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Int. J. Electron. Commun. (AEÜ) 59 (2005) 370–373
International Journal of Electronics and Communications AEÜ
www.elsevier.de/aeue

LETTER

The circuit realization of Mexican Hat wavelet function

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Received 23 January 2004; received in revised form 19 June 2004

Abstract

A wavelet network circuit implementation for Mexican Hat mother wavelet has been proposed for nonlinear function approximation which can also be used for the realization of the algebraic nonlinear components. The Mexican Hat mother wavelet function has been implemented with discrete circuit components and it has been observed that the experimental waveform obtained from the realized circuit is approximately same as the Spice simulation of the original function. The circuit simulations of exemplar functions implemented in Spice are also given.

Keywords: Wavelets; Circuit realization; Function approximation

1. Introduction

The time–frequency domain methods and especially the wavelet transform have been extensively used in the analysis of nonstationary signals in many different branches of the science because of its multiresolution property. It is known from the wavelet theory that any finite energy multivariate function can be approximated by wavelets. The wavelet network inspired by both feedforward neural networks and wavelet decompositions has been proposed in [1]. The identification of static and dynamical systems using wavelet network have attracted many researchers since the wavelet analysis has been successfully applied for analyzing signals both in time and frequency domain with different resolution levels [2–5]. The circuit implementation of a wavelet network for static systems has been accomplished in [6] by using the sigmoidal wavelet function proposed in [7]. In this study, a circuit implementation for the Mexican Hat mother wavelet has been proposed because of its better approximation properties compared to the sigmoidal mother wavelet function.

2. Wavelet network

When the input–output pairs measured from the system to be modelled is given as

$$\{x(t_k),\, y(t_k) \,|\, y(t_k) = f(x(t_k)) + \varepsilon_k,\, k = 1, \ldots, K\}, \tag{1}$$

where ε_k is the measurement error and $f(\cdot) : \mathbb{R}^d \to \mathbb{R}$, the problem is to minimize the mean-squared error between the actual output and the output of the wavelet network

$$e \triangleq \frac{1}{2} E \left\{ [y - f_w(x)]^2 \right\}, \tag{2}$$

where the output of the wavelet network is defined as

$$f_w(x) = \sum_{i=1}^{N} w_i \Psi(D_i x - b_i) + c^{\mathrm{T}} x + \overline{b}, \tag{3}$$

where N is the number of d-*dimensional* wavelons, w_i is the wavelet coefficients for each d-*dimensional* wavelon,

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doi:10.1016/j.aeue.2004.11.023

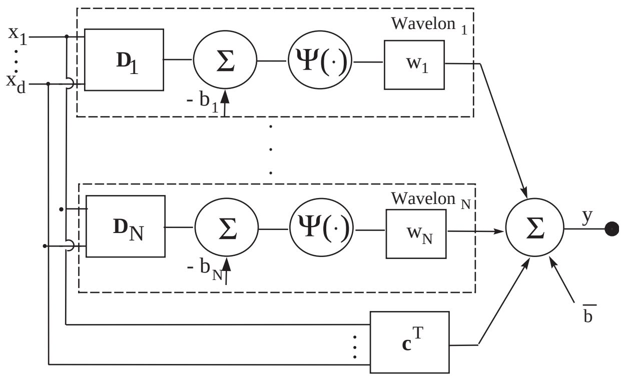

Fig. 1. The block diagram of the static wavelet network.

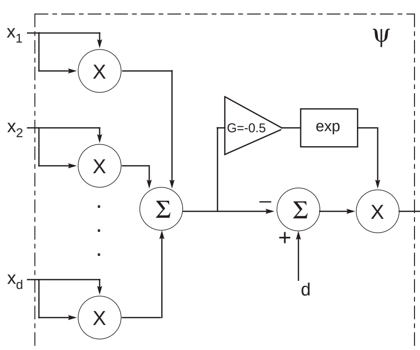

Fig. 2. The block diagram of Mexican hat mother wavelet circuit.

