VIBRATION ANALYSIS OF A PLATE WITH PIEZOELECTRIC LAYERS

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ABSTRACT

VIBRATION ANALYSIS OF A PLATE WITH PIEZOELECTRIC LAYERS

In this thesis, vibration analyses of a rectangular plate with piezoelectric layers are studied by using ANSYS. Computer codes for various cases related with the geometry of the piezoelectric layers are developed by using ANSYS Parametric Design Language (APDL) in ANSYS with SOLID5 and SOLID45 for piezoelectric patches and plate, respectively. The effect of the piezoelectric layer on active control of vibrations of plate is investigated.

ÖZET

PİEZOELEKTRİK KATMANLI BIR PLAĞIN TİTREŞİM ANALİZİ

Bu tezde, piezoelektrik tabakaları olan dikdörtgen bir plakanın titreşim analizleri ANSYS kullanılarak çalışılmıştır. Piezoelektrik katmanların geometrisi ile ilgili çeşitli durumlar için bilgisayar kodları, ANSYS de piezoelektrik yamalar ve plaka için sırasıyla SOLID5 ve SOLID45 elemanlarının ANSYS Parametrik Tasarım Dili (APDL) kullannılması ile geliştirilmiştir. Piezoelektrik tabakanın plaka titreşimlerinin aktif kontrolü üzerindeki etkisi araştırılmıştır.

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a, b	Dimensions of the plate
A	Cross-sectional area of piezoelectric material
$[B_u]$	Strain-displacement matrix
$[B_V]$	Electrical field-electrical potential matrix
c	Young's modulus
$[C]$	Damping matrix
d	Piezoelectric strain coefficient
d^a	Moment arm of piezoelectric layer
D	Bending stiffness of plate, Dielectric displacement
e	Electric displacement-strain relationship
E	Modulus of elasticity, Electric field
$f(x, y, t)$	Transverse load
F	Force
$\{F\}$	Nodal force vector
g	Piezoelectric voltage or charge constant
G	Transfer function of block diagram shown in Figure 2.7.
G_s	Transfer function of smart structure
h	Height of the plate
K_c	Transfer function of controller
K_p	Proportional gain coefficient
K_v	Transfer function of voltage amplifier
$[K]$	Structural stiffness matrix
$[K_d]$	Dielectric conductivity
$[K_z]$	Piezoelectric coupling matrix
L	Length of piezoelectric material
$\{L\}$	Nodal change vectors
M_x, M_y	Bending moments about x and y axes
$[M]$	Mass matrix
$[N^u]$	Shape function matrix for displacements
N_i	Shape function for node i

s	Inverse of Young's modulus
s^E	s in constant electric field
S	Strain
t_p	Thickness of the piezoelectric layer
T	Stress
u_z	Displacement of controlled point M
$\{u\}$	Nodal displacement vector
$\{V\}$	Nodal electric potential vector
vol	Volume
V_a	Applied voltage
V_s	Output or sensor voltage
W_k	Modal participation factor
x , y	Co-ordinate axes in the neutral plane
Y	Young's modulus
w	Transverse displacement of plate
z	Co-ordinate axis perpendicular to plate
α	Mass matrix coefficient
β	Stiffness matrix coefficient
ε	Dielectric permittivity
ε^T	ε in constant stress
ε^S	ε in constant strain
λ	Nondimensional frequency parameter
ν	Poisson ratio
ρ	Density
ω	Natural frequency

CHAPTER 1

GENERAL INTRODUCTION

Vibration analysis of plates is related with the plate theory used for analysis. Thin and thick plate theories are two major topics. On the other hand, laminated plates are used for having more strength plates. The theoretical backgrounds on the topics mentioned above are highly available in the literature as papers and textbooks. However, vibration analysis of plates with piezoelectric layers is not become classical topic. After finding the piezoelectricity, structural members such as beam, plate and shells are combined with piezoelectric layers to control their vibration behaviors. As know, piezoelectric layers may have two functions as sensing and actuating.

There are studies on the plates with piezoelectric layers. Some of these are summarized in this chapter.

Allik and Hughes (1970) formulated a general finite element formulation for electroelastic analysis of elastic system having the piezoelectric or electroelastic effect. As an application of the presented formulation, a tetrahedral finite element is presented for A tetrahedral finite element is presented, implementing the theorem for application to problems of three-dimensional electroelasticity.

Tzou and Tseng (1990) derived a piezoelectric finite element formulation for plate or shell structure having distributed piezoelectric sensor and actuator. Their finite element has internal degrees of freedom. They used Guyan reduction for the degrees of freedoms related with the electrical potential to improve computation efficiency.

Lee and Moon (1990) developed a kind of distributed sensors/actuators by using piezoelectric laminate theory for structural vibration of a plate.

Dimitriadis et al (1991) analytically investigated the behavior of piezoelectric patches bonded to thin plate surface as vibration exciter. In order to estimate the load generated by the piezoelectric actuator, static analysis is done. Then, the theory is extended for dynamic model for time response of a rectangular plate with piezoelectric patch in three different configurations of actuators shown in Figure 1.1. They demonstrated that the geometry of the piezoelectric patch affects the response modes.

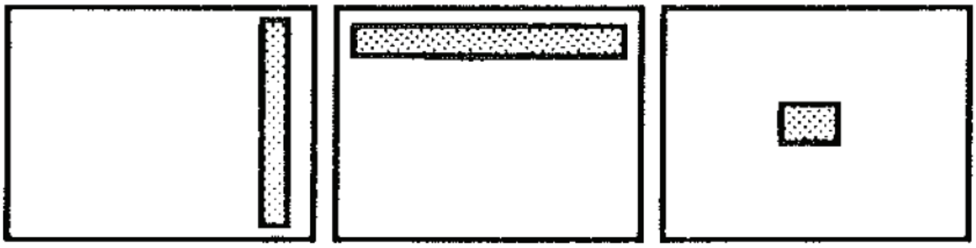


Figure 1.1. Three different configurations of actuators
(Source: Dimitriadis et al 1991)

Crawley and Lazarus (1991) presented the derivations of equations for induced strain actuation of isotropic and anisotropic plates and also verified their equations by their experimental studies. Moreover, they formulated a general procedure based on Rayleigh-Ritz method for more complex geometries and also considered complicated boundary conditions.

Lee et al (1991) derived modal sensor/actuator pairs based on classical laminate plate theory with piezoelectricity to sense and control the vibration of a one-dimensional cantilever plate. Their derivation is verified by experimental studies. They showed that critical damping of a mode is related with saturating level of the modal actuator. They stated that velocity feedback control can be used without any tuner due to the proportionality of the sensor signal with the modal coordinate time derivative.

Tzou and Fu (1994) published a study on segmentation of distributed piezoelectric sensors/actuators in two parts as Part I and Part II related with theoretical analysis and parametric study and active vibration controls, respectively. In Part I, they presented analytical formulations for the distributed actuator and distributed sensor shown in Figure 1.2 and the segmented distributed actuator and distributed sensor shown in Figure 1.3. Then, they concluded that

1. A symmetrically distributed single sensor layer is ineffective for sensing of all even modes due to the cancellation of the charges which are generated locally on the effective sensor surface.
2. First conclusion is also valid for controlling of all even modes.
3. Quarterly segmented sensors/actuators have sensing and controlling ability for most of the modes, but not quadruple modes.
4. Segmentation of distributed piezoelectric sensors/actuators improve the performance of the sensing and controlling of the plate.

In Part II, they presented parametric study for the single-piece symmetrically distributed actuator/sensor and multi-piece segmented actuators/sensors.

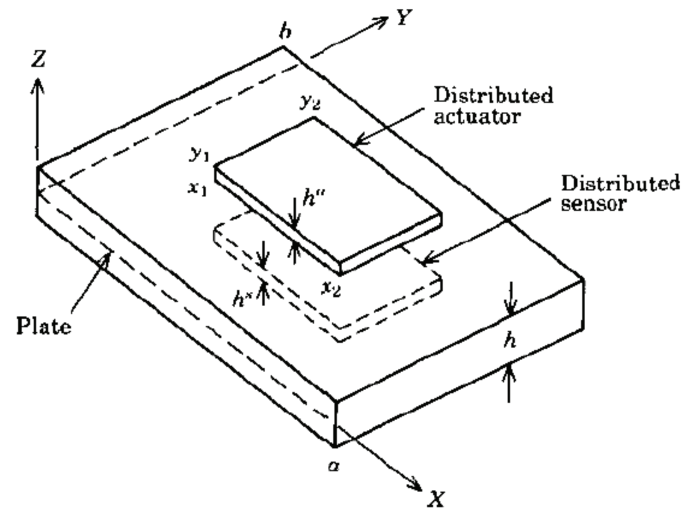


Figure 1.2. Plate having distributed actuators and sensors
(Source: Tzou and Fu 1994)

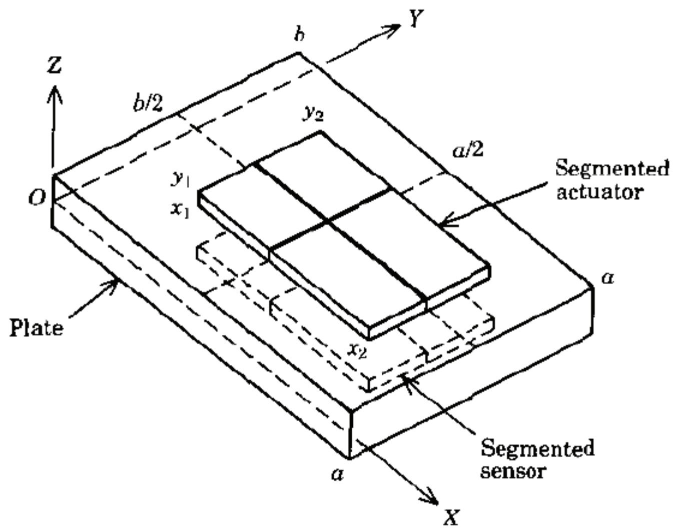


Figure 1.3. Plate having segmented actuators and sensors
(Source: Tzou and Fu 1994)

Zhou et al (1996) developed a model theoretically and then verified experimentally for two-dimensional active structures with piezoelectric patch actuators by using the input impedance of actuator and the mechanical impedance of the host structures. This model is named as the impedance-based model and then applied to thin plates and thin .shells, and to beams. They concluded that their model offers insight into the dynamic coupling of the piezoelectric patch and substructures system.

Kim et al (1996) described the behavior of a smart cantilever plate having piezoelectric sensor and actuator by finite element model. They demonstrated the accuracy of their model with experimental study that carried out with the smart cantilever beam shown in Figure 1.4. They used 3D finite element for the piezoelectric material, flat-shell elements for the plate structure, and transition elements for connecting the elements to model PZT and plate.

