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Effective stress principle for saturated fractured porous media

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Abstract. An effective stress principle for saturated fractured porous media is proposed based on the double-porosity representation. Both the solid grains and the fractured porous medium are assumed to be linearly elastic materials. The derivation employs volume averaging technique to obtain macroscopic scale expressions. Two parameters, the bulk modulus of the fractured medium and bulk modulus of the porous matrix, are introduced in the formulation. The final expression reduces to the one obtained by *Biot and Willis* [1957], *Skempton* [1960], *Nur and Byeerle* [1971], and *Verruijt* [1984] when the volume fraction of the fractures vanishes, that is, for a nonfractured porous medium.

Introduction

The effective stress principle is extensively used in many disciplines, such as groundwater hydrology and soil and rock mechanics. *Terzaghi* [1925] terms the pore water pressure as the "neutral stress" and effective stress as the "excess" over the neutral stress. It has been shown by various researchers that the original form of Terzaghi's effective stress expression has two fundamental assumptions: solid grains are incompressible, and the shear deformation of the solid matrix is independent of the pore water pressure [*Skempton*, 1960]. A derivation of the effective stress principle was presented by *Bear and Pinder* [1983] and *Bear et al.* [1984] for a porous medium constituted by incompressible grains.

The extension of the effective stress principle to porous media with compressible grains has been studied by *Biot and Willis* [1957], *Skempton* [1960], *Nur and Byeerle* [1971], and *Verruijt* [1984]. Although these researchers used different methodologies, their conclusions are similar. For a porous medium with compressible grains, the form of the effective stress principle is the same as that of Terzaghi's effective stress principle except a correction factor for the pore fluid pressure. This correction factor is a function of the bulk moduli of the solid grains and the porous matrix. When the bulk modulus of the grains is very large compared with the bulk modulus of the matrix, this coefficient converges to unity, and the expression reduces to that of Terzaghi. A discussion on the role of Terzaghi's effective stress in linearly elastic deformation was presented by *Carroll and Katsube* [1983]. Carroll and Katsube showed that the total strain is the sum of the average strain and a component due to change in the pore geometry that is determined by Terzaghi's effective stress. We should note that all expressions are limited to linearly elastic materials and are not applicable to inelastic media.

The single-porosity models are shown to be fairly successful to describe the behavior of porous materials. However, single-porosity models are not suitable for fractured (or fissured) porous materials (Figure 1). In such systems, although most of the fluid mass is stored in the pores, the fracture permeability is much higher than the permeability of the pores. This leads to two distinct pressure fields: one in the fractures and the other in the pores. *Barenblatt et al.* [1960] appear to be first researchers proposing a double-porosity model to represent naturally fractured porous media. A double-porosity model can be considered as a three-phase system, that is, solid phase, fluid phase in the pores, and fluid phase in the fractures, with fluid mass exchange between the pores and fractures. Although flow in fractured porous media has been studied quite extensively [*Barenblatt et al.*, 1960; *Barenblatt*, 1963; *Warren and Root*, 1963; *Kazemi*, 1969; *Bear and Berkowitz*, 1987], the work on deformable fractured porous media is limited. This might be attributed to lack of an effective stress principle in a double-porosity medium. Such an expression would allow us to model the behavior of deformable fractured porous media by using the drained material parameters.

In this study, we investigate the effective stress principle for a fractured porous medium based on the double-porosity representation. We assume that both the solid grains and the fractured medium are linearly elastic. These assumptions allow application of the superposition principle. Two macroscopic material constants, drained (frame) bulk modulus of the fractured porous medium and drained bulk modulus of the solid matrix, are introduced in the formulation. The later is equivalent to the bulk modulus of a nonfractured porous medium extracted from the fractured medium. Following Terzaghi and others, we assume that shear deformation is not effected by fluid pressures. Our final expression reduces to the one obtained by *Biot and Willis* [1957], *Skempton* [1960], *Nur and Byeerle* [1971], and *Verruijt* [1984] when the volume fraction of the fractures vanishes.

