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Abstract
Two different realizations of a symmetry principle that impose a zero
cosmological constant in an extra-dimensional set-up are studied. The
symmetry is identified by multiplication of the metric by minus one. In the fist
realization of the symmetry this is provided by a symmetry transformation that
multiplies the coordinates by the imaginary number i. In the second realization
this is accomplished by a symmetry transformation that multiplies the metric
tensor by minus one. In both realizations of the symmetry the requirement
of the invariance of the gravitational action under the symmetry selects out
the dimensions given by $D = 2(2n + 1), n = 0, 1, 2 \ldots$, and forbids a bulk
cosmological constant. Another attractive aspect of the symmetry is that it
seems to be more promising for quantization when compared to the usual scale
symmetry. The second realization of the symmetry principle is more attractive
in that it is possible to make a possible brane cosmological constant zero in a
simple way by using the same symmetry, and the symmetry may be identified
by reflection symmetry in extra dimensions.

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The universe at cosmic scales may be described by a homogeneous and isotropic ideal fluid.
The 00-component of the corresponding Einstein equations results in [1, 2]

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{\rm N}}{3}(\rho + 3p) \tag{1}$$

where $a = a(t) > 0$ is the scale factor for the expansion of the universe related to the
Hubble parameter H by $H = \frac{\dot{a}}{a}$, $G_{\rm N}$ is the Newton's constant, ρ is the energy density
and p is the pressure of the ideal fluid (modelling our universe at cosmic scales). Recent
cosmological observations [3] suggest that $\ddot{a} > 0$ while the standard matter and radiation
(e.g. stars and electromagnetic radiation) requires $\ddot{a} < 0$. This combined with the amount
of the standard matter and radiation requires a form of energy density with $p \simeq -\rho$, which,
in turn, may be identified with vacuum energy density ρ_v of value $\simeq (2.3 \times 10^{-3} \text{ eV})^4$ [4].

This is the most standard explanation for acceleration of the universe although there are alternative ways of explanation as well [5]. Vacuum energy density results in a stress–energy tensor that may be identified by a cosmological constant through the relation $\rho_v = \frac{\Lambda}{8\pi G_N}$. However the value of the theoretical contributions to vacuum energy density $\simeq (100 \text{ MeV})^4 - (10^{19} \text{ GeV})^4$ is extremely larger than its measured value $(\simeq 10^{-3} \text{ eV})^4$ [2, 6]. Most of the so-called cosmological constant problems (i.e. what is the source of the huge discrepancy between the theoretical and the observational values of Λ, why is Λ so small?, why is Λ not exactly equal to zero?) are variations of this fact In this talk, I study only one of these cosmological constant problems, namely, why is Λ so small?. In the literature there are many different schemes that deal with this problem [6, 7]; symmetries (i.e. supersymmetry, supergravity, superstrings, conformal symmetry, invariant length reversal symmetry), anthropic considerations, adjustment mechanisms, changing gravity, quantum cosmology, diluting through extra dimensions. In this study, a symmetry principle in an extra-dimensional set-up is employed to make the cosmological constant zero. The accelerating expansion of the universe then may either be attributed to breaking of the symmetry by a small amount through the usual symmetry arguments or may be attributed to the alternative mechanisms of the acceleration [5]. I consider two different realizations of this symmetry. The fist realization employs a symmetry transformation that multiplies the coordinates by the imaginary number i [7–9]. The second realization is implemented by signature reversal that multiplies the metric tensor by -1 [10–12]. In both realizations the requirement of the (non-vanishing and the) invariance of the gravitational action restricts the number of spacetime dimensions to $D = 2(2n + 1)$, $n = 0, 1, 2, \ldots$ (or stating more precisely; to spaces that have a $2(2n + 1)$-dimensional subspace whose metric being odd under the signature reversal and the metric of the remaining part of the space being even under signature reversal). The symmetry forbids a bulk cosmological constant in the allowed dimensions. A brane cosmological constant confined to the usual 4-dimensional space is forbidden by the symmetry because $D = 4$ does not satisfy the rule $D = 2(2n + 1)$. However an effective 4-dimensional cosmological constant may be induced through the part of the curvature scalar, that depends only on the extra dimensions. In order to forbid such a contribution to the cosmological constant one needs an extra mechanism in the first realization while in the second realization this can be achieved by putting the usual 4-dimensional space at the intersection of two $2(2n+1)$-dimensional spaces and then imposing the same symmetry i.e. the signature reversal symmetry to both spaces as will be shown later in this talk. I also find that the form of the matter Lagrangian and the transformation rule for fields (other than gravitation) obtained under the requirement of the corresponding action functional are almost the same in both realizations. The transformation rules for the fields suggest that this symmetry is more promising for quantization than the usual scaling symmetry.

