

Physical Layer Network Coding Enabled NOMA with Multiple Antennas

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Abstract—In this work, a combination of non-orthogonal multiple access (NOMA) with multiple antennas and physical layer network coding (PNC) scheme is proposed to increase the overall data rate. In the proposed scheme, we employ higher-order modulations for the users with relatively high signal-to-noise ratio (SNR) in the PNC-NOMA pair to increase the data rate. Meanwhile, lower-order modulations are chosen for the users with relatively lower SNR values in the PNC-NOMA pair. We showed the results in terms of bit error rate (BER) for different number of antennas and users in the proposed PNC-NOMA scheme.

Index Terms—PNC, NOMA, Multiple Antennas Systems.

I. INTRODUCTION

High data rate, ultra-low latency, large connectivity while enhancing the overall spectral efficiency are some of the requirements of the sixth generation (6G) wireless communication systems. Non-orthogonal multiple access (NOMA) with multiple antennas is one of the key enabling technologies for improving spectral efficiency.

In the NOMA scheme, the base station (BS) serves multiple users simultaneously at the same frequency and at the same time [1]. In the power-domain NOMA, the BS exploits the power difference to serve more than one user while using successive interference cancellation (SIC) at the receiver side to eliminate the inter-user interference [2].

In the physical layer network coding (PNC) scheme, there are one destination node and two terminal nodes that exchange data within two time slots instead of four time slots [3]. Firstly, two terminal nodes send their signals to the destination node, simultaneously. The destination node, then, broadcasts, the network coded symbol (NCS) to both terminal nodes. In the corresponding terminal node, the other node's data is obtained by operating the network coding operation of the NCS and its own data. For binary cases, network-coding operation can be considered as an exclusive-OR (XOR).

In [4], the combination of PNC and multiple antenna has been examined to improve the system performance by using a sum difference matrix and log-likelihood ratio (LLR) after performing zero-forcing (ZF) or minimum mean squared error (MMSE) equalization. The PNC scheme has been further extended to the multi-user massive multiple input multiple output (MIMO) by using M-ary Quadrature Amplitude Modulations (M-QAMs) in [5].

On the other hand, recent studies have examined the combination of NOMA and PNC. For example, a cognitive radio-inspired NOMA and PNC combination has been presented to increase the spectral efficiency further in [6], while a joint NOMA and PNC scheme has been considered through clustering algorithm by employing ZF to eliminate inter-cluster interference in [7]. In [8], [9], the decoding techniques for the PNC with multiple access have been examined to improve the throughput. However, the complexity of these decoding schemes is increased exponentially at high data rates. In order to address this problem, in [10], a combination of the NOMA and PNC has been provided by using the deep neural network (DNN) which has lower complexity compared to the traditional brute force approach. Uplink NOMA-based PNC system with multiple antennas through a novel user set selection has been presented in [11] where the same modulation scheme for all users in a NOMA pair is employed.

In this paper, we propose PNC enabled NOMA with multiple antennas scheme based on the LLR detection which does not obtain the users' symbol separately. We summarize the contributions of this work as:

- We propose a novel LLR-based detection for a PNC enabled NOMA scheme with multiple antennas.
- We detect NCS directly in each PNC pair by employing different modulation schemes for strong and weak users in a NOMA pair.

The remainder of this paper is organized as follows. The considered system model is given in Section 2. Next, the proposed PNC for NOMA with multiple antennas is given in Section 3. Section 4 gives the simulation results. The conclusion is given in Section 5.

II. SYSTEM MODEL

An uplink NOMA-PNC system with multiple antennas is considered as shown in Fig. 1. There is a single BS with N antennas, serving K_u users where each user is equipped with a single antenna. It is assumed that the BS has perfect channel state information (CSI) and all users transmit their signals simultaneously to the BS under perfect synchronization, using M-QAM.

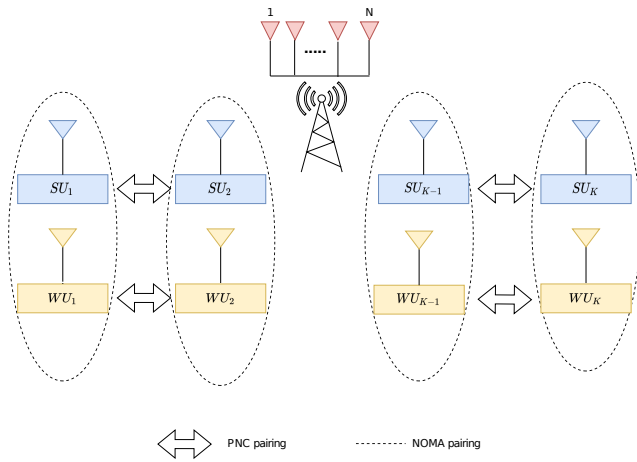


Fig. 1: System model for uplink NOMA-PNC with multiple antennas.

