

# Resource Allocation for Underlying Device-to-Device Communications Using Maximal Independent Sets and Knapsack Algorithm

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**Abstract**—In this paper, we address the resource allocation problem of device-to-device (D2D) communications underlying orthogonal frequency division multiple access (OFDMA) based cellular systems by exploiting the efficiency that comes from an ensemble of graph theory and Knapsack problem. It is possible to construct the conflict graph of the D2D pairs by finding the maximal independent sets. Then, we use those independent sets as inputs to Knapsack problem iteratively in order to find D2D groups which allocate the subchannels. In Knapsack problem, we consider a maximum interference level that the base station is exposed at each subchannel. We illustrate that the proposed resource allocation method significantly outperforms graph coloring in terms of average data rate for the high number of underlying D2D pairs in cellular systems.

**Index Terms**—D2D Communications; OFDMA; Conflict graph; Maximal Independent Set; Knapsack Problem

## I. INTRODUCTION

The need for higher data rate is exponentially increasing due to very high throughput services such as high quality video streaming and augmented reality. The need for high throughput encourages us to allocate given resources efficiently. It is known that device-to-device (D2D) communications can be used in cellular communication to increase the data rate where the goal is to maximize D2D data rate without affecting the cellular data rate.

D2D communications is recognized as one of the technology components of evolving 5G architecture by the European Union project METIS. D2D enables users to communicate directly with others without any need for the network infrastructure. Such direct communication has significant benefits such as decreased latency, increased data rate and decreased power consumption. Moreover, the spectrum can be shared by several D2D pairs if such pairs are far enough to have negligible mutual interference. Of course, not all mobile users may

be involved in direct communication. Some users will still communicate through the BS. Spectrum sharing is allowed between cellular and D2D communications dynamically; however, adding D2D communications should not generate any regression of cellular transmissions. D2D communications are then underlaid in cellular transmissions. The application areas of D2D communications include emergency communication, internet-of-things enhancement and localization.

The problem of resource block allocation in the underlaid D2D networks has been studied recently. The approaches mainly differ in being distributed or centralized. For instance, Katsinis and Tsiropoulou [1] have tackled the problem in a distributed manner, whereas we aim to maximize sum-rate of the network in a centralized manner. Phunchongharn et al. [2] have described the general scheme of D2D communications in LTE-Advanced networks. They presented an algorithm where the goal is to maximize the spectrum utilization by finding the minimum transmission length in terms of time slots for D2D links while protecting the cellular users from harmful interference and guaranteeing the quality of service of D2D links. The objective of maximizing the sum rate or the energy efficiency has been considered in [3] and [4] where each D2D pair is only allocated one resource block (RB) and several D2D pairs are multiplexed with a cellular user. The multiplexing of D2D pairs with cellular users has been written as a graph coloring problem in [5], [6]. Hoang et al. [7] has used an algorithm that relies on iterative solutions to maximize the weighted sum rate while achieving proportional fairness. In [8], authors have addressed the resource allocation by using the applications of game-theoretic models. In this paper, we propose a resource allocation method using maximal independent sets and knapsack algorithm for underlaid D2D cellular networks.

The remainder of the paper is organized as follows.

We describe the system model in Section II. We explain the proposed approach to construct the conflict graph and clarify how we make use of maximal independent sets in the Knapsack problem for resource allocation in Section III. The simulation results are illustrated in Section IV and we conclude the paper in Section V.

## II. SYSTEM MODEL

We consider uplink orthogonal frequency division multiple access (OFDMA) based systems having  $N$  subchannels. There are  $K_c$  cellular users and  $K_d$  D2D pairs in a single cell single input single output (SISO) system.