In this talk, I consider a symmetry whose effect is to multiply the metric by minus one, that is,

$$\mathrm{d}s^2 = g_{AB}\, \mathrm{d}x^A\, \mathrm{d}x^B \to -\mathrm{d}s^2. \tag{2}$$

The fist realization of this symmetry is through the transformation (that multiplies the coordinates x_A by i) [7–9]

$$x_A \to \mathrm{i}x_A, \qquad A = 0, 1, 2, \ldots, D - 1 \tag{3}$$

where D is the dimension of the space. The requirement of the invariance of physics under the symmetry transformation (3) may be imposed in two ways; either through the requirement of the covariance of the Einstein field equations or by the requirement of the invariance of the corresponding action functional under the symmetry transformation given in (3). The

application of the first approach to the gravitation (i.e. the requirement of the covariance of the Einstein field equations under the transformation (3)) results in the conclusion that the cosmological constant breaks the covariance of the Einstein equations and hence, is not allowed [7, 9]. This conclusion is independent of the number of dimensions of the space. Hence one may take the space be the usual 4-dimensional space. The second approach [8] will be followed here and it leads to a restriction on the number of dimensions. In this approach we require the invariance of the gravitational action functional

$$S_R = \frac{1}{16\pi G} \int \sqrt{g} R \, \mathrm{d}^D x \tag{4}$$

under (3). Here $g = (-1)^s \det(g)$, $s = 0$ or 1 so that $\sqrt{g}$ gives a real number contribution to the 4-dimensional action after integration over extra dimensions. One notes that

$$R \to -R, \qquad \sqrt{g} \, \mathrm{d}^D x \to (\pm \mathrm{i})^D \sqrt{g} \, \mathrm{d}^D x \qquad \text{as} \quad x_A \to \mathrm{i} x_A. \tag{5}$$

So only the number of dimensions given by

$$D = 2(2n + 1), \qquad n = 0, 1, 2, 3, \ldots \tag{6}$$

are allowed by the invariance of (4) under (3). A bulk cosmological constant is forbidden in the dimensions given in (6) since the corresponding action functional

$$S_C = \frac{1}{16\pi G} \int \sqrt{g} \Lambda \, \mathrm{d}^D x \tag{7}$$

is not invariant under the symmetry transformation (3). However a possible contribution to the 4-dimensional cosmological constant through the part of curvature scalar, that depends only on the extra dimensions, is not forbidden in this realization of the symmetry; one needs an additional symmetry to forbid it. Such a symmetry was employed in [8] for a 6-dimensional metric. The second realization of the symmetry is more promising in this respect because the same symmetry may be also employed to forbid a possible contribution to the 4-dimensional cosmological constant through curvature scalar as we will see.

The symmetry transformation for the second realization of this symmetry [11, 12] is given by

$$g_{AB} \to -g_{AB}. \tag{8}$$

The curvature scalar R and the invariant volume element $\sqrt{g} \, \mathrm{d}^D x$ transform exactly in the same way as in the first realization (5). So the second realization as well selects out the dimensions $D = 2(2n + 1)$ and forbids a bulk cosmological constant. In fact it is not essential that the dimension of space is $2(2n + 1)$ to have the symmetry be applicable. The essential point is that the space should contain a subspace whose metric tensor transforms like (8) while the metric tensor of the remaining part of the space is invariant under the symmetry transformation. However such a choice would be ad hoc.

