Fuzzy-Syllogistic Systems: A Generic Model for Approximate Reasoning

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Abstract. The well known Aristotelian syllogistic system $\mathbb{S}$ consists of 256 moods. We have found earlier that 136 moods are distinct in terms of equal truth ratios that range in $\tau = [0,1]$. The truth ratio of a particular mood is calculated by relating the number of true and false syllogistic cases that the mood matches. The introduction of (n − 1) fuzzy existential quantifiers, extends the system to fuzzy-syllogistic systems $^{n}\mathbb{S}$, $1 < n$, of which every fuzzy-syllogistic mood can be interpreted as a vague inference with a generic truth ratio, which is determined by its syllogistic structure. Here we introduce two new concepts, the relative truth ratio $^{r}\tau = [0,1]$ that is calculated from the cardinalities of the syllogistic cases of the mood and fuzzy-syllogistic ontology (FSO). We experimentally apply the fuzzy-syllogistic systems $^{2}\mathbb{S}$ and $^{6}\mathbb{S}$ as underlying logic of a FSO reasoner (FSR) and discuss sample cases for approximate reasoning.

Keywords: Syllogistic reasoning · Fuzzy logic · Approximate reasoning

1 Introduction

Multi-valued logics were initially introduced by Łukasiewicz [13], as an extension to propositional logic, which was then generalised by Zadeh using fuzzy sets [22] to fuzzy logic. After he had introduced approximate reasoning [23], he proposed fuzzy-syllogistic reasoning as a theory of common sense [24] and discussed fuzzy quantifiers again in the context of fuzzy logic [25]. However, these initial fuzzifications of syllogistic moods were experimentally applied to only a few true moods and did not cover all moods systematically, in terms of the four syllogistic figures. Only fuzzy quantifications based on interval arithmetic [6] comply to some extend with traditional figures [12]. The first systematic application of multi-valued logics on syllogisms were intermediate quantifiers and their reflection on the square of opposition [17]. However only set-theoretic representation of moods as syllogistic cases allow analysing the fuzzy-syllogistic systems $^{n}\mathbb{S}$ mathematically exactly, such as by calculating truth ratios of moods [8] and their algorithmic usage in fuzzy inferencing [9]. Here we present a sample application of $^{n}\mathbb{S}$ for fuzzy-syllogistic ontology reasoning.

Learning from scratch can be modelled probabilistically, as objects and their relationships need to be first synthesised from a statistically significant number of perceived instances of similar objects. This leads to probabilistic ontologies [4, 14, 18], in which attributes of objects may be synthesised also as objects.

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There are more probabilistic ontology reasoners than fuzzy or possibilistic ones and most of them reason with probabilist ontologies [10]. Several ontology reasoners employ possibilistic logic and reason with fuzzy ontologies. The most popular reasoning logic being hyper-tableau, for instance in HermiT [15]. Other experimental reasoning logics are also interesting to analyse, such as fuzzy rough sets and Łukasiewicz logic [3] in FuzzyDL [1], Zadeh and Gödel fuzzy operators in DeLorean [2], Mamdani inference in HyFOM [21] or possibilistic logic in KAON [18]. Fuzzy-syllogistic reasoning (FSR) can be seen as a generalisation of both, fuzzy-logical and possibilistic reasoners.

A fuzzy-syllogistic ontology (FSO) extends the concept of ontology with the quantities that led to the ontological concepts. A FSO is usually generated probabilistically, but does not preserve any probabilities like probabilistic ontologies [14] or probabilistic logic networks [7] do. A FSO can be a fully connected and bidirectional graph.

Several generic reasoning logics are discussed in the literature, like probabilistic, non-monotonic or non-axiomatic reasoning [20]. Fuzzy-syllogistic reasoning in its basic form [26] is possibilistic, monotonic and axiomatic.

Syllogistic reasoning reduced to the proportional inference rules deduction, induction and abduction are employed in the Non-Axiomatic Reasoning System (NARS) [19]. Whereas FSR uses the original syllogistic moods and their fuzzified extensions [27].

There is one implementation mentioned in the literature that is close to the concept of syllogistic cases: Syllogistic Epistemic REAsoner (SEREA) implements poly-syllogisms and generalised quantifiers that are associated with combinations of distinct spaces, which are mapped onto some interval arithmetic. Reasoning is then performed with concrete quantities, determined with the interval arithmetic [16].

First the fuzzy-syllogistic systems ${}^{n}\mathbb{S}$ are discussed, thereafter fuzzy-syllogistic reasoning is introduced, followed by its sample application on a fuzzy-syllogistic ontology and the introduction of relative truth ratios ${}^{r}\tau$.

2 Fuzzy-Syllogistic Systems

The fuzzy-syllogistic systems ${}^{n}\mathbb{S}$, with $1 < n$ fuzzy quantifiers, extend the well known Aristotelian syllogisms with fuzzy-logical concepts, like truth ratio for every mood and fuzzy quantifiers or in general fuzzy sets. We discuss first the systems ${}^{n}\mathbb{S}$ and introduce them further below as the basic reasoning logic of FSR.

