Three-Dimensional Electromagnetic Scattering from Flat Plates by Using Sinc-Type Basis Functions in Method of Moments

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Abstract— **Sinc functions are used in the basis and testing procedures in the conventional method of moments (MoM) formulation. But unlike conventional MoM, simple pointwise meshing is enough and no integral computation is required due to the mathematical properties of the sinc function. The simulation results of surface current densities and radar cross section for a few wavelength square flat plate are obtained by using our own codes. The results are compared with those of the rooftop basis functions. The CPU-time decreases by half of the time of the conventional method. The error that occurs from the approximated integral of the sinc function in the computation of main matrix elements is very small and decreases while the bandwidth of the sinc function increases.**

I. Introduction

Method of moments (MoM) is used for the numerical solution of the electromagnetic scattering problems by using different types of basis and testing functions [1]. In this method, the induced current on the surface of the scatterer is first found from the numerical solution of integral equation (IE) and these IEs are obtained by imposing the boundary conditions. Once the current density is solved, the scattered electromagnetic field can be found by using radiation integrals. Large antenna problems can be solved by using entire domain basis functions like studied in [2]. Electromagnetic scattering by arbitrarily shaped geometries are investigated by evaluating electric-field integral equation (EFIE) with MoM in the study [3]. Parallel computations have been employed to determine the electromagnetic scattering from a conducting electrically large rectangular plate in the study [4].

Sinc functions are used in the numerical solution of ordinary differential equations (ODE's), partial differential equations (PDE's) and IEs for solving the initial and boundary value problems [5]. They are defined in the category of quasi-localized bandlimited basis functions as given in the literature [6] and easily implemented in the problems having singularities and a good accuracy is obtained by their usage. Therefore, sinc functions are effectively used in engineering and applied physics. The investigation of sinc functions to reduce the computation of Hallen's IE to some linear algebraic equations is presented in [7] (i.e., sinc collocation). Furthermore, the approximate surface current densities and the scattered fields are obtained for finite strip grating array by using sinc based MoM for a two-dimensional electromagnetic scattering problem in [8] and [9].

In this study, three-dimesional scattering problems are investigated by using sinc-type basis and testing functions in the application of MoM. The approximate analytical properties of the sinc function are taken an important role in the formulation. The important analytical property of the sinc function such as the convolution property is also used in the formulation.The approximated integral of the sinc functions is used in deriving MoM matrix elements. The error that occurs from the approximated integral of sinc function is calculated numerically by using Simpson method [10]. Additionally, the relative error is calculated with the ratio of the error to the exact value of the integral that is the integral of Green function with the sinc function. The number of integrals which is computed is decreased with the analytical properties of the sinc function so it computes faster current density for vertical and horizontal polarizations and radar cross section (RCS) for co-polarized and cross-polarized cases of a few wavelengths square flat plate. The current density for vertical and horizontal polarizations and RCS for co-polarized and cross-polarized cases are obtained for 2λ, 4λ and 6λ square flat plates. The results obtained from the proposed method are compared with the ones obtained by the application of the well-known and widely used rooftop basis functions. Our own conventional MoM codes use:

- Direct, LU decomposition by inverting full matrix
- No fast multipole in the filling of the matrix elements
- No parallelization, single computer is used.

The simulation results are compared with the solutions in the studies [11] and [12] and it considers that they are very similar each other. Additionally, it was observed that the computation time in the new method is decreased by 50% when compared with that of the conventional rooftop basis dependent codes written in the same computer environment. The proposed method can be easily applied for arbitrary planar surfaces. The CPU time can also be decreased by using preconditioning techniques by taking the inverse of the full matrix in MoM [13]. A reasonable initial condition and a suitable precondioning matrix will be used in this algorithm

