NEW CASIMIR ENERGY CALCULATIONS

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New Casimir energy results for massless scalar field in some 3 -dimensional cavities are presented. We attempted to discuss the correlation between the sign and the magnitude of the energy and the shape of the cavities.

1. Introduction

Sign of the Casimir energy known to be dependent on the dimension, topology and the shape of the geometry. In this note we present some new exact results for massless scalar fields in three dimensional cavities with the trivial topology. We then compare the known Casimir energy values for several three dimensional cavities. The conclusion we arrived is that the existence of the corners lowers the vacuum energy.

2. New Casimir Energy Results in Some 3-dimensional Cavities

In this section we present Casimir Energies for massless scalar field in some 3-dimensional cavities. These cavities are rather special regions, for all of them are fundamental domains for some crystallographic group generated by reflections with respect to the boundary walls. This property enables us to obtain the wave functions satisfying the Dirichlet boundary conditions and then the correct energy spectrum.

(i) A Pyramidal Cavity

The region is defined by the planes

$$P_1 : z = x, \;\; P_2 : \;\; y = 0, \quad P_3 : y = z, \quad P_4 : z = a. \tag{1}$$

This is the fundamental domain of the group of order 48 generated by the reflection with respect to the above planes [1]. The Casimir energy for massless scalar field in this cavity is (in $\hbar = c = 1$ units)

$$E_{pyr} \simeq \frac{0.069}{a} > 0 \tag{2}$$

(ii) A Conical Cavity

The conical cavity we consider is the one with height $h = a$ and with a very special opening angle $\beta = \arcsin \frac{1}{3}$ [2]. The crystallographic group which admits this cavity as the fundamental domain is the Tetrahedral group. The Casimir energy due to the fluctuation of the massless scalar field is [2]

$$E_{con} \simeq \frac{0.080}{a} > 0 \tag{3}$$

(iii) Triangular Cylinders

Three kind of triangle are the fundamental domain of some crystallographic groups in the plane. These are equilateral, right-angled isosceles and the right-angled triangle which is the half of the equilateral one [3]. Here we give the results for a cylindrical cavity of height b and with equilateral triangular cross-section of edges a. Three possibilities are distinguished:

a) For $b > a$

$$E_{tri} \simeq -\frac{0.053}{a} + \frac{(0.029)b}{a^2}. \tag{4}$$

b) For $a > b > \frac{a}{\sqrt{2}} \simeq 0,7a$

$$E_{tri} \simeq \frac{1}{2}(-\frac{0.013}{b} + \frac{(0.011)a}{b^2} + \frac{0.093}{a} - \frac{(0.048)b}{a^2}. \tag{5}$$

c) For $b < \frac{a}{\sqrt{2}}$

$$E_{tri} \simeq -\frac{0.039}{b} + \frac{(0.014)a}{b^2}. \tag{6}$$

The energy for height a is

$$E_{tri} \simeq \frac{0,022}{a}. \tag{7}$$

3. Some Known Casimir Energy Results for 3-dimensional Cavities

(i) Casimir energy for the spherical cavity of radius a is [4]

$$E_{ball} \simeq \frac{0,045}{a}. \tag{8}$$

(ii) Coming to the cylinders with rectangular cross-sections we list the results for the ones of square cross-section of edges a and of height b [5]:

$$E_{rect} \simeq -\frac{0.013}{a} + \frac{(0.011)b}{a^2} \quad for\ b > a \tag{9}$$

and

$$E_{rect} \simeq -\frac{0.013}{b} + \frac{(0.011)a}{b^2} \quad for\ b < a \tag{10}$$

For the cube of edges a we have

$$E_{cub} \simeq -\frac{0.002}{a} < 0 \tag{11}$$

which is very small.

4. Discussion

To have a meaningful comparison of the results we consider the cavities of equal volumes. Such an approach may help us to understand the shape dependence of the energy. The positive energies for the pyramidal, conical and triangular cylinder cavities can be expressed in terms of energy of spherical cavity of the same volume as:

$$E_{pyr} \simeq 0,51 E_{sph} \tag{12}$$

$$E_{con} \simeq 0,54 E_{sph} \tag{13}$$

$$E_{tr} \simeq 0,11 E_{sph} \tag{14}$$

The energy for the cube (which is negative but very close to zero) of the same volume is

$$E_{cub} \simeq -0,0003 E_{sph} \tag{15}$$

It seems that corners of the cavity reduces the energy. If we think in terms of path integral picture we can say that the paths hitting the corners cannot bounce back, but disappear. Thus we can think that corners in the cavities reduces the phase space volume, and then reduces the vacuum energy.

Acknowledgments The authors H. Ahmedov and I. H. Duru thanks the Turkish Academy of Sciences (TUBA) for its support.

References

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[5] See for example V.M. Mostapanenko and N.N. Trunov "The Casimir Effect and its Applications" Oxford Univ. Press, New York (1997) and references therein.