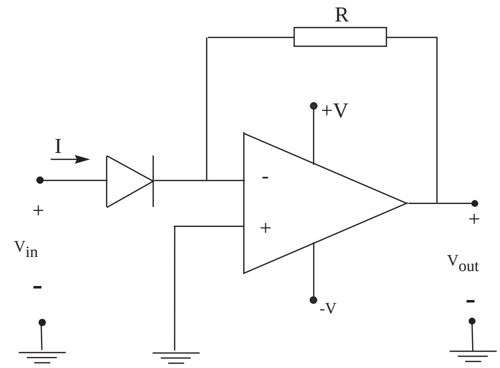

Fig. 3. The antilog amplifier.

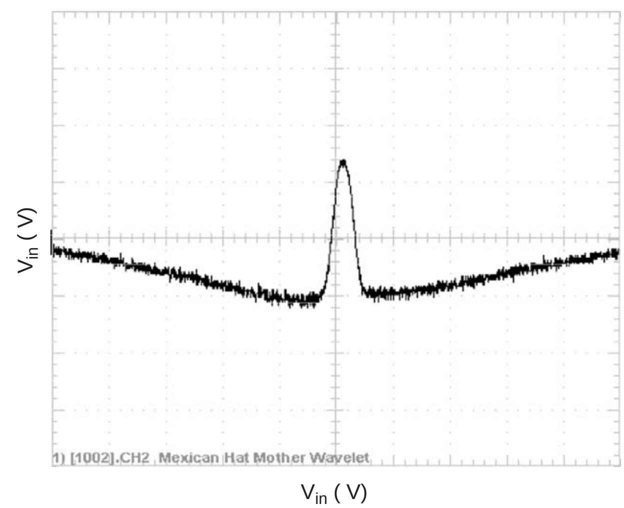

Fig. 4. The Mexican hat mother wavelet.

$D_i = \mathrm{diag}(d^i_{11}, \ldots, d^i_{mm}) \in \mathbb{R}^{d\times d}$ where $d^i_{jj} = 1/a_{ij}$ and a_{ij} is the dilation coefficient, $\Psi(\cdot) : \mathbb{R}^d \to \mathbb{R}$ is the mother wavelet function, $b_i \in \mathbb{R}^d$ is the translation coefficient vector, $c \in \mathbb{R}^d$ represents the coefficient of the linear term and $\bar{b}$ is the bias term to approximate the functions with nonzero mean. The block diagram of the wavelet network is shown in Fig. 1. The optimum parameter set and the number of wavelons are to be determined for the construction of the wavelet network. The selection of suitable wavelons has been implemented by the "Stepwise Selection by Orthogonalization" algorithm proposed in [5].

The Mexican Hat mother wavelet is one of the most commonly used real mother wavelet because of its good approximation ability. The mother wavelet defined as

$$\Psi(x) = (d - \|x\|^2)\exp\left(-\frac{\|x\|^2}{2}\right) \quad \Psi(\cdot) : \mathbb{R}^d \to \mathbb{R}, \quad (4)$$

where $\|x\|^2 = x^{\mathrm{T}}x$.

The block diagram of the *d-dimensional* Mexican Hat mother wavelet circuit is shown in Fig. 2. The adders and amplifiers have been implemented with operational amplifiers. The four channel four quadrant analog multiplier MLT04 of Analog Devices has been used for multiplication. The exponential function has been implemented with an antilog amplifier which is contained in the real time analog computational unit AD538 IC of Analog Devices. The antilog amplifier is shown in Fig. 3.

Assuming that the current of the diode connected to the input of the operational amplifier is

$$I = I_{\mathrm{s}} \exp\left(\frac{V_{\mathrm{in}}}{V_{\mathrm{t}}}\right), \quad (5)$$

where $I_{\mathrm{s}}, V_{\mathrm{in}}, V_{\mathrm{t}}$ are reverse saturation current, the input voltage and threshold voltage, respectively, where $V_{\mathrm{t}} = kT/q$ where k is the Boltzman constant, T is temperature and q is the electron charge. The output voltage is obtained as

$$V_{\mathrm{out}} = -RI_{\mathrm{s}} \exp\left(\frac{V_{\mathrm{in}}}{V_{\mathrm{t}}}\right). \quad (6)$$

The measured input–output voltage pair of the Mexican Hat mother wavelet circuit is shown in Fig. 4.