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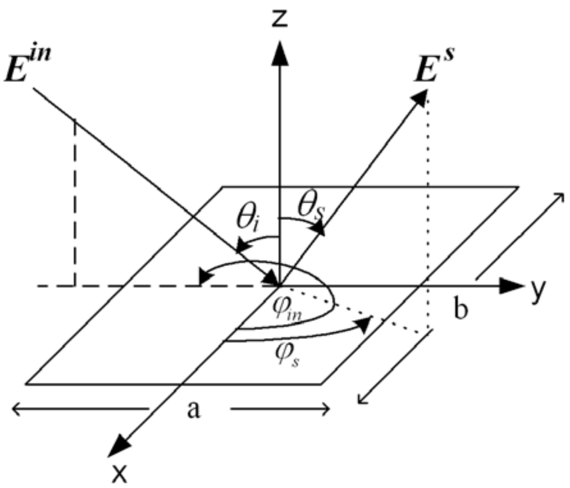


Fig. 1. Geometry of the flat plate.

in order to converge the solution as fast as possible.

II. Formulation

The geometry of the problem, the flat plate illuminated by a plane wave, is located in the $z = 0$ plane and shown in Fig.1. The dimension of the plate is $a \times b$. The radiated field can be evaluated by using electromagnetic potential equation to solve the EFIE. The components of the electric field can be determined from the formula

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\Phi. \quad (1)$$

Equation (1) can be expanded by using convolution of the current density and the green function and by using the continuity equation. Then, the final form is given as follows

$$E_x = -j\omega\mu[J_x * G] - \frac{1}{\varepsilon}\frac{\partial}{\partial x}[\rho * G] \quad (2)$$

$$E_y = -j\omega\mu[J_y * G] - \frac{1}{\varepsilon}\frac{\partial}{\partial y}[\rho * G], \quad (3)$$

where the Green function is $G(x - x', y - y') = e^{-jkR}/4\pi R, R(x-x', y-y') = \sqrt{(x - x')^2 + (y - y')^2}$, and the charge density can be solved from the continuity equation $\rho = -\nabla \cdot \mathbf{J}/j\omega$. The wavelength λ is taken as 0.015 m. The approximate of the current densities J_x and J_y are the superposition of the basis functions and the expansion of these basis functions gives

$$J_x = \sum_{s=1}^{Q} a_s \underbrace{\mathrm{sinc}(2W_x x - p(s))\mathrm{sinc}(2W_y y - q(s))}_{B_{xn(s)}(x,y)} \quad (4)$$

$$J_y = \sum_{s=1}^{Q} b_s \underbrace{\mathrm{sinc}(2W_y y - q(s))\mathrm{sinc}(2W_x x - p(s))}_{B_{yn(s)}(x,y)}, \quad (5)$$

where $Q = (N \times N - 2 \times N) \times 2$, $N = 15$ (per λ), $2W_x$ and $2W_y$ are the bandwidths of the sinc functions and $p(s)/2W_x$ and $q(s)/2W_y$ are the positions of the sinc functions.

By testing both sides of the EFIE's with T_{xm} and T_{ym} that are same with basis functions, the following matrix equations can be obtained

$$\begin{aligned}\langle E_x, T_{xm(o)}\rangle = & -j\omega\mu \sum_{s=1}^{N} a_s \langle B_{xn(s)}(x', y') * G, T_{xm(o)}\rangle \\ & + \frac{1}{j\omega\varepsilon}\sum_{s=1}^{N} a_s \langle \frac{\partial}{\partial x}(\frac{\partial B_{xn(s)}(x', y')}{\partial x'} * G), T_{xm(o)}\rangle \\ & + \frac{1}{j\omega\varepsilon}\sum_{s=1}^{N} b_s \langle \frac{\partial}{\partial x}(\frac{\partial B_{yn(s)}(x', y')}{\partial y'} * G), T_{xm(o)}\rangle, \\ & o = 1, 2, ..., Q. \quad (6)\end{aligned}$$

$$\begin{aligned}\langle E_y, T_{ym(o)}\rangle = & -j\omega\mu \sum_{s=1}^{N} b_s \langle B_{yn(s)}(x', y') * G, T_{ym(o)}\rangle \\ & + \frac{1}{j\omega\varepsilon}\sum_{s=1}^{N} a_s \langle \frac{\partial}{\partial y}(\frac{\partial B_{xn(s)}(x', y')}{\partial x'} * G), T_{ym(o)}\rangle \\ & + \frac{1}{j\omega\varepsilon}\sum_{s=1}^{N} b_s \langle \frac{\partial}{\partial y}(\frac{\partial B_{yn(s)}(x', y')}{\partial y'} * G), T_{ym(o)}\rangle, \\ & o = 1, 2, ..., Q. \quad (7)\end{aligned}$$

