

**ANALYSIS OF ELECTROMECHANICAL  
BEHAVIOR OF PIEZOELECTRIC SMART  
CURVED BEAMS**

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**by  
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# ABSTRACT

## ANALYSIS OF ELECTROMECHANICAL BEHAVIOR OF PIEZOELECTRIC SMART CURVED BEAMS

In this study, electromechanical behavior of piezoelectric smart curved beam with variable radius of curvature is investigated using Finite Element Method. Firstly, a background of beam theories is provided as well as a discussion on the history of piezoelectric materials and the development of smart beams and smart curved beams. The deformations of curved beams, the differential equations for in-plane bending of curved beam with and without piezoelectric patch are presented. Finite element modeling of smart curved beam is done using ANSYS. The mesh size correctness and numerical accuracy is controlled by an equivalent straight cantilever beam having the same length, cross-section and loaded from its tip with the same amount of force that will be applied to the curved beam model. Tip displacement comparisons are done using an analytical and finite elements approach. The model is verified by comparing the analytical results of piezoelectric constitutive equations with results of finite elements method for a rectangular prism shaped piezoelectric patch. After the verification is complete, the consequent tip displacements on x direction and generated electric fields are observed after the loads of different magnitudes applied to the tip of piezoelectric curved beam. Finally, results are discussed.

## ÖZET

### PIEZOELEKTRİK AKILLI EĞRİ ÇUBUKLARIN ELEKTROMEKANİK DAVRANIŞ ANALİZLERİ

Bu çalışmada, değişken eğrilik yarı çaplı akıllı eğri çubukların elektromekanik analizi Sonlu Elemanlar Yöntemi ile incelenmiştir. İlk önce, giriş teoremleri ve piezoelektrik malzemelerin tarihi ele alınmıştır. Ardından piezoelektrik malzemelerin tarihi, piezoelektrik akıllı çubuk ve akıllı eğri çubukların gelişimi anlatılmıştır. Piezoelektrik eğri çubukların deformasyonu, piezoelektrik bantsız veya bantlı eğri çubukların düzlem içi bükülme diferansiyel denklemleri tanıtılmıştır. Akıllı eğri çubuğun sonlu eleman modellemesi ANSYS ile yapılmıştır. Eğri çubuk ile aynı uzunluğa, keside sahip ve aynı boyutta uç yükle yüklenmiş eş değer düz çubuk ile mesh boyutları ve numerik doğruluğu kontrol edilmiştir. Dikdörtgenler prizması şeklindeki piezoelektrik bant için, temel piezoelektrik denklemlerinden sonuçlar ile sonlu elemanlar metodunun sonuçları karşılaştırılarak; model doğrulanmıştır. Doğrulamanın ardından farklı büyüklükteki kuvvetlerin piezoelektrik eğri çubuğa uygulanması sonucunda, çubuk ucundaki x eksenini yönündeki yer değiştirmeler ve oluşturulan elektrik alanlar gözlenmiştir. En sonunda, sonuçlar değerlendirilmiştir.

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## LIST OF SYMBOLS

$a$	Coefficient of Archimedean spiral
$A$	Cross-sectional area of curved beam
$b_p$	Width of piezoelectric patch
$B$	Bending rigidity of curved beam
$[B_u]$	Strain-displacement matrix
$[B_V]$	Electrical field-electrical potential matrix
$c$	Young's modulus (IEEE standard)
$[c]$	Young's modulus matrix
$d$	Piezoelectric constant without mechanical stress (IEEE standard)
$[d]$	Piezoelectric constant matrix (IEEE standard)
$[d]^T$	Transpose of piezoelectric constant matrix (IEEE standard)
$D$	Electric displacement (IEEE standard)
$\{D\}$	Dielectric displacement matrix (IEEE standard)
$e$	Electric displacement and strain relationship (IEEE standard)
$[e]$	Electric displacement and strain relationship matrix (IEEE standard)
$[e]^T$	Transpose of e matrix (IEEE standard)
$E$	Electric field (IEEE standard)
$\{E\}$	Electric field matrix (IEEE standard)
$E_p$	Elastic modulus of piezoelectric patch
$\{F\}$	Nodal force vector
$h$	The moment arm between the centroidal distance of cross-section of curved beam and piezo patch
$k$	Coupling coefficient of piezoelectric material
$l_{xp}$	Length of the verification piezoelectric layer
$l_{yp}$	Height of the verification piezoelectric layer
$[K]$	Structural stiffness matrix
$[K^d]$	Dielectric conductivity
$[K^z]$	Piezoelectric coupling matrix

$[K^z]^T$	Transpose of piezoelectric coupling matrix
$L$	Length of piezoelectric layer
$\{L\}$	Nodal change vectors
$L_p$	Length of piezoelectric patch
$M_y$	Bending moment about y axis
$m_y$	Distributed bending moment about y axis
$N$	Internal shear force
$N_i$	Shape function for node i
$[N^u]$	Shape function matrix for displacements
$[N^u]^T$	Transpose of shape function matrix for displacements
$\{N^V\}$	Shape function matrix for electric potential
$\{N^V\}^T$	Transpose of shape function matrix for displacements
$n$	Number of disks in the stack
$N_i$	Shape function for node I
$q_n$	Distributed internal axial force
$q_t$	Distributed internal shear force
$r$	Radius of Archimedean spiral
$s$	Inverse of Young's modulus (IEEE standard)
$[s]$	Inverse of Young's modulus matrix (IEEE standard)
$s^E$	s in constant electric field (IEEE standard)
$S$	Strain (IEEE standard)
$\{S\}$	Strain Matrix (IEEE standard)
$t$	Thickness of piezoelectric layer
$t_p$	Thickness of the verification piezoelectric layer
$T$	Stress (IEEE standard)
$T$	Internal axial force
$\{T\}$	Stress matrix (IEEE standard)
$u$	Displacement on curved beam in x direction
$w$	Displacement on curved beam in z direction
$\{u\}$	Nodal displacement vector
$\{u_c\}$	Displacement vector
$U$	Voltage (IEEE standard)

