z}_i^{(p)} + \beta_i(\underline{Y} - \sum_{j=1}^{2}\underline{Z}_j^{(p)}) \tag{16}$$

M-Step:

$$\underline{\hat{h}}_i^{(p+1)} = \arg\min_{\underline{\hat{h}}_i^{(p)}}\{\|\underline{\hat{Z}}_i^{(p)} - \mathbf{X}_i\mathbf{V}\underline{\hat{h}}_i^{(p)}\|^2\} \tag{17}$$

where $p = 1, 2, ...$ gives the number of iterations. When solving (17), it is obtained:

$$\underline{\hat{h}}_i^{(p+1)} = \mathbf{V}^H\mathbf{X}_i^{-1}\underline{\hat{Z}}_i^{(p)}, \qquad i = 1, 2. \tag{18}$$

The selection of β_i is related to the arbitrarily decomposition of the independent noise components $\underline{N}_i$. The only constraint is $\sum_{i=1}^{2}\beta_i = 1$. This algorithm can only be applied if the transmitted symbols are known such as pilot symbols and it can be carried out twice in each iteration since STBC-OFDM systems use two OFDM symbols. Finally, we can say that this algorithm updates channel coefficients iteratively in the time domain as seen in Equation (18). In this algorithm, the matrix inversion increase the complexity of this algorithm and the selection of β value is very critical since it directly influences the performance. Thus, we propose a frequency domain EM algorithm for STBC-OFDM in the next section.

V. Proposed Frequency domain EM-based channel estimation for STBC-OFDM

We propose to estimate the channel coefficients in the frequency domain iteratively using EM algorithm for STBC-OFDM systems. The estimation process is applied to each subcarrier separately. In this algorithm, there is no need to decompose the signal and noise. Moreover, we do not have to know transmitted symbols in order to estimate the channel coefficients as time domain EM algorithm described in section IV. We propose to decide these unknown transmitted symbols from *antenna 1* and *antenna 2* by computing their coupled probabilities, i.e., probabilities of two symbol combinations.

Using the Gaussian noise assumption, we can calculate the probability density function (pdf) of $\underline{Y}$ given $\mathbf{X}$ and $\underline{H}$ for two concatenated time as:

Time n:

$$f(Y(n,k)|\mathbf{X},\underline{H}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}|\Phi(1)|^2\right\} \quad (19)$$

Time $n+1$:

$$f(Y(n+1,k)|\mathbf{X},\underline{H}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}|\Phi(2)|^2\right\} \quad (20)$$

where

$$\underline{\Phi} = \underline{Y} - \mathbf{X}\underline{H}$$

with

$$\underline{Y} = \begin{bmatrix} Y(n,k) \\ Y(n+1,k) \end{bmatrix}, \; \underline{H} = \begin{bmatrix} H_1(k) \\ H_2(k) \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} X_1(n,k) & X_2(n,k) \\ -X_2^*(n,k) & X_1^*(n,k) \end{bmatrix}.$$

In Equations (19) and (20), we need to know $\mathbf{X}$, which is a matrix contains transmitted symbols from *antenna 1* and *antenna 2*, respectively. These transmitted symbols from *antenna 1* and *antenna 2* at the same time can take *C* different values from an alphabet. The value of *C* is determined using the modulation type of the system. Thus, we can say that *C* is the set of all probable modulated symbol pairs. For example, if we use QPSK modulation, the constellation size is 4 and $C = 16$.

Using these all probable transmitted symbols sent from two antennas, we can find the pdf of $Y(n,k)$ and $Y(n+1,k)$ given $\underline{H}$, respectively as:

Time n:

$$f(Y(n,k)|\underline{H}) = \sum_{l=1}^{C} \frac{1}{\sqrt{2\pi\sigma^2}C} \exp\left\{-\frac{1}{2\sigma^2}|\Phi_l(1)|^2\right\} \quad (21)$$

Time $n+1$:

$$f(Y(n+1,k)|\underline{H}) = \sum_{l=1}^{C} \frac{1}{\sqrt{2\pi\sigma^2}C} \exp\left\{-\frac{1}{2\sigma^2}|\Phi_l(2)|^2\right\} \quad (22)$$

where

$$\underline{\Phi_l} = \underline{Y} - \mathbf{X}_l\underline{H}$$

and $\mathbf{X}_l$ is a matrix which take l different values through a set contains all probable symbols. $(l = 1, ..., C)$

We can apply EM algorithm to improve the channel estimation accuracy. Each iteration $p = 0, 1, 2, ...$ process in the EM algorithm at time n for estimating $\underline{H}$ from $Y(n,k)$ consists of the following two steps:

E-Step:

$$\Theta(\underline{H}|\underline{H}^{(p)}) = E_{\underline{\mathbf{X}}}\{\log f(Y(n,k),\mathbf{X}|\underline{H})|Y(n,k),\underline{H}^{(p)}\} \quad (23)$$

M-Step:

$$\underline{\hat{H}}^{(p+1)} = \arg\max_{\underline{H}} \Theta(\underline{H}|\underline{H}^{(p)}) \quad (24)$$

where

$$\Theta(\underline{H}|\underline{H}^{(p)}) = \sum_{l=1}^{C} \log\left\{\frac{1}{C} f(Y(n,k)|\underline{H},\mathbf{X}_l)\right\} \times \frac{f(Y(n,k)|\underline{H}^{(p)},\mathbf{X}_l)}{Cf(Y(n,k)|\underline{H}^{(p)})}. \quad (25)$$

The same procedure is also applied at time $n+1$ for estimating $\underline{H}$ from $Y(n+1,k)$. Then, the expression given in (24) is differentiated with respect to $\underline{H}$ for two concatenated times and set to zero in order to obtain updated channel coefficients. Then, combining these updated channel coefficients obtained at time n and $n+1$, we have:

For each $k = 0, 1, 2, ..., K-1$:

$$\underline{\hat{H}}^{(p+1)} = \left[\sum_{l=1}^{C} \mathbf{X}_l^*\mathbf{X}_l\alpha_l\right]^{-1} \left[\sum_{l=1}^{C} \mathbf{X}_l^*\underline{Y}\ \alpha_l\right] \quad (26)$$

where α_l is a scalar value and $\underline{\hat{H}}^{(p+1)} = [\hat{H}_1^{(p+1)} \quad \hat{H}_2^{(p+1)}]^T$.

In Equation (26), the value of α_l is important since the channel coefficients can not be converged the actual values properly. It is chosen through $\underline{G_l}$ which is the vector of normalized pdf's. This vector is computed using Equations (19-22) as given below:

$$\underline{G_l} = \left[\frac{f(Y(n,k)|\underline{H}^{(p)},\mathbf{X}_l)}{f(Y(n,k)|\underline{H}^{(p)})} \quad \frac{f(Y(n+1,k)|\underline{H}^{(p)},\mathbf{X}_l)}{f(Y(n+1,k)|\underline{H}^{(p)})}\right]^T \quad (27)$$

In order to decide α_l, we propose to use:

$$\alpha_l = min\{G_l(1), G_l(2)\} \qquad l = 1, ..., C. \quad (28)$$

After updating the channel coefficients in Equation (26), the channel coefficients are passed through a low pass filter as explained in Equations (11-13) to eliminate the noise on the estimated coefficients.

VI. Simulation Results

In this section, we evaluate the proposed iterative EM-based channel estimation method for uncoded STBC-OFDM systems and compare with the time domain EM-based channel estimation algorithm in terms of mean square error (MSE) and bit error rate (BER). Furthermore, the number of iterations of two algorithms are given in order to make a comparison on their computational complexities. The entire channel bandwidth is chosen 800 kHz, and is divided into 64 sub-carriers. The symbol duration is chosen $100\mu s$ including $20\mu s$ guard interval that is used to avoid ISI due to channel delay-spread. The time varying frequency fading channel has 8 taps and the Doppler shift is chosen to be $100Hz$. Each taps is Rayleigh distributed and the conventional exponential decay multipath channel model is used for power-delay profile. The channel is assumed to be constant in one OFDM frame.

We use a scenario which has 64 OFDM symbols in one OFDM frame and 64 subcarriers in one OFDM symbol. In order to find an initial channel coefficient values, the pilots are distributed in the time-frequency lattice. The distance between pilot symbols in the frequency domain, D_f and in the time domain, D_t are chosen 8 and 9, respectively.

We use Alamouti STBC-OFDM system with two transmit and one receive antenna. The simulation results are obtained for QPSK modulation technique.

In Figure 3, the BER performance are shown. It is indicated that the proposed algorithm gives $2dB$ better results than the pilot-based initial channel estimation and it gives the same performance with the time domain EM algorithm.