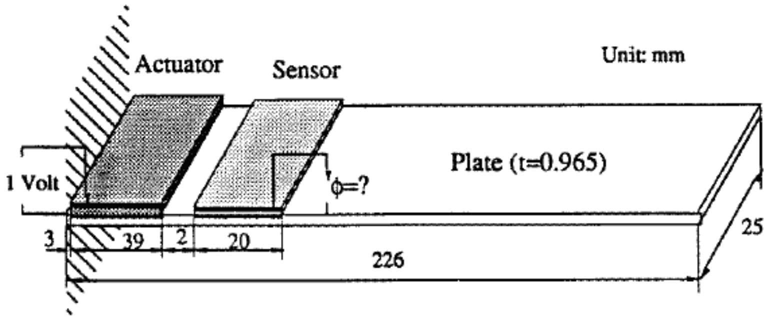


Figure 1.4. Smart cantilever plate
(Source: Kim et al 1996)

Lim (2003) simulated the vibration control of smart plate shown in Figure 1.5 by

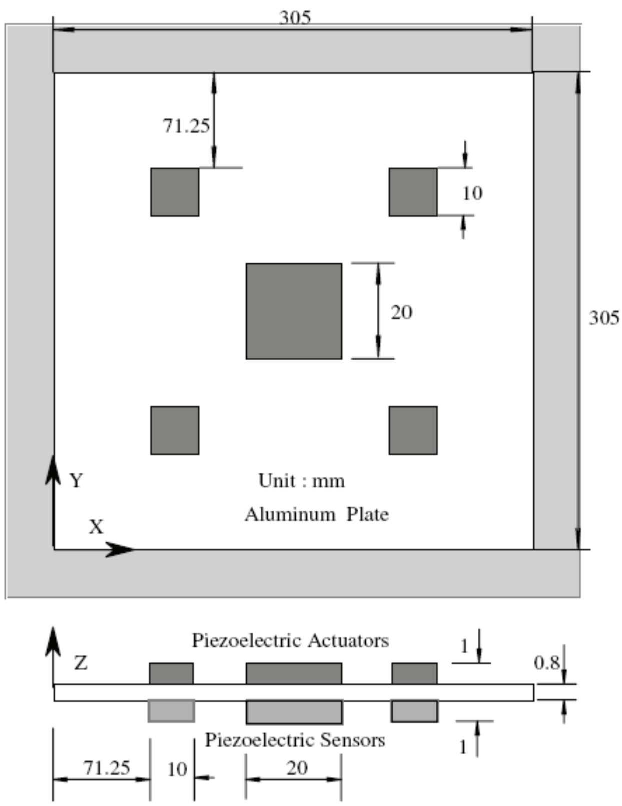


Figure 1.5. Smart clamped plate
(Source: Lim 2003)

ANSYS that is a finite element software. He used SOLID45 for metal part with 305x305x0.8 mm and SOLID5 for piezoelectric part of the system with 4 patches with 10x10x1 mm and 1 path with 20x20x1 mm. He located sensor/actuator pairs strategically to control the several modes of the plate. His strategy is based on the selecting the points where maximum strain occurs in the frequency and time domains.

Quek et al (2003) presented vibration control of laminated composite plates by placing of piezoelectric patches optimally. They employed the classical direct pattern search method to determine the local optimum by considering two performance indices based on modal and system controllability. The pattern search is started at a point having maximum normal strain.

Agneni et al (2003) developed a procedure to increase the passive damping for vibration of elastic systems having shunted piezoelectric devices. They applied their procedure for aero-elastic systems, and then proposed an optimal strategy for tuning the electrical load of the piezo devices as function of the flight speed

Narayanana and Balamurugan (2003) investigated active vibration control by developing finite elements for various structural components such as beam, plate and shells for laminated elastic bodies with piezoelectric sensors and actuator layers. They also considered the temperature effects on the electrical and mechanical properties. They used Timoshenko beam theory and first order shear deformation theory for beam and plate, respectively. In active vibration control, the structure is subjected to harmonic, impact, and random excitations. They used different feedback algorithms, and then found that the linear quadratic regulator approach is more effective in vibration.

Peng et al (2005) proposed a performance criterion based on minimizing the energy requirement for vibration control of flexible plate structures by locating the piezoelectric patch actuators optimally. They utilized genetic algorithm for optimization and ANSYS to model the system.

Ren et al (2010) presented a dynamic model of simply supported plate with segmented piezoelectric patches which produces the sensor signals and control moments to analyze the vibration properties by using the finite difference method.

Smart materials with piezoelectric layers were presented by various textbooks written by Preumont (2002), Smith (2005), and Leo (2007).

In this thesis, vibration analyses of a plate with piezoelectric layers are studied by using ANSYS which is a commercial finite element package. Computer codes for various cases are developed by using ANSYS Parametric Design Language (APDL) in ANSYS with SOLID5 and SOLID45 for piezoelectric patches and plate, respectively. The effect of the piezoelectric layer on vibrations of plate is investigated.

CHAPTER 2

THEORIES

2.1. Introduction

This chapter prepares the groundwork for the present study. The equation of motion of thin plate is introduced in next section as background of the smart plate. Piezoelectric materials and their constitutive equations are presented as essential topic. Then, finite element method is summarized for vibration of structures having piezoelectric components due to the selection for solution of the present problem. Therefore, geometric modeling and vibration analysis in ANSYS is given.

2.2. Equation of Motion of Thin Plate

The thin rectangular plate having thickness h shown in Figure 2.1 is considered. The equation of motion of this plate under transverse load $f(x,y,t)$ acting on unit area of plate is given by Rao (2007) as

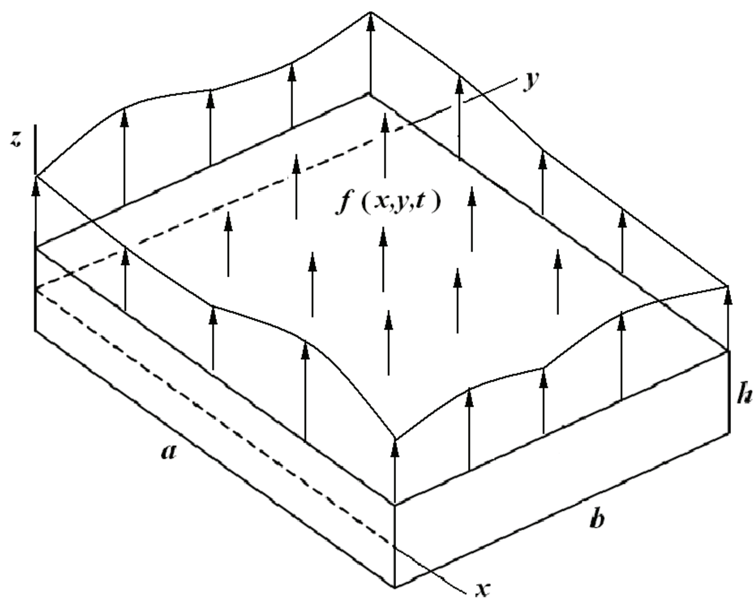


Figure 2.1. The rectangular plate

$$D\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right) + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = f(x,y,t) \tag{2.1}$$

where $w(x,y,t)$ and ρ are transverse displacement and mass density of plate, respectively. Also, D is bending stiffness of the plate and given as

$$D = \frac{Y h^3}{12(1-\upsilon^2)} \tag{2.2}$$

in which Y and υ are the Young's modulus and the Poisson ratio of plates, respectively.

2.3. Piezoelectric Materials and Constitutive Equations

The Curie brothers, shown in Figure 2.2, discovered the direct piezoelectric effect in 1880. In this effect, the applied force to piezoelectric material produces the voltage. They described the producing the piezoelectric material as follows: If the material is heated above a certain temperature that is called as Curie temperature and applied a strong electric field, and then cool down, piezoelectric material is obtained. The main point in this process is poling of material in the direction of applied voltage.

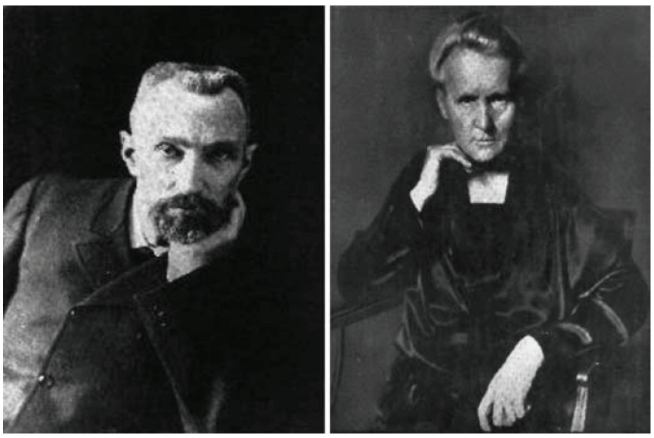
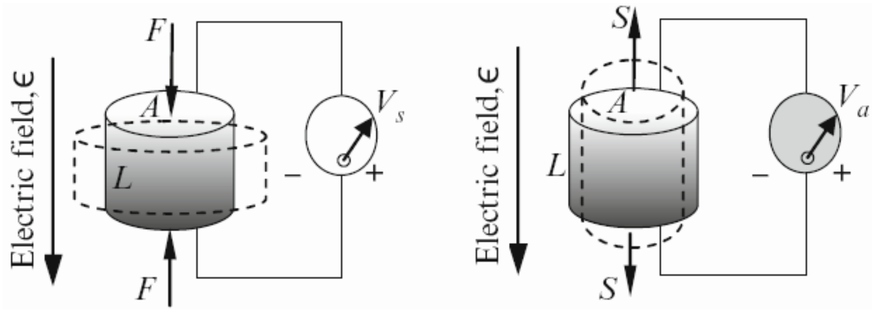


Figure 2.2. Pierre and Jacques Curie (1880)
(Source: Jalili 2010)

Direct piezoelectric effect is illustrated in Figure 2.3.a. Later, converse piezoelectric effect demonstrated schematically in Figure 2.3.b was found. It is seen from Figure 2.3.b that the applied voltage to piezoelectric material produces the strain.



a. Direct effect b. Converse effects

Figure 2.3. Direct and converse piezoelectric effects
(Source: Jalili 2010)

For direct piezoelectric effects, the output voltage V_s is expressed mathematically as (Jalili, 2010)

$$V_s = -g(L/A)F \tag{2.3}$$

where g is the piezoelectric voltage or charge constant, L and A are the length and cross-sectional area of the piezoelectric material along the poling direction as shown in Figure 2.2, respectively. Also, for converse piezoelectric effect are expressed mathematically as (Jalili, 2010)

$$S = d(1/L)V_a \tag{2.4}$$

where d (C/N) is the piezoelectric strain coefficient and V_a is applied voltage.

The direct and converse piezoelectric effects are also called as mechanical-to-electrical coupling and the electrical-to-mechanical coupling, respectively. Due to these couplings, applied force F (N) gives two results as strain S (m/m) and electric displacement D (C/m^2) in direct piezoelectric effect and also applied voltage gives the same results in converse piezoelectric effect. Therefore, the following equations are written for first coupling effect as (Leo, 2007)

$$S = \frac{1}{Y} T = s\,T \tag{2.5}$$

where Y is the Young's modulus of the material, s (m^2/N) is mechanical compliance, and T (N/m^2) is stress due to applied force F. Moreover,

$$D = d\,T \tag{2.6}$$

For second coupling effect, the following equations are written

$$S = d\,E \tag{2.7}$$

where E (V/m) is the electric field applied to piezoelectric material. Additionally,

$$D = \varepsilon\,E \tag{2.8}$$

where ε (F/m) is dielectric permittivity of the piezoelectric material.