Our objective is to maximize the sum data rate of both cellular users and the D2D pairs. Then, the optimization problem is expressed by,

$$\begin{aligned} \max_{\substack{\mathbf{P}_d, \mathbf{P}_c \\ \mathbf{V}_d, \mathbf{V}_c}} & \sum_{k_c=1}^{K_c} \sum_{n \in \mathbf{V}_{k_d}} \log_2 \left( 1 + \frac{P_{k_c}^n G_{0,k_c}^n}{N_0^n + I_0^n} \right) \\ & + \sum_{k_d=1}^{K_d} \sum_{n \in \mathbf{V}_{k_d}} \log_2 \left( 1 + \frac{P_{k_d}^n G_{k_d,k_d}^n}{N_{k_d}^n + I_{k_d}^n + I_{k_d,k_c}^n} \right) \end{aligned} \quad (1)$$

$$\text{s.t.} \quad \sum_{n \in \mathbf{V}_{k_c}} P_{k_c}^n \leq P^{\max}, \quad k_c \in \{1, \dots, K_c\} \quad (2)$$

$$\text{s.t.} \quad \sum_{n \in \mathbf{V}_{k_d}} P_{k_d}^n \leq P^{\max}, \quad k_d \in \{1, \dots, K_d\} \quad (3)$$

$$\text{s.t.} \quad I_0^n \leq I_0^{\max}, \quad \forall n \in \{1, \dots, N\} \quad (4)$$

where s.t. stands for 'subject to',  $\mathbf{V}_{k_d}$  is the set of subchannels allocated to D2D transmitter  $k_d$ ,  $\mathbf{V}_{k_c}$  is the set of subchannels allocated to cellular user  $k_c$ .  $N_0^n$  is the noise power at the BS,  $N_{k_d}^n$  is the noise power at D2D receiver  $k_d$ , both for subchannels  $n$ ,  $I_0^{\max}$  is the maximum allowed interference at the BS per subchannel and  $P^{\max}$  is the maximum total transmit power from cellular users or D2D pairs.

$\mathbf{P}_d = [P_{k_d}^n]_{\substack{k_d \in \{1, \dots, K_d\} \\ n \in \{1, \dots, N\}}}$  is the matrix of transmit powers of D2D transmitters in all subchannels and  $\mathbf{P}_c = [P_{k_c}^n]_{\substack{k_c \in \{1, \dots, K_c\} \\ n \in \{1, \dots, N\}}}$  is the transmitted power of cellular users in all subchannels, where  $P_{k_d}^n$  and  $P_{k_c}^n$  are the transmitted powers per each subchannel for D2D transmitter  $k_d$  and cellular user  $k_c$ , respectively.  $G_{k_d,j_d}^n$  is the channel gain between D2D transmitter  $k_d$  and D2D receiver  $j_d$  in subchannel  $n$  including path loss, shadowing and fading.  $G_{0,j}^n$  is the channel gain between node  $j$  (either cellular user or D2D transmitter) and the BS including path loss, shadowing and fading.

$I_0^n$  is the received interference from the allocated D2D transmitters at the BS in subchannel  $n$ , which must be

lower than maximum allowed interference  $I_0^{\max}$  and is given by,

$$I_0^n = \sum_{k_d=1}^{K_d} P_{k_d}^n G_{k_d,0}^n; \quad \text{if } n \in \mathbf{V}_{k_d} \quad (5)$$

$I_{k_d}^n$  is the received interference from all other D2D transmitters by D2D receiver  $k$  in subchannel  $n$  and is determined by,

$$I_{k_d}^n = \sum_{\substack{j=1 \\ j \neq k_d}}^{K_d} P_j^n G_{k_d,j}^n \quad (6)$$

Finally,  $I_{k_d,k_c}^n$  is the received interference from the cellular user  $k_c$  to the  $k_d$ th D2D receiver and determined by,

$$I_{k_d,k_c}^n = P_{k_c}^n G_{k_d,k_c}^n \quad (7)$$

where  $G_{k_d,k_c}^n$  is the channel gain between cellular user  $k_c$  and D2D receiver  $k_d$ .