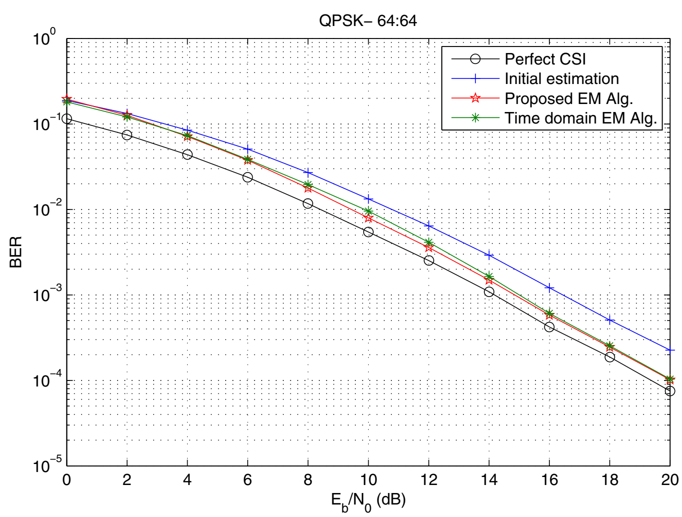

Fig. 3. BER performance for QPSK/STBC-OFDM

In Figure 4, the MSE performances of these algorithms are given. The proposed algorithm again gives the same performance with the time domain algorithm. Both algorithms are better than initial channel estimation and satisfies the Cramer Rao Lower Bound (CRLB) for high E_b/N_0 values.

In Figure 5, we compare the iteration numbers of both EM-based iterative algorithms in order to comment on computational complexities. It is seen that proposed algorithm needs more iteration for low E_b/N_0 values. However, when E_b/N_0 is higher than $4dB$, the number of iterations for proposed algorithm decreases considerably. While time domain EM based channel estimation algorithm requires almost 8 iterations for $E_b/N_0 = 10dB$, the proposed algorithm requires only 3 iterations. It is seen that the proposed algorithm gives the same performance with less iteration process.

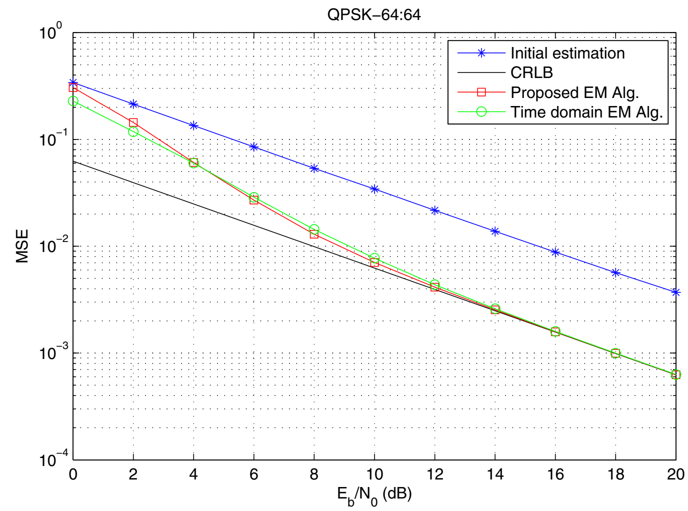

Fig. 4. MSE performance for QPSK/STBC-OFDM

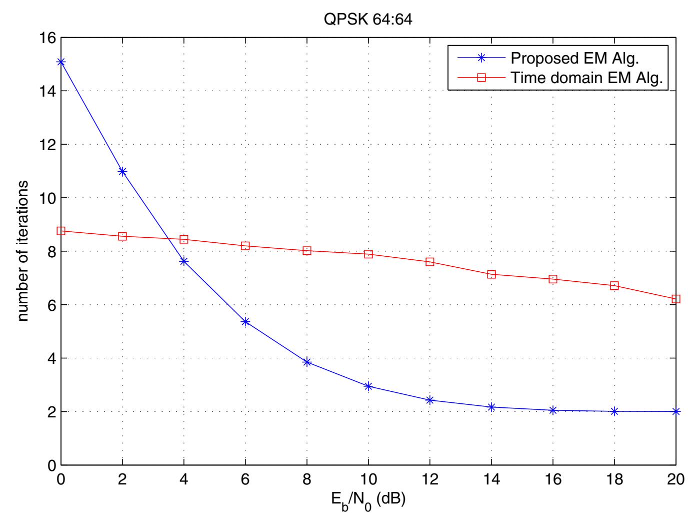

Fig. 5. Iteration number comparison for QPSK/STBC-OFDM

VII. Conclusion

In this paper, an iterative EM-based channel estimation algorithm for STBC-OFDM system is proposed. The channel coefficients are estimated for each subcarrier in the frequency domain. While estimating the channel coefficients, signal or noise decomposition is not used as in the time domain EM based channel estimation algorithm. Moreover, the matrix inversion process required in the time domain algorithm is eliminated using the frequency domain EM based algorithm. The simulation results indicate that the computational complexity of the proposed algorithm is reduced without any performance loss.

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