HIGGS BOSONS OF GAUGE-EXTENDED SUPERSYMMETRY AT THE LHC

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Hale SERT

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We approve the thesis of **Hale SERT**

Prof. Dr. Durmuş Ali DEMİR
Supervisor

Prof. Dr. Oktay PASHAEV
Committee Member

Assoc. Prof. Dr. Kerem CANKOÇAK
Committee Member

24 December 2010

Prof. Dr. Nejat BULUT
Head of the Department of
Physics

Prof. Dr. Durmuş Ali DEMİR
Dean of the Graduate School of
Engineering and Sciences

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ABSTRACT

HIGGS BOSONS OF GAUGE-EXTENDED SUPERSYMMETRY AT THE LHC

This thesis work is devoted to a detailed phenomenological analysis of the Higgs sector of gauge-extended supersymmetry in light of the most recent experimental bounds. Such extra gauge symmetries, obtained by adding an extra Abelian symmetry $U(1)'$ to the gauge structure of the Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM) which have the same gauge structure, are urged by the μ problem of the MSSM, and they also arise frequently in low-energy supersymmetric models stemming from GUTs and strings.

We analyze the Higgs boson masses and their dependencies on various model parameters. In particular, we compute masses of all the Higgs bosons, and confront the mass of the lightest one with the LEP and Tevatron experiments. Then we indicate the restrictions from LEP and Tevatron bounds on the masses and remaining model parameters. We analyze correlations among various model parameters, and determine excluded regions by both scanning the parameter space and examining certain likely parameter values. Furthermore, we make educated projections for LHC measurements in light of the LEP and Tevatron restrictions on the parameter space.

As a result of this thesis work we find that μ-problem motivated generic low-energy $U(1)'$ model yields lightest Higgs masses as large as $\sim 200\ \mathrm{GeV}$, and violates the Tevatron bounds for certain ranges of parameters. However, we find that $U(1)'$ model stemming from E(6) breaking elevate Higgs boson mass into Tevatron's forbidden band when $U(1)'$ gauge coupling takes larger values than the one corresponding to one-step GUT breaking. We also obtain that the Tevatron bounds put strong restrictions on certain parameters of the $U(1)'$ model and they lead to determinations of certain parameter ranges before the LHC measurements.

ÖZET

AYAR GENİŞLETMELİ SÜPERSİMETRİK MODELLERİN LHC'DEKİ HİGGS SİNYALLERİ

Bu tez çalışması ayar genişletmeli süpersimetrik modellerin Higgs sektörünün detaylı fenomenolojik analizine dayanır. Standart modelin ve aynı ayar yapısına sahip Minimal Süpersimetrik Standart Modelin (MSSM) ayar yapısına extra abelyen $U(1)'$ simetrisi ekleyerek elde edilen bu extra ayar simetrileri MSSM'in μ problemini çözmek için ileri sürülmüştür, ve büyük birleşim teorisi ile sicim teorilerinden kaynaklanan düşük enerjili süpersimetrik modellerde de ortaya çıkarlar.

Bu çalışmada Higgs bozon kütleleri ve bu kütlelerin çeşitli model parametrelerine bağlılığını analiz ettik. Özellikle tüm Higgs bozonlarının kütlelerini hesapladık, ve en hafif Higgs kütlesini LEP ve Tevatron deneyleri ile karşılaştırdık. Sonra LEP ve Tevatron sınırlarından gelen, Higgs kütleleri ve geri kalan model parametreleri üzerindeki sınırlandırmaları gösterdik. Çeşitli model parametreleri arasındaki ilişkileri analiz ettik ve parametre uzayını tarayarak ve muhtemel parametre değerlerini inceleyerek dışlanan bölgeleri belirledik. Daha sonra parametre uzayı üzerindeki LEP ve Tevatron sınırlandırmaları ışığında LHC ölçümleri için tahminde bulunduk.

Bu tez çalışmasının sonucu olarak μ-probleminin çözümünden kaynaklanan düşük enerjili $U(1)'$ modelinin en hafif Higgs kütlesinin ~ 200 GeV kadar büyük değerler alabilmesine olanak sağladığını ve belli parametre bölgesi için Tevatron sınırını ihlal ettiğini bulduk. Bununla birlikte E(6) grubunun kırılmasından kaynaklanan $U(1)'$ modelinin, Higgs bozonu kütlesini Tevatronun yasak bandına, ancak $U(1)'$ ayar birleşme katsayısının büyük birleşim teorisinin tek adımda kırılmasına karşılık gelen ayar birleşim katsayısından büyük değerler aldığında yükseldiğini bulduk. Aynı zamanda Tevatron sınırının $U(1)'$ modelinin belli parametreleri üzerine güçlü sınırlandırmalar koyduğunu belirledik ve bu da bizi LHC ölçümleri öncesinde belli parametre aralıklarını belirlemeye yönlerdirdi.

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CHAPTER 1

INTRODUCTION

The curiosity is the most important and efficient factor which lead people to search. Before the 1960, two questions “What is the world made of?” and “What holds it together?” make scientists wonder about the answer for these questions. As a result of this curiosity the Standard Model (SM) is proposed by Weinberg, Salam and Glashow (Glashow, 1961; Weinberg, 1967). The Standard Model describes the fundamental particles in universe and how they interact with each other. According to SM, elementary particles which constitute the matter are called fermions. Fermions are particles with spin fractional namely in this case 1/2 and obey to the Fermi-Dirac statistics. These elementary particles include six leptons (electron(e), muon(μ), tau(τ) and their neutrinos (ν_e, ν_μ, ν_τ)) and six quarks (up(u), down(d), charm(c), strange(s), top(t) and bottom(b)). The combination of these quarks form the known baryons (consisting of three quarks) and mesons (consisting of one quark and one anti-quark) such as; proton($p \approx uud$), neutron($n \approx udd$), pion($\pi^+ \approx u\bar{d}$). At the same time the Standard Model describes three fundamental interactions which are strong, weak and electromagnetic interactions. There is also one more fundamental interaction called gravitation which is not determined by the SM. Each interaction has a mediator carrying the physical forces. These particles with spin 1 are bosons which obey the Bose-Einstein statistics. The mediators of the strong, weak and electromagnetic interactions are, respectively, gluon, $W^\pm$, Z^0 bosons and photon. To each of the forces corresponds the gauge symmetry group and the theory exhibits an exact invariance under the combination of these symmetries. Therefore it is stated that the Standard Model is a gauge theory.

The theory is based on $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry group with subscripts C, L, Y that refer to color, left chirality and weak hypercharge. These subscripts represent the characteristic properties of the groups; for instance, weak hypercharge (Y) is the charge of the particles under $U(1)_Y$ symmetry group. Components of this gauge group stand for the strong, weak and electromagnetic interactions, respectively. Each gauge group has generators and for each generator a “vector field” arises. These vector fields come from the necessities of the Lagrangian to be invariant under the local group transformations called gauge invariance. These vector fields are also called as “gauge fields” which can be thought of as the carriers of physical forces. The gauge fields of the each group are given in Chapter 2.

The gauge invariance forbids the mass terms of the fermions and gauge bosons in the SM Lagrangian. Hence it seems that these should be massless, however from the experimental results it is known that these particles have mass . To give mass to the particles it is considered that the vacuum is filled by a Higgs field which has a nonzero value at the vacuum state (at the minimum of the Higgs potential energy). When the Higgs field acquires its vacuum expectation value (VEV), electroweak symmetry is spontaneously broken. By breaking of the symmetry when one massive Higgs boson arises, three massless Goldstone bosons occur. These Goldstone bosons are eaten by the massless gauge fields and give to gauge bosons their masses. The fermions also interact with the Higgs field and when the Higgs field acquire its VEV, the fermions get their masses. The mechanism that cause the spontaneous symmetry breaking (SSB) and give mass to particles is called as the “Higgs Mechanism”, and detailed explanation about the SSB and Higgs mechanism is given in Appendix A.

The Standard Model explains lots of experimental facts, however it has some problems, such as “the hierarchy problem” which is explained in Chapter 2. To overcome this problem we need extension of the SM. The supersymmetry is one of the theories proposed to solve this problem by J. Wess and B. Zumino in 1974 and by some other scientists independently. The Minimal Supersymmetric Standard Model (MSSM) which is the first realistic supersymmetric version of the SM was proposed in 1981 by Howard Georgi and Savas Dimopoulos, is the minimal supersymmetric extension of the SM since these are the same gauge structure as seen in Chapter 3 with detailed explanation. Besides this, we give a discussion about the MSSM in regard to its symmetries, gauge structure as well as particle and super-partner spectrum in Chapter 3. For example, the Higgs sector of the MSSM can be summarized as following. There are two Higgs doublets in the MSSM contrary to the SM. While in the SM there is only one Higgs boson, in the MSSM five Higgs bosons arise: two of them are neutral and CP even scalar Higgs bosons (h and H), two of them are charged Higgs bosons ($H^{\pm}$) and the rest is the neutral and CP odd pseudoscalar Higgs boson (A).

Although the MSSM solves some problems of the SM, in Chapter 4 we make an observation that the superpotential of the MSSM contains a dimensionful parameter - the μ parameter - which can be of arbitrary scale, while the natural coefficient should be dimensionless and at the electroweak scale. This problem can be solved naturally if one considers an extra $U(1)'$ which is spontaneously broken at the soft-breaking scale. Such extra $U(1)'$ symmetries are also predicted in stringy scenarios and supersymmetric

GUTs. We discuss generic features of $U(1)'$ models, and explain their differences from the MSSM. For instance, in the $U(1)'$ model with one extra singlet Higgs scalar there is one extra Higgs boson that is neutral, CP even scalar (H'). When we consider the gauge boson sector, we see that there is also one extra neutral gauge boson (Z') in the $U(1)'$ model.

Having set up the $U(1)'$ model, in Chapter 5 we go on to study its Higgs sector. We compute quantum corrections to its Higgs potential at one loop level by including quantum fluctuations of top quark, bottom quark, scalar top quark, and scalar bottom quark. We adopt effective potential approximation with a renormalization scale around the top quark mass.

After design this setup, in Chapter 6 we present details of our works. In our work we analyze the Higgs boson masses and their parametric dependencies on various model parameters in order to determine the allowed regions under the LEP and Tevatron bounds for certain selected $U(1)'$ models and to make projections for LHC measurements in light of these restrictions. In our work we compute, especially, masses of all the Higgs bosons, and compare the mass of the lightest one with the LEP and Tevatron experiments which, respectively, state that a light scalar with standard couplings to quarks and leptons cannot weigh below ~ 114 GeV, and in between the 159 GeV and 167 GeV. We analyze correlations among various model parameters, and determine excluded regions by both scanning the parameter space and by examining certain likely parameter values.

In the last Chapter we conclude this thesis work and its findings by stating that the Tevatron and LEP bounds guide to expectations at the LHC for the $U(1)'$ model.

CHAPTER 2

STANDARD MODEL IN BRIEF

2.1. The Structure of the Model

The Standard Model (SM) is a gauge theory that describes the fundamental particles and their interactions and it is based on the following gauge group structure:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{2.1}$$

where to each group corresponds a fundamental interaction; strong, weak and electromagnetic, respectively. The gauge fields of each group which arise from making the Lagrangian invariant under the local gauge transformation are considered as the carriers of the corresponding interaction.

The number of the gauge fields is equal to the number of generators of each group. There are "$n^2 - 1$" generators for a non-Abelian group $SU(n)$ while "$n^2 = 1$" for an Abelian group $U(1)$. Abelian groups have commuting generators with each other while generators of non-Abelian groups anticommute. In the Standard Model there are 12 gauge fields: 1 gauge field for an abelian $U(1)$ group, 3 and 8 gauge fields for nonabelian $SU(2)$ and $SU(3)$ groups, respectively. These gauge fields and their properties are given in Table 2.1.

As mentioned in Chapter 1, the SM has 12 fermions considered as fundamental

Table 2.1. Properties of the Gauge Groups

Gauge Groups	Gauge Fields	Properties	Number of Generators
$SU(3)_C$	G^a_μ, $a = 1, 2..., 8$	Color	$n^2 - 1 = 8$
$SU(2)_L$	W^i_μ, $i = 1, 2, 3$	Isospin	$n^2 - 1 = 3$
$U(1)_Y$	B_μ	Hypercharge	$n^2 = 1$

particles. These fermions can be written in a 3-fold family structure (Pich, 2005),

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix}, \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix}, \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix} \tag{2.2}$$

where each family has the same properties except for their mass and their flavor quantum number. Here d', s' and b' stand for the weak eigenstates while d, s and b stand for the mass eigenstates. We prefer the representation with prime for these quarks since there is a mixing between the mass eigenstates. The relation between these two eigenstates is given by

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{2.3}$$

where the 3×3 unitary matrix is called Cabibbo- Kobayashi- Maskawa matrix V that expresses the quark mixing.

Representation in (2.2) gives us a general information about the particle content of the SM. To obtain more information about the particles we should examine some properties of the fermions such as their property to be a Dirac spinor. Dirac spinors are written as right-handed and left-handed by means of their helicities. Right and left handed particles mean that directions of their spin and the motion are the same and opposite, respectively.

According to gauge structure of the SM the left handed fermions should be represented as a doublet since they are invariant under $SU(2)_L$ and $U(1)_Y$ symmetries. However, right handed fermions should be represented as singlet since they are only invariant under $U(1)_Y$ symmetry.

$$L_\ell = \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L \quad \text{and} \quad Q_q = \begin{pmatrix} q_u \\ q_{d'} \end{pmatrix}_L \quad \text{and} \quad \ell_R^-,\ q_{uR},\ q_{dR} \tag{2.4}$$

where there is no ν_R particle in the SM because neutrino is considered as massless. Here left handed and right handed fermions transform differently since they are represented differently. Here it should be noted that the mixing between quark mass states exist only in the left handed representation since mass state mixing arises to be invariant under

$SU(2)_L$ symmetry.

For the mass terms of the fermions, mixing of the left-right handed fermions is necessary, $mf\bar{f} = m(\bar{f}_R f_L + \bar{f}_L f_R)$, however, it is not possible since it violates gauge invariance of the Lagrangian. Therefore mass terms of fermions are forbidden in the Lagrangian. Nevertheless, experiments show that the fermions are massive. To obtain mass terms of the fermions to be included in the Lagrangian we introduce a new complex, scalar doublet Higgs field, $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$. The Higgs field has a vacuum expectation value (VEV) which is constant throughout all space. When the Higgs field acquires its VEV due to its desire to be at the minimum potential energy, the electroweak symmetry is spontaneously broken. Then, massless fermions get their masses by interacting with the Higgs field, such as $h_e(\bar{L}_e H e_R + h.c.)$ where h_e is an arbitrary coupling of interaction. Since L_e and H are doublets, $\bar{L}_e H$ becomes a singlet and then we can multiply this by the right handed singlet. When the VEV of the Higgs field are put into $h_e(\bar{L}_e H e_R + h.c.)$ we obtain the mass term of the fermions which are gauge invariance like $(h_e v/\sqrt{2})(\bar{f}_R f_L + \bar{f}_L f_R)$. This mechanism is called the Higgs mechanism and detailed explanation of it is given in Appendix.

All symmetry groups have a charge under the related symmetry group, such as electric charge of a particle under electromagnetic $U(1)_{EM}$ symmetry group. The charge of the particles under the $U(1)_Y$ symmetry group called hypercharge is determined by using the Gell-Mann-Nishijima relation (Novaes, 1999):

$$Q = T_3 + \frac{1}{2}Y \tag{2.5}$$

where Q, T_3 and Y represent electromagnetic charge, third component of the isospin and hypercharge of a particle. So the hypercharges of the fermions and Higgs field are $Y_{L_\ell} = -1$, $Y_{Q_q} = \frac{1}{3}$, $Y_{\ell_R} = -2$, $Y_{q_{uR}} = \frac{4}{3}$, $Y_{q_{dR}} = -\frac{2}{3}$ and $Y_H = 1$.

To explain it more clear it is better to go on by giving an example. The Dirac Lagrangian density for a free fermion is given as

$$\mathcal{L} = i\overline{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - m\overline{\psi}(x)\psi(x). \tag{2.6}$$

Since the first term of the Lagrangian above is not invariant under the local gauge transformations, the covariant derivative replaces the partial derivative to make the Lagrangian invariant. For simplicity the local gauge transformation under $U(1)$ gauge symmetry group can be considered. For this symmetry transformation, covariant derivative is defined as follows

$$D_\mu = \partial_\mu + iqA_\mu \tag{2.7}$$

where A_μ is a vector field, introduced to construct a covariant derivative and, q is the fermion's electric charge, which is generator of the $U(1)$ symmetry group. The transformation rules for the fermion and gauge fields are given by

$$\psi' = e^{i\theta(x)}\psi \tag{2.8}$$

$$A'_\mu = A_\mu + \frac{1}{q}\partial_\mu\theta(x). \tag{2.9}$$

Now, the Lagrangian in (2.6) becomes

$$\mathcal{L} = i\overline{\psi}(x)\gamma^\mu D_\mu\psi(x) - m\overline{\psi}(x)\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{2.10}$$

where $F_{\mu\nu}$ is the field strength tensor of the $U(1)$ symmetry group. This term represents the kinetic energy term of the gauge field A_μ and it is written as follows in terms of the gauge field:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.11}$$

Terms of the Lagrangian in (2.10) are invariant under the local gauge transformation. However, the mass term ($\frac{1}{2}M_A^2 A_\mu A^\mu$) of the gauge field is forbidden in the Lagrangian since this term is not invariant. If one generalizes this situation, it can be stated that the gauge fields should be massless, however it is known that the gauge bosons of the weak interaction $W^\pm$ and Z are massive, while photon which is the gauge boson of the electromagnetic interaction remains massless (Table 2.2). To get rid of this contradiction, a Higgs field that is also necessary for the fermions masses can be introduced. According

Table 2.2. Gauge Bosons correspond to Gauge Groups

Gauge Groups	Gauge Bosons	Massive/Massless
$SU(3)_C$	g^a $a = 1, 2, ..8$	Massless
$SU(2)_L$	$W^{\pm}$, Z^0	Massive
$U(1)_Y$	A_μ	Massless

to Higgs mechanism, massless gauge fields interact with this Higgs field and as a result of this interaction the gauge bosons and one Higgs boson (h) which have physical mass states arise. While some of the gauge bosons acquire mass, some of them remain massless. After the Higgs field is defined, the Lagrangian in (2.10) becomes as follows:

$$\mathcal{L} = i\overline{\psi}(x)\gamma^\mu D_\mu \psi(x) - m\overline{\psi}(x)\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu H)^\dagger (D^\mu H) - V(H) \quad (2.12)$$

where $(D_\mu H)^\dagger (D^\mu H)$ shows the kinetic energy of the Higgs field while $V(H)$ shows the potential energy. The potential energy term is given by $V(H) = -\mu^2|H|^2 + \lambda|H|^4$ where $-\mu^2$ is proportional to the mass terms of the Higgs boson and λ is quartic gauge coupling.

The Lagrangian in (2.12) is a total Lagrangian. Now, let us examine the electroweak theory and construct the Lagrangian in this theory step by step. The electroweak theory based on the $SU(2) \otimes U(1)$ gauge group has the following Lagrangian:

$$\mathcal{L}_{SU(2)\otimes U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{fermion} + \mathcal{L}_{Yukawa} \quad (2.13)$$

The electroweak Lagrangian can be represented as above, but actually this representation is not correct since there are mixings among the terms; for example, there are also gauge fields in the scalar part (within the covariant derivative). The kinetic energy of the gauge fields are

$$\mathcal{L}_{gauge} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} \quad (2.14)$$

where $B_{\mu\nu}$ and $W^i_{\mu\nu}$ are the field strength tensors of the $U(1)_Y$ and $SU(2)_L$ gauge groups, respectively. The field strength tensors are given in terms of the gauge fields of the related gauge group as follows:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.15}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_2 \epsilon_{ijk} W^j_\mu W^k_\nu \tag{2.16}$$

where the g_2 is gauge coupling of the $SU(2)$ group and ϵ_{ijk} are the structure constants in the form of absolute antisymmetric Levi-Civita tensor. The structure of the field strength tensors should be like above to be invariant under the gauge transformations.

The scalar part of the SM Lagrangian,

$$\mathcal{L}_{scalar} = (D_\mu H)^\dagger (D^\mu H) - V(H), \tag{2.17}$$

where $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ is the complex and scalar Higgs field introduced to give mass to the particles and gauge bosons. The covariant derivative is

$$D_\mu = \partial_\mu + i\frac{g_Y}{2} Y_H B_\mu + i g_2 T^i W^i_\mu \tag{2.18}$$

$$D_\mu = \partial_\mu + i\frac{g_Y}{2} B_\mu + i g_2 \frac{\sigma^i}{2} W^i_\mu \tag{2.19}$$

where the hypercharge of the Higgs field is $Y_H = +1$ and $T^i = \frac{\sigma^i}{2}$ are the generators of the $SU(2)$ group. Here the gauge coupling of the $U(1)$ group is taken by $\frac{g_Y}{2}$ due to the simplicity of the calculation. The first term in (2.17) gives us the three and four point interactions between the gauge and scalar fields. The second term, $V(H)$ is the Higgs potential given by

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2. \tag{2.20}$$

In this term λ must be positive ($\lambda > 0$) to satisfy the vacuum stability. When $\mu^2 < 0$, the

Higgs field acquire its VEV

$$\langle H \rangle = \sqrt{-\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}} \tag{2.21}$$

and then electroweak symmetry is spontaneously broken ($SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}$). After that, in addition to one massive Higgs boson, three massless Higgs bosons also arise. These so called Goldstone bosons are eaten by the massless gauge fields and become the third polarization state which is longitudinal. Then massive gauge bosons arise in the theory.