## III. PROPOSED RESOURCE ALLOCATION

The proposed resource allocation for underlying D2D in OFDMA based cellular networks includes the allocation of cellular users firstly and then the allocation of D2D pairs. The detailed explanations of each step are given in the following.

### A. RA for cellular users

The optimization problem is re-written to maximize sum rate by assuming no interference coming from D2D transmitters.

$$\max_{\mathbf{P}_c, \mathbf{V}_c} \sum_{k_c=1}^{K_c} \sum_{n \in \mathbf{V}_{k_c}} \log_2 \left( 1 + \frac{P_{k_c}^n G_{k_c,0}^n}{N_0^n} \right) \quad (8)$$

The RA algorithm is performed to allocate only one cellular user having the best channel gain at each subchannel.

- Initialization:  $\mathbb{N}_1 = \{1, \dots, K_c\}$ ,  $P_{k_c}^n = \frac{P^{\max}}{N}$ ;  $\forall k_c, \forall n$ ,  $R_{k_c} = 0$ ;  $\forall k_c$ .
- for  $n = 1 : N$ 
  - Find the cellular user that has the maximum channel gain:
$$k_c^* = \arg \max G_{k_c,0}^n \quad (9)$$
  - Update  $\mathbf{V}_{k_c^*} = \mathbf{V}_{k_c^*} \cup n$  and  $\mathbb{N}_1 = \mathbb{N}_1 \setminus k_c^*$ .
- End

Then, the transmit power of each cellular user is determined by using open loop power control in order to reduce the interference power on the D2D receivers:

$$P_{k_c}^n = \frac{\gamma_{k_c}^n (N_0^n + I_0^{\max})}{G_{k_c,0}^n} \quad (10)$$

where  $\gamma_{k_c}^n$  is the target SINR for each cellular user in subchannel  $n$ .

## B. RA for D2D pairs

The RA for D2D pairs is performed with the construction of conflict graph based on the signal-to-interference-noise (SINR) values, finding the maximal independent set and defining Knapsack problem considering the interference level at the BS. Beforehand, the open loop power control for each D2D pair is performed to reduce the interference at the BS.

$$P_{k_d}^n = \frac{\gamma_{k_d}^n N_{k_d}^n}{G_{k_d, k_d}^n} \quad (11)$$

where  $\gamma_{k_c}^n$  is the target SINR for each D2D pair in subchannel  $n$ .

1) *Conflict Graph*: In order to decide interference level between any two D2D pairs, say pair  $k_{d_i}$  and pair  $k_{d_j}$ , we use average SINR levels in receivers as decision metric. Suppose pair  $k_{d_i}$  is communicating while  $k_{d_j}$  is simultaneously making a transmission to its receiver. Then, the communications of both pairs will be successful if and only if inequalities (12) and (13) are satisfied:

$$S\bar{I}\bar{N}R_{k_{d_i}} \geq S\bar{I}\bar{N}R^{th} \quad (12)$$

$$S\bar{I}\bar{N}R_{k_{d_j}} \geq S\bar{I}\bar{N}R^{th} \quad (13)$$

where  $S\bar{I}\bar{N}R_{k_{d_i}}$  and  $S\bar{I}\bar{N}R_{k_{d_j}}$  are the average SINR in the receivers of pair  $k_{d_i}$  and  $k_{d_j}$ , respectively.

Under simultaneous transmission of D2D pairs  $k_{d_i}$  and  $k_{d_j}$ , the average SINR at each receiver is calculated as follows:

$$S\bar{I}\bar{N}R_{k_{d_i}} = \frac{\bar{P}_{k_{d_i}} \bar{G}_{k_{d_i}, k_{d_i}}}{N_{k_{d_i}} + \bar{P}_{k_{d_j}} \bar{G}_{k_{d_i}, k_{d_j}}} \quad (14)$$

and

$$S\bar{I}\bar{N}R_{k_{d_j}} = \frac{\bar{P}_{k_{d_j}} \bar{G}_{k_{d_j}, k_{d_j}}}{N_{k_{d_j}} + \bar{P}_{k_{d_i}} \bar{G}_{k_{d_j}, k_{d_i}}} \quad (15)$$

where  $\bar{G}_{i,j}$  is the average channel gain between the transmitter of D2D  $i$  and the receiver of D2D  $j$  and  $\bar{P}_j$  is the average transmitted power for D2D transmitter  $j$ .

