GAUGED AND GEOMETRIC VECTOR FIELDS AT THE MeV SCALE

A Thesis Submitted to
the Graduate School of
İzmir Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of

DOCTOR OF PHILOSOPHY

in Physics

by
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July 2020
İZMİR

ACKNOWLEDGMENTS

I would like to express my deepest appreciation to my supervisor Prof. Dr. Durmuş Ali Demir who has been continually supportive to me. It is an honour to work with such a guide who teaches and works Physics with imcomparable passion and enthusiasm. I would have to say that it is him who has most greatly inspired me in Physics.

I would like to thank to the committee members Prof. Dr. Ramazan Tuğrul Senger and Assoc. Prof. Dr. Levent Selbuz for joining to the thesis watch seminars and making fruitful disccusions. I am also thankful to Prof. Dr. İsmail Turan and Assoc. Prof. Dr. Fatih Erman for joining the thesis defense seminar as committee members.

I would like to express my thanks to my friends and all the people who have provided a nice environment in the department.

I would like to thank my family for always being by my side.

ABSTRACT

GAUGED AND GEOMETRIC VECTOR FIELDS AT THE MeV SCALE

In this thesis, we have studied gauged and geometric vector fields at the MeV scale in two main parts. The basic framework of these two parts are given briefly as follows.

In the first part (Chapter 2), we have built a family-nonuniversal $U(1)'$ model populated by an MeV-scale sector with a minimal new field content which explains the recent anomalous beryllium decays. Excited beryllium has been observed to decay into electron-positron pairs with a $6.8\ \sigma$ anomaly. The process is properly explained by a 17 MeV proto-phobic vector boson. In this thesis, we consider a family-nonuniversal $U(1)'$ that is populated by the $U(1)'$ gauge boson Z' and a scalar field S. The kinetic mixing of Z' with the hypercharge gauge boson, as we show by a detailed analysis, generates the observed beryllium anomaly. We show that beryllium anomaly can be explained by an MeV-scale sector with a minimal new field content.

In the second part (Chapter 3), we have shown how a light vector particle can arise from metric-affine gravity and how this particle fits the current data and constraints on the dark matter. We show that, metric-affine gravity , which involves metric tensor and affine connection as two independent fields, dynamically reduces, in its minimal form, to the usual gravity plus a massive vector field. The vector Y_μ is neutral and long-living when its mass range lies in the range $9.4\ \mathrm{MeV} < M_Y < 28.4\ \mathrm{MeV}$. Its scattering cross section from nucleons, which is some 60 orders of magnitude below the current bounds, is too small to facilitate direct detection of the dark matter. This property provides an explanation for whys and hows of dark matter searches. We show that due to its geometrical origin the Y_μ couples only to fermions. This very feature of the Y_μ makes it fundamentally different than all the other vector dark matter candidates in the literature. The geometrical dark matter we present is minimal and self-consistent not only theoretically but also astrophysically in that its feebly interacting nature is all that is needed for its longevity.

ÖZET

MeV ÖLÇEĞİNDE AYARLI VE GEOMETRİK VEKTÖR ALANLAR

Bu tezde MeV ölçeğinde ayarlı ve geometrik vektör alanları iki ana bölümde çalıştık. Bu iki bölümün ana hatları kısaca aşağıdaki gibidir.

İlk kısımda (Bölüm 2) , yakın zamandaki olağan dışı berilyum bozunumlarını her yönüyle açıklayan, minimum yeni alan içeren MeV ölçeğindeki bir sektör ile desteklemiş bir model oluşturduk. Uyarılmış berilyumun elektron-positron çiftine olağan dışı bozunumu $6.8\ \sigma$ düzeyinde gözlemlendi. Bu proses 17-MeV'lik proto-phobic bir vektör bozonla açıklanmıştır. Bu tezde, bir $U(1)'$ ayar bozonu Z' ve $U(1)'$ altında yüklü Standard Model (SM) ayar simetrisi altında singlet olan bir S skaler alanından oluşan, fermion ailelerinde evrensel olmayan bir $U(1)'$ ele alıyoruz. SM kiral fermiyon ve skaler alaları $U(1)'$ altında yüklüdür ve biz bu yüklerin anomaliden ari olmalarını sağlıyoruz. Cabibbo-Kobayashi-Maskawa (CKM) matrisi, S tarafından sağlanan yüksek boyutlu Yukawa etkileşimleri ile doğru bir şekilde üretilmektedir. Z''ın birinci fermiyon jenerasyonuna olan vektör ve aksiyal-vektör akım kuplajları çeşitli deneylerden gelen tüm kısıtlamaları sağlamaktadır. Z' bozonu hiperyük ayar bozonu ile kinetik karışım yapabilir ve S SM benzeri Higgs alanına doğrudan kuplaj yapabilir. Detaylı bir analiz ile gösterceğimiz üzere, Z''ın hiperyük ayar bozonu ile kinetik karışımı gözlemlenmiş olan berilyum anomalisini meydana getirir. Bulduğumuz o ki, berilyum anomalisi minimal yeni alan içeren MeV ölçeğindeki bir sektör ile her yönüyle açıklanabilir. Oluşturduğumuz bu minimal model çeşitli olağan dışı SM bozunumlarının tartışılabileceği bir yapıdadır.

İkinci kısımda (Bölüm 3), hafif bir vektör parçacığın metrik-afin kütleçekiminden nasıl elde edilebileceğini ve eldeki karanlık madde verilerine ve kısıtlamalarına nasıl fit ettiğini gösterdik. Düz rotasyon eğrilerinden yapı oluşumuna kadar çeşitli olaylar için gerekli olan karanlık madde sadece nötral ve uzun ömürlü değil aynı zamanda bilinen maddeden oldukça izole gibi görünüyor. Burada gösteriyoruz ki metrik tensörü ve afin bağlantısını iki bağımsız alan olarak içeren minimal formdaki metrik-afin kütleçekimi dinamik olarak bilinen kütleçekim artı kütleli bir vektör alana indirgenir. Yalnızca quark, lepton ve kütleçekimle etkileşen Y_μ vektörü kütle aralığı $9.4\ \mathrm{MeV} < M_Y < 28.4\ \mathrm{MeV}$ aralığında iken nötral ve uzun ömürlüdür (evrenin yaşından daha uzun). Karanlık maddenin günümüz kıstlamalarından 60 mertebe daha aşağıda olan nükleonlardan saçılma

tesir kesiti doğrudan keşfedilmeye olanak sağlaması için çok fazla küçüktür. Bu özellik karanlık madde araştırmalarının nedenleri ve nasıllarına bir açıklama sağlamaktadır. Gösteriyoruz ki geometrik doğasından dolayı Y_μ skaler ve bozonlara kuplaj yapmamaktadır. Y_μ yalnızca fermiyonlara kuplaj yapmaktadır. Y_μ'nün bu belirgin özelliği kendisini litertürdeki diğer tüm vektör karanlık madde adaylarından farklı kılar. Sunduğumuz geometrik karanlık madde sadece teorik açıdan minimal ve kendi içinde uyumlu değil aynı zamanda astrofiziksel olarak da öyledir, öyle ki zayıf etkileşen doğası uzun ömürlü olması için gerekli olan tek şeydir.

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CHAPTER 1

INTRODUCTION

We have studied gauged and geometric vector fields at the MeV scale in this thesis. The light, weakly-coupled new particles have been in consideration especially for the dark matter conundrum. They have been considered as the mediators between the visible and the dark sectors or the dark matter itself. The possible existence of these kinds of light new particles have provided motivation to probe the new physics at the intensity frontier.

One of the low energy experiments, the Atomki experiment, has recently observed anomolous decays of the beryllium which leads to existence of a new light particle. The Atomki experiment has recently observed a $6.8\,\sigma$ anomaly (Krasznahorkay et al. (2016)) (see also (Krasznahorkay, A. J. et al. (2017); Krasznahorkay, A.J. et al. (2017); Krasznahorkay et al. (2018))) in excited ^8Be nuclear decays, $^8Be^* \rightarrow \ ^8Be\, e^+e^-$, in both the distributions of the opening angles and the invariant masses of the electron-positron pairs (IPC). The SM predicts the angular correlation between the emitted e^+e^- pairs to drop rapidly with the seperation angle. However, the experiment observed a bump with a high significance at a large angle of $\simeq 140^o$ which is consistent with creation and subsequent decay of a new particle with an invariant mass of $m_{e^+e^-} = 16.7 \pm 0.35(\text{stat}) \pm 0.5(\text{sys})$ MeV. In (Krasznahorkay et al. (2019)), they observed also a peak in e^-e^+ angular correlations at 115^o with $7.2\,\sigma$ in 21.01 MeV $0^- \rightarrow 0^+$ transition of ^4He and it is described with a light particle with a mass of $m_x c^2 = 16.84 \pm 0.16(\text{stat}) \pm 0.20(\text{sys})$. It is likely the same particle with the one that is observed in (Krasznahorkay et al. (2016)).

In recent interpretations of the experiment (Feng et al. (2016, 2017)), possible particle physics interpretations of the ^8Be anomalous decays are examined and concluded that a proto-phobic, spin-1 boson with a mass of ≈ 17 MeV fit the anomaly. They determine the bounds on the vector current couplings of the new gauge boson to the first generation of the SM fermions via combination of the relevant experimental data. They propose two particle physics models, $U(1)_B$ and $U(1)_{B-L}$ models, that are not initially anomaly-free therefore they add a new matter content to cancel the anomalies. Another recent interpretation (Gu and He (2017)) makes an extension of the SM with two gauge groups, $U(1)_{Y'} \times U(1)_X$, and they add a new matter content to get rid of the $Z - Z'$ mass mixing. In (Delle Rose et al. (2017)), they present a $U(1)'$ extended 2-Higgs doublet model for ^8Be anomalous decays. In Ellwanger and Moretti (2016) a pseudoscalar and in (Kozaczuk et al. (2017)) an axial vector candidates are presented. The extension of the minimal supersymmetric standard model (MSSM) by an extra $U(1)'$ is discussed (Demir et al. (2005)) with $U(1)'$ charges of the fields to be family-dependent and satisfy the anomaly-free conditions.

In this thesis, we extend the standard model (SM) with a family-nonuniversal $U(1)'$ with its associated light gauge boson Z' and a singlet scalar S charged under $U(1)'$ and singlet under the SM gauge symmetry. In the model, there are two mixings with the SM: the gauge kinetic mixing of the hypercharge gauge boson and the Z' boson, and the quartic scalar mixing of the SM-like Higgs and the extra scalar. The masses of the gauge bosons are generated dynamically through spontaneous symmetry breaking (SSB) via vacuum expectation values (vev) of the scalar fields.

Our intention in the first part of this thesis is to construct the framework of an anomaly-free, family-nonuniversal $U(1)'$ model that fits the Atomki signal with a minimal field content. The model we present is able to explain the Atomki signal with a protophobic gauge boson with a mass of ≈ 17 MeV. We find the couplings of the Z' boson to the first generation of the SM fermions via the family-nonuniversal charges of the chiral fields that satisfy the anomaly-free conditions. We show that with these couplings we are able to explain the Atomki signal. The vector and axial-vector current couplings of the Z' boson to the first generation of fermions do satisfy all the bounds from the various experimental data. The minimal model we construct forms a framework in which various anomalous SM decays can be discussed.

In Chapter 3, we have studied a vector particle as a dark matter candidate. This vector particle is originally different from all the known dark matter candidates, such that it is generated from metric-affine gravity and due to the geometrical feutures of the vector particle its interactions provide it to be a viable dark matter candidate that satisfy all the constraints and bounds on dark matter. We introduced the notion of "geometric dark matter". We have shown that it is a viable dark matter candidate when its mass range is at the MeV scale. The geometry of the model provides it to be the true minimal dark matter model as we will show in detail in Chapter 3.

