



The Ablowitz–Ladik lattice system by means of the extended (G'/G) -expansion method

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ABSTRACT

We analyzed the Ablowitz–Ladik lattice system by using the extended (G'/G) -expansion method. Further discrete soliton and periodic wave solutions with more arbitrary parameters are obtained. We observed that some previously known results can be recovered by assigning special values to the arbitrary parameters.

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1. Introduction

Nonlinear differential–difference equations (NDDs) is a subject of immense breadth. Its spectrum ranges from the abstract through numerical techniques to many interesting applications. These applications range from such theoretical applications as the use of NDDs in mechanical engineering, biophysics, condensed matter physics, and to many practical applications in such fields as atomic chains, molecular crystals, currents in electrical networks [1–4], etc. Many integrable NDDs [5–8] are proposed in the literature. Contrary to difference equations which are being fully discretized, NDDs are semi-discretized with some (or all) of their space variables discretized while time is usually kept continuous.

On the other hand, a considerable number of well-known analytic methods are successfully extended to NDDs by the researchers [9–17]. However, no method obeys the strength and the flexibility for finding all solutions to all types of NDDs. Recently, the (G'/G) -expansion method [18] has become popular in the research community, and there has been a number of studies refining the initial idea [19–30]. The method is powerful in the sense that it takes full advantage of linear theory. It is reliable, efficient, and entirely algorithmic. Moreover, the (G'/G) -expansion method delivers three types of exact solutions; hyperbolic, trigonometric, and rational functions. Not long ago, Zhang et al. [31] and Aslan [32] have successfully generalized the (G'/G) -expansion method to some physically important NDDs.

The objective of this study is to illustrate the applicability of the (G'/G) -expansion method to the famous Ablowitz–Ladik lattice system. In order to get as widest results as possible, we add extra terms with negative powers to the ansatz considered in the standard (G'/G) -expansion method. The rest of this paper is organized as follows. In Section 2, we describe our methodology for NDDs, and state the main steps. In Section 3, we analyze the Ablowitz–Ladik lattice system. Finally, we give some concluding remarks in Section 4.

2. The extended (G'/G) -expansion method for NDDs

Consider a system of M polynomial NDDs in the form

$$\mathbf{P}(\mathbf{u}_{n+p_1}(\mathbf{x}), \dots, \mathbf{u}_{n+p_k}(\mathbf{x}), \dots, \mathbf{u}'_{n+p_1}(\mathbf{x}), \dots, \mathbf{u}'_{n+p_k}(\mathbf{x}), \dots, \mathbf{u}_{n+p_1}^{(r)}(\mathbf{x}), \dots, \mathbf{u}_{n+p_k}^{(r)}(\mathbf{x})) = 0, \quad (1)$$

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where the dependent variable \mathbf{u}_n have M components $u_{i,n}$ and so do its shifts, the continuous variable \mathbf{x} has N components x_i , the discrete variable \mathbf{n} has Q components n_j , the k shift vectors $\mathbf{p}_i \in \mathbb{Z}^Q$, and $\mathbf{u}^{(r)}(\mathbf{x})$ denotes the collection of mixed derivative terms of order r .

Step 1. For constructing traveling wave solutions to Eq. (1), we consider the wave transformation

$$\mathbf{u}_{\mathbf{n}+\mathbf{p}_s}(\mathbf{x}) = \mathbf{U}_{\mathbf{n}+\mathbf{p}_s}(\xi_{\mathbf{n}}), \quad \xi_{\mathbf{n}} = \sum_{i=1}^Q d_i n_i + \sum_{j=1}^N c_j x_j + \zeta, \quad (s = 1, 2, \dots, k), \tag{2}$$

where the coefficients $c_1, c_2, \dots, c_N, d_1, d_2, \dots, d_Q$ and the phase ζ are all constants. Then, Eq. (1) becomes

$$P(\mathbf{U}_{\mathbf{n}+\mathbf{p}_1}(\xi_{\mathbf{n}}), \dots, \mathbf{U}_{\mathbf{n}+\mathbf{p}_k}(\xi_{\mathbf{n}}), \dots, \mathbf{U}'_{\mathbf{n}+\mathbf{p}_1}(\xi_{\mathbf{n}}), \dots, \mathbf{U}'_{\mathbf{n}+\mathbf{p}_k}(\xi_{\mathbf{n}}), \dots, \mathbf{U}^{(r)}_{\mathbf{n}+\mathbf{p}_1}(\xi_{\mathbf{n}}), \dots, \mathbf{U}^{(r)}_{\mathbf{n}+\mathbf{p}_k}(\xi_{\mathbf{n}})) = 0. \tag{3}$$