$\{V\}$	Nodal electric potential vector
$(vol)$	Volume
$V_c$	Electric potential
$\Delta L$	Length change of piezoelectric layer
$\varepsilon$	Strain in s direction
$\varepsilon$	Dielectric constant (IEEE standard)
$[\varepsilon]$	Dielectric constant matrix (IEEE standard)
$\varepsilon^T$	$\varepsilon$ in constant stress (IEEE standard)
$\varepsilon^S$	$\varepsilon$ in constant strain (IEEE standard)
$\theta$	Polar angle of Archimedean spiral
$\kappa'_0$	Initial radius of curvature of curved beam
$\kappa'_1$	Deformed radius of curvature of curved beam
$\rho_0$	Initial radius of curved beam

# CHAPTER 1

## GENERAL INTRODUCTION

### 1.1. Introduction

A discussion on the background of beam theories, including the discovery of piezoelectricity and piezoelectric effects and the pioneers who combined piezoelectric effects with beam theories and directly related studies essential to understand the subject of this thesis is provided in the following sections.

### 1.2. Beam Theories

A beam is a 3D structure (Labuschagne et al. 2009) which has one larger dimension compared to the remaining two. The axis along the larger dimension specifies the beam's axis. It is assumed that the cross-section normal to the axis smoothly changes along the length.

There are many application sites of beams. In civil engineering, there are structures consisting of I, T or square cross-section assembled beams (Bauchau and Craig, 2009a). Many machine parts such as jacks and shafts are also beam-like structures. Furthermore, it is possible to consider aerospace structures of wings and fuselages as thin-walled beams.

Beam Theory is a common and simplified theory referring to beams in solid mechanics. Because of their simplicity in analyzing several structures, they occupy a significant place in structural analysis. Beam models provide significant insight into the behavior of structures. For this reason, they are frequently used in pre-design stages. Their results are also beneficial for validating computational solutions such as finite elements method which is a more sophisticated tool for stress analysis.

Several beam theories were defined based on different assumptions with different levels of accuracies. The first theory, Euler-Bernoulli, dates back to the 18th century (Han et al. 1999). Jacob Bernoulli found out that the curvature of an elastic

beam at any point is proportional to the bending moment of specified point. In later years, Leonhard Euler investigated the shape of elastic beams under different loading conditions and supported the findings of Jacob Bernoulli's. Therefore, the theory is named by joining their surnames. It may also be referred as the Euler beam theory, Bernoulli beam theory or the classical beam theory. Overall, the theory has three assumptions. The cross-section of the beam is infinitely rigid in its plane which allows no deformation. For this reason, in-plane displacement field can be represented by two rigid body translation and one rigid body rotation. Moreover; in case of deformation, the cross-section is assumed to remain plane and normal to the deformed axis. But, experimental results state that these assumptions are only eligible for long, slender beams which have material isotropy. Without those specifications, the Euler-Bernoulli beam theorem becomes inaccurate. Also, the theory overestimates the natural frequencies, especially for higher modes.

The overestimation of natural frequencies in Euler-Bernoulli theorem was attempted to be removed by John William Strutt, 3rd Baron of Rayleigh who added the effect of rotation of the cross-section and he established the Rayleigh Beam Theorem on 1877 (Rayleigh and Lindsay, 1945). While the effect of rotation can be expressed as the angular acceleration of beam elements, this theory gives more satisfying results than Euler-Bernoulli's. However, natural frequencies are still overestimated.

The assumption that the cross-section of the beam remains normal to the deformed axis makes the transverse shear effect disappear in Euler-Bernoulli's theorem. If shear distortion is combined with the bending effect of the Euler-Bernoulli's theorem, Shear Beam theory is reached. Thus, the estimation of natural frequencies improves.

In addition to the theorems mentioned above, Stephen Timoshenko established his own beam theorem by combining the effect of shear and rotation to Euler-Bernoulli's theorem in 1921 and 1922 (Bauchau and Craig, 2009b). Shear or rotary effects are not negligible in this model. Thus, analyses of non-slender beams and its high-frequency responses are improved.

### 1.3. History of Piezoelectric Materials

After the introduction of beam theories, the foundations of piezoelectric studies and their history should be investigated. The word piezoelectric originates from the Greek words "piezin" which means "to press" and "electron" which has different meanings as "an amber", "a form of currency", "a solar divinity" (Olcott, 2013). Before expressing the historical background, phenomena of direct and inverse piezo electric effects illustrated in Figure 1 should be mentioned. The direct piezoelectric effect is generating an electrical charge in proportion to an externally applied force. This effect is used as sensor. However, inverse piezoelectric effect is an expansion of the material due to electric field applied to material and this effect used as actuator.

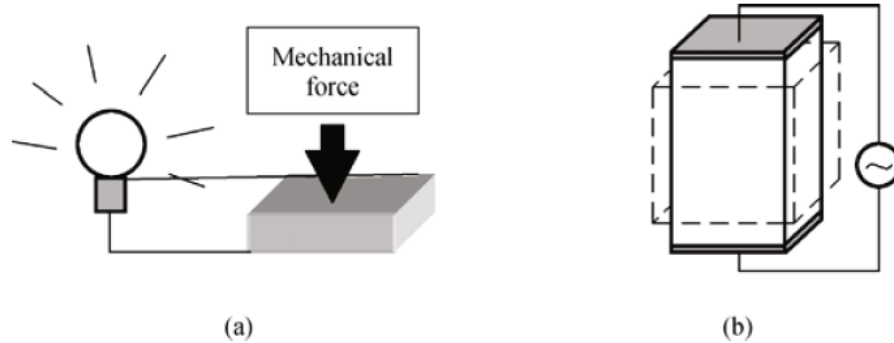


Figure 1.1. Direct and inverse piezoelectric effects: a) Direct effect, b) Inverse effect (Source: Qui 2013)