The parameters s and o in (6) and (7), which defines source and observation respectively, scan all the input coordinates of the basis functions location on the scatterer surface in an arbitrary order. Therefore, this will bring us to simulate arbitrary plate geometries. The first term of (6) can be found by approximating the convolution integral

$$B_{xn(s)}(x', y') * G(u, v) = \underbrace{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathrm{sinc}(2W_x x' - p(s))\mathrm{sinc}(2W_y y' - q(s))G(u, v)\, dx'dy'}_{H(x,y,p(s),q(s))} \quad (8)$$

where $u = x - x'$, $v = y - y'$ and $H(x, y, p(s), q(s))$ can be expanded in a double series with the coefficients h_s^o and the coefficients can be determined by taking the inverse of H function using complete orthogonal set of sinc functions. This is performed by integrating the both sides of (8) with using sinc functions. Then one can use that convolution property of sinc functions and it again gives a sinc function approximately. The approximate solution of the coefficients generally can be written as in (9).

$$h_s^o = \begin{cases} G(|L|t_x, |M|t_y), & \text{otherwise} \\ \iint_{-\infty}^{+\infty} G(u, v)\mathrm{sinc}(2W_u u - r'(o) + p(s)), & |L| < 2 \\ \mathrm{sinc}(2W_v v - t'(o) + q(s))\, du\, dv & |M| < 2 \end{cases} \quad (9)$$

where $L = r'(o) - p(s)$, $M = t'(o) - q(s)$ and $t_x = t_y = 1/2W$.

TABLE I
RELATIVE ERROR IN THE COMPUTED MATRIX ELEMENTS

W $(\lambda = 0.015m)$ $t_x = t_y = 1/2W$	L=2 M=2	L=2 M=3	L=3 M=2	L=3 M=3
250	0.00096	0.00068	0.00068	0.00055
500	0.00107	0.00051	0.00051	0.00043
750	0.00120	0.00052	0.00052	0.00055
1000	0.00113	0.00122	0.00122	0.00096

III. ERROR ANALYSIS

The approximation of the integral of the sinc functions with Green function is used by calculating the matrix elements in MoM. The approximated integral is expressed as

$$\underbrace{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(x,y)B_{xn(s)}\,dx\,dy}_{I_1} = \underbrace{\frac{1}{4W_xW_y}G(|L|t_x,|M|t_y)}_{I_2} + \text{error}. \quad (10)$$

The error is calculated with the difference between the terms I_1 and I_2. The relative error is the ratio of this difference to the exact value of the integral I_1.

Numerical integration technique studied in [10], Simpson rule with $n = 5000$ subdivisions is used to solve the left hand side integral equation I_1 over the interval $[-0.5, 0.5]$. The subdivision of the numerical integration n, is increased at constant interval $[-0.5, 0.5]$ and the amount of the error value changes 9×10^{-9} while the relative error changes in the amount of 10^{-3}. If you take the integral in the broader interval you must increase the subdivision n, proportionally. While W is increasing from 250 to 1000 more subdvisions must be taken in numerical calculations. Numerical solutions for different L and M values are given in Table I. The bandwidth of the sinc function is taken $W_x = W_y = W$ and changes between 250 and 1000. The parameters t_x and t_y are equal to $1/2W$ each other. The relative error in the computational process of the main matrix elements for $L > 1$ and $M > 1$ are given in Table I. The relative error decreases for W is between 250 and 1000, while L and M increases. In the simulations $W = 500$ is used so it is shown from the table that while L and M increases less relative error is obtained in the computational results.