Dark matter, needed for various phenomena ranging from flat rotation curves to structure formation, seems to be not only neutral and long-living but also highly secluded from the ordinary matter. Dark matter, which is roughly 5 times more than the baryonic matter (Aghanim et al. (2018)), has been under intense theoretical (Lin (2019); Feng (2010)) and experimental (Schumann (2019); Liu et al. (2017)) studies since its first inference (Rubin et al. (1976)). The particle dark matter which weighs around the weak scale and which has electroweak-size couplings to the known particles (WIMP) has always been the core of the dark matter paradigm. It has been modeled in supersymmetry (Baer et al. (2009); Peskin (2014)), extra dimensions (Arrenberg et al. (2013); Hooper and Profumo (2007)), and various other contexts. It has, however, revealed itself neither in direct searches (Schumann (2019); Liu et al. (2017)) nor in collider searches (Boveia and Doglioni (2018)). This negative result possibly means that dark matter falls outside the WIMP domain in that it interacts with known matter (proton, neutron and leptons, for instance) exceedingly weakly.

It is known that among the well-motivated candidates for vector dark matter are also hidden sector $U(1)$ gauge bosons. They have been studied in a variety of scenarios as hidden vector dark matter which interacts with the standard model fields through kinetic mixing with the photon (Chen et al. (2009); Redondo and Postma (2009); Arias et al. (2012); Ringwald (2012)) and through Higgs portal (Lebedev et al. (2012); Djouadi et al. (2012); Farzan and Akbarieh (2012); Baek et al. (2013); Choi et al. (2013); Baek et al. (2014); Ko et al. (2014); Chao (2015); Duch et al. (2015); DiFranzo et al. (2016); Belyaev et al. (2017)). In view of these interactions, those vector dark matter models face stringent constraints from their stability (their lifetimes must be longer than the age of the Universe and their annihilation to the standard model particles must be consistent with experimental data).

In an attempt to understand such a dark matter scheme, we explore geometrical fields beyond the general relativity (GR). To this end, we exercise the metric-affine gravity (MAG) (Vitagliano et al. (2011); Vitagliano (2014)) – an extension of GR in which the metric $g_{\mu\nu}$ and connection $\Gamma^{\lambda}_{\mu\nu}$ are independent geometrodynamical variables. One reason for this choice is that MAG is known to admit decomposition into scalars, vectors and tensors (Karahan et al. (2013)). Another reason is that attempts to understand electroweak stability via gravitational completion leads to MAG (Demir (2019)), showing that MAG could be the gravity sector necessitated by a UV-safe quantum field theory. Our analysis shows that MAG, in its simplest ghost-free form, decomposes into GR plus a massive vector field Y_μ, which couples only to fermions (quarks and leptons) such that lighter the Y_μ smaller the couplings. This geometric vector acquires a lifetime longer than that of the Universe if its mass range is $9.4 \text{ MeV} < M_Y < 28.4 \text{ MeV}$ and its scattering cross section from nucleons is some 60 orders of magnitude below the current bounds (Schumann (2019); Liu et al. (2017)). The Y_μ qualifies therefore a viable dark matter candidate, well satisfying the existing bounds.

In the recent paper (Jiménez and Maldonado Torralba (2020)), a pseudoscalar dark matter candidate is studied in MAG such that its derivative couplings to fermions arise through its couplings to the axial vector part of the torsion. The properties of the scalar depends on various model parameters due to the decomposition of the full connection. It is claimed that the coherent oscillations of the pseudoscalar can give rise to an ultra light dark matter of mass $\approx 10^{-22}$ eV. In the present work, we study MAG in the Palatini formalism (torsion is zero) in which decomposition of the full connection into the Levi-Civita connection plus a rank (1,2) tensor field leads to the massive vector Y_μ. Our torsion-free minimal framework leads to the geometric dark matter Y_μ which depends on a single parameter. The coupling of Y_μ to fermions follows from spin connection and is universal with the same coupling parameter. The vector dark matter (geometric dark matter) Y_μ in the present work is entirely different than the candidates (Chen et al. (2009); Redondo and Postma (2009); Arias et al. (2012); Ringwald (2012)) and (Lebedev et al. (2012); Djouadi et al. (2012); Farzan and Akbarieh (2012); Baek et al. (2013); Choi et al. (2013); Baek et al. (2014); Ko et al. (2014); Chao (2015); Duch et al. (2015); DiFranzo et al. (2016); Belyaev et al. (2017)) as well as the the approach in (Jiménez and Maldonado Torralba (2020)). In particular, the geometric dark matter is not a $U(1)$ gauge boson; it stems from geometry of the spacetime. It does not couple to scalars and gauge bosons. It couples only to fermions. These features stem from its geometrical origin, and make it fundamentally different than the other known vector dark matter candidates. We show that due to the geometric nature of our dark matter, there is no interaction with the photon (or any other gauge boson). Therefore, we do not need to impose any selection rule (like the well-known Z_2 symmetry) to prevent the decay of the Y_μ into photons. Moreover, we show that the Y_μ is a geometric vector which is generated by the affine connection as a massive vector. We do not therefore need to deal with interactions due to Higgs or Stueckelberg mechanisms. It is easy to see that this keeps the present model minimal as there is no need for additional scalars which would lead to some constraints due to annihilation of vector dark matter into standard model particles through the Higgs portal or invisible decays of the standard model Higgs.

CHAPTER 2

A FAMILY-NONUNIVERSAL $U(1)'$ MODEL FOR EXCITED BERYLLIUM DECAYS

In this chapter, we construct a family-nonuniversal $U(1)'$ model,with a minimal field content, which is able to explain the Atomki signal. The model is populated with a 17 MeV $Z^{'}$ and a scalar S which is charged under $U(1)'$ and singlet under the SM gauge symmetry. The minimality of the model is provided by the family-nonuniversality of the SM charges under $U(1)'$. This feature makes it different from all the other models for the beryllium decay as we will show in a detailed analysis. We present a model that explains the Atomki signal with a proto-phobic 17 MeV Z' with couplings to the first generation of the SM fermions which satisfy all the experimental data.

Chapter 2 is organized as follows. We give the basic structure of the scalar and gauge sectors of the SM in Section 2.1. In Section 2.2 we construct the framework of the family-nonuniversal $U(1)'$ model. We summarize the experimental bounds in Section 2.3. We give the vector and axial-vector current couplings of the $Z^{'}$ boson to the first generation of the SM fermions in Section 2.4. We show that the CKM matrix is properly obtained in the model in Section 2.5. In Section 2.6, we consider the LHC bound on the decays of the SM Higgs. We summarize the model and discuss future prospects in Section 2.7.

2.1. The Scalar and Gauge Sectors of the Standard Model

In the family-nonuniversal $U(1)'$ model, there is an extra scalar S and a Z' gauge boson hence in this section, we give briefly the scalar and gauge sectors of the SM. As it will be analyzed in detail in the family-nonuniversal $U(1)'$ model, here we give basic structures of the relevant SM gauge and scalar sectors.

The SM is based on a $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry. The

SM scalar Lagrangian reads as

$$\mathcal{L}_{Higgs}^{SM} = |\mathcal{D}_\mu H|^2 + \mu^2|H|^2 - \lambda|H|^4 \tag{2.1}$$

where the Higgs field is parametrized as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + h + i\phi_3 \end{pmatrix}. \tag{2.2}$$

The covariant derivative in (2.1) is given by

$$\mathcal{D}_\mu = \partial_\mu + +ig_s t^a G_\mu^a + igT^i W_\mu^i + ig' Q_Y B_\mu \tag{2.3}$$

where $G_\mu (a = 1, ..., 8)$ are the eight gluon fields, g_s is the QCD coupling constant and $t^a = \frac{\lambda^a}{2}$ where λ^a are the Gell-Mann matrices; $T^i = \frac{1}{2}\sigma^i$ is the third component of isospin where σ^i $(i = 1, 2, 3)$ are the Pauli spin matrices; W_μ^i $(i = 1, 2, 3)$ and B_μ are the $SU(2)_L$ and the $U(1)_Y$ gauge fields with the corresponding couplings constants g and g', respectively. The weak hypercharge is denoted by Q_Y.

For $\mu^2, \lambda > 0$, the Higgs potential is minimized at a nonvanishing vev

$$v = \frac{\mu}{\sqrt{\lambda}} \tag{2.4}$$

and the Higgs mass reads as

$$m_h^2 = 2\lambda v^2. \tag{2.5}$$

The SM gauge kinetic Lagrangian is given by

$$\mathcal{L}_{gauge} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}. \tag{2.6}$$

where the field strength tensors are given by

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c, \\ W_{\mu\nu} &= \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon^{ijk} W_\mu^j W_\nu^k, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{2.7}$$

where f^{abc} and ϵ^{ijk} are the structure constants.

The mass eigenstates of the gauge bosons are obtained by the following redefinitions of the fields

$$\begin{aligned} W_\mu^\pm &= \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}, \\ \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} &= \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \end{aligned} \tag{2.8}$$

where θ_W is the Weinberg angle. The masses of the gauge bosons read as

$$\begin{aligned} M_W^2 &= \frac{1}{4} g^2 v^2, \\ M_Z^2 &= \frac{1}{4} (g^2 + g'^2) v^2, \\ M_A &= 0. \end{aligned} \tag{2.9}$$

We have given briefly the basic structure of the SM scalar and gauge sectors which are relevant to the family-nonuniversal $U(1)'$ model which in Section 2.2 we will study its every sector in detail. The model involves a new Z' gauge boson and an extra scalar field S and there are mixings with the SM fields.

2.2. The Family-nonuniversal $U(1)'$ Model

In this section, we present the framework of the family-nonuniversal $U(1)'$ model. We extend the SM gauge symmetry by an extra $U(1)'$ symmetry

$$G_{SM} \times U(1)'. \tag{2.10}$$

The $U(1)'$ quantum number assignment to chiral fermion and scalar fields is given in Tab. 2.1.

Table 2.1. The gauge quantum numbers of the fields in the family-nonuniversal $U(1)'$ model for $i = 1, 2, 3$ which refers to the three generations of matter.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q_i	3	2	$1/6$	Q_{Q_i}
u_{R_i}	3	1	$2/3$	$Q_{u_{Ri}}$
d_{R_i}	3	1	$-1/3$	$Q_{d_{Ri}}$
L_i	1	2	$-1/2$	Q_{L_i}
e_{R_i}	1	1	-1	$Q_{e_{R_i}}$
$\hat{H}$	1	2	$1/2$	Q_H
$\hat{S}$	1	1	0	Q_S

2.2.1. Mixing of Higgs Bosons

The Lagrangian of the scalars in the family-nonuniversal $U(1)'$ model is given by

$$\mathcal{L}_{Higgs} = \mathcal{L}^{SM}_{Higgs} + \mathcal{L}^{S}_{Higgs} + \mathcal{L}^{mix}_{Higgs}; \tag{2.11}$$

$$\mathcal{L}^{SM}_{Higgs} = |\mathcal{D}_\mu \hat{H}|^2 + \mu^2 |\hat{H}|^2 - \lambda |\hat{H}|^4, \tag{2.12}$$

$$\mathcal{L}^{S}_{Higgs} = |\mathcal{D}_\mu \hat{S}|^2 + \mu_s^2 |\hat{S}|^2 - \lambda_s |\hat{S}|^4, \tag{2.13}$$

$$\mathcal{L}^{mix}_{Higgs} = -\kappa |\hat{H}|^2 |\hat{S}|^2 \tag{2.14}$$

where the last equation contains a mixing term with a scalar mixing parameter κ. The hatted fields are used since we will use the fields without hat in the mass-basis.

We parametrize the SM-like Higgs $\hat{H}$ and the extra scalar $\hat{S}$, respectively as

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \hat{h} + i\phi_3 \end{pmatrix}, \qquad \hat{S} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_s + \hat{s} + i\phi_s \end{pmatrix} \tag{2.15}$$

where ϕ_1, ϕ_2, ϕ_3 and ϕ_s are the Goldstone bosons; v and v_s are vevs of the scalar fields that are real and positive.

The scalar potential is bounded from below provided that

$$\lambda > 0, \quad \lambda_s > 0 \quad \texttt{and} \quad 4\lambda\lambda_s - \kappa^2 > 0. \tag{2.16}$$

For both nonvanishing values of vevs, the minimum of the potential occurs at

$$\frac{v^2}{2} = \frac{2\lambda_s \mu^2 - \kappa \mu_s^2}{4\lambda\lambda_s - \kappa^2}, \tag{2.17}$$

$$\frac{v_s^2}{2} = \frac{2\lambda \mu_s^2 - \kappa \mu^2}{4\lambda\lambda_s - \kappa^2}. \tag{2.18}$$

These solutions are physical for $v^2 > 0$ and $v_s^2 > 0$ which leads to $\lambda_S \mu^2 > \kappa \mu_S^2/2$ and $\lambda \mu_S^2 > \kappa \mu^2/2$ if (2.16) is satisfied. One can realize that for both nonvanishing vevs there are solutions for

- $\mu^2, \mu_s^2 > 0$ for both signs of κ,
- $(\mu^2 > 0, \mu_s^2 < 0)$ or $(\mu^2 < 0, \mu_s^2 > 0)$ for only $\kappa < 0$.
- There are not any solutions for $\mu^2, \mu_s^2 < 0$.