2.1 Aristotelian Syllogistic System $\mathbb{S}$

The Aristotelian syllogistic system $\mathbb{S}$ consists of inclusive existential quantifiers ψ, i.e. I includes A and O includes E as one possible case:

$$\textbf{Universal affirmative } \psi = \textbf{A : All S are P : } \{\mathbf{x} |\ \mathbf{S} - \mathbf{P} = \varnothing \wedge \mathbf{x} \in \mathbf{S} \cap \mathbf{P}\}$$

$$\textbf{Universal negative } \psi = \textbf{E : All S are not P : } \{\mathbf{x} |\ \mathbf{x} \in \mathbf{S} - \mathbf{P} \wedge \mathbf{P} \cap \mathbf{S} = \varnothing\}$$

$$\textbf{Inclusive existential affirmative } \psi = \textbf{I : Some S are P :}$$
$$\mathbf{A} \cup \{\mathbf{x} |\ \mathbf{x} \in \mathbf{S} \cap \mathbf{P} \vee (\mathbf{x} \in \mathbf{S} \cap \mathbf{P} \wedge \mathbf{P} - \mathbf{S} = \varnothing)\} \Leftrightarrow \mathbf{A} \cup \{\mathbf{x} |\ \mathbf{x} \in \mathbf{S} \cap \mathbf{P} \vee \mathbf{P} - \mathbf{S} = \varnothing\}$$

$$\textbf{Inclusive existential negative } \psi = \textbf{O : Some S are not P :}$$
$$\mathbf{E} \cup \{\mathbf{x} |\ \mathbf{x} \in \mathbf{S} - \mathbf{P} \vee (\mathbf{x} \in \mathbf{S} - \mathbf{P} \wedge \mathbf{P} - \mathbf{S} = \varnothing)\} \Leftrightarrow \mathbf{E} \cup \{\mathbf{x} |\ \mathbf{x} \in \mathbf{S} - \mathbf{P} \vee \mathbf{P} - \mathbf{S} = \varnothing\}$$

A categorical syllogism $\psi_1\psi_2\psi_3$F is an inference schema that concludes a quantified proposition $\Phi_3 = S\psi_3P$ from the transitive relationship of two given quantified proportions $\Phi_1 = \{M\psi_1P, P\psi_1M\}$ and $\Phi_2 = \{S\psi_2M, M\psi_2S\}$:

$$\boldsymbol{\psi_1\psi_2\psi_3}\mathbf{F} = (\boldsymbol{\Phi_1} = \{\mathbf{M}\boldsymbol{\psi_1}\mathbf{P}, \mathbf{P}\boldsymbol{\psi_1}\mathbf{M}\}, \boldsymbol{\Phi_2} = \{\mathbf{S}\boldsymbol{\psi_2}\mathbf{M}, \mathbf{M}\boldsymbol{\psi_2}\mathbf{S}\}, \boldsymbol{\Phi_3} = \mathbf{S}\boldsymbol{\psi_3}\mathbf{P})$$

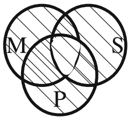
where F = {1, 2, 3, 4} identifies the four possible combinations of Φ_1 with Φ_2, namely syllogistic figures. Every figure produces $4^3 = 64$ moods and the whole syllogistic system $\mathbb{S}$ has $4 \times 64 = 256$ moods.

2.2 Syllogistic-Cases

Syllogistic cases are an elementary concept of the fuzzy-syllogistic systems ${}^n\mathbb{S}$, for calculating truth ratios [8] of the moods algorithmically [9].

For three sets, 7 distinct spaces δ_i, i = [1, 7] are possible, which can be easily identified in a Venn diagram (Table 1). There are in total j = 96 distinct combinations of the spaces $\Delta_j = \delta_1\delta_2\delta_3\delta_4\delta_5\delta_6\delta_7$, j = [1,96] [27], which constitute the universal set of syllogistic moods. Within that universe, we determine for every mood true and false matching space combinations (Fig. 1).

Table 1. Binary coding of the 7 possible distinct spaces for three sets.

Sample Syllogistic Case $\Delta_j=\delta_1\delta_2\delta_3\delta_4\delta_5\delta_6\delta_7$*; Δ_{95}=1111110#	
Venn Diagram	Space Diagram+
	 δ_1 δ_2 δ_3 δ_4 δ_5 δ_6 δ_7

*Binary coding of all possible distinct space combinations Δ_j, j=[1,96] that can be generated for three sets.
#δ_i=0: space i is empty; δ_i=1: space i is not empty; i=[1,7].
+Every circle of a space diagram represents exactly one distinct sub-set of M∪P∪S.

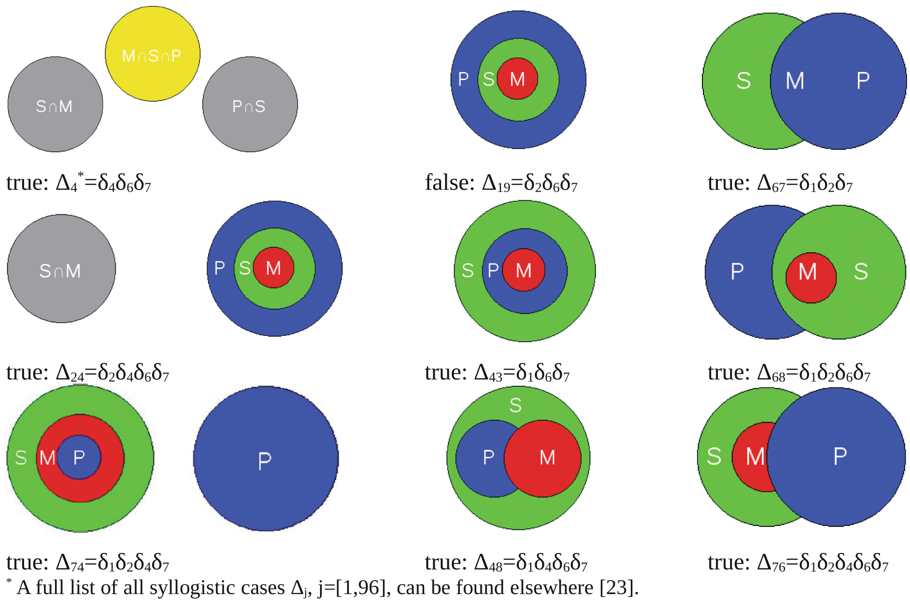


Fig. 1. 9 syllogistic cases Δ_j of the mood ${}^{2/1}\mathrm{IA}^1\mathrm{I4}$ of the fuzzy-syllogistic systems ${}^2\mathbb{S}$.