The main advantage of the realization is that the same symmetry may be used to forbid a possible contribution to the 4-dimensional cosmological constant, after integration over extra dimensions, through the piece of the curvature scalar that depends only on extra dimensions. To this end I take two $2(2n + 1)$-dimensional spaces, say, one with 6 dimensions and the other with 10 dimensions, and the usual 4-dimensional space is taken at the intersection of these spaces. I require that the transformations of the metric tensors of each space under the signature reversal (8) leave the action invariant, both under the separate and the simultaneous transformations on the two spaces. The requirement of the invariance of the action under the signature reversal of the metric tensors of each space separately guarantees the absence of bulk cosmological constants while the requirement of the invariance of the action under

the simultaneous signature reversals of the metrics of both spaces guarantees the absence of any possible contribution to the 4-dimensional cosmological constant through the part of the curvature scalar that depends only on the extra dimensions. This mechanism may be better seen through the following example. Consider the metric describing the union of two spaces of dimensions D' and D''

$$ds^2 = \Omega_1(y)\Omega_2(z)g_{\mu\nu}(x)\,dx^\mu\,dx^\nu + \Omega_1(y)g_{ab}(w)\,dx^a\,dx^b + \Omega_2(z)g_{cd}(w)\,dx^c\,dx^d$$

$$\text{where} \quad x = x^\mu, \qquad y = x^a, \qquad z = x^c, \qquad w = y, z \tag{9}$$

$$\Omega_1(y) = \Omega_1(y_1) = \cos k_1 x_{5'}, \qquad \text{and} \qquad \Omega_2(z) = \Omega_2(z_1) = \cos k_2 x_{6''} \tag{10}$$

$$\mu, \nu = 0, 1, 2, 3, \qquad a, b = 4', 5', \ldots D' - 1, \qquad c, d = 4'', 5'', \ldots D'' - 1 \tag{11}$$

$$D' = 2(2n + 1), \qquad D'' = 2(2m + 1), \qquad n, m = 1, 2, 3, \ldots. \tag{12}$$

The overall dimension of the space is $D = 2n + 2m$. We note that $\Omega_1(y)$, $\Omega_2(z)$ are odd functions of y, z, respectively, under the reflection about the point $k_{1(2)}x_{5'(6'')} = \frac{\pi}{2}$,

$$k_{1(2)}x_{5'(6'')} \to \pi - k_{1(2)}x_{5'(6'')}. \tag{13}$$

The application of (13) to each of the $2(2n + 1)$-dimensional spaces induces a transformation of the metric tensor of that space exactly in the same way as given in (8). Hence the curvature scalar and the invariant volume element transform exactly in the same way as given in (5). On the other hand, the application of (13) to both spaces simultaneously results in

$$g_{\mu\nu} \to g_{\mu\nu}, \qquad g_{ab} \to -g_{ab}, \qquad g_{cd} \to -g_{cd} \tag{14}$$

where the indices μ, ν, a, b, c, d run as given in (11). In fact (14) is not specific to this example and is the general transformation rule for the metric tensor of a space that consists of the union of two $2(2n+1)$-dimensional spaces where there is signature reversal symmetry in each space. The 4-dimensional part of the curvature scalar $R_4 = g^{\mu\nu}R_{\mu\nu}$, the extra-dimensional part of the curvature scalar, $R_e = g^{ab}R_{ab}$, and the invariant volume element $\sqrt{g}\,d^D x$ transform under the simultaneous applications of the two transformations in (14) as

$$R_4 \to R_4, \qquad R_e \to -R_e, \qquad \sqrt{g}\,d^D x \to \sqrt{g}\,d^D x. \tag{15}$$

The transformation rule for the metric under (14) becomes

$$ds^2 = g_{MN}\,dx^M\,dx^N = g_{\mu\nu}\,dx^\mu\,dx^\nu + g_{ab}\,dx^a\,dx^b \to g_{\mu\nu}\,dx^\mu\,dx^\nu - g_{ab}\,dx^a\,dx^b. \tag{16}$$

It is evident from (15) that the contribution due to R_e vanishes and that due to R_4 survives so that we reach our goal of eliminating any contribution to the cosmological constant through the part of the curvature scalar that depends only on extra dimensions. In fact this conclusion is true for any metric in a space formed of two $2(2n + 1)$-dimensional spaces so that the usual 4-dimensional space is at their intersection, and that obeys (14), and has 4-dimensional Poincare invariance [13] (since the 4-dimensional Poincare invariance insures the metric tensors of the extra dimensions depend only on extra dimensions). For a more detailed discussion and calculations one may refer to [10]. In other words the requirement of the invariance of the action functional under the application of the signature reversal on each $2(2n + 1)$-dimensional space separately (through transformations of the form of (8)) insures the absence of bulk cosmological constant while the requirement of the invariance of the action functional under the application of signature reversal on both spaces simultaneously (through transformations of the form of (15)) insures the absence of any contribution to the 4-dimensional cosmological constant through the extra-dimensional piece of the curvature scalar.

