Truth Ratios of Syllogistic Moods

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***Abstract*—The syllogistic system consists of 256 moods, of which only 24 have been recognized as true. From a set-theoretical point of view, a mood can be represented with three sets and their possible relationships. Three sets can have up to seven sub-sets or spaces. In an earlier work we have used 41 permutations of the spaces, out of which every mood matches an individual number as true or false cases. The truth ratio of a mood is then calculated, by relating the true and false cases with each other. In this work we revise the previously presented properties of the moods and the syllogistic system, this time by using the maximum possible cover, which consists of 96 distinct space permutations. Our results mostly verify our previous findings, like the additional true mood anasoy, the inherently symmetric truth distribution of the moods. Additionally we have revealed some new properties, like the equivalence of some moods, which reduces the system to 136 distinct moods.**

Keywords—categorical syllogism; syllogistic system, syllogistic reasoning

I. INTRODUCTION

The earliest known works related to formal logic belongs to Aristotle. Aristotle's logic was built on the principle of deduction (sullogismos) [1]. In order to understand a deduction and its content it is necessary to investigate Aristotle's whole theory. According to Aristotle, syllogism is "discourse in which, certain things being stated something other than what is stated follows of necessity from their being so" [5]. But in practice he specified the syllogism as a structure, that contains two premises and a conclusion, each of which is a categorical proposition. The subject and predicate of the conclusion each occur in only one of the premises, together with a middle term that is found in both premises but not in the conclusion. A syllogism thus argues that because subject and predicate are related in certain ways to middle term in the premises, they are related in a certain way to one another in the conclusion.

In recent years various approaches to essential logical problems, in particular, to classical syllogistics, were considered in philosophy, linguistics and certain applications of artificial intelligence. However, there are still many unresolved problems in this field, such as issues with weakness and strictness of universal end extensional quantifiers [6], reasoning with unlimited numbers of terms, relative quantifiers and so on.

The current approaches to the extension of the syllogistic reasoning can be divided into two independently developing groups:

- approaches, based on introducing new crisp [14] or fuzzy quantifiers [12] (such as much, many, few...), in additional to the classical ones.
- approached, based on the expanding of number of terms and premises (N), consisting syllogism (N >2) without introducing new quantifiers [15].

As far as we know, there is no general framework, dealing with the two approaches at the same time. Particularly, reasoning with the large numbers of premises with crisp and fuzzy quantifiers is considered in [16]. However, their methods are not well formalized and designed framework looks very abstract and difficult to use. Thus, syllogistic reasoning remains incomplete tack in terms of general solution for reasoning.

The reasoning with unlimited numbers of terms and premises in the reasoning scheme is beyond the scope of the current work. In this work we present generic solution for the problem of classical syllogistic reasoning with the possible extension of developed scheme by intermediate quantifiers, discussed in detail in [14], [2], [4].

This paper presents an algorithm for deciding syllogistic cases that can be used in various implementations of automated reasoning. Firstly, categorical syllogisms are discussed briefly. Thereafter a mathematical model for the representation of syllogistic cases is proposed along with an algorithmic approach for syllogistic reasoning.

II. CLASSICAL SYLLOGISMS

In this section a brief description of the structure of categorical syllogism, that is core of proposed syllogistic system.

A. Categorical Syllogisms

As an inference scheme, a syllogism may generally be expressed in the form:

$$\begin{array}{c} \psi_1 A\text{'s are } B\text{'s} \\ \underline{\psi_2 C\text{'s are } D\text{'s}} \\ \psi_3 E\text{'s are } F\text{'s} \end{array}$$

where ψ_1, ψ_2 and ψ_3 are numerical, or more general, fuzzy quantifiers (e.g. few, many, most [10]), and A, B, C, D, E and F are crisp or fuzzy predicates. The predicates A, B, … F are assumed to be related in a specific way, giving rise to different types of syllogisms [12].

A categorical syllogism can be defined as a logical argument that is composed of two categorical propositions for deducing a logical conclusion, where the propositions and the conclusion each consist of a quantified relationship between two objects [3].