In the considered system model, there are two users sets called strong set and weak set. Each set includes K users with $K = K_u/2$. The strong set includes the users which have relatively higher SNR values than the users in the weak set. The channel matrices of the strong and the weak sets are defined by \mathbf{H}_1 and \mathbf{H}_2 , respectively, according to

$$\mathbf{H}_1 = [\mathbf{h}_{1,1} \dots \mathbf{h}_{k,1} \dots \mathbf{h}_{K,1}], \quad (1)$$

$$\mathbf{H}_2 = [\mathbf{h}_{1,2} \dots \mathbf{h}_{k,2} \dots \mathbf{h}_{K,2}], \quad (2)$$

where $\mathbf{h}_{k,r} = \mathbf{g}_{n,r} \sqrt{L_{k,r}}$ is the uplink channel vector with dimension of $K \times 1$ between each user and the BS with $k = 1, 2, \dots, K$ and $r = 1, 2$. The path-loss coefficient is given by $L_{k,r}$ and $\mathbf{g}_{k,r}$ is the fading coefficient modelled by $\mathcal{CN}(\mathbf{0}_{N \times 1}, \mathbf{I}_{N \times 1})$.

The received signal at the BS is:

$$\mathbf{y} = \mathbf{H}_1 \mathbf{s}^s + \mathbf{H}_2 \mathbf{s}^w + \mathbf{n}, \quad (3)$$

where \mathbf{n} is the additive white Gaussian noise (AWGN) vector with dimension of $N \times 1$ whose elements are complex Gaussian random variables with σ_n^2 variance and zero mean, \mathbf{s}^s and \mathbf{s}^w are transmit signal vectors of strong and weak users, respectively.

The composite transmit symbol vector $\mathbf{s} \in \mathbb{C}^{2K \times 1}$ is obtained through the concatenation of \mathbf{s}^s and \mathbf{s}^w according to

$$\mathbf{s} = [\mathbf{s}_1^s \ \mathbf{s}_2^s \ \dots \ \mathbf{s}_K^s \ \mathbf{s}_1^w \ \mathbf{s}_2^w \ \dots \ \mathbf{s}_K^w]^T. \quad (4)$$

where $[\cdot]^T$ is the transpose operator.

The received vector $\mathbf{y} \in \mathbb{C}^{N \times 1}$ can be re-written as:

$$\mathbf{y} = \mathbf{C} \mathbf{s} + \mathbf{n}, \quad (5)$$

where $\mathbf{C} = [\mathbf{H}_1 \ \mathbf{H}_2]$ is the composite channel matrix.

In the considered NOMA-PNC scheme, the paired users, where each SU is paired with another SU and each WU is paired with another WU, exchange their data via the BS using the PNC scheme. This scenario is called strong-strong and weak-weak (SS-WW) user pairing [11].

The NCS vector is obtained for the considered scenario as:

$$\mathbf{s}_R = \begin{bmatrix} \mathbf{s}_1^s \oplus \mathbf{s}_2^s \\ \mathbf{s}_3^s \oplus \mathbf{s}_4^s \\ \dots \\ \mathbf{s}_{K-1}^s \oplus \mathbf{s}_K^s \\ \mathbf{s}_1^w \oplus \mathbf{s}_2^w \\ \mathbf{s}_3^w \oplus \mathbf{s}_4^w \\ \dots \\ \mathbf{s}_{K-1}^w \oplus \mathbf{s}_K^w \end{bmatrix}. \quad (6)$$

We use the sum-difference matrix defined in [11] as:

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_{2 \times 2} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{D}_{2 \times 2} \end{bmatrix}, \quad (7)$$

where $\mathbf{D}_{2 \times 2} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

Then, the received vector can be rewritten as:

$$\mathbf{y} = (\mathbf{C} \mathbf{D}^{-1}) (\mathbf{D} \mathbf{s}) + \mathbf{n} \quad (8)$$

$$= \hat{\mathbf{C}} \hat{\mathbf{s}} + \mathbf{n}, \quad (9)$$

where $\hat{\mathbf{C}} \in \mathbb{C}^{N \times 2K}$ is manipulated composite channel matrix and $\mathbf{D} \mathbf{s} = \hat{\mathbf{s}} \in \mathbb{C}^{2K \times 1}$ is the target estimated vector.