Derivation of the Effective Stress Principle

The development is based on the double-porosity representation of a fractured porous medium which is characterized by three volume fractions: α_s, α_p, and α_f corresponding to the solid phase, fluid phase in the pores, and fluid phase in the fractures, respectively. We assume that the solid phase is linearly elastic, isotropic, homogeneous, and experiencing small deformations. Then the microscopic constitutive relations are given as

$$\boldsymbol{\tau}_s = K_s \nabla \cdot \mathbf{u}_s \mathbf{I} + G_s\left(\nabla \mathbf{u}_s + (\nabla \mathbf{u}_s)^T - \frac{2}{3}\nabla \cdot \mathbf{u}_s\right) \qquad (1)$$

where $\mathbf{u}_s$, $\boldsymbol{\tau}_s$, K_s, G_s, and $\mathbf{I}$ are the displacement vector, incremental stress tensor, bulk modulus, and shear modulus of the

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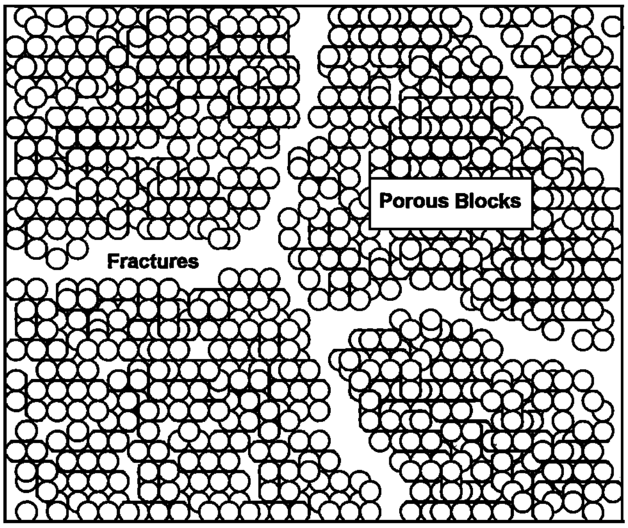

Figure 1. Conceptual model of a saturated fractured porous medium.

solid phase, and the unit tensor, respectively. The superscript T denotes the transpose operator. Applying volume-averaging technique to (1) (see appendix for the averaging rules), we obtain

$$\langle \boldsymbol{\tau}_s \rangle = K_s \left[\nabla \cdot (\alpha_s \bar{\mathbf{u}}_s) + \frac{1}{V} \int_{S_{si}} \mathbf{u}_s \cdot n \, dA \right] \mathbf{I} + G_s \left[\nabla(\alpha_s \bar{\mathbf{u}}_s) + (\nabla(\alpha_s \bar{\mathbf{u}}_s))^T - \frac{2}{3} \nabla \cdot (\alpha_s \bar{\mathbf{u}}_s) + \frac{1}{V} \int_{S_{si}} \mathbf{u}_s n_s + n_s \mathbf{u}_s - \frac{2}{3} \mathbf{u}_s \cdot n_s \, dA \right] \qquad i = f, p \tag{2}$$

where angle brackets and overbars indicate volume-averaged and intrinsic-averaged quantities, respectively. Notation is described in the appendix. If we assume that there is no mass exchange between the solid and fluid phases, then the velocity of the solid/fluid interface is equal to the velocity of a point at the interface, that is, material surface. By employing (A8), we can write

$$\frac{1}{V} \int_{S_{si}} \mathbf{u}_s \cdot n_s \, dA = \alpha_s - \alpha_s^0 = \Delta \alpha_s \qquad i = f, p \tag{3}$$

where superscript 0 refers to a reference configuration and $\Delta\alpha_s$ is the change in volume fraction of the solid phase from the reference configuration. Since the displacements $\bar{\mathbf{u}}_s$ are assumed to be small, by definition

$$\bar{\mathbf{u}}_s \cdot \nabla \alpha_s \approx 0 \tag{4}$$

Then volume-averaged constitutive relations for the solid phase (2) simplify to

$$\alpha_s \bar{\boldsymbol{\tau}}_s = K_s(\alpha_s \nabla \cdot \bar{\mathbf{u}}_s + \Delta \alpha_s)\mathbf{I} + \alpha_s G_s \left[\nabla \bar{\mathbf{u}}_s + (\nabla \bar{\mathbf{u}}_s)^T - \frac{2}{3} \nabla \cdot \bar{\mathbf{u}}_s + \frac{1}{\alpha_s V} \int_{S_{si}} \mathbf{u}_s n_s + n_s \mathbf{u}_s - \frac{2}{3} \mathbf{u}_s \cdot n_s \, dA \right] \qquad i = f, p \tag{5}$$