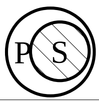
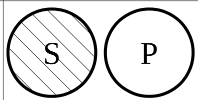
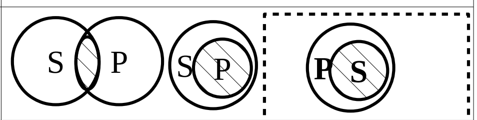
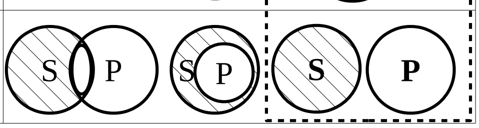
B. Syllogistic Propositions

A syllogistic proposition or synonymously categorical proposition specifies a quantified relationship between two classes. We shall denote such relationships with the operator ψ. Four different types are distinguished $\psi=\{A, E, I, O\}$:

A	Universal Affirmative	All A are P	**E**	Universal Negative	All A are not P
I	Particular Affirmative	Some S are P	**O**	Particular Negative	Some S are not P

The set – theoretical representation of the syllogistic propositions by Euler diagrams is presented on Table I. Propositions I and O have three cases respectively. The cases, bounded by the dashed line are controversial in the literature. Some do not consider them as valid [3] and some do [11]. The additional case for proposition I is similar to the case for the proposition A, so we can consider A as a special case for I. In the same way, E is a special case for O. Inclusion of additional cases for the propositions I and O is related with inclusive logic, exclusion of those cases refers to the exclusive logic. As was shown in our work, only in case of using of inclusive logic, the result corresponds with the classical syllogistic.

TABLE I. SYLLOGISTIC PROPOSITIONS CONSIST OF QUANTIFIED OBJECT RELATIONSHIPS.

Opera	**Propositio**	**Set-Theoretic Representation of Logical Cases**
A	All S are P	
E	All S are not P	
I	Some S are P	
O	Some S are not P	

C. Syllogistic Figures

A syllogism consists of the three propositions: major premise, minor premise and conclusion. The first proposition consist of a quantified relationship between the classes M and P, the second proposition of S and M, the conclusion of S and P (see Table II). The letter S is the subject of the conclusion, P is the predicate of the conclusion, and M is the middle term. Note the symmetrical combinations of the classes. Since the proposition operator may have four values for ψ, 64 syllogistic moods are possible for every figure and 256 moods for all four figures in total.

TABLE II. SYLLOGISTIC FIGURES

Figure Name	**I**	**II**	**III**	**IV**
Major Premise	$M\psi_1P$	$P\psi_1M$	$M\psi_1P$	$P\psi_1M$
Minor Premise	$S\psi_2M$	$S\psi_2M$	$M\psi_2S$	$M\psi_2S$
Conclusion	$S\psi_3P$	$S\psi_3P$	$S\psi_3P$	$S\psi_3P$

III. Algorithmic Representation

Our previous algorithmic calculation of the truth ratios of moods was based on 41 space permutations [7], [8] and the 256 moods matched them in total 2624 times. Our current calculations are based on 96 space permutations, which the moods match 6144 times. The algorithm is explained here ones more, along with the revised properties.

A. Set-Theoretical Analysis

For three symmetrically intersecting sets there are in total 7 possible sub-sets in a Venn diagram (Fig. 1). If symmetric set relationships are relaxed and the three sets are named, for instance with the syllogistic terms P, M and S, then 109 set relationships are possible [9]. Excluding sets where at least 2 sets are equivalent we end up with 96 sets. These 96 relationships are distinct, but re-occur in the 256 moods as basic syllogistic cases. The 7 sub-sets in case of symmetric relationships and the 96 distinct set relationships in case of relaxed symmetry are fundamental for the design of an algorithmic decision of syllogistic moods.

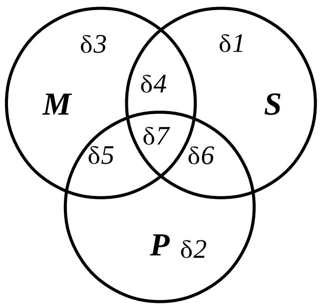


Fig. 1. Venn – diagram for 3 symmetrically intersecting sets.

We have pointed out earlier that, including the additional cases for the syllogistic propositions I and O (inclusive logic), is required by the algorithm to calculate correctly according to the classical notion (6 true moods for each figure). Without these cases (exclusive logic), there are 11 valid moods in total. In case of inclusive logic, there are 25 valid moods. Found solutions are fully consistent with the known valid syllogistic moods, but additionally we have found out that AAO for Figure 4 is also true. The obtained system is absolutely symmetrical (there are 25 valid and 25 fully invalid moods,

moods are symmetrical by number of true/false cases). The reason for that is that the syllogistic propositions are basically a symmetric sub-set of the in total 16 distinct set relationships between two named sets. Therefore the additional cases for I and O are required to complement the symmetric relationships between the syllogistic propositions.