Step 2. We initially predict the structure of the solutions $\mathbf{U}_n = \mathbf{U}_n(\xi_{\mathbf{n}})$ to Eq. (3) in the finite series form

$$\mathbf{U}_n = \sum_{l=-m}^m a_l \left(\frac{G'}{G}\right)^l, \tag{4}$$

where m (a positive integer) and a_l s are constants to be determined and the function $G = G(\xi_{\mathbf{n}})$ satisfies the auxiliary equation

$$G'' + \lambda G' + \mu G = 0, \tag{5}$$

where λ and μ are constants and the primes denote derivatives with respect to $\xi_{\mathbf{n}}$. We note that the general solution of Eq. (5) is well known for us.

Step 3. A straightforward calculation leads to the identity

$$\xi_{\mathbf{n}+\mathbf{p}_s} = \xi_{\mathbf{n}} + \varphi_s, \quad \varphi_s = p_{s1}d_1 + p_{s2}d_2 + \dots + p_{sQ}d_Q, \tag{6}$$

where p_{sj} is the j th component of the shift vector \mathbf{p}_s . Thus, we can derive the expressions for the shift functions as follows:

$$\mathbf{U}_{\mathbf{n}+\mathbf{p}_s} = \sum_{l=-m}^m a_l \left(\frac{\frac{\lambda}{2} + \frac{G'}{G} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \varphi_s\right)}{1 + \frac{2}{\sqrt{\lambda^2 - 4\mu}} \left(\frac{\lambda}{2} + \frac{G'}{G}\right) \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \varphi_s\right)} - \frac{\lambda}{2} \right)^l, \quad \lambda^2 - 4\mu > 0, \tag{7a}$$

$$\mathbf{U}_{\mathbf{n}+\mathbf{p}_s} = \sum_{l=-m}^m a_l \left(\frac{\frac{\lambda}{2} + \frac{G'}{G} - \frac{\sqrt{4\mu - \lambda^2}}{2} \tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2} \varphi_s\right)}{1 + \frac{2}{\sqrt{4\mu - \lambda^2}} \left(\frac{\lambda}{2} + \frac{G'}{G}\right) \tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2} \varphi_s\right)} - \frac{\lambda}{2} \right)^l, \quad \lambda^2 - 4\mu < 0, \tag{7b}$$

$$\mathbf{U}_{\mathbf{n}+\mathbf{p}_s} = \sum_{l=-m}^m a_l \left(\frac{\frac{\lambda}{2} + \frac{G'}{G}}{1 + \left(\frac{\lambda}{2} + \frac{G'}{G}\right) \varphi_s} - \frac{\lambda}{2} \right)^l, \quad \lambda^2 - 4\mu = 0. \tag{7c}$$

Step 4. From Eq. (3), we can easily determine the degree m of the ansatz (4) and the expressions (7a–c) by balancing the highest order nonlinear term (s) and the highest-order derivative term in \mathbf{U}_n . Since $\mathbf{U}_{\mathbf{n}+\mathbf{p}_s}$ can be interpreted as being of degree zero in (G'/G) , the leading terms of $\mathbf{U}_{\mathbf{n}+\mathbf{p}_s}$ ($\mathbf{p}_s \neq 0$) will not effect the balance.

Step 5. Finally, substituting the ansatz (4) and the expressions (7a–c) along with (5) into Eq. (3), setting the coefficients of $(G'/G)^l$ ($l = 0, 1, 2, \dots$) to zero, we obtain a system of nonlinear algebraic equations from which the undetermined constants a_l, d_i, c_j, λ and μ can be found explicitly. Then, by substituting these values into the ansatz (4), we derive traveling wave solutions to Eq. (1).