2.3 Fuzzy-Syllogistic Moods

We extend the ancient binary truth classification of moods, to a fuzzy classification with truth values in [0,1]. For this purpose, first the above set-theoretical definitions of the quantifiers of a particular mood are compared against the set of all syllogistic cases Δ_j, j = [1,96], in order to identify true and false matching cases:

$$\textbf{True syllogistic cases}: \boldsymbol{\Lambda}^t = {}_{j=1}\cup^{96} \boldsymbol{\Delta}_j \in \left(\boldsymbol{\Phi}_1^\Delta \cap \boldsymbol{\Phi}_2^\Delta\right) \rightarrow \boldsymbol{\Delta}_j \in \boldsymbol{\Phi}_3^\Delta$$

$$\textbf{False syllogistic cases}: \boldsymbol{\Lambda}^f = {}_{j=1}\cup^{96} \boldsymbol{\Delta}_j \in \left(\boldsymbol{\Phi}_1^\Delta \cap \boldsymbol{\Phi}_2^\Delta\right) \rightarrow \boldsymbol{\Delta}_j \notin \boldsymbol{\Phi}_3^\Delta$$

where Λ^t and Λ^f is the set of all true and false matching cases of a particular mood, respectively (Fig. 1) and Φ^Δ is a proposition in terms of its true and false matching syllogistic cases. For instance, the two premisses Φ_1 and Φ_2 of the mood IAI4 of the syllogistic system $\mathbb{S}$, match the 10 syllogistic cases $\Lambda^t = \{\Delta_4, \Delta_{19}, \Delta_{67}, \Delta_{24}, \Delta_{43}, \Delta_{46}, \Delta_{68}, \Delta_{74}, \Delta_{48}, \Delta_{76}\}$, which are all true for the conclusion Φ_3 as well. Thus the mood has no false cases $\Lambda^f = \emptyset$.

The truth ratio of a mood is then calculated by relating the amounts of the two sets Λ^t and Λ^f with each other. Consequently the truth ratio becomes either more true or more false $\tau \in \{\tau^f, \tau^t\}$:

$$\textbf{More true}: \tau^t = 1 - |\Lambda^f|/(|\Lambda^t| + |\Lambda^f|) = [0.545, 1] \textbf{ for } |\Lambda^f| < |\Lambda^t|$$

$$\textbf{More false}: \tau^f = |\Lambda^t|/(|\Lambda^t| + |\Lambda^f|) = [0, 0.454] \textbf{ for } |\Lambda^t| < |\Lambda^f|$$

where $|\Lambda^t|$ and $|\Lambda^f|$ are the numbers of true and false syllogistic cases, respectively. A fuzzy-syllogistic mood is then defined by assigning an Aristotelian mood $\psi_1\psi_2\psi_3F$ the structurally fixed truth ratio τ:

$$\textbf{Fuzzy-syllogistic mood}: (\psi_1\psi_2\psi_3F, \ \tau)$$

The truth ratio identifies the degree of truth of a particular mood, which we will associate further below in fuzzy-syllogistic reasoning with generic vagueness of inferencing with that mood.

The analysis of the Aristotelian syllogistic system $\mathbb{S}$ with these concepts reveals several interesting properties, like $\mathbb{S}$ has 136 distinct moods, 25 true moods $\tau = 1$, of which 11 are distinct, and 25 false moods $\tau = 0$, of which 11 are distinct, and that $\mathbb{S}$ is almost point-symmetric on syllogistic cases and truth ratios of the moods [11, 27].

2.4 Fuzzy-Syllogistic System $^2\mathbb{S}$

In the fuzzy-syllogistic system (FSS) $^2\mathbb{S}$, the universal cases A and E are excluded from the existential quantifiers I and O, respectively:

$$\textbf{Exclusive existential affirmative}: \textbf{Some S are P}: \psi = \textbf{I}: \{\textbf{x} | \textbf{x} \in \textbf{S} \cap \textbf{P} \vee \textbf{P} - \textbf{S} = \emptyset\}$$

$$\textbf{Exclusive existential negative}: \textbf{Some S are not P}: \psi = \textbf{O}: \{\textbf{x} | \textbf{x} \in \textbf{S} - \textbf{P} \ \vee \ \textbf{P} - \textbf{S} = \emptyset\}$$

For instance the mood IAI4 of $\mathbb{S}$, becomes $^{2/1}IA^1I4$ in $^2\mathbb{S}$. Because of the exclusive existential quantifier $^{2/1}I$, the case Δ_{46} is no more matched by the first premiss Φ_1 and the conclusion Φ_3 becomes false for the case Δ_{19} (Fig. 1).

The analysis of the FSS $^2\mathbb{S}$ shows that $^2\mathbb{S}$ has 70 distinct moods, 11 true moods $\tau = 1$, of which 5 are distinct, and 40 false moods $\tau = 0$, of which 13 are distinct, and that $^2\mathbb{S}$ is not point-symmetric [11, 27].