Construction of conflict graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  consists of the steps below:

C1.1) We can denote communicating pair  $v$  modeling as a vertex,  $v \in \mathcal{V}$ .

C1.2) Say  $v_1, v_2 \in \mathcal{V}$ , then  $\{v_1, v_2\} \in \mathcal{E}$  if  $v_1$  and  $v_2$  cannot be scheduled simultaneously.

Let  $v_1, v_2 \in \mathcal{V}$  be in the conflict graph.  $\{v_1, v_2\} \in \mathcal{E}$  if any of the conditions (12) or (13) is satisfied.

2) *Maximal Independent Set*: After setting the conflict graph of the communicating D2D pairs, the next task is to find the maximal independent set (MIS) of the graph. By this way, we find the maximum number of D2D pairs that we are allowed to assign the same subchannel.

Given an undirected graph  $G = (\mathcal{V}, \mathcal{E})$ , an independent set is a subset of nodes  $\mathcal{U} \subseteq \mathcal{V}$ , such that no two nodes in  $\mathcal{U}$  are adjacent. An independent set is maximal if no node can be added without violating independence. Finding a maximal independent set is an easy task. We scan the nodes in an arbitrary order and add  $u$  to the set if it does not violate independence. Otherwise, we simply discard node  $u$ . A maximal independent set is not a subset of any other independent set, so there is no vertex outside the maximal independent set that may join it. Its complexity is  $O(K_d)$  where  $K_d$  is the number of vertices, the number of D2D pairs in our case.

We can possibly calculate maximum independent set in  $O(K_d^2 \log K_d)$  time by making conflict graph claw-free by adding extra constraints for better results [9] but maximal independent set is enough here in terms of performance.

3) *The Knapsack Problem*: After finding the maximal independent set, we use this set into Knapsack problem to see if the BS can work properly while all D2D pairs of the set are communicating. In this problem, our goal is to maximize the number of communicating D2D pairs subject to their cumulative interference at the BS. We have two  $K$ -tuples of positive numbers where  $K$  is the number of D2D pairs contained in the maximal independent set. One is the value tuple which gives the importance degrees of each D2D communication. In our case, we set  $v_i = 1$ , for  $i = \{1, 2, \dots, K\}$ , making all communications equal in significance. The other tuple contains the weights of D2D transmissions, which, in our case, has the interference levels of each D2D communication at the BS. We assume that BS can support such D2D pairs if their total interference does not exceed a specified level  $W$ .

Our optimization problem is given below:

$$\text{maximize: } \sum_{i=1}^K v_i x_i$$

$$\text{subject to: } \sum_{i=1}^K w_i x_i \leq W$$

$$x_i \in \{0, 1\}, 1 \leq i \leq K$$

Is in set  $x_i$  show the D2D pairs that can be communicated simultaneously. This output set of D2D pairs are then excluded from the current conflict graph and the steps are repeated until conflict graph remains empty. In the end, we derive different sets to assign subchannels. Assume that we have  $M$  subchannels to assign to D2D communications. The decision on the number of subchannels that will be assigned to a set of D2D pairs is given regarding on the number of D2D pairs in the set. In other words, we assign higher number of subchannels to a set of D2D pairs if it contains higher number of D2D pairs.