The Curie brothers, Pierre and Jacques Curie announced direct piezoelectric effect on Academie des Sciences in 2 August 1880 (Gautschi, 2002). This announcement was based on their systematic studies with the crystal symmetry of materials such as zinc blende, sodium chlorate, boracite, tourmaline, cane sugar, rochelle salt and observing their effects. Polar electricity developed due to variations in pressure as a consequence of their experiments. One year later, the inverse piezoelectric effect was predicted by Gabriel Lipmann. He declared that a crystal will deform if an electrical voltage is applied to certain opposite faces of the crystal. On the same year the prediction of Lipmann was confirmed by Curie brothers with their experiments. During their first declaration in 1880, the term piezoelectric was not yet used by Curie brothers. Wilhelm Gottlieb Hankel in 1881 was the first one to propose and the name was immediately accepted in the field (Dineva et al. 2014).

Although the studies by Curie brothers spread to the outside of France and famous scientists of this era such as Woldemar Voigt, Eduard Riecke initiated their own studies of the newly established topic; piezoelectricity remained an object of curiosity instead of practical application for more than 30 years. However, a need for detection of submarines during the First World War changed the situation. Paul Langevin used inverse piezoelectric effect of quartz to create mechanical vibration to emit ultrasonic sound waves underwater. When the created ultrasonic waves were reflected back by objects at the bottom of or in the water, those waves could be captured by same quartz plates before submarines detected by direct piezoelectric effect.

#### **1.4. Developments of Piezoelectric Materials on Smart Beams**

Many applications, literatures and patents appeared combining piezoelectric and beam theories after the First World War. But, beam as a word generally represents a line of sound or light in earlier studies. Wente (1922) patented “piezo-electrical voltage indicator” which contained rigidly supported piezoelectric crystalline blades. Seventeen years later, Mason (1939) patented “piezoelectric apparatus and circuits”. He became one of the pioneers who used the word beam as a structural member in piezoelectric applications. These included structures made of low density material, for example aluminum, for weight saving and piezoelectric elements bonded by adhesive material such as shellac. Koren (1949) studied metal strips with thin plate of ceramic soldered on one or both sides which was an early example of active structures for transducer. A combination of piezoelectricity and plate theory was done by Mindlin (1952). Thurston (1953) examined corner loaded, uniformly loaded and central loaded bimorph transducer by theory of elasticity.

Between 1960 and 1970, various patents that contained piezoelectric beams such as piezoelectric ignition system of Crownover (1966), 3D accelerometer device of Igarashi and Chiku (1967), force transducers of Rogallo (1967) and Norris (1969) were designed. Furthermore, Cain et al. (1967) used transducer to measure the heartbeats of embryo of poultries. Berry (1972) used the forces generated by bending or twisting of piezoelectric material during acceleration and deceleration to design a vehicle speed synthesizer which was intended to be used in antilock brake systems in automobiles. One of the first biomedical application of piezoelectric beam was studied by Williams

and Breger (1975). Rauch and Witter (1976) used a piezoceramic beam for their device that wrote with liquid ink.

The term active structures was established by the 1980's. Also, aerospace applications were initiated. On the other hand, theoretical works appeared. Lee and Marcus (1981) worked on a cantilever mounted piezoelectric bimorph and unimorph structures. Cucci (1982) patented vibrating beam pressure sensor. Bailey and Hubbard (1985) designed and analyzed an active damper for a thin cantilever beam using piezoelectric polymers.

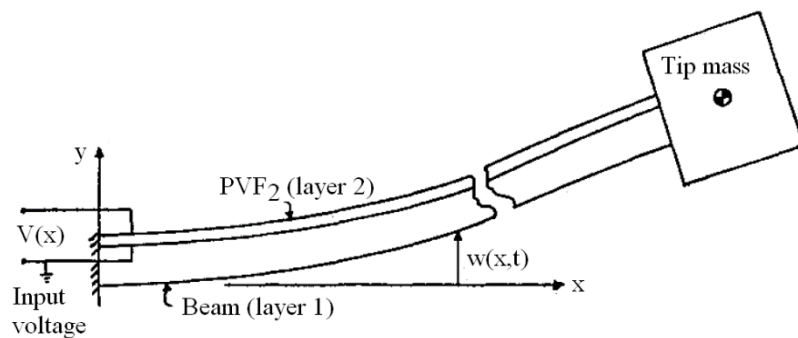


Figure 1.2. Active damper model  
(Source: Bailey and Hubbard 1985)

Crawley and De Luis (1987) are the pioneers who used "intelligent structures" by defining them as structures with many actuators, sensors and processing networks. They tested a cantilever beam made of composite and isotropic materials with piezoelectric actuators bonded on them. Tzou (1989) completed various studies such as the development of robot end effector using bimorph. Im and Atluri (1989) investigated the effect of piezoelectric liners on a deformed beam for the usage of model in space structures in which the effect of transverse shear, axial forces and bending moment were investigated.

Tzou and Zhong (1990) simplified the piezoelectric shell to a piezoelectric bimorph cantilever beam. The design, experiments and feasibility of piezoelectric actuators mounted on helicopter blades were accomplished by Hall and Spangler (1990). Lee and Moon (1990) developed modal sensors/actuators for beams and plates.

Studies consisting of smart materials having classical beam models were published during the last decade of 20th century. Inman (1991) finalized his studies between 1987 and 1990 by developing the Timoshenko model of layered piezoelectric devices. Tzou and Howard (1992) extended the studies on smart structures by adding



piezo-thermo elasticity using shell and beam for creating a simplified model. Koconis et al. (1994a) calculated the changes of shape under applied voltage for the fiber reinforced structural elements such as beam, curved beam, rectangular plate with different boundary conditions. Shen (1995) proposed a modeling technique which does not require of integration of piezoelectric devices into the governing equation. Lee (1996) implemented neural network system on smart structures. Lee et al. (1997) concentrated not only on performance of smart material structures, but also their reliability by detailed stress analysis and simulation of crack growth. Aldraihem et al. (1997) compared the Timoshenko and Euler-Bernoulli beam models for suppressing the beam vibration by using piezoelectric sensor and actuator layers. Krommer and Irschik (1999) used laminated beams, plates and shells to investigate the effect of electric field on free transverse vibrations. The study of Wang et al. (1999) was one of the examples which concentrated on sensing capacity of various types of layered beam.