IV. NUMERICAL RESULTS

The geometry of the flat plate illuminated by a plane wave is located in the $z = 0$ plane and shown in Fig.1. The elevation angle of the incident electric field is θ^{inc} and is equal to $45°$ and the incident azimuth angle is φ^{inc} and is equal to $0°$. In this work, the sinc functions are used alike subdomain basis functions and 15 basis functions are taken for each wavelength of the flat plate. Simulations are carried out on a Genuine Intel(R) 2160 1.8 GHz processor with 2.49 GB of RAM under Matlab 6.5 and Fortran90 program packages on the windows operating systems. The simulation results of the approximate current densities in the x directions for vertical and horizontal polarizations obtained with sinc-type basis and testing functions are given in Fig. 2 for $P1 = 2\lambda \times 2\lambda$ and $P2 = 4\lambda \times 4\lambda$ square flat plates respectively. The horizontal polarization component of the x directed current density is smaller than vertical one. The change in current density is increasing while the dimension of the plate increases. The bistatic case of the radar cross section is observed in Fig. 3. The scattered field is increasing while the dimension of the plate increases. The observation angle, ϕ^{obs}, is $90°$ cross polarized (vertical-horizontal) for the $P1$ flat plate and $0°$ co polarized (vertical-vertical) for the $P2$ flat plate. The incident elevation and azimuth angles, $\theta^{inc}$and ϕ^{inc}, are same with given above values. The simulation results obtained with sinc type basis functions are compared with those of the rooftop basis functions. They are nearly same each other. The

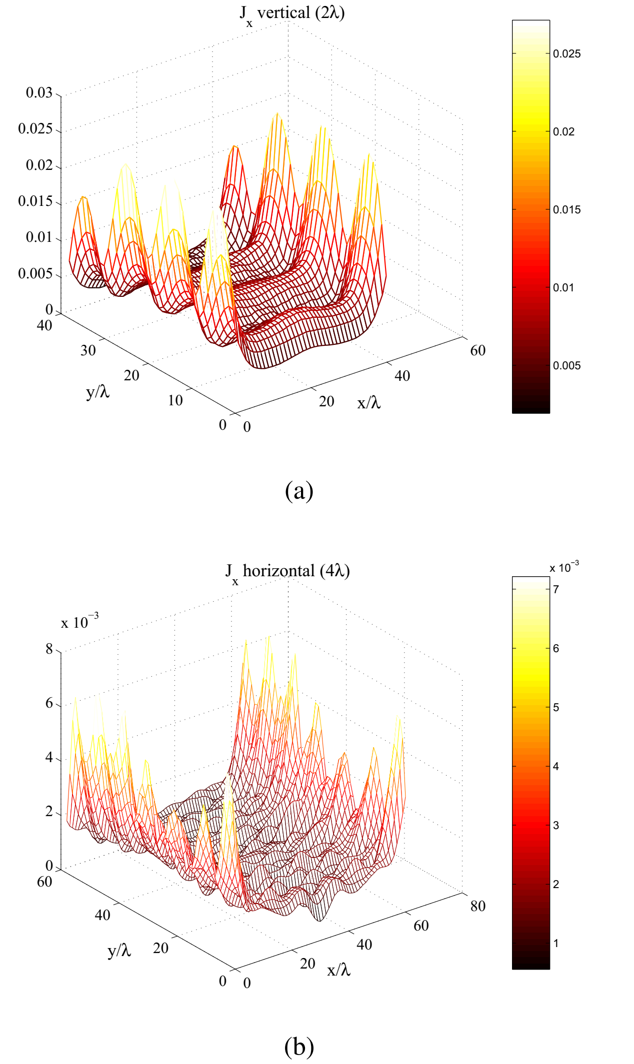
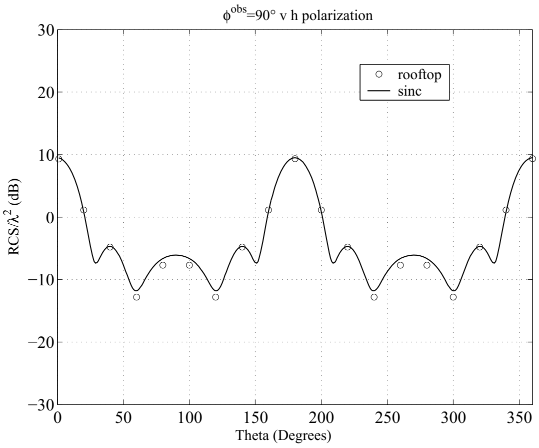
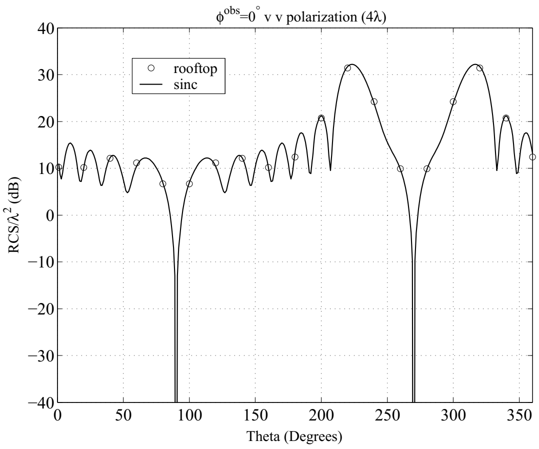