The scalar mass Lagrangian is given by

$$\mathcal{L}_{scalar}^{mass} = -V_{scalar} = -\frac{1}{2}\begin{pmatrix}\hat{h} & \hat{s}\end{pmatrix}\begin{pmatrix} 2\lambda v^2 & \kappa v v_s \\ \kappa v v_s & 2\lambda_s v_s^2 \end{pmatrix}\begin{pmatrix}\hat{h} \\ \hat{s}\end{pmatrix}. \tag{2.19}$$

We go to the mass basis, (h, s), via transformation

$$\begin{pmatrix}\hat{h} \\ \hat{s}\end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} h \\ s \end{pmatrix} \tag{2.20}$$

where the mixing angle is given by

$$\tan 2\alpha = -\frac{\kappa v v_s}{\lambda v^2 - \lambda_s v_s^2}. \tag{2.21}$$

The masses of the SM-like Higgs h and the extra scalar s are given by

$$m_{h,s}^2 = \lambda v^2 + \lambda_s v_s^2 \pm \sqrt{(\lambda v^2 - \lambda_s v_s^2)^2 + \kappa^2 v^2 v_s^2} \tag{2.22}$$

where $\lambda v^2 > \lambda_s v_s^2$. In the limit of no scalar mixing, $\kappa \to 0$, the masses of the scalars in (2.22) reduce to

$$m_{h^0}^2 = 2\lambda v^2, \quad m_{s^0}^2 = 2\lambda_s v_s^2. \tag{2.23}$$

2.2.2. Mixing of Gauge Bosons

The $U(1)'$ symmetry couples to the SM hypercharge symmetry $U(1)_Y$ through the kinetic mixing which leads to the most general gauge Lagrangian of $U(1)_Y \times U(1)'$

$$\mathcal{L}_{gauge} = \mathcal{L}_{gauge}^{SM} + \mathcal{L}_{gauge}^{Z'} + \mathcal{L}_{gauge}^{mix}; \tag{2.24}$$

$$\mathcal{L}_{gauge}^{SM} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}, \tag{2.25}$$

$$\mathcal{L}_{gauge}^{Z'} = -\frac{1}{4}\hat{Z'}_{\mu\nu}\hat{Z}'^{\mu\nu}, \tag{2.26}$$

$$\mathcal{L}_{gauge}^{mix} = -\frac{1}{2}\sin\chi\, \hat{B}_{\mu\nu}\hat{Z}'^{\mu\nu} \tag{2.27}$$

where $\hat{B}_{\mu\nu}$ and $\hat{Z}'_{\mu\nu}$ are the field strength tensors of $U(1)_Y$ and $U(1)'$, respectively. The last equation contains a mixing term with a gauge kinetic mixing parameter χ.

We diagonalize the field strength terms via a $GL(2,R)$ transformation

$$\begin{pmatrix} \tilde{Z}'_\mu \\ \tilde{B}_\mu \end{pmatrix} = \begin{pmatrix} \sqrt{1-\sin^2\chi} & 0 \\ \sin\chi & 1 \end{pmatrix} \begin{pmatrix} \hat{Z}'_\mu \\ \hat{B}_\mu \end{pmatrix} \tag{2.28}$$

where $\tilde{Z}'_\mu$ and $\tilde{B}_\mu$ are not the mass eigenstates yet.

In this basis, the general covariant derivative is given by

$$\mathcal{D}_\mu = \partial_\mu + igT^i W^i_\mu + ig' Q_Y \tilde{B}_\mu + i(e\tilde{g}Q' + \eta g' Q_Y)\tilde{Z}'_\mu \tag{2.29}$$

where $T^i = \frac{1}{2}\sigma^i$ is the third component of isospin in which σ^i are the Pauli spin matrices with $i = 1,2,3$; W_μ is the $SU(2)_L$ gauge field; g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively.

In (2.29), we have introduced

$$\tilde{g} \equiv \frac{\hat{g}}{\cos\chi}, \quad \eta \equiv -\tan\chi \tag{2.30}$$

where $\hat{g}$ is the normalized $U(1)'$ gauge coupling

$$\hat{g} \equiv \frac{g_{U(1)'}}{e}. \tag{2.31}$$

The mass squared matrix of the gauge bosons in the $(\tilde{B}_\mu, \tilde{Z}'_\mu)$ gauge-basis is given by

$$\mathcal{L}^{mass}_{gauge} = \frac{1}{2}\begin{pmatrix} \tilde{B}^\mu & W^{3\mu} & \tilde{Z}'^\mu \end{pmatrix}$$
$$\cdot \begin{pmatrix} \frac{1}{4}v^2 g'^2 & -\frac{1}{4}v^2 gg' & \frac{1}{2}g'v^2(\frac{g'\eta}{2} + e\tilde{g}Q_H) \\ -\frac{1}{4}v^2 gg' & \frac{1}{4}v^2 g^2 & -\frac{1}{2}gv^2(\frac{g'\eta}{2} + e\tilde{g}Q_H) \\ \frac{1}{2}g'v^2(\frac{g'\eta}{2} + e\tilde{g}Q_H) & -\frac{1}{2}gv^2(\frac{g'\eta}{2} + e\tilde{g}Q_H) & v^2(\frac{g'\eta}{2} + e\tilde{g}Q_H)^2 + Q_S^2 v_s^2 e^2 \tilde{g}^2 \end{pmatrix}$$
$$\cdot \begin{pmatrix} \tilde{B}_\mu \\ W^3_\mu \\ \tilde{Z}'_\mu \end{pmatrix}. \tag{2.32}$$

The mass eigenstates of the neutral gauge bosons are obtained via the transformation

$$\begin{pmatrix} \tilde{B}_\mu \\ W^3_\mu \\ \tilde{Z}'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W\cos\varphi & \sin\theta_W\sin\varphi \\ \sin\theta_W & \cos\theta_W\cos\varphi & -\cos\theta_W\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} \tag{2.33}$$

where θ_W is the Weinberg angle and and φ is the gauge mixing angle which is given by

$$\tan 2\varphi = \frac{2(g'\eta + 2e\tilde{g}Q_H)\sqrt{g^2+g'^2}}{(g'\eta + 2e\tilde{g}Q_H)^2 + 4(\frac{v_s}{v})^2 Q_s^2 e^2 \tilde{g}^2 - g^2 - g'^2}. \tag{2.34}$$

The masses of the physical gauge bosons read as

$$\begin{aligned} M_A &= 0, \\ M^2_{Z,Z'} &= \frac{1}{2}\left\{ M^2_{Z^0} + M^2_{Z'^0} + \Delta^2 \pm \sqrt{(M^2_{Z^0} - M^2_{Z'^0} - \Delta^2)^2 + 4M^2_{Z'^0}\Delta^2} \right\} \end{aligned} \tag{2.35}$$

where

$$\begin{aligned} M^2_{Z^0} &= \frac{1}{4}(g^2 + g'^2)v^2, \\ M^2_{Z'^0} &= e^2\tilde{g}^2 Q_S^2 v_s^2, \\ \Delta &= v(\frac{g'\eta}{2} + e\tilde{g}Q_H). \end{aligned} \tag{2.36}$$

It is clear that if

$$\left(\frac{g'\eta}{2} + e\tilde{g}Q_H\right) = 0, \tag{2.37}$$

the gauge mixing angle in (2.34) vanishes identically. This ensures zero mixing between the Z and the $Z^{'}$ bosons so that the $Z^{'}$ mass is set by the vev v_s of the extra scalar

$$M^2_{Z'} = e^2\tilde{g}^2 Q_S^2 v_s^2. \tag{2.38}$$

The condition in (2.37) can be relaxed. We know that the mixing of the Z and the $Z^{'}$ can be at most at the level of the $Z^{'}$ mass

$$\frac{1}{2}g'v^2\left(\frac{g'\eta}{2} + e\tilde{g}Q_H\right) \lesssim M^2_{Z'} \tag{2.39}$$

which gives

$$\left(\frac{g'\eta}{2} + e\tilde{g}Q_H\right) \lesssim 10^{-8} \tag{2.40}$$

for a Z' mass of $M_{Z'} = 17$ MeV which implies $\tan 2\varphi \lesssim 10^{-8}$. The current limit on the $Z - Z'$ mixing angle from the LEP data is about $|\varphi| = 10^{-3} - 10^{-4}$ (Erler et al. (2009)). It is thus clear that the $Z - Z'$ mixing angle in our family-nonuniversal $U(1)'$ model is well below the limit from the electroweak precision data.

2.2.3. Leptons and Quarks

The kinetic Lagrangian of the fermions is given by

$$\mathcal{L}^{kinetic}_{fermion} = i\bar{Q}_i\gamma^\mu\mathcal{D}_\mu Q_i + i\bar{u}_{Ri}\gamma^\mu\mathcal{D}_\mu u_{Ri} + i\bar{d}_{Ri}\gamma^\mu\mathcal{D}_\mu d_{Ri} + i\bar{L}_i\gamma^\mu\mathcal{D}_\mu L_i + i\bar{e}_{Ri}\gamma^\mu\mathcal{D}_\mu e_{Ri} \tag{2.41}$$

where $i = 1, 2, 3$ is the family index, Q_i is for the left-handed quark doublets and (u_{Ri}, d_{Ri}) are for the right-handed quark singlets

$$Q = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, u_{Ri}, d_{Ri}, \tag{2.42}$$

and L is for the left-handed lepton doublet and e_{Ri} is for the right-handed lepton singlet

$$L = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}, e_{Ri}. \tag{2.43}$$

The Yukawa Lagrangian is

$$\mathcal{L}^{Yukawa}_{fermion} = -Y_u\bar{Q}\tilde{\hat{H}}u_R - Y_d\bar{Q}\hat{H}d_R - Y_e\bar{L}\hat{H}e_R + h.c. \tag{2.44}$$

where (Y_u, Y_d, Y_e) are the Yukawa matrices and $\tilde{\hat{H}} = i\sigma_2\hat{H}^*$. The gauge invariance conditions from the diagonal elements of the Yukawa interactions in (2.44) are given by

$$\begin{aligned} Q_{u_{R_i}} &= Q_{Q_i} + Q_H, \\ Q_{d_{R_i}} &= Q_{Q_i} - Q_H, \\ Q_{e_{R_i}} &= Q_{L_i} - Q_H. \end{aligned} \tag{2.45}$$

It is clear that the conditions in (2.45) involve only the diagonal elements of the Yukawa interactions. Actually, they are general enough to cover also conditions coming from off-diagonal Yukawa entries. One will realize in Sec.(2.4) that the $U(1)'$ charges give rise to a specific mass matrix structure. The first two families of the up and down-type quarks have the same $U(1)'$ charges while the third family has a different charge, which implies that $(M_u)_{13}, (M_u)_{31}, (M_u)_{23}, (M_u)_{32}$ and $(M_d)_{13}$, $(M_d)_{31}, (M_d)_{23}, (M_d)_{32}$ all vanish. These zeroes leave no Yukawa interactions between the first two families and the third family of the up and down-type quarks. There can arise thus no non-trivial gauge invariance conditions in these sectors. The general Yukawa interactions between the first two families are trivial in that their $U(1)'$ charges are universal. Moreover, leptons have family-universal $U(1)'$ charges. It therefore is clear that (2.45) covers all cases.