Pseudocode of the Knapsack Algorithm can be seen

below:

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Input :
    v → Values of the D2D pairs.
    w → Weights (converted to integer) of the D2D pairs.
    K → Number of D2D pairs that will be evaluated.
    W → Maximum weight (converted to integer) that
    can be supported.
1 for w = 0 to W do
2   V[0, w] = 0
3 end
4 for i = 1 to K do
5   for w = 0 to W do
6     if (wi ≤ w) and (vi + V[i - 1, w - wi] ≥ V[i -
7     1, w]) then
8       V[i, w] = vi + V[i - 1, w - wi]
9       keep[i, w] = 1
10    end
11    else
12      V[i, w] = V[i - 1, w]
13      keep[i, w] = 0
14    end
15  end
16 end
17 T = W
18 for i = K downto 1 do
19   if (keep[i, T] == 1) then
20     output i
21     T = T - wi
22   end
23 end
24 return V[K, W]

```

#### IV. SIMULATION RESULTS

We consider a BS in the cell with radius  $R = 0.5\text{km}$ . The noise is modelled as additive white Gaussian noise (AWGN) with power spectrum density  $-174\text{dBm/Hz}$ . The number of subchannels is equal to  $N = 50$  and the bandwidth of each subchannel is chosen  $180\text{kHz}$ . The number of cellular users are also selected as  $K_c = 50$ . Therefore, each cellular user is allocated at least one subchannel to establish a transmission with target SINR of  $\gamma_{k_c}^q = 20\text{dB}; \forall k_c$ . D2D transmitters are either cell edge or cell center region with target SINR of  $\gamma_k^q = 10\text{dB}; \forall k$ . The maximum transmit power is  $21\text{dBm}$  for both D2D transmitters and the cellular users. The channel is modelled by using Rayleigh fading and shadowing follows log-normal distribution. The path loss and shadowing standard deviation both depend on whether the receiver is BS or a device. For the case that the receiver is the BS, the path loss model is  $L = 128.1 + 37.6 \log_{10}(d)$  where  $d$  (in km) is the distance between the transmitter and receiver. For the case that the receiver is a device, the path loss model is  $L = 140 + 36.8 \log_{10}(d)$  where  $d$  (in km) is the distance between the transmitter and receiver. The threshold average SINR is set to  $\text{SINR}^{th} = 0\text{dB}$  and the interference level for each D2D pair at the BS is determined as  $W = 10^{-11}$ .

The performance results based on average sum data rate can be seen in Fig. 1. The proposed resource allocation algorithm provides much better results when the number of D2D pairs is higher than 20. In Fig. 2,

