Towards the solution of cosmological constant and zero point energy problems through metric reversal symmetry

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Abstract. In this talk I review my studies on metric reversal symmetry and their further implications. The talk is mainly concentrated on the relevance of the metric reversal symmetry to the solutions of the cosmological constant and zero point energies. However the use of metric reversal symmetry to hide higher Kaluza-Klein modes at the scales larger than the size of extra dimensions is also discussed, and speculations on its possible relevance to Pauli-Villars and Lee-Wick model are also briefly mentioned.

1. Introduction

The (old) Cosmological constant problem [1] is a fundamental problem of physics that exists independent of the expansion of the universe being in an accelerated fashion [2] or not although the observation of the expansion rate of the universe made this problem more popular because a positive cosmological constant in Einstein field equations is the most standard explanation for the accelerated expansion of the universe. The problem can be summarized as follows: There are many contributions to the cosmological constant from quantum physics and high energy physics. These contributions must be extremely fine tuned so that they cancel or are hidden from gravity either exactly or almost exactly. These contributions should cancel exactly if the accelerated expansion of the universe is attributed to alternative mechanisms such as quintessence, phantom fields, or modified gravity [3, 4] while the cancellation of these contributions must be extremely fine tuned or be hidden from gravity in a fine tuned manner if the accelerated expansion of the universe is attributed to a positive cosmological constant [5]. The (dark) energy density [3, 4] corresponding to the observed value of the expansion rate of the universe is $\sim 10^{-3}$ eV [6] if the accelerated expansion of the universe is attributed to cosmological constant or other dark energy sources while the theoretically expected contribution to the energy density of cosmological constant due to the renormalized zero point energy of an electron is $\sim 10^{33}$ times, the contribution due to the OCD vacuum is $\sim 10^{44}$ times, the contribution due to the vacuum expectation value of the Higgs field is $\sim 10^{55}$ times the observed value. This is the old cosmological constant problem. There are other cosmological constant problems as well; cosmic coincidence (i.e. the energy densities of matter and dark energies being of the same order of magnitude) and if cosmological constant vary with time, etc.. In this talk what I mean by the cosmological constant problem is the old cosmological problem and it is one of the main topics of this talk.

There are many schemes that intend to solve the (old) cosmological constant problem such as symmetries, anthropic principle, string theory landscape, diluting through extra dimensions, adjustment mechanisms etc, and numerous models in each scheme [1, 7]. However none have been wholly successful up to now. In this talk I consider a symmetry that may be called metric reversal symmetry as a possible tool to make some progress in the direction of the solution of the (old) cosmological constant problem [8, 9, 10, 12], and to give an alternative mechanism to the Kaluza-Klein modes at length scales larger than the size of the extra dimension(s) [13]. The main approach in this study is to implement the metric reversal symmetry to forbid (or eliminate) a cosmological constant and then to attribute the accelerated expansion of the universe to other mechanism such as quintessence, phantom field, or modified gravity. However a non-vanishing cosmological constant may be accommodated into this scheme by breaking the metric reversal symmetry while setting the cosmological constant to zero seems me more aesthetic in this scheme. In the first part of the talk I consider the implementation of the symmetry in the context of classical field theory [8, 9, 10]. In the second part I consider extension of the symmetry to quantum domain by studying the contribution of the quantum zero modes to the cosmological constant problem [11, 12]. In the last part I present a scheme that uses metric reversal symmetry to hide higher Kaluza-Klein modes at the present length scales accessible to experiments [13]. I also briefly speculate on further implications of the symmetry in connection to Pauli-Villars regularization [14], and Lee-Wick model [15].

2. Metric reversal symmetry and the cosmological constant problem

I name the multiplication of the infinitesimal line element by minus one as metric reversal, that is,

$$ds^2 \;=\; g_{AB}dx^A\,dx^B \;\rightarrow\; \; -\,ds^2 \tag{1}$$

There are two ways to realize (1) given by [8, 7, 16]

$$x^A \rightarrow i\,x^A \quad , \quad g_{AB} \rightarrow g_{AB} \quad . \tag{2}$$

and [17, 9, 18]

$$x^A \rightarrow x^A \quad , \quad g_{AB} \rightarrow -g_{AB} \quad . \tag{3}$$

The transformations (2) and (3) may be regarded as the first and the second realizations of the metric reversal symmetry, respectively. In fact these two realizations are wholly equivalent at the level of gravity while become somewhat different when matter is included. For example the transformation rule of gauge fields under these two realizations are different.