The fermion part of the Lagrangian,

$$\mathcal{L}_{fermion} = \mathcal{L}_{leptons} + \mathcal{L}_{quarks} \tag{2.22}$$

$$\mathcal{L}_{leptons} = \bar{L}_\ell i\gamma^\mu D^L_\mu L_\ell + \bar{\ell}_R i\gamma^\mu D^R_\mu \ell_R \tag{2.23}$$

$$\mathcal{L}_{quarks} = \bar{Q}_q i\gamma^\mu D^L_\mu Q_q + \bar{q}_{uR} i\gamma^\mu D^R_\mu q_{uR} + \bar{q}_{dR} i\gamma^\mu D^R_\mu q_{dR} \tag{2.24}$$

where the covariant derivatives are

$$D^L_\mu = \partial_\mu + i\frac{g_Y}{2} Y_{L_\ell} B_\mu + i g_2 \frac{\sigma^i}{2} W^i_\mu \tag{2.25}$$

$$D^R_\mu = \partial_\mu + i\frac{g_Y}{2} Y_{\ell, q_R} B_\mu \tag{2.26}$$

There is no mass term in the fermion part. So one can write the Yukawa interaction terms to determine the mass of the fermions.

$$\mathcal{L}_{Yukawa} = -h_\ell \bar{L} \cdot H \ell_R - h_{q_d} \bar{Q} \cdot H q_{dR} - h_{q_u} \bar{Q} \cdot \widetilde{H} q_{uR} + h.c. \tag{2.27}$$

The dot products in the Lagrangian can be rewritten as $\bar{L} \cdot H = \epsilon^{ab} L_a H_b$ where $L =$

$\begin{pmatrix} L_a \\ L_b \end{pmatrix}$, $H = \begin{pmatrix} H_a \\ H_b \end{pmatrix}$ and ϵ^{ab} is the completely antisymmetric SU(2) tensor with $\epsilon^{12} = 1$. To give mass to the up quark (for all family) one needs a different representation of the Higgs field defined. This representation should have $Y_{\widetilde{H}} = -1$ hypercharge. It is represented like $\widetilde{H} \equiv i\sigma_2 H^\dagger = \begin{pmatrix} H^0 \\ -H^- \end{pmatrix}$.

When we examine the kinetic term of the Higgs (scalar) Lagrangian and work out analytical calculations, we can see that there is a mixing between the gauge fields. To diagonalize the mass matrix obtained from this Lagrangian, the new fields are introduced,

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3 \tag{2.28}$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3. \tag{2.29}$$

where the θ_W is the weak (Weinberg) angle defined by

$$\sin\theta_W = \frac{g_2}{\sqrt{g_Y^2 + g_2^2}}, \qquad \cos\theta_W = \frac{g_Y}{\sqrt{g_Y^2 + g_2^2}}. \tag{2.30}$$

As seen from above equations, the third $SU(2)$ gauge field W_μ^3 and $U(1)$ gauge field B_μ come together to form the neutral gauge bosons photon A_μ and Z_μ. The combination of the first W_μ^1 and second W_μ^2 gauge fields of the $SU(2)$ gauge group are also defined as the charged gauge bosons,

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2). \tag{2.31}$$

By going on to work out the analytical calculations using the new introduced fields we obtain the masses of $W^\pm, Z$ weak gauge bosons and also we see that the mass of the photon is zero,

$$M_W = \frac{g_2 v}{2} \tag{2.32}$$

$$M_Z = \sqrt{g_2^2 + g_Y^2}\,\frac{v}{2} = \frac{M_W}{\cos\theta_W} \tag{2.33}$$

$$M_A = 0 \tag{2.34}$$

The experimental value of the above parameters are $M_W \sim 80$ GeV, $M_Z \sim 90$ GeV and $\sin^2\theta_W \sim 0.22$ (Novaes, 1999).

The second term of the scalar Lagrangian, that is potential energy term gives us the mass of the Higgs boson. We can obtain this by applying necessary transformations to make Lagrangian invariant under local gauge transformations. As a result of this calculation the mass of the Higgs boson is derived as

$$m_h = \sqrt{-2\mu^2} = \sqrt{2\lambda}v. \tag{2.35}$$

where $v = \sqrt{-\frac{\mu^2}{\lambda}}$. The μ^2 and λ parameters are unknown in SM, therefore the value of the Higgs mass is not determined in the SM. However the ratio of two parameter (VEV of the Higgs field) can be determined by using the experimental value of the vector bosons as

$$v \simeq 246 \quad \text{GeV}. \tag{2.36}$$

While the exact value of the λ parameter is not known in the SM, it is known that its value is approximately smaller than unity ($\lambda < 1$). Therefore, the approximate value of the Higgs mass is considered as $m_h^2 \approx (100\,\text{GeV})^2$.

We can obtain that there arise one massive Higgs boson after the spontaneous symmetry breaking (SSB) in the SM by working out the analytical calculations as above. In addition to this method we can determine how many massive Higgs bosons arise by comparing the number of the degree of freedom (dof) of the states before and after the

Table 2.3. The Gauge and Higgs fields of the SM Before the SSB and Their Corresponding Gauge and Higgs Bosons After the SSB, also Their Degree of Freedom

Before the SSB			After the SSB		
Name	**Fields**	**DOF**	**Name**	**Bosons**	DOF
Gauge Fields	B_μ W^i_μ $(i=1,2,3)$	2 dof $2\times 3=6$ dof	**Gauge Bosons**	A_μ $W^\mp$, Z^0	2 dof $3\times 3=9$ dof
Higgs Field	$H=\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	$2\times 2=4$ dof	**Higgs Boson**	h	**1 dof**
Total		12 dof	**Total**		12 dof

SSB. While the number of dof is being determined, there are two points to be taken into account.

(a)The massless gauge field or gauge bosons have two transverse polarization states, that is these have two degree of freedom.

(b)The massive gauge bosons have three polarization state. Two of them are the transverse polarization states, the rest is the longitudinal state.

When we calculate and compare the number of the total dof before and after the SSB, we can see that after the SSB, there is one extra dof and this dof belongs to a Higgs boson which arise after the SSB as seen from Table 2.3.

2.2. The Problems of the Model

While the Standard Model explains almost all experimental facts, there are lots of unexplained arbitrary parameters and unsolved problems in the SM. Therefore this theory is not a complete theory. The problems of the SM can be summarized as follows:

- **Baryon Asymmetry Problem** : It is considered that the amounts of matter and antimatter were equal in early times -at high energies- after the Big Bang. Since as the temperature decreases and the matter interacts with antimatter and they annihilate, the amount of matter and antimatter will decrease due to the annihilation. In this situation one expects that the amounts of the matter and antimatter should be less than before and equal at low energies; however, the amounts are not equal in the universe. There are more matter than antimatter, everything in the universe consists of matter. This difference is known as "*baryon asymmetry problem*" and the SM can not explain the causes of this problem. It can be solved by the CP violation in the quark sector, but it is too small to explain this (Quigg, 2009).

- **Fermion Problem** : In the Standard Model it is not determined how many fermion families there are. While it is known that there are three fermion families, only first fermion family (ν_e, e^-, u, d) exists in nature. The SM does not explain the presence of the second (ν_μ, μ^-, c, s) and third (ν_τ, τ^-, t, b) fermion families, the heavier copies of the first family and does not predict their quantum numbers. Moreover, the SM can not predict the fermion masses precisely. It seems that fermion masses can be explained via the Higgs mechanism; however, the value of the masses depends on the arbitrary coupling of the Higgs boson to the fermions which can not be determined in the SM (Quigg, 2009). Furthermore, the mixing angles that parametrize the mismatch between flavor eigenstates and mass eigenstates also depend on this coupling and so the mixing angles can not be also determined in the SM (Quigg, 2009).

- **Unification Problem** : The Standard Model states that there are three single symmetry groups and gauge couplings. Nevertheless, according to the Grand Unification Theory (GUT) these three groups should be combined at high energies and there must be one gauge coupling. The SM can not explain this unification. Moreover, the SM does not contain the fourth fundamental force, gravity and so gives no information concerning gravitational interaction.

- **Quantization of Electric Charge** : The Standard Model does not explain why all particles have the quantized charges which are multiples of $e/3$. Since this property allows the electrical neutrality of atoms, it is important for stability of matter (Langacker, 2009).

- **Cosmological Constant Problem** : The cosmological constant can be thought of as the energy of the vacuum. However, the spontaneous symmetry breaking (SSB) also generates a vacuum expectation value (VEV) of the Higgs field at the minimum of the Higgs potential. When the theory is coupled to the gravity, the VEV of the Higgs field contributes to the cosmological constant (Langacker, 2009). Then the cosmological constant becomes

$$\Lambda_{cosm} = \Lambda_{bare} + \Lambda_{SSB} \tag{2.37}$$

where $\Lambda_{bare} = 8\pi G_N V(0)$ is the vacuum energy in the absence of the SSB. While in the absence of the SSB the value of the observed constant is approximate to the bare value ($\Lambda_{obs} \sim \Lambda_{bare}$), when one takes into account the SSB, the value of the Λ_{SSB} will become $|\Lambda_{SSB}| \sim 10^{56}\Lambda_{obs}$. It is 10^{56} times larger in magnitude than the observed value. This difference can not be explained in the SM.

- **Dark Matter and Dark Energy Problem** : According to the cosmological observations it is noted that the standard model is able to explain only about 4% of the matter present in the universe. This observation states that about 24% of the missing 96% should be dark matter, while the rest should be dark energy. Dark matter behaves just like the other matter we know, but it interacts only weakly with the standard model fields. Dark energy is a constant energy density for the vacuum. Although we have known these experimentally, the SM can not explain the amount of dark matter. Attempts to explain the dark energy in terms of vacuum energy of the standard model lead to a mismatch of 120 orders of magnitude as explained above.

- **Strong CP Problem**: When we take the charge conjugate of a particle (change a particle with its antiparticle or vice versa) and apply a parity symmetry (swap left and right), if the laws of physics remain the same, we can say that CP symmetry is conserved. Theoretically it can be found that the standard model should contain a term that break CP symmetry in the strong interaction sector (QCD). However, experimentally there is no observation related to such violation, implying the coefficient of this term is very close to zero. This fine tuning is also considered unnatural.

- **Neutrino Masses and Mixings** : According to the standard model, the neutrinos are massless particles. However, neutrino oscillation experiments have shown that neutrinos must have mass. This is also a problem of the SM.

In addition to these, there is another important problem, known as the “Hierarchy Problem”, about quantum corrections to the Higgs mass.

2.2.1. The Hierarchy Problem

Neutral part of the Higgs field of the Standard Model has a classical potential given as follows:

$$V = \mu^2|H|^2 + \lambda|H|^4 \tag{2.38}$$

According to the SM if $\mu^2 < 0$ and $\lambda > 0$ conditions are satisfied, this neutral Higgs field will have a nonvanishing vacuum expectation value (VEV) at the minimum of the potential. Value of this VEV is determined by using the extremum condition -take first derivative of the Higgs potential and equal to zero-. The determined value is $\langle H \rangle = \nu = \sqrt{-\mu^2/2\lambda}$.

Experimental value of the μ^2 standing for the mass of the Higgs boson is approximately $-(100\ \text{GeV})^2$. This is the classical value, when one considers the quantum effects, this value will change. The correction from the quantum effects will be larger than the classical value and this is known as the “hierarchy problem”.

Quantum effects can be considered as loop corrections (Figure 2.1). While the incoming and outgoing particles and their properties such as momentum is known, it is not known what happens in the loop. Every particle that couples to the Higgs field directly or indirectly contributes to the quantum corrections. Particles with spin 0 and 1 have different contributions and these are represented as Figure 2.1. In this figure dashed lines represent scalar bosons (Higgs or another scalar boson) while solid lines represent fermions. To calculate the loop contribution, propagators which give the probability am-

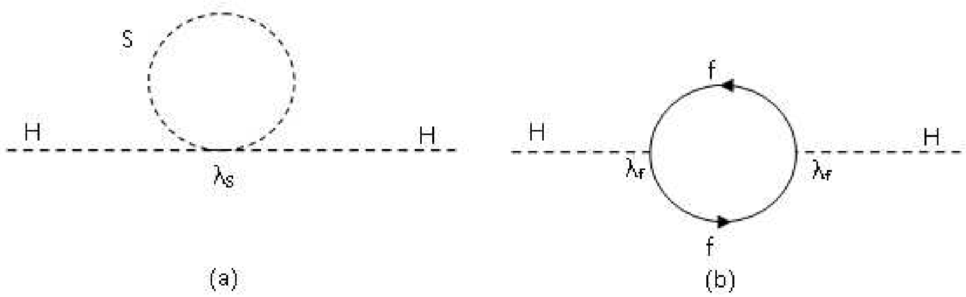

Figure 2.1. One loop radiative corrections to the Higss mass-squared value m_H^2 for (a) an interaction with a scalar (b) an interaction with a fermion.

plitude for a particle to travel with a certain energy and momentum can be used. For example, propagators for a scalar and fermion are given by

$$G_{scalar} = \frac{i}{p^2 - m_s^2 + i\epsilon}, \qquad G_{fermion} = \frac{i}{\gamma^\mu p_\mu - m_f + i\epsilon} \tag{2.39}$$

Total propagator of the process is

$$G(p) = G_0(p) + G_0(p)\frac{\sum_{int}(p)}{i}G_0(p) + G_0(p)\frac{\sum_{int}(p)}{i}G_0(p)\frac{\sum_{int}(p)}{i}G_0(p) + ... \tag{2.40}$$

where the first term shows the propagator for the free particle, $G_0(p) = i/(p^2 - m_h^2)$, other terms represent the interaction propagators. $\sum_{int}(p)$ stands for the amplitude of the process and at the same time this shows the self energy of particle which represents the contribution to the Higgs field's energy due to interactions between the particle and the Higgs field.

The right hand side in (2.40) can be rewritten as

$$G(p) = G_0(p)(1 + \frac{\sum_{int}(p)}{i}G_0(p) + \frac{\sum_{int}(p)}{i}G_0(p)\frac{\sum_{int}(p)}{i}G_0(p) + ...) \tag{2.41}$$

$$G(p) = G_0(p)(1 - \frac{\sum_{int}(p)}{i}G_0(p))^{-1} = (G_0(p)^{-1} - \frac{\sum_{int}(p)}{i})^{-1} = \frac{i}{p^2 - m_h^2 - \sum_{int}}. \tag{2.42}$$

To derive the second line from the first one, the series expansion rule is used. As can be deduced from the last equation by analogy to the propagator for free particle, to find the correction to the mass it is enough to calculate the amplitude of the process.

$$m^2_{h(corrected)} = m_h^2 + \sum_{int}(p) \tag{2.43}$$

When the amplitude is calculated, one can find the corrected Higgs mass as following,

$$m^2_{h(corrected)} = m_h^2\left(1 - \frac{\lambda_s}{16\pi^2}\frac{m_s^2}{m_h^2}ln\left(\frac{\Lambda^2 + m_s^2}{m_s^2}\right)\right) + \frac{\lambda_s}{16\pi^2}\Lambda^2 \tag{2.44}$$

where Λ is called the ultraviolet cutoff used to avoid the infinity of the loop integral. It is the upper scale in which the SM is valid. This equation represents the interaction of the scalar with the Higgs field. The interaction of a fermion with the Higgs field gives the following contribution:

$$m^2_{h(corrected)} = m^2_h \left(1 + \frac{\lambda^2_f}{8\pi^2}\frac{m^2_f}{m^2_h} ln\left(\frac{\Lambda^2 + m^2_f}{m^2_f}\right)\right) - \frac{\lambda^2_f}{8\pi^2}\Lambda^2. \tag{2.45}$$

Generally Λ is equal to the scale of the Planck Mass, $\Lambda = M_P \cong 10^{18} GeV$. When Λ acquire this value, while logarithmic term has a logical contribution which is approximate to the tree level mass, other correction term changing with Λ^2 will be quadratically divergent. It is approximately 30 order of the magnitude larger than the required value of $m^2_h \sim (100GeV)^2$ (Martin, 1997). This problem known as the "Hierarchy Problem" is one of the very important problems that need new theories beyond the SM.

CHAPTER 3

SUPERSYMMETRIC STANDARD MODEL IN BRIEF

3.1. Basics of Supersymmetry

The “Hierarchy Problem” of the Standard Model is the most important problem about the Higgs mass stabilization. Supersymmetry (SUSY) is one of the theories suggested to solve this problem (Martin, 1997). Supersymmetry states that if there is a boson partner for every fermion and vice versa, the quadratic corrections of the Higgs mass cancel each other for the fermion and corresponding boson partner which is called superpartner. To satisfy this there must be a relation between the coupling constants ($\lambda_s = 2\lambda_f^2$). If $m_S^2 = m_f^2$, the logarithmic corrections also cancel each other. There is no corrections in this case which is expressed as unbroken supersymmetry.

The particles and their superpartners have the same quantum numbers except for their spins. The spins of the particle and superpartner differ by $1/2$ unit. When the connection between the fermion and its superpartner boson is examined, it is noted that supersymmetry transforms a fermionic state into a bosonic state or vice versa. Thus, SUSY transformations are given by

$$\widehat{Q}|Fermion >= |Boson >, \qquad \widehat{Q}|Boson >= |Fermion > \tag{3.1}$$

where $\widehat{Q}$ is an operator generating such transformations. We will represent SUSY generator which is an operator as Q instead of $\widehat{Q}$. Since there is $1/2$ unit difference between spins of the fermions and bosons, generator Q must be spinorial.

Generators of any symmetry are charges of the associated symmetry such as the electric charge which is generator of the electromagnetic symmetry group ($U(1)_{EM}$). Therefore charge of supersymmetry Q must be commute with the Hamiltonian of the system,

$$[Q_a, H] = 0 \tag{3.2}$$

where Q_a is one of two components of spinorial charge Q. The generators Q and $Q^\dagger$ must satisfy the below anticommutation and commutation relations as a result of the Coleman-Mandula theorem (Coleman, 1967)

$$\{Q\,, Q^\dagger\} \propto P^\mu, \tag{3.3}$$

$$\{Q\,, Q\,\} = \{Q^\dagger, Q^\dagger\} = 0, \tag{3.4}$$

$$[P^\mu, Q\,] = [P^\mu, Q^\dagger] = 0 \tag{3.5}$$

where P^μ is the four- momentum generator of space-time translations and we have suppressed the spinor indices on Q and $Q^\dagger$. Supersymmetry is an extension of the Poincare group which contains the Lorentz transformations and translation. While P^μ is generator of the space-time translation, $M^{\mu\nu}$ is generator of the Lorentz transformation. There are also some relations about these generators, for the detailed information related to the SUSY algebra one can look at (Aitchison, 2007; Wess, 1992).

Supersymmetry solves not only the hierarchy problem, but also gauge coupling unification and dark matter problem. How Supersymmetry solves these problem will be explained later.

There are some supersymmetric extensions of the SM, the minimal extension of it is based on the same gauge symmetry group ($SU(3) \otimes SU(2) \otimes U(1)$) and is called the Minimal Supersymmetric Standard Model (MSSM). The number of MSSM particles is minimum within supersymmetric models.

Before giving the information about the structure of the MSSM, it can be instructive to give some definitions about supersymmetry representation and algebra.

3.1.1. Supermultiplets

In the SM left handed and right handed fermions are represented by doublets and singlets, respectively. However, in the SUSY all particles, fields and their superpartners are combined in multiplets so-called “supermultiplets” according to their some properties. One of the properties of particles in a supermultiplet is that the number of the degree of freedom of the bosonic and fermionic states should be same ($n_F = n_B$). There are two

Table 3.1. Chiral(Matter) Supermultiplets in the MSSM

$\mathbf{Names}$	$\mathbf{Superfields}$	**Spin 0**	**Spin 1/2**	$\mathbf{SU(3)_C, SU(2)_L, U(1)_Y}$
$\mathbf{Squarks, Quarks}$	$\widehat{Q}_i$	$(\widetilde{u}_{L_i}\ \widetilde{d}_{L_i})$	$(u_{L_i}\ d_{L_i})$	$(3,\ 2,\ \ 1/3)$
	$\widehat{u}^c_{R_i}$	$\widetilde{u}^*_{R_i}$	$u^\dagger_{R_i}$	$(\bar{3},\ 1, -4/3)$
	$\widehat{d}^c_{R_i}$	$\widetilde{d}^*_{R_i}$	$d^\dagger_{R_i}$	$(\bar{3},\ 1,\ \ 2/3)$
$\mathbf{Sleptons, Leptons}$	$\widehat{L}_i$	$(\widetilde{\nu_{L}}_i\ \widetilde{e}_{L_i})$	$(\nu_{L_i}\ e_{L_i})$	$(1,\ 2, -1)$
	$\widehat{e}^c_{R_i}$	$\widetilde{e}^*_{R_i}$	$e^\dagger_{R_i}$	$(1,\ 1,\ \ \ 2)$
$\mathbf{Higgs, Higgsinos}$	$\widehat{H}_u$	$(H^+_u\ H^0_u)$	$(\widetilde{H}^+_u\ \widetilde{H}^0_u)$	$(1,\ 2, +1)$
	$\widehat{H}_d$	$(H^0_d\ H^-_d)$	$(\widetilde{H}^0_d\ \widetilde{H}^-_d)$	$(1,\ 2, -1)$

known supermultiplets in SUSY.