2.5 Fuzzy-Syllogistic System $^n\mathbb{S}$

By using (n − 1) fuzzy-existential quantifiers, the total number of fuzzy-syllogistic moods of the FSS $^n\mathbb{S}$ increases to $(2n)^3$. For instance the mood IAI4 of $\mathbb{S}$ can be generalised in $^n\mathbb{S}$ to $^{n/k1}IA^{k2}I4$, k_1, k_2 = [2, n]. $^{n/k1}IA^{k2}I4$ consists of $(n-1)^2$ fuzzy-moods, all having the very same 9 syllogistic cases (Fig. 1).

Same linguistic terms used in different FSSs do not necessarily equal each other. For instance, "most" may have different value ranges in the FSSs $^3\mathbb{S}$, $^4\mathbb{S}$, $^5\mathbb{S}$, $^6\mathbb{S}$ and therefore are in general not equal $^{3/2}I \neq {}^{4/3}I \neq {}^{5/3}I \neq {}^{6/4}I$, respectively. Likewise for "half" in $^4\mathbb{S}$ and $^6\mathbb{S}$ the quantifiers may not exactly equal $^{4/2}\mathrm{I} \neq {}^{6/3}\mathrm{I}$, respectively (Table 2).

Table 2. Value ranges of affirmative fuzzy quantifiers[#] of n fuzzy-syllogistic systems $^n\mathbb{S}$

<table>
<tr><td colspan="2">Syllogistic System</td><td colspan="61">Fuzzy Quantifier ψ*</td></tr>
<tr><td>Aristotelian</td><td>$\mathbb{S}$</td><td>A=all</td><td colspan="60">I=some (including A)</td></tr>
<tr><td rowspan="6">Fuzzy</td><td>$^2\mathbb{S}$</td><td>A=all</td><td colspan="60">$^{2/1}$I=some (excluding A)</td></tr>
<tr><td>$^3\mathbb{S}$</td><td>A=all</td><td colspan="30">$^{3/2}$I=most</td><td colspan="30">$^{3/1}$I=several</td></tr>
<tr><td>$^4\mathbb{S}$</td><td>A=all</td><td colspan="20">$^{4/3}$I=most</td><td colspan="20">$^{4/2}$I=half</td><td colspan="20">$^{4/1}$I=several</td></tr>
<tr><td>$^5\mathbb{S}$</td><td>A=all</td><td colspan="15">$^{5/4}$I=many</td><td colspan="15">$^{5/3}$I=most</td><td colspan="15">$^{5/2}$I=several</td><td colspan="15">$^{5/1}$I=few</td></tr>
<tr><td>$^6\mathbb{S}$</td><td>A=all</td><td colspan="12">$^{6/5}$I=many</td><td colspan="12">$^{6/4}$I=most</td><td colspan="12">$^{6/3}$I=half</td><td colspan="12">$^{6/2}$I=several</td><td colspan="12">$^{6/1}$I=few</td></tr>
<tr><td>$^n\mathbb{S}$</td><td>A=all</td><td colspan="10">$^{n/n-1}$I</td><td colspan="40">...</td><td colspan="10">$^{n/1}$I</td></tr>
</table>

[#] Negative quantifiers are arranged analogously.
[*] Column breadths are not drawn proportional to the overall value range or to other quantifiers.

3 Fuzzy-Syllogistic Ontology

A fuzzy-syllogistic ontology (FSO) is an extended semantic network, whose concept relationships consist of possibly multiple bi-directional fuzzy quantifiers ψ. A FSO may be obtained in two ways, by extending an existing crisp ontology or by extending a probabilistically learned ontology. Here we provide a definition for FSO and discuss learning FSOs, along with some distinguishing properties of FSOs.

3.1 Definition

A FSO consists of concepts, their relationships and assertions on them, whereby all quantities are given with fuzzy quantifications:

$$\textbf{Fuzzy-syllogistic ontology : FSO} = {}^{\mathbf{k}}(\mathbf{C}, \mathbf{R}, \mathbf{A})$$

where C is the set of all concepts, R is the set of all binary relationships between the concepts, A is the set of all assertions and k is a particular FSS $^k\mathbb{S}$, k = [2, n]. A FSO is in compliance with a particular FSS $^k\mathbb{S}$, if all quantifiers ψ of the FSO comply with $^k\mathbb{S}$ (Fig. 2).

3.2 Learning Fuzzy Quantifiers

Although existing learning approaches generate ontological concepts and their relationships through probabilistic analysis of domain data [4, 14, 18], the quantities that

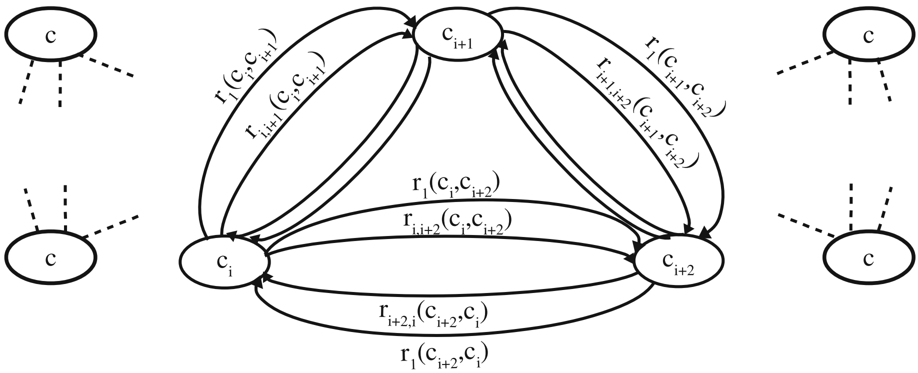


Fig. 2. Poly-bi-directional fuzzy-quantified binary concept relationships of a fuzzy-syllogistic ontology (FSO).

actually imply the concepts and relationships, are not preserved in probabilistic ontologies [10]. Exactly this data, ie the quantities of the samples, needs to be preserved in the learning phase of a FSO, as they imply the fuzzy quantifiers ψ of the fuzzy moods $(\psi_1\psi_2\psi_3F, \tau)$.