The infinitesimal volume element, dV and the scalar curvature, R transform under the metric reversal as

$$dV \;=\; \sqrt{(-1)^S g}d^Dx \;\rightarrow\; (-1)^{\frac{D}{2}}dV \quad , \quad R \rightarrow -R \tag{4}$$

where S denotes the number of spatial dimensions and D the (total) dimension of the space. The requirement of gravitational action

$$S_R = \frac{1}{16\pi\,G}\int \sqrt{(-1)^S g}\,R\,d^Dx \tag{5}$$

be invariant under metric reversal symmetry constraints the number of dimensions by

$$D = 2(2n+1) \quad , \quad n = 0, 1, 2, 3, ... \quad . \tag{6}$$

On the other hand the metric reversal symmetry in $D = 2(2n+1)$ dimensions forbids a cosmological constant term

$$S_C = \frac{1}{8\pi G} \int \sqrt{g}\,\Lambda\, d^D x. \tag{7}$$

One may reach a similar conclusion by requiring covariance of Einstein equations under metric reversal while in this case there is no constraint on the dimension of the space. However the formulation of the symmetry by action functional is more promising for the extension of the analysis to quantum domain. Moreover the use of an extra dimensional setting enables the implementation of the symmetry through extra dimensional reflections. For example one may take the metric and the extra dimensional reflection, respectively, as

$$ds^2 = \cos(kx_5)\, g_{AB} dx^A dx^B \ , \quad A, B = 0, 1, 2, ..., 5 \tag{8}$$
$$ds^2 \to -ds^2 \quad \text{as} \quad kx_5 \to \pi - kx_5 \ , \quad g_{AB} \to g_{AB} \tag{9}$$

In this way the orientation reversal induced through the infinitesimal volume element may be obtained in a more standard setting because in this way the orientation change in the space takes place only when one travels through the extra dimensions while there is no orientation change as one travels through the usual 4-dimensional space. This is the approach I adopt in this study for the second realization of the symmetry Eq.(3). The source of the first realization may be attributed to some mechanism at the level of quantum gravity as given in [19].

It is evident that the metric reversal symmetry forbids a bulk cosmological constant (CC). Further one may form models where both a possible brane CC or a CC that may be induced by the part of the cosmological constant that only depends on extra dimensions is forbidden. One may refer to references [8, 9, 10] for more detail and some specific models of this type. However the metric reversal symmetry in its simple form seems to be a classical symmetry that can not be extended into quantum domain. This is due to the complication brought by the inclusion of the matter into theory. This can be seen either by showing the failure of the symmetry to be compatible with correct boundary conditions necessary for quantum mechanics as shown in [16] or by studying the transformation properties of the classical fields under the symmetry. In the latter approach as shown in [12] one observes that either both the classical matter Lagrangian and the vacuum expectation value of energy-momentum are allowed or both are forbidden simultaneously because the energy-momentum tensor follows from the matter Lagrangian. However one can extend the symmetry to quantum domain if one realizes the symmetry through extra dimensional reflections and allows mixture of different Kaluza-Klein modes since some of the modes are even and the others are odd under the extra dimensional reflections. These main ideas were put into a concrete form in [12]. The resulting model does not have zero point energy problem of quantum fields. After diagonalization of the kinetic terms the result is a spectrum of fields consisting of the usual fields plus ghost-like fields. This is essentially Linde's model [20, 21]. This scheme automatically follows from metric reversal symmetry while Linde's model was put forward in an ad hoc way. In this way it is easy to see that the problem with the incorrect boundary conditions under the symmetry transformation is removed because the transformed boundary conditions belong to the ghost-like copy of the universe and vice versa. To see the situation more clearly let us summarize the work of [12] below.