B. Data Structure for Case Representation

Based on theses 7 sub-sets, we have proposed a data - structure for modeling of the syllogistic cases. Each case is presented as a sequence of 7 bits. Each bit is related with a particular sub-set in a Venn – diagram (Table III). This structure is efficient in case of memory consumption and processing, 7 bits are minimal set that allows to fully recover the correspondent Venn - diagram.

TABLE III. IDENTIFICATION OF THE SEVEN POSSIBLE SUBSETS OF THREE SETS AS DISTINCT SPACES.

Sub-Set Number	δ1	δ2	δ3	δ4	δ5	δ6	δ7
Syllogistic Case	S-M-P	P-S-M	M-S-P	(M∩S)-P	(S∩P)-M	(M∩P)-S	S∩P∩M

C. Case generation

The efficient way of cases generation is using of recursive procedure, with recursion depth is equivalent to number of sub-sets. On each level of recursion one bit is added into the described data structure. On the last level the verification procedure is performed, and if current case passed the verification, it is added to the resulting set of cases.

In our approach the goal is to generate all possible combinations (cases) from 7 elements, where elements are from {0; 1} (the total number such of combinations is 2^7=128). Each case from resulting set must contain at least one non-zero subset from each M, P, S sets, and each set (M, P or S) must be not equivalent to others.

Obviously if at least one variable from {δ1, δ4, δ6, δ7} is set to 1, case consist elements from M. It gives us a simple criterion for case evaluation:

$\forall\Delta$ from Cases[128]: $\exists\Delta$: $\delta1_\Delta \vee \delta4_\Delta \vee \delta6_\Delta \vee \delta7_\Delta \rightarrow \Delta$ consist elements from M

In the same way:

$\forall\Delta$ from Cases[128]: $\exists\Delta$: $\delta2_\Delta \vee \delta5_\Delta \vee \delta6_\Delta \vee \delta7_\Delta \rightarrow \Delta$ consist elements from P

$\forall\Delta$ from Cases[128]: $\exists\Delta$: $\delta3_\Delta \vee \delta4_\Delta \vee \delta5_\Delta \vee \delta7_\Delta \rightarrow \Delta$ consist elements from S.

Applying given criteria at the same time, we obtain set of 109 elements, which consists subsets from S, M, P together. However, resulting set consists of cases, such as M=P=S (0000001) or M=P etc. To exclude equivalent sets we propose next criterion to evaluate equivalent sets (two sets are equivalent if they have only the common subsets):

S = P: $\forall\Delta$ from Cases[109]: $\exists\Delta$: $(\delta6_\Delta \vee \delta7_\Delta) \wedge \neg\delta1_\Delta \wedge \neg\delta2_\Delta$

P = M: $\forall\Delta$ from Cases[109]: $\exists\Delta$: $(\delta5_\Delta \vee \delta7_\Delta) \wedge \neg\delta2_\Delta \wedge \neg\delta3_\Delta$

M = S: $\forall\Delta$ from Cases[109]: $\exists\Delta$: $(\delta4_\Delta \vee \delta7_\Delta) \wedge \neg\delta1_\Delta \wedge \neg\delta3_\Delta$

The final set consist of 96 cases Δ=[1,96] (see Appendix A) which are essential data of for deciding syllogistic moods.

D. Algorithmic Decision

The basic idea of the algorithm for determining the true and false cases of a given mood is based on selecting the possible set relationships that satisfy premises for that mood and splitting resulting set into 2 sets of false and true cases according to the conclusion meaning, out of all 96 possible set relationships.

As was discussed above each mood is presented if form of triple of syllogistic quantifiers {A, E, I, O} and number of related figure and total number of moods for all figures is 256.

The validation process includes the validation each of 3 syllogistic statements (2 premises and conclusion). Since there are 4 syllogistic quantifiers, 4 possible situations for validation procedure are possible:

```
AllAre(set1, set2, Δ)
AllAreNot(set1, set2, Δ)
SomeAre(set1, set2, Δ)
SomeAreNot(set1, set2, Δ),
```

where set1 and set2 represent sets from {M, P, S}, case is related with element from Δ[96].