2.3. Constraints from Experiments

It is argued that the new boson is likely a vector boson (Feng et al. (2016, 2017)) that couples to the SM fermion currents as

$$\mathcal{L} \supset iZ'_\mu J^\mu = iZ'_\mu \sum_{i=u,d,e,\nu_e,\ldots} \varepsilon_i^v e J_i^\mu \ , \ \ J_i^\mu = \bar{f}_i \gamma^\mu f_i \tag{2.46}$$

where ε^v is the vector current couplings of the Z' with superscript 'v' referring to 'vector'. It is showed that the vector current couplings of the Z' to the SM fermions are constrained by several experimental data (Feng et al. (2016, 2017)). The Atomki signal (Krasznahorkay et al. (2016)), the neutral pion decay, $\Pi^0 \to X\gamma$, by NA$48/2$ experiment (Batley et al. (2015); Raggi (2016)), the SLAC E141 experiment (Riordan et al. (1987); Bjorken et al. (2009); Essig et al. (2013)), constraint via the electron anomalous magnetic dipole moment $(g-2)_e$ (Davoudiasl et al. (2014)) and the $\bar{\nu}_e - e$ scattering by TEXONO (Deniz et al. (2010)) put constraints on the vector current couplings of the Z' to the first generation of the SM fermions

$$\begin{aligned}
|\varepsilon_p^v| &\lesssim 1.2 \times 10^{-3}, \\
|\varepsilon_n^v| &= (2-10) \times 10^{-3}, \\
|\varepsilon_e^v| &= (0.2-1.4) \times 10^{-3}, \\
\sqrt{\varepsilon_e^v \varepsilon_{\nu_e}^v} &\lesssim 7 \times 10^{-5}.
\end{aligned} \tag{2.47}$$

The constraints on the couplings of the Z' from the neutral pion decay (Feng et al. (2016, 2017)) require it to be proto-phobic, *i.e.* it has a suppressed coupling to the proton compared with the neutron

$$-0.067 < \frac{\varepsilon_p^v}{\varepsilon_n^v} < 0.078 \tag{2.48}$$

where the nucleon couplings are explicitly given by

$$\varepsilon_p^v = 2\varepsilon_u^v + \varepsilon_d^v,$$
$$\varepsilon_n^v = \varepsilon_u^v + 2\varepsilon_d^v. \tag{2.49}$$

2.4. $Z^{'}$ Couplings

In this section, we find the vector and axial-vector current couplings of the Z' that are able to explain the Atomki anomaly. First, we show the vector and axial-vector current couplings of the Z' to the first generation of the fermions in terms of the model parameters including the $U(1)'$ charges of the related chiral fermions in Tab. 2.2.

Table 2.2. The Z' couplings to the first generation of fermions in terms of the model parameters including the $U(1)'$ charges of the related chiral fermions.

$\varepsilon_u^v = \frac{1}{2}\epsilon + \frac{2}{3}\delta + \cos\varphi\tilde{g}\left(\frac{Q_{Q_1}+Q_{u_{R_1}}}{2}\right)$	$\varepsilon_u^a = \frac{1}{2}\epsilon + \cos\varphi\tilde{g}\left(\frac{Q_{Q_1}-Q_{u_{R_1}}}{2}\right)$
$\varepsilon_d^v = -\frac{1}{2}\epsilon - \frac{1}{3}\delta + \cos\varphi\tilde{g}\left(\frac{Q_{Q_1}+Q_{d_{R_1}}}{2}\right)$	$\varepsilon_d^a = -\frac{1}{2}\epsilon + \cos\varphi\tilde{g}\left(\frac{Q_{Q_1}-Q_{d_{R_1}}}{2}\right)$
$\varepsilon_e^v = -\frac{1}{2}\epsilon - \delta + \cos\varphi\tilde{g}\left(\frac{Q_{L_1}+Q_{e_{R_1}}}{2}\right)$	$\varepsilon_e^a = -\frac{1}{2}\epsilon + \cos\varphi\tilde{g}\left(\frac{Q_{L_1}-Q_{e_{R_1}}}{2}\right)$
$\varepsilon_{\nu_e}^v = \frac{1}{2}\epsilon + \cos\varphi\tilde{g}\frac{Q_{L_1}}{2}$	$\varepsilon_{\nu_e}^a = \frac{1}{2}\epsilon + \cos\varphi\tilde{g}\left(\frac{Q_{L_1}}{2}\right)$

In Tab. 2.2, we have introduced

$$\epsilon \equiv -\frac{1}{2}\left((\cot\theta_W + \tan\theta_W)\sin\varphi + \frac{\cos\varphi}{\cos\theta_W}\eta\right), \tag{2.50}$$

$$\delta \equiv \tan\theta_W \sin\varphi + \frac{\cos\varphi}{\cos\theta_W}\eta. \tag{2.51}$$

The SM chiral fermion and scalar fields are charged under $U(1)'$. We determine the couplings by providing the charges to satisfy the anomaly-free and the gauge invariance conditions. In order to avoid gauge and gravitational anomalies, the $U(1)'$ charges of the chiral fields must satisfy the following anomaly-free conditions

$$
\begin{aligned}
U(1)' - SU(3) - SU(3): \quad 0 &= \sum_i (2Q_{Q_i} - Q_{u_{R_i}} - Q_{d_{R_i}}), \\
U(1)' - SU(2) - SU(2): \quad 0 &= \sum_i (3Q_{Q_i} + Q_{L_i}), \\
U(1)' - U(1)_Y - U(1)_Y: \quad 0 &= \sum_i (\frac{1}{6}Q_{Q_i} - \frac{1}{3}Q_{d_{R_i}} - \frac{4}{3}Q_{u_{R_i}} + \frac{1}{2}Q_{L_i} - Q_{e_{R_i}}), \\
U(1)' - \text{graviton - graviton}: \quad 0 &= \sum_i (6Q_{Q_i} - 3Q_{u_{R_i}} - 3Q_{d_{R_i}} + 2Q_{L_i} - Q_{e_{R_i}}), \\
U(1)' - U(1)' - U(1)_Y: \quad 0 &= \sum_i (Q^2_{Q_i} + Q^2_{d_{R_i}} - 2Q^2_{u_{R_i}} - Q^2_{L_i} + Q^2_{e_{R_i}}), \\
U(1)' - U(1)' - U(1)': \quad 0 &= \sum_i (6Q^3_{Q_i} - 3Q^3_{d_{R_i}} - 3Q^3_{u_{R_i}} + 2Q^3_{L_i} - Q^3_{e_{R_i}}) \qquad (2.52)
\end{aligned}
$$

There are 16 charges and 6 anomaly-free conditions with additional conditions from Yukawa interactions such that as we show in Tab. 2.3 one could express 12 charges in terms of 4 free charges

$$Q_H,\ Q_{Q_2},\ Q_{Q_3} \text{ and } Q_{L_3}.$$

Table 2.3. The $U(1)'$ charge solutions of the chiral SM fermions by the gauge invariance and the anomaly-free conditions.

$Q_{Q_1} = Q_H - Q_{Q_2} - Q_{Q_3}$	$Q_{u_{R_1}} = 2Q_H - Q_{Q_2} - Q_{Q_3}$	$Q_{d_{R_1}} = -Q_{Q_2} - Q_{Q_3}$
	$Q_{u_{R_2}} = Q_{Q_2} + Q_H$	$Q_{d_{R_2}} = Q_{Q_2} - Q_H$
	$Q_{u_{R_3}} = Q_{Q_3} + Q_H$	$Q_{d_{R_3}} = Q_{Q_3} - Q_H$
$Q_{L_1} = -Q_H$	$Q_{e_{R_1}} = -2Q_H$	
$Q_{L_2} = -2Q_H - Q_{L_3}$	$Q_{e_{R_2}} = -3Q_H - Q_{L_3}$	
	$Q_{e_{R_3}} = Q_{L_3} - Q_H$	

We parametrize the vector current coupling of the Z' boson to the proton as

$$\varepsilon^v_p = 2\varepsilon^v_u + \varepsilon^v_d = \delta' \qquad (2.53)$$

where we introduce parameter δ' which obeys the bound

$$|\delta'| \lesssim 10^{-3}. \qquad (2.54)$$

Then, by (2.53), we get

$$\delta = \delta' - \frac{1}{2}\epsilon + \cos\varphi \tilde{g} \left(3Q_{Q_2} + 3Q_{Q_3} - \frac{7}{2}Q_H\right) \qquad (2.55)$$

which together with the charge solutions in Tab. 2.3 lead to the couplings in Tab. 2.4 with a vanishing gauge mixing angle, $\cos\varphi \to 1$. We apply the zero $Z - Z'$ mixing limit from now on.

Table 2.4. The Z' couplings after using the charge solutions in Tab. 2.3 and parametrization of the vector current coupling of the Z' boson to the proton as $\varepsilon_p^v = 2\varepsilon_u^v + \varepsilon_d^v \equiv \delta'$ with $|\delta'| \lesssim 10^{-3}$. Consideration of other constraints reduce the couplings in this table to the couplings in Tab. 2.5.

$\varepsilon_u^v = \frac{1}{6}\epsilon + \frac{2}{3}\delta' + \tilde{g}(Q_{Q_2} + Q_{Q_3} - \frac{5}{6}Q_H)$	$\varepsilon_u^a = \frac{1}{2}\epsilon - \frac{1}{2}\tilde{g}Q_H$
$\varepsilon_d^v = -\frac{1}{3}\epsilon - \frac{1}{3}\delta' - \tilde{g}(2Q_{Q_2} + 2Q_{Q_3} - \frac{5}{3}Q_H)$	$\varepsilon_d^a = -\frac{1}{2}\epsilon + \frac{1}{2}\tilde{g}Q_H$
$\varepsilon_e^v = -\delta' - \tilde{g}(3Q_{Q_2} + 3Q_{Q_3} - 2Q_H)$	$\varepsilon_e^a = -\frac{1}{2}\epsilon + \frac{1}{2}\tilde{g}Q_H$
$\varepsilon_{\nu_e}^v = \frac{1}{2}\epsilon - \frac{1}{2}\tilde{g}Q_H$	$\varepsilon_{\nu_e}^a = \frac{1}{2}\epsilon - \frac{1}{2}\tilde{g}Q_H$

The Lagrangian of the axial-vector current interaction of the Z' boson is given by

$$\mathcal{L} \supset iZ'_\mu \sum_{i=u,d,e,\nu_e} \varepsilon_i^a e \bar{f}_i \gamma^\mu \gamma^5 f_i \tag{2.56}$$

where ε^a is the axial-vector current coupling with superscript 'a' referring to 'axial-vector'.

We obtain the solutions of the free charges Q_{Q_2}, Q_{Q_3}, Q_H and Q_{L_3} as follows.

- In the limit of minimal flavor violation there holds the relation $\varepsilon_s^v = \varepsilon_d^v$ by which we obtain the solution

$$Q_{Q_3} = Q_H - 2Q_{Q_2}. \tag{2.57}$$

- Next, we parametrize the vector current coupling of the Z' boson to the neutron

$$\varepsilon_n^v = \varepsilon_u^v + 2\varepsilon_d^v \equiv \epsilon' \tag{2.58}$$

where parameter ϵ' satisfies

$$|\epsilon'| \approx (2 - 10) \times 10^{-3}. \tag{2.59}$$

Then, by (2.57) and (2.58), we obtain the solutions of Q_{Q_2} and Q_{Q_3}

$$Q_{Q_2} = \frac{1}{3\tilde{g}}(\epsilon + \epsilon'),$$
$$Q_{Q_3} = \frac{1}{3\tilde{g}}(\epsilon - 2\epsilon'). \tag{2.60}$$

- The axial-vector coupling to the electron vanishes, $\varepsilon_e^a = 0$, identically via the zero $Z - Z'$ mixing condition in (2.37) as well as the the axial-vector current couplings to the up and down quarks $\varepsilon_u^a = \varepsilon_d^a = 0$; the vector and axial-vector current couplings to the electron neutrino, $\varepsilon_{\nu_e}^v = \varepsilon_{\nu_e}^a = 0$ with the following $U(1)'$ charge of the SM-like Higgs boson

$$Q_H = \frac{\epsilon}{\tilde{g}}. \tag{2.61}$$

  Using the solution of Q_H in 2.61, we get $\eta \lesssim 10^{-4}$, which well agrees with the bounds.