The total stress is the sum of the volume averaged stresses of individual phases

$$\langle \boldsymbol{\tau}_t \rangle = \alpha_s \bar{\boldsymbol{\tau}}_s + \alpha_p \bar{\boldsymbol{\tau}}_p + \alpha_f \bar{\boldsymbol{\tau}}_f \tag{6}$$

where $\bar{\boldsymbol{\tau}}_f$ and $\bar{\boldsymbol{\tau}}_p$ are the intrinsic averaged incremental stress tensor in the fractures and pores, respectively. For an inviscid fluid, $\bar{\boldsymbol{\tau}}_f$ and $\bar{\boldsymbol{\tau}}_p$ are equal to the fluid pressures in the fractures, $\bar{P}_f$, and in the pores, $\bar{P}_p$, respectively. Consequently, we assume that all shear resistance is provided by the solid matrix. In other words, the fluid pressures affect the volume change behavior of the fractured porous medium only. We define a mean solid stress $\bar{P}_s$ as

$$-\alpha_s \bar{P}_s = \frac{\text{trace}(\langle \boldsymbol{\tau}_s \rangle)}{3} = K_s(\alpha_s \nabla \cdot \bar{\mathbf{u}}_s + \Delta \alpha_s) \tag{7}$$

In this study we take the stress tensor positive for tension and fluid pressures positive for compression.

To incorporate $\bar{P}_s$, $\bar{P}_f$, and $\bar{P}_p$ in the effective stress expression, we analyze three different stress state conditions individually. In each of these cases we obtain an expression for the dilatation of the solid matrix $\nabla \cdot \bar{\mathbf{u}}_s$ by introducing macroscopic material coefficients when necessary. Then we will superpose these expressions to obtain a relation for $\nabla \cdot \bar{\mathbf{u}}_s$ when $\bar{P}_s$, $\bar{P}_f$, and $\bar{P}_p$ are simultaneously present. Superposition is justified by the linearity of the system.

In the first case, we consider a drained porous medium, that is, $\bar{P}_f = \bar{P}_p = 0$ (In this study the term "drained" refers to the complete dissipation of excess pore pressures). Introducing the drained bulk modulus of the fractured porous medium K_{fr}, we write

$$-\alpha_s \bar{P}_s = K_{fr} \nabla \cdot \bar{\mathbf{u}}_s \tag{8}$$

By substituting (8) in (7), we obtain

$$\Delta \alpha_s = -\left(\frac{\alpha_s}{K_s} - \frac{\alpha_s^2}{K_{fr}} \right) \bar{P}_s \tag{9}$$

K_{fr} can be evaluated experimentally by testing a drained fractured porous sample with standard techniques.

In the second case we consider a stress state where $\bar{P}_s = \bar{P}_f = \bar{P}_p$. This case corresponds to a fractured porous medium immersed in a fluid subjected to external pressure. Because of the homogeneity and isotropy of the medium, all volume fractions remain constant, and (7) yields

$$-\bar{P}_s = K_s \nabla \cdot \bar{\mathbf{u}}_s \tag{10}$$

In the third case, which is a thought experiment, we assume that the volume fraction change of the fractures is zero and furthermore that $\bar{P}_p = 0$. As in the second case, the fractured porous sample is immersed in a fluid. The pressure in the fractures $\bar{P}_f$ is equal to the applied pressure. Then from (6) and (7) we can write

$$\bar{P}_f = \alpha_s \bar{P}_s + \alpha_f \bar{P}_f \qquad \text{or} \qquad \bar{P}_s = \frac{1 - \alpha_f}{\alpha_s} \bar{P}_f \tag{11}$$

Introducing K_{fr}^m as the drained bulk modulus of the nonfractured porous matrix, we write

$$-\alpha_s \bar{P}_s = K_{fr}^m \nabla \cdot \bar{\mathbf{u}}_s \tag{12}$$

Experimentally, K_{fr}^m can be determined by extracting a nonfractured sample from the fractured porous medium. The extracted sample of porous medium can be tested by standard techniques to determine K_{fr}^{m} experimentally. The third case will be discussed in more detail after the superposition of three cases. The volume fraction change of the solid phase can be solved from (7) and (12) as

$$\Delta\alpha_s = -\left(\frac{\alpha_s}{K_s} - \frac{\alpha_s^2}{K_{fr}^m}\right)\bar{P}_s \tag{13}$$