The manipulated composite channel matrix is provided by,

$$\hat{\mathbf{C}} = [\hat{\mathbf{H}}_1 \ \hat{\mathbf{H}}_2], \quad (10)$$

where $\hat{\mathbf{H}}_r = \mathbf{H}_r \mathbf{D}^{-1}$, $\forall r \in \{1, 2\}$ is the manipulated composite channel matrix of either the strong or the weak set. After the definition of the manipulated composite channel matrix, MMSE or ZF equalization is performed to detect the NCSs for the SUs and WUs pairs, separately.

For the ZF solution, the equalization matrix of either strong or weak users set $\hat{\mathbf{W}}_r$, $\forall r \in \{1, 2\}$ is given by,

$$\hat{\mathbf{W}}_r = (\hat{\mathbf{H}}_r^H \hat{\mathbf{H}}_r)^{-1} \hat{\mathbf{H}}_r^H, \quad (11)$$

where $(\cdot)^H$ is the Hermitian transpose operator.

For the MMSE solution, the equalization matrix of either the strong or the weak users set is defined by,

$$\hat{\mathbf{W}}_r = (\hat{\mathbf{H}}_r^H \hat{\mathbf{H}}_r + \mathbf{I}_{K \times K} \sigma_n^2)^{-1} \hat{\mathbf{H}}_r^H. \quad (12)$$

Then, the equalized vector of the SUs is obtained by:

$$\mathbf{r}_1 = \hat{\mathbf{W}}_1 \mathbf{H}_1 \mathbf{s}^s + \hat{\mathbf{W}}_1 \mathbf{H}_2 \mathbf{s}^w + \hat{\mathbf{W}}_1 \mathbf{n}. \quad (13)$$

After performing SIC, the equalized vector of the WUs is presented by

$$\mathbf{r}_2 = \hat{\mathbf{W}}_2 \mathbf{H}_2 \mathbf{s}^w + \hat{\mathbf{W}}_2 \mathbf{n}. \quad (14)$$

Then, the equalized vectors are concatenated as

$$\mathbf{r} = [\mathbf{r}_1^T, \mathbf{r}_2^T]^T. \quad (15)$$

III. PROPOSED SCHEME

In this paper, we propose an LLR-based detection method to obtain PNC symbols at the BS in the case that the users in the NOMA pairs employ different QAM schemes while either strong users or weak users in PNC pairs exchange data through NCS.

Let $\mathbf{t}_j \in \mathbb{C}^{1 \times J}$ be the sum vector whose elements are the sum signals for the j^{th} possible NCS and $\mathbf{d}_j \in \mathbb{C}^{1 \times J}$ is the difference vector whose elements are the difference signals for the j^{th} possible NCS where the number of possible NCS values is $J = \log_2 M$, with M indicating the modulation size.

Thereby, the log likelihood [5] is defined by:

$$\text{LL}_{k,j} = \sum_{p=1}^J \ln \left(e^{-(r_m - (\mathbf{t}_{j,p}))^2 - (r_{m+1} - (\mathbf{d}_{j,p}))^2)} \right) + V_n, \quad (16)$$

where $\text{LL}_{k,j}$ is the log likelihood for the j^{th} possible NCS value of k^{th} pair, r_m is the m^{th} element of the equalized vector in (15) with $m \in \{1, 3, \dots, 2K - 1\}$ and $k = \frac{m+1}{2}$, V_n is the log likelihood term related to the noise variances after equalization and can be obtained as

$$V_n = \ln \left(\frac{1}{\sqrt{2\pi\sigma_m^2}} \right) + \ln \left(\frac{1}{\sqrt{2\pi\sigma_{m+1}^2}} \right), m = 1, 3, \dots, 2K - 1 \quad (17)$$

where σ_m^2 and σ_{m+1}^2 where m and $m + 1$ indices denote a pair in the equalized vector are the noise variances after the equalization obtained by:

$$\sigma_\ell^2 = \begin{cases} \{\hat{\mathbf{W}}_1 \hat{\mathbf{W}}_1^H\}_{\ell,\ell} \sigma_n^2 + \{\mathbf{A} \mathbf{A}^H\}_{\ell,\ell} \mathbf{P}_w, & \forall \ell \in \{1, 2, \dots, K\} \\ \{\hat{\mathbf{W}}_2 \hat{\mathbf{W}}_2^H\}_{\ell-K, \ell-K} \sigma_n^2, & \forall \ell \in \{K + 1, \dots, 2K\} \end{cases} \quad (18)$$

where $\mathbf{A} = \hat{\mathbf{W}}_1 \mathbf{H}_2$ is multiplier matrix and $\mathbf{P}_w = \mathbb{E}\{(\mathbf{s}^w)(\mathbf{s}^w)^H\}$ is the autocorrelation of the transmit symbol vector of the weak set and $\mathbb{E}\{\cdot\}$ is the expectation operator.