3. Solutions to the Ablowitz–Ladik lattice system

The Ablowitz–Ladik lattice system [8,10,33,34] is known as

$$\begin{cases} \frac{\partial u_n}{\partial t} = (\alpha + u_n v_n)(u_{n+1} + u_{n-1}) - 2\alpha u_n, \\ \frac{\partial v_n}{\partial t} = -(\alpha + u_n v_n)(v_{n+1} + v_{n-1}) + 2\alpha v_n, \end{cases} \tag{8}$$

where α is a constant. Eq. (8) was proposed by using inverse scattering method and has a rich mathematical structure. To look for traveling wave solutions of Eq. (8), we first let

$$u_n = U_n(\xi_n), \quad v_n = V_n(\xi_n), \quad \xi_n = dn + ct + \zeta, \tag{9}$$

where c, d are constants to be determined and ζ denotes the phase shift. Then, Eq. (8) can be reduced to

$$\begin{cases} cU'_n - (\alpha + U_n V_n)(U_{n+1} + U_{n-1}) + 2\alpha U_n = 0, \\ cV'_n + (\alpha + U_n V_n)(V_{n+1} + V_{n-1}) - 2\alpha V_n = 0, \end{cases} \tag{10}$$

where $U_n = U_n(\xi_n), V_n = V_n(\xi_n)$ and the prime denotes derivative with respect to ξ_n . The balancing procedure in (10) leads to $m = 1$. Thus, we assume that the solution of Eq. (10) is in the form

$$\begin{cases} U_n = a_0 + a_1 \left(\frac{G'}{G}\right) + a_{-1} \left(\frac{G'}{G}\right)^{-1}, \\ V_n = b_0 + b_1 \left(\frac{G'}{G}\right) + b_{-1} \left(\frac{G'}{G}\right)^{-1}, \end{cases} \tag{11}$$

where $a_0, a_1, a_{-1}, b_0, b_1,$ and b_{-1} are constants to be specified.

Case 1: When $\lambda^2 - 4\mu > 0$, from (7a), we obtain

$$U_{n\pm 1} = \sum_{l=-1}^1 a_l \left(\frac{\frac{\lambda}{2} + \frac{G'}{G} \pm \frac{\sqrt{\lambda^2 - 4\mu}}{2} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} d\right)}{1 \pm \frac{2}{\sqrt{\lambda^2 - 4\mu}} \left(\frac{\lambda}{2} + \frac{G'}{G}\right) \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} d\right)} - \frac{\lambda}{2} \right)^l, \tag{12a}$$

$$V_{n\pm 1} = \sum_{l=-1}^1 b_l \left(\frac{\frac{\lambda}{2} + \frac{G'}{G} \pm \frac{\sqrt{\lambda^2 - 4\mu}}{2} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} d\right)}{1 \pm \frac{2}{\sqrt{\lambda^2 - 4\mu}} \left(\frac{\lambda}{2} + \frac{G'}{G}\right) \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} d\right)} - \frac{\lambda}{2} \right)^l. \tag{12b}$$

Substituting the ansatz (11) and the expressions (12a–b) along with Eq. (5) into Eq. (10), clearing the denominator and equating the coefficients of $(G'/G)^l (0 \leq l \leq 8)$ to zero, we obtain a system of algebraic equations for $a_0, a_1, a_{-1}, b_0, b_1, b_{-1}, c, d, \lambda,$ and μ . Solving the resulted algebraic system (we will omit to display them for simplicity) simultaneously, we get the following solution sets and the corresponding hyperbolic function solutions to Eq. (8):

Case 1.1.

$$\left\{ c = \mp 2\alpha \sinh^2(d\sqrt{-\mu})/\sqrt{-\mu}, a_0 = \alpha \sinh^2(d\sqrt{-\mu})/b_0, b_{-1} = \pm b_0 \sqrt{-\mu}, \right. \tag{13}$$

$$\left. \begin{aligned} & a_{-1} = \mp \sqrt{-\mu} \alpha \sinh^2(d\sqrt{-\mu})/b_0, a_1 = 0, b_1 = 0, b_0 = b_0, d = d, \lambda = 0, \mu = \mu \right\}, \\ & \left\{ \begin{aligned} u_{n,1}^\mp(t) &= \frac{\alpha \sinh^2(d\sqrt{-\mu})}{b_0} \left(1 \mp \frac{C_1 \sinh(\sqrt{-\mu}\xi_n) + C_2 \cosh(\sqrt{-\mu}\xi_n)}{C_1 \cosh(\sqrt{-\mu}\xi_n) + C_2 \sinh(\sqrt{-\mu}\xi_n)} \right), \\ v_{n,1}^\mp(t) &= b_0 \left(1 \pm \frac{C_1 \sinh(\sqrt{-\mu}\xi_n) + C_2 \cosh(\sqrt{-\mu}\xi_n)}{C_1 \cosh(\sqrt{-\mu}\xi_n) + C_2 \sinh(\sqrt{-\mu}\xi_n)} \right), \end{aligned} \right. \tag{14}$$

where $\xi_n = dn \mp (2\alpha \sinh^2(d\sqrt{-\mu})/\sqrt{-\mu})t + \zeta$ and $\mu < 0, \zeta, b_0, d, C_1, C_2$ are arbitrary constants.