- The axial-vector current coupling of the Z' boson to the electron is constrained via the neutral pion decay process, $\Pi^0 \to e^+e^-$ (Abouzaid et al. (2007)). The matrix element of this process is proportional to $\varepsilon_e^a(\varepsilon_u^a - \varepsilon_d^a)$ (Kahn et al. (2008)). However, in our model the axial-vector current coupling of the Z' to the electron vanishes, $\varepsilon_e^a = 0$, as well as the axial-vector current couplings to the up and down quarks $\varepsilon_u^a = \varepsilon_d^a = 0$. Therefore this rare process imposes no constraints on the axial-vector current coupling of the Z'. The axial-vector current coupling of the Z' to the electron is constrained also by the atomic parity violation (Porsev et al. (2009)) and the parity-violating Moller scattering (Anthony et al. (2005)) which constrain the products $\varepsilon_e^a \varepsilon_q^v$ and $\varepsilon_e^a \varepsilon_e^v$, respectively. It is obvious that due to vanishing ε_e^a, there arise no constraints from these processes.

  As a result of these, the vector and axial-vector current couplings of the Z' to the first generation of the SM fermions take the forms in Tab. 2.5.

Table 2.5. The Z' couplings to the first generation of the SM fermions that fit the Atomki signal with $\varepsilon_p^v = 2\varepsilon_u^v + \varepsilon_d^v \equiv \delta'$, $|\delta'| \lesssim 10^{-3}$ and $\varepsilon_n^v = \varepsilon_u^v + 2\varepsilon_d^v \equiv \epsilon'$, $|\epsilon'| \approx (2 - 10) \times 10^{-3}$. The couplings of the $Z^{'}$ are proto-phobic, (2.48), and satisfy the experimental constraints in (2.47).

$\varepsilon_u^v = \frac{2}{3}\delta' - \frac{1}{3}\epsilon'$	$\varepsilon_u^a = 0$
$\varepsilon_d^v = -\frac{1}{3}\delta' + \frac{2}{3}\epsilon'$	$\varepsilon_d^a = 0$
$\varepsilon_e^v = \epsilon' - \delta'$	$\varepsilon_e^a = 0$
$\varepsilon_{\nu_e}^v = 0$	$\varepsilon_{\nu_e}^a = 0$

- In Tab. 2.5, we present the Z' couplings to the first generation of the SM fermions that fit the Atomki signal. The couplings of the $Z^{'}$ are proto-phobic, (2.48), and satisfy the experimental constraints in (2.47) with $\varepsilon_p^v = 2\varepsilon_u^v + \varepsilon_d^v \equiv \delta'$, $|\delta'| \lesssim 10^{-3}$ and $\varepsilon_n^v = \varepsilon_u^v + 2\varepsilon_d^v \equiv \epsilon'$, $|\epsilon'| \approx (2-10) \times 10^{-3}$.

- As one can realize, our model is proto-phobic in both vector and axial-vector current interactions. The axial-vector current couplings to up and down quarks vanish identically via the zero $Z-Z'$ mixing condition in (2.37) so the Z' has purely vector current interactions with up and down quarks.

- The vector current coupling to the electron does not vanish as it should not for the IPC and it is able take value satisfying the experimental constraints. The axial-vector current coupling to the electron vanishes identically via the zero $Z-Z'$ mixing condition in (2.37).

- The experimental constraints require the vector current coupling to the electron neutrino to be significantly below the vector current coupling to the neutron. The vector and axial-vector current couplings to the electron neutrino vanish identically with zero $Z-Z'$ mixing condition in (2.37) and this obviously satisfies the experimental data.

- In order to have universal charges in the lepton sector, we assume

$$Q_{L_3} = -Q_H. \tag{2.62}$$

As a result of these, the first two families of the quarks have the same $U(1)'$ charges which are different from the third family charge and the leptons have universal $U(1)'$ charges, as we show in Tab. 2.6.

Table 2.6. The $U(1)'$ charges of the chiral SM fermions. One obtains the Z' couplings in Tab. 2.5 if these charge solutions are put into the couplings in Tab. 2.4.

$Q_{Q_1} = Q_{Q_2} = \frac{1}{3\tilde{g}}(\epsilon + \epsilon')$	$Q_{u_{R_1}} = Q_{u_{R_2}} = \frac{1}{3\tilde{g}}(4\epsilon + \epsilon')$	$Q_{d_{R_1}} = Q_{d_{R_2}} = \frac{1}{3\tilde{g}}(-2\epsilon + \epsilon')$
$Q_{Q_3} = \frac{1}{3\tilde{g}}(\epsilon - 2\epsilon')$	$Q_{u_{R_3}} = \frac{2}{3\tilde{g}}(2\epsilon - \epsilon')$	$Q_{d_{R_3}} = -\frac{2}{3\tilde{g}}(\epsilon + \epsilon')$
$Q_{L_1} = Q_{L_2} = Q_{L_3} = -\frac{\epsilon}{\tilde{g}}$	$Q_{e_{R_1}} = Q_{e_{R_2}} = Q_{e_{R_3}} = -\frac{2\epsilon}{\tilde{g}}$	

2.5. CKM Matrix

There are several texture-specific quark mass matrices in the literature (Rasin (1998); Branco et al. (2000); Fritzsch and Xing (2000); Xing and Zhang (2004); Branco et al. (2009); Gupta and Ahuja (2011, 2012)). The goal has always been avoiding the large number of parameters in these mass matrices. Some elements of these matrices are assumed to be zero and they are generally referred to as 'texture zero matrices'. These kind of matrices provide a viable framework to obtain the flavor mixing matrix, the CKM matrix, which is compatible with the current data (Patrignani et al. (2016)).

For definiteness, we focus here on the texture-specific quark mass matrices in (Fritzsch and Xing (1997, 2000))

$$M_{u,d} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} \tag{2.63}$$

which are known to reproduce the CKM matrix. The viability of these mass matrices are analyzed in (Ahuja (2016)) by showing the compatibility with the CKM matrix.

In our model the Higgs field leads to $(M_{u,d})_{13} = 0, (M_{u,d})_{31} = 0$ and $(M_{u,d})_{23} = 0, (M_{u,d})_{32} = 0$. In order to match to (2.63), we need to induce matrix elements $(M_{u,d})_{23} \neq 0$ and $(M_{u,d})_{32} \neq 0$. One way to do this is by higher-dimensional operators (Buchmuller and Wyler (1986); Barger et al. (2003); Grzadkowski et al. (2010); Murdock et al. (2011)). Then, as a minimal approach that fits to our $U(1)'$ set up, we introduce the Yukawa interactions

$$\mathcal{L} \supset \lambda_u^{23} \left(\frac{S}{\Lambda}\right)^{\delta_u^{23}} \bar{Q}_2 \tilde{\hat{H}} t_R + \lambda_u^{23} \left(\frac{SS^*}{\Lambda^2}\right)^{\delta_d^{23\prime}} \left(\frac{S}{\Lambda}\right)^{\delta_d^{23}} \bar{Q}_2 \hat{H} b_R + h.c. \tag{2.64}$$

where λ_u^{23} is the Yukawa coupling, Λ is the mass scale for flavor physics, $\delta_{u,d}^{23}$ and $\delta_d^{23\prime}$ are parameters that will be determined below. From (2.64), we get the gauge invariance conditions

$$\begin{aligned} -Q_{Q_2} - Q_H + Q_{u_{R_3}} + \delta_u^{23}.Q_S = 0, \\ -Q_{Q_2} + Q_H + Q_{d_{R_3}} + \delta_d^{23}.Q_S = 0 \end{aligned} \tag{2.65}$$

which lead to

$$\delta_u^{23} = \delta_d^{23} = \frac{\epsilon'}{Q_S \tilde{g}} \tag{2.66}$$

after using the solutions of the charges in Tab. 2.6. This method of generating the hierarchy can be extended to the other Yukawa entries (in terms of their 33 entries or few other entries) (Buchmuller and Wyler (1986); Barger et al. (2003); Grzadkowski et al. (2010); Murdock et al. (2011)).

The parameters δ_u^{23} and δ_d^{23} are positive integers so that we adopt $Q_S = \frac{\epsilon'}{\tilde{g}}$ to obtain $\delta_u^{23} = \delta_d^{23} = 1$. This solution of Q_S leads to $v_s \approx \mathcal{O}(10)$ GeV for a 17 MeV Z' boson. The charge of the extra scalar $\hat{S}$ is $Q_S \approx \mathcal{O}(10^{-2})$ for the coupling $\tilde{g} \approx \mathcal{O}(10^{-1})$. If we use the optimized values of the matrix elements of $(M_{u,d})_{23}$ from (Ahuja (2016)), we find that $\delta_d^{23'} \approx 2$ for $\Lambda \approx \mathcal{O}(10)$ GeV and $\lambda_{u,d}^{23} = 1$.

The solutions via (2.64) are not necessarily specific to the texture in (2.63). One can consider different textures and generate the same CKM structure by modifications or extensions of (2.64).

In the present model in the interaction basis the couplings of the Z' to the SM quarks are diagonal but nonuniversal. This nonuniversality gives rise to flavor changing neutral currents (FCNCs). From $B^0 - \bar{B}^0$ mixing there arise stringent constraints for these FCNCs (Bećirević et al. (2016); Kumar and London (2019))

$$|\epsilon^{L(R)}| \lesssim 10^{-6} \tag{2.67}$$

where $\epsilon^{L(R)}$ are the chiral couplings of the Z' to the $\bar{s}\gamma^{\mu}b$ current.

In the present model the chiral couplings in the down quark sector are given by

$$g_{d_L} \equiv diag(g_{d_L}^1, g_{d_L}^1, g_{d_L}^3), \tag{2.68}$$

$$g_{d_R} \equiv diag(g_{d_R}^1, g_{d_R}^1, g_{d_R}^3) \tag{2.69}$$

where $g_{d_L}^1 = g_{d_R}^1 = \frac{\epsilon'}{3}, g_{d_L}^3 = g_{d_R}^3 = -\frac{2\epsilon'}{3}$. If we introduce the CKM matrix, in the quark mass eigenstate basis the chiral couplings become

$$\epsilon_{sb}^L \equiv (V_{CKM} g_{d_L} V_{CKM}^{\dagger})_{23}, \tag{2.70}$$

$$\epsilon_{sb}^R \equiv (V_{CKM}^{\dagger} g_{d_R} V_{CKM})_{23}. \tag{2.71}$$

Then one obtains the following condition from both of the chiral couplings above

$$|\epsilon'| = 2 \times 10^{-3}. \tag{2.72}$$

2.6. LHC bound

In our family-nonuniversal $U(1)'$ model, the SM-like Higgs boson is charged under $U(1)'$ which leads to decay of ($h \to Z^{'}Z^{'}$) that should be sufficiently small such that the branching fraction of the SM-like Higgs to the Z' boson pairs has to be $BR(h \to Z^{'}Z^{'}) \lesssim 10\%$ (Curtin et al. (2014); Lee and Sher (2013)).

The decay rate of this process is given by

$$\Gamma(h \to Z'Z') = \frac{3}{32\pi m_h}\xi^2\left(1-\frac{4M_{Z'}^2}{m_h^2}\right)^{1/2}\left(1-\frac{m_h^2}{3M_{Z'}^2}+\frac{m_h^4}{12M_{Z'}^4}\right) \tag{2.73}$$

where we have introduced

$$\xi \equiv 4\left[\cos\alpha\sin^2\theta_W\eta^2\frac{M_Z^2}{v}-\sin\alpha\frac{M_{Z'}^2}{v_s}-\frac{\cos\alpha}{2\cos\theta_W}v\left(g'-\frac{e}{2\cos\theta_W}\right)e\eta^2\right]. \tag{2.74}$$

In Fig.(2.6), we show the region where the partial decay width $\Gamma(h \to Z'Z')$ is less than 10% of the SM Higgs total decay width

$$BR(h \to Z'Z') = \frac{\Gamma(h \to Z'Z')}{\Gamma_{total}^{SM}(h)+\Gamma(h \to Z'Z')} \lesssim 0.10 \tag{2.75}$$

where $\Gamma_{total}^{SM}(h) = 4.07 \times 10^{-3}$ GeV (Andersen et al. (2013)).

The scalar mixing angle is $\sin\alpha \sim \mathcal{O}(10^{-3})$ and accordingly the scalar mixing parameter is $\kappa \sim \mathcal{O}(10^{-3})$ required for $BR(h \to Z'Z') \lesssim 10\%$ for the SM Higgs boson mass of $m_h = 125.09$ GeV (Patrignani et al. (2016)) and $\eta = 10^{-4}$. The scalar mixing remains at the same order for different values of the kinetic mixing $\eta = 10^{-5}, 10^{-6}$.