Furthermore, there are remarkable studies published during the late 90's and early 2000's about smart materials with finite element method (hereafter FEM) to be denoted. Söderkvist (1997) used FEM to cope with the difficulty and cost of the experiments of micro-machined structures to obtain the analytical results of piezoelectric cantilever beam. Mackerle (1998) listed all the smart material and structure studies that employed finite element methods between 1986 and 1997. He (2003) expanded his bibliography by adding studies from the same area between 1997 and 2002. Benjeddou (2000) published his research on the piezoelectric adaptive structures modelled by finite elements and concentrating on various specifications such as elements types, their shapes, nodal degree of freedoms, formulations, and assumptions. He also suggested where further developments should be made. Piefort (2001) gave detailed information on his PhD. thesis about history of piezoelectricity, its equations, abilities of different designs, background of finite elements and implementations in smart materials, as well as possible application areas of sensing or actuation capabilities and controlling.

During the first decade of millennium, the number of studies including piezoelectric and beam skyrocketed compared to the previous decade. In fact, there were 427 studies between 1990 and 2000, 1540 from 2000 to 2010 and 2910 in last 7 years from the internet until publishing this thesis. Those values show that the power of the topic has not diminished. Therefore, articles having the highest citations present the trends and developments on smart materials are discussed in the following paragraphs.

As the micro electromechanical systems (hereafter MEMS) became popular, usage of “energy harvesting” from piezoelectric systems instead of external batteries emerged. One of the earlier example was by Elvin et al. (2001). He used a simply supported beam with a strain sensor to generate its power and showed that the frequency and applied force affected the measured strain. Glynne-Jones et al. (2001) showed the voltage generation using a cantilever beam layered with smart materials and carrying mass on tip. The similar study was performed by Lu et al. (2004) using numbers of piezoelectric layers and types. duToit et al. (2005) investigated energy harvesting from vibrations and detailed them for one dimensional harvester model, mass mounted cantilever beam model, harvesting modes and their analytical equations.

Roundy et al. (2005) published a paper on generating more power with vibration based harvesters known as “scavengers” in their research. They concluded that an efficient harvester should have maximum piezoelectric response with respect to the given input, a smaller stress concentration, and also a simple manufacturing method. Moreover, they considered different geometries and boundary conditions, tuning of frequency to enlarge bandwidth, thinner piezoelectric structures, and micro-manufacturing processes which are easily attainable. In the same year, Baker et al (2005) accomplished experimental studies using trapezoid beam for energy harvesting due to showing superiority of trapezoid beam over rectangular beam. They concluded that the output increased 30% and frequency tuning was obtained via bi-stable boundary conditions. Shu and Lien (2006) investigated the optimal power generation for a rectified piezoelectric device. Challa et al. (2008) presented an approach to enlarge the bandwidth of energy harvesting from a cantilever beam that having tip mass between the magnets which reduced the original natural frequency of the system.

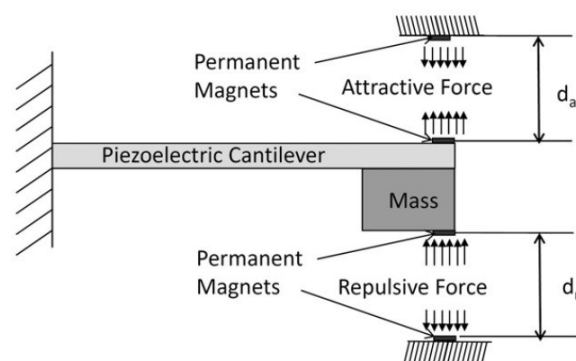


Figure 1.3. Cantilever beam for energy harvesting  
(Source: Challa et al. 2008)

In order to correct the past studies, Erturk and Inman (2008) published a new mathematical modeling that considered resonance, true form of piezoelectric coupling and forced vibration terms, true usage of sensing and actuation equations and dynamic deflection patterns. They validated their mathematical model by experimenting a cantilever bimorph beam too (2009). Shen et. al (2008) manufactured a beam with multilayered piezoelectric materials which was in the form of silica wafer for vibration with low frequencies and high amplitudes. Lee et al. (2009) manufactured a composite material by spraying powder of PZT on silica wafer in order to obtain high quality thin piezoelectric film.

Cottone et al. (2009) worked on an inverted pendulum under magnetic excitation at both its base in lateral direction and tip in axial direction. Thus, they proposed a method for stochastic nonlinear oscillators. A similar method was proposed by Ferrari et al. (2010). They used a horizontal cantilever beam instead of an inverted pendulum. The pendulum was excited from its root mechanically, while the tip of it was under magnetic force in the axial direction. Their experimental results showed that the power output increased 88% with respect to linear systems. Also, Stanton et al. (2010) studied nonlinearity concept analytically and experimentally following the former studies. Various bandwidth enlargement techniques were investigated by Tang et al. (2010) according to their advantages and disadvantages. Tabesh and Frenchette (2010) provided energy-harvesting circuit with low power dissipation that was critical for ac/dc voltage conversion.

Wang (2008) was one of the first researcher on usage of nano wires for energy harvesting. The effects of surface stresses of Euler- Bernoulli beam model of nano wires was investigated by Wang and Feng (2010). Sun et al. (2010) tested nano wires statically with different materials and shapes in order to show which generates power the best and generate guideline in future works.