Case 1.2.

$$\left\{ c = \pm \sqrt{-\mu} \alpha \sinh^2(2d\sqrt{-\mu})/\mu, a_0 = \mp \sqrt{-\mu} \alpha \sinh^2(2d\sqrt{-\mu})/2\mu b_1, a_{-1} = -\alpha \sinh^2(2d\sqrt{-\mu})/4b_1, \right. \tag{15}$$

$$\left. \begin{aligned} & b_0 = \pm 2\sqrt{-\mu} b_1, b_{-1} = -\mu b_1, a_1 = \alpha \sinh^2(2d\sqrt{-\mu})/4\mu b_1, b_1 = b_1, d = d, \lambda = 0, \mu = \mu \right\}, \\ & \left\{ \begin{aligned} u_{n,2}^\mp(t) &= \frac{-\alpha \sinh^2(2d\sqrt{-\mu})}{4\sqrt{-\mu} b_1} \left(\frac{C_1 \sinh(\sqrt{-\mu}\xi_n) + C_2 \cosh(\sqrt{-\mu}\xi_n)}{C_1 \cosh(\sqrt{-\mu}\xi_n) + C_2 \sinh(\sqrt{-\mu}\xi_n)} + \frac{C_1 \cosh(\sqrt{-\mu}\xi_n) + C_2 \sinh(\sqrt{-\mu}\xi_n)}{C_1 \sinh(\sqrt{-\mu}\xi_n) + C_2 \cosh(\sqrt{-\mu}\xi_n)} \mp 2 \right), \\ v_{n,2}^\mp(t) &= \sqrt{-\mu} b_1 \left(\frac{C_1 \sinh(\sqrt{-\mu}\xi_n) + C_2 \cosh(\sqrt{-\mu}\xi_n)}{C_1 \cosh(\sqrt{-\mu}\xi_n) + C_2 \sinh(\sqrt{-\mu}\xi_n)} + \frac{C_1 \cosh(\sqrt{-\mu}\xi_n) + C_2 \sinh(\sqrt{-\mu}\xi_n)}{C_1 \sinh(\sqrt{-\mu}\xi_n) + C_2 \cosh(\sqrt{-\mu}\xi_n)} \pm 2 \right), \end{aligned} \right. \tag{16}$$

where $\xi_n = dn \pm (\sqrt{-\mu} \alpha \sinh^2(2d\sqrt{-\mu})/\mu)t + \zeta$ and $\mu < 0, \zeta, b_1, d, C_1, C_2$ are arbitrary constants.

Remark 1. The expressions (14) and (16) represent abundant traveling wave solutions to Eq. (8). If we let “ $C_1 \neq 0$ and $C_2 = 0$ ” or “ $C_1 = 0$ and $C_2 \neq 0$ ” in (14) respectively, then we get formal solitary wave solutions to Eq. (8) as follows

$$\begin{cases} u_{n,3}^\mp(t) = \frac{\alpha \sinh^2(d\sqrt{-\mu})}{b_0} (1 \mp \tanh(\sqrt{-\mu}\xi_n)), \\ v_{n,3}^\mp(t) = b_0 (1 \pm \tanh(\sqrt{-\mu}\xi_n)), \end{cases} \tag{17}$$

$$\begin{cases} u_{n,4}^\mp(t) = \frac{\alpha \sinh^2(d\sqrt{-\mu})}{b_0} (1 \mp \coth(\sqrt{-\mu}\xi_n)), \\ v_{n,4}^\mp(t) = b_0 (1 \pm \coth(\sqrt{-\mu}\xi_n)), \end{cases} \tag{18}$$

where ξ_n is as in (14). Moreover, we observe that our results (17) and (18) include the results of Baldwin et al. [10] and Wang [35] respectively. Thus, the solutions (14) and (16) are wider in the sense that they contain more arbitrary parameters.