The criteria for AllAre() quantifier is very similar to described in previous section for equivalent sets. For example, for sets S, P and given case Δ=[1,96]:

```
boolean AllAre(S, P, Δ)
{if[(δ6Δ∨δ7Δ)∧¬δ1Δ]return true}
```

Verification procedure for AllAreNot() quantifiers can be implemented as simple negotiation of AllAre() quantifier. Another possible implementation is based on idea that sets are fully non-equivalent if those sets have not common elements in total. So, for sets S, P and given case Δ=[1,96]:

```
boolean AllAreNot(S, P, Δ)
{ if [¬(δ6Δ∨δ7Δ)]return true}
```

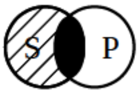
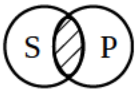
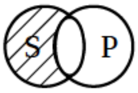
The implementation of SomeAre() and SomeAreNot() quantifiers in case if exclusive logic is identical. Actually, this 2 quantifiers can be considered as intermediate state between exact quantifiers All and None, so if case not satisfy AllAre() and AllAreNot() quantifiers, it must satisfy SomeAre(Not) quantifier. It is shown in the Table IV more clearly.

So, we need to check all possibilities where two sets have non- empty compliment and intersection at the same time. At this point for sets S, P and given case Δ=[1,96] we can write:

```
boolean SomeAre_SomeAreNot(S, P, Δ)
if
(δ6Δ∧δ1Δ)∨(δ6Δ∧δ4Δ)∨ (δ7Δ∧δ1Δ)∨(δ7Δ∧δ4Δ)
return true
```

It should be noted again that described procedure for SomeAre(Not) quantifiers is valid only for exclusive logic. Since our purpose to develop the universal approach, we can define the type of used logic optionally and implement the quantifiers as follow by adding appropriate special cases respectively:

TABLE IV. IDENTIFICATION OF THE SEVEN POSSIBLE SUBSETS OF THREE SETS AS DISTINCT SPACES.

Operator ψ	Venn-diagram for logical cases	Conditions	
		Exclusive logic	Inclusive logic
A		$S \setminus P = \varnothing$ $S \cap P \neq \varnothing$	
E		$S \setminus P \neq \varnothing$ $S \cap P = \varnothing$	
I		$S \setminus P \neq \varnothing$ $S \cap P \neq \varnothing$	$S \cap P \neq \varnothing$
O		$S \setminus P \neq \varnothing$ $S \cap P \neq \varnothing$	$S \setminus P \neq \varnothing$

```
boolean SomeAre(set1, set2, Δ)
if (SomeAre_SomeAreNot(set1, set2, Δ))
    return true
    if (use_inclusive_logic)
    {return AllAre(set1, set2, Δ);}
    return false

boolean SomeAreNot(set1, set2, Δ)
if (SomeAre_SomeAreNot(set1, set2, Δ))
    return true
    if (use_inclusive_logic)
    {return AllAreNot(set1,set2, Δ);}
    return false
```

After implementation verification functions we can propose general algorithm for calculating truth/false cases Δ for a given mood:

INPUT: mood: sequence if 3 quantifiers from {A, I, E, O} and figure's number

OUTPUT: Δ_true[], Δ_false[]: 2 lists, containing true and false cases for given mood

ALGORITHM:

1. GENERATE 96 possible set combinations with 7 relationships into a list of cases Δ[96]

2. VALIDATE premission_1 of given mood with AllAre(), AllAreNot(), SomeAre(), SomeAreNot() according to the quantifier and figure's number for all cases from Δ; construct list of cases Δ_1[] , that satisfy premise_1

3. VALIDATE premission_2 of given mood with AllAre(), AllAreNot(), SomeAre(), SomeAreNot() according to the quantifier and figure's number for all cases from Δ_1[]; construct list of cases Δ_2[] , that satisfy premise_2

4. VALIDATE conclusion of given mood with AllAre(), AllAreNot(), SomeAre(), SomeAreNot() according to the quantifier and figure's number for all cases from Δ_2[]; construct Δ_true[] and Δ_false[], that satisfy and not satisfy conclusion respectively.

E. *Truth ratio of the particular mood*

Since we can calculate the number of true and false cases for each mood, we can introduce a measure of truth for particular mood. We defined a truth ratio τ as:

$$\tau = \frac{number\ of\ (TC)}{number\ of\ (TC+FC)}$$

where *number_of(TC)* corresponds with with the quantity of true cases for given mood and *number_of(TC+FC)* corresponds with the sum of quantity of true and false cases for given mood respectively. Thus, the truth ratio τ becomes a real number, normalized within [0.0, 1.0].

Knowing truth ratio τ we can identify absolutely true (τ = 1.0) and false (τ = 0.0) moods. Absolutely true moods coincide with known valid forms of categorical syllogisms, however we have found out that there is an additional valid mood (AAO4) that was not considered in modern literature.

The truth ratio τ of a mood allows us now to define the concept of a fuzzy-syllogistic mood:

Fuzzy-syllogistic mood: ($\psi_1\psi_2\psi_3$, τ).