The decay process of ($h \to ZZ'$) would also be relevant however, the $(hZZ^{'})$ vertex factor, which is given by

$$hZZ' : -\frac{\cos\alpha}{\sin 2\theta_W}ve\Big(\frac{g'\eta}{2}+e\tilde{g}Q_H\Big) \tag{2.76}$$

is proportional to the left-hand side of the zero $Z - Z'$ mixing condition in (2.37). Therefore this vertex is zero and there arise no constraints from this decay.

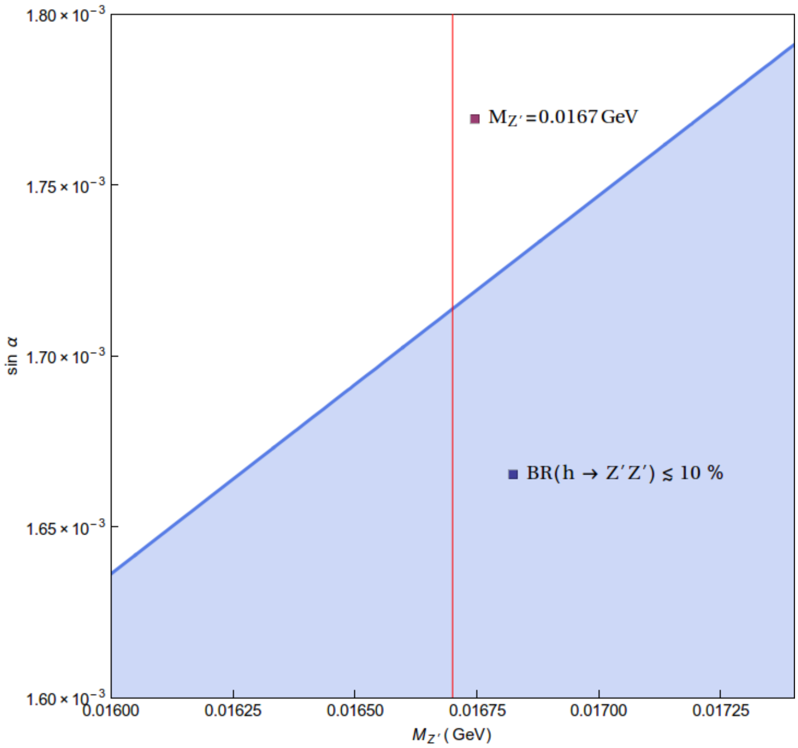

Figure 2.1. We show the region where the partial decay width $\Gamma(h \to Z'Z')$ is less than 10% of the SM Higgs total decay width $BR(h \to Z'Z') \lesssim 10\%$. The Higgs mixing angle is $\sin\alpha \sim \mathcal{O}(10^{-3})$ for $m_h = 125.09$ GeV and $\eta = 10^{-4}$. The vertical red line is for the Z' boson mass $M_{Z'}$ determined via the experimental data.

2.7. Summary and Outlook

In Chapter 2, we have constructed the framework of a family-nonuniversal $U(1)'$ model, which is a minimal, anomaly-free extension of the SM that is able to explain the $6.8\,\sigma$ anomaly in ^8Be nuclear decays at the Atomki pair spectrometer experiment.

One possible interpretation of the Atomki signal is a spin-1, proto-phobic gauge boson with a mass of ≈ 17 MeV. We present a family-nonuniversal $U(1)'$ model with its associated Z' boson with a mass of ≈ 17 MeV which fullfills all the experimental constraints on its vector and axial-vector current couplings to the first generation of fermions that are necessary to explain the ^8Be anomalous decays.

The previously proposed models have a large new field content. However, we have a minimal field content with the Z' boson and the extra scalar. Our family-nonuniversal

$U(1)'$ model is an anomaly-free extension of the SM with a minimum field content that can explain the observed beryllium anomaly.

The CKM matrix is reproduced correctly by higher-dimensional Yukawa interactions facilitated by S. The model provides new couplings to probe new physics at low energies. It may provide framework for anomalous SM decays and forms a framework in which various low-energy phenomena can be addressed. The low-energy phenomena such as $ss^* \to f\bar{f}$ and $Z'Z' \to f\bar{f}$ can be relevant for phenomenological as well as astrophysical (dark matter) purposes.

CHAPTER 3

GEOMETRIC DARK MATTER

In this chapter, we exercise the MAG in which the metric $g_{\mu\nu}$ and connection $\Gamma^{\lambda}_{\mu\nu}$ are independent geometrodynamical variables. We show that the MAG, in its minimal form, decomposes into GR plus a massive vector field Y_μ, which couples only to fermions (quarks and leptons) and gravity. It does not couple to scalars and gauge bosons. This interaction feature of the Y_μ stems from the geometric nature of it and it is generated from the affine connection as a massive vector. These very features of the Y_μ provide to keep the model minimal. The model involves only one free parameter and the Y_μ feebly interacts with the fermions. This interaction feature of the Y_μ is the only thing necessary for its longetivity as a dark matter candidate. We show in detail that the Y_μ qualifies a viable dark matter candidate, well satisfying the existing bounds. We introduced the notion of "geometric dark matter" and under all these aspects we have shown that this Y_μ geometric dark matter is the true dark matter model which differs from all the known dark matter candidates.

In Chapter 3, Section 3.1 gives brief introduction of the metric and Palatini formulations. Section 3.2 explains the physical necessity of affine connection, and Section 3.3 builds on it by structuring the most minimal ghost-free MAG. Section 3.4 quantizes Y_μ in the flat metric limit. Section 3.5 shows that Y_μ possesses all the features required of a dark matter particle. Section 3.6 concludes.

3.1. Notes on Metric and Palatini Formulations

In this section we give introductory notes on the metric and Palatini formulations briefly as it will be detailed within the concept in the next section.

The GR is based on metric formulation (or second-order formalism). The metric $g_{\mu\nu}$ measures distances between points of the manifold and angles between vectors in the tangent space. In pure metric gravity the connection is the Levi-Civita connection with components ${}^{g}\Gamma^{\lambda}_{\mu\nu}$ and it defines parallel transport of tensor fields along a given curve in spacetime. It is characterised such that the connection is compatible with the metric $\nabla_\lambda g_{\mu\nu} = 0$ and has no torsion ${}^g\Gamma^\lambda_{\mu\nu} = {}^g\Gamma^\lambda_{\nu\mu}$. The curvature of the spacetime is determined by the Riemann tensor given by

$$\mathbb{R}^\lambda_{\mu\sigma\nu}({}^g\Gamma) = \partial_\sigma {}^g\Gamma^\lambda_{\mu\nu} - \partial_\nu {}^g\Gamma^\lambda_{\sigma\mu} + {}^g\Gamma^\lambda_{\sigma\rho} {}^g\Gamma^\rho_{\mu\nu} - {}^g\Gamma^\lambda_{\nu\rho} {}^g\Gamma^\rho_{\sigma\mu}, \tag{3.1}$$

and its dynamics is described by the principle of minimal action. Riemann and Ricci tensors are directly given in terms of the metric tensor. As will be detailed in the next section, the curvature tensor involves second derivatives of the metric tensor and it requires adding an extrinsic curvature to cancel the surface terms (York (1972); Gibbons and Hawking (1977)).

In the Palatini formulation (Palatini (1919); Einstein (1925); Deser (2006)) (or first-order formalism) the metric tensor $g_{\mu\nu}$ and the connection coefficients $\Gamma^\lambda_{\mu\nu}$ are a priori independent geometrical variables. In this formalism the connection is not necessarily the Levi-Civita connection, it is not a predetermined quantity; its form is determined through the requirement of dynamics. Riemann and Ricci tensors do depend only on the connection and the curvature tensor involves only first derivatives of the connection which requires no extrinsic curvature. We will discuss the two formalisms in the next section such that one will see clearly that the Palatini formalism is the right generalized form of the GR to obtain the Einstein field equations.

3.2. Necessity of Affine Connection

The GR, whose geometry is based on the metric tensor $g_{\mu\nu}$ and its Levi-Civita connection

$${}^g\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right), \tag{3.2}$$

is defined by the Einstein-Hilbert action

$$S\left[g\right] = \int d^4x \sqrt{-g} \frac{M^2_{Pl}}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}({}^g\Gamma) \tag{3.3}$$

as a purely metrical theory of gravity. The problem is that this action is known not to lead to the Einstein field equations. It needs be supplemented with exterior curvature (York

(1972); Gibbons and Hawking (1977)) because the Ricci curvature of the Levi-Civita connection

$$\mathbb{R}_{\mu\nu}({}^g\Gamma) = \partial_\lambda {}^g\Gamma^\lambda_{\mu\nu} - \partial_\nu {}^g\Gamma^\lambda_{\lambda\mu} + {}^g\Gamma^\rho_{\rho\lambda} {}^g\Gamma^\lambda_{\mu\nu} - {}^g\Gamma^\rho_{\nu\lambda} {}^g\Gamma^\lambda_{\rho\mu}, \tag{3.4}$$

obtained from the Riemann tensor as $\mathbb{R}_{\mu\nu}({}^g\Gamma) \equiv \mathbb{R}^\lambda_{\mu\lambda\nu}({}^g\Gamma)$, involves second derivatives of the metric. The need to exterior curvature disrupts the action principle for GR.

The remedy, long known to be the Palatini formalism (Palatini (1919); Einstein (1925); Deser (2006)), is to replace the Levi-Civita connection ${}^g\Gamma^\lambda_{\mu\nu}$ with a (symmetric) affine connection $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$ and restructure the Einstein-Hilbert action (3.3) accordingly

$$S\left[g, \Gamma\right] = \int d^4x \sqrt{-g} \frac{M^2_{Pl}}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) \tag{3.5}$$

to find that $\Gamma^\lambda_{\mu\nu}$ reduces to ${}^g\Gamma^\lambda_{\mu\nu}$ dynamically because $S\left[g, \Gamma\right]$ can stay stationary against variations in $\Gamma^\lambda_{\mu\nu}$ only if the nonmetricity vanishes, that is, only if ${}^\Gamma\nabla_\lambda g_{\mu\nu} = 0$. This ensures that the Palatini action (3.5) is the right framework for getting the Einstein field equations.

3.3. Metric-Affine Gravity

The Palatini formalism, a signpost showing the way beyond the purely metrical geometry of the GR, evolves into a dynamical theory if the affine connection $\Gamma^\lambda_{\mu\nu}$ acquires components beyond the Levi-Civita connection. In this context, spread of $\Gamma^\lambda_{\mu\nu}$ into the curvature (Karahan et al. (2013)) and matter (Bauer and Demir (2008, 2011)) sectors, for instance, leads to the MAG. The MAG is described by the action

$$S\left[g, \Gamma, \digamma\right] = \int d^4x \sqrt{-g} \Big\{ \frac{M^2_{Pl}}{2} g^{\mu\nu} \mathbb{R}_{\mu\nu}(\Gamma) - \frac{\xi}{4} \overline{\mathbb{R}}_{\mu\nu}(\Gamma) \overline{\mathbb{R}}^{\mu\nu}(\Gamma) + \mathcal{L}\left(g, \Gamma, \digamma\right) \Big\} + \Delta S \tag{3.6}$$

in which $\mathbb{R}_{\mu\nu}(\Gamma)$ is the Ricci curvature obtained from (3.4) by replacing ${}^g\Gamma$ with Γ, $\mathcal{L}$ is the Lagrangian of the matter fields $\digamma$ with Γ kinetics, and $\overline{\mathbb{R}}_{\mu\nu}(\Gamma)$ is the second Ricci curvature

$$\overline{\mathbb{R}}_{\mu\nu}(\Gamma) = \partial_\mu \Gamma^\lambda_{\lambda\nu} - \partial_\nu \Gamma^\lambda_{\lambda\mu} \tag{3.7}$$

obtained from the Riemann tensor as $\overline{\mathbb{R}}_{\mu\nu}(\Gamma) \equiv \mathbb{R}^\lambda_{\lambda\mu\nu}(\Gamma)$. It equals the antisymmetric part of $\mathbb{R}_{\mu\nu}(\Gamma)$, and vanishes identically in the metrical geometry, $\overline{\mathbb{R}}_{\mu\nu}({}^g\Gamma) \equiv 0$.