For any two concepts c_i and c_{i+1} of a FSO, all binary relationships have to be stored with the FSO. Hence, all possible bi-directional binary relationships between all concepts of a FSO constitute a poly-bi-directional graph (Fig. 2):

$$\begin{aligned}&\textbf{Poly-uni-directional binary relationships:}\\ &\mathbf{R}_{\mathbf{ci},\mathbf{ci+1}} = \{\mathbf{r_1}(\mathbf{c_i},\mathbf{c_{i+1}}),\ \ldots,\ \mathbf{r}_{\mathbf{i},\mathbf{i+1}}(\mathbf{c_i},\mathbf{c_{i+1}})\};\ \mathbf{2} < \mathbf{i} \leq \mathbf{o}\end{aligned}$$

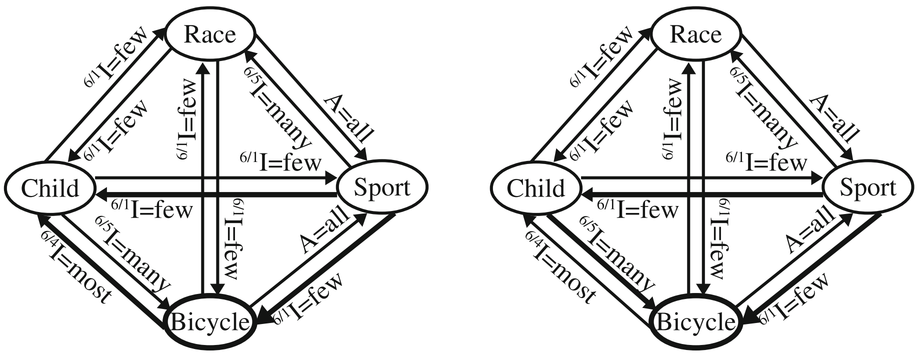
$$\textbf{Poly-bi-directional binary relationships:}\ \mathbf{R}_{\mathbf{ci},\mathbf{ci+1}} \cup \mathbf{R}_{\mathbf{ci+1},\mathbf{ci}};\ \mathbf{2} < \mathbf{i} \leq \mathbf{o}$$

Since the data may imply for any three concept multiple bi-directional relationships, multiple ternary relationships may be generated for those concepts. Every ternary relationship of a FSO = $^k(C, R, A)$ may be interpreted as a fuzzy-syllogistic mood:

$$\begin{aligned}&\textbf{Ternary relationships/fuzzy-syllogistic moods:}\\ &(\boldsymbol{\psi_1\psi_2\psi_3}\mathbf{F},\ \boldsymbol{\tau}) = \big(\boldsymbol{\psi_1} \in \{\mathbf{r}_{\mathbf{i},\mathbf{i+1}}(\mathbf{c_i},\mathbf{c_{i+1}}), \mathbf{r}_{\mathbf{i+1},\mathbf{i}}(\mathbf{c_{i+1}},\mathbf{c_i})\}\\ &\boldsymbol{\psi_2} \in \{\mathbf{r}_{\mathbf{i+1},\mathbf{i+2}}(\mathbf{c_{i+1}},\mathbf{c_{i+2}}), \mathbf{r}_{\mathbf{i+2},\mathbf{i+1}}(\mathbf{c_{i+2}},\mathbf{c_{i+1}})\}\boldsymbol{\psi_3} = \mathbf{r}_{\mathbf{i},\mathbf{i+2}}(\mathbf{c_i},\mathbf{c_{i+2}}),\ \boldsymbol{\tau}\big)\end{aligned}$$

Ones concepts and relationships of a FSO = $^k(C, R, A)$ are learned, the final step is to determine the most appropriate fuzzy quantifier system, i.e. FSS, $^k\mathbb{S}$. This is achieved by matching the average quantity distributions between all concepts of the FSO to the closest FSS $^k\mathbb{S}$.

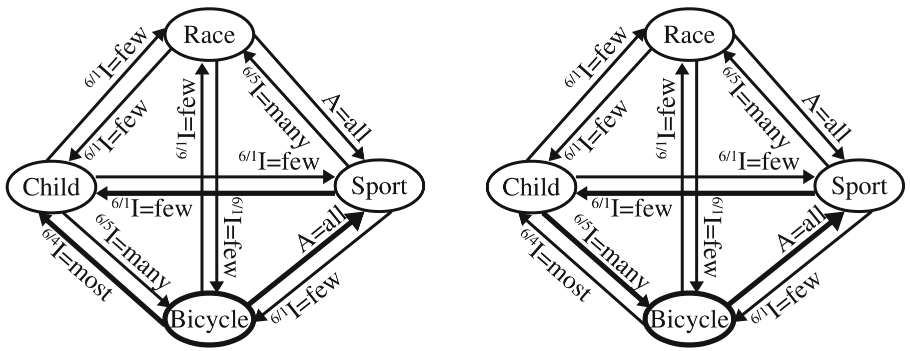
Since fuzzy quantifiers are calculated by accumulating samples, new samples can continuously be learned by cumulatively updating the quantifiers. The most appropriate FSS $^k\mathbb{S}$ out of $^n\mathbb{S}$, k = [2, n] can be re-calculating, if necessary. For instance, in case of significant amounts of quantifier updates, which change the quantity distributions.