Akaydin et al. (2010) explored energy harvesting from unsteady turbulent fluid flow using piezoelectric generator. On the other hand, Clair et al. (2010) studied an energy harvester based on a cantilever beam subjected to air blow.

Gu and Livermore (2011) presented a double beamed harvester which consisted of one resonant beam that carried mass and one piezoelectric layered beam that vibrates due to impact of mass of the other one. Thus, the system resulted in better power output and increased bandwidth comparing with the conventional one. Zhou et al. (2012)

proposed a cantilever beam with two portions as thick and thin form, namely serially connected. Both portions had mass at the end of them.

Masana and Daqaq (2012) focused on energy harvesting in the super harmonic frequency region of a twin-well (bistable) oscillator. Harne and Wang (2013) concentrated on the mechanical and magnetic bistable systems that generated more power comparing with the linear ones in steady state environment. But their potential was uncertain across the random conditions. Pellegrini et al. (2013) schematically investigated and grouped the bistable harvesters, and summarized nonlinearity, dimensionless effectiveness equations. They stated that the harvesters based on linear oscillators were more effective comparing with the bistable oscillators because of their nonlinear responses despite their high power output capacity.

Li et al. (2014) not only concentrated on conventional beams, but also cymbal type harvesters and curved multilayered harvesters. Zhou et al. (2014) used a model consist of cantilever beam carried two magnet masses on tip. Also, another two extra magnets had optimum position and angle that positioned at right and left side of the tip of the beam. Thus, tri-stability condition was created. Leadenham and Erturk (2014, 2015) designed and experimented a shallow M-shape structure having two inclined bimorph beams as its legs. Those were mounted on two shakers with a proof mass in the middle of the structure to analyze nonlinear energy harvesting for very low base accelerations: primary and secondary resonances.

Cao et al. (2015) also studied the tri-stability condition. He showed how different angles of rested masses at the right side and left side of the beam affected the frequency range and voltage output. Another bi-stable and tri-stable comparison was done by Haitao et al. (2016). However, their beam carried just one magnet on tip.

The daily practical applications of energy harvester became popular due to the developments in that field. Mateu and Moll (2005) studied energy harvesting from daily walking by using various beam shapes with different boundary conditions and alignments. Elvin et al. (2006) focused on civil engineering structures under excitation forces such as traffic, earthquake, and wind for vibration which could be used for structural monitoring based on piezo harvesters.

Anton and Inman (2008) designed a radio controlled UAV (Unmanned Aerial Vehicles) which powered from solar energy and piezoelectricity. They examined the power output of the photovoltaic and piezoelectric devices. Renaud et al. (2009) studied piezoelectric energy harvesting system by motion of human limbs.

Sirohi and Mahadik (2011) proposed a wind energy-harvesting device as an alternate power source. Their system had two parallel cantilever beams with piezoelectric sheets which were attached to a common mass body excited by incident wind. They (2012) also studied a similar topic using a single cantilever beam. Bryant and Garcia (2011) derived a linearized analytical model of a novel piezoelectric energy harvester device driven by flutter vibration of flap pinned at the end of a cantilever beam with piezoelectric patches. Weinstein et al. (2012) used heating, ventilating and air conditioning flow to excite the cantilever beam with piezoelectric energy harvester. Bibo et al. (2015) developed and validated a piezoelectric cantilever-type energy harvester by combining base excitation and aerodynamic forces in their numerical and experimental study.

Li et al. (2013) introduced an acoustic energy harvester that was a quarter-wave length straight-tube acoustic resonator with PVDF piezoelectric cantilever beams placed inside. Bowen and Arafa (2014) summarized the tire pressure monitoring system based on piezoelectric cantilever beams. Cao et al. (2015) used to test the application of bi-stable nonlinear cantilever beam with PZT under magnetic field for power generation from human motion and showed its superiority comparing with the linear and mono-stable harvesters. Wang et al. (2015) theoretically studied energy harvesting from railway systems. Their model was based on Euler-Bernoulli beam on a Winkler foundation. Ansari and Karami (2016) used fan-folded piezoelectric bimorph cantilevers to generate more power from heartbeat vibrations for powering leadless pacemakers.

The general aspects of piezoelectric energy harvesters and a comparison of their models, explanations, and application examples between 2003 and 2006 were reviewed by Anton and Sodano (2007). Another review was published by Khaligh et al. (2010). At their review, Kim et al. (2011) offered a brief explanation in theories and models behind energy harvesting, summarized recently used materials; presented examples of harvesting applications such as usage in shoes, plants and backpacks; explained circuits and storage of harvesters; and offered the future improvements. Wang (2012) reviewed piezoelectric nanogenerators.

Other applications of the piezoelectric beams related with sensing and vibration suppressing were studied by researchers. Lee et al. (2001) used piezoelectric beam for sensing the motion and acceleration in order to investigate the sleep and activity patterns of women diseased by HIV/AIDS. Jean-Louis et al. (2001) used a motion

sensor mounted on the wrist and took accurate results for sensing whether awake and sleeping.

Narayanan and Balamurugan (2003) developed beam, plate, and shell type finite elements for piezoelectric laminates. They also considered the temperature effects in their formulations. Maurini et al. (2004) studied on control of beam vibration by using piezoelectric networks acting as distributed vibration absorbers. Kumar and Narayanan (2008) interested in active vibration control of beams with optimal placement of piezoelectric sensor/actuator pairs.

## **1.5. Developments of Piezoelectric Material on Smart Curved Beams**

Curved beams can be seen in various engineering areas, for example, aerospace, mechanical, civil, etc. Longitudinal axis of curved beam can be two or three dimensional. The simplest planar curve is an arc which has constant radius. However, in some cases, it has variable curvature such as Archimedean spiral.

Two dimensional curved beams can deform in two fashions depending on the load action on it. Those deformations are in-plane and out-of-plane deformations. An in-plane deformation of curved beam has two coupled simple deformations: bending and axial deformation. On the other hand, out-of-plane deformation of curved beam has bending and torsion.