Equations from (2.5) to (2.8) can be written in matrix form as follows:

$$\begin{Bmatrix} S \\ D \end{Bmatrix} = \begin{bmatrix} s & d \\ d & \varepsilon \end{bmatrix} \begin{Bmatrix} T \\ E \end{Bmatrix} \tag{2.9}$$

The coordinate system shown in Figure 2.4 is used to generalize Equation (2.9).

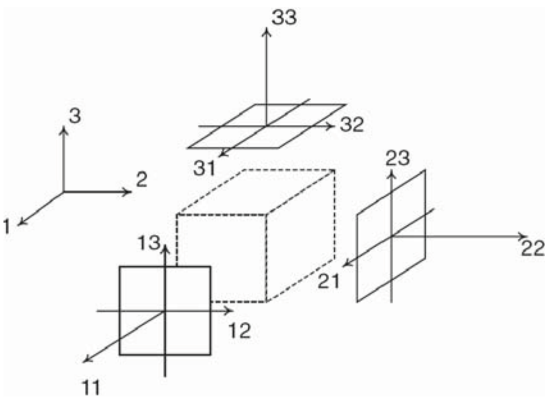


Figure 2.4. The coordinate axes for three-dimensional case
(Source: Leo 2007)

The following definitions are used for constitutive equations of piezoelectric material in three-dimensional case (Leo, 2007).

$$
\begin{aligned}
S_1 &= S_{11} & T_1 &= T_{11} \\
S_2 &= S_{22} & T_2 &= T_{22} \\
S_3 &= S_{33} & T_3 &= T_{33} \\
S_4 &= S_{23} + S_{32}, & T_4 &= T_{23} + T_{32} \\
S_5 &= S_{31} + S_{13} & T_5 &= T_{31} + T_{13} \\
S_6 &= S_{12} + S_{21} & T_6 &= T_{12} + T_{21}
\end{aligned}
\tag{2.10}
$$

Therefore, the general constitutive equations of piezoelectric material can be written as (Leo, 2007)

$$
\begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{13} & s_{23} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{Bmatrix} + \begin{bmatrix} 0 & 0 & d_{13} \\ 0 & 0 & d_{23} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}
\tag{2.11}
$$

$$
\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{13} & d_{23} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{Bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}
\tag{2.12}
$$

The polarization direction of piezoelectric materials is axis 3. The operating modes of piezoelectric patches shown in Figure 2.5 are 31 and 32. First and second number in operating mode refers the polarization direction and strain, respectively. Figure 2.5 is given to illustrate this concept.

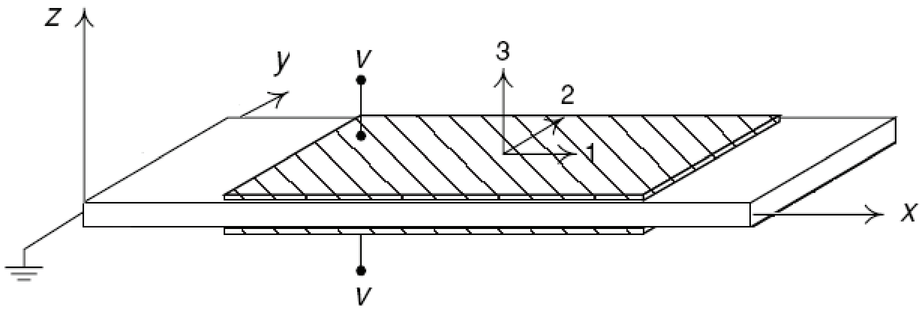


Figure 2.5. Smart plate with sensor and actuator
(Source: Leo 2007)

The general constitutive equations of piezoelectric material given in Equations (2.11) and (2.12) can be reduced for operating mode 31 by using the assumptions $T_2=T_3=T_4=T_5=T_6=E_1=E_2=0$ and also not considering S_2 and S_3 to the following equations (Leo, 2007)

$$S_1 = \frac{1}{Y_1^E} T_1 + d_{13} E_3 \tag{2.13}$$

$$D_3 = d_{13} T_1 + \varepsilon_{33}^T E_3 \tag{2.14}$$

where superscripts E and T denote that the electric field and the stress are constant, respectively. The constitutive equations for operating mode 32 can be expressed in similar manner as follows

$$S_2 = \frac{1}{Y_2^E} T_2 + d_{23} E_3 \tag{2.15}$$

$$D_3 = d_{23} T_2 + \varepsilon_{33}^T E_3 \tag{2.16}$$

2.4. Equations of Smart Plates

The thin rectangular plate with piezoelectric layers as sensor and actuator shown Figure 2.6 is used for equations of smart plate.

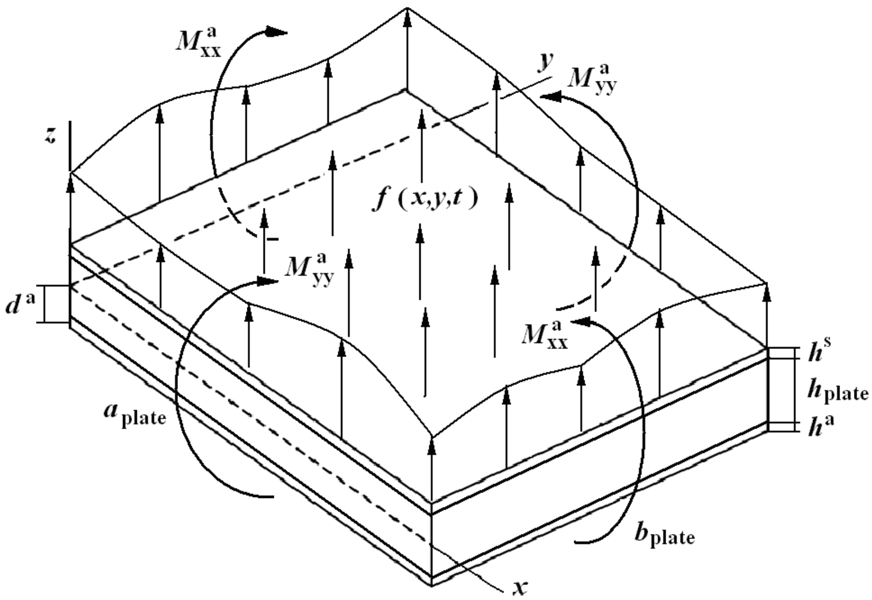


Figure 2.6. Smart plate under transverse load and actuator moments

The equation of motion of this smart plate is given by Tzou (1991) as

$$
\begin{aligned}
& D\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right) \\
& +\rho h\frac{\partial^2 w(x,y,t)}{\partial t^2}+\frac{\partial^2 M_{xx}^{a}}{\partial x^2}+\frac{\partial^2 M_{yy}^{a}}{\partial y^2}=f(x,y,t)
\end{aligned}
\tag{2.17}
$$

where M_{xx}^{a} and M_{yy}^{a} are control moments applied by piezoelectiric layer depending on the control type. For open-loop control, M_{xx}^{a} and M_{yy}^{a} are given by Tzou (1991) as

$$M_{xx}^{a}=d^{a}d_{31}Y_{p}V_{a} \tag{2.18}$$

$$M_{yy}^{a}=d^{a}d_{32}Y_{p}V_{a} \tag{2.19}$$

in which d^a is moment arm of the piezoelectric layer, namely the distance between neutral planes of plate and piezoelectric layer as shown in Figure 2.6. Also Y_p is modulus of elasticity of piezoelectric material and V_a is applied voltage to piezoelectric material.

On the other hand, the feedback control voltage V_a for closed-loop control can be written as (Tzou, 1991)

$$V_a = K_p \; V_s \tag{2.20}$$

where K_p and V_s are the control gain and the sensor output voltage, respectively. The following equations are obtained by substituting Equation (2.20) into Equations (2.18) and (2.19),

$$M_{xx}^a = K_p \, d^a d_{31} Y_p V_s \tag{2.21}$$

$$M_{yy}^a = K_p \, d^a d_{32} Y_p V_s \tag{2.22}$$

where V_s is given by Tzou (1991) as

$$V_s = -\frac{h^s}{A^s} \int_{A^s} \left\{ \left[h_{31} d^s \frac{\partial^2 w}{\partial x^2} \right] + \left[h_{32} d^s \frac{\partial^2 w}{\partial y^2} \right] \right\} dx\, dy \tag{2.23}$$

where h^s and A^s are the thickness and the surface area of the piezoelectric sensor layer as shown in Figure 2.6. Also, h_{31} and h_{32} are the piezoelectric coefficients which relate the open-circuit voltage at a given strain . Equation (2.23) can be expressed in modal expansion by using the following relation

$$w = \sum_{k=1}^{\infty} \lambda_k W_k \tag{2.24}$$

where λ_k and W_k are the k^{th} modal participation factor and k^{th} modal function.

2.5. Finite Element Method

Finite element method is an approximate numerical solution method based on Rayleigh-Ritz method. It has been used commonly since development of high speed computer technology, almost half century ago. The method basically is described by dividing the body with complex geometry into the simple components called as finite elements in one, two, or three dimensional simple geometrical shapes having solution analytically such as bar, straight beam, curved beam, membrane, plate, shell, etc. Division of the body having complex geometrical shape to finite elements is called as meshing.

In Rayleigh-Ritz method, approximate solution is used in whole domain. However, in finite element method, the shape functions similar to approximate solution is used in each finite element. Connectivity conditions of finite elements at their connecting points which are called as nodes are used to combine all finite elements in order to have the characteristics properties such as stiffness or mass of the complex geometry in matrix form.

Depending on the physical problem on hand, nodal variables such as displacements, slopes, curvatures, temperatures, pressures, current and voltage are selected. The simplest finite element in one dimensional problem has at least two nodes and one nodal freedom, for example axial displacement in bar element. Therefore, nodal freedoms are represented by nodal freedom vector. Accordingly, characteristics properties of finite elements in solid mechanics are stiffness and mass matrices.

After modeling the body having complex geometry by finite elements, boundary conditions are applied to the global characteristics matrices in various methods such as elimination or penalty methods.

The critical points in finite element modeling are selecting the finite element types and their sizes. These are accomplished by understanding the theory of physical problem and convergence study performing with different element sizes (Yardimoglu, 2015).

There are a lot of textbooks on finite element method. Considering their publication time, the books written by Cook et al (1989), Petyt (1990), and Reddy (1993) can be referred for more details.

2.6. Geometric Modeling and Vibration Analysis in ANSYS

Kohnke (2004) edited the theory reference for ANSYS having the finite element formulations of piezoelectric materials referring the study of Allik and Hughes (1970). The equations in this section mainly are taken from Kohnke (2004) and Allik and Hughes (1970).