Chiral (Matter) Supermultiplets: All chiral particles which have left and right handed parts and their superpartners are combined in one supermultiplet in terms of left handed particles. So there must be one chirality in the SUSY, there is no right handed particle in the particle content of the SUSY, yet the conjugates of the right handed particles is included as seen in Table 3.1. This supermultiplets are called "chiral (matter) supermultiplets" (Table 3.1). To determine the properties of the fermions and boson partners, equality property of the number of degree of freedom can be used. According to this it is stated that if a fermion is a two component Weyl spinor, then the corresponding boson partner should be complex scalar to have two dof, it can not be a real scalar having one dof. The particles in a chiral supermultiplet have the above properties (Martin, 1997).

Gauge (Vector) Supermultiplets: Like chiral fermions, the boson particles and their superpartners are also combined in one supermultiplet so-called "gauge (vector) supermultiplets" Table 3.2. These supermultiplets consist of one spin 1 massless vector boson (two dof) and a massless spin 1/2 two component Weyl spinor (Martin, 1997).

If superpartners of particles are bosons and fermions, they are called by prepending an "s" and appending "-ino" to the name of the SM particle as seen in Table 3.1 and 3.2, respectively. For example, the boson partner of the quarks or leptons are called squarks or sleptons which mean scalar quarks and leptons, the fermion partner of bosons are called Higgsino, gluino, wino or bino. There are also superpartners of the gauge bosons, photon and Z boson which arise by mixing of the W^0 and B^0 after the electroweak breaking, photino and zino.

Table 3.2. Gauge(Vector) Supermultiplet in the MSSM

Names	Superfields	Spin 1/2	Spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
Gluino, Gluons	$\widehat{G}_a$	$\widetilde{g}$	g	(8, 1, 0)
Winos, W bosons	$\widehat{W}$	$\widetilde{W}^{\mp}$ $\widetilde{W}^0$	$W^{\mp}$ W^0	(1, 3, 0)
Bino, B boson	$\widehat{B}$	$\widetilde{B}^0$	B^0	(1, 1, 0)

3.1.2. Superfields and Superspace

As seen in Table 3.1 and Table 3.2 the combination of a fermion or boson and their superpartners is shown as a "superfield". A superfield is denoted by $\widehat{\Phi}$ and its relation with the bosonic and fermionic fields can be represented by

$$\widehat{\Phi}(x,\theta) = \phi(x) + \theta\psi(x) + ... \tag{3.6}$$

where ϕ and ψ shows a spin $= 0$ boson, spin $= 1/2$ two component Weyl fermion, respectively.θ is a spinor parameter. This spinor parameter is necessary to obtain a field with integer spin from a fermion with half-integer spin. The components of this spinor parameter are anticommuting parameters ($\{\theta_\alpha, \theta_\beta\} = 0$) which are "Grassmann numbers" and these numbers have some specific properties (Dress, 2004), for example; the square of a Grassmann number equals to zero $\theta_\alpha^2 = 0$.

Above representation is fundamental, actually there are extra terms obtained by applying the power series expansion in spinor parameter θ and its conjugate $\bar{\theta}$. General superfield is given by

$$\begin{aligned}\widehat{\Phi}(x,\theta,\theta^*) &= \phi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) + \theta\sigma^m\bar{\theta}V_(x) \\ &+ \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\varphi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x)\end{aligned} \tag{3.7}$$

where ($\theta\theta$) can be rewritten as ($\epsilon_{\alpha\beta}\theta^\beta\theta^\alpha$) and ϵ is an antisymmetrical tensor ($\epsilon_{12} = 1, \epsilon_{21} = -1$). The higher order terms than above are vanish because of the property of the Grassmann numbers. Here the component fields $\phi(x), m(x)$ and $n(x)$ are complex scalar or pseudoscalar fields, $\psi(x)$ and $\varphi(x)$ are left handed Weyl spinors, $\bar{\chi}(x)$ and $\bar{\lambda}(x)$ are right handed Weyl spinor fields, $V(x)$ is a four-vector field and $d(x)$ is a scalar.

General representation takes different forms with respect to the type of the superfields. There are two type of superfields corresponding to supermultiplets, chiral and vector superfields (Wess, 1992).

Chiral superfields do not include the conjugate of a spinor parameter and they should satisfy the condition $\bar{D}\widehat{\Phi} = 0$ where $\bar{D} = -\frac{\partial}{\partial\theta} + i\bar{\theta}\gamma^{\mu}\partial_{\mu}$ is called covariant derivative.

The chiral superfield is

$$\widehat{\Phi} = \phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) \tag{3.8}$$

where F(x) is an auxiliary field to make the Lagrangian invariant under supersymmetry transformation when the classical equations of motions are not satisfied (off-shell condition). There is also D-term which introduced to make the vector supermultiplets invariant. Detailed explanation about them in terms of components of superfields will be explained in Section 3.1.3. Here we give a brief definition about these in terms of superfield formalism.

The product of two superfields is again a chiral superfield while the product of a superfield with a conjugate superfield is not a chiral superfield. We can see this by constructing the combination of the superfields.

$$\begin{aligned}\widehat{\Phi}_i(y,\theta)\widehat{\Phi}_j(y,\theta) = \phi_i(y)\phi(y)_j + \sqrt{2}\theta[\psi_i(y)\phi_j(y) + \phi_i(y)\psi_j(y)] \\ + \theta\theta[\phi_i(y)F_j(y) + \phi_j(y)F_i(y) - \psi_i(y)\psi_j(y)]\end{aligned} \tag{3.9}$$

This product represents interaction terms in the theory which form components of the Superpotential. As we can see, the structure of the product of two chiral superfields is the same with the original chiral superfields. By integrating this two times we can obtain the F term of the Lagrangian that is the Lagrangian of the auxiliary field. However, we see that below combination is not the same structure with the chiral superfield.

$$\begin{aligned}\widehat{\Phi}_i(y,\theta)\widehat{\Phi}_j^{\dagger}(y,\theta) &= \phi_i(y)\phi_j^*(y) + \sqrt{2}\theta\psi_i(y)\phi_j^*(y) + \sqrt{2}\bar{\theta}\bar{\psi}_i(y)\phi_j(y) + +\theta\theta\phi_j^*(y)F_i(y) \\ &+ \bar{\theta}\bar{\theta}F_j(y)^*\phi_i(y) + 2\bar{\theta}\bar{\psi}_j(y)\theta\psi_i(y) + \sqrt{2}\theta\theta\bar{\theta}\bar{\psi}_j(y)F_i(y) \\ &+ \sqrt{2}\bar{\theta}\bar{\theta}\theta\psi_j(y)\bar{F}_i^*(y) + \bar{\theta}\bar{\theta}\theta\theta F_j^*(y)F_i(y) \end{aligned} \quad (3.10)$$

Equation 3.10 stands for the kinetic term of the theory which is called the Kähler Potential. From this product we can obtain the D-term contribution since it behaves like a vector field. The coefficient of the $\bar{\theta}\bar{\theta}\theta\theta$ term gives us D-term contribution to the scalar Lagrangian described in following section.

The vector superfield which is the other kind of the superfield includes both the spinor parameter and its conjugate, and they have a vector field. Vector superfields satisfy the condition $V = V^{\dagger}$. General representation for a vector field can be given by

$$\widehat{V}(x,\theta,\bar{\theta}) = -\theta\sigma^{\mu}\bar{\theta}V_{\mu} + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D \quad (3.11)$$

where V_{μ} is a vector field, λ is fermionic superpartner of vector field and D is an auxiliary field mentioned before. The detailed explanation is given in reference (Wess, 1992).

As a result, it can be stated that a superfield is a field which depends not only on space-time coordinates x^{μ} corresponding to bosonic degrees of freedom, but also on fermionic degrees of freedom- specifically, spinor parameter and its conjugate θ and θ^* (Aitchison, 2007).

While the ordinary space-time coordinates x^{μ} correspond to bosonic coordinates, a spinor parameter and its conjugate mentioned above correspond to the fermionic ones in SUSY. In total, there are four fermionic coordinates (θ_1, θ_2, θ_1^*, θ_2^*) corresponding to each of bosonic states. A space involving these bosonic and fermionic coordinates (x^{μ}, θ, θ^*) is called the “superspace”.

Superpotential : Superpotential is an analytic function which includes interactions of the chiral superfields such as Yukawa interactions of the SM and the mass term to obtain three mass dimensions [W]=3. Since superpotential is an analytic function, it does not include the complex conjugate of a superfield.

$$\widehat{W}(\widehat{\phi}) = a\widehat{\phi} + b\widehat{\phi}^2 + c\widehat{\phi}^3 \quad (3.12)$$

where $\widehat{\phi}$ is a chiral superfield containing a scalar or a fermion and their superpartner and mass dimensions of the a,b,c parameters are [a]= 2, [b]=1, [c]=0. Superpotential is used to obtain the scalar potential of superfields.

3.1.3. The Lagrangian for the Supersymmetric Standard Model

The Supersymmetric Standard Model Lagrangian is composed of the Lagrangian of the chiral and gauge superfields. To determine the supersymmetric Lagrangian it is instructive to begin with the chiral and gauge Lagrangian.

- **Chiral (Matter) Lagrangian**

If the Lagrangian is written using components of the superfield instead of superfield notation, it is easier to compare the MSSM Lagrangian with the SM Lagrangian (Shah, 2003).

$$\mathcal{L}_{free} = \partial^{\mu}\phi^{*i}\partial_{\mu}\phi_i + i\psi^{\dagger i}\bar{\sigma}^{\mu}\partial_{\mu}\psi_i + F^{*i}F_i \tag{3.13}$$

where ϕ and ψ are the scalar and fermion components of a chiral superfield, F is the auxiliary field which is introduced to get the supersymmetry algebra to work off shell (when the classical equations of motion are not satisfied). This auxiliary field is a complex scalar field which does not have a kinetic term. Since the lagrangian density must be 4 mass dimensions, the new auxiliary field must have 2 mass dimensions according to $\mathcal{L}_{auxiliary} = F^{*}F$. This Lagrangian is invariant under the supersymmetry transformations (Martin, 1997). The interaction part of the chiral Lagrangian is given by

$$\mathcal{L}_{int} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^{i}F_i\right) + c.c. \tag{3.14}$$

where W is the superpotential, W^i and W^{ij} are the first and second derivatives of the superpotential with respect to scalar components of the superfields.

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}\phi_i\phi_j\phi_k \tag{3.15}$$

$$W^i = \frac{\partial}{\partial \phi_i} W \quad (3.16)$$

$$W^{ij} = \frac{\partial^2}{\partial \phi_i \partial \phi_j} W \quad (3.17)$$

The classical equation of motion of the auxiliary field F gives $F_i = -W_i^*$ and $F^{*i} = -W^i$. Using these the total chiral Lagrangian can be derived as follows:

$$\mathcal{L}_{chiral} = \mathcal{L}_{free} + \mathcal{L}_{int} \quad (3.18)$$

$$\mathcal{L}_{chiral} = \partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2}\left(W^{ij}\psi_i\psi_j + W_{ij}^* \psi^{\dagger i}\psi^{\dagger j}\right) - F^{*i}F_i \quad (3.19)$$

- **Gauge (Vector) Lagrangian**

The gauge Lagrangian in SM can be written in the supersymmetric model as

$$\mathcal{L}_{gauge} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + i\lambda^{\dagger a}\bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a \quad (3.20)$$

where $F_{\mu\nu}^a$, λ^a, $D_\mu \lambda^a$ are a field strength tensor, a gaugino field, the covariant derivative of the gaugino field, respectively and D^a is a real bosonic auxiliary field introduced in order for supersymmetry to be consistent off-shell. This auxiliary field has no kinetic term and has 2 mass dimension like the fermionic auxiliary field F^i.

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \quad (3.21)$$

$$D_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A_\mu^b \lambda^c \quad (3.22)$$

where g is gauge coupling constant and f^{abc} are structure constants which are antisymmetric in all indices and differs according to symmetry group.

- **Gauge Interactions with the Components of the Chiral Superfields**

The chiral and gauge Lagrangians described above are invariant under the supersymmetric transformations; however, the chiral Lagrangian is not invariant under the gauge transformations while the gauge Lagrangian is invariant. To make the Lagrangian invariant we must define the covariant derivatives instead of ordinary derivatives as defined in the SM. These covariant derivatives are

$$D_\mu \phi_i = \partial_\mu \phi_i - igA^a_\mu (T^a \phi)_i \tag{3.23}$$

$$(D_\mu \phi)^*_i = \partial_\mu \phi^*_i + igA^a_\mu (\phi^* T^a)_i \tag{3.24}$$

$$D_\mu \psi_i = \partial_\mu \psi_i - igA^a_\mu (T^a \psi)_i . \tag{3.25}$$

where T^a are generators of the symmetry groups, for example for $U(1)$ and $SU(2)$ symmetries, T^a stand for hypercharge and pauli spin matrices, respectively.

As seen from the equations above gauge bosons couple to scalars and fermions in the chiral superfields. In addition to these there are some interaction terms that can be seen below between the other gauge fields (gaugino and bosonic auxiliary field D^a) and components of the superfields (scalars and fermions).

$$(\phi^* T^a \psi)\lambda^a, \quad \lambda^{\dagger a}(\psi^\dagger T^a \phi), \ \text{and} \ (\phi^* T^a \phi) D^a \tag{3.26}$$

Now, the total supersymmetric Lagrangian which is invariant under the supersymmetry and gauge transformations can be written as follows:

$$\begin{aligned}\mathcal{L}_{susy} =& D^\mu \phi^{*\,i} D_\mu \phi_i + i\psi^{\dagger\,i} \bar{\sigma}^\mu D_\mu \psi_i - \frac{1}{2}\left(W^{ij}\psi_i\psi_j + W^*_{ij}\psi^{\dagger i}\psi^{\dagger j}\right) - F^{*\,i}F_i \\ & - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - i\lambda^{\dagger a}\bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2}D^a D^a \\ & - \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi \dagger T^a \phi) + g(\phi^* T^a \phi)D^a \end{aligned} \tag{3.27}$$

The equation of motion for the D^a term gives the value of the bosonic auxiliary field in terms of scalar fields,

$$D^a = -g(\phi^* T^a \phi). \tag{3.28}$$

Substituting Equation (3.28) into the total Lagrangian (3.27) and organizing it we get

$$\begin{aligned}\mathcal{L}_{susy} =& (D^\mu \phi^i)^* D_\mu \phi_i + i\psi^{\dagger\, i} \bar{\sigma}^\mu D_\mu \psi_i - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i\lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a \\ & - \sqrt{2} g (\phi^* T^a \psi) \lambda^a - \sqrt{2} g \lambda^{\dagger a} (\psi^\dagger T^a \phi) \\ & - \frac{1}{2} \left(W^{ij} \psi_i \psi_j + W^*_{ij} \psi^{\dagger i} \psi^{\dagger j} \right) \\ & - F^{*\, i} F_i - \frac{1}{2} D^a D^a \end{aligned} \tag{3.29}$$

where the first line is the gauge-invariant kinetic energies for the components of the chiral and gauge superfields. The next line describes the interactions of the gauginos with the scalar and fermion components of the chiral superfields, that is, these terms describe how gauginos couple matter fermions to their superpartner, or Higgs bosons to their superpartners. The third line describes the non-gauge, superpotential interactions of matter and Higgs fields as well as fermion mass terms. The last line describes the scalar potential which consist of two distinct contribution (Baer, 2006). The first term is called the F-term contribution that arise from the superpotential. The second term is related to the gauge interactions and it is called the D-term contribution (Kazakov, 2001).

$$V_{scalar} = V_F + V_D \tag{3.30}$$

where

$$V_F = F^{*\, i} F_i \qquad \text{and} \qquad V_D = \frac{1}{2} D^a D^a \tag{3.31}$$

$$F^{*\, i} = -\frac{\partial W}{\partial \phi_i} \qquad \text{and} \qquad D^a = -g(\phi^* T^a \phi). \tag{3.32}$$

The supersymmetric Lagrangian in (3.29) is the total Lagrangian when the Supersymmetry is not broken. However, it is known that the supersymmetry is broken. Because if SUSY were an exact symmetry, the sparticles could have been the same mass with the original particles, and these sparticles could have been seen in nature. Since there has been no sparticles in nature, it can be stated that the SUSY is broken symmetry. This breaking is called “soft supersymmetry breaking” as the symmetry is broken by keeping the cancellation of the quadratic divergences at the SM Higgs mass. Due to this soft breaking of the SUSY there must be additional terms to the supersymmetric Lagrangian in (3.29),

$$\mathcal{L}_{soft} = -(m^2)^i_j \phi^{j*}\phi_i - \frac{1}{2}(M_a\lambda^a\lambda^a + h.c) + \left(\frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i + h.c.\right) \tag{3.33}$$

where the $\mathcal{L}_{soft}$ contains the mass-squared terms $(m^2)^i_j$ of the scalars, the gaugino masses M_a for each gauge group, trilinear and bilinear scalar couplings a^{ijk} and b^{ij}, respectively and linear (tadpole) couplings t^i. The linear coupling term exist only if ϕ_i is a gauge singlet. The terms in $\mathcal{L}_{soft}$ break the supersymmetry since they involve only scalars and gauginos and not their respective superpartners. These soft terms gives the masses to the scalars and gauginos in a theory even if the gauge bosons and fermions in chiral supermultiplets are massless or relatively light (Martin, 1997). Soft terms of the Lagrangian are defined as a third contribution to the scalar potential. Now, the scalar potential can be rewritten as follows:

$$V_{scalar} = V_F + V_D + V_{soft} \tag{3.34}$$

The potential terms V_F and V_D are given in Equation 3.31 and V_{soft} is represented as Lagrangian in Equation 3.33.

As mentioned before minimal extension of the Standard Model is Minimal Supersymmetric Standard Model (MSSM). Since we gave the basic information about SUSY, now we are ready to examine the structure of this minimal model.

3.2. The Structure of the Minimal Supersymmetric Standard Model

Minimal supersymmetric model is the minimal extension of the SM because they have the same gauge groups, $SU(3) \otimes SU(2) \otimes U(1)$. Since there are superpartners of each bosons and fermions, the number of the particle in MSSM is double of the SM's. Particle content of the MSSM is given in Table 3.1 and 3.2 as a chiral and gauge supermultiplets. While there is one Higgs doublet in the SM, one extra Higgs doublet is necessary in the MSSM as seen in Table 3.1 and its superpartner . Why do we need two Higgs doublets?

The first reason of this is to cancel the anomaly which occur when the superpartner of the Higgs field is taken into account. In the SM there is no anomaly, that is SM satisfy the condition for cancellation of gauge anomalies, $Tr[T_3^2 Y] = Tr[Y^3] = 0$, where T_3 and Y are the third component of the weak isospin and the weak hypercharge, respectively (Martin, 1997). The hypercharge of the particles can be computed by using the Gell-Mann Nishijima formula. When the Higgs fields' superpartner with the hypercharge $Y_{\widetilde{H}} = 1$ is considered, the above condition are not satisfied, $Tr[T_3^2 Y] = Tr[Y^3] \neq 0$. Therefore to get rid of this anomaly it is necessary to introduce a new Higgs field.

The second reason is about the structure of the supersymmetry. As seen above, the Higgs doublet in the SM is not sufficient to give mass to up quark. While in the SM the complex conjugate of the Higgs field can be defined, in the SUSY it can not be defined since the superpotential which is the only source of Yukawa interactions used to give mass to the fermions must be analytic. Complex conjugate of any parameters is not allowed in superpotential, therefore, second Higgs field is defined in the SUSY and this field is represented by

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \; H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}. \tag{3.35}$$

To satisfy the conservation of charge, the hypercharge of the new Higgs doublet (H_d) is found $Y_{H_d} = -1$ by applying the Gell-Mann Nishijima formula.