Transformation rules for fields (other than gravitation) under the symmetry is another important issue to be discussed because it is decisive in the invariance properties of n-point correlation functions of quantum field theory. We require the invariance of the action functional

$$S_L = \int \sqrt{g}\, \mathrm{d}^D x \mathcal{L} \tag{17}$$

where $\mathcal{L}$ denotes the Lagrangian for the fields other than gravitation. This gives us transformation rule for the Lagrangian and this transformation rule, in turn, is used to determine the transformation rule for the fields by using the requirement of the invariance of the kinetic terms of the Lagrangian. In the first realization of the symmetry

$$\sqrt{g}\, \mathrm{d}^D x \rightarrow (i)^D \sqrt{g}\, \mathrm{d}^D x \qquad \text{so in } 2(2n+1) \text{ dimensions this imposes } \mathcal{L} \rightarrow -\mathcal{L}. \tag{18}$$

In the second realization the transformation rule for $\mathcal{L}$ is the same as the first realization (18) when the transformation is applied to the metric tensor of each space separately while the transformation rules for the invariant volume element and the Lagrangian when the transformation is applied to the metric tensors of both spaces simultaneously are

$$\sqrt{g}\, \mathrm{d}^D x \rightarrow (\mathrm{i})^{4n} \sqrt{g}\, \mathrm{d}^D x = \sqrt{g}\, \mathrm{d}^D x, \qquad \text{so} \quad \mathcal{L} \rightarrow \mathcal{L}. \tag{19}$$

where the transformation rule (14) is used. Hence in both realizations one obtains the same transformation for the scalars

$$\phi \rightarrow \pm \phi \tag{20}$$

and the extra-dimensional piece of the kinetic term drops out in the second realization if the space is taken as the union of two spaces where the usual 4-dimensional space lies at the intersection. The transformation rule for gauge fields in both realizations are different. In the first realization only $U(1)$ gauge fields B_A are allowed and transform as

$$F_{AB} \rightarrow \pm \mathrm{i} F_{AB}, \qquad \text{and} \qquad B_A \rightarrow B_A \tag{21}$$

while in the second realization all gauge fields are allowed and transform as

$$F_{AB} \rightarrow F_{AB}, \qquad \text{and} \qquad B_A \rightarrow B_A. \tag{22}$$

In the first realization, fermions are allowed only on $(2n+1)$-dimensional spaces. The situation is essentially the same in the second realization as well. Fermions ψ in both realizations transform (in $2n + 1$ dimensions) as

$$\psi \rightarrow e^{\alpha} \psi \tag{23}$$

where α is an overall constant phase. Moreover it was shown in [11] that the part of the fermionic Lagrangian that depends only on extra dimensions does not pose a problem for cosmological constant problem in the second realization since it cancels out after integration over extra dimensions.

Once the transformation properties of the fields are determined one can discuss the invariance properties of the n-point (correlation) functions of quantum field theory

$$\langle 0 | \varphi_1(x_1) \varphi_2(x_2) \cdots \varphi_n(x_n) | 0 \rangle \tag{24}$$

where $|0\rangle$, and $\varphi_k(x_k)$ stand for the vacuum state, and a general field at the position x_k in a D-dimensional space. It is evident from equations (20)–(22) that the basic building blocks for Feynman diagrams, two-point functions (propagators) are always invariant and arbitrary n-point functions are invariant in most of the cases under the symmetry discussed here.