$^{6}\mathbb{S}$: ($^{6/4}I^{1}I^{1}$I1, 40/47=0.851)
Φ_1: Most **bicycles** are good for children
Φ_2: Few sports are good for **bicycles**
Φ_3: Few **sports** are good for **children**

$^{6}\mathbb{S}$: ($^{6/5}I^{1}I^{1}$I2, 40/48=0.833)
Φ_1: Many children have **bicycles**
Φ_2: Few sports are good for **bicycles**
Φ_3: Few **sports** are good for **children**

Fig. 3. Sample fuzzy-syllogistic ontology with affirmative relationships and the best matching fuzzy-syllogistic moods from the syllogistic Figs. 1 and 2.



$^{6}\mathbb{S}$: ($^{6/4}IA^{1}$I3, 6/6=1.0)
Φ_1: Most **bicycles** are good for children
Φ_2: All **bicycles** are good for sports
Φ_3: Few **sports** are good for **children**

$^{6}\mathbb{S}$: ($^{6/5}IA^{1}$I4, 8/9=0.888)
Φ_1: Many children have **bicycles**
Φ_2: All **bicycles** are good for sports
Φ_3: Few **sports** are good for **children**

Fig. 4. Sample fuzzy-syllogistic ontology with affirmative relationships and the best matching fuzzy-syllogistic moods from the syllogistic Figs. 3 and 4.

3.3 Relative Truth Ratio

Relative truth ratios are calculated from the exact quantities of all syllogistic cases of a particular mood, rather than from just the amount of the cases:

$$\textbf{Relative true}: {}^{r}\tau^{t} = \lambda^{f}/(\lambda^{f}+\lambda^{t}) \ \textbf{for} \ \lambda^{f} < \lambda^{t}$$

$$\textbf{Relative false}: {}^{r}\tau^{f} = \lambda^{t}/(\lambda^{t}+\lambda^{f}) \ \textbf{for} \ \lambda^{t} < \lambda^{f}$$

where $\lambda^t =_{j=1} \sum^{|\Lambda t|} |\Delta_j^t|$ and $\lambda^f =_{j=1} \sum^{|\Lambda f|} |\Delta_j^f|$ is the total number of elements accumulated over all true and false syllogistic cases, respectively. Where $|\Lambda^t|$ and $|\Lambda^f|$ is the number of true and false cases of the mood, respectively. Accordingly, we re-define the truth of a fuzzy-syllogistic mood in terms of relative truth ratio ${}^r\tau$:

$$\textbf{Fuzzy-syllogistic mood with relative truth ratio} : (\psi_1\psi_2\psi_3 F, {}^r\tau)$$

The structural truth ratio τ of a particular mood represents the generic vagueness of the mood and is constant, whereas the relative truth ratio ${}^r\tau$ adjusts τ by weighting every case of the mood with its actual quantity.

The concept of a relative truth ratio is a set-theoretic representation of a weighted logic, which is not new in the literature [5].

3.4 Mood Semantics

Truth ratios τ, ${}^r\tau$ of a mood provide solely structural evaluations for any propositions loaded on the mood. The semantics of a mood for given sample propositions can be determined with following two principle approaches:

- Top-down: Specifying from existing knowledge; possibilistic.
- Bottom-up: Learning from data sources; probabilistic.

In either approach the objective is to determine for reasonable concepts {M, P, S}, reasonable propositions $\Phi_1 \in \{M\psi_1 P, P\psi_1 M\}$, $\Phi_2 \in \{S\psi_2 M, M\psi_2 S\}$ with reasonable fuzzy quantifiers ψ_1, ψ_2, ψ_3 and to find the most reasonable concluding proposition Φ_3 = (S, P), which is the one with the highest truth ratio (Fig. 5).

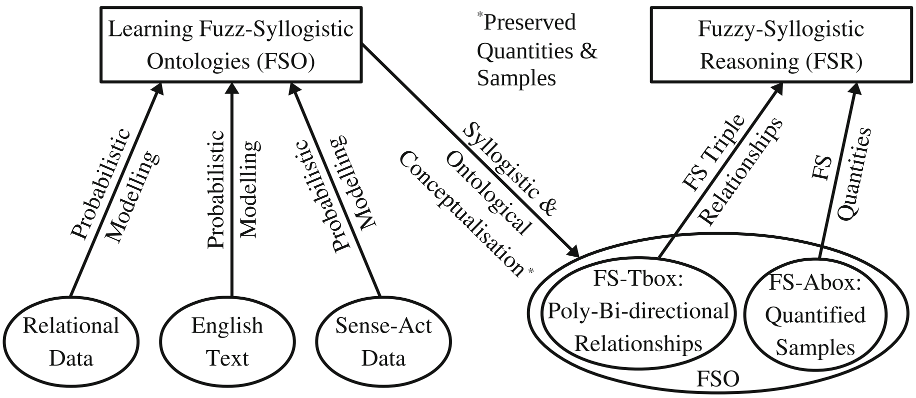


Fig. 5. Learning fuzzy syllogistic ontologies (FSO) from various sources and fuzzy-syllogistic reasoning with FSOs.

The specification of a sample FSO is sketched for the below discussed reasoning examples (Figs. 3 and 4). The quantifier distributions of the FSO represent personal perceptions from that domain. Therefore relative truth ratios cannot be calculated for the moods.

In bottom-up approaches, concepts and their relationships are synthesised from probability distributions that are calculated from source data. Such a process is typically associated with learning ontologies [10, 14]. In case of statistically sufficient numbers of samples found in the source data for ever concept and every concept relationship, we can assume that a learned FSO sufficiently represents the domain. and that uncertainties become increasingly more tolerable and eventually neglectable. Since source data is available in this approach, relative truth ratios ${}^{r}\tau$ can be calculated for the moods.