Several finite elements are capable to model the piezoelectric material in ANSYS. These are given below:

- SOLID5: coupled-field quadrilateral solid,
- PLANE13: coupled-field brick,
- SOLID98: coupled-field tetrahedron
- PLANE223: coupled-field 8-node quadrilateral,
- SOLID226: coupled-field 20-node brick,
- SOLID227: coupled-field 10-node tetrahedron.

The nodal displacements vector $\{u\}$ and nodal electrical potential vector $\{V\}$ of a finite element are expressed as

$$\{u\} = \{Ux_1 \quad Uy_1 \quad Uz_1 \quad \cdots \quad Ux_n \quad Uy_n \quad Uz_n\}^T \tag{2.25}$$

$$\{V\} = \{V_1 \quad V_2 \quad \cdots \quad V_n\}^T \tag{2.26}$$

The strain $\{S\}$ is related with the displacement $\{u\}$ as

$$\{S\} = [B_u]\{u\} \tag{2.27}$$

where $[B_u]$ is strain-displacement matrix. Also, the electric field $\{E\}$ is related with the electrical potential $\{V\}$ as

$$\{E\} = -[B_V]\{V\} \tag{2.28}$$

where $[B_V]$ is electric field-electrical potential matrix.

The electromechanical constitutive equations in matrix form are written as

$$\{T\} = [c]\{S\} - [e]\{E\} \tag{2.29}$$

$$\{D\} = [e]^T\{S\} + [\varepsilon]\{E\} \tag{2.30}$$

where $\{T\}$, $\{S\}$, $\{E\}$, and $\{D\}$ are stress, mechanical strain, electric field, and electric flux density vectors, respectively. Additionally, $[c]$, $[e]$, and $[\varepsilon]$ are the matrices for elastic stiffness at constant electric field, the piezoelectric parameter, and the dielectric parameter at constant mechanical strain, respectively.

By substituting Equations (2.27) and (2.28) into Equations (2.29) and (2.30), the following equations are obtained

$$\{T\} = [c][B_u]\{u\} + [e][B_V]\{V\} \tag{2.31}$$

$$\{D\} = [e]^T[B_u]\{u\} - [\varepsilon][B_V]\{V\} \tag{2.32}$$

The equation of motion of electromechanical system in terms of the nodal displacements vector $\{u\}$ and nodal electrical potential vector $\{V\}$ is given by Allik and Hughes (1970) as

$$\begin{aligned} &\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}\} \\ \{\ddot{V}\} \end{Bmatrix} + \begin{bmatrix} [C] & [0] \\ [0]^T & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{u}\} \\ \{\dot{V}\} \end{Bmatrix} \\ &+ \begin{bmatrix} [K] & [K^z] \\ [K^z]^T & [K^d] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{V\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{L\} \end{Bmatrix} \end{aligned} \tag{2.33}$$

where $[M]$, $[C]$, $\{F\}$, and $\{L\}$ are mass matrix, damping matrix, nodal force, and nodal charge vectors, respectively. The dots in Equation (2.33) are used to denote time derivatives.

The mass matrix $[M]$ is written as

$$[M] = \iiint_{vol} \rho [N_u]^T [N_u] dvol \tag{2.34}$$

where ρ is density of material and $[N_u]$ is displacement shape functions matrix.

Proportional damping model can be used for damping matrix $[C]$ as

$$[C] = \alpha[M] + \beta[K] \tag{2.35}$$

where α is mass matrix coefficient and β is stiffness matrix coefficient. Furthermore, $[K]$, $[K^z]$ and, $[K^d]$ are stiffness, piezoelectric coupling, and dielectric conductivity matrices, respectively, and given by Kohnke (2004) as follows

$$[K] = \iiint_{vol} [B_u]^T [c][B_u] dvol \tag{2.36}$$

$$[K^z] = \iiint_{vol} [B_u]^T [e][B_V] dvol \tag{2.37}$$

$$[K^d] = \iiint_{vol} [B_V]^T [\varepsilon][B_V] dvol \tag{2.38}$$

By considering the harmonic motion in the vibrating system, Equation (2.33) is reduced to the following generalized eigenvalue problem

$$[A]\{X\} = \omega^2 \, [B]\{X\} \tag{2.39}$$

where ω is natural frequency and $\{X\}$ is mode shape vector.

Solutions of this equation are possible by various numerical methods. Block Lanczos is default method in ANSYS. Also other methods such as reduced and subspace are available in ANSYS.

2.7. Active Vibration Control of Smart Plate

Block diagram of active vibration control of a smart plate can be drawn as in Figure 2.7. In this diagram, the position feedback is used by measuring the displacement of the amplitude of a point having maximum displacement. That is depended on the mode shape of the plate.

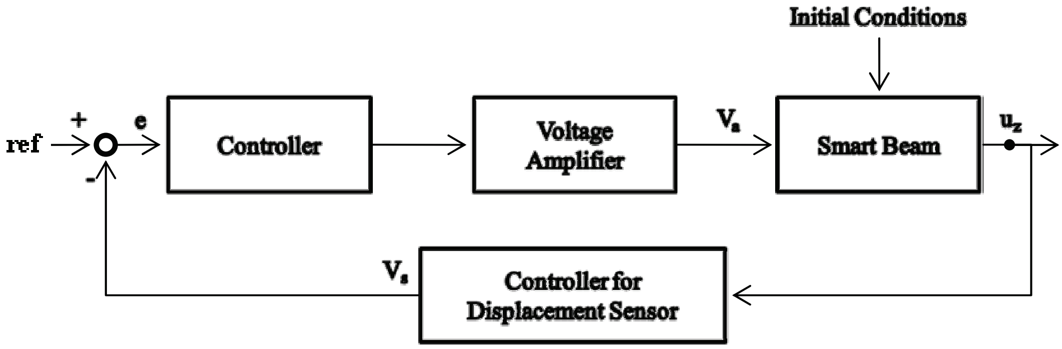


Figure 2.7. Block diagram of the active control system
(Source: Kavuncu, 2018)

In finite element model study in ANSYS, the control algorithm is applied by using by using transient analysis. Initial displacement is applied to proper nodes in smart plate to initiate the transient analysis. Feed-back is applied in each time increment determined depending on the natural frequency of the structure in ANSYS.

In order to express the transfer function of the block diagram shown in Figure 2.7, the following parameter can be defined for compact expression

$$K_p = K_c\, K_v \tag{2.40}$$

where K_c and K_v are transfer functions of controller and voltage amplifier, respectively. Thus, the transfer function is written as

$$G = \frac{u_z}{ref} = \frac{K_p\, G_s}{1 + K_p K_s G_s} \tag{2.41}$$

where u_z, *ref*, K_s and G_s are displacement of the control point, reference voltage, transfer function of displacement sensor and the transfer function of the smart structure, respectively.

CHAPTER 3

NUMERICAL STUDIES

3.1. Modeling of the System

The geometrical model of the plate with piezoelectric layers shown in Figure 2.6 is considered. Plate material is selected as plexiglas that has the following material properties (Tzou, 1993):

Young's modulus Y	: 3.1028 GPa
Mass density ρ	: 1190 kg/m^3
Poisson's ratio υ	: 0.3

Geometrical properties of the plate referring to Figure 2.6 are chosen as follows,

Length of the plate a_{plate}	: 100 mm
Width of the plate b_{plate}	: 100 mm
Thickness of the plate t_{plate}	: 3.1 mm

Piezoelectric material is selected as PVDF (polyvinylidene fluoride) which has the following material and electromechanical characteristics (Schwartz, 2002):

Young's modulus Y_1, Y_2	: 2.56 GPa, 2.6 GPa
Mass density ρ	: 1780 kg/m^3
Poisson's ratio υ_{21}, υ_{21}	: 0.3
Piezoelectric strain coefficients d_{31}, d_{32}, d_{33}	: 21.4 pC/N, 2.3 pC/N, -31 pC/N
Piezoelectric strain coefficients d_{24}, d_{15}	: -35 pC/N, -27 pC/N
Dielectric permittivity ε	: 106 pF/m, (ε_0=8.854 pF/m)

The thickness of the piezoelectric layers h^a and h^s are chosen as 40 μm.

The piezoelectric material matrix elements are in the order of x, y, z, yz, xz, xy. However, input order in ANSYS is x, y, z, xy, yz, xz. Because of this reason, the data are arranged in ANSYS accordingly (ANSYS. 2009).

The plate and piezoelectric layers are meshed by SOLID45 and SOLID5, respectively. One edge of the plate is fixed to model a cantilever plate.

In order to decide for proper number of element to model the plate and piezoelectric material system, non-dimensional first natural frequency parameters of the cantilever plate $\lambda_1 = \omega a_{plate}^2 \sqrt{12(1-\upsilon^2)\rho / E t_{plate}^3}$ are found then plotted as in Figure 3.1.

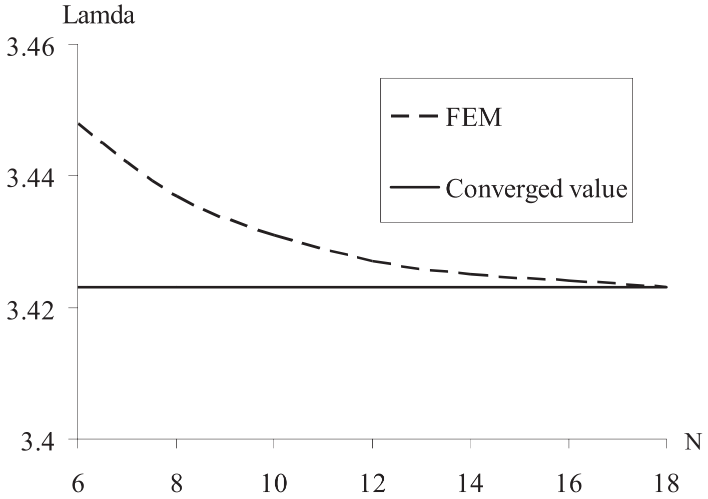


Figure 3.1. Convergence curve of lamda

On the other hand, present finite element results are given in Table 3.1 to compare with its exact value λ_1 available in Leissa (1993).

Table 3.1. Non-dimensional first natural frequencies λ_1

Analytical from Leissa (1993)	3.44
Experimental from Leissa (1993)	3.33
Numerical by FEM with 12 elements	3.427
Numerical by FEM with 14 elements	3.425
Numerical by FEM with 16 elements	3.424
Numerical by FEM with 18 elements	3.423

It is decided from Figure 3.1 and Table 3.1 that the number of finite element along the square plate edge *N* can be taken as 16 in order to keep the simulation time as possible as minimum.

Finite element model of plate, plate with piezoelectric layer are shown in Figures 3.2 and 3.3, respectively. Also, application of boundary condition for finite element model of cantilever plate with piezoelectric layer is illustrated in Figure 3.4.