In this study, I have reviewed a symmetry that insures a zero cosmological constant. The acceleration of the universe either may be attributed to breaking of the symmetry by a small amount [8] or to that of the alternatives ways such as quintessence, phantom(ghost)

etc [5]. This symmetry has more attractive aspects compared to other symmetries employed. Supersymmetry and supergravity theories are broken by a large amount when compared to the upper bound on the observational value of the cosmological constant while there is no such problem for this symmetry. Conformal symmetry is also employed to make cosmological constant zero in the literature. However quantization of conformal field theories is troublesome [14] while this symmetry seems to be more promising in this aspect as well, as we have seen. Usually signature reversal is accompanied with the existence of ghost fields. However the signature reversal symmetry here may be identified by reflections in extra dimensions, that is, the so-called ghosts and the usual particles do not share the same position so they do not cause the usual troubles caused by the presence of ghosts (in addition to the usual particles). Therefore it does not suffer from the problems of *E*-parity models [15–17] that use a usual particle—ghost particle symmetry to eliminate the cosmological constant problem. I think these points make the signature reversal symmetry (introduced in the context of extra-dimensional models) an attractive possibility of being considered further.

References

[1] Carroll S 2003 *Spacetime and Geometry: An Introduction to General Relativity* (San Francisco: Addison-Wesley) (one may also see Lecture Notes on General Relativity on web, http://preposterousuniverse.com/grnotes by the same author)
[2] Carroll S 2001 The cosmological constant *Living Rev. Rel.* **4** 1 (*Preprint* astro-ph/0004075)
[3] Riess A G *et al* (Supernova Search Team) 1998 Observational evidence from supernovae for an accelerating universe and a cosmological constant *Astron. J.* **116** 1009 (*Preprint* astro-ph/9805201)
    Perlmutter S *et al* (Supernova Cosmology Project) 1999 Measurement of omega and lambda from 42 high redshift supernovae *Astrophys. J.* **517** 565 (*Preprint* astro-ph/9812133)
    Astier P *et al* 2006 The supernova legacy survey: measurement of ω_m and ω_λ and w from the first year data set *Astron. Astrophys.* **447** 31 (*Preprint* astroph/0510447)
    Tegmark M *et al* (SDSS) 2004 Cosmological parameters from SDSS and WMAP *Phys. Rev.* D **69** 103501 (*Preprint* astro-ph/0310723)
[4] Yao W-M *et al* 2006 Review of particle physics *J. Phys. G: Nucl. Part. Phys.* **33** 1
[5] Copeland E J, Sami M and Tsujikawa S 2006 Dynamics of dark energy *Preprint* hep-th/0603057 and the references there in
[6] Weinberg S 1989 The cosmological constant problem *Rev. Mod. Phys.* **61** 1
[7] Nobbenhuis S 2004 Categorizing different approaches to the cosmological constant problem *Preprint* gr-qc/0411093
[8] Erdem R 2005 A symmetry for vanishing cosmological constant in an extra dimensional toy model *Phys. Lett.* B **621** 11 (*Preprint* hep-th/0410063)
[9] 't Hooft G and Nobbenhuis S 2006 Invariance under complex transformations, and its relevance to the cosmological constant problem *Class. Quantum Grav.* **23** 3819 (*Preprint* gr-qc/0602076)
[10] Bonelli G and Boyarsky A 2000 Six-dimensional topological gravity and the cosmological constant problem *Phys. Lett.* B **490** 147 (*Preprint* hep-th/0004058)
[11] Erdem R 2006 A symmetry for vanishing cosmological constant: another realization *Phys. Lett.* B **639** 348 (*Preprint* gr-qc/0603080)
[12] Duff M J and Kalkkinen J 2006 Signature reversal invariance *Nucl. Phys.* B **758** 161 (*Preprint* hep-th/0605273)
[13] Rubakov V A and Shaposhnikov M E 1983 Extra space-time dimensions: towards a solution to the cosmological constant problem *Phys. Lett.* B **125** 139
[14] Coleman S 1985 *Aspects of Symmetry: Selected Erice Lectures* (New York: Cambridge University Press)
[15] Linde A D 1988 The universe multiplication and the cosmological constant problem *Phys. Lett.* B **200** 272
    Linde A D 2003 Inflation, quantum cosmology and the anthropic principle *Science and Ultimate Reality* ed J D Barrow (Cambridge: Cambridge University Press) pp 426–58
[16] Kaplan D E and Sundrum R *et al* 2006 A symmetry for the cosmological constant *J. High Energy Phys.* JHEP0607(2006)042 (*Preprint* hep-th/0505265)
[17] Moffat J W 2005 Charge conjugation invariance of the vacuum and the cosmological constant problem *Phys. Lett.* B **627** 9 (*Preprint* hep-th/0507020)