Case 2: When $\lambda^2 - 4\mu < 0$, from (7b), we obtain

$$U_{n\pm 1} = \sum_{l=-1}^1 a_l \left(\frac{\frac{\lambda}{2} + \frac{G'}{G} \mp \frac{\sqrt{4\mu-\lambda^2}}{2} \tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}d\right)}{1 \pm \frac{2}{\sqrt{4\mu-\lambda^2}}\left(\frac{\lambda}{2} + \frac{G'}{G}\right) \tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}d\right)} - \frac{\lambda}{2} \right)^l, \tag{19a}$$

$$V_{n\pm 1} = \sum_{l=-1}^1 b_l \left(\frac{\frac{\lambda}{2} + \frac{G'}{G} \mp \frac{\sqrt{4\mu-\lambda^2}}{2} \tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}d\right)}{1 \pm \frac{2}{\sqrt{4\mu-\lambda^2}}\left(\frac{\lambda}{2} + \frac{G'}{G}\right) \tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}d\right)} - \frac{\lambda}{2} \right)^l. \tag{19b}$$

Substituting the ansatz (11) and the expressions (19a–b) along with Eq. (5) into Eq. (10), clearing the denominator and equating the coefficients of $(G'/G)^l (0 \leq l \leq 8)$ to zero, we derive a system of algebraic equations for $a_0, a_1, a_{-1}, b_0, b_1, b_{-1}, c, d, \lambda,$ and μ . Solving the resulted algebraic system simultaneously, we get the following solution sets and the corresponding trigonometric function solutions to Eq. (8):

Case 2.1.

$$\left\{ \begin{aligned} c &= \mp 2i\alpha \sin^2(d\sqrt{\mu})/\sqrt{\mu}, \quad a_0 = -\alpha \sin^2(d\sqrt{\mu})/b_0, \quad b_0 = b_0, \quad a_{-1} = \pm i\sqrt{\mu}\alpha \sin^2(d\sqrt{\mu})/b_0, \\ a_1 &= 0, \quad b_1 = 0, \quad b_{-1} = \pm i\sqrt{\mu}b_0, \quad d = d, \quad \lambda = 0, \quad \mu = \mu, \quad i^2 = -1 \end{aligned} \right\}, \tag{20}$$

$$\left\{ \begin{aligned} u_{n,5}^{\mp}(t) &= \frac{\alpha \sin^2(d\sqrt{\mu})}{b_0} \left(-1 \pm i \frac{C_1 \cos(\sqrt{\mu}\xi_n) + C_2 \sin(\sqrt{\mu}\xi_n)}{C_2 \cos(\sqrt{\mu}\xi_n) - C_1 \sin(\sqrt{\mu}\xi_n)} \right), \\ v_{n,5}^{\mp}(t) &= b_0 \left(1 \pm i \frac{C_1 \cos(\sqrt{\mu}\xi_n) + C_2 \sin(\sqrt{\mu}\xi_n)}{C_2 \cos(\sqrt{\mu}\xi_n) - C_1 \sin(\sqrt{\mu}\xi_n)} \right), \end{aligned} \right. \tag{21}$$

where $\xi_n = dn \mp (2i\alpha \sin^2(d\sqrt{\mu})/\sqrt{\mu})t + \zeta$ and $\mu > 0, \zeta, b_0, d, C_1, C_2$ are arbitrary constants.

Case 2.2.

$$\left\{ \begin{aligned} c &= \pm i\alpha \sin^2(2d\sqrt{\mu})/\sqrt{\mu}, \quad a_0 = \mp i\alpha \sin^2(2d\sqrt{\mu})/2\sqrt{\mu}b_1, \quad a_{-1} = \alpha \sin^2(2d\sqrt{\mu})/4b_1, \quad b_0 = \mp 2i\sqrt{\mu}b_1 \\ d &= d, \quad b_{-1} = -\mu b_1, \quad a_1 = -\alpha \sin^2(2d\sqrt{\mu})/4\mu b_1, \quad b_1 = b_1, \quad \lambda = 0, \quad \mu = \mu, \quad i^2 = -1 \end{aligned} \right\}, \tag{22}$$

$$\left\{ \begin{aligned} u_{n,6}^{\mp}(t) &= -\frac{\alpha \sin^2(2d\sqrt{\mu})}{4\sqrt{\mu}b_1} \left(-\frac{C_1 \cos(\sqrt{\mu}\xi_n) + C_2 \sin(\sqrt{\mu}\xi_n)}{C_2 \cos(\sqrt{\mu}\xi_n) - C_1 \sin(\sqrt{\mu}\xi_n)} + \frac{C_2 \cos(\sqrt{\mu}\xi_n) - C_1 \sin(\sqrt{\mu}\xi_n)}{C_1 \cos(\sqrt{\mu}\xi_n) + C_2 \sin(\sqrt{\mu}\xi_n)} \pm 2i \right), \\ v_{n,6}^{\mp}(t) &= \sqrt{\mu} b_1 \left(-\frac{C_1 \cos(\sqrt{\mu}\xi_n) + C_2 \sin(\sqrt{\mu}\xi_n)}{C_2 \cos(\sqrt{\mu}\xi_n) - C_1 \sin(\sqrt{\mu}\xi_n)} + \frac{C_2 \cos(\sqrt{\mu}\xi_n) - C_1 \sin(\sqrt{\mu}\xi_n)}{C_1 \cos(\sqrt{\mu}\xi_n) + C_2 \sin(\sqrt{\mu}\xi_n)} \mp 2i \right), \end{aligned} \right. \tag{23}$$