Applications of curved beams and rectangular shaped piezoelectric layers were popular between 1960 and 1980's, such as acoustic transducer modelling and experiments of Royster's (1970), injection pressure measurement device of Heggie (1977) and sandwich curved beam vibration experiment of Vaswani et al (1988).

Designing and modeling semicircular shaped piezoelectric layered curved beams emerged as actuators in the 1990's. The study of Brei and Blechschmidt (1992) was one of the early examples. Three years later, Brei (1995) proposed "C-Block actuators" in the shape of semicircle to be used in different arrangements such as series, arrays, and parallel. He investigated their comparisons with straight beam actuators. Seeley et al (1996) continued to study C-block as offered by Brei to find the hybrid usage of C-blocks in optimum fashion. Moskalik and Brei (1997) presented deflection-voltage model and experimental results for polymeric piezoelectric C-block actuators shown in

Figure 1.4. They (1999) extended their C-block actuator studies to see the force-deflection behavior of the system.

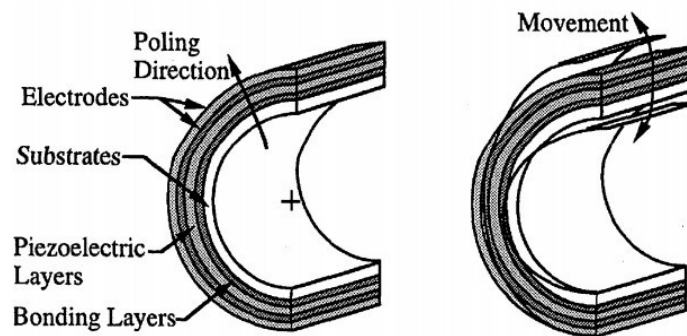


Figure 1.4. Polymeric piezoelectric C-block actuators  
(Source: Moskalik and Brei 1997)

Lee (2000) compared straight bimorph actuator and C-block actuator considering their force and displacement output capacities with respect to same amount of given voltage by using finite elements method, and showed that C-Block actuators have advantages in output forces, but disadvantages in output displacements.

Another application area of curved piezoelectric layers is shape controlling. Koconis et al. (1994b) published a highly cited paper on the stated topic. Another shape control application was the study of Saggere and Kota (1999) which included camber shaping of wing structures. Yoon et al. (1998, 2000) examined single and double curved antennas with PZT actuators.

Piezoelectric micro-sensors that embedded on the curved shaped composite structures of ship hulls and planes for structural health monitoring were studied by Krantz et al. (1998, 1999).

Application of the pairing of piezoelectricity and curved beam were widened after 2000. Lobontiu et al. (2001) designed a piezoelectric-driven inchworm locomotion device. Borgen et al. (2003) used two piezoelectric curved beams to design a miniature swimming vehicle. Yoon et al. (2005) developed initially curved piezoelectric unimorph beams with different dimensions under different forces in order to model the energy harvesting from walking. Susanto and Yang (2007) modeled a piezoelectric curved beam actuator for meso/micro grasping. Chen et al. (2008) fabricated a helical actuator designed in ANSYS. Magoteaux et al. (2008) compared the performance differences between UAV without harvester, UAV with curved beam harvester at landing gear, and

UAV with straight beam harvester at landing gear. Results showed that the curved beam harvesters have better power output in high frequencies. Zhou et al. (2010) developed spiral curved beam as a compact energy harvester by using ANSYS. Acoustic energy harvesting from piezoelectric curved beams in the cavity of sonic crystal was studied by Wang et al. (2010). A. Maha and S.-S. Pang (2012) studied on the shape recovery model of a curved beam using piezoelectric actuation. In biomedical area, Kadooka et al. (2015) used multiple connected curved beams in order to create artificial muscle. Yang et al. (2017) compared three energy harvester structures. Those are (i) curved beam in the shape of S connected to straight beam, (ii) C-shaped curved beam connected to straight beam, and (iii) full straight beam. They concluded that the S-shaped curved beam has superiority.

## **1.6. Aim of the Thesis**

In this study, modeling and analyses of curved beams with variable curvature and piezoelectric layer as smart material are studied by using a commercial finite element package that is ANSYS Mechanical. Modeling and solution procedure are coded using APDL (ANSYS Parametric Design Language) in ANSYS. Curved beam is modeled with SOLID45 elements while piezoelectric layer is represented by SOLID5 elements. After validation of the present model, the effects of applied forces to the tip displacement and generated electric field are observed by parametric studies.



## CHAPTER 2

### THEORETICAL VIBRATION ANALYSIS

#### 2.1. Introduction

In this chapter, the deformations of curved beams are summarized. Based on this section, the differential equations for in-plane bending of the curved beam without piezo patch are presented. Next, the differential equations of curved beam with piezo patch are discussed. Finally, finite element modeling of smart curved beam by using APDL (ANSYS Parametric Design Language) in ANSYS is outlined.

#### 2.2. Geometry of Curved Beams Axis

In order to study a curved beam with variable radius of curvature, an Archimedean spiral  $r=a\theta$  is selected for the neutral axis of the curved beam as shown in Figure 2.1.

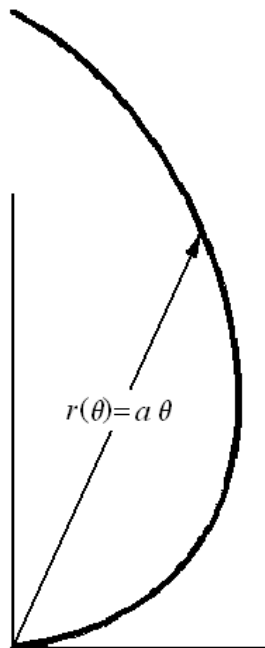


Figure 2.1. Curved beam with variable radius of curvature

### 2.3. Differential Equations for In-Plane Bending of the Curved Beam

The part of a curved beam with variable radius of curvature and its internal axial force  $T$ , internal shear force  $N$ , and bending moment  $M_y$  about axis  $y$  are shown in Figure 2.2. The coordinate system and equations are based on Love (1944).