3. Metric reversal symmetry and zero point energy problem: extension of the symmetry to the quantum domain

Consider a space consisting of the sum of 2(2n+1) and 2(2m+1) (e.g 6 and 2) dimensional subspaces with the metric

$$ds^2 \;=\; g_{AB} dx^A dx^B + g_{A'B'} dx^{A'} dx^{B'}$$

$$
\begin{aligned}
&= \Omega_z(z)[g_{\mu\nu}(x)\,dx^\mu dx^\nu + \tilde{g}_{ab}(y)\,dy^a dy^b] + \Omega_y(y)\tilde{g}_{A'B'}(z)\,dz^{A'}dz^{B'} && (10)\\
&\Omega_y(y) = \cos k|y|) \;,\quad \Omega_z(z) = \cos k'|z| && (11)\\
A,B &= 0,1,2,3,5,....N \;,\quad N=2(2n+1) \;,\quad A',B'=1',2',,....N' \;,\quad N'=2(2m+1)\\
&\mu\nu = 0,1,2,3 \;,\quad a,b=1,2,...,N-4 \;,\quad n,m=0,1,2,3...... \;.
\end{aligned}
$$

The usual four dimensional space is embedded in the first space $g_{AB}dx^A\,dx^B$. I take the action be invariant under both realizations of the metric reversal symmetry, that is,

$$
\begin{aligned}
&ds^2 \to -ds^2 \quad \text{as} \quad x^A \to i\,x^A,\;\; x^{A'} \to i\,x^{A'},\;\; g_{AB} \to g_{AB},\;\; g_{A'B'} \to g_{A'B'} && (12)\\
&\Rightarrow\;\; \Omega_z \to \Omega_z,\;\; \Omega_y \to \Omega_y,\;\; g_{\mu\nu} \to g_{\mu\nu},\;\; \tilde{g}_{ab} \to \tilde{g}_{ab},\;\; \tilde{g}_{A'B'} \to \tilde{g}_{A'B'} && (13)
\end{aligned}
$$

and

$$
\begin{aligned}
&ds^2 \to -ds^2 \quad \text{as} \quad ky \to \pi - ky\,,\;\; k'z \to \pi - k'z\,,\;\; x^A \to x^A,\;\; x^{A'} \to x^{A'} && (14)\\
&\Rightarrow\;\; \Omega_z \to -\Omega_z,\;\; \Omega_y \to -\Omega_y,\;\; g_{\mu\nu} \to g_{\mu\nu},\;\; \tilde{g}_{ab} \to \tilde{g}_{ab},\;\; \tilde{g}_{A'B'} \to \tilde{g}_{A'B'} \;. && (15)
\end{aligned}
$$

The requirements of the homogeneity and isotropy of the 4-dimensional space together with the equations (12-15) set $g_{\mu\nu}$ to the Minkowski metric $\eta_{\mu\nu} = diag(1,-1,-1,-1)$.

To see the essentials of the scheme consider the zero point energy induced by the kinetic Lagrangian of a scalar field

$$
\begin{aligned}
S_{\phi k} &= \int dV\,\mathcal{L}_{\phi k} && (16)\\
dV &= \sqrt{(-1)^S g}\sqrt{(-1)^{S'} g'}\,d^Dx\,d^Dx'\\
\mathcal{L}_{\phi k} &= \frac{1}{2}g^{AB}\partial_A\phi\partial_B\phi && (17)
\end{aligned}
$$

After writing ϕ in terms of its Kaluza-Klein modes and integrating over extra dimensions one obtains

$$
\begin{aligned}
S_{\phi k} = \;& \frac{1}{8}(LL')^2\int d^4x\,\{4\partial_\mu[\,\phi_{1,2}(x) + \phi_{1,0}(x)]\,\partial_\nu(\,\phi_{0,0}(x)\,)\\
&+4\partial_\mu[\,\phi_{0,2}(x) + \phi_{0,0}(x) + \phi_{2,2}(x) + \phi_{2,0}(x)]\,\partial_\nu(\,\phi_{1,0}(x)\,)\\
&+4\eta^{\mu\nu}\sum_{r=1,s=1}^{\infty}\partial_\mu[\,\phi_{|r-1|,|s-2|}(x) + \phi_{|r-1|,s+2}(x)\\
&+2\phi_{|r-1|,s}(x) + \phi_{r+1,|s-2|}(x) + \phi_{r+1,s+2}(x) + 2\phi_{r+1,s}(x)\,]\partial_\nu(\,\phi_{r,s}(x)\,)\\
&-4k^2\sum_{r=1,s=0} r[\,(|r-1|)(\,\phi_{|r-1|,|s-2|}(x) + \phi_{|r-1|,s+2}(x) + 2\,\phi_{|r-1|,s}(x)\,)\\
&+(r+1)(\,\phi_{r+1,|s-2|}(x) + \phi_{r+1,s+2}(x) + 2\,\phi_{r+1,s}(x)\,) - \phi_{r+1,s}(x)\,)\,]\phi_{r,s}(x)\\
&-4\frac{1}{2}k'^2\sum_{r=0,s=1} s\,[\,(|s-3|)\phi_{r,|s-3|}(x) + (s+3)\,\phi_{r,s+3}(x)\\
&+3(|s-1|)\,\phi_{r,|s-1|}(x) + 3(s+1)(\,\phi_{r,s+1}(x)\,]\,\phi_{r,s}(x)\} \;. \qquad (18)
\end{aligned}
$$