The general superpotential term for the MSSM is given by

$$\widehat{W} = \mu \widehat{H_u} \cdot \widehat{H_d} + h_u \widehat{Q} \cdot \widehat{H_u}\, \widehat{u}_R^c + h_d \widehat{Q} \cdot \widehat{H_d}\, \widehat{d}_R^c + h_e \widehat{L} \cdot \widehat{H_d}\, \widehat{e}_R^c \tag{3.36}$$

where $H_u,\ H_d,\ Q,\ L,\ u_R^c,\ d_R^c,\ e_R^c$ denotes the superfields, h_u, h_d and h_e are dimensionless Yukawa coupling constants. μ parameter refers to the supersymmetric version of the SM Higgs mass. Here the gauge(color and weak isospin) and family indices are suppressed. The dot product of two doublet superfields can be written by using an antisymmetric parameter $\epsilon^{\alpha\beta}$. For instance, the first term of the superpotential is written as $\mu \widehat{H_u} \cdot \widehat{H_d} = \mu \epsilon^{\alpha\beta} (H_u)_\alpha (H_d)_\beta$ and the second term is written as $h_u \widehat{Q} \cdot \widehat{H_u}\, \widehat{u}_R^c = h_u \epsilon^{\alpha\beta} Q_{ia\alpha} (H_u)_\beta u_{R_i}^{ca}$ where $i = 1, 2, 3$ is a family index, $a = 1, 2, 3$ is a color index and $\alpha, \beta = 1, 2$ are the weak indices. Another notation which can be used for the dot products is $\mu \widehat{H_u} \cdot \widehat{H_d} = \widehat{H}_u^T (i\sigma_2) \widehat{H}_d$.

The hypercharge of each term in the superpotential is conserved, hence it is said that the superpotential is invariant under $U(1)_Y$. The above superpotential also satisfy the conservation of the Baryon and Lepton number. In addition to the above terms in the superpotential there can be extra terms which is gauge invariant and renormalizable but violate the baryon B or lepton L symmetry.

$$\widehat{W}' = \mu'\widehat{L}\cdot\widehat{H}_u + \lambda_1\widehat{L}\cdot\widehat{L}\,\widehat{e}^c_R + \lambda_2\widehat{L}\cdot\widehat{Q}\,\widehat{d}^c_R + \lambda_3\widehat{u}^c_R\,\widehat{d}^c_R\,\widehat{d}^c_R \tag{3.37}$$

where the lepton and baryon numbers are $L = +1$ for L_i, $L = -1$ for e^c_R and $L = 0$ for all others, $B = +1/3$ for Q_i, $B = -1/3$ for u^c_R, d^c_R and $B = 0$ for all others. The first three term violates the conservation of lepton number since these have "1" lepton number while the last term violates the conservation of the baryon symmetry because this has "1" baryon number. Nevertheless baryon and lepton number violating interactions have never been seen experimentally. If both violating interactions were present, the proton could decay rapidly. To keep the proton sufficiently stable a new symmetry (R-parity) is introduced.

$$P_R = (-1)^{3B+2S+L} \tag{3.38}$$

where B, L and S represent the baryon, lepton numbers and spin of the each particle. R parity condition states that the scalar and fermion (spinor or vector) components of a chiral scalar (spinor) superfield have opposite R parities due to the $(-1)^{2S}$ dependence. So, while all of the Standard Model particles and the Higgs bosons have even R-parity ($P_R = +1$), all of the superpartners of the SM particles and fields have odd parity ($P_R = -1$). Since this symmetry is not conserved in (3.37) superpotential, these terms are forbidden while the superpotential in (3.36) is allowed.

If R parity is exactly conserved, then there can be no mixing between the SM particles ($P_R = +1$) and their superpartners with opposite R parity ($P_R = -1$). Moreover, every interaction in the allowed superpotential (3.36) contains an even number of odd R parity sparticles ($P_R = -1$). This property has important phenomenological consequences:

- Experimentally sparticles can only be produced in pairs.
- The lightest sparticle called the "lightest supersymmetric particle" (LSP) must be

stable, it can not decay at all. If the LSP is electrically neutral, it could be an attractive candidate for non-baryonic dark matter.

- Each particle produced in experiments must decay into a state that contains an odd number of other sparticles and any number of the SM particles. At the last step there must be at least one LSP.

After giving a general information about structure of the minimal supersymmetric model let us examine the Higgs sector of the MSSM in detail.

3.2.1. Higgs Sector

Minimal Supersymmetric Standard Model contains two Higgs doublets with hypercharges $Y_{H_u} = +1$, $Y_{H_u} = -1$.

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \; H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}. \tag{3.39}$$

To determine the Higgs mass firstly we must find the Higgs potential and we can do this by working out the scalar potential for the Higgs scalar at the tree level.

$$V_{tree} = V_F + V_D + V_{soft} \tag{3.40}$$

Then the F-term, D-term and soft term contributions to the scalar potential can be obtained as

$$V_F = |\mu|^2(|H_u|^2 \; + \; |H_d|^2) \tag{3.41}$$

$$V_D = \frac{G^2}{8}\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{g_2^2}{2}\left(|H_u|^2|H_d|^2 - |H_u \cdot H_d|^2\right) \tag{3.42}$$

$$V_{soft} = m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + B\mu(H_u \cdot H_d + h.c.) \tag{3.43}$$

where $G^2 \; = \; g_2^2 + g_Y^2$. Here g_2 and g_Y are the gauge couplings of the gauge groups of

Table 3.3. The Gauge and Higgs fields of the MSSM Before the SSB and Their Corresponding Gauge and Higgs Bosons After the SSB, also Their Degree of Freedom

Before the SSB			After the SSB		
Name	**Fields**	**DOF**	**Name**	**Bosons**	**DOF**
Gauge Fields	B_μ W^i_μ $(i = 1, 2, 3)$	2 dof $2 \times 3 = 6$ dof	**Gauge Bosons**	A_μ $W^\mp$, Z^0	2 dof $3 \times 3 = 9$ dof
Higgs Fields	H_u, H_d	$2 \times 4 = 8$ dof	**Higgs Bosons**	$h, H, A, H^\pm$	**5 dof**
Total		20 dof	**Total**		20 dof

$SU(2)_L$ and $U(1)_Y$, respectively. $m^2_{H_u}$, $m^2_{H_d}$ are mass-squared terms of the scalars and B is bilinear scalar coupling constant.

$$V_{tree} = (m^2_{H_u} + \mu^2)|H_u|^2 + (m^2_{H_d} + \mu^2)|H_d|^2 + B\mu(H_u \cdot H_d + h.c.) + \frac{G^2}{8}\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{g_2^2}{2}\left(|H_u|^2|H_d|^2 - |H_u \cdot H_d|^2\right) \qquad (3.44)$$

Since the neutral part of the Higgs field has a non-vanishing value at the vacuum state (the minimum of the potential), the fields can be expanded as follows:

$$H_u = \frac{1}{\sqrt{2}}\begin{pmatrix} H_u^+ \\ v_u + \phi_u + i\varphi_u \end{pmatrix}, \qquad H_d = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d + \phi_d + i\varphi_d \\ H_d^- \end{pmatrix}. \qquad (3.45)$$

Electroweak symmetry is spontaneously broken when the Higgs fields acquire their vacuum expectation values at the minimum,

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}}, \quad \langle H_u^+ \rangle = 0, \quad \langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}}, \quad \langle H_d^- \rangle = 0, \qquad (3.46)$$

Then Higgs fields interact with fermions and gauge fields. At the end of this interaction while the fermions get their masses, the gauge and Higgs fields also acquire their physical mass states. After the SSB, 5 Higgs bosons arise as seen in Table (3.3). Two of them are the CP even (scalar) neutral Higgs bosons(h and H), one of them is CP odd (pseudoscalar) neutral Higgs boson (A) and two of them are charged Higgs bosons ($H^\pm$).

After giving the general properties of the MSSM and its Higgs sector, now let's determine the masses of the gauge bosons. To derive the masses of the gauge bosons of the MSSM, the kinetic energy terms of the Higgs field is used that is written as follows:

$$\mathcal{L} \ni |D_\mu H_u|^2 + |D_\mu H_d|^2 \tag{3.47}$$

where covariant derivatives can be determined as seen below to make the Lagrangian under the $U(1)_Y$ and $SU(2)_L$ symmetries.

$$D_\mu H_u = (\partial_\mu + ig_2 \frac{\sigma_i}{2} W_\mu^i + i\frac{g_Y}{2} B_\mu) H_u, \tag{3.48}$$

$$D_\mu H_d = (\partial_\mu + ig_2 \frac{\sigma_i}{2} W_\mu^i - i\frac{g_Y}{2} B_\mu) H_d. \tag{3.49}$$

When the Higgs fields acquire their VEVs at the minimum of the Higgs potential energy as in 3.46, gauge bosons acquire their masses by following the same way as in the SM,

$$M_W^2 = \frac{g_2^2}{2}(v_u^2 + v_d^2) \quad \text{and} \quad M_Z^2 = \frac{g_Y^2 + g_2^2}{2}(v_u^2 + v_d^2). \tag{3.50}$$

If we determine the Higgs potential energy, we can get the formula for the mass-squared matrix of the Higgs bosons by taking the second derivative of the Higgs potential with respect to the components of the Higgs fields. Because the conservation of electric charge one states that there must be no mixing between the charges and neutral components of the Higgs fields. So we consider these part separately.

Since we examine the components of the Higgs fields, writing the Higgs potential in terms of component fields may be helpful.

$$\begin{aligned} V_{MSSM} &= (m_{H_u}^2 + \mu^2)(|H_u^0|^2 + |H_u^+|^2) + (m_{H_d}^2 + \mu^2)(|H_d^0|^2 + |H_d^-|^2) \\ &- B\mu(H_u^0 H_d^0 - H_u^+ H_d^- + h.c.) \\ &+ \frac{g_Y^2}{8}\left[(|H_u^+|^2 - |H_u^0|^2 + |H_d^0|^2 - |H_d^-|^2)^2 + 4|H_u^+|^2|H_u^0|^2 + 4|H_d^0|^2|H_d^-|^2\right] \\ &- \frac{g_Y^2}{2}(H_u^{+*} H_d^{-*} H_u^0 H_d^0 + H_u^{0*} H_d^{0*} H_u^+ H_d^-) \\ &+ \frac{g_2}{8}\left[|H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2\right]^2 \end{aligned} \tag{3.51}$$

where we renamed the tree level potential of the MSSM as MSSM potential (V_{MSSM}).

Firstly, we can examine the charged fields. The Lagrangian should have following form to include the mass terms of the charged Higgs fields:

$$\mathcal{L} \ni \begin{pmatrix} H_u^{+*} & H_d^- \end{pmatrix} M_{H^\pm}^2 \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} \tag{3.52}$$

where

$$M_{H^\pm}^2 = \begin{pmatrix} \frac{\partial^2 V_{MSSM}}{\partial H_u^+ \partial H_u^{+*}} \big|_{h_i \to v_i} & \frac{\partial^2 V_{MSSM}}{\partial H_u^{+*} \partial H_d^{-*}} \big|_{h_i \to v_i} \\ \frac{\partial^2 V_{MSSM}}{\partial H_u^+ \partial H_d^-} \big|_{h_i \to v_i} & \frac{\partial^2 V_{MSSM}}{\partial H_d^- \partial H_d^{-*}} \big|_{h_i \to v_i} \end{pmatrix} \tag{3.53}$$

Now, derivatives of the tree level scalar potential with respect to above components give us the mass squared matrix of the charged Higgs fields as seen below (Baer, 2006). It should be noted that these derivatives must be taken at the VEV of the Higgs field.

$$M_{H^\pm}^2 = \begin{pmatrix} B\mu \cot\beta + \frac{g_2^2}{2} v_d^2 & -B\mu + \frac{g_2^2}{2} v_u v_d \\ -B\mu + \frac{g_2^2}{2} v_u v_d & B\mu \tan\beta + \frac{g_2^2}{2} v_u^2 \end{pmatrix} \tag{3.54}$$

Diagonalizing this matrix we can obtain the masses of the charges Higgs bosons,

$$m_{G^\pm} = 0 \qquad \text{and} \qquad m_{H^\pm}^2 = B\mu(\cot\beta + \tan\beta) + M_W^2 \tag{3.55}$$

where $tan\beta = g_Y/g_2$ and $M_W^2 = g_2(v_u^2 + v_d^2)/2$. The massless $G^\pm$ bosons are Goldstone bosons which are eaten by the charged gauge bosons $W^\pm$.

For the neutral part the mass squared matrix can be found by using the below formula:

$$\mathcal{M}_{ij}^2 = \left(\frac{\partial^2}{\partial \Psi_i \partial \Psi_j} V \right)_0 \tag{3.56}$$

with $\Psi_i \in \{\phi_u, \phi_d, \varphi_u, \varphi_d\}$. Instead of finding the (4×4) mass squared matrix as above, we can decomposes into to (2×2) matrices by using CP invariance property. CP invariance of the Higgs sector states that there is also no mixing between the real and imaginary part of the Higgs fields.

For examining the real and imaginary part of the neutral Higgs fields, the Lagrangian should include the following part:

$$\mathcal{L} \ni \frac{1}{2} \begin{pmatrix} \varphi_u & \varphi_d \end{pmatrix} M^2_{H^0_I} \begin{pmatrix} \varphi_u \\ \varphi_d \end{pmatrix}, \qquad \mathcal{L} \ni \frac{1}{2} \begin{pmatrix} \phi_u & \phi_d \end{pmatrix} M^2_{H^0_R} \begin{pmatrix} \phi_u \\ \phi_d \end{pmatrix}. \tag{3.57}$$

The mass matrices of the imaginary part of the neutral Higgs fields and its value are given by

$$M^2_{H^0_I} = \begin{pmatrix} \frac{\partial^2 V_{MSSM}}{\partial\varphi_u \partial\varphi_u} & \frac{\partial^2 V_{MSSM}}{\partial\varphi_u \partial\varphi_d} \\ \frac{\partial^2 V_{MSSM}}{\partial\varphi_d \partial\varphi_u} & \frac{\partial^2 V_{MSSM}}{\partial\varphi_d \partial\varphi_d} \end{pmatrix}_{h_i \to v_i}, \quad M^2_{H^0_I} = \begin{pmatrix} B\mu \cot\beta & B\mu \\ B\mu & B\mu \tan\beta \end{pmatrix} \tag{3.58}$$

The eigenvalues of this mixing matrix give the physical masses of the Higgs bosons (Aitchison, 2007; , Baer, 2006).

$$m^0_G = 0 \quad \text{and} \quad m^2_A = B\mu(\cot\beta + \tan\beta) \tag{3.59}$$

where G^0 is also Goldstone boson which are eaten buy the neutral gauge boson Z^0 and A is the pseudoscalar (CP odd) Higgs boson.

The real part of the neutral Higgs fields has following mass squared matrix

$$M^2_{H^0_R} = \begin{pmatrix} \frac{\partial^2 V_{MSSM}}{\partial\phi_u \partial\phi_u} & \frac{\partial^2 V_{MSSM}}{\partial\phi_u \partial\varphi_d} \\ \frac{\partial^2 V_{MSSM}}{\partial\phi_d \partial\phi_u} & \frac{\partial^2 V_{MSSM}}{\partial\phi_d \partial\phi_d} \end{pmatrix}_{h_i \to v_i} \tag{3.60}$$

$$M^2_{H^0_R} = \begin{pmatrix} m^2_A \cos^2\beta + M^2_Z \sin^2\beta & -(m^2_A + M^2_Z)\sin\beta\cos\beta \\ -(m^2_A + M^2_Z)\sin\beta\cos\beta & m^2_A \sin^2\beta + M^2_Z \cos^2\beta \end{pmatrix} \tag{3.61}$$

Masses of the neutral Higgs bosons are found as follows:

$$m_{h,H}^2 = \frac{1}{2}\left[(m_A^2 + M_Z^2) \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}\right] \tag{3.62}$$

where h and H shows the lighter and heavier neutral Higgs bosons.

The relation between the bosons which have physical mass states and fields can be found from the mixing matrix by deriving the eigenvectors,

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} H_d^{-*} \\ H_u^+ \end{pmatrix} \tag{3.63}$$

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} \varphi_u \\ \varphi_d \end{pmatrix}, \begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_u \\ \phi_d \end{pmatrix} \tag{3.64}$$

where α and β are the mixing angles with

$$\tan\alpha = \frac{(m_A^2 - M_Z^2)\cos 2\beta + \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos 2\beta}}{(m_A^2 + M_Z^2)\sin 2\beta}. \tag{3.65}$$

When we compare the masses of the physical Higgs bosons, the first result we obtain is $m_{H^\pm}^2 = m_A^2 + M_W^2$, so $m_{H^\pm}$ is larger than the masses of the pseudoscalar Higgs boson m_A and gauge boson m_W. The second result is related to the lightest neutral Higgs boson mass. While the masses m_A, m_H and $m_{H^\pm}$ are unconstraint, the mass m_h is bounded. Since M_Z parameter is known from the experiments, the mass term of lightest Higgs boson in (3.62) include 2 unknown parameter, m_A and $\cos\beta$. If we examine the conditions when the m_A^2 is small and large, we obtain the maximum value of the mass

$$m_h \leq M_Z|\cos 2\beta| \leq M_Z \tag{3.66}$$

where the maximum value of $\cos 2\beta = 1$. According to this result m_h should be equal or smaller than $M_Z \approx 90$ GeV ($m_h \leq M_Z$). Nevertheless, experimental lower bound from the LEP experiment is

$$m_h \geq 114.4\,\text{GeV} \tag{3.67}$$

at the % 95 Confidence Level. Therefore we can say that the one-loop quantum corrections is significant in the MSSM. Under the radiative correction from the top quark lightest Higgs boson mass becomes

$$m_h^2 \leq M_Z^2 \cos^2 2\beta + \frac{3m_t^4 \sin^4 \beta}{2\pi^2(v_u^2 + v_d^2)} \ln\left(\frac{\overline{m}_{\tilde{t}}}{m_t}\right) \tag{3.68}$$

where m_t is the top quark mass and $\overline{m}_{\tilde{t}}^2 = (m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2$ is the average of the squared masses of two scalar top quarks (Aitchison, 2007; , Baer, 2006). As seen in (3.68) these corrections shift the upper limit of the lightest MSSM Higgs boson mass and satisfy the LEP limit. But despite of the quantum corrections, the mass of the lightest Higgs boson can not exceed 135 GeV ($m_h \leq 135$ GeV).

3.3. The Successes and Problems of the MSSM

Supersymmetry and also Minimal Supersymmetric Standard Model are introduced mainly to stabilize the Higgs sector which is unstable to the quantum corrections. Besides this, there are other motivations for the MSSM. One of them is to unify the gauge coupling constant.

As seen from in Figure (3.1), SM can not explain the unification of the three fundamental forces. That is, there is no a mass scale or interaction scale at which the electromagnetic, weak and strong interactions have the same strength. In this figure Y-axis is the fine-structure constants ($1/\alpha_i$) which is related to square of the gauge coupling constant ($\alpha_i \propto g_i^2$). X-axis shows the mass or energy on a logarithmic scale. The indices $1, 2, 3$ stand for the $U(1)_Y$, $SU(2)_L$, $SU(3)_C$ couplings, respectively. While this unification can not explained by the SM, MSSM unify these three forces at high energies.

The other motivation for the MSSM is to explain the Dark Matter problem. Standard Model can explain the small amount of total matter in the universe, however the amount of dark matter is much larger than the matter explained by the SM. The additional particle content and R-parity properties of the MSSM help us to explain the dark

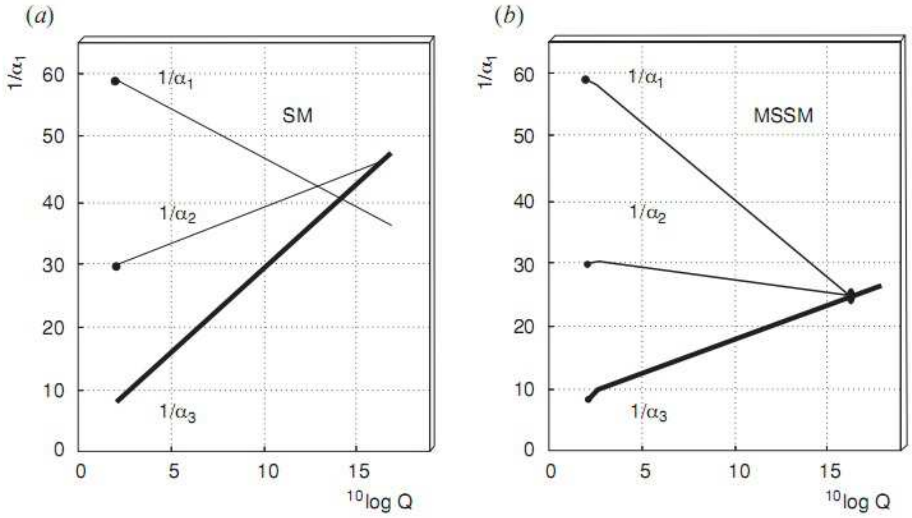

Figure 3.1. Gauge coupling unification (a) in the SM and (b) in the MSSM (Aitchison, 2007).

matter. As explained before the Lightest Supersymmetric Particle (LSP) must be stable as a result of the R-parity. It is also weakly interacting massive particle (WIMP) that is in the $10^2 - 10^3$ GeV range with only weak interaction strength couplings, it does not have electromagnetic or strong interactions. Because of having the similar properties to the dark matter, LSP might be best candidate for the dark matter.

Although the Minimal supersymmetric model solve some important problem of the SM mentioned above, most of them remain unsolved. New ones are also introduced.