The ΔS in (3.6), containing two– and higher-derivative terms, has the structure

$$\Delta S\left[g,\Gamma\right]=\int d^4x\,\sqrt{-g}\Big\{A\left(g^{\mu\nu}\mathbb{R}_{\mu\nu}(\Gamma)\right)^2+B\mathbb{R}_{\mu\nu}(\Gamma)\mathbb{R}^{\mu\nu}(\Gamma)+C\mathbb{R}_{\mu\nu\alpha\beta}(\Gamma)\mathbb{R}^{\mu\nu\alpha\beta}(\Gamma)+\cdots\Big\} \tag{3.8}$$

in which the leading terms, weighted by dimensionless coefficients A, B, C, are of similar size as the ξ term in (3.6). These terms, excepting A, are, however, dangerous in that they give ghosts in the metrical part. This is so because ΔS involves at least four derivatives of the metric. (The ξ term in (3.6) has no metrical contribution and remains always two-derivative.) We will hereon drop B, C and all higher-order terms on the danger of ghosts. The A term and terms containing higher powers of $g^{\mu\nu}\mathbb{R}_{\mu\nu}(\Gamma)$ are known to lead collectively to a scalar degree of freedom in excess of the GR (Sotiriou and Faraoni (2010)). In principle, there is no harm in keeping them but we drop them as they do not have any distinctive effect on the vector dark matter we shall construct. They can be included to study vector dark matter in scalar-tensor theories (Quiros (2019)), and this can indeed be an interesting route.

Now, we continue with (3.6) with ΔS completely dropped. The Palatini formalism implies that MAG can always be analyzed via the decomposition

$$\Gamma^\lambda_{\mu\nu}={}^g\Gamma^\lambda_{\mu\nu}+\Delta^\lambda_{\mu\nu} \tag{3.9}$$

where $\Delta^\lambda_{\mu\nu}=\Delta^\lambda_{\nu\mu}$ is a symmetric tensor field. Under (3.9), the two Ricci curvatures split as

$$\begin{aligned}\mathbb{R}_{\mu\nu}(\Gamma) &= \mathbb{R}_{\mu\nu}({}^g\Gamma)+\nabla_\lambda\Delta^\lambda_{\mu\nu}-\nabla_\nu\Delta^\lambda_{\lambda\mu}+\Delta^\rho_{\rho\lambda}\Delta^\lambda_{\mu\nu}-\Delta^\rho_{\nu\lambda}\Delta^\lambda_{\rho\mu},\\ \overline{\mathbb{R}}_{\mu\nu}(\Gamma) &= \partial_\mu\Delta^\lambda_{\lambda\nu}-\partial_\nu\Delta^\lambda_{\lambda\mu}\end{aligned} \tag{3.10}$$

to put the MAG action in (3.6) (with ΔS dropped) into the form

$$\begin{aligned}S\left[g,\Delta,\digamma\right] &= \int d^4x\,\sqrt{-g}\Big\{\frac{M_{Pl}^2}{2}g^{\mu\nu}\mathbb{R}_{\mu\nu}({}^g\Gamma)\\ &- \frac{1}{4}\xi g^{\mu\alpha}g^{\nu\beta}\left(\partial_\mu\Delta^\lambda_{\lambda\nu}-\partial_\nu\Delta^\lambda_{\lambda\mu}\right)\left(\partial_\alpha\Delta^\rho_{\rho\beta}-\partial_\beta\Delta^\rho_{\rho\alpha}\right)\\ &+ \frac{M_{Pl}^2}{2}g^{\mu\nu}\left(\Delta^\rho_{\rho\lambda}\Delta^\lambda_{\mu\nu}-\Delta^\rho_{\nu\lambda}\Delta^\lambda_{\rho\mu}\right)+\mathcal{L}\left(g,{}^g\Gamma,\Delta,\digamma\right)\Big\}\end{aligned} \tag{3.11}$$

where ∇_α is the covariant derivative with respect to the Levi-Civita connection ($\nabla_\alpha g_{\mu\nu}=0$).

In the action (3.11), the kinetic term, proportional to ξ, pertains only to the vector field $\Delta^\lambda_{\lambda\mu}$ but the quadratic term, proportional to M^2_{Pl}, involves all components of $\Delta^\lambda_{\mu\nu}$. In fact, it shrinks to a consistent vector field theory if the quadratic term reduces to mass term of $\Delta^\lambda_{\lambda\mu}$, and this happens only if $\Delta^\lambda_{\mu\nu}$ enjoys the decomposition

$$\Delta^\lambda_{\mu\nu} = \frac{1}{2}\left(\Delta^\rho_{\rho\mu}\delta^\lambda_\nu + \delta^\lambda_\mu\Delta^\rho_{\rho\nu} - 3g^{\lambda\alpha}\Delta^\rho_{\rho\alpha}g_{\mu\nu}\right) \tag{3.12}$$

under which the action (3.11) takes the form

$$\begin{aligned} S[g,Y,\digamma] &= \int d^4x\sqrt{-g}\Big\{\frac{M^2_{Pl}}{2}R(g) - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - \frac{3M^2_{Pl}}{4\xi}Y_\mu Y^\mu \\ &- \frac{3}{2\sqrt{\xi}}\overline{f}\gamma^\mu f Y_\mu + \overline{\mathcal{L}}(g, {}^g\Gamma, \digamma)\Big\} \end{aligned} \tag{3.13}$$

where $R(g) \equiv g^{\mu\nu}\mathbb{R}_{\mu\nu}({}^g\Gamma)$ is the metrical curvature scalar, $Y_\mu \equiv \sqrt{\xi}\Delta^\lambda_{\lambda\mu}$ is a vector field generated by the affine connection, and $\overline{\mathcal{L}}\left(g, {}^g\Gamma, \digamma\right)$ is part of the matter Lagrangian that does not involve Y_μ. This action exhibits two crucial facts about the geometrical vector Y_μ:

1. First, it is obliged to be massive if gravity is to attract with the observed strength. Indeed, the Newton's constant ($G_N = (8\pi M^2_{Pl})^{-1}$) and the Y_μ mass ($M^2_Y = \frac{3}{2\xi}M^2_{Pl}$) are both set by the Planck scale M_{Pl}. This action represents a rather rare case that Planck's constant sets both the gravitational scale and a particle mass.

2. Second, it couples only to fermions $f \subset \digamma$. And its couplings, originating from the spin connection through the decomposition in (3.12), are necessarily flavor-universal. It couples to the known (leptons and quarks in the SM) and any hypothetical (say, the dark matter particle χ) fermion in the same way, with the same strength.

3.4. Quantization

The classical setup in (3.13) involves two distinct fields: The metric tensor $g_{\mu\nu}$ which leads to gravity as is the GR, and the geometrical vector Y_μ which gives rise to a fifth force that affects fermions universally. These two may well be quantized but, given the difficulties with the quantization of gravity, it would be reasonable to keep $g_{\mu\nu}$

classical yet let Y_μ be quantized. In the flat limit, for which $g_{\mu\nu}$ nears the flat metric $\eta_{\mu\nu}$, quantum field theory is full force and effect so that Y_μ, along with the other fields in $\overline{\mathcal{L}}\left(g, {}^{g}\Gamma, F\right)$, changes to the field operator (see, for instance, Ramond (1981, 1989); Peskin and Schroeder (1995))

$$\hat{Y}^\mu(x) = \sum_{\lambda=0}^{3} \int \frac{d^3\vec{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega(\vec{p})}} \left\{ \hat{a}(\vec{p},\lambda)\epsilon^\mu(\vec{p},\lambda)e^{-ip\cdot x} + \hat{a}^\dagger(\vec{p},\lambda)\epsilon^{\mu\star}(\vec{p},\lambda)e^{ip\cdot x} \right\} \tag{3.14}$$

in which the operator $\hat{a}(\vec{p},\lambda)$, with the commutator $\left[\hat{a}(\vec{p},\lambda), \hat{a}^\dagger(\vec{p}', \lambda')\right] = i\delta^4\left(p-p'\right)\delta_{\lambda\lambda'}$, annihilates a spin-1 boson of momentum $\vec{p}$, energy $\omega(\vec{p}) = (M_Y^2 + \vec{p}\cdot\vec{p})^{1/2}$, polarization direction λ, and polarization sum $\sum_{\lambda=1}^{3}\epsilon^\mu(\vec{p},\lambda)\epsilon^{\nu\star}(\vec{p},\lambda) = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{M_Y^2}$. The Y_μ-quanta can be converted into or created from any fermion f and its anti-fermion f^c, as will be analyzed in the next section.

3.5. Geometric Dark Matter

In this section we will study Y_μ to determine if it can qualify as dark matter. (The vector dark matter, as an Abelian gauge field, has been studied in (Chen et al. (2015)).) To this end, the crucial factor is its lifetime. The rate of Y_μ into a fermion f and its anti-particle f^c decay is obtained from the following differential rate

$$d\Gamma = N_c^f \frac{1}{32\pi^2} < |M|^2 > \frac{|\vec{p_1}|}{M_Y^2} d\Omega \tag{3.15}$$

for the following amplitude

$$M = -i\frac{3}{2\sqrt{\xi}}\bar{u}(\vec{p_1})\gamma^\mu v(\vec{p_2})\epsilon_\mu(\vec{p_1}+\vec{p_2}). \tag{3.16}$$

In fact, as follows from (3.13) with (3.14), it decays into a fermion f and its anti-particle f^c with a rate

$$\Gamma\left(Y \to f f^c\right) = \frac{N_c^f}{8\pi}\left(\frac{3}{2\xi}\right)^{\frac{3}{2}}\left(1 + \frac{4\xi m_f^2}{3M_{Pl}^2}\right)\left(1 - \frac{8\xi m_f^2}{3M_{Pl}^2}\right)^{\frac{1}{2}} M_{Pl} \xrightarrow{\xi m_f^2 \ll M_{Pl}^2} \frac{N_c^f}{8\pi}\left(\frac{3}{2\xi}\right)^{\frac{3}{2}} M_{Pl} \tag{3.17}$$

where m_f is the mass of the fermion and N_c^f is the number of its colors. Then, summing over SM quarks (up and down) and leptons (electrons and neutrinos) its lifetime turns out to be

$$\tau_Y = \frac{1}{\Gamma_{tot}} = \frac{4\pi}{5}\left(\frac{2}{3}\right)^{3/2}\frac{\xi^{3/2}}{M_{Pl}} \tag{3.18}$$

which is larger than the age of the Universe $t_U = 13.8 \times 10^9$ years (Ade et al. (2016)) if $\xi > 1.1 \times 10^{40}$. On the other side, the decay rate is prevented to go imaginary if $\xi < 1 \times 10^{41}$. These two bounds lead to the allowed mass range

$$9.4\ \mathrm{MeV} < M_Y < 28.4\ \mathrm{MeV} \tag{3.19}$$

across which Y_μ lifetime ranges from $4.4 \times 10^{17}\ s$ to $1.2 \times 10^{19}\ s$. This means that Y_μ exists today to contribute to galactic dynamics and other phenomena (Rubin et al. (1976); Schumann (2019); Liu et al. (2017)). Its relic density

$$\rho_{relic} = \rho_{primordial}\ e^{-\Gamma_{\rm tot} t_U} \tag{3.20}$$

ranges from $\rho_{primordial}/e$ (for $M_Y = 28.4\ \mathrm{MeV}$) to near $\rho_{primordial}$ (for $M_Y = 9.4\ \mathrm{MeV}$).

The question of if Y_μ can be detected in direct searches is a crucial one. To see this, it is necessary to compute rate of scattering from nucleons. The relevant diagrams are depicted in Fig. 3.1 below.

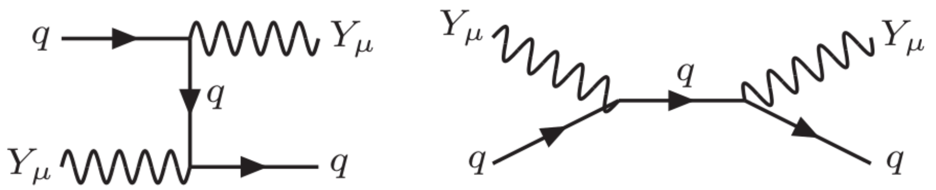

Figure 3.1. The $Y_\mu q \to Y_\mu q$ scattering. The quark q belongs to the nucleon.