One observes that Eq.(18) contains only off-diagonal mixing of Kaluza-Klein modes. So the corresponding 4-dimensional energy momentum tensor

$$
T_\mu^\nu = \int dV\,\Omega_z^{-1}\{g^{\nu\tau}\partial_\tau\phi\partial_\mu\phi - \frac{1}{2}\delta_\mu^\nu g^{\rho\tau}\partial_\rho\phi\partial_\tau\phi\} \qquad (19)
$$

also consists of off-diagonally coupled Kaluza-Klein modes. Hence the vacuum expectation value of T_μ^ν consists of the following type of terms

$$< 0|T_\mu^\nu|0 > \;\propto\; < 0|\,a_{n,m}a_{r,s}^\dagger|0 >= 0\,,\quad < 0|\,a_{r,s}^\dagger a_{r,s}|0 >= 0 \qquad n \neq r \quad \text{and/or} \quad m \neq s \tag{20}$$

(because $a_{r,s}|0 >= 0$, and $[\,a_{n,m}, a_{r,s}^\dagger\,] = 0$ for $n \neq r$ and/or $m \neq s$) where $a_{n,m}$, $a_{n,m}^\dagger$ are the creation and annihilation operators in the expansion of $\phi_{n,m}$. Similar results apply to the extra dimensional piece of the kinetic term and bulk mass terms for ϕ, and to other fields such as fermions and gauge bosons. Any other power of ϕ also necessarily contains at least one pair of off-diagonal mixings of Kaluza-Klein modes due to the symmetry. The details about these points may be found in [12]. Therefore this scheme eliminates any non-zero contribution to the cosmological constant due to quantum zero modes.

One can see the situation in another way as well by diagonalizing the action. To see the main points in the derivation consider the second term in (18)

$$\begin{aligned} S_{\phi k} &= \frac{1}{2}(LL')^2\int d^4x\, 2\eta^{\mu\nu}\partial_\mu\phi_{1,0}\partial_\nu\phi_{0,0} \\ &= \frac{1}{2}(LL')^2\int d^4x\,[\,\eta^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 \,-\, \eta^{\mu\nu}\partial_\mu\phi_2\partial_\nu\phi_2\,] \end{aligned} \tag{21}$$

The other terms in the kinetic terms and in bulk mass terms may be diagonalized in similar way. In other words this scheme corresponds to a scheme where the field spectrum consists of the usual particles and their ghost-like copies with a possible interaction between these two sets through gravity, and trilinear and higher order terms (e.g. ϕ^4) in the Lagrangian. This is essentially a variant of Linde's model. However the usual and the ghost universe picture emerges automatically as a consequence of the metric reversal symmetry in this scheme while Linde's model is formulated in an ad hoc way.

The gravitational action in this space can not be taken simply as the Einstein-Hilbert action given by (5) because it cancels out since the scalar curvature consists of two parts one due to the $2(2n+1)$ dimensional subspace (that is even under $kz \to \pi - kz$) and the other due to the $2(2m+1)$ dimensional subspace (that is even under $ky \to \pi - ky$) while the volume element is odd under both. Instead one may adopt an R^2 or $R\phi$ type of term for the gravitational Lagrangian. It is shown in [12] that one may take a R^2 term for the gravitational sector and it reduces to the usual Einstein-Hilbert term after integration over extra dimensions.