Then, $\mathbf{v} \in \mathbb{C}^{1 \times J}$ is a vector whose elements are possible NCS values as,

$$\mathbf{v} = [v_1, v_2, \dots, v_J]. \quad (19)$$

Then, the NCS of the k^{th} pair is chosen based on the LLR value. Thereby, the index of the NCS value is determined by

$$j_k^* = \underset{1 \leq j \leq J}{\text{argmax}} (\text{LL}_{k,j}); \quad k = 1, \dots, K. \quad (20)$$

Finally, the NCSs are obtained as:

$$\tilde{s}_{Rk} = v_{j_k^*}; \quad k = 1, \dots, K. \quad (21)$$

TABLE I: Possible transmit, sum, difference signals and NCS values for 4-QAM.

s_1	s_2	$s_1 + s_2$	$s_1 - s_2$	$s_R = s_1 \oplus s_2$
-1(0)	-1(0)	-2	0	-1(0)
1(1)	-1(0)	0	2	1(1)
-1(0)	1(1)	0	-2	1(1)
1(1)	1(1)	2	0	-1(0)

For 4-QAM, the possible signal values, sum signal values, difference signal values and NCS values are given in Table I.

In this case, with $J = 2$, each sum signals vector \mathbf{t}_j and each difference signals vector \mathbf{d}_j are given by,

$$\mathbf{t}_1 = [0, 0], \quad \mathbf{d}_1 = [-2, 2], \quad (22)$$

$$\mathbf{t}_2 = [-2, 2], \quad \mathbf{d}_2 = [0, 0], \quad (23)$$

Then, the possible NCS values vector \mathbf{v} is given by:

$$\mathbf{v} = [v_1, v_2] = [1, -1] \quad (24)$$

After that, k^{th} NCS value is determined by using (20) and (21).

For 16-QAM with $J = 4$, the possible signal values, sum signal values, difference signal values and NCS values are given in Table II.

TABLE II: Possible transmit, sum, difference signals and NCS values for 16-QAM.

s_1	s_2	$s_1 + s_2$	$s_1 - s_2$	$s_R = s_1 \oplus s_2$
1(11)	1(11)	2	0	-3(00)
1(11)	-1(01)	0	2	3(10)
1(11)	3(10)	4	-2	-1(01)
1(11)	-3(00)	-2	4	1(11)
-1(01)	1(11)	0	-2	3(10)
-1(01)	-1(01)	-2	0	-3(00)
-1(10)	3(10)	2	-4	1(11)
-1(01)	-3(00)	-4	2	-1(01)
3(10)	1(11)	4	2	-1(01)
3(10)	-1(01)	2	4	1(11)
3(10)	3(10)	6	0	-3(00)
3(10)	-3(00)	0	6	3(10)
-3(00)	1(11)	-2	-4	1(11)
-3(00)	-1(01)	-4	-2	-1(01)
-3(00)	3(10)	0	-6	3(10)
-3(00)	-3(00)	-6	0	-3(00)

Then, for 16-QAM, each sum signals vector \mathbf{t}_j and each difference signals vector \mathbf{d}_j are given as:

$$\begin{aligned} \mathbf{t}_1 &= [0, 0, 0, 0], & \mathbf{d}_1 &= [2, -2, 6, -6] \\ \mathbf{t}_2 &= [2, -2, 6, -6], & \mathbf{d}_2 &= [0, 0, 0, 0], \\ \mathbf{t}_3 &= [-2, 2, 2, -2], & \mathbf{d}_3 &= [4, -4, 4, -4], \\ \mathbf{t}_4 &= [4, -4, 4, -4], & \mathbf{d}_4 &= [-2, 2, 2, -2]. \end{aligned} \quad (25)$$