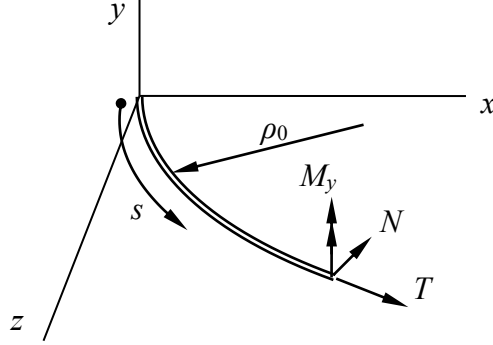


Figure 2.2. A curved beam with variable radius of curvature

The deformed curvature  $\kappa'_1$  in  $x$ - $z$  plane is written as

$$\kappa'_1 = \kappa'_0 + \frac{d}{ds} \left( \frac{du}{ds} + \kappa'_0 w \right) \quad (2.1)$$

where  $u$  and  $w$  are displacements of any point on curved beam in  $x$  and  $z$  directions, respectively. Also  $\kappa'_0$  is initial curvature. If a curve is represented by for example  $r=a\theta$ , then curvature is obtained by

$$\kappa'_0 = \frac{\left| r^2 + 2 \left( \frac{dr}{d\theta} \right)^2 - \frac{d^2 r}{d\theta^2} \right|}{\left[ r^2 + \left( \frac{dr}{d\theta} \right)^2 \right]^{3/2}} \quad (2.2)$$

The axial force  $T$  is expressed as

$$T = E A \varepsilon \quad (2.3)$$

where  $\varepsilon$  is strain in direction of  $s$  due to tension and given by

$$\varepsilon = \frac{dw}{ds} - \kappa'_0 u \quad (2.4)$$

The internal bending moment  $M_y$  is expressed as

$$M_y = B(\kappa'_1 - \kappa'_0) \quad (2.5)$$

where  $B$  is original notation used by Love (1944) and represents bending rigidity of curved beam material, which is known as  $EI$ .

Equilibrium equations of curved beam for in-plane deformations are written from Love (1944)

$$\frac{dN}{ds} + T\kappa'_1 + q_n = 0 \quad (2.6)$$

$$\frac{dT}{ds} - N\kappa'_1 + q_t = 0 \quad (2.7)$$

$$\frac{dM_y}{ds} + N + m_y = 0 \quad (2.8)$$

where  $q_n$ ,  $q_t$  are distributed force in normal and tangential directions. Also,  $m_y$  is the distributed moment about  $y$  axis.

The boundary conditions can be stated as follows:

$$\text{Either } M_y=0 \text{ (pinned or free), or } w''=0 \text{ (clamped)} \quad (2.9)$$

$$\text{Either } N=0 \text{ (free), or } w'=0 \text{ (pinned or clamped)} \quad (2.10)$$

$$\text{Either } M_y=0 \text{ (pinned or free), or } w=0 \text{ (pinned or clamped)} \quad (2.11)$$

## 2.4. Constitutive Equations of Piezoelectric Materials

This section is based on the textbook written by Preumont (2002). The notations of IEEE standard on piezoelectricity (1988) are used in this section. First, one dimensional dielectric medium is presented to discuss the constitutive equation of piezoelectric materials. Then, the three dimensional medium is outlined.

The relationship between the electric field  $E$  (V/m) and the electric displacement  $D$  (Coulomb/m<sup>2</sup>) for an unstressed dielectric medium is given as

$$D = \varepsilon E \quad (2.12)$$

where  $\varepsilon$  is the dielectric constant or dielectric permittivity (F/m or Coulomb/(Volt×m)) of the material. If elastic body has zero electric field, the relationship between the strain  $S$  (-) and the stress  $T$  (N/m<sup>2</sup>) is written as

$$S = sT \quad (2.13)$$

where  $s$  inverse of the Young's modulus of the material which is also called mechanical compliance of the material. Mathematically,  $s=1/c$ .

The constitutive equations of a piezoelectric material are related with the mechanical and electrical effects. They are expressed as

$$S = s^E T + d E \quad (2.14)$$

$$D = d T + \varepsilon^T E \quad (2.15)$$

In Equation (2.14), superscript  $E$  denotes that the electric field is constant. Also,  $d$  (m/V or Coulomb/Newton) is the piezoelectric strain coefficient. In Equation (2.15), superscript  $T$  denotes that the stress is constant. Equations (2.14) and (2.15) can be transformed into the following ones

$$T = \frac{1}{s^E} S - \frac{d}{s^E} E \quad (2.16)$$

$$D = \frac{d}{s^E} S + \varepsilon^T \left( 1 - \frac{d^2}{s^E \varepsilon^T} \right) E \quad (2.17)$$

Moreover, the last two equations usually expressed as

$$T = c^E S - e E \quad (2.18)$$

$$D = e S + \varepsilon^S E \quad (2.19)$$

where  $e$  (Coulomb/m<sup>2</sup>) relates the electric displacement with the strain for constant electric field or short-circuited electrodes and expresses as

$$e = d / s^E \quad (2.20)$$

and

$$\varepsilon^S = \varepsilon^T (1 - k^2) \quad (2.21)$$

in which superscript  $S$  indicates that the strain is constant and  $k^2 = d^2 / (s^E \varepsilon^T)$  is called the coupling coefficient of the piezoelectric material.

