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Short note

A supplement to the paper of Zayed et al. [Optik, 170 (2018) 339–341]

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ABSTRACT

It seems that the results obtained by the so-called Khater method contain computational or print errors. We look at this issue from a different point of a view, namely, from a theoretical side. We prove our claim by a formal direct approach instead of back substitution (trial and error) approach.

1. Introduction

Recently, Zayed et al. [1] made comments on the results of the so-called Khater method (KM) which was proposed for finding exact traveling wave solutions of nonlinear evolution equations. They stated that the new auxiliary equation method (KM) is wrong and consequently, all the exact solutions obtained in [2,3] are all wrong. In [1], they supported their claim by substituting the first four solutions (4), (7), (10), (13) into the auxiliary equation of the KM and checked the left/right hand sides for whether they match or not. They concluded that all exact solutions obtained by KM are all wrong. Finally, they made a call to the research community for not using KM in their studies. However, since the involved expressions are large and complicated, we think that their approach (trial and error) is also open to incorrect computations. We believe that a much more convincing approach needs to be demonstrated to resolve this serious issue. That is why, in this note, we give a formal direct treatment to show that the published results of the so-called Khater method may contain computational or print errors.

Indeed, the so-called KM is noting but the well-known generalized Riccati equation mapping method (GREMM). First of all, the KM [2,3] assumes that the solution of a nonlinear evolution equation can be written in the form

$$u(\xi) = \sum_{i=0}^{N} a_i (a^{f(\xi)})^i, \tag{1}$$

where $f(\xi)$ satisfies the auxiliary equation

$$f'(\xi) = \frac{1}{\text{In}(a)}(\alpha a^{-f(\xi)} + \beta + \sigma a^{f(\xi)}), \tag{2}$$

in which a_i, α, β, σ and a are arbitrary constants with $a_N \neq 0$, $a > 0$, $a \neq 1$. Now, we show that Eq. (2) can always be reduced to a Riccati equation. To do so, the substitution that is needed is

$$F(\xi) = a^{f(\xi)}. \tag{3}$$

Then, we have

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$$F'(\xi) = f'(\xi)a^{f(\xi)}\text{In}(a) = f'(\xi)F(\xi)\text{In}(a), \quad (4)$$

$$f'(\xi)\text{In}(a) = \frac{F'(\xi)}{F(\xi)}. \quad (5)$$

Substitution of Eq. (5) into Eq. (2) gives

$$\frac{F'(\xi)}{F(\xi)} = \alpha(F(\xi))^{-1} + \beta + \sigma(F(\xi))$$

which can be written as a Riccati equation of the form

$$F'(\xi) = \alpha + \beta F(\xi) + \sigma F^2(\xi). \quad (6)$$

Thus, a solution of Eq. (2) will lead to a solution of Eq. (6) and vice versa. The right side of Eq. (6) is quadratic in the unknown function and its special solutions (obtained by quadrature, i.e., a simple integration) are as follows:

$$F(\xi) = -\frac{\beta}{2\sigma} - \frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2\sigma}\tanh\left(\frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2}\xi\right), \beta^2 - 4\alpha\sigma > 0, \quad (7)$$

$$F(\xi) = -\frac{\beta}{2\sigma} - \frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2\sigma}\coth\left(\frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2}\xi\right), \beta^2 - 4\alpha\sigma > 0, \quad (8)$$

$$F(\xi) = -\frac{\beta}{2\sigma} + \frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2\sigma}\tan\left(\frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2}\xi\right), \beta^2 - 4\alpha\sigma < 0, \quad (9)$$

$$F(\xi) = -\frac{\beta}{2\sigma} - \frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2\sigma}\cot\left(\frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2}\xi\right), \beta^2 - 4\alpha\sigma < 0, \quad (10)$$

$$F(\xi) = -\frac{\beta}{2\sigma} - \frac{1}{\sigma\xi + c}, \beta^2 - 4\alpha\sigma = 0, \quad (11)$$

where c is an arbitrary constant. On the other hand, the ansatz function (1) becomes

$$u(\xi) = \sum_{i=0}^{N} a_i(F(\xi))^i, \quad (12)$$

where $F(\xi)$ is the solution of the Riccati Eq. (6). Now, substituting (3) into (7)–(11), we get special solutions of the auxiliary equation of the so-called KM. This proves that the so-called KM is an another form of the well-known GREMM. Even though twenty-seven solutions of the Riccati Eq. (6) are presented in some studies (see, for example [4]), those solutions (except (7)–(11)) are generated from the special solutions (7)–(11) via easily proved identities of trigonometric functions, hyperbolic functions and exponential functions. We refer the interested readers to the most recent study [5] in which a systematic treatment of the equivalent solutions of the Riccati Eq. (6) has been demonstrated via Bäcklund transformations.

As a result, in the light of our analysis, we think that there are computational or print errors in the studies which is based on the so-called KM. Contrary to the comments made in [1], we presented a direct formal approach to prove our claim. We suggest that those authors check their results and submit a correction note to the corresponding journals. We believe that our observation will be a good supplement to the literature.

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