The two diagrams in Fig. 3.1 result in the amplitude

$$\mathcal{M} = -i\frac{9}{4\xi}\bar{u}(k')\left(\gamma^\nu\frac{\not{k} - \not{p}' + m_q}{(k-p')^2 - m_q^2}\gamma^\mu + \gamma^\mu\frac{\not{k} + \not{p} + m_q}{(k+p)^2 - m_q^2}\gamma^\nu\right)u(k)\epsilon^*_\mu(p')\epsilon_\nu(p) \tag{3.21}$$

where $1/\xi$ in front follows from the Y_μ coupling to quark q in (3.13). In the nonrelativistic limit, Y_μ momenta become $p = p' = (M_Y, 0, 0, 0)$. Moreover, the quark momentum reduces to $k = (E_q, 0, 0, 0)$ after neglecting its motion in the nucleon. The total amplitude then takes the form

$$\mathcal{M} = -i\frac{9}{4\xi M_Y}\bar{u}(k')(\gamma^\mu\gamma^0\gamma^\nu - \gamma^\nu\gamma^0\gamma^\mu)u(k)\epsilon^*_\mu(p')\epsilon_\nu(p) \tag{3.22}$$

after imposing $E_q \approx m_q < M_Y$ in (3.21). As a result, the spin-dependent scattering cross section for a vector dark matter scattering off a proton (Chen et al. (2015)) becomes

$$\sigma_p^{SD} = \frac{1}{2\pi}\frac{m_p^2}{(M_Y + m_p)^2}a_p^2 \tag{3.23}$$

where m_p is the proton mass and a_p is the effective spin-spin interaction of the dark matter Y_μ and the proton

$$a_p = \frac{9}{4\xi M_Y}\sum_{q=u,d,s}\Delta_q^p \tag{3.24}$$

where $\Delta_u^p = 0.84, \Delta_d^p = -0.44$ and $\Delta_s^p = -0.03$ (Cheng and Chiang (2012)). We plot the spin-dependent cross section (3.23) in Fig. 3.2 in the allowed range (3.19) of M_Y.

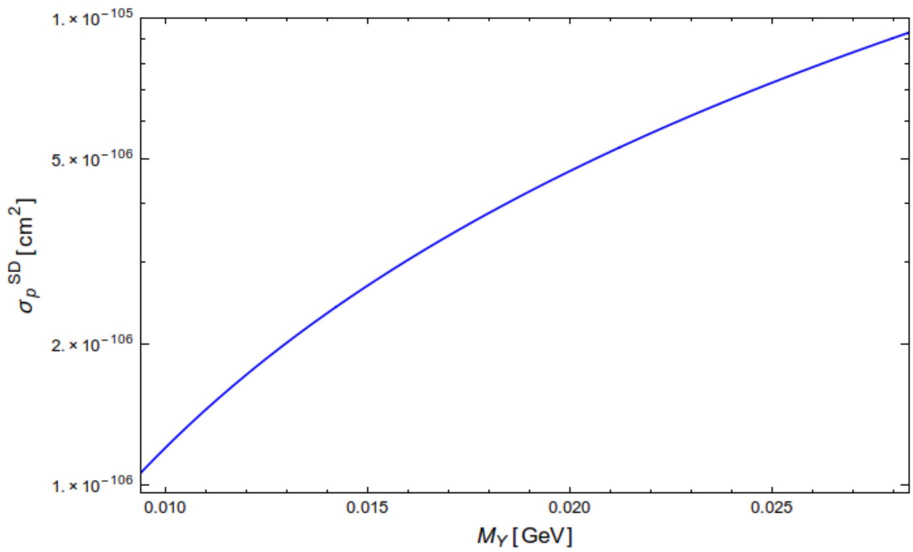

Figure 3.2. The spin-dependent Y_μ–proton cross section as a function of M_Y.

Direct search experiments like COUPP (Behnke et al. (2012)), SIMPLE (Felizardo et al. (2014)), XENON100 (Aprile et al. (2016)), PICO-2L (Amole et al. (2016)), PICO-60 (Amole et al. (2017)), PandaX-II (Fu et al. (2017)), PICASSO (Behnke et al. (2017)) and LUX (Akerib et al. (2016)) have put stringent upper limits on the spin-dependent cross section for scattering of dark matter off the SM particles. They mainly exclude the WIMPs. The most stringent limit is around $\sigma_p^{SD} \sim \mathcal{O}(10^{-41})\ \mathrm{cm}^2$. It is clear that the Y_μ–proton spin-dependent cross section given in Fig. 3.2 is at most $\mathcal{O}\left(10^{-106}\right)\ \mathrm{cm}^2$, which is too small to be measurable by any of the current experiments. The Fig. 3.2 can be taken as the explanation of why dark matter has so far not been detected in direct searches. It might, however, be measured in future experiments though it is hard to imagine what future technology can provide access to such tiny cross section. One possibility, speculatively speaking, would be variants of the (laser, SQUID, etc) technology that led to the detection of gravitational waves.

3.6. Conclusion

In this second part of the thesis, we have set forth a new dark matter candidate, which seems to agree with all the existing bounds. In accordance with its signatures, it reveals itself only gravitationally. Our candidate particle, a genuinely geometrical field provided by the metric-affine gravity, is a viable dark matter candidate, and explains the current conundrum by its exceedingly small scattering cross section from nucleons. It can be difficult to detect it with today's technology but future experiments (plausibly extensions of gravitational wave detection technology) might reach the required accuracy.

We propose a fundamentally different vector dark matter candidate from all the other vector dark matter candidates in the literature. We show that due to its geometrical origin the geometric vector dark matter Y_μ does not couple to scalars and gauge bosons. It couples only to fermions. It must be emphasized that there is no need to impose any Z_2 symmetry to prevent the gauge kinetic interaction in the vector portal. Its feebly interacting nature is all that is needed for its longevity. Morover, we should note that since the Y_μ is generated by the affine connection as a massive vector, we do not need to deal with interactions due to Higgs or Stueckelberg mechanisms. This keeps the present model minimal as there is no need for additional scalars conceptually. On the basis of the above-mentioned basic features it is obvious that the geometric dark matter Y_μ is the truly minimal model of dark matter.

The model can be extended in various ways. As already mentioned in the text, one possibility is to include quadratic and higher-order terms in curvature tensor. This kind of terms, even after discarding the ghosty terms, can cause, among other things, the rank-3 tensor to be fully dynamical. The theory is then a tensor theory involving dynamical fields beyond Y_μ, where the excess degrees of freedom may contribute to dark energy and inflation.

Before closing, it proves useful to emphasize that quantization of Y_μ is actually quantization of the geometry. But, what is done here is a partial quantization in that metric tensor is kept classical. This is certainly not the long-sought quantum gravity but it might be a glimpse of the fact that what is to be quantized may not be the metric (measurement toolbox) but the connection (the source of curvature and dark matter).

CHAPTER 4

CONCLUDING REMARKS AND OUTLOOK

Here, we give a brief summary of the thesis as well as some future prospects that may be relevant for a future study.

As for the astrophysical implications, it is worth to mention the Big Bang nucleosynthesis (BBN). It is known that the BBN, which is based on the standard model physics (Wagoner et al. (1967)) in which the spacetime is described by the GR, cosmology is based on the ΛCDM model and the particle physics is based on the SM. The BBN gives predictions of the abundances of the light elements $D,^3He,^4He$ and ^7Li which depend only on one parameter namely the baryon-to-photon ratio $\eta = n_b/n_\gamma$ (or equivalently the baryon density $\Omega_b h^2 \equiv \omega_b$). The baryon-to-photon ratio has been determined to be $\eta = 6.10 \pm 0.004$ (or $\omega_b = 0.002225 \pm 0.00016$) via measurements of the microwave background anisotropies by Planck (Ade et al. (2016)). The BBN takes place from ~ 1 s to ~ 3 min after the Big Bang and roughly at the end of the three minutes the abovementioned light elements were synthesized. The BBN-predicted abundances are in good agreement with the observational data except the lithium abundance (Schramm and Turner (1998); Steigman (2007); Iocco et al. (2009); Cyburt et al. (2016)). The BBN-predicted lithium abundance is about a factor of 3 higher than the abundance determined in low-metalicity halo stars. There is no solution to the problem from nuclear aspects hence it is yet a possibility that new physics during or after the nucleosynthesis might explain it. However, the BBN as a probe of the early universe puts stringent constraints on physics beyond the SM (Sarkar (1996); Jedamzik and Pospelov (2009); Pospelov and Pradler (2010); Fields (2011)). Morover, it is important to note that all the known forces; strong, weak, electroweak and gravitational, are effective during the synthesis of the light elements hence the precise determinations of light element abundances in the early Universe put constraints on the possible new physics in gravity and particle physics. Therefore there may be some astrophysical implications of this picture for our work which is out of scope of this thesis. However, it may be a future work study to analyze the effects of such MeV-scale particles in the early Universe.

In the first part (Chapter 2) of the thesis, we have constructed a family-nonuniversal $U(1)'$ model which explains the recent anomalous beryllium decays. The beryllium decays can be explained by a neutral, proto-phobic gauge boson. It is exciting that the new neutral gauge boson might refer to the fifth force. The model we construct is populated with the 17 MeV Z' and the scalar field S which is singlet under the SM and charged under $U(1)'$. It is worth to mention that the framework represents a minimal model with its minimal field content which makes it different from all the other models in the literature for the beryllium decays. This minimality is provided by the nonuniversality of the $U(1)'$ charges of the SM chiral fermions that satisfy the anomaly-free conditions. It is shown that the CKM matrix is reproduced correctly by higher-dimension Yukawa interactions facilitated by the S. It is presented by a detailed analysis that the vector and axial-vector coupligs of the Z' to the first generation of the fermions satisfy all the experimental constraints. The S scalar couples directly to the SM-like Higgs; and the Z' has a kinetic mixing with the hypercharge gauge boson which generates the observed beryllium anomaly as we have shown by a detailed analysis. We have constructed a family-nonuniversal $U(1)'$ model, with a minimal field content, and show that the beryllium anomaly can be explained by an MeV-scale sector. Under all these aspects, one can see clearly that the model we have constructed explains the beryllium anomalous decays properly and it differs from all the other models.

In the second part (Chapter 3) of the thesis , we have introduced the notion of "geometric dark matter". It is basically a light vector particle Y_μ that arises from the MAG. In this set up, we have shown that the MAG dynamically reduces to the usual gravity plus a massive vector field Y_μ. The Y_μ interacts with only quarks, leptons and gravity due to the decomposition of the affine connection. It is neutral and long-living when its mass range lies in the range $9.4\,\mathrm{MeV} < M_Y < 28.4\,\mathrm{MeV}$. As we have explained in detail its longevity is already provided by its interaction nature and it is generated by the affine conection as a massive vector. These very futures of the Y_μ keeps the present model minimal, self-consistent and different from all the other dark matter models in the literature. It is remarkable under all these features that the geometric dark matter Y_μ is the minimal model of dark matter. Its spin-dependent scattering cross section in Fig. 3.2, which corresponds to the diagrams in Fig. 3.1, is at most $\mathcal{O}\left(10^{-106}\right)\ \mathrm{cm}^2$ which is too small to be measurable by any of the current experiments. The most stringent bound is around $\sigma_p^{SD} \sim \mathcal{O}(10^{-41})\ \mathrm{cm}^2$. This small scattering cross section of the Y_μ gives an explanation of why dark matter has not been detected in direct search experiments yet. We have shown that Y_μ is a viable dark matter candidate that satisfies all the experimental data and constraints on the dark matter and it differs from all the other dark matter candidates.

We have studied vector fields at the MeV scale and after all it is clearly realized that the physics at the low energy is the frontier worth to probe; as it seems the recent experimental results as well as the geometric dark matter model that we have constructed point the low energy physics. The new physics needed might be at low energies.

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- Demir, Durmuş and Puliçe, Beyhan (2020), "Geometric Dark Matter", *Journal of Cosmology and Astroparticle Physics*, 04 (2020) 051 (SCI),

  [arXiv:2001.06577 [hep-ph]].

Research Projects:

2019 – , TÜBİTAK 1001 Project encoded 118F387,

İzmir Institute of Technology, Sabancı University.