One of them is the "**little hierarch problem**". The MSSM predicts a light Higgs boson near the Z mass at the tree level, while the experimental lower bound is 114 GeV according to the LEP experiment. Whereas this LEP bound can not be satisfied at the tree level, one loop radiative correction from the top quark may be used to satisfy it. To satisfy it, the mass of the top quark must be taken to be ~ 1 TeV. Thus supersymmetry(SUSY) must be broken above the weak scale, recreant in fine-tuning of $\sim 1\%$ or worse in the soft SUSY-breaking parameters in order to reproduce the observed value of the weak scale. This is how the little hierarchy problem appears in the context of the MSSM.

The other and most important problem of the MSSM is the μ **problem.** When the following general superpotential term for the MSSM is examined,

$$\widehat{W} = \mu\widehat{H_u} \cdot \widehat{H_d} + h_u\widehat{Q} \cdot \widehat{H_u}\widehat{u^c_R} + h_d\widehat{Q} \cdot \widehat{H_d}\widehat{d^c_R} + h_e\widehat{L} \cdot \widehat{H_d}\widehat{e^c_R} \qquad (3.69)$$

it is realized the first term contains μ parameter with a mass dimension which is not restricted to be at the electroweak scale, the soft supersymmetry breaking mass parameters' scale. This is known as "μ problem" of the MSSM. Because of these problems we need to extend the Minimal Supersymmetric Standard Model. Although there are some theories for extension, we will explain one of them in the next chapter.

CHAPTER 4

GAUGE-EXTENDED SUPERSYMMETRIC MODEL

4.1. Motivations for the U(1)′ Model

While gauge structure of the Standard Model and Minimal Supersymmetric Standard Model is given by $SU(3) \otimes SU(2) \otimes U(1)$, the simplest gauge extension of the MSSM is found by expanding its gauge group with an additional Abelian factor and this model is called $U(1)'$ model. There are two main motivations for this model. The most direct motivation for such an extra group factor is the need to solve the μ problem of the MSSM (Kim, 1984; Giudice, 1998). Indeed, the mass term of the Higgsinos

$$\widehat{W}_{MSSM} \ni \mu \widehat{H}_u \cdot \widehat{H}_d \tag{4.1}$$

involves a dimensionful parameter μ and this parameter can be totaly arbitrary scale. However, the μ parameter should be dimensionless like natural coefficients and should be at the electroweak scale that is the scale of mass parameters of the theory determined by the soft supersymmetry breaking. To overcome this μ problem, the μ parameter can be replaced by a new SM chiral superfield $\widehat{S}$ (Cvetic, 1997). When the scalar component of this superfield acquire its vacuum expectation value (VEV) via the spontaneous symmetry breaking, effective μ parameter is induced,

$$\mu_{eff} = h_s \langle S \rangle. \tag{4.2}$$

Then Equation (4.1) becomes

$$\widehat{W} \ni h_s \widehat{S} \widehat{H_u} \cdot \widehat{H_d} \tag{4.3}$$

with h_s being a Yukawa coupling. Gauge invariance of the superpotential under the $U(1)'$ symmetry requires that total charges of each term in the new superpotential for the $U(1)'$

model should be zero (the charges of the particle and field is given in Table (4.1)). That is, for the Higgsino mass term of the superpotential gauge invariance condition is given by

$$Q_S + Q_{H_u} + Q_{H_d} = 0 \tag{4.4}$$

where $Q_S \neq 0$ (Sert, 2010). These conditions forbid a bare μ term as in (4.1) completely, and μ parameter is deemed to arise from the VEV of S via (4.2). In addition to these constraints arising from the gauge invariance, there are also some constraints arising to avoid quantum-induced trilinear mixing among the gauge bosons that causes the triangle anomalies in the gauge sector and so gauge coupling non-unification. The anomalies can be cancelled either by introducing family non-universal charges (Demir, 2005; Hayreter, 2007) or by adding exotics to the models descending from E(6) and other GUT groups (Langacker, 1998). In the present work we shall assume that anomalies are cancelled by additional matter falling outside the reach of LHC experiments.

Table 4.1. The gauge quantum numbers of chiral superfields where i=1,2,3 stands for the family index

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
$\widehat{Q}_i$	**3**	**2**	$1/3$	Q_{Q_i}
$\widehat{u}^c_{R_i}$	$\bar{\mathbf{3}}$	**1**	$-4/3$	Q_{u_i}
$\widehat{d}^c_{R_i}$	$\bar{\mathbf{3}}$	**1**	$2/3$	Q_{d_i}
$\widehat{L}_i$	**1**	**2**	-1	Q_{L_i}
$\widehat{e}^c_{R_i}$	**1**	**1**	2	Q_{e_i}
$\widehat{H}_u$	**1**	**2**	1	Q_{H_u}
$\widehat{H}_d$	**1**	**2**	-1	Q_{H_d}
$\widehat{S}$	**1**	**1**	0	Q_S

The μ problem mentioned above is one of the motivations for introducing $U(1)'$ model. In addition to this, such extra gauge symmetries arise in low energy supersymmetric models stemming from GUTs and strings (Barr, 1985; Hewett, 1989; Cvetic, 1996; Cleaver, 1998; Ghilencea, 2002). As an example we can examine the E(6) GUT (King, 2006; Diener, 2009), the breaking pattern of the E(6) groups is given

$$E(6) \to SO(10) \otimes U(1)_\psi \to SU(5) \otimes U(1)_\chi \otimes U(1)_\psi \to G_{SM} \otimes U(1)'. \tag{4.5}$$

In this chain each arrow corresponds to spontaneous symmetry breaking at a specific energy scale and after these breaking two extra $U(1)$ symmetries occur, $U(1)_\psi$ and $U(1)_\chi$. $U(1)'$ at the last step is a linear combination of these extra symmetries like that

$$U(1)' = \cos\theta_{E(6)}\, U(1)_\psi - \sin\theta_{E(6)}\, U(1)_\chi \tag{4.6}$$

which is a light $U(1)'$ invariance broken near the TeV scale whereas the other orthogonal combination $U(1)'' = \cos\theta_{E(6)}\, U(1)_\chi + \sin\theta_{E(6)}\, U(1)_\psi$ is broken at a much higher scale not accessible to LHC experiments. The angle $\theta_{E(6)}$ (mixing angle) designates the breaking direction in $U(1)_\psi \otimes U(1)_\chi$ space and it is a function of the associated gauge couplings and VEVs that realize the symmetry breaking. For all different values of the $\theta_{E(6)}$ mixing angle, there are various $U(1)'$ models based on E(6) groups. For instance, in ψ Model $\theta_{E(6)} = 0$, in η Model $\theta_{E(6)} = \arcsin\sqrt{\frac{3}{8}}$, in I Model $\theta_{E(6)} = -\arcsin\sqrt{\frac{5}{8}}$, in N Model $\theta_{E(6)} = \arcsin\frac{1}{4}$, in S Model $\theta_{E(6)} = \arcsin\sqrt{\frac{27}{32}}$. We excluded χ model ($\theta_{E(6)} = -\frac{\pi}{2}$) as it does not lead to a solution for μ problem (the singlet S acquires vanishing $U(1)'$ charge) (Barr, 1985; Hewett, 1989; Cvetic, 1996; Cleaver, 1998; Ghilencea, 2002).

4.2. The Structure of the $\mathrm{U(1)'}$ Model

$U(1)'$ model is obtained by adding an extra abelian $U(1)$ group to the gauge group of the SM or MSSM. The gauge structure of the model can be represented as follows:

$$SU(3) \otimes SU(2) \otimes U(1) \otimes U(1)' \tag{4.7}$$

The extra $U(1)$ group requires an extra gauge boson Z' and gauge fermion $\widetilde{Z}'$ with respect to MSSM. Also the Higgs sector of such models differ from those of the SM and MSSM (Spira, 1998). Firstly, there are an extra Higgs field that can be represented as a chiral, singlet SM superfield $\widehat{S}$ in addition to two Higgs doublets H_u and H_d of the MSSM. The Higgs fields can be given by

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_u^- \end{pmatrix} \text{ and } \; S. \tag{4.8}$$

Table 4.2. Gauge Fields, Higgs Fields and Their Corresponding Bosons in the Models where $i = 1, 2, 3$ and $a = 1, 2, .., 8$.

Fields/Bosons	**SM**	**MSSM**	$\mathbf{U(1)'}$ **Model**
Gauge Fields	$G^a_\mu,\ W^i_\mu,\ B_\mu$	$G^a_\mu,\ W^i_\mu,\ B_\mu$	$G^a_\mu,\ W^i_\mu,\ B_\mu,\ B'_\mu$
Higgs Fields	H	$H_u,\ H_d$	$H_u,\ \ H_d,\ S$
Gauge Bosons	$g^a,\ W^\pm,\ Z,\ A_\mu$	$g^a,\ W^\pm,\ Z,\ A_\mu$	$g^a,\ W^\pm,\ Z,\ Z',\ A_\mu$
Higgs Bosons	h	$h, H, A\,H^\pm$	$h, H, H', A\,H^\pm$

where doublets and singlet fields are complex scalar fields. Moreover, the modifications in the masses and couplings of the Higgs bosons are also different from the MSSM and SM (Demir, 2004). The additional fields and bosons may be summarized as in Table 4.2 by comparing the other models. All the particles and fields are charged under this extra $U(1)$ symmetry. Quantum numbers of the $U(1)'$ model particle contents are given in Table (4.1).

If Higgs sector of the $U(1)'$ Model is considered again, Table 4.3 may be helpful to comprehend how many Higgs boson arises in this model. As seen in Table 4.3 there are six Higgs boson arising in $U(1)'$ Model after the spontaneously electroweak symmetry breaking. One extra Higgs boson is a CP even (scalar) neutral boson heavier than other neutral bosons. Table (4.3) gives information not only about Higgs sector, but also about gauge sector. As seen from the Table (4.3), there are 5 gauge bosons arising; $W^\pm, Z, Z'$ and photon(A_μ). However, the neutral gauge bosons Z and Z' exhibit nontrivial mixing (Langacker, 2008) and $Z - Z'$ mass-squared matrix is given by

$$(M_{Z-Z'})^2 = \begin{pmatrix} M_Z^2 & \delta^2_{Z-Z'} \\ \delta^2_{Z-Z'} & M^2_{Z'} \end{pmatrix} \tag{4.9}$$

where

$$M_Z^2 = \frac{G^2}{4}[\upsilon_u^2 + \upsilon_d^2] \tag{4.10}$$

$$M^2_{Z'} = g_Y'^2[Q^2_{H_u}\upsilon_u^2 + Q^2_{H_d}\upsilon_d^2 + Q_S^2\upsilon_s^2] \tag{4.11}$$

Table 4.3. The Gauge and Higgs fields of the $U(1)'$ Model Before the SSB and Their Corresponding Gauge and Higgs Bosons After the SSB, also Their Degree of Freedom

Before the SSB			**After the SSB**		
Name	**Fields**	**DOF**	**Name**	**Bosons**	**DOF**
Gauge Fields	B_μ W_μ^i $(i=1,2,3)$ B'_μ	2 dof $2\times 3=6$ dof 2 dof	**Gauge Bosons**	A_μ $W^\mp$, Z^0 Z'^0	2 dof $3\times 3=9$ dof 3 dof
Higgs Fields	H_u, H_d S	$2\times 4=8$ dof 2 dof	**Higgs Bosons**	$h, H, A\, H^\pm$ H'	**5 dof** **1 dof**
Total		20 dof	**Total**		20 dof

$$\delta^2 = \frac{g'_Y G}{2}[Q_{H_u}\upsilon_u^2 - Q_{H_d}\upsilon_d^2] \tag{4.12}$$

These values can be derived from the kinetic part of the Higgs Lagrangian for the $U(1)'$ Model. The two eigenvalues of this matrix give the masses of the physical massive vector bosons,

$$M^2_{Z_1,Z_2} = \frac{1}{2}\left[M_Z^2 + M_{Z'}^2 \mp \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4\delta^4_{Z-Z'}}\,\right] . \tag{4.13}$$

If there is no mixing M_{Z_1} will be a mass of the SM Z boson. The mixing angle of the mixing matrix in (4.9) can be found from diagonalization of this matrix. To diagonalize (4.9) the rotation matrix below is used

$$R = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \tag{4.14}$$

Using this rotation matrix and $R^\dagger \cdot (M_{Z-Z'})^2 \cdot R = diag(M^2_{Z_1}, M^2_{Z_2})$ diagonalization condition, the mixing angle can be derived by equaling the off-diagonal terms to zero as follows (Ali, 2009),

$$\alpha_{Z-Z'} = \frac{1}{2}\arctan\left(\frac{2\delta^2_{Z-Z'}}{M^2_{Z'} - M^2_Z}\right) . \tag{4.15}$$

The value of the mixing angle $\alpha_{Z-Z'}$ must be a few 10^{-3} according to the LEP experiments. This puts a bound on the Z_2 boson mass. In particular, in generic E(6) models m_{Z_2} must weigh nearly a TeV or more according to the Tevatron measurements (Erler, 2009).

Another important aspect of this model is the Higgs sector which constitute the main structure of this thesis work. Therefore, the Higgs Sector of this model at the tree level and one loop level will be explained in the following chapter.

CHAPTER 5

HIGGS SECTOR OF THE U(1)′ MODEL

5.1. Higgs Sector of the U(1)′ Model at the Tree Level

The Higgs sector of the model, as mentioned before, involves the singlet Higgs S and the electroweak doublets H_u and H_d. All of them are charged under U(1)′ gauge group. The Higgs fields expand around the vacuum state as follows

$$\begin{aligned} H_u &= \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H_u^+ \\ v_u+\phi_u+i\varphi_u \end{pmatrix},\ H_d=\frac{1}{\sqrt{2}}\begin{pmatrix} v_d+\phi_d+i\varphi_d \\ \sqrt{2}H_d^- \end{pmatrix}, \\ S &= \frac{1}{\sqrt{2}}\left(v_s+\phi_s+i\varphi_s\right) \end{aligned} \tag{5.1}$$

where H_u^+ and H_d^- span the charged sector involving the charged Goldstone eaten up by the $W^\pm$ boson as well as the charged Higgs boson. The remaining ones span the neutral degrees of freedom: $\phi_{u,d,s}$ are scalars and $\varphi_{u,d,s}$ are pseudoscalars.

When the local gauge symmetry is broken, the Higgs fields gets the vacuum expectation values (VEVs) in the vacuum given by

$$\frac{v_u}{\sqrt{2}} \equiv \langle H_u^0\rangle\,,\quad \frac{v_d}{\sqrt{2}} \equiv \langle H_d^0\rangle\,,\quad \frac{v_s}{\sqrt{2}} \equiv \langle S\rangle \tag{5.2}$$

and then the $W^\pm$, Z and Z' bosons all acquire masses. Besides the gauge bosons, the Higgs bosons get also their masses. Masses of the Higgs bosons are determined by taking the second derivative of the scalar potential with respect to the components of the Higgs field, scalar and pseudoscalar fields:

$$\mathcal{M}_{ij}^2 = \left(\frac{\partial^2}{\partial\Psi_i\partial\Psi_j}V\right)_0 \tag{5.3}$$

with $\Psi_i \in \{\phi_u,\phi_d,\phi_s,\varphi_u,\varphi_d,\varphi_s\}$.

At the tree level the potential in (5.3) is the tree level scalar potential of the Higgs fields composed of F term, D term and soft breaking pieces.

$$V_{tree} = V_F + V_D + V_{soft} \tag{5.4}$$

with

$$V_F = |h_s|^2 \left[|H_u \cdot H_d|^2 + |S|^2(|H_u|^2 + |H_d|^2)\right] \,, \tag{5.5}$$

$$\begin{aligned} V_D =& \frac{G^2}{8}\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{g_2^2}{2}\left(|H_u|^2|H_d|^2 - |H_u \cdot H_d|^2\right) \\ &+ \frac{{g'_Y}^2}{2}\left(Q_{H_u}|H_u|^2 + Q_{H_d}|H_d|^2 + Q_S|S|^2\right)^2 \,, \end{aligned} \tag{5.6}$$

$$V_{soft} = m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_s^2|S|^2 + (h_s A_s S H_u \cdot H_d + h.c.) \tag{5.7}$$

where $G^2 = g_2^2 + g_Y^2$ (Sert, 2010; Cincioglu, 2010). Here g_2, g_Y and g'_Y are the gauge couplings of the gauge groups of $SU(2)_L, U(1)_Y$ and $U(1)'$, respectively. Soft masses of the scalar Higgs $m_{H_u}^2\,,\; m_{H_d}^2\,,\; m_S^2$ are obtained by taking the first derivative of the potential with respect to scalar components of the Higgs fields (ϕ_i) and equaling zero, that is applying the condition for finding the extremum points.

$$\left(\frac{\partial V}{\partial \Psi_i}\right)_0 = 0 \tag{5.8}$$

These soft masses are obtained as follows

$$(\bar{m}_{H_u}^2) = m_0^2 \cot\beta + \frac{1}{8}G^2 v^2 \cos 2\beta - \frac{1}{2}g_Y'^2 Q_{H_u}(\bar{Q}_H v^2 + Q_S v_s^2) - \frac{1}{2}h_s^2(v^2\cos^2\beta + v_s^2)$$

$$(\bar{m}_{H_d}^2) = m_0^2 \tan\beta - \frac{1}{8}G^2 v^2 \cos 2\beta - \frac{1}{2}g_Y'^2 Q_{H_d}(\bar{Q}_H v^2 + Q_S v_s^2) - \frac{1}{2}h_s^2(v^2\sin^2\beta + v_s^2)$$

$$(\bar{m}_S^2) = m_0^2\frac{v^2}{v_s^2}\sin\beta\cos\beta - \frac{1}{2}g_Y'^2 Q_S(\bar{Q}_H v^2 + Q_S v_s^2) - \frac{1}{2}h_s^2 v^2 \,, \tag{5.9}$$

where $(\bar{m}^2_{H_u}), (\bar{m}^2_{H_d})$ and $(\bar{m}^2_S)$ stand for $(m^2_{H_u})_{tree}$, $(m^2_{H_d})_{tree}$ and $(m^2_S)_{tree}$ and $m^2_0 = (h_s/\sqrt{2})A_s,\ \bar{Q}_H = Q_{H_u}\sin^2\beta + Q_{H_d}\cos^2\beta,\ v^2 = v_u^2 + v_d^2$ and $\tan\beta = v_u/v_d$.

Once the mass states of the Higgs fields are derived by using the tree level potential, we obtain $(6x6)$ mass-squared matrix of the Higgs fields. Diagonalizing this matrix we get one massive pseudoscalar Higgs boson, 3 massive scalar Higgs bosons and 2 massless Goldstone bosons which are eaten by the neutral gauge bosons Z and Z'. The mass-squareds of the Higgs bosons at the tree level are derived by

$$m^2_{A^0} = \frac{\sqrt{2}Ah_s v_s}{\sin 2\beta}\left[1 + \frac{v^2}{4v_s^2}\sin^2 2\beta\right], \tag{5.10}$$

which is never negative, and

$$m^2_{h^0_1} \le M^2_Z\cos^2 2\beta + \frac{1}{2}h_s^2 v^2 sin^2 2\beta + g_1'^2(Q_{H_d}\cos^2\beta + Q_{H_u}\sin^2\beta)^2 v^2 \tag{5.11}$$

$$m^2_{H^\pm} = M^2_W + \frac{\sqrt{2}Ah_s v_s}{\sin 2\beta} - \frac{1}{2}h_s^2 v^2 \tag{5.12}$$

$m^2_{H^\pm}$ could be lighter than the W boson due to the negative third contribution. It could be negative for some choices of the parameters (Cvetic, 1997).

5.2. Higgs Sector of the U(1)$'$ Model at the One Loop Level

Due to the soft breaking of supersymmetry, the Higgs boson masses shift in proportion to particle–sparticle mass splitting under quantum corrections. Though all particles which couple to the Higgs fields S, H_u and H_d contribute to the Higgs boson masses, the largest correction comes from the top quark and its superpartner scalar top quark (and to a lesser extent from the bottom quark multiplet). Including top and bottom quark superfields, the superpotential takes the form

$$\widehat{W} \ni h_s\widehat{S}\widehat{H}_u\cdot\widehat{H}_d + h_t\widehat{Q}\cdot\widehat{H}_u\widehat{t}^c_R + h_b\widehat{Q}\cdot\widehat{H}_d\widehat{b}^c_R \tag{5.13}$$

where h_t and h_b are top and bottom Yukawa couplings . This superpotential encodes the dominant couplings of the Higgs fields which determine the F term contributions.