Using the conception of truth ratio, we can introduce the system of possibilistic argument (in this case, the possibilistic argument stands for truth ratio τ). After utilization of symmetric distributions of truth ratios, we can define membership function FuzzySyllogisticMood(x) with a symmetrical possibility distribution. FuzzySyllogisticMood(x) is determined by the linguistic variables such as Certainly/Likely/Uncertainly/Unlikely/Certainly_Not with corresponding cardinalities (see Fig. 2).

F. *Sample Reasoning*

Let us consider the example of the syllogistic reasoning. As input data, we have data sets M, P, S that consist the following elements (see Table V):

TABLE V. INPUT DATA: ELEMENTS OF SETS M, P AND S

Set	Elements
M	#1, #2, #3, #4, #5, #6, #7, #10
P	#1, #3, #6, #7, #8, #10
S	#2, #4, #6, #9

According to the given cases, we can calculate the syllogistic case that satisfies input data. The corresponding case (case #94, see Appendix A) is shown in Table VI.

TABLE VI. SYLLOGISTIC CASE THAT SATISFIES INPUT DATA FROM TABLE V.

Sub-Set Number	**δ1**	**δ2**	**δ3**	**δ4**	**δ5**	**δ6**	**δ7**
Syllogistic Case	S-M-P	P-S-M	M-S-P	(M∩S)-P	(S∩P)-M	(M∩P) -S	S∩P∩ M
Case	1	1	1	1	1	0	1

Since we can calculate the true cases for each mood, we can perform reverse operation such as calculating of moods, containing given case. Moods, matching considering case are presented in Table VII.

TABLE VII. MOODS, CONTAINING CASE #96

Mood(s)	τ	Mood(s)	τ
OIO-3; OIO-1	0.915	IIO-4; IIO-3; IIO-2; IIO-1	0.857
OOO-1	0.910	IOO-2; IOO-1	0.850
IOI-2; IOI-1	0.895	IOI-4; IOI-3	0.845
III-4; III-3; III-2; III-1	0.885	OII-3; OII-1	0.845
OOO-3	0.871	IOO-3; IOO-4	0.816
OOI-1	0.865	OOI-3	0.814

The moods with maximal truth ratio (OIO-3 and OIO-1) can be considered as the most suitable moods for given data set and represents the following syllogisms respectively:

Some M are not P	Some M are not P
Some M are S	Some S are M
Some S are P	Some S ere not P

IV. STATISTICS ABOUT THE SYLLOGISTIC SYSTEM

The introduced algorithm enables revealing various interesting statistics about the structural properties of the syllogistic system. Some of them are presented now.

For each mood we have calculated the truth ratio (see Appendix B). Note the symmetric distribution of the moods according their truth values (see Fig. 2). 25 moods have $\tau = 0.0$ (absolutely false) and 25 have $\tau = 1.0$ (absolutely true). 103 moods have τ between 0.0 and 0.5 and between 0.5 and 1.0 respectively. No mood has τ of exactly 0.5.

Every mood has from 0 to 65 true and false cases respectively, which is a real sub-set of the 96 distinct cases. The total number of true or false cases varies from one mood to another, from 1 to 72 cases. For instance, mood AAA1 has only 1 true and 0 false cases, whereas mood AAA2 has 1 true and 5 false cases. Hence the truth ratio of AAA1 is 1.0 and that of AAA2 is 1/6. The algorithm calculates 6144 syllogistic cases in total, since all cases of the 256 moods map the 96 distinct cases multiple times. Interesting is also that for any given figure the total number of all true cases is equal to all false cases, ie 768 true and 768 false cases. Thus we get for all 4 syllogistic figures the total number of 768 x 2 x 4 = 6144 cases.

A. Distinct moods

Since we have 256 moods, there are only 136 distinct moods, in terms of identical true and false cases matched per mood and equal truth ratios. Thus, the syllogistic system consists of 136 inference rules in total for inclusive logic (see Appendix B).