where $\xi_n = dn \pm (i\alpha \sin^2(2d\sqrt{\mu})/\sqrt{\mu})t + \zeta$ and $\mu > 0, \zeta, b_1, d, C_1, C_2$ are arbitrary constants.

Remark 2. As a special example, if we let “ $C_1 \neq 0$ and $C_2 = 0$ ” or “ $C_1 = 0$ and $C_2 \neq 0$ ” in the expression (21) respectively, then we get formal trigonometric function solutions to Eq. (8) as follows

$$\left\{ \begin{aligned} u_{n,7}^{\mp}(t) &= \frac{\alpha \sin^2(d\sqrt{\mu})}{b_0} (-1 \mp i \cot(\sqrt{\mu}\xi_n)), \\ v_{n,7}^{\mp}(t) &= b_0 (1 \mp i \cot(\sqrt{\mu}\xi_n)), \end{aligned} \right. \tag{24}$$

$$\left\{ \begin{aligned} u_{n,8}^{\mp}(t) &= \frac{\alpha \sin^2(d\sqrt{\mu})}{b_0} (-1 \pm i \tan(\sqrt{\mu}\xi_n)), \\ v_{n,8}^{\mp}(t) &= b_0 (1 \pm i \tan(\sqrt{\mu}\xi_n)), \end{aligned} \right. \tag{25}$$

where ξ_n is as in (21). In addition, we note that our results (24) and (25) are not presented in [10,35].

Case 3: When $\lambda^2 - 4\mu = 0$, from (7c), we obtain

$$U_{n\pm 1} = \sum_{l=-1}^1 a_l \left(-\frac{\lambda}{2} + \left(\frac{G'}{G} + \frac{\lambda}{2} \right) / \left(1 \pm \left(\frac{G'}{G} + \frac{\lambda}{2} \right) d \right) \right)^l, \tag{26a}$$

$$V_{n\pm 1} = \sum_{l=-1}^1 b_l \left(-\frac{\lambda}{2} + \left(\frac{G'}{G} + \frac{\lambda}{2} \right) / \left(1 \pm \left(\frac{G'}{G} + \frac{\lambda}{2} \right) d \right) \right)^l. \tag{26b}$$

Substituting the ansatz (11) and the expressions (26a–b) along with Eq. (5) into Eq. (10), clearing the denominator and equating the coefficients of $(G'/G)^l (0 \leq l \leq 8)$ to zero, we derive a system of algebraic equations for $a_0, a_1, a_{-1}, b_0, b_1, b_{-1}, c, d, \lambda,$ and μ . Solving the resulted algebraic system simultaneously, we get the following solution set and the corresponding rational solution to Eq. (8):

$$\{c = 0, a_1 = -d^2 \alpha / b_1, a_{-1} = 0, a_0 = 0, b_{-1} = 0, b_0 = 0, b_1 = b_1, d = d, \lambda = 0, \mu = 0\}, \quad (27)$$

$$\begin{cases} u_{n,9}(t) = \frac{A_1}{n+A_2}, \\ v_{n,9}(t) = \frac{A_3}{n+A_2}, \end{cases} \quad (28)$$

where A_1, A_2 , and A_3 are arbitrary constants.

Remark 3. The expression (28) is a non-constant steady-state (time independent) solution to Eq. (8) and not derived in [10,35].

4. Conclusion

By means of the extended (G'/G)-expansion method, we obtained various kinds of exact discrete solutions to the Ablowitz–Ladik lattice system. Our approach provides traveling wave solutions from which one can easily construct solitary and periodic waves by setting special values to the parameters. We checked the correctness of the solutions by putting them back into the original equation with the aid of MATHEMATICA, this provides an extra measure of confidence in the results. We predict that the extended (G'/G)-expansion method will be a promising method for exactly solving NDDs.

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