The constitutive equations of a three dimensional piezoelectric materials are given in matrix form from Equations (2.14) and (2.15) as follows:

$$\{S\} = [s]\{T\} + [d]\{E\} \quad (2.22)$$

$$\{D\} = [d]^T \{T\} + [\varepsilon]\{E\} \quad (2.23)$$

where superscript  $T$  is used to represent the transpose of the matrix. Equations (2.22) and (2.23) are known as actuation and sensing equations of which details are given below:

$$\{S\} = \begin{Bmatrix} S_{11} \\ S_{22} \\ S_{33} \\ 2S_{23} \\ 2S_{31} \\ 2S_{12} \end{Bmatrix} \quad (2.24)$$

$$\{T\} = \begin{Bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{31} \\ T_{12} \end{Bmatrix} \quad (2.25)$$

$$\{E\} = \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (2.26)$$

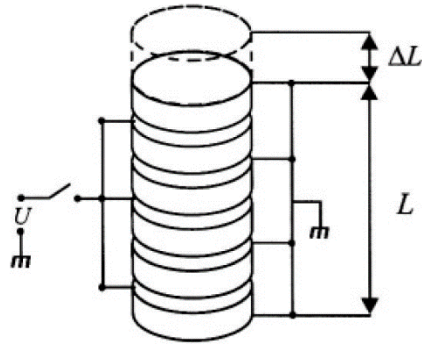
$$\{D\} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} \quad (2.27)$$

$$[s] = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{13} & s_{23} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \quad (2.28)$$

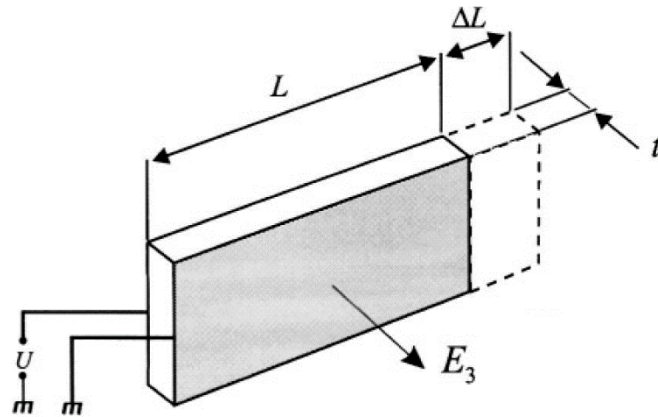
$$[d] = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.29)$$

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \quad (2.30)$$

The two examples of piezoelectric actuators are shown in Figure 2.3.



(a) Stacked design



(b) Laminar design

Figure 2.3. The two examples of piezoelectric actuators

The stacked design is related with  $d_{33}$ . The change of length due the voltage  $U$  applied as shown in Figure 2.3.a is given by

$$\Delta L = d_{33} nU \quad (2.31)$$

where  $n$  is the number of disks in the stack and  $U$  is the applied voltage.

The laminar design is related with  $d_{31}$ . The change of length due the voltage  $U$  applied as shown in Figure 2.3.b is given by

$$\Delta L = E_3 L d_{31} \quad (2.32)$$

where

$$E_3 = \frac{U}{t} \quad (2.33)$$

in which  $U$  is the applied voltage and  $t$  is the thickness of the piezoelectric layer.

## 2.5. Differential Equations for Smart Curved Beam

The differential equations for in-plane bending of the curved beam with variable curvature are given in Section 2.3 for the applied distributed forces  $q_n, q_t$  and distributed moment  $m_y$ . If a thin piezoelectric patch is mounted to lateral surface of the curved beam as shown in Figure 2.4 by light gray area, it causes distributed forces in tangential direction which also generates bending moment along the  $y$  axis.

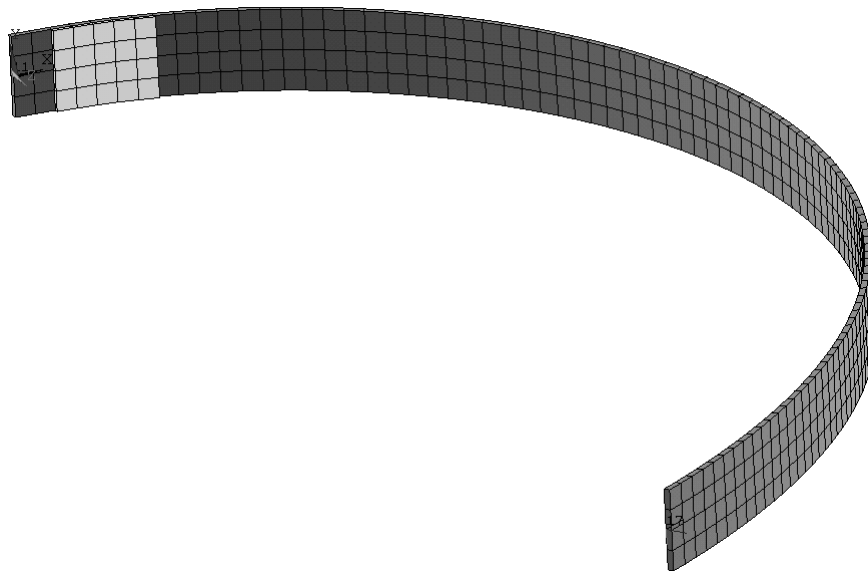


Figure 2.4. Curved beam with piezoelectric patch



Thus, distributed bending moments  $m_y$  can be treated in two parts: an externally applied distributed moment denoted by  $m_{ye}$  and a distributed moment applied by piezoelectric patch denoted as  $m_{ye}$ . The second part can be expressed by using the concentrated moment provided by the textbook written by Preumont (2002) as

$$m_{ye} = E_p d_{13} b_p h U / L_p \quad (2.34)$$

where  $E_p$ ,  $b_p$  and  $L_p$  are the Young's modulus, the width and length of the piezoelectric patch.  $h$  is the moment arm between the centroidal distance of cross-section of curved beam and piezo patch and  $U$  is the applied voltage, respectively.

## 2.6. Finite Element Modeling of Smart Curved Beam

Equations (2.16) and (2.17) can be written for multidimensional case as follows for finite element formulation (Allik and Hughes, 1970):

$