3. Applications

The circuit implementations of the sample wavelet networks with Mexican Hat mother wavelet have been introduced. For the given networks, the normalized sensitivities of the output voltages with respect to the wavelet network parameters have also been calculated in this section.

The normalized sensitivity of the output function with respect to the parameter β_i is defined in [8] as

$$S(f_w, \beta_i) = \frac{\partial \ln f_w}{\partial \ln \beta_i}. \tag{7}$$

The multiparameter sensitivity of the output function is given in [9] by

$$S(f_w, \underline{\beta}) = \left[\frac{\partial \ln f_w}{\partial \ln \beta_1} \quad \frac{\partial \ln f_w}{\partial \ln \beta_2} \cdots \frac{\partial \ln f_w}{\partial \ln \beta_K}\right] = [S(f_w, \beta_1) S(f_w, \beta_2) \cdots S(f_w, \beta_K)], \tag{8}$$

where

$$\underline{\beta} = [\beta_1 \ \beta_2 \ \cdots \ \beta_K]. \tag{9}$$

When the multiparameter sensitivity vector is known, then the normalized changes in the parameters can be found from

$$\frac{\Delta f_w}{f_w} = S(f_w, \underline{\beta}) \Delta\hat{\beta}^{\mathrm{T}}, \tag{10}$$

where T denotes transpose and

$$\Delta\hat{\beta} = \left[\frac{\Delta\beta_1}{\beta_1} \ \frac{\Delta\beta_1}{\beta_2} \cdots \frac{\Delta\beta_1}{\beta_K}\right]. \tag{11}$$

Example 1. The function to be approximated is given by

$$y = \begin{cases} -2.186x - 1.2864 & -1 < x \leqslant -0.2, \\ 4.246x & -0.2 < x \leqslant 0, \\ \mathrm{e}^{-0.5x-0.5} \sin(3x^2 + 7x) & 0 < x \leqslant 1 \end{cases} \tag{12}$$

and 6 wavelons have been used for the approximation. The circuit has been simulated in Spice. The parameters of the wavelet network have been calculated as in Table 1 . The target function and the simulation results have been shown in Fig. 5.

The normalized change in the output is calculated for the parameters with 1% tolerances using the normalized sensitivities in Table 1 as

$$\frac{\Delta f_w}{f_w} = 0.041\,\mathrm{mV/percent} \tag{13}$$

Example 2. The 2-dimensional function

$$y = f(x_1, x_2) = \begin{cases} 2\mathrm{e}^{x_1^2 - x_2^2} \cos(2x_1)\sin(3x_2), & 0 < x_1 \leqslant 1, \ 0 < x_2 \leqslant 1, \\ 0 & \text{elsewhere.} \end{cases} \tag{14}$$

has been approximated by five $2-D$ wavelons. The parameters of the wavelet network and the Spice simulation result have been given in Table 2 and in Fig. 6, respectively.

Assuming the tolerances of the parameters used in approximation is 1%, the normalized change in the output using the normalized sensitivities can be found

Table 1. The wavelet network parameters of Example 1

Parameter name	Parameter value	Normalized sensitivity (mV/percent)
D_1	8.4706	0.007671
D_2	1.8578	−0.1579
D_3	4.2705	−1.181
D_4	1.8304	−5.370
D_5	4.9995	−0.4220
D_6	3.5911	8.862
b_1	−0.2071	0.07681
b_2	−0.9480	−0.1579
b_3	0.8648	−1.181
b_4	−0.4371	−5.373
b_5	0.7565	−0.4219
b_6	0.2365	8.853
w_1	0.1544	−0.6890
w_2	−0.2271	1.012
w_3	−0.7417	0.1025
w_4	0.4897	1.274
w_5	−0.3318	0.03457
w_6	−0.7728	−1.505
c	−0.5848	-1.931×10^{-5}
$\overline{b}$	0.0383	0.3830