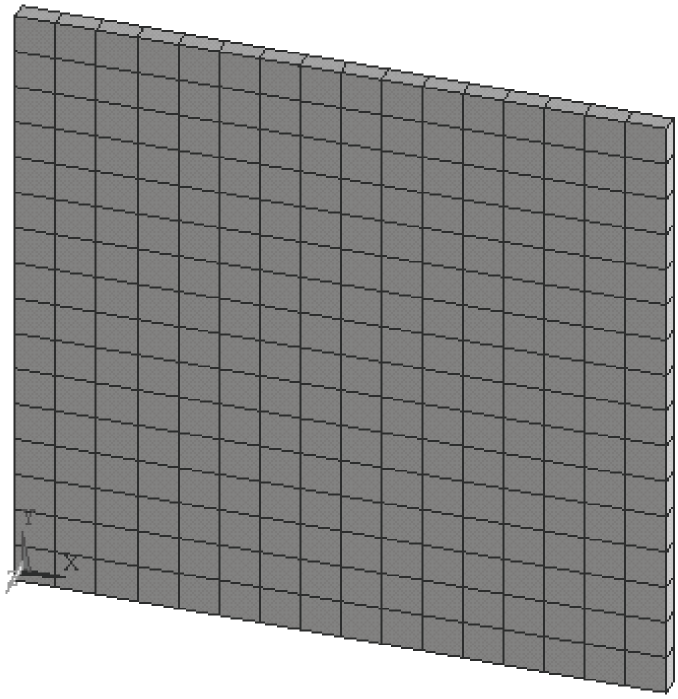


Figure 3.2. Finite element model of plate

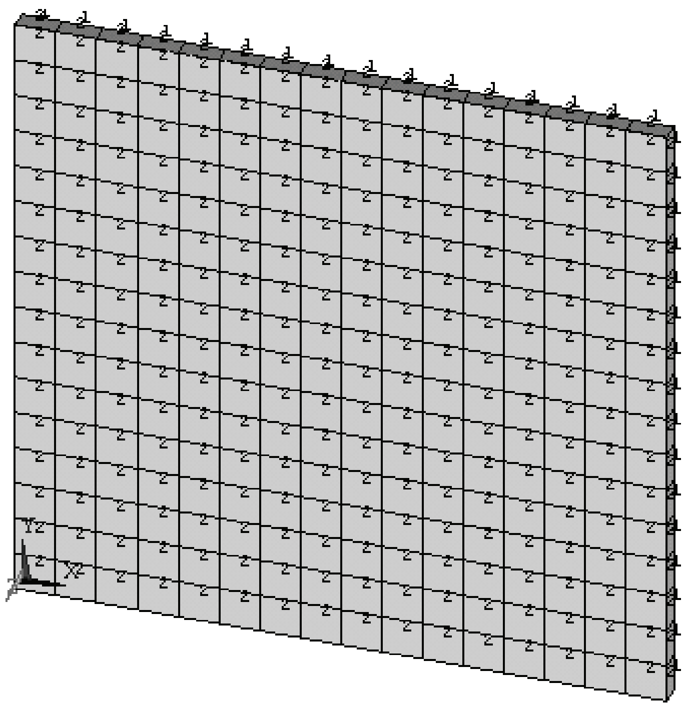


Figure 3.3. Finite element model of plate with piezoelectric layer

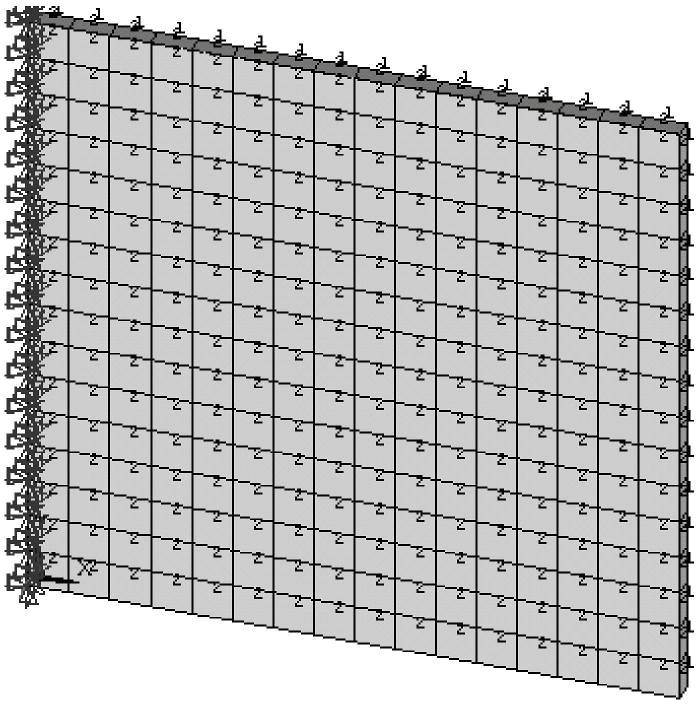


Figure 3.4. Finite element model of cantilever plate with piezoelectric layer

First mode shape of cantilever plate is illustrated in Figure 3.5. It is seen from Figure 3.5 that opposite of fixed edge has the same displacement. Therefore, any point in this edge can be used for displacement measurement in closed-feed back control.

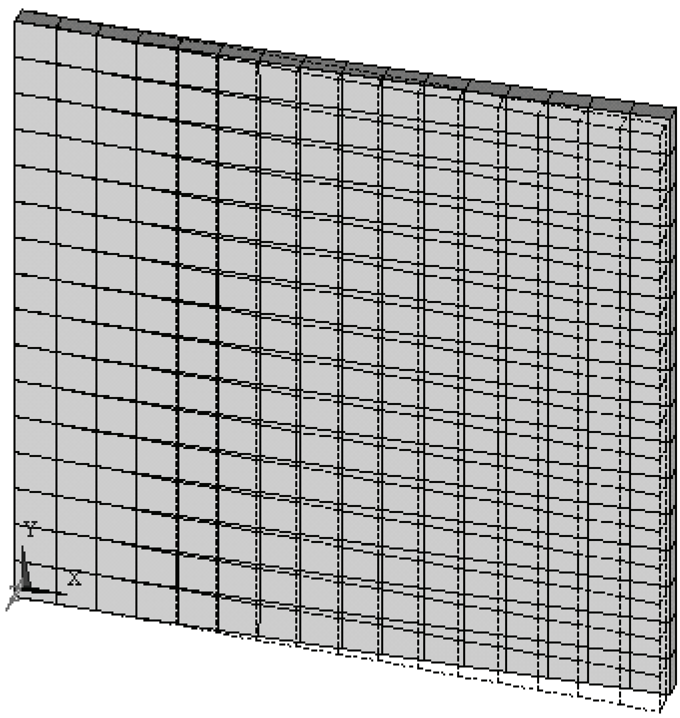


Figure 3.5. First mode shape of cantilever plate

3.2. Numerical Results

The smart plate modeled in Section 3.1 is considered in this section. In order to investigate the active vibration control given in Section 2.7, contact surface of piezoelectric layer is coupled for voltage value. Similarly, free surface of piezoelectric layer is also coupled for voltage value as shown in Figure 3.6.

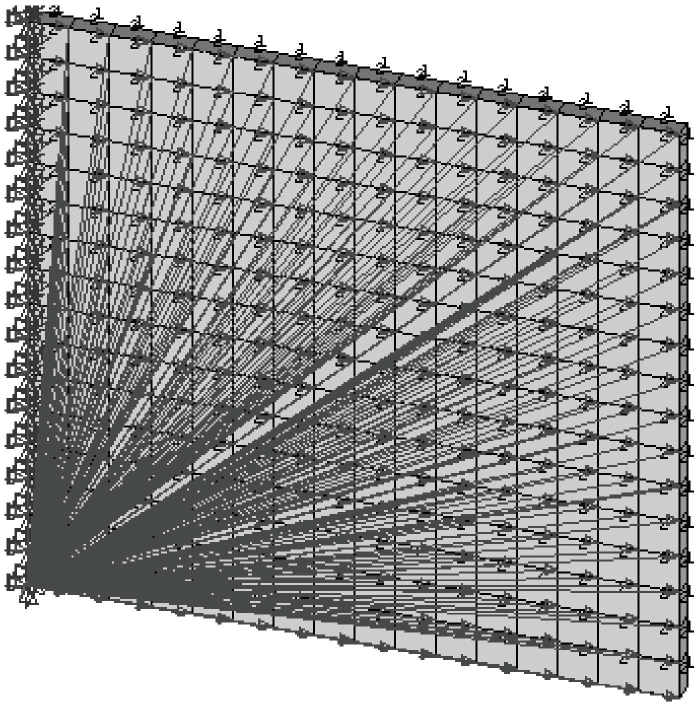


Figure 3.6. Finite element model of smart plate with coupled electrical field

In order to see the effect of piezoelectric material damping on smart plate, proportional damping that is introduced in Section 2.6 is not considered, namely mass matrix coefficient α and stiffness matrix coefficient β are taken as zero in computer code developed.

The active vibration control is accomplished by using the transverse displacement of mid point M shown in Figure 3.7. Smart plate is initially excited by an impulsive distributed tip edge load having the total value of 10 N for 4 times of integration time which is used in transient analysis. Integration time is taken as the one tenth of first natural period.

Displacements of the controlled point M of the smart plate under distributed tip edge load are found in ANSYS for K_s=10000 and different K_p by using *ref*=0.

The results are given are given in Figures 3.8-3.21.