Thus mood semantics of FSOs are learned from the source data. The semantic of a sample syllogism is determined by the FSR, by searching for the fuzzy mood with the highest truth ratio τ.

4 Fuzzy-Syllogistic Reasoning

The fuzzy-syllogistic systems $\mathbb{S}$, ${}^{2}\mathbb{S}$ and ${}^{6}\mathbb{S}$ are currently implemented experimentally as the reasoning logic of the fuzzy-syllogistic reasoner (FSR), for reasoning over FSOs [26]. Our objective is to generalise the logic of the reasoner to ${}^{n}\mathbb{S}$ and to use it as a cognitive primitive for modelling other cognitive concepts within a cognitive architecture. We now sketch the algorithmic design of the FSR.

4.1 Sample Reasoning Processes

For any directly connected ternary concept relationship of the FSO, seven distinct relationships are possible (Table 1). FSR is concerned with identifying for any given concept $c \in C$, all possible ternary concept relationships $r \in R$, $r = \{M,P,S\}$, of a given FSO = ${}^{k}(C, R, A)$ and to reason with the most appropriate fuzzy-syllogistic moods of the FSS ${}^{k}\mathbb{S}$, $k = [2, n]$. Whereby associated assertions $a \in A$ may be used for exemplifying a particular reasoning.

For instance, for the concept c = Bicycle, multiple ternary relationships r = {Bicycle, Child, Sports} exist in the sample FSO = ${}^{6}(C, R, A)$ (Fig. 3). The reasoner iterates for all moods of the FSS ${}^{6}\mathbb{S}$ and matches the moods with the closest fuzzy-syllogistic quantities of relationships r. As best match the mood ${}^{6/k1}IA^{k2}I4$, $0 < k_1,k_2 < 6$ is found for this particular relationship r.

In the below example with $\mathbb{S}$, I in Φ_3 may include A and therefore is less true. Whereas in ${}^{3}\mathbb{S}$, ${}^{3/1}I$ in Φ_3 is still too general. The best matching quantifiers are found in ${}^{6}\mathbb{S}$ (Fig. 4).

4.2 Reasoning Algorithm

For a FSO = $^k(C, R, A)$ and any given directly connected three concepts $c_1, c_2, c_3 \in C$, the FSR searches all fuzzy moods for the highest matching truth ratio τ. That mood is determined as the most reasonable syllogism for the given ternary concepts.

The steps of the reasoning algorithm are as follows:

```
For given c₁, c₂, c₃∈C
For all fuzzy moods in ᵏ𝕊
Match the fuzzy mood having maximum truth ratio τ or ʳτ
```

4.3 Less Reasonable FSOs

For the sample domain (Figs. 3 and 4), the FSSs $\mathbb{S}$ and $^3\mathbb{S}$ are less reasonable than $^6\mathbb{S}$, because their quantifiers cover a too broad range or the coverage of the quantifiers of $^6\mathbb{S}$ are closer to the domain data.

In the Aristotelian system $\mathbb{S}$, the existential quantifier I may include A, thus in the below example, the proposition Φ_1 is wrong for the A case. Φ_1 is further wrong for all bicycles having larger wheel sizes than those sizes more suitable for children.

```
𝕊: (IAI3, 10/10=1.0)
Φ₁: Some bicycles are good for children
Φ₂: All bicycles are good for sports
Φ₃: Some sports are good for children
```

The below example is based on $^3\mathbb{S}$. The quantifier $^{3/2}I$ = Most of the proposition Φ_1 expresses a closer quantity representation of the domain data than I = Some, since it excludes the above mentioned quantity ranges. Although, the quantity $^{3/2}I$ = Several in Φ_3, covers the domain quantity closer than I = Some, it matches weaker than $^{6/4}I$ = Few of $^6\mathbb{S}$. Here we assume that in reality only few sports are actually suitable for children, ie most spots become suitable for children only in simplified versions.

```
³𝕊: (³ᐟ²IA¹I3, 6/6=1.0)
Φ₁: Most bicycles are good for children
Φ₂: All bicycles are good for sports
Φ₃: Several sports are good for children
```

In general, the closer a particular FSS $^k\mathbb{S}$ to the quantity distributions of the domain data, the more realistic conclusions can be expected from the FSR.

4.4 Cognitive Primitive

The FSS $^n\mathbb{S}$ serves as the underlying logic of the FSO and the FSR. The current implementation of the components comprises further learning FSOs, which is also based on the same logic (Fig. 4).

Learning FSOs is the emergent component of the symbolic FSR. Therefore, the depicted architecture is a hybrid cognitive architecture, in which the FSS serves as a cognitive primitive logic.

5 Conclusion

The FSS ${}^{n}\mathbb{S}$ was introduced as the fundamental logic of the FSR and its application to approximate reasoning on FSOs was shown on the sample FSSs ${}^{2}\mathbb{S}$ and ${}^{6}\mathbb{S}$. The relative truth ratio ${}^{r}\tau$ of a mood was introduced, which adapts the structural truth ratio τ of a mood to the cardinalities of the syllogistic cases of the mood. FSR with FSOs has been proposed as a generic possibilistic reasoning approach, since the underlying logic ${}^{n}\mathbb{S}$ is structurally generic. We have further proposed learning mood semantics statistically.

Currently we are generalising the FSS analysis tool, such that all reasonable systems ${}^{n}\mathbb{S}$ can be exploited algorithmically for realistic numbers of quantifiers [2, n]. That will enable us to implement a comprehensive FSR. Currently we are further developing applications for learning FSOs from English text sources and from robotic sense-act data relationships. Our ultimate goal is to employ the components learning FSOs and FSR.