The stress states can be summarized as

Case 1

$$\bar{P}_s = P_1 \qquad \bar{P}_p = 0 \qquad \bar{P}_f = 0 \tag{14a}$$

Case 2

$$\bar{P}_s = P_2 \qquad \bar{P}_p = P_2 \qquad \bar{P}_f = P_2 \tag{14b}$$

Case 3

$$\bar{P}_s = \frac{1-\alpha_f}{\alpha_s}P_3 \qquad \bar{P}_p = 0 \qquad \bar{P}_f = P_3 \tag{14c}$$

where subscripts 1, 2, and 3 refer to cases 1, 2, and 3, respectively. Since we seek expressions when $\bar{P}_s$, $\bar{P}_p$ and $\bar{P}_f$ are simultaneously present in the system, P_1, P_2 and P_3 must satisfy

$$P_1 + P_2 + \frac{1-\alpha_f}{\alpha_s}P_3 = \bar{P}_s$$
$$P_2 = \bar{P}_p \tag{15}$$
$$P_3 + P_2 = \bar{P}_f$$

Solution of (15) for P_1, P_2, and P_3 yields

$$P_1 = \bar{P}_s - \bar{P}_p - \frac{1-\alpha_f}{\alpha_s}(\bar{P}_f - \bar{P}_p)$$
$$P_2 = \bar{P}_p \tag{16}$$
$$P_3 = \bar{P}_f - \bar{P}_p$$

In the third case, we observe that $P_3 = \bar{P}_f - \bar{P}_p$. Hence case 3 corresponds to the dilatation of the fractured solid matrix due to the pressure difference between the pores and fractures. In other words, there are three components of the matrix dilatation associated with three bulk moduli, K_s, K_{fr}, and K_{fr}^{m}.

The dilatation of the fractured solid matrix is obtained by superposing (8), (10), and (12) and substituting the expressions for $\bar{P}_s$ (equation (14)) as

$$\nabla\cdot\bar{\mathbf{u}}_s = -\frac{\alpha_s P_1}{K_{fr}} - \frac{P_2}{K_s} - \frac{\alpha_s}{K_{fr}^m}\frac{(1-\alpha_f)}{\alpha_s}P_3 \tag{17}$$

Substitution of (16) in (17) yields

$$\nabla\cdot\bar{\mathbf{u}}_s = -\frac{\alpha_s}{K_{fr}}\left(\bar{P}_s - \bar{P}_p - \frac{1-\alpha_f}{\alpha_s}(\bar{P}_f - \bar{P}_p)\right)$$
$$-\frac{\bar{P}_p}{K_s} - \frac{1-\alpha_f}{K_{fr}^m}(\bar{P}_f - \bar{P}_p) \tag{18}$$

Similarly, $\Delta\alpha_s$ is obtained from (9) and (13) as

$$\Delta\alpha_s = -\left(\frac{\alpha_s}{K_s} - \frac{\alpha_s^2}{K_{fr}}\right)\left(\bar{P}_s - \bar{P}_p - \frac{1-\alpha_f}{\alpha_s}(\bar{P}_f - \bar{P}_p)\right)$$
$$-\left(\frac{\alpha_s}{K_s} - \frac{\alpha_s^2}{K_{fr}^m}\right)\frac{1-\alpha_f}{\alpha_s}(\bar{P}_f - \bar{P}_p) \tag{19}$$

Employing the definition of total stress given in (6), (18) can be rewritten as

$$\frac{\text{trace}(\langle\tau_t\rangle)}{3} + \beta_p\bar{P}_p + \beta_f\bar{P}_f = K_{fr}\nabla\cdot\bar{\mathbf{u}}_s \tag{20}$$

where

$$\beta_p = \frac{(1-\alpha_f)K_{fr}}{K_{fr}^m} - \frac{K_{fr}}{K_s} \qquad \beta_f = 1 - \frac{(1-\alpha_f)K_{fr}}{K_{fr}^m} \tag{21}$$

We recall that K_{fr} is the drained bulk modulus of the fractured porous medium. Then we can write the following equation for a drained fractured porous medium

$$\frac{\text{trace}(\langle\boldsymbol{\tau}_s\rangle)}{3} = K_{fr}\nabla\cdot\bar{\mathbf{u}}_s \tag{22}$$

Comparing (20) and (22), we conclude that the effective stress is given by

$$\langle\boldsymbol{\tau}_{eff}\rangle = \langle\boldsymbol{\tau}_t\rangle + \beta_p\bar{P}_p + \beta_f\bar{P}_f \tag{23}$$