B. Point – symmetric moods

As was noticed before, in case of inclusive logic, the obtained system is fully symmetric in terms of the truth ratio of syllogistic moods. Actually, all these moods are pairwise point-symmetric in terms of the syllogistic cases they match and

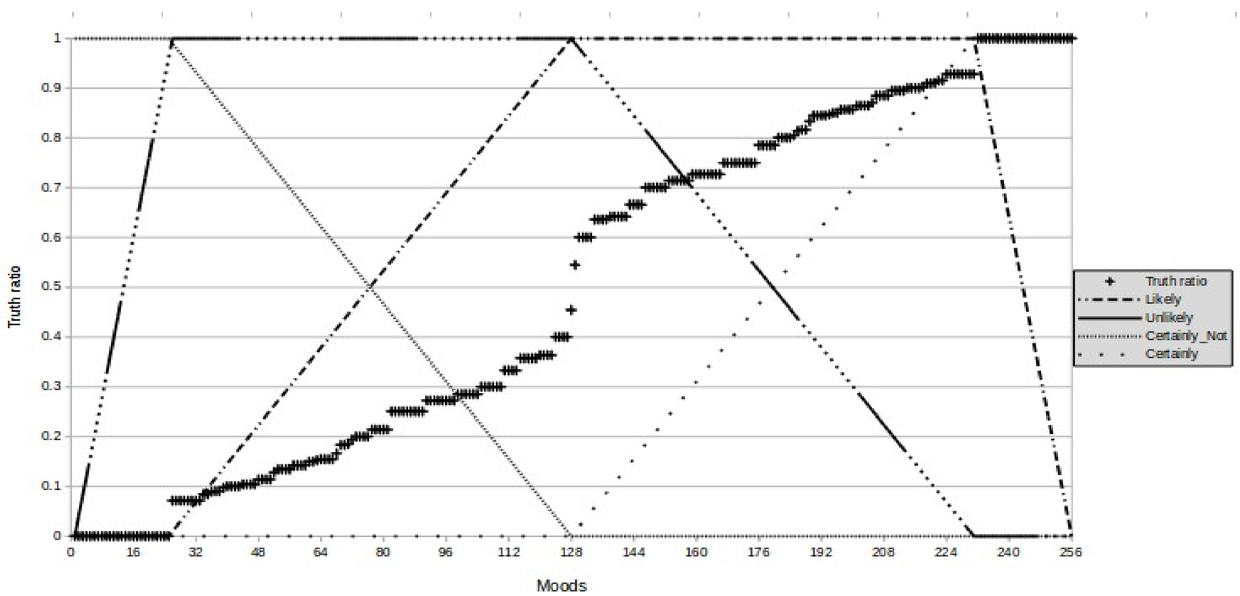


Fig. 2. 256 moods of the fuzzy-syllogistic system sorted in ascending order by their truth ratio with distribution of membership function FuzzySyllogisticMood(x)

respectively in terms of their truth ratios too. The list of the first 10 point-symmetric moods is shown in Table VII.

TABLE VII . FIRST 10 POINT- SYMMETRIC MOODS (IN DECREASING ORDER OF TRUTH RATIO)

#	Mood	τ	t	f	Mood	τ	t	f
1	EIO4	1.0	11	0	EIA4	0.0	0	11
2	EIO3	1.0	11	0	EIA3	0.0	0	11
3	EIO2	1.0	11	0	EIA2	0.0	0	11
4	EIO1	1.0	11	0	EIA1	0.0	0	11
5	OAO3	1.0	11	0	OAA3	0.0	0	11
6	AII3	1.0	10	0	AIE3	0.0	0	10
7	AII1	1.0	10	0	AIE1	0.0	0	10
8	IAI4	1.0	10	0	IAE4	0.0	0	10
9	IAI3	1.0	10	0	IAE3	0.0	0	10
10	AOO2	1.0	9	0	AOA2	0.0	0	9

Pairs have equal propositional quantifiers, but shifting concluding quantifiers. Almost all moods (250), shift from O to A, in total 63 pairs, or from I to E, in total 62 pairs.

V. FUTURE WORK

For now, we are working under extension of the designed system by intermediate quantifiers. We have introduced fuzzy versions of quantifiers I and O and modified the list of quantifiers, proposed by Peterson in [14], by including the new quantifiers as Half and Several and excluding Some.

Based on the semantics of the intermediate statements and the fuzzy-logical graph of opposition, defined similarly to the classical square of opposition, we are trying to determine the valid (and non-valid) intermediate syllogisms and propose the algorithmic solution for the calculation of their structural properties.

We are also testing the whole system as one complex approach for approximate reasoning [13].

VI. CONCLUSIONS

We have presented the syllogistic system that consists of 256 moods, of which are 136 distinct, since some moods match exactly the same syllogistic cases and have equal truth ratios. We have presented the maximum possible 96 distinct space permutations for three sets and how they are matched by every mood. We have presented the exact truth ratios of the moods and grouped equal moods.