One may allow the emergence of the Robertson-Walker metric from the Minkowski metric in (21) by breaking of the first realization of the symmetry given in (12,13) so that the accelerated expansion of the universe is accommodated while the second realization of the metric reversal symmetry given by (14,15) remains unbroken to insure the vanishing of the zero point energies. In this scheme the accelerated expansion of the universe is either should be attributed to other mechanism such as quintessence, phantom, modified gravity etc. or one may break the metric reversal through the introduction of a classical term (other than zero point energies) that breaks the first realization of the metric reversal symmetry. For example one may introduce a classical Lagrangian, that breaks the symmetry, given by

$$\mathcal{L}_{cl} \,=\, \alpha\, v_{1,0}v_{0,1}\cos k\,y\,\cos k'\,z \tag{22}$$

where $\alpha << 1$ is a constant that reflects that $\mathcal{L}_{cl}$ is small since it corresponds to the breaking of the $x^A \to i\,x^A$, $x^{A'} \to i\,x^{A'}$ symmetries separately by a small amount, and $v_{1,0}$, $v_{0,1}$ are some constants.

4. Finite number of Kaluza-Klein modes through metric reversal symmetry

The significance of metric reversal symmetry is not restricted to the cosmological constant problem. For example one may employ the metric reversal symmetry to hide some of the Kaluza-Klein modes at usual length scales even when they have already been excited. This may give an alternative way to explain the absence of the Kaluza-Klein modes at current accelerator energies while the usual prescription is to take the Kaluza-Klein modes be too heavy to be excited at current energies. In this section I give a simple scheme to hide Kaluza-Klein modes at the current length scales accessible in experiments by using metric reversal symmetry. The details may be found in [13].

To illustrate the scheme consider the free fermions in the following 5-dimensional metric

$$ds^2 \;=\; \cos k\, z\, [\, g_{\mu\nu}(x)\, dx^\mu dx^\nu \,-\, dz^2\,] \qquad \mu,\nu \;=\; 0,1,2,3 \tag{23}$$

where the extra dimension is taken to be compact and have the size L, and $k = \frac{2\pi}{L}$. The action for (free) fermionic fields for this space is

$$\begin{aligned} S_f \;&=\; \int (\cos kz)^{\frac{5}{2}}\, \mathcal{L}_f\, d^4x\, dz \\ &=\; \int (\cos kz)^2\; i\bar{\chi}\gamma^a(\,\partial_a \,+\, \frac{1}{16}\tan kz\,[\,\gamma_a\;,\;\gamma_5]\,)\chi\; d^4x\, dz \;+\; H.C. \\ &\{\gamma^a,\gamma^b\} \;=\; 2\eta^{ab} \quad , \quad (\eta^{ab}) = diag(1,-1,-1,-1,-1) \end{aligned} \tag{24}$$

where $H.C.$ stands for Hermitian conjugate, and the second term is the spin connection term. The Kaluza-Klein decomposition of χ is

$$\chi = \sum\, [a_n \cos n\frac{kz}{2} + b_n \sin n\frac{kz}{2}]\chi_n(x) \tag{25}$$

I take the action (24) be invariant under an extra dimensional reflection realization of metric reversal symmetry given by

$$k\, z \;\rightarrow\; \pi \;+\; k\, z. \tag{26}$$

This transformation is essentially equivalent to (9) while (9) is more restrictive in constraining the conformal factors and more applicable to orbifold constructions. I also impose the action be invariant under the first realization of the metric reversal symmetry or its subgroup given by

$$x^a \;\rightarrow\; -\, x^a \qquad\qquad a \;=\; 0,1,2,3,4 \tag{27}$$

where the Kaluza-Klein modes χ_n are required to transform as

$$\chi_n(x) \;\rightarrow\; \epsilon^{\lambda_n}\, \mathcal{CPT}\, \chi_n(-x) \qquad , \qquad \lambda_n = \frac{i}{2}(-1)^{\frac{n}{2}} \tag{28}$$

where ϵ is some constant, and $\mathcal{CPT}$ denotes the usual 4-dimensional CPT operator (acting on the spinor part of the field). I impose χ satisfy anti-periodic boundary conditions i.e. $\chi(z=0) = -\chi(z=L)$. This sets n in (25,28) be odd.