At the one loop there is a contribution to the tree level Higgs potential due to the radiative corrections and this contribution can be computed by using the effective potential method. In fact, the radiatively corrected potential is written as

$$V_{total}\left(H\right) = V_{tree}\left(H\right) + \Delta V\left(H\right) . \tag{5.14}$$

The contributions of the quantum fluctuations in (5.14) read as

$$\Delta V = \frac{1}{64\,\pi^2}\text{Str}\left[\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{\Lambda^2} - \frac{3}{2}\right)\right] \tag{5.15}$$

where Str $\equiv \sum_J (-1)^{2J}(2J+1)$Tr is the usual supertrace which generates a factor of 6 for squarks and -12 for quarks (Demir, 2004; Sert, 2010). The number of the factor can be calculated by multiplying the number of color (3 for quarks and squarks), the number of spin ($2(+1/2, -1/2)$ and 0 for the quarks and squarks, respectively), the number of charges ($2(+,-)$ for the quarks and squarks) and $(-1)^{2J}$. Λ is the renormalization scale and $\mathcal{M}$ is the field-dependent mass matrix of quarks and squarks (we take $\Lambda = m_t + m_{Z_2}/2$). The dominant contribution comes from top quark (and bottom quark, to a lesser extent) multiplet. More explicitly we can write the radiative correction to the tree level scalar potential as follows:

$$\Delta V = \frac{6}{64\pi^2}\left[\sum_{k=1,2}(m^2_{(\tilde{t},\tilde{b})_k})^2\left[\ln\left(\frac{m^2_{(\tilde{t},\tilde{b})_k}}{\Lambda^2}\right) - \frac{3}{2}\right] - 2(m^2_{(t,b)_k})^2\left[\ln\left(\frac{m^2_{(t,b)_k}}{\Lambda^2}\right) - \frac{3}{2}\right]\right] \tag{5.16}$$

The required top and bottom quark field-dependent masses read as

$$m_t^2\left(H\right) = h_t^2\left|H_u^0\right|^2 \ , \quad m_b^2\left(H\right) = h_b^2\left|H_d^0\right|^2 . \tag{5.17}$$

The mass-squareds of their superpartners are also necessary to calculate (5.15), the mass-squareds of squarks are obtained by diagonalizing the mass-squared matrix below

$$m^2_{\tilde{f}} = \begin{pmatrix} M^2_{\tilde{f}LL} & M^2_{\tilde{f}LR} \\ M^2_{\tilde{f}RL} & M^2_{\tilde{f}RR} \end{pmatrix} \tag{5.18}$$

where $f = t$ or b. For instance, the entries of the stop mass-squared matrix read to be

$$\begin{aligned} M^2_{\tilde{t}LL} =& m^2_{\tilde{Q}} + m^2_t - \frac{1}{12}\left(3g_2^2 - g_Y^2\right)\left(|H_u^0|^2 - |H_d^0|^2\right) \\ & + g'^2_Y\, Q_Q\left(Q_{H_u}|H_u|^2 + Q_{H_d}|H_d|^2 + Q_S|S|^2\right) \end{aligned}$$

$$\begin{aligned} M^2_{\tilde{t}RR} =& m^2_{\tilde{t}_R} + m^2_t - \frac{1}{3}g_Y^2(|H_u^0|^2 - |H_d^0|^2) \\ & + g'^2_Y\, Q_U\left(Q_{H_u}|H_u|^2 + Q_{H_d}|H_d|^2 + Q_S|S|^2\right) \end{aligned}$$

$$M^2_{\tilde{t}LR} = M^2_{\tilde{t}RL} = h_t\left(A_t H_u^0 - h_s S H_d^0\right) \tag{5.19}$$

These entries are obtained by taking the second derivative of the general tree level potential including all the scalars. The coefficients of the quadratic fields after the derivatives give us above entries.

Insertion of the top and bottom mass matrices into (5.15) generates the full one-loop effective potential. Radiatively corrected Higgs masses and mixings are computed from the effective potential (Demir, 2004). Now, Higgs potential in (5.3) and (5.8) becomes the radiatively corrected effective potential,

$$\left(\frac{\partial V_{total}}{\partial \Psi_i}\right)_0 = 0\,, \qquad \mathcal{M}^2_{ij} = \left(\frac{\partial^2}{\partial\Psi_i\partial\Psi_j}V_{total}\right)_0 \tag{5.20}$$

with $\Psi_i \in \{\phi_u, \phi_d, \phi_s, \varphi_u, \varphi_d, \varphi_s\}$. The soft masses of the Higgs scalars at one loop include additional terms arising from the radiative correction, and these contribution is expressed as

$$m_{H_u}^2 = (m_{H_u}^2)_{tree} - \frac{1}{v_u}\left(\frac{\partial \Delta V}{\partial \phi_u}\right)_0$$

$$m_{H_d}^2 = (m_{H_d}^2)_{tree} - \frac{1}{v_d}\left(\frac{\partial \Delta V}{\partial \phi_d}\right)_0$$

$$m_S^2 = (m_S^2)_{tree} - \frac{1}{v_s}\left(\frac{\partial \Delta V}{\partial \phi_s}\right)_0 , \quad (5.21)$$

where $(m_{H_u}^2)_{tree}$, $(m_{H_d}^2)_{tree}$ and $(m_S^2)_{tree}$ are given in (5.9).

The mass-squared matrix of the Higgs bosons can be formed by substituting above values into the total scalar potential and taking the below derivatives of the total scalar potential:

$$M^2 = \begin{pmatrix}
\left(\frac{\partial^2 V_{total}}{\partial\phi_u\partial\phi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_u\partial\phi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_u\partial\phi_s}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_u\partial\varphi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_u\partial\varphi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_u\partial\varphi_s}\right)_0 \\
\left(\frac{\partial^2 V_{total}}{\partial\phi_d\partial\phi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_d\partial\phi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_d\partial\phi_s}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_d\partial\varphi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_d\partial\varphi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_d\partial\varphi_s}\right)_0 \\
\left(\frac{\partial^2 V_{total}}{\partial\phi_s\partial\phi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_s\partial\phi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_s\partial\phi_s}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_s\partial\varphi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_s\partial\varphi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\phi_s\partial\varphi_s}\right)_0 \\
\left(\frac{\partial^2 V_{total}}{\partial\varphi_u\partial\phi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_u\partial\phi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_u\partial\phi_s}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_u\partial\varphi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_u\partial\varphi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_u\partial\varphi_s}\right)_0 \\
\left(\frac{\partial^2 V_{total}}{\partial\varphi_d\partial\phi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_d\partial\phi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_d\partial\phi_s}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_d\partial\varphi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_d\partial\varphi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_d\partial\varphi_s}\right)_0 \\
\left(\frac{\partial^2 V_{total}}{\partial\varphi_s\partial\phi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_s\partial\phi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_s\partial\phi_s}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_s\partial\varphi_u}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_s\partial\varphi_d}\right)_0 & \left(\frac{\partial^2 V_{total}}{\partial\varphi_s\partial\varphi_s}\right)_0
\end{pmatrix} \quad (5.22)$$

in the $(\phi_u, \phi_d, \phi_s, \varphi_u, \varphi_d, \varphi_s)$ basis. Above matrix can be considered as a combination of the scalar part, pseudoscalar part and mixing parts,

$$M^2 = \begin{pmatrix} M_{SS}^2 & M_{SP}^2 \\ M_{PS}^2 & M_{PP}^2 \end{pmatrix} . \quad (5.23)$$

Since there is no mixing between the scalar and pseudoscalar parts in the CP conserving limit ($M_{SP}^2 = M_{PS}^2 = 0$), we can examine these parts separately. Firstly let's examine pseudoscalar part of the mass-squared matrix in the $(\varphi_u, \varphi_d, \varphi_s)$ basis. After we find the entries of pseudoscalar matrix we must diagonalize it to find the physical mass states. This matrix is a 3×3 matrix, hence diagonalization condition should be applied two times,

$$R_1 = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & \cos\alpha & 0 \end{pmatrix} \tag{5.24}$$

where $\tan\beta = v_u/v_d$ and $\cot\alpha = (v\sin\beta\cos\beta)/v_s$ is found after some calculations. Using the above rotation matrix and diagonalization condition ($R^\dagger \cdot M^2_{PP} \cdot R$), it is found that there is one massive Higgs state called pseudoscalar Higgs boson (A) and two massless Goldstone bosons which are eaten by the neutral Z and Z' bosons in the ($G_{Z'}, G_Z, A$) basis while these bosons acquire their masses. The relation between the basis states can be found by multiplying the rotation matrices:

$$\begin{pmatrix} \varphi_u \\ \varphi_d \\ \varphi_s \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & \cos\alpha & 0 \end{pmatrix} \begin{pmatrix} G_{Z'} \\ G_Z \\ A \end{pmatrix} \tag{5.25}$$

Firstly the matrix are multiplied and then orthogonality condition ($M^\dagger M = 1$) is used to find the second basis in terms of the first basis and below result is obtained.

$$\begin{pmatrix} G_{Z'} \\ G_Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta\cos\alpha & \sin\beta\cos\alpha & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \cos\beta\sin\alpha & \sin\beta\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \varphi_u \\ \varphi_d \\ \varphi_s \end{pmatrix} \tag{5.26}$$

From this matrix the physical states of the pseudoscalar CP-odd Higgs bosons are obtained as below:

$$G_Z = -\sin\beta\varphi_u + \cos\beta\varphi_d\,, \tag{5.27}$$

$$G_{Z'} = \cos\beta\cos\alpha\varphi_u + \sin\beta\cos\alpha\varphi_d - \sin\alpha\varphi_s \tag{5.28}$$

$$A = \cos\beta\sin\alpha\varphi_u + \sin\beta\sin\alpha\varphi_d + \cos\alpha\varphi_s \tag{5.29}$$

After the relation between the basis states is obtained as above, by finding eigenvalues of the mixing mass squared matrix for the pseudoscalar part we can obtain masses of the pseudoscalar Higgs bosons which have physical mass states as mentioned above. Two of them are found to be equal zero corresponding to Goldstone bosons and the value of the rest is found as follows:

$$M_P^2 = \frac{M_A^2}{\sin^2 \alpha} \tag{5.30}$$

$$M_A^2 = M_0^2 \left(1 + \frac{3h_t^2}{32\pi^2} \frac{A_t}{A_s} \mathcal{F}\right) \tag{5.31}$$

where M_0^2 is a mass parameter introduced for simplicity and $\mathcal{F}$ is a loop function depends explicitly on the renormalization scale, and their explicit forms are as the following

$$M_0^2 = \frac{h_s A_s v_s}{\sqrt{2} \sin\beta \cos\beta}, \tag{5.32}$$

$$\mathcal{F}(\Lambda^2, m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) = -2 + \ln\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{\Lambda^4}\right) + \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right). \tag{5.33}$$

Now, let's examine the scalar part of the mass-squared matrix in the (ϕ_u, ϕ_d, ϕ_s) basis. After the define the entries of the matrix which is given below, we must diagonalize it to find the mass states of the CP-even scalar Higgs bosons (Demir, 2004).

$$M_{SS}^2 = \begin{pmatrix} M_{uu}^2 + M_A^2 \cos^2\beta & M_{ud}^2 - M_A^2 \sin\beta\cos\beta & M_{us}^2 - M_A^2 \cot\alpha\cos\beta \\ M_{ud}^2 - M_A^2 \sin\beta\cos\beta & M_{dd}^2 + M_A^2 \sin^2\beta & M_{ds}^2 - M_A^2 \cot\alpha\sin\beta \\ M_{us}^2 - M_A^2 \cot\alpha\cos\beta & M_{ds}^2 - M_A^2 \cot\alpha\sin\beta & M_{ss}^2 + M_A^2 \cot^2\alpha \end{pmatrix} \tag{5.34}$$

The explicit form of M_A^2 is given in (5.31), the mass parameters $M_{ij}^2 (i, j = u, d, s)$ may be represented as

$$M_{ij}^2 = v_i v_j \{\bar{\lambda}_{ij} + \frac{3}{(4\pi)^2}[\frac{(\rho_i \widetilde{m}_j^2 + \widetilde{m}_i^2 \rho_j)}{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}(2 - \mathcal{G}) + \left(\rho_i\rho_j + \zeta_i\zeta_j + \delta_{id}\delta_{js}\frac{h_t^2 h_s^2}{4}\right)\mathcal{F}$$
$$+ \left(\rho_i\rho_j + \frac{\widetilde{m}_i^2 \widetilde{m}_j^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2}\right)\mathcal{G} - \delta_{iu}\delta_{ju} h_t^4 \ln\left\{\frac{m_t^4}{Q^4}\right\}]\} \tag{5.35}$$

where $\bar{\lambda}_{ij} = \lambda_{ij}$ for $i \neq j$, $\bar{\lambda}_{ij} = 2\lambda_i$ for $i = j$ which are given by

$$\lambda_{u,d} = \frac{1}{8}G^2 + \frac{1}{2}Q^2_{(H_u,H_d)}\, {g_Y}'^2, \qquad \lambda_s = \frac{1}{2}Q^2_{H_s}\, {g_Y}'^2, \tag{5.36}$$

$$\lambda_{ud} = -\frac{1}{4}G^2 + Q_{H_u}Q_{H_d}\, {g_Y}'^2 + h_s^2, \qquad \lambda_{us,ds} = Q_{H_s}Q_{(H_u,H_d)}\, {g_Y}'^2 + h_s^2 \tag{5.37}$$

and $\mathcal{G}$ is the loop function which is independent of the renormalization scale and has the following form

$$\mathcal{G}(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) = 2 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right) \tag{5.38}$$

For simplicity we have introduced some quantities which is dimensionless,

$$\rho_u = h_t^2 - \lambda_u, \quad \rho_d = (h_s^2 - \lambda_{ud})/2, \quad \rho_s = (h_s^2 - \lambda_{us})/2, \tag{5.39}$$

and

$$\zeta_u = -\frac{1}{8}(g_2^2 - \frac{5}{3}g_Y^2) + \frac{1}{2}(Q_Q - Q_{u^c})Q_{H_u}\, g_Y', \tag{5.40}$$

$$\zeta_d = \frac{1}{8}(g_2^2 - \frac{5}{3}g_Y^2) + \frac{1}{2}(Q_Q - Q_{u^c})Q_{H_d}\, g_Y', \tag{5.41}$$

$$\zeta_s = -\,(\zeta_u + \zeta_d) \tag{5.42}$$

where the Equations (5.40)-(5.42) are D-term contributions, and also dimensionful

$$\widetilde{m}_u^2 = \zeta_u \delta + h_t^2 A_t (A_t - \mu_{eff} \cot\beta) \tag{5.43}$$

$$\widetilde{m}_d^2 = \zeta_d \delta + h_t^2 \mu_{eff} (\mu_{eff} - A_t \tan\beta) \tag{5.44}$$

$$\widetilde{m}_s^2 = \zeta_s \delta + \frac{v_d^2}{v_s^2} h_t^2 \mu_{eff} (\mu_{eff} - A_t \tan\beta) \tag{5.45}$$

with $\delta = M_{\tilde{Q}}^2 - M_{\tilde{u}^c}^2 + \zeta_u v_u^2 + \zeta_d v_d^2 + \zeta_s v_s^2$.

When we diagonalize (5.34) we see that there are three massive scalar Higgs bosons h, H and H'. The approximate values of these masses and their variations against some model parameters have been computed by doing numerical calculations and steps of this analysis and results are given in the next chapter.

CHAPTER 6

TEVATRON BOUNDS AND EXPECTATIONS FOR THE LHC

At the wake of LHC experiments, it is convenient to study the Higgs boson masses in $U(1)'$ models. The existing bounds from the LEP and Tevatron experiments given in Figure (6.1) can guide one to more likely regions of the parameter space. The LEP experiments (Barate, 2003) have ended with a clear preference for the lightest Higgs boson mass:

$$m_h > 114.4 \,\mathrm{GeV}\,. \tag{6.1}$$

The knowledge of the Higgs mass has recently been further supported by the Tevatron results (Aaltonen, 2010; Dominguez, 2009) which state that the lightest Higgs boson cannot have a mass in the range

$$159 \,\mathrm{GeV} < m_h < 168 \,\mathrm{GeV}\,. \tag{6.2}$$

It is clear that LEP bound influences the parameter spaces of the SM, MSSM and its extensions like $U(1)'$ models. The reason is that the LEP range is covered by all these models of electroweak breaking. However, it is obvious that the Tevatron bound has almost no impact on the MSSM parameter space within which m_h cannot exceed $\sim 135 \,\mathrm{GeV}$. However, the Tevatron bounds can be quite effective for extensions of the MSSM whose lightest Higgs bosons can weigh above $2M_W$. This is the case in $U(1)'$ models (Demir, 2004).

In this work we shall analyze $U(1)'$ models in regard to their Higgs mass predictions and constrained parameter space under the LEP as well as Tevatron bounds by assuming that the Higgs boson searched by D∅ and CDF corresponds to that of the $U(1)'$ models. In course of the analysis, we shall consider the $U(1)'$ model achieved by low-energy considerations as well as by high-energy considerations (the GUT and stringy $U(1)'$ models mentioned before). In each case we shall scan the parameter space to deter-

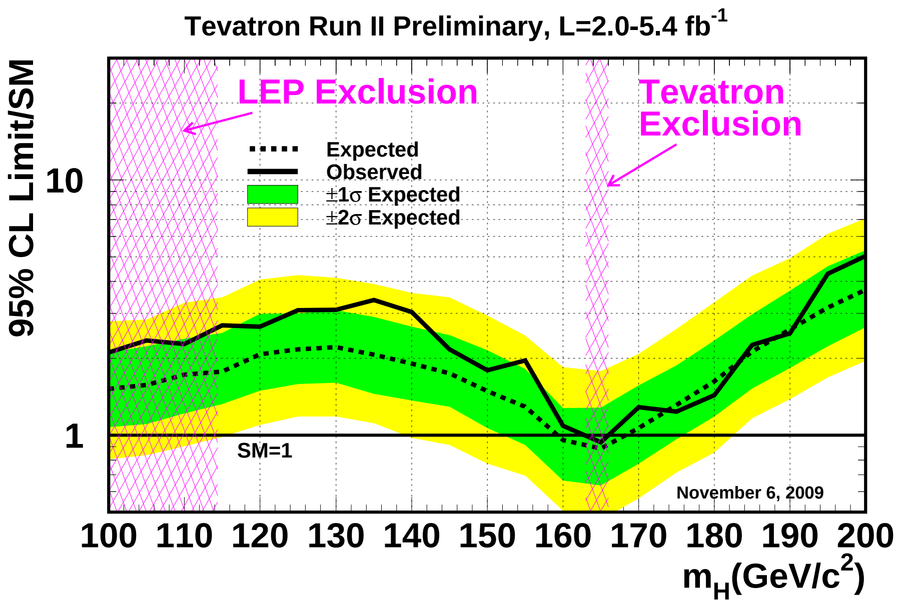

Figure 6.1. Bounds of the Higgs mass arising from the LEP and Tevatron experiments. For the LEP experiment the mass of the Higgs boson should be $m_H > 114,4$ GeV while for the Tevatron experiment the mass range excluded at 95 C.L. for a SM Higgs is $163 < m_H < 166$ GeV , with an expected exclusion of $159 < m_H < 168$ GeV (CDF and D0 Collaboration, 2009).

mine the bounds on the model parameters by imposing the bounds from direct searches.

6.1. Analysis

In this section we shall perform a numerical analysis of Higgs boson masses in order to determine the allowed regions under the LEP and Tevatron bounds (Sert, 2010). In the following we will first discuss the parameter space to be employed, and then we shall provide a set of figures each probing certain parameter ranges in the $U(1)'$ models considered.

6.1.1. Parameters

In course of the analysis, we shall partly scan the parameter space and partly analyze certain parameter regions which best exhibit the bounds from the Higgs mass measurements. We first list down various parameter values to be used in the scan.

U(1)′ Gauge Coupling : The $U(1)'$ models we consider are inherently *unconstrained* in that, irrespective of their low–energy or high-energy origin, we let $U(1)'$ gauge coupling g'_Y to vary in a reasonable range in units of the hypercharge gauge coupling. We thus call all the models we investigate as 'Unconstrained $U(1)'$ Models', or, $UU(1)'$ Models, in short.

We shall be dealing with four different $UU(1)'$ models:

- UU(1)′ from E(6) supersymmetric GUT: The η, N and ψ Models. These models can be obtained from equation 4.6 with the angles $\theta_{E(6)} = \arcsin\sqrt{\frac{3}{8}}$ in η Model, $\theta_{E(6)} = \arcsin\frac{1}{4}$ in N Model, $\theta_{E(6)} = 0$ in ψ Model.
- UU(1)′ from low-energy (solution of the μ problem): This is the low-energy model obtained by taking $Q_{H_u} = Q_{H_d} = Q_{Q_L} = -1$ and hence $Q_{t^c_R} = Q_{d^c_R} = Q_S = 2$, and we call this model as the X Model.

The charge assignments of E(6)-based models can be found in Table 6.1. We use the same symbols with these models but mutate them by giving up the typically-assumed value $g'_Y = \sqrt{\frac{5}{3}(g_2^2 + g_Y^2)}\sin\theta_W$ (obtained by one-step GUT breaking), and changing it in the range g_Y to $2g_Y$. The motivation behind this mutation of the E(6)-based $U(1)'$ groups is that one-step GUT breaking is too unrealistic to follow; the GUT group is broken at various steps as indicated in (4.5). By varying the g'_Y we will treat E(6)-based models as some kind of specific UU' models in which we can probe the impact of different g'_Y values on the lightest Higgs mass.