Fig. 2. Current densities on the square flat plate for $\theta^{inc} = 45°$, $\phi^{inc} = 0°$, (a) 2λ and (b) 4λ.



(a)



(b)

Fig. 3. RCS/λ^2 of square flat plate for $\theta^{inc} = 45°$, $\phi^{inc} = 0°$, (a) 2λ and (b) 4λ.

maximum difference between these results is less than 1%. The incident angles of the plane wave are changed to 45° and the bistatic RCS solution of the $P3 = 6\lambda \times 6\lambda$ is observed for φ^{obs} is equal to 90°. The obtained RCS results with sinc type basis functions are compared with those of the rooftop basis functions and the MoM solutions given in [11]. The results are similar with rooftop basis functions and the study [11]. The CPU time of the code that is developed with sinc type basis functions are 50% shorter than the compared one. It can be developed by using preconditioning technique in our own code.

V. Conclusions

The approximate current densities and radar cross section parameters for 2λ, 4λ, and 6λ square flat plates are investigated by using sinc type basis functions in MoM technique in this study. The simulation results based on sinc type basis functions are compared with those of the rooftop basis functions. The computation time to calculate approximate current densities and RCS decreases 50% by using sinc type basis functions other than rooftop basis functions. Additionally, the

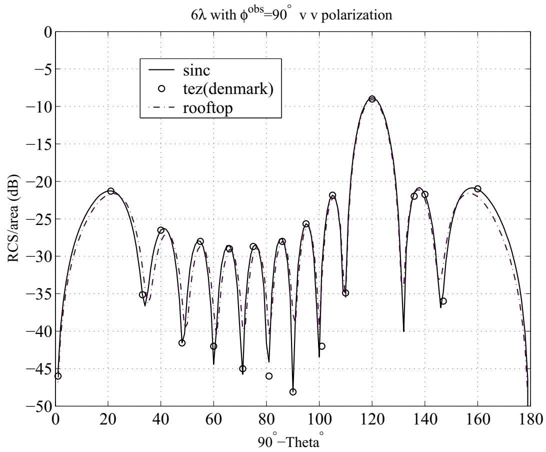


Fig. 4. RCS/area (dB) of a 6λ square flat plate for $\theta^{inc} = 45°$, $\phi^{inc} = 45°$ and $\phi^{obs} = 90°$.

computation time can be decreased by using preconditioning techniques and FMM in our own proposed conventional MoM code in the future. Larger geometries can be simulated with high computer facilities. The Fortran code can be developed for planar arbitrary shapes.

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