the average data rate per D2D pair results are illustrated and it is shown that the graph coloring works better for low number of operating D2Ds. This is explained by the conservatism of the Knapsack algorithm in order not to deteriorate the performance of BS, otherwise both algorithms would perform nearly the same. Starting from 20 D2Ds underlying in the cellular networks, the proposed resource allocation method significantly outperforms graph coloring algorithm by providing 25% higher data rate in average. In fact, it is not only outperforming graph coloring in terms of D2D data rate, but also preserving the quality of service at the BS. As a result of allocating more D2D pairs thanks to the proposed algorithm, the data rate of cellular users is slightly decreased, 8% in average, as seen in Fig. 3. The average transmit power per D2D pairs is around  $2.2\text{dBm}$  as illustrated in Fig. 4 as a result of open loop power control mechanism. The average transmitted power of D2D pairs can be further reduced by employing iterative power allocation algorithms to avoid the slight reduction on the cellular data.

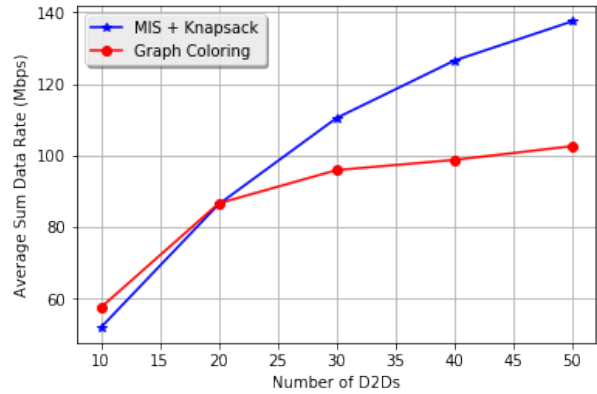


Fig. 1: Average Sum Data Rate Comparison of MIS + Knapsack and Graph Coloring Algorithms

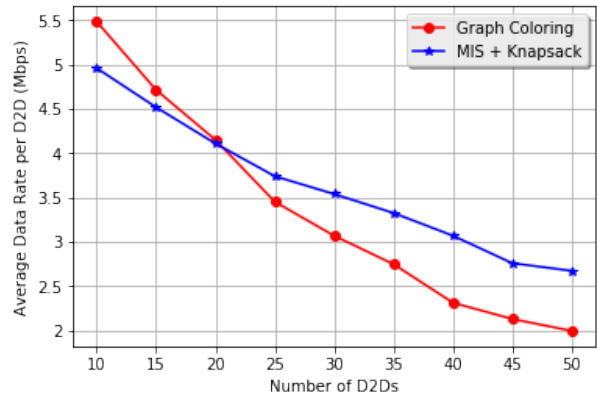


Fig. 2: Comparison on Average Data Rate per D2D Pair

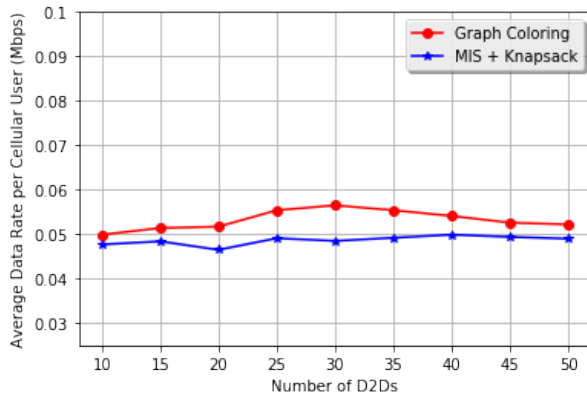


Fig. 3: Comparison on Average Data Rate per Cellular User

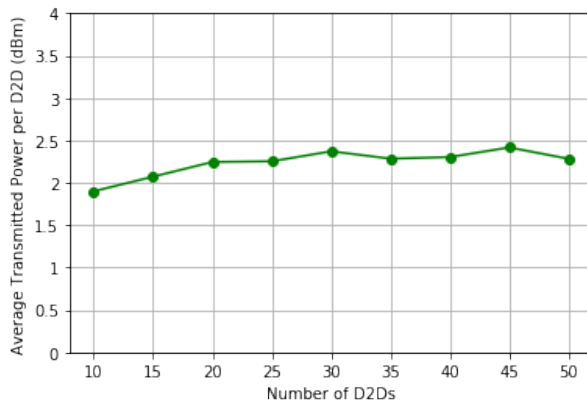


Fig. 4: Transmitted power of D2D pairs

## V. CONCLUSION

In this paper, we have examined the resource allocation problem in underlying D2D cellular communications. In order to address for the sum data rate maximization, we have proposed MIS + Knapsack approach. First, we have constructed the conflict graph of the D2D pairs. We have found maximal independent sets in the graph

and have given this set as an input to Knapsack algorithm where the weights are given according to interference levels that BS is exposed to. Since we have implemented this algorithm iteratively, we have guaranteed nearly the best possible D2D data rate without leading to a harmful level of interference in the BS. In the end, we have showed that the proposed algorithm clearly outperforms Graph Coloring assuming that we have sufficient number of D2D pairs in the system.

For future work, the multiple antennas case can be considered to limit uplink interference to even lower levels. Besides, we can first make the conflict graph claw-free and then find the maximum independent set instead of finding maximal independent set.

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