References

1. Bobillo, F., Straccia, U.: fuzzyDL: an expressive fuzzy description logic reasoner. In: Fuzzy Systems (FUZZ-IEEE). IEEE Computer Society (2008)
2. Bobillo, F., Delgado, M., Gomez-Romero, J.: DeLorean: a reasoner for fuzzy OWL 1.1. In: Uncertainty Reasoning for the Semantic Web (URSW), LNAI. Springer (2008). http://ceur-ws.org/Vol-423
3. Bobillo, F., Straccia, U.: Reasoning with the finitely many-valued Łukasiewicz fuzzy description logic SROIQ. Inf. Sci. **181**, 758–778 (2011)
4. Cimiano, P., Völker, J.: Text2Onto: a framework for ontology learning and data-driven change discovery. In: Montoyo, A., Muńoz, R., Métais, E. (eds.) NLDB 2005. LNCS, vol. 3513, pp. 227–238. Springer, Heidelberg (2005)
5. Dubois, D., Godo, L., Prade, H.: Weighted logics for artificial intelligence – an introductory discussion. Int. J. Approx. Reason. **55**, 1819–1829 (2014)
6. Dubois, D., Godo, L., Prade, H.: On fuzzy syllogisms. Comput. Intell. **4**, 171–179 (1988)
7. Goertzel, B., Iklé, M., Goertzel, I.L.F., Heljakka, A.: Probabilistic Logic Networks: A Comprehensive Conceptual, Mathematical and Computational Framework for Uncertain Inference. Springer, New York (2008)
8. Kumova, Bİ., Çakir, H.: The fuzzy syllogistic system. In: Hernández Aguirre, A., Reyes García, C.A., Sidorov, G. (eds.) MICAI 2010, Part II. LNCS (LNAI), vol. 6438, pp. 418–427. Springer, Heidelberg (2010)
9. Kumova, Bİ., Çakır, H.: Algorithmic decision of syllogisms. In: García-Pedrajas, N., Herrera, F., Fyfe, C., Benítez, J.M., Ali, M. (eds.) IEA/AIE 2010, Part II. LNCS, vol. 6097, pp. 28–38. Springer, Heidelberg (2010)

10. Kumova, Bİ.: Generating ontologies from relational data with fuzzy-syllogistic reasoning. In: Kozielski, S., Mrozek, D., Kasprowski, P., Małysiak-Mrozek, B., Kostrzewa, D. (eds.) BDAS 2015. CCIS, vol. 521. Springer, Switzerland (2015)
11. Kumova, B.İ.: Properties of the syllogistic system. J. Rev. (2016)
12. Lui, Y., Kerre, E.E.: An overview of fuzzy quantifiers. (II). Reasoning and applications. Fuzzy Sets Syst. **95**, 135–146 (1998)
13. Łukasiewicz, J.: O logice trójwartościowej (translation from Polish: On three-valued logic). Ruch filozoficzny 5 (1920)
14. Lukasiewicz, T., Straccia, U.: Managing uncertainty and vagueness in description logics for the SemanticWeb. Web Semant.: Sci. Serv. Agents WWW **6**, 291–308 (2008). Elsevier
15. Motik, B., Shearer, R., Horrocks, I.: Hypertableau reasoning for description logics. J. Artif. Intell. Res. **36**, 165–228 (2009). AAAI
16. Pereira-Fariña, M., Vidal, J.C., Díaz-Hermida, F., Bugarín, A.: A fuzzy syllogistic reasoning schema for generalized quantifiers. Fuzzy Sets Syst. **234**, 79–96 (2014)
17. Peterson, P.: Intermediate Quantifiers: Logic, Linguistics, and Aristotelian Semantics. Ashgate, Farnham (2000)
18. Qi, G., Pan, J.Z., Ji, Q. A possibilistic extension of description logics. In: Description Logics (DL). Sun SITE Central Europe (CEUR) (2007)
19. Wang, P.: From inheritance relation to non-axiomatic logic. Int. J. Approx. Reason. **7**, 281–319 (1994)
20. Wang, P.: Reference classes and multiple inheritances. Int. J. Uncertain. Fuzziness Knowl. Based Syst. **3**(1), 79–91 (1995)
21. Yaguinuma, C.A., Magalhães Jr., W.C., Santos, M.T., Camargo, H.A., Reformat, M.: Combining fuzzy ontology reasoning and mamdani fuzzy inference system with HyFOM reasoner. In: Hammoudi, S., Cordeiro, J., Maciaszek, L.A., Filipe, J. (eds.) ICEIS 2013. LNBIP, vol. 190, pp. 174–189. Springer, Heidelberg (2014)
22. Zadeh, L.A.: Fuzzy Sets. Inf. Control **8**, 338–353 (1965)
23. Zadeh, L.A.: Fuzzy logic and approximate reasoning. Syntheses **30**, 407–428 (1975)
24. Zadeh, L.A.: A theory of commonsense knowledge. In: Skala, H.J., Trillas, E., Termini, S. (eds.) Aspects of Vagueness. Reidel, Dordrecht (1984)
25. Zadeh, L.A.: Syllogistic reasoning in fuzzy logic and its application to usuality and reasoning with dispositions. IEEE Trans. Syst. Man Cybern. **15**(6), 754–763 (1985)
26. Zarechnev, M., Kumova, Bİ.: Ontology-based fuzzy-syllogistic reasoning. In: Ali, M., Kwon, Y.S., Lee, C.-H., Kim, J., Kim, Y. (eds.) IEA/AIE 2015. LNCS, vol. 9101. Springer, Switzerland (2015)
27. Zarechnev, M., Kumova, B.İ.: Truth ratios of syllogistic moods. In: IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). IEEE Xplore (2015)