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APPENDIX A. 96 DISTINCT SYLLOGISTIC CASES

Case	Space Combi-nation	Case	Space Combina-tion	Case	Space Combina-tion	Case	Space Combi-nation
1	0000110	25	0101100	49	1001100	73	1101000
2	0000111	26	0101101	50	1001101	74	1101001
3	0001010	27	0101110	51	1001110	75	1101010
4	0001011	28	0101111	52	1001111	76	1101011
5	0001100	29	0110001	53	1010001	77	1101100
6	0001101	30	0110010	54	1010010	78	1101101
7	0001110	31	0110011	55	1010011	79	1101110
8	0001111	32	0110101	56	1010100	80	1101111
9	0010101	33	0110110	57	1010101	81	1110000
10	0010110	34	0110111	58	1010110	82	1110001
11	0010111	35	0111000	59	1010111	83	1110010
12	0011001	36	0111001	60	1011001	84	1110011
13	0011010	37	0111010	61	1011010	85	1110100
14	0011011	38	0111011	62	1011011	86	1110101
15	0011100	39	0111100	63	1011100	87	1110110
16	0011101	40	0111101	64	1011101	88	1110111
17	0011110	41	0111110	65	1011110	89	1111000
18	0011111	42	0111111	66	1011111	90	1111001
19	0100011	43	1000011	67	1100001	91	1111010
20	0100101	44	1000110	68	1100011	92	1111011

Case	Space Combination	Case	Space Combination	Case	Space Combination	Case	Space Combination
21	0100110	45	1000111	69	1100100	93	1111100
22	0100111	46	1001001	70	1100101	94	1111101
23	0101010	47	1001010	71	1100110	95	1111110
24	0101011	48	1001011	72	1100111	96	1111111

Appendix B. Distinct moods with their truth ratios

#	Truth ratio, τ	Moods	Moods in group	t	f
1	1.000	EIO4; EIO3; EIO2; EIO1	4	11	0
2	1.000	OAO3	1	11	0
3	1.000	AII3; AII1	2	10	0
4	1.000	IAI4; IAI3	2	10	0
5	1.000	AOO2	1	9	0
6	1.000	EAO4; EAO3	2	5	0
7	1.000	AAI3	1	4	0
8	1.000	AAI1; AAA1	2	1	0
9	1.000	AAO4; AAI4	2	1	0
10	1.000	AEO4; AEO2; AEE4; AEE2	4	1	0
11	1.000	EAO2; EAO1; EAE2; EAE1	4	1	0
12	0.928	AIO4; AIO2	2	13	1
13	0.928	AOI1	1	13	1
14	0.928	AOO4	1	13	1
15	0.928	EOO2; EOO1	2	13	1
16	0.928	OAI4	1	13	1
17	0.928	OAO4	1	13	1
18	0.915	OIO3; OIO1	2	65	6
19	0.910	OOO1	1	61	6
20	0.909	AOI3	1	10	1
21	0.909	OAI3	1	10	1
22	0.902	OOI2	1	65	7
23	0.900	EOO4; EOO3	2	9	1
24	0.900	IAO4; IAO3	2	9	1
25	0.895	IOI2; IOI1	2	60	7
26	0.895	OII4; OII2	2	60	7
27	0.885	III4; III3; III2; III1	4	62	8
28	0.871	OOO3	1	61	9
29	0.865	OIO4; OIO2	2	58	9
30	0.865	OOI1	1	58	9
31	0.865	OOI4	1	58	9
32	0.857	IIO4; IIO3; IIO2; IIO1	4	60	10
33	0.850	IOO2; IOO1	2	57	10
34	0.847	OOO2	1	61	11
35	0.845	IOI4; IOI3	2	60	11
36	0.845	OII3; OII1	2	60	11
37	0.833	AAO2	1	5	1
38	0.816	IOO4; IOO3	2	58	13
39	0.814	OOI3	1	57	13
40	0.805	OOO4	1	54	13
41	0.800	AEI3; AEI1	2	4	1
42	0.800	EAI4; EAI3	2	4	1
43	0.785	EOI2; EOI1	2	11	3
44	0.785	OAO1	1	11	3
45	0.785	OEI4; OEI2	2	11	3
46	0.750	AAO3	1	3	1
47	0.750	EEI4; EEI3; EEI2; EEI1	4	3	1
48	0.750	EEO4; EEO3; EEO2; EEO1	4	3	1
49	0.727	EII4; EII3; EII2; EII1	4	8	3
50	0.727	IEI4; IEI3; IEI2; IEI1	4	8	3
51	0.714	AII4; AII2	2	10	4
52	0.714	IAI2; IAI1	2	10	4
53	0.714	IAO2; IAO1	2	10	4
54	0.700	EOI4; EOI3	2	7	3
55	0.700	OEI3; OEI1	2	7	3
56	0.700	OEO3; OEO1	2	7	3
57	0.666	AOI2	1	6	3
58	0.666	OAI2	1	6	3
59	0.666	OAO2	1	6	3
60	0.666	AAI2	1	4	2
61	0.642	AOI4	1	9	5
62	0.642	AOO1	1	9	5
63	0.642	OAI1	1	9	5
64	0.642	OEO4; OEO2	2	9	5
65	0.636	IEO4; IEO3; IEO2; IEO1	4	7	4
66	0.600	AIO3; AIO1	2	6	4
67	0.600	AEO3; AEO1	2	3	2
68	0.545	AOO3	1	6	5
69	0.454	AOA3	1	5	6
70	0.400	AEA3; AEA1	2	2	3
71	0.400	AIA3; AIA1	2	4	6
72	0.363	IEA4; IEA3; IEA2; IEA1	4	4	7
73	0.357	AOA1	1	5	9
74	0.357	AOE4	1	5	9
75	0.357	OAE1	1	5	9
76	0.357	OEA4; OEA2	2	5	9
77	0.333	AAE2	1	2	4
78	0.333	AOE2	1	3	6