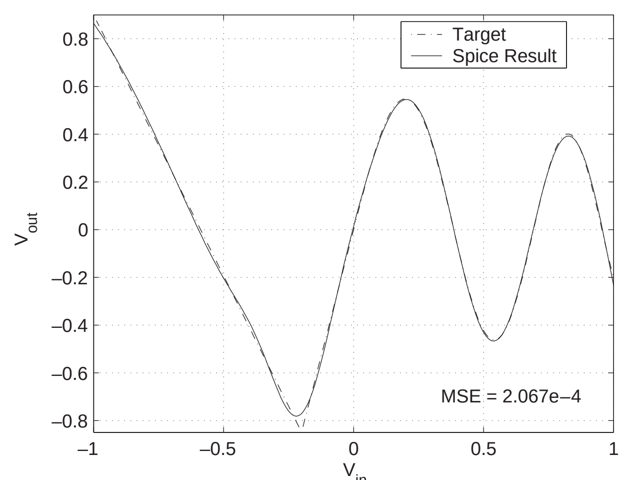

Fig. 5. The result of approximation for Example 1 with wavelet network.

Table 2. The wavelet network parameters of Example 2

i	D_i	b_i	w_i
1	diag(2.2398, 2.8578)	$[-0.0076\ 0.5240]^{\mathrm{T}}$	−2.5851
2	diag(4.2121, 2.8351)	$[-1.4035\ 0.9603]^{\mathrm{T}}$	−8.1595
3	diag(1.5585, 2.1494)	$[-0.0745\ \ -0.5048]^{\mathrm{T}}$	0.9558
4	diag(4.3670, 2.6324)	$[-0.9103\ 0.9273]^{\mathrm{T}}$	−0.8347
5	diag(2.6742, 3.0105)	$[-0.0122\ 0.5462]^{\mathrm{T}}$	2.0968

$c = [-0.2638 \ \ -0.0830]^{\mathrm{T}}$ $\overline{b} = 0.2113$

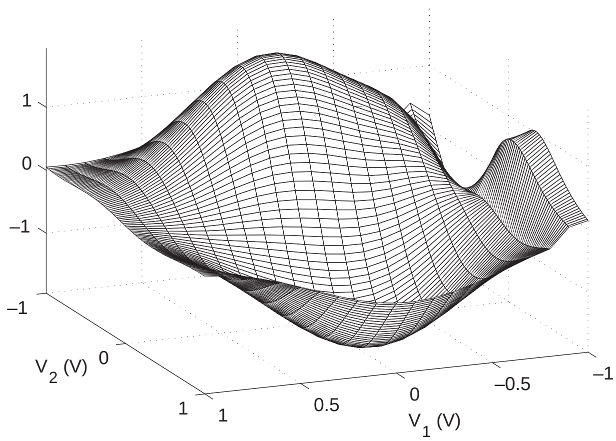

Fig. 6. The result of approximation for Example 2 with wavelet network.

from the Eq. (10) as

$$\frac{\Delta f_w}{f_w} = 0.18\,\mathrm{mV/percent}, \qquad (15)$$

where the normalized sensitivities with respect to 28 parameters have been omitted for the sake of brevity.

4. Conclusions

In this study, the circuit implementation of Mexican Hat mother wavelet has been given and the static systems have been modelled with the wavelet network. The proposed method may not be suitable for the implementation with the discrete components because of the difficulties in obtaining the correct values of the components and the number of the components of the circuit could be large. On the other hand, the wavelet network contains only operational amplifiers, diodes and passive circuit components and it is suitable for VLSI manufacturing because of the systematic procedure and also the VLSI manufacturing allows the accurate realization. From the examples, the sensitivity analysis for the static systems show that the changes in the parameters of the network result in acceptable change in the output voltage. The identification of the dynamical systems using the proposed wavelet network is of the concern of the future works.

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