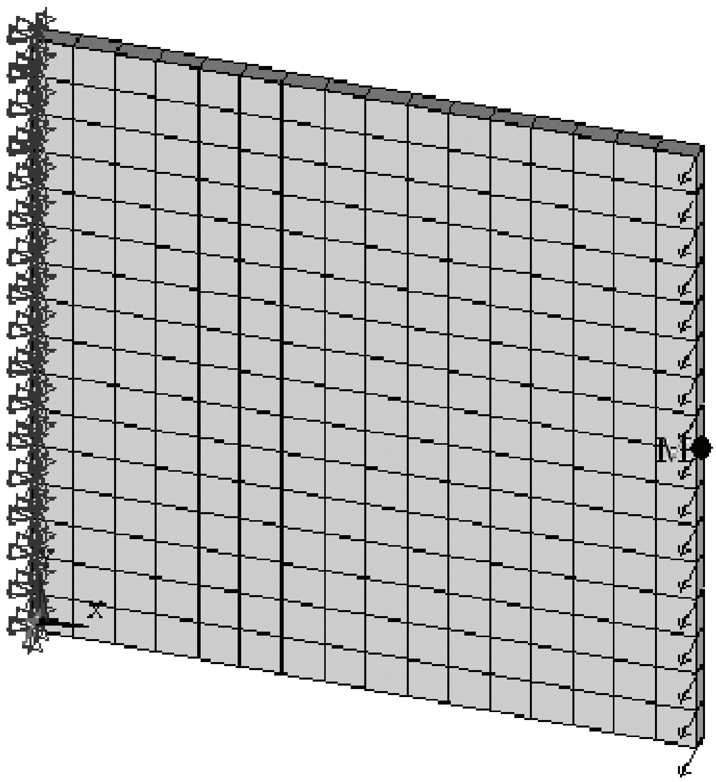


Figure 3.7. Finite element model of smart plate under edge load

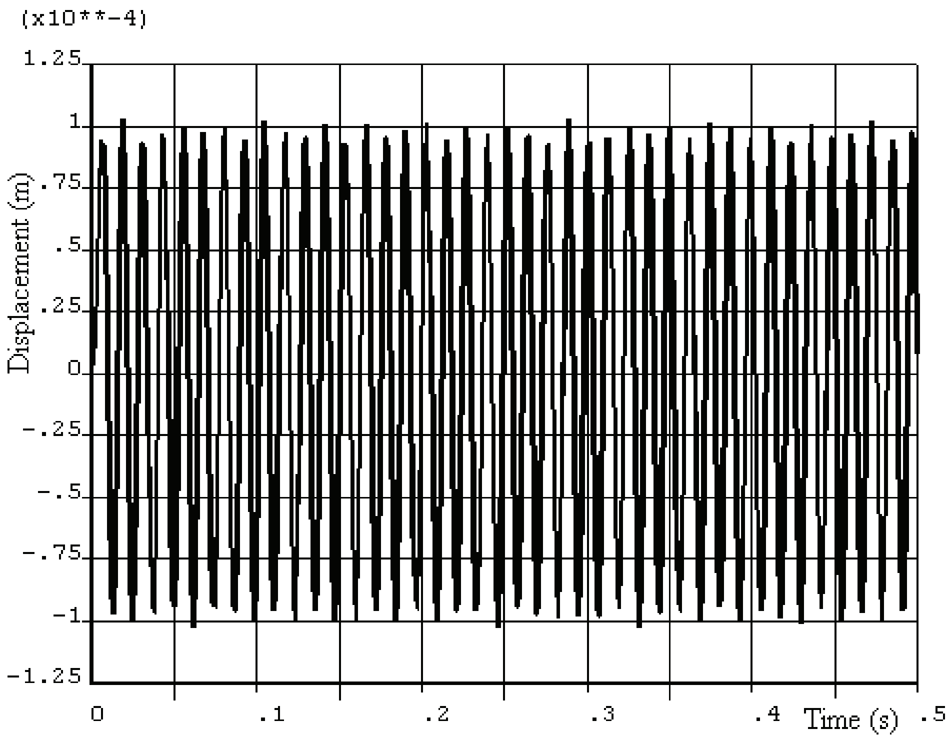


Figure 3.8. Displacement without control

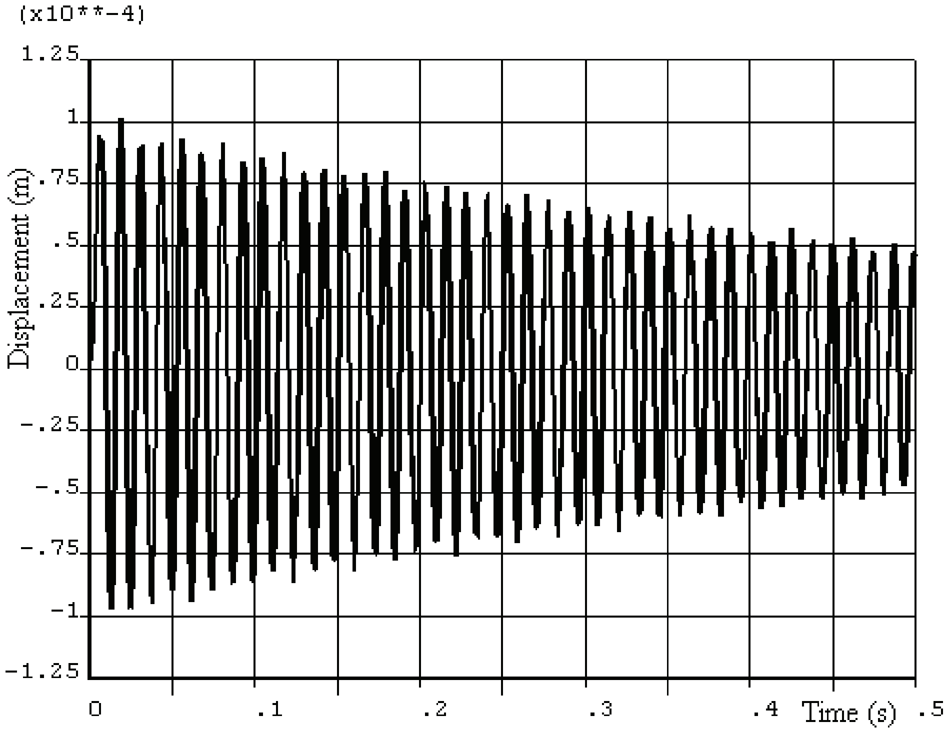


Figure 3.9. Displacement for K_p=1000

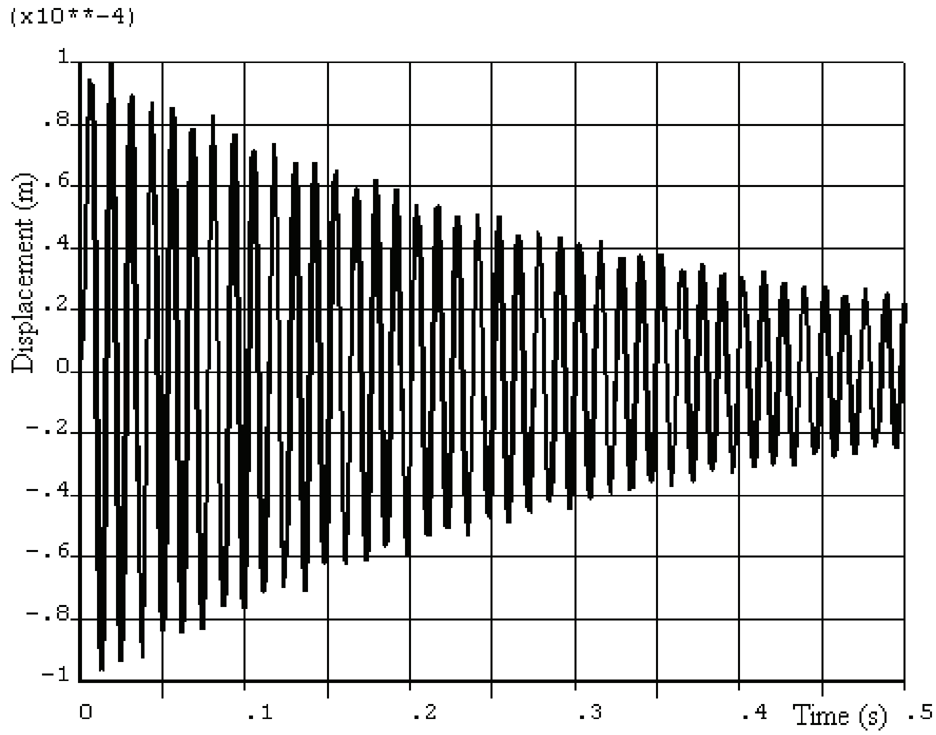


Figure 3.10. Displacement for K_p=2000

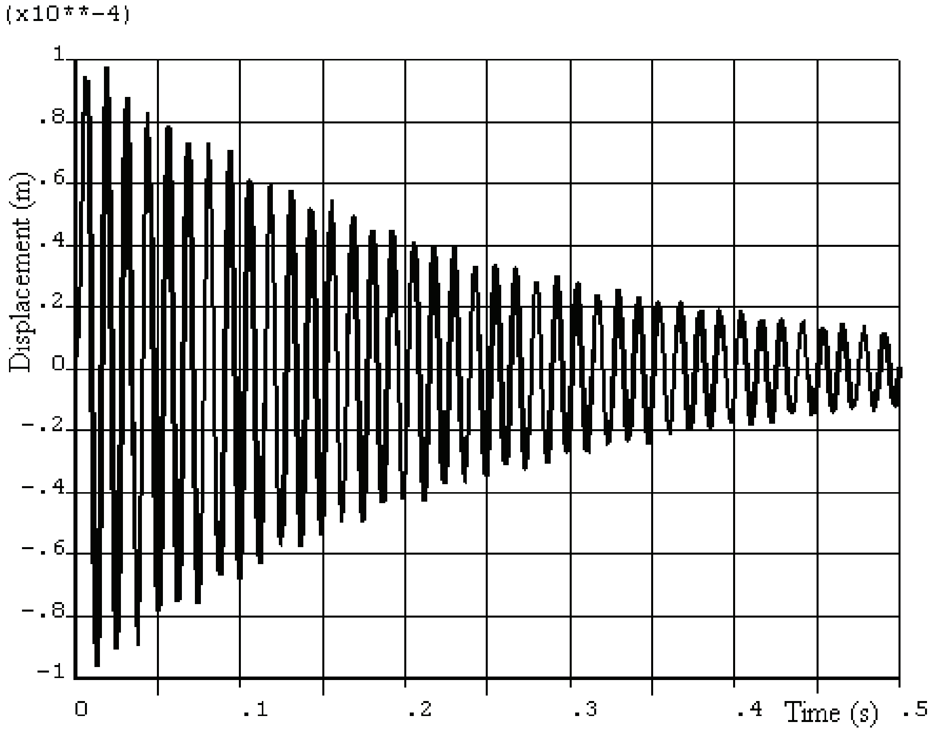


Figure 3.11. Displacement for K_p=3000

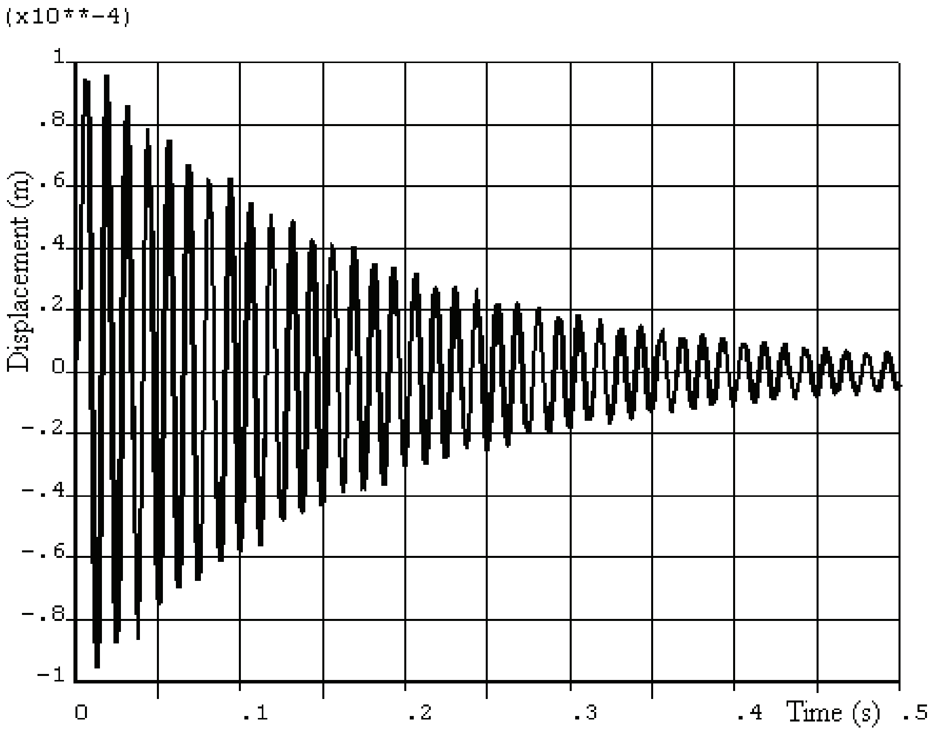


Figure 3.12. Displacement for K_p=4000

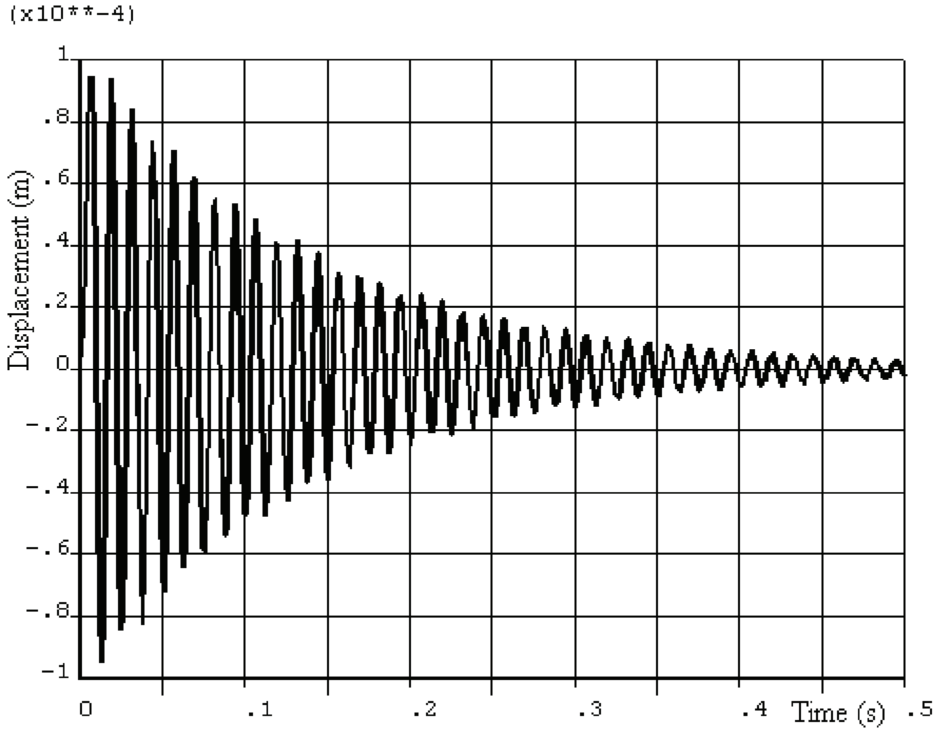


Figure 3.13. Displacement for K_p=5000

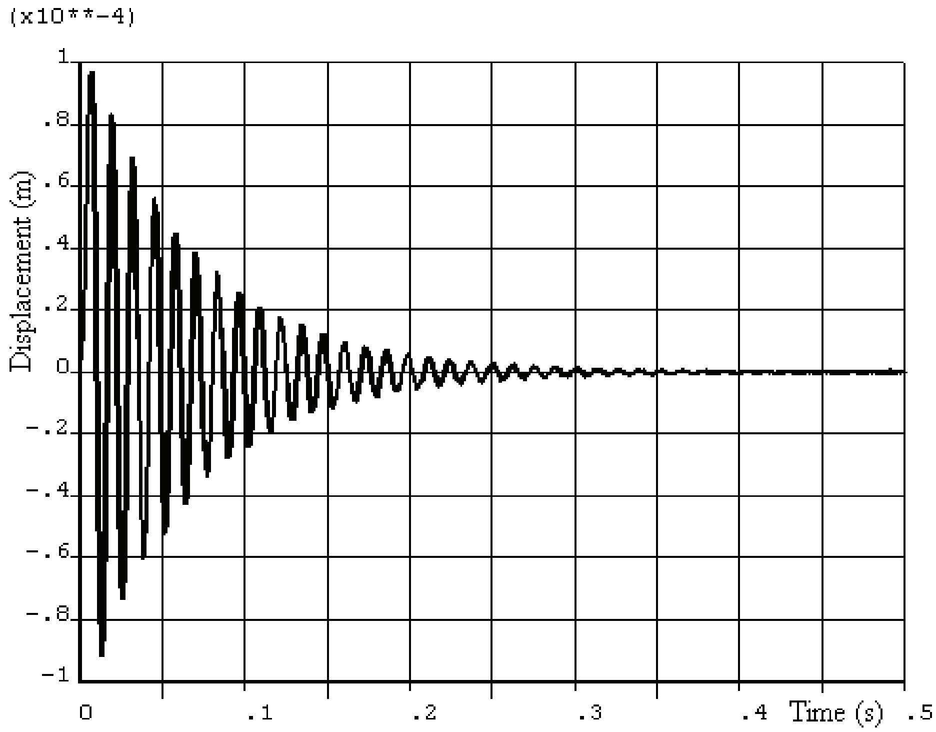


Figure 3.14. Displacement for K_p=10000

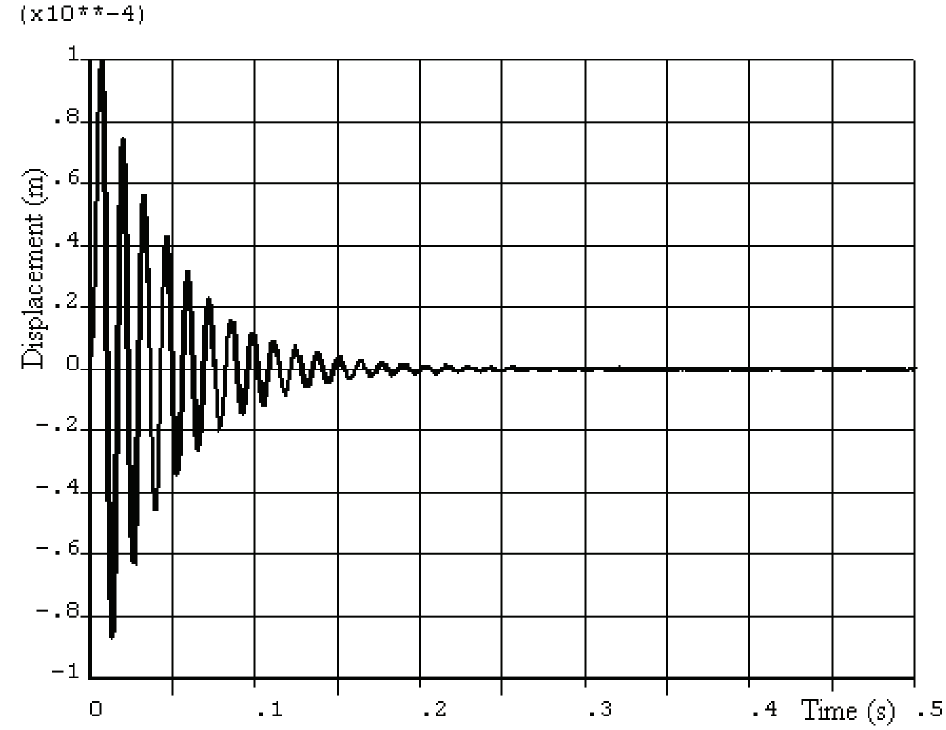


Figure 3.15. Displacement for K_p=15000

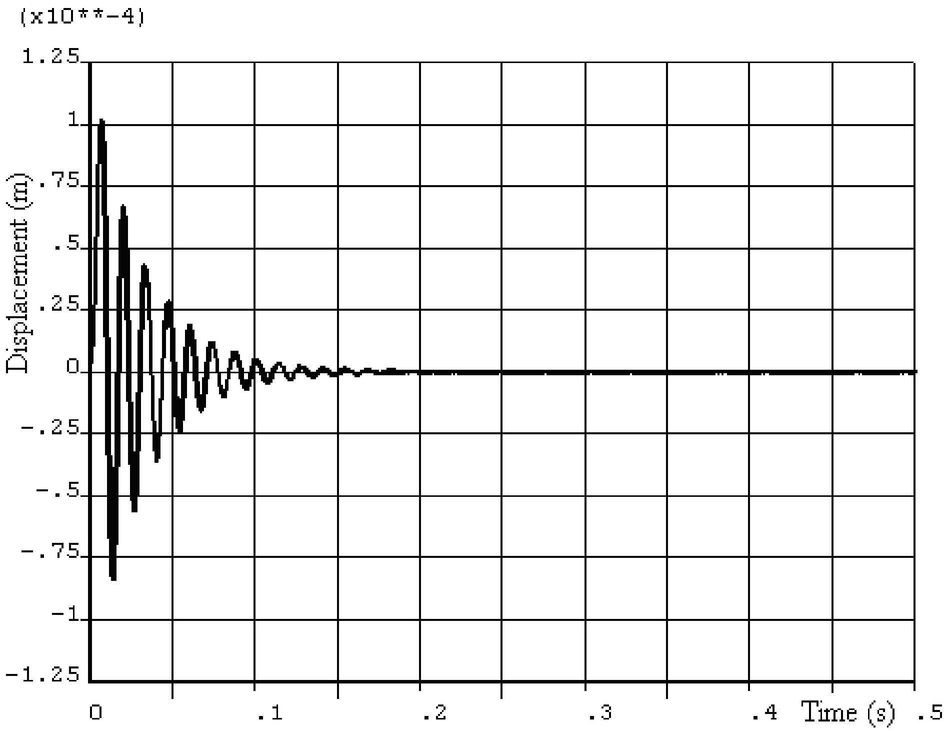


Figure 3.16. Displacement for K_p=20000