The requirement of the invariance of the action (24) under (26), (27) (and (28)) results in the following expression for the 4-dimensional part of S_f

$$\begin{aligned} &\sum_{r,s=0}^{\infty} (\int\, d^4x\, [\, i\bar{\chi}_{(2|r|+1)}\gamma^\mu\partial_\mu\chi_{(2|s|+1)}\, \int\, dz\, (\cos kz)^2 \\ &\times [\, f^*_{2|r|+1}g_{2|s|+1} + (\cos\frac{2|r|+1}{2}kz \,+\, \sin\frac{2|r|+1}{2}kz)(\cos\frac{2|s|+1}{2}kz \,-\, \sin\frac{2|s|+1}{2}kz) \end{aligned}$$

$$
\begin{aligned}
&+g^*_{2|r|+1}f_{2|s|+1})(\cos\frac{2|r|+1}{2}kz-\sin\frac{2|r|+1}{2}kz)(\cos\frac{2|s|+1}{2}kz+\sin\frac{2|s|+1}{2}kz)]+H.C.\\
=\;&\sum_{r,s=0}^{\infty}(\int d^4x\,[2\,i\bar{\chi}_{(2|r|+1)}\gamma^\mu\partial_\mu\chi_{(2|s|+1)}\int dz\,(\cos kz)^2\,(f^*_{2|r|+1}g_{2|s|+1}+g^*_{2|r|+1}f_{2|s|+1})\\
&\times[\cos\frac{2|r|+1}{2}kz\,\cos\frac{2|s|+1}{2}kz\;-\;\sin\frac{2|r|+1}{2}kz\,\sin\frac{2|s|+1}{2}kz]\;+\;H.C.\\
=\;&\frac{1}{2}\sum_{r,s=0}^{\infty}(f^*_{2|r|+1}g_{2|s|+1}+g^*_{2|r|+1}f_{2|s|+1})\int d^4x\,i\bar{\chi}_{(2|r|+1)}\gamma^\mu\partial_\mu\chi_{(2|s|+1)}\\
&\times\int_0^L dz\,[\cos(|r|+|s|-1)kz]\;+\;H.C.
\end{aligned}
\tag{29}
$$

where $2r+1=4l+1$, $2s+1=4p+3$ (l,p=0,1,2,....) or vice versa, , and $f_n=\frac{1}{2}(a_n+b_n)$, $g_n=\frac{1}{2}(a_n-b_n)$. Because of the periodicity of cosine function the terms in (29) give non-zero contributions after integration over z only if the arguments of cosines are zero. This is possible only when

$$|r|+|s|-1=0\quad\Rightarrow\qquad r=0\;\;,\;\;s=1\quad\text{ or }\quad s=1\;\;,\;\;r=0 \tag{30}$$

The result of z integration in (8) is

$$\frac{L}{2}\int d^4x\quad[\;\;i\bar{\chi}_1\gamma^\mu\partial_\mu\chi_0\;+\;i\bar{\chi}_0\gamma^\mu\partial_\mu\chi_1\,]\;+\;H.C. \tag{31}$$

where the term $(f_1^*g_0+g_1^*f_0)$ is absorbed into redefinition of χ_1, χ_0. The diagonalization of (31) gives

$$\frac{1}{2}L\int d^4x[i\bar{\psi}\gamma^\mu\partial_\mu\psi\;-\;i\bar{\tilde{\psi}}\gamma^\mu\partial_\mu\tilde{\psi}\,]\;+\;H.C. \tag{32}$$

$$\psi\;=\;\frac{1}{\sqrt{2}}(\,\chi_1\;+\;\chi_0\,)\quad,\quad\tilde{\psi}\;=\;\frac{1}{\sqrt{2}}(\,\chi_1\;-\;\chi_0\,) \tag{33}$$

Hence there are one usual fermion and one ghost fermion in the spectrum at the large length scales where the extra dimension can not be detected. All other Kaluza-Klein modes will be hidden at these length scales even when they are produced in high energy experiments.

5. Conclusion

I have reviewed the relevance of extra dimensional metric reversal symmetry to the solutions of cosmological constant, zero point energy problems, and to a scheme where higher Kaluza-Klein modes may be hidden at current length scales accessible to experiments even when they have already been produced. The results seem to be encouraging for the solutions of these problems in a quite natural framework. Moreover the implementation of metric reversal symmetry (or orientation reversal symmetry) through the use of extra dimensions makes the symmetry plausible. The emergence of a ghost field for each usual field due to the mixture of Kaluza-Klein modes of the same field implies that this framework may also be relevant to Pauli-Villars regularization and Lee-Wick model. These points will be studied in my future work.

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