Unlike the E(6)-based models, we adopt the value of g'_Y from one-step GUT breaking in analyzing the X model. In X model, by the need to cancel the anomalies, we assume that there exist an unspecified sector of fairly light chiral fields, and normalization of the charge and other issues depend on that sector (Cvetic, 1997). Our analysis will be indicative of a generic $U(1)'$ model stemming from mainly the need to evade the naturalness problems associated with the μ problem of the MSSM.

The Gauge and Yukawa Couplings : In $U(1)'$ models, at the tree level one can write $m_h^2 \lesssim a_i + b_i\, h_s^2$ as deduced from Equation (5.11) where a_i, b_i are some constants to be determined from the given value of $\tan\beta$, charge assignments as well as the soft

Table 6.1. Charges of the particles under $U(1)'$ models

	Q_X	$2\sqrt{15}Q_\eta$	$2\sqrt{10}Q_N$	$2\sqrt{6}Q_\psi$
Q_{H_u}	-1	4	-2	-2
Q_{H_d}	-1	1	-3	-2
Q_S	2	-5	5	4
Q_{Q_L}	-1	-2	1	1
$Q_{t_R^c}$	2	-2	1	1
$Q_{d_R^c}$	2	1	2	1

supersymmetry-breaking sector. Hence, for sufficiently large b_i/a_i ratios, one can expect $m_h \propto h_s$. At one-loop level, it is interesting to probe if such a relation also exists for the gauge coupling, Yukawa coupling and other important model parameters. We will be dealing with this issue numerically, by changing the value of g'_Y as stated above.

The Z-Z$'$ Mixing : We shall always require the $Z-Z'$ mixing to obey the bound $|\alpha_{Z-Z'}| < 10^{-3}$ for consistency with current measurements (Abazov , 2008). The collider analyses (Kotwal, 2008) constrain m_{Z_2} to be nearly a TeV or higher with the assumption that Z_2 boson decays exclusively into the SM fermions. However, inclusion of decay channels into superpartners increases the Z_2 width, and hence, decreases the m_{Z_2} lower bound by a couple of $100\ \mathrm{GeVs}$ (Langacker, 2008). But, for simplicity and definiteness, we take $m_{Z_2} \geq 1\ \mathrm{TeV}$ as a nominal value.

Ratio of the Higgs VEVs $\tan\beta$: We fix $\tan\beta$ from the knowledge of $\alpha_{Z-Z'}$ (Demir, 2004). Since the value of the mixing angle $\alpha_{Z-Z'}$ is small as mentioned above, we can determine the value of the $\tan\beta = v_u/v_d$ by using the small angle approximation $\tan(2\alpha_{Z-Z'}) \approx 2\alpha_{Z-Z'}$ as: $\tan^2\beta = F_d/F_u$ where

$$F_u = (2g'_Y/G)Q_{H_u} + \alpha_{Z-Z'}(-1 + (2g'_Y/G)^2(Q_{H_u}^2 + Q_S^2(v_s^2/v^2)))\,,$$

$$F_d = (2g'_Y/G)Q_{H_d} - \alpha_{Z-Z'}(-1 + (2g'_Y/G)^2(Q_{H_d}^2 + Q_S^2(v_s^2/v^2)))\,. \qquad (6.3)$$

Using this expression we find that $\tan\beta$ stays around 1 (this is true as far as v_s is not very large), and thus, we scan $\tan\beta$ values from 0.5 to 5 in E(6)-based models, and in the X Model. The post-LEP analyses of the MSSM disfavors $\tan\beta \sim 1$ yet in U(1)$'$ models there is no such conclusive result. One can in fact, consider $\tan\beta$ values significantly smaller than unity, as a concrete example η model favors $\tan\beta = 0.5$.

The Higgsino Yukawa Coupling : Our analysis respects $h_s = 1/\sqrt{2}$ in our X model; this value is suggested by the RGE analysis of (Cvetic, 1997). However, not only for our X model but also for our mutated E(6) models we allow h_s to vary from 0.1 to 0.8 for determining its impact on the Higgs boson masses. The Higgsino Yukawa coupling h_s determines the effective μ parameter in units of the singlet VEV v_s as in Equation (4.2).

The Squark Soft Mass-Squareds : We scan each of $m_{\tilde{Q}}, m_{\tilde{t}_R}$ and $m_{\tilde{b}_R}$ in $[0.1, 1]$ TeV range. Following the PDG values (Amsler , 2008), we require light stop and sbottom to weigh appropriately: $m_{\tilde{t}_1} > 180$ GeV and $m_{\tilde{b}_1} > 240$ GeV. These bounds follow from direct searches at the Tevatron and other colliders.

Singlet VEV v_s : We scan v_s in $[1, 2]$ TeV range so that m_{Z_2} can be larger than 1 TeV. In doing this we set $\mu_{eff} < 1$ TeV as the upper limit of this parameter. Larger values of μ_{eff} are more fine-tuned in such models than the MSSM (Barger, 2006). Such keen values of v_s and μ_{eff} turn out to be necessary for keeping the mentioned models at the low energy region and also for satisfying the aforementioned constraints.

Trilinear Couplings : In the general scan we vary each of A_t, A_b, A_s in $[-1, 1]$ TeV range, independently. This is followed by a specific scan regarding Tevatron bounds where the trilinears and soft masses of the scalar quarks are assigned to share some common values. We do this for all of the models we are considering.

These parameter regions will be employed in scanning the parameter space for determining the allowed domains. In addition to and agreement with these, we shall select out certain parameter values to illustrate how strong or weak the bounds from Higgs mass measurements can be. The results are displayed in a set of figures in the following subsection.

6.1.2. Scan of the Parameter Space

In this subsection we present our scan results for various model parameters in light of the Tevatron and LEP bounds on the lightest Higgs mass. We start the analysis with a general scan using the inputs mentioned in the previous subsection. This will allow us to perform a specific search concentrated around the Tevatron exclusion limits. In both of the scans we will present the results for X model first, which is followed by the E(6)-based models η, N and ψ models.

Related with the general scan we present Figure 6.2 wherein h_s, g'_Y and μ_{eff} are variables on the surface (The only exception is X model for which g'_Y is taken at

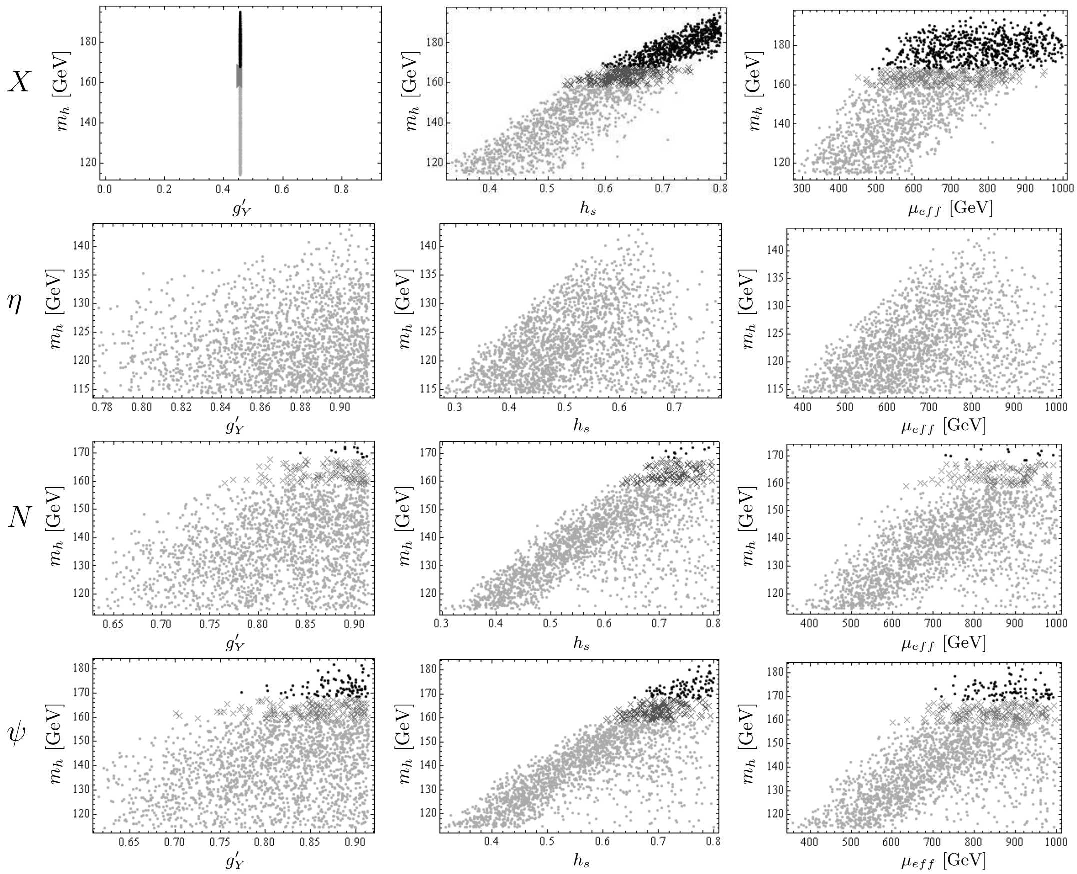

Figure 6.2. The plots for the X,η, N and ψ models (from top to bottom). The mass of the lightest Higgs boson against the gauge coupling g'_Y (left-panels), Higgsino Yukawa coupling h_s (middle-panels), and effective μ parameter (right-panels).

its GUT normalized value.). The remaining variables, whose ranges were mentioned in the previous section, vary in the background. In Figure 6.2, shown are the variations of the lightest Higgs boson mass against the gauge coupling g'_Y (left-panels), Higgsino Yukawa coupling h_s (middle-panels), and the effective μ parameter μ_{eff} (right-panels). The shading convention is such that the points giving $m_h > 168$ GeV are shown by black dots, those yielding 114.4 GeV $\leq m_h \leq 159$ GeV by grey dots, and those yielding 159 GeV $\leq m_h \leq 168$ GeV by grey crosses.

As are seen from the left panels of Figure 6.2, increase in the g'_Y gives rise to higher upper bounds on m_h for E(6)-based models. The same behavior, though not shown explicitly, occurs in the X model (which already yields m_h values as high as 195 GeV). Excepting the η model, the E(6)-based models are seen to accommodate Higgs boson masses larger than the Tevatron upper bound when g'_Y rises to extreme values above ~ 0.8.

Needless to say, the regions with grey dots are followed by regions with grey crosses (the forbidden region), as expected from the dependence of the Higgs boson mass on g'_Y. The η model does not touch even the Tevatron lower bound of the excluded region for the parameter values considered.

Depicted in the middle panels of Figure 6.2 is the variation of the Higgs boson mass with the Higgsino Yukawa coupling for the models considered. Clearly, h_s parameter is more determinative than g'_Y in that m_h tends to stay in a strip of values for the entire range of h_s. Indeed, upper bound on m_h (and its lower bound, to a lesser extent) varies linearly with h_s for X, N and ψ models. This is also true for the η model at least up to $h_s \sim 0.65$. In general, Tevatron bounds divide h_s values into two disjoint regions separated by the forbidden region yielding m_h values excluded by the Tevatron results. One keeps in mind that, in this and following figures, the η model serves to illustrate E(6)-based models yielding a genuine light Higgs boson: The Higgs boson stays light for the entire range of parameter values considered. At least for the X model, one can write

$$159 \gtrsim m_h \gtrsim 114.4 \;\Rightarrow\; h_s \;\in [0.3, 0.7] \;\text{ and }\; m_h \gtrsim 168 \;\Rightarrow\; h_s \;\in [0.6, 0.8] \qquad (6.4)$$

from the distribution of the allowed regions (top middle panel). More precisely, the Higgsino Yukawa coupling largely determines the ranges of the Higgs mass in that while m_h barely saturates the lower edge of the Tevatron exclusion band for $h_s < 0.52$, it takes values above the Tevatron upper edge for $h_s > 0.58$. In other words, Tevatron bound divides h_s ranges into two regions in relation with m_h values: The h_s values for low m_h ($114.4 \text{ GeV} \leq m_h \leq 158 \text{ GeV}$) and those for high m_h ($m_h > 168 \text{ GeV}$). This distinction is valid for all the variables we are analyzing.

The variation of the Higgs boson mass with the effective μ parameter is shown in the right-panels for Figure 6.2, for each model. It is clear that $\mu_{eff} \gtrsim 300\,\text{GeV}$ for the LEP bound to be respected. On the other hand, one needs $\mu_{eff} \gtrsim 500\,\text{GeV}$ for m_h to touch the lower limit of the Tevatron exclusion band in the X model. Similar conclusions hold also for the mutated E(6) models: $\mu_{eff} \gtrsim 700 \text{ GeV}$ for ψ and N models (while the forbidden Tevatron territory is never reached in the η model). The η model is bounded by LEP data only (at least within the input values assumed for which we considered $v_s \leq 2$ TeV).

From the scans above we conclude that:

- All models are constrained by the LEP bound, that is, each of them predict Higgs masses below 114.4 GeV for certain ranges of parameters.

- The X model, a genuine low-energy realization of UU(1)$'$ models based solely on the solution of the μ problem, yields large m_h values, and thus, violated the Tevatron forbidden band low values of g'_Y, h_s and μ_{eff} compared to the mutated E(6)-based models. The latter require typically large values of g'_Y, h_s and μ_{eff} for yielding m_h values falling within the Tevatron territory (Meanwhile, this can happen only if $g'_Y \gtrsim 0.77$ in N model and $g'_Y \gtrsim 0.7$ in ψ model with a Yukawa coupling saturating $h_s \gtrsim 0.62$). In fact, the η model does not even approach to the 159 GeV border so that it does not feel Tevatron bounds at all. There is left only a small parameter space wherein m_h exceeds 159 GeV for ψ and N models. One can safely say that for 'small' g'_Y and h_s the E(6)-based models predict m_h to be low, significantly below 159 GeV. In other words, Tevatron bounds shows tendency to rule out non-perturbative behavior of E(6)-based models.

- One notices that heavy Higgs limit typically require large μ_{eff} (close to TeV domain) and thus one expects Higgsinos to be significantly heavy in such regions. The LSP is to be dominated by the gauginos, mainly. In such regions, one expects the physical neutralino corresponding to $\widetilde{Z}'$ to be also heavy due to the fact that $\widetilde{Z}'$ mixes with $\widetilde{S}$ by a term proportional to $h_s v_s$ (Ali, 2009). Therefore, the light neutralinos are to be dominantly determined by the MSSM gauginos.

Using the grand picture reached above, we now perform a point-wise search aiming to cover critical points wherein Tevatron exclusion is manifest. We project implications of these exclusions to scalar fermions and other neutral Higgs bosons. But, for doing this we first fix certain variables, and by doing so, we get rid of overlapping regions (seen in surface parameters while others running in the background).

From Figure 6.2, we find it sufficient to consider values around $h_s \sim 0.7$ and $g'_Y \sim 2g_Y$. More precisely, we consider Higgsino Yukawa couplings as $h_s = 0.65, 0.5, 0.7$ and 0.7 for X, η, N and ψ models, respectively. We set $g'_Y = 1.9 g_Y$ for all three mutated E(6) models, while we keep it as in Figure 6.2 for the X model.

Figure 6.3 shows variations of the m_h and scalar top quark masses ($m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$) with μ_{eff} and M_{Z_2}. Our shading convention is the same as in Figure 6.2. The inputs are selected as: $m_{common} = m_{\tilde{Q}} = m_{\tilde{t}_R} = m_{\tilde{b}_R} = -A_t = -A_b = -A_s = 0.2$ to $1\,\mathrm{TeV}$ with increments 200 GeV in N and ψ models. In X and η models we scan m_{common} from 0.5 to $1\,\mathrm{TeV}$ with increments 100 GeV. These inputs are also used in the following figure. In any panel of the figures we observe a hierarchy such that largest m_{common} value

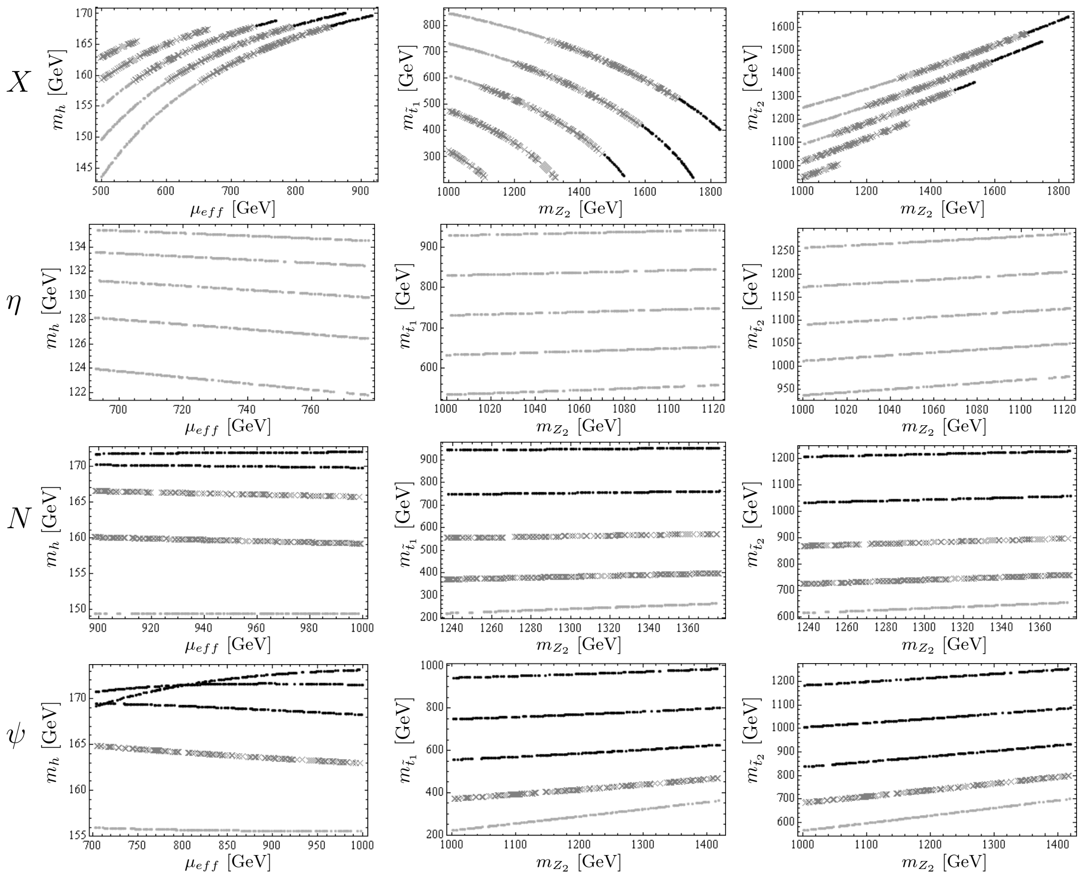

Figure 6.3. The mass of the lightest Higgs boson against the effective μ parameter (left-panels), the mass of the light scalar top $m_{\tilde{t}_1}$ against the mass of the Z_2 boson (middle-panels), and the mass of the heavy scalar top $m_{\tilde{t}_2}$ against the mass of the Z_2 boson (right-panels) in X, η, N and ψ models (top to bottom).

corresponds to the largest m_h value (topmost data lines) which is fixed at 1 TeV.This is the targeted search. Now, as can be seen from the left panels of Figure 6.3, the effective μ parameter should satisfy $\mu_{eff} > 500$ GeV in X model, while others demanding higher values. This is due to already fixed h_s parameter value. In this figure, the impact of Tevatron exclusions is seen clearly (gray-crosses) on scalar fermions (middle and right panels of X, N and ψ models), too. It is interesting to check model dependent issues for this sector because the scalar fermions shall be important for discriminating among the supersymmetric models (even among the U(1)$'$ models) at the LHC and ILC. The goal of Figure 6.3 is to serve this aim, in which scalar quark masses are plotted against varying Z_2 boson mass (middle and right-panels). The correlation between sfermion masses and M_{Z_2} comes mainly from the U(1)$'$ D-term contributions (proportional to $g_{Y'}^2 v_s^2$) to the LL and

RR entries of the sfermion mass-squared matrices. There are also F-term contributions proportional to $h_s v_s$ to LR entries but their effects are much smaller compared to those in the LL and RR entries (see Eq. (5.19) for details). This is an important effect not found in the minimal model: Variation of sfermion masses with μ probes only the LR entry in the MSSM given by (Baer, 2006)

$$\begin{aligned} M^2_{\tilde{t}LL} &= m^2_{\tilde{Q}} + m^2_t + M^2_Z \cos 2\beta(\frac{1}{2} - \frac{2}{3} sin^2\theta_W) \\ M^2_{\tilde{t}RR} &= m^2_{\tilde{t}_R} + m^2_t + M^2_Z \cos 2\beta(\frac{2}{3} sin^2\theta_W) \\ M^2_{\tilde{t}LR} &= M^2_{\tilde{t}RL} = m_t(-A_t + \mu \cot\beta) \end{aligned} \tag{6.5}$$

It is in such extensions of the MSSM that one finds explicit dependence on μ_{eff} in not only the LR entries but also in LL and RR entries; effects of μ_{eff} are more widespread than in the minimal model where μ is regarded as some external parameter determined from the electroweak breaking condition.