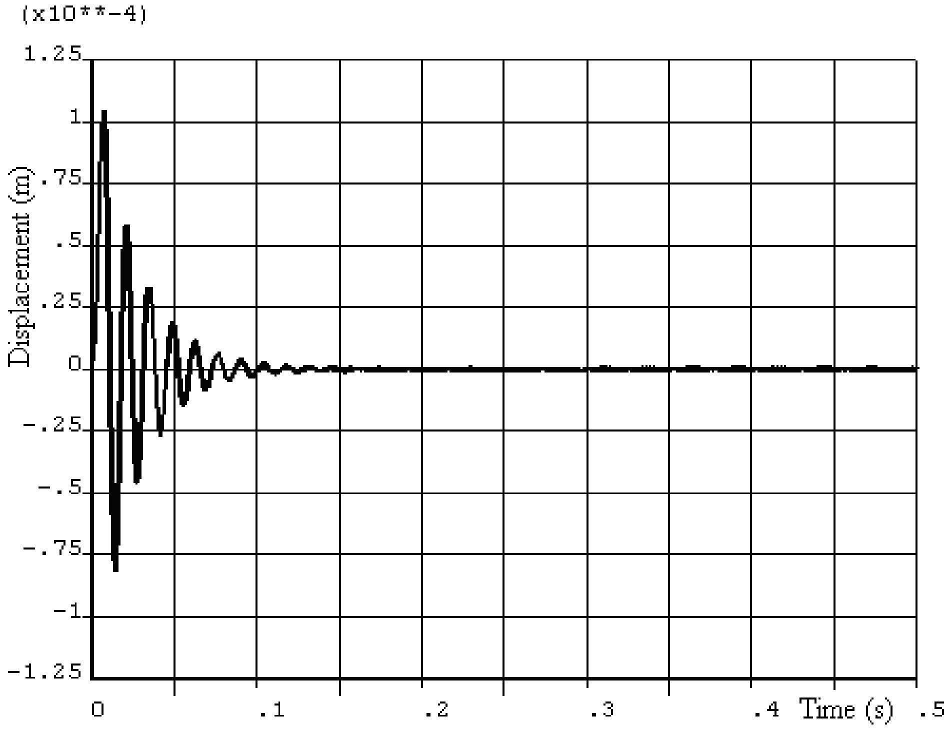


Figure 3.17. Displacement for K_p=25000

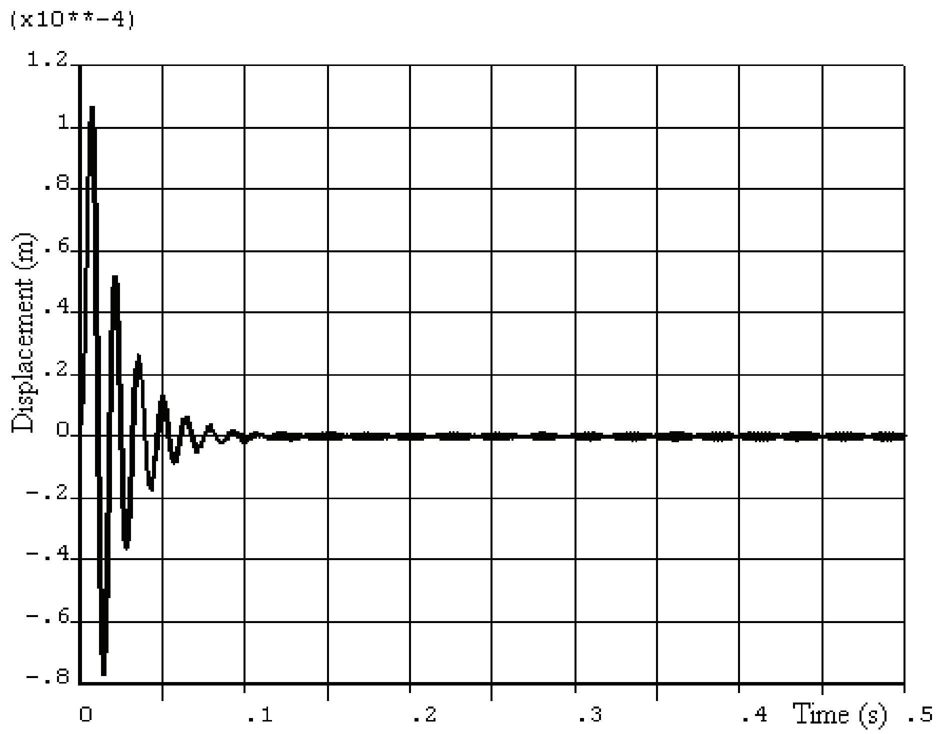


Figure 3.18. Displacement for K_p=30000

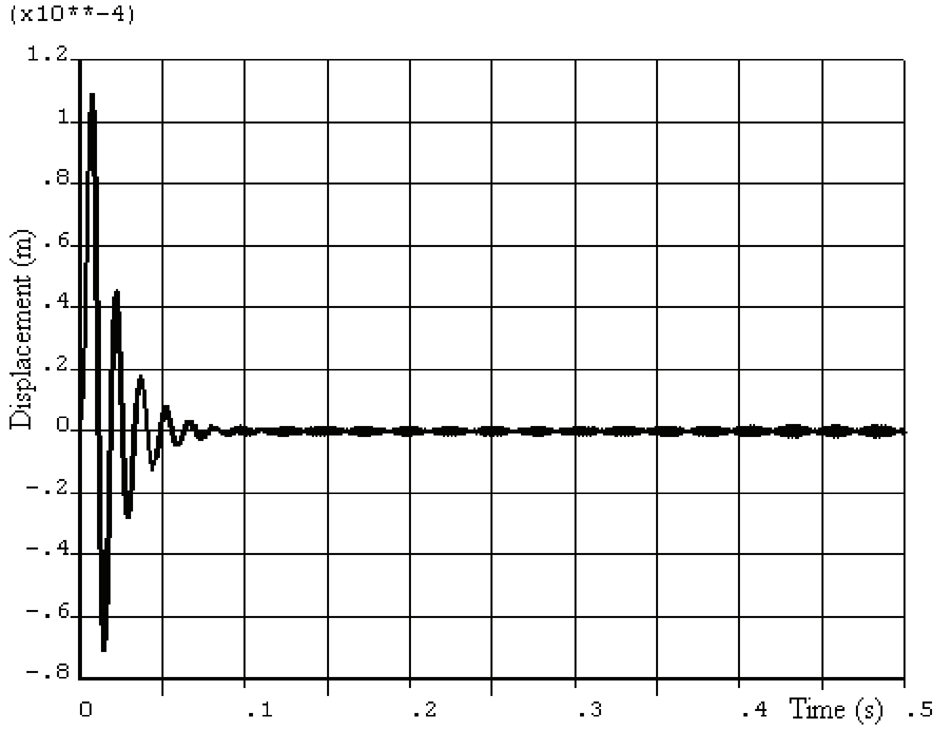


Figure 3.19. Displacement for K_p=35000

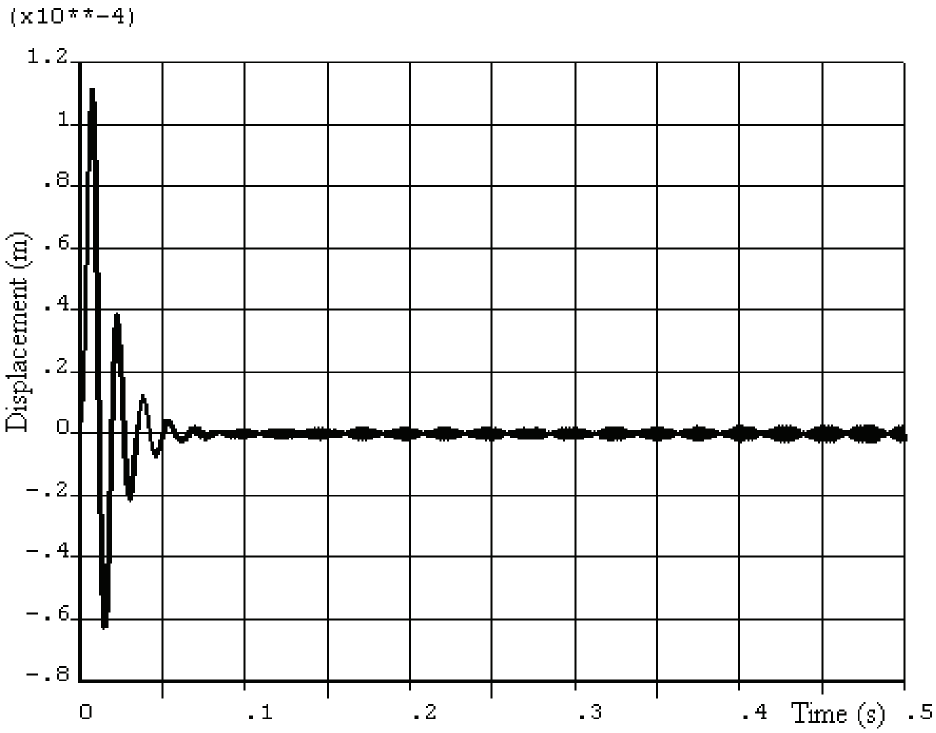


Figure 3.20. Displacement for K_p=40000

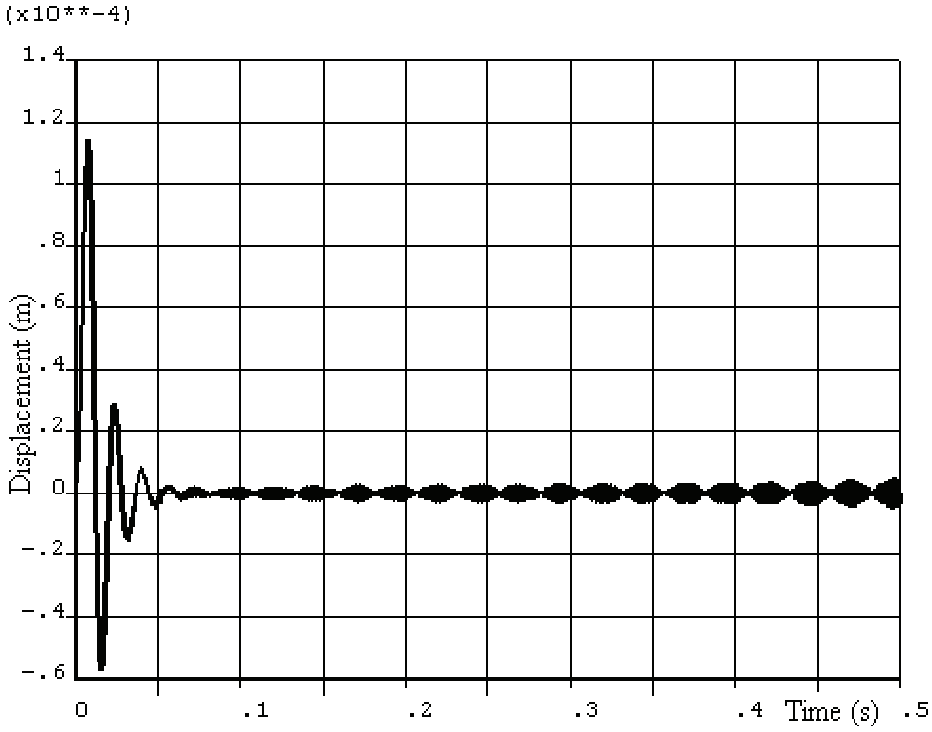


Figure 3.21. Displacement for K_p=45000

3.3. Discussion of Results

In order to determine the control parameters in numerical methods, trial error method can be used. After performing several iterations, the results shown in Figures 3.8-3.21 are plotted. It can be seen from Figures 3.8-3.21 that the value of K_p which is defined in Equation (2.40) as multiplication of transfer functions of controller and voltage amplifier is critical. For a desired active control of vibration, the settling time of the displacements should be short as possible as, however this is limited with instability in progressing time.

CHAPTER 4

CONCLUSIONS

Finite element modeling and active vibration control of rectangular plate having piezoelectric layer is studied by using finite element software ANSYS. The usage of APDL (ANSYS Parametric Design Language) in ANSYS is preferred instead of graphical user interface in modeling and analysis in computer. Therefore, a parametric computer code for active vibration of plate with piezoelectric layer is developed. This code is based on the displacement feed-backs in transient analysis. Results show that control parameter has limitations due to the settling time and instability in progressing time.

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