#	Truth ratio, τ	Moods	Moods in group	t	f
79	0.333	OAA2	1	3	6
80	0.333	OAE2	1	3	6
81	0.300	EOE4; EOE3	2	3	7
82	0.300	OEA3; OEA1	2	3	7
83	0.300	OEE3; OEE1	2	3	7
84	0.285	AIE4; AIE2	2	4	10
85	0.285	IAA2; IAA1	2	4	10
86	0.285	IAE2; IAE1	2	4	10
87	0.272	EIE4; EIE3; EIE2; EIE1	4	3	8
88	0.272	IEE4; IEE3; IEE2; IEE1	4	3	8
89	0.250	AAA3	1	1	3
90	0.250	EEA4; EEA3; EEA2; EEA1	4	1	3
91	0.250	EEE4; EEE3; EEE2; EEE1	4	1	3
92	0.214	EOE2; EOE1	2	3	11
93	0.214	OAA1	1	3	11
94	0.214	OEE4; OEE2	2	3	11
95	0.200	AEE3; AEE1	2	1	4
96	0.200	EAE4; EAE3	2	1	4
97	0.194	OOA4	1	13	54
98	0.185	OOE3	1	13	57
99	0.183	IOA4; IOA3	2	13	58
100	0.166	AAA2	1	1	5
101	0.154	IOE4; IOE3	2	11	60
102	0.154	OIE3; OIE1	2	11	60
103	0.152	OOA2	1	11	61
104	0.149	IOA2; IOA1	2	10	57
105	0.142	IIA4; IIA3; IIA2; IIA1	4	10	60
106	0.134	OIA4; OIA2	2	9	58
107	0.134	OOE1	1	9	58
108	0.134	OOE4	1	9	58
109	0.128	OOA3	1	9	61
110	0.114	IIE4; IIE3; IIE2; IIE1	4	8	62
111	0.104	IOE2; IOE1	2	7	60
112	0.104	OIE4; OIE2	2	7	60
113	0.100	EOA4; EOA3	2	1	9
114	0.100	IAA4; IAA3	2	1	9
115	0.097	OOE2	1	7	65
116	0.090	AOE3	1	1	10
117	0.090	OAE3	1	1	10
118	0.089	OOA1	1	6	61
119	0.084	OIA3; OIA1	2	6	65
120	0.071	AIA4; AIA2	2	1	13
121	0.071	AOA4	1	1	13
122	0.071	AOE1	1	1	13
123	0.071	EOA2; EOA1	2	1	13
124	0.071	OAA4	1	1	13
125	0.071	OAE4	1	1	13
126	0.000	EAI1; EAA1; EAI2; EAA2	4	0	1
127	0.000	AEI2; AEA2; AEI4; AEA4	4	0	1
128	0.000	AAO1; AAE1	2	0	1
129	0.000	AAE4; AAA4	2	0	1
130	0.000	AAE3	1	0	4
131	0.000	EAA3; EAA4	2	0	5
132	0.000	AOA2	1	0	9
133	0.000	IAE3; IAE4	2	0	10
134	0.000	AIE1; AIE3	2	0	10
135	0.000	OAA3	1	0	11
136	0.000	EIA1; EIA2; EIA3; EIA4	4	0	11