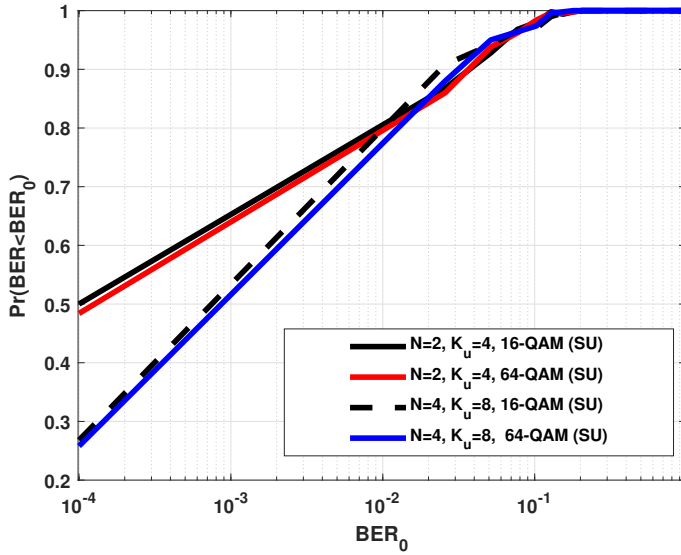


Fig. 2: BER performances for the proposed PNC-NOMA with MMSE equalization when WUs use 4-QAM and various modulation schemes are employed for SUs.

Thereby, the possible NCS values vector \mathbf{v} is given by:

$$\begin{aligned} \mathbf{v} &= [v_1, v_2, v_3, v_4] \\ &= [3, -3, 1, -1] \end{aligned}$$

Then, k^{th} NCS value is determined by (20) and (21).

IV. PERFORMANCE RESULTS

This section gives the performance results in wireless channel modeled by Rayleigh distribution for different number of users, antennas and modulation types. We obtain the cumulative distribution function (CDF) of the bit error rate (BER) for strong and weak users. The SNR values of users in the weak set are uniformly distributed between 5dB and 15dB and the SNR values of users in the strong set are uniformly distributed between 35dB and 45dB.

In Fig.2, the comparison results are given based on the CDF of BER versus the target BER for the proposed PNC-NOMA through different modulation schemes and various number of users. The SUs employ either 64-QAM or 16-QAM while the WUs use 4-QAM. It is observed that when the data rate of the SUs is increased, the BER is not degraded by employing the proposed scheme. It is also shown that BER performance is slightly reduced when the number of users is increased.

In Fig. 3, the conventional NOMA and the proposed PNC-NOMA are compared by using CDF of BER for NCS versus target BER in which 4-QAM for WUs and 16-QAM for SUs are employed. For conventional NOMA, MMSE equalization and SIC are employed in the uplink system with multiple antennas while allocating the same number of users as the proposed NOMA-PNC scheme. However, in the conventional NOMA, the PNC detection is not performed and each user's data is decoded separately. Meanwhile, for the proposed PNC-NOMA, NCS is detected for each PNC pair. As given in

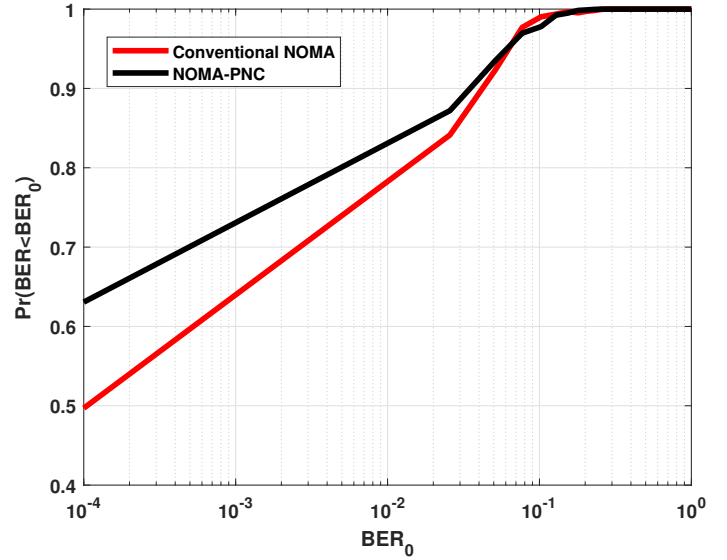


Fig. 3: Comparison of the proposed PNC-NOMA and conventional NOMA with $N = 2, K_u = 4, 16\text{-QAM}$ for SUs and 4-QAM for WUs.

Fig. 3, the proposed scheme improves the BER performance significantly compared to the conventional NOMA.

V. CONCLUSION

In this work, the NOMA-based PNC with multiple antennas has been proposed through a novel LLR based detection scheme. We employ different modulation schemes for NOMA pairs in each PNC pair to increase the overall data rate. It is shown that our proposed NOMA-PNC gives better BER performance when compared with the conventional NOMA. As a future work, the proposed method will be combined with an efficient user selection to achieve multiuser diversity gain.

ACKNOWLEDGMENT

This work has been funded by the European Union Horizon 2020, RISE 2018 scheme (H2020-MSCA-RISE-2018) under the Marie Skłodowska-Curie grant agreement No. 823903 (RECENT).

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