Substituting (21) in (23) and rearranging, we obtain

$$\langle\boldsymbol{\tau}_t\rangle = \langle\boldsymbol{\tau}_{eff}\rangle - \left(\frac{(1-\alpha_f)K_{fr}}{K_{fr}^m} - \frac{K_{fr}}{K_s}\right)\bar{P}_p$$
$$-\left(1 - \frac{(1-\alpha_f)K_{fr}}{K_{fr}^m}\right)\bar{P}_f \tag{24}$$

Conclusions

Equation (24) is the effective stress principle for a saturated fractured porous medium. It expresses the macroscopic total stress $\langle\boldsymbol{\tau}_t\rangle$ in terms of effective stress $\langle\boldsymbol{\tau}_{eff}\rangle$, pore fluid pressure $\bar{P}_p$, fracture fluid pressure $\bar{P}_f$ and bulk moduli of nonfractured porous blocks (K_{fr}^{m}), fractured porous medium (K_{fr}), and solid grains (K_s). These three bulk moduli can be determined experimentally by employing standard soil mechanics tests. As can be seen in (24), when $\bar{P}_f = \bar{P}_p$ or the volume fraction of the fractures is zero, that is, $\alpha_f = 0$, $K_{fr}^{m} = K_{fr}$, (24) reduces to

$$\langle\boldsymbol{\tau}_t\rangle = \langle\boldsymbol{\tau}_{eff}\rangle - \left(1 - \frac{K_{fr}}{K_s}\right)\bar{P}_p \tag{25}$$

Equation (25) is the same expression obtained by *Biot and Willis* [1957], *Skempton* [1960], *Nur and Byeerle* [1971], and *Verruijt* [1984] for a saturated nonfractured porous medium. When the bulk modulus of the grains is much greater than the bulk modulus of the frame, that is, $K_{fr}/K_s \to 0$, (25) reduces to Terzaghi's effective stress principle.

Appendix: Volume-Averaging Theorems

Let L, l and r be the characteristic lengths of the macroscopic scale, averaging volume, and pore scale, respectively. The required condition for the volume averaging is [*Slattery*, 1981]

$$r \ll l \ll L \tag{A1}$$

In this study, we assume that this requirement is satisfied. We continue with the definitions used in volume averaging literature. Let B_i be a field quantity of phase i; then volume average of B_i is defined as

$$\langle B_i \rangle = \frac{1}{V} \int_{R_i} B_i \, dV \tag{A2}$$

where V is the averaging volume and R_i is the region occupied by phase i. The intrinsic volume average of B_i, that is, the mean value of B_i in R_i, is given by

$$\bar{B}_i = \frac{1}{V_i} \int_{R_i} B_i \, dV \tag{A3}$$

where V_i is the volume of phase i in the averaging volume. These two averages are related by

$$\langle B_i \rangle = \alpha_i \bar{B}_i \tag{A4}$$

where α_i is the volume fraction of phase i. Now, we set the volume average theorem for a gradient and a time derivative [*Slattery*, 1981]

$$\langle \nabla B_i \rangle = \nabla \langle B_i \rangle + \frac{1}{V} \int_{S_{ij}} B_i n_i \, dA \tag{A5}$$

$$i \neq j \qquad j = 1, \cdots, N$$

$$\left\langle \frac{\partial B_i}{\partial t} \right\rangle = \frac{\partial \langle B_i \rangle}{\partial t} - \frac{1}{V} \int_{S_{ij}} B_i \mathbf{u} \cdot n_i \, dA \tag{A6}$$

$$i \neq j \qquad j = 1, \cdots, N$$

where S_{ij} is the interface between phase i and phase j, n_i is the outward normal of S_{ij} and $\mathbf{u} \cdot n_i$ is the speed of displacement of S_{ij} into other phases. The theorem of volume average of a divergence is stated as

$$\langle \nabla \cdot B_i \rangle = \nabla \cdot \langle B_i \rangle + \frac{1}{V} \int_{S_{ij}} B_i n_i \, dA \tag{A7}$$

$$i \neq j \qquad j = 1, \cdots, N$$

If B is taken to be a constant, (A6) takes the following forms

$$\frac{\partial \alpha_i}{\partial t} = + \frac{1}{V} \int_{S_{ij}} \mathbf{u} \cdot n_i \, dA \qquad i \neq j \qquad j = 1, \cdots, N \tag{A8}$$

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