From Figure 6.3 one concludes that variations of m_h and $m_{\tilde{t}_{1,2}}$ are much more violent in X model than in the E(6)-based models. In the X model changes in M_{Z_2} and μ_{eff} influence Higgs and stop masses violently so that allowed and forbidden regions are seen rather clearly. In E(6)-based models what we have nearly constant strips, and thus, m_h and $m_{\tilde{t}_{1,2}}$ remain essentially unchanged with μ_{eff} and M_{Z_2}. Moreover, in mutated E(6) models the forbidden regions and allowed regions fall into distinct strips, signalling thus the aforementioned near constancy of the Higgs and stop masses.

From Figure 6.3 it is possible to read out certain likely ranges for stop and Higgs boson masses, which will be key observables in collider experiments like LHC and ILC. Indeed, in X model one deduces that

- Higgs in low-mass region $\Longrightarrow m_{\tilde{t}_1} \in [600, 800]$ GeV and $m_{Z_2} \in [1.0, 1.3]$ TeV,
- Higgs in high-mass region $\Longrightarrow m_{\tilde{t}_1} \in [200, 550]$ GeV and $m_{Z_2} \in [1.5, 1.8]$ TeV.

Therefore, in principle, taking the X model as the underlying setup, one can determine if Higgs is in the low- or high-mass domains by a measurement of the scalar top quark masses. For instance, if collider searches exclude low-mass light stops up to ~ 600 GeV then one immediately concludes that the Higgs boson should be light, *i. e.* below $2M_W$.

Contrary to model X, E(6)-based models N and ψ allow the Z' mass to be more confined, *i.e.* the mass of the Z_2 boson is in $\sim [1, 1.4]$ TeV range within these two models. Furthermore, these two models can rule out $m_{\tilde{t}_1}$ around $\sim$ $[300, 500]$ GeV (One keeps in mind, however, that in these models low (high) stop mass values are related with low (high) m_h values, in contradiction with the X model). Besides this, all three of X, N and ψ models exploration of ***high-mass*** region demands larger values for $m_{\tilde{t}_2}$. One notices that largest (smallest) splitting between $m_{\tilde{t}_2}$ and $m_{\tilde{t}_1}$ is observed in X (ψ) model.

As an extension of the MSSM, the present model predicts 3 CP-even Higgs bosons: h, H and H'. There is no analogue of H' in the MSSM. The model predicts one single pseudoscalar Higgs boson A as in the MSSM. In the decoupling regime *i. e.* when heavier Higgs bosons decouple from h one expects the mass hierarchy $m_{H'} \sim m_{Z_2} \gg m_H \sim m_A \gg m_h$. It is thus convenient to analyze the model in regard to its Higgs mass spectra to determine in what regime the model is working. To this end, we depict variations of m_h with m_H, m'_H and m_{Z_2} in Figure 6.4. The notation is such that m_A and $m_{H'}$ are denoted by grey dots, m_H and m_{Z_2} by black dots. For quantifying the analysis we define the ratios $R_1 \equiv \frac{m_H}{m_A}$, $R_2 \equiv \frac{m_{Z_2}}{m_{H'}}$ which are, respectively, shown by gray and black dots in Figure 6.4. The input parameters are taken as in Figure 6.3.

In Figure 6.4, shown in the leftmost column are variations of m_h with m_H (black dots) and with m_A (grey dots). It is clear that, the X and N models are well inside the decoupling regime for the parameter ranges considered. On the other hand, the ψ and η models, especially the η model, are far from their decoupling regime. In this regime, the lightest Higgs can weigh well above its lower bound. One notices that, A and H bosons exhibit no sign of degeneracy in the η model.

The variations of m_h with m'_H and m_{Z_2} are shown in the middle column of Figure 6.4. One observes that grand behavior is similar to those in the first column. One, however, makes the distinction that m_h depends violently on m'_H and m_{Z_2} in X and η models while it stays almost completely independent for ψ and N models.

All the properties summarized above are quantified in the third column wherein m_h is plotted against R_1 and R_2. The degree to which $R_{1,2}$ measure close to unity give a quantitative measure of how close the parameter values are to the decoupling regime. One notices that they differ significantly from unity in η and ψ models. In summary, m_A/m_H ratio drops to $\sim$ 0.8 in η model. This is also true for $m_{Z_2}/m_{H'}$. It is interesting to observe that R_1 and R_2 behave very similar in most of the parameter space. This figure depicts the heavy model dependency of neutral Higgs masses.

Experiments at the LHC and ILC will be able to measure all these Higgs boson

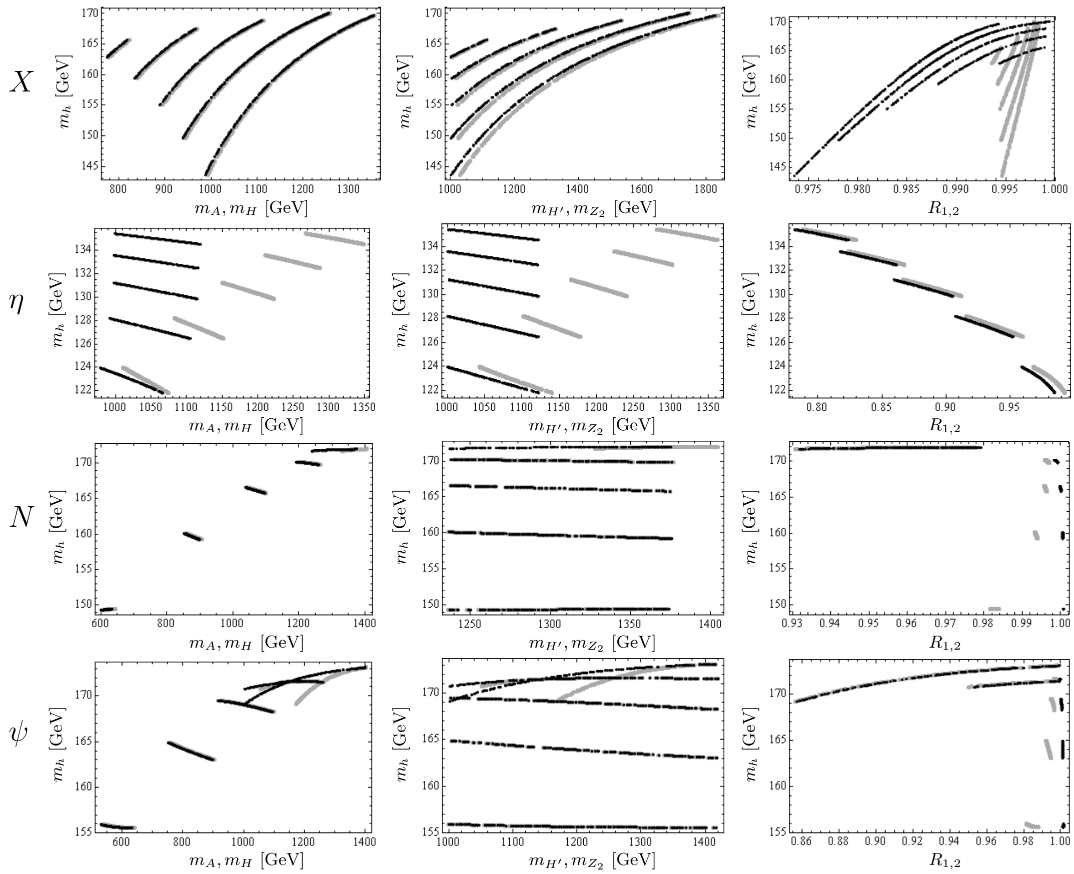

Figure 6.4. Variations of the lightest Higgs boson mass m_h with those of the heavy CP-even Higgs scalars H, H' and of the CP-odd scalar A. Also given is the dependence of m_h on the Z_2 boson mass. In the decoupling region, $m_H \sim m_A$ and $m_{H'} \sim m_{Z_2}$.

masses, couplings and decay modes (Barger, 2006). Clearly, N and ψ (especially ψ) model yield lightest of H, A among all the models considered. In course of collider searches, these two models will be differentiated from the others by their relatively light heavy-Higgs sector.

CHAPTER 7

CONCLUSION

In this thesis work we examined the Higgs sector in unconstrained $U(1)'$ model. $U(1)'$ model is a gauge-extended minimal supersymmetric model which contains an extra gauge field (B'_μ) and a singlet, chiral SM Higgs field (S). Therefore, this model has a neutral Higgs boson (H') in addition to the neutral Higgs bosons of the MSSM (h and H) and an extra gauge boson Z' to the $W^\pm$ and Z gauge bosons found in the MSSM. We renamed this model as unconstrained $U(1)'$ model ($UU(1)'$) since we let gauge coupling of the $U(1)'$ model g'_Y to vary in a reasonable range in unit of the hypercharge gauge coupling g_Y.

Within this thesis content before $U(1)'$ model, in Chapter 2 and 3 we explained the general structure of the Standard Model (SM) and minimal supersymmetric extension of the SM (MSSM) including their Higgs sector and their problems to understand why we need $U(1)'$ models . Minimal supersymmetric model is introduced to solve the Hierarchy problem of the SM which states that Higgs boson is unstable under quantum corrections. Although MSSM can solve this problem, it suffer from the μ problem which is the main motivation of the $U(1)'$ model. Then we gave detailed explanation for the motivation of the $U(1)'$ model and its structure in Chapter 4. Moreover, Higgs sector of the $U(1)'$ model which is related to our main work was explained in Chapter 5 with details.

In Chapter 6 we analyzed the lightest Higgs boson mass against various model parameters and particles masses. Firstly, we examined the variation of the lightest Higgs mass against the gauge coupling of the $U(1)'$ model g'_Y, Higgsino Yukawa coupling constant h_s and effective μ parameter μ_{eff} as seen in Figure 6.2. All these variables belong to the $U(1)'$ model. MSSM does not contain these, only μ parameter is included instead of μ_{eff} parameter. From Figure 6.2 we can say that the LEP bound is satisfied for all model we have considered as in the MSSM. We can also obtain this result for the MSSM from the analytical calculation. Contrary to the LEP bound, we can conclude that the Tevatron bound is satisfied by the three model X, ψ and N since η model gets smaller values than lower bound of the Tevatron exclusion region. While the X model is the most sensitive model to the Tevatron bound, the ψ and N model satisfy it only at the higher values of the variables. However, MSSM is not sensitive to the Tevatron bound since the maximum mass of the lightest Higgs boson in the MSSM is 135 GeV inspite of radiative corrections.

We can conclude from the Figure 6.2 that Tevatron bound divide all model parameters into two distinct regions, low-mass region and high-mass region for the mass of the lightest Higgs boson. For instance, in the X model for the low-mass region, Higgsino Yukawa coupling should have smaller values than 0.52 while for the high-mass region its value should be larger than 0.58.

Using conclusions of Figure 6.2 which had been obtained by scanning the parameter space, we selected some specific values and performed our desired work which is shown in Figure 6.3 and 6.4. These certain parameter ranges were chosen from ranges satisfying higher values than upper limit of the Tevatron bound. As a result of this analysis we obtained more precise values which show excluded parameter regions satisfying the Tevatron exclusion bound clearly. We can state that the X model exhibit the best behavior in this sense. The ψ and N model also show the excluded and allowed region clearly, however, the η model can not reach even the Tevatron lower limit. Therefore, we can conclude that certain UU' models such as the η model can be the first one to be ruled out.

Figure 6.4 which represents the variations between the masses of the scalar top quarks and mass of the Z' boson denotes that we can determine whether the lightest Higgs mass is in the above or below the Tevatron exclusion region by measuring the mass of the scalar top quarks except for the η model. We can deduce from this figure this situation is also valid for the mass of the Z' boson only in the X model. From the last figure we can see that in the X model the variation of the m_h depends on violently the variation of the heavier Higgs bosons and Z' boson masses. However, in the N and ψ models m_h remains almost stable with changes of the variables. Therefore, we deduce that the relation among masses of the neutral Higgs bosons depends on the model.

Experimental data we have used for our analysis belongs to data obtained on 6 November 2009. After our analysis new results are obtained as seen in Figure 7.1. According to this figure not only the previous excluded region by the Tevatron is extended, but also Tevatron excludes additional region around the LEP limit. Now the lightest Higgs mass can not get the values in the below ranges:

$$158 < m_h < 175 \text{ GeV} \quad \text{and} \quad 100 < m_h < 109 \text{ GeV} \tag{7.1}$$

in the 95% confidence level. The expected value of the first bound is $156 < m_h < 173$ GeV.

As a result of this thesis work we restricted the allowed regions for the parameters

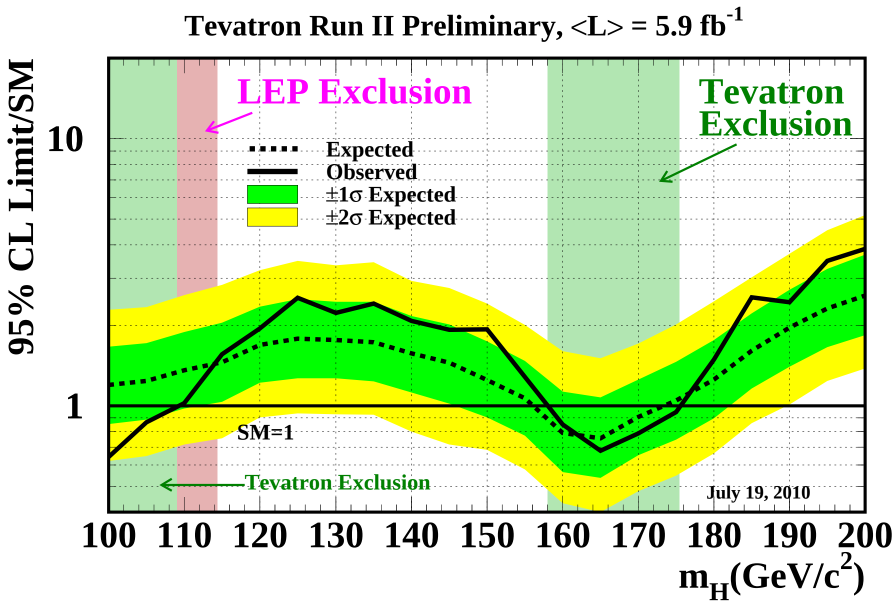

Figure 7.1. Bounds of the Higgs mass arising from the LEP and Tevatron experiments. For the LEP experiment the mass of the Higgs boson should be $m_H > 114, 4$ GeV while for the Tevatron experiment the mass ranges excluded at 95% C.L. for a SM Higgs are $100 < m_h < 109$ GeV and $158 < m_H < 175$ GeV , with an expected exclusion of $156 < m_H < 173$ GeV (CDF and D0 Collaboration, 2010).

which we had considered. The results of this thesis, though unavoidably carry a degree of model dependence, can be directly tested at the LHC (and at the ILC with much higher precision). If measurements of the Higgs mass at the LHC get large values like $130 - 140$ GeV or above, we can interpret this situation as presence of extensions of the MSSM like $UU(1)'$ models. Depending on the new exclusion limits, we might find more regions of parameter space excluded.

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APPENDIX A

SPONTANEOUS SYMMETRY BREAKING (SSB)

According to gauge theories the mass terms of the gauge bosons and chiral fermions are not allowed. However, experimentally it is known that fermions and gauge bosons such as, electron and $W^{\pm}$, Z^0 bosons have mass.

Therefore, in order to generate masses gauge invariance must be spontaneously broken. To explain how the symmetry is spontaneously consider the Lagrangian below:

$$\mathcal{L} = \partial_\mu H^* \partial^\mu H - V(H^* H) \tag{A.1}$$

with

$$V(H) = \mu^2 |H|^2 + \lambda |H|^4 \tag{A.2}$$

where λ which is quartic coupling constant should be larger than zero $\lambda > 0$ in order to have a ground state for the potential.

This Lagrangian has two main properties (Pich, 2005):

1. It is invariant under a group G of transformations,

2. It has two degenerate states with minimal energy as in Figure A.1 if $\mu^2 < 0$. The vacuum expectation value of the Higgs fields corresponding to these energies are calculated by using the minimization condition $\frac{\partial V}{\partial H} = 0$. If one of these degenerate states is chosen, one says that the symmetry is spontaneously broken.

Firstly let's examine the condition when H is a scalar and the invariance of the vacuum states under $H \rightarrow -H$ symmetry which Lagrangian is invariant.

There are two possibilities with respect to the sign of the μ^2 parameter:

1. If $\mu^2 > 0$ the potential has one minimum at $H = 0$ and this vacuum condition is invariant under $H \rightarrow -H$ symmetry as seen from Figure A.1.(a).

2. If $\mu^2 < 0$ the potential has two degenerate minimum at $\langle H^0 \rangle = \pm\sqrt{-\frac{\mu^2}{2\lambda}} = \pm\frac{v}{\sqrt{2}}$ and vacuum conditions are not invariant under $H \rightarrow -H$ symmetry as in Figure A.1.b. When we choose one of the vacuum states (for example the positive one, $\langle H^0 \rangle =$

Figure A.1. Shape of scalar potential when (a) $\mu^2 > 0$, (b) $\mu^2 < 0$ in two dimension.

$\sqrt{-\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$) we cause breaking of the symmetry. To overcome of this non-invariance we expand the Higgs field around vacuum state as follows:

$$H^0(x) = h(x) + \langle H^0 \rangle \tag{A.3}$$

where $h(x)$ represents the fluctuation around vacuum state as we can see from Figure A.2.a. We put this expansion into the potential energy and then we obtain that there arise one "massive Higgs Boson (h)" with mass $m_h = \sqrt{-2\mu^2}$. The calculation about these can be found in reference (Abers, 1973).

This condition is valid for discrete symmetries in two dimensions. When we consider continuous symmetry in three dimensions, we should take into account the phase transformations which are continuous.

A.1. Global Phase Transformations

Lagrangian is invariant under the global phase transformation, $H(x) \rightarrow e^{i\theta} H(x)$ transformation where θ is independent of x. In three dimension the Higgs field should be complex and in this condition one of the axis represents the real parts of the Higgs fields, while one of them shows the imaginary part of them as seen in Figure A.2.b.

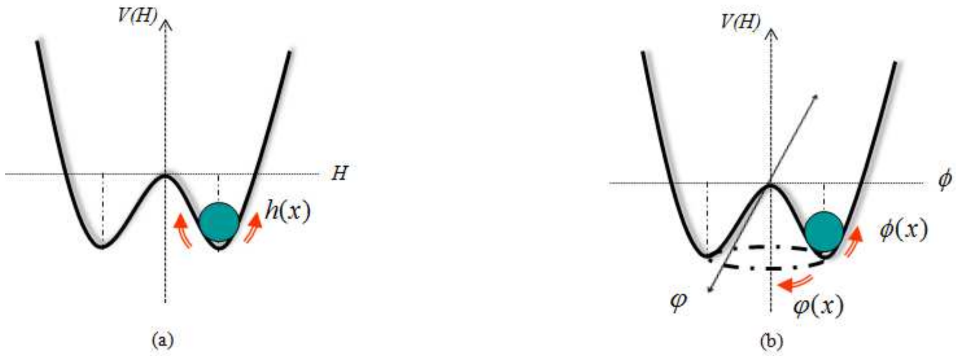

Figure A.2. (a) Fluctuations in two dimension, (b) Shape of scalar potential and fluctuations in three dimension for $\mu^2 < 0$.

The Higgs field can be expanded as below at the vacuum state:

$$H^0(x) = \frac{1}{\sqrt{2}}[v + \phi(x) + i\varphi(x)] \tag{A.4}$$

When we do the same calculation as explained above we see that ϕ becomes a “massive Higgs Boson (h)” and φ arise as a ”massless *Goldstone Boson*”.

Goldstone Theorem: If a Lagrangian is invariant under a continuous symmetry group G, but the vacuum is only invariant under a subgroup $H \subset G$, then there must exist as many massless spin-0 particles (Goldstone bosons) as broken generators (generators of G which do not belong to H) (Pich, 2005).

A.2. Local Phase Transformations

Lagrangian is not invariant under the local phase transformations given by

$$H(x) \rightarrow e^{i\theta(x)} H(x). \tag{A.5}$$

Firstly we must obtain an invariant Lagrangian under the given symmetry. To make Lagrangian invariant ”Covariant Derivative” is defined instead of partial derivative.

$$D_\mu = \partial_\mu - igTA_\mu \quad \text{with} \quad A'_\mu = A_\mu - \partial_\mu \theta(x) \tag{A.6}$$

where g, T and A_μ are gauge coupling, generators corresponding to symmetry groups and a vector field, respectively. For the detailed explanation look at (Abers, 1973). After Lagrangian becomes invariant, we can consider the vacuum states: there are also two vacuum states and Higgs fields can be expanded around a chosen vacuum state as it is in the global phase transformation. When we examine the potential energy and do some calculations, we see that there arise one “massive Higgs Boson (h)” and one “massive Vector Field (A_μ)” while there is no massless Goldstone Boson. This mechanism is known as “**Higgs Mechanism**”. Massless Goldstone boson is eaten by vector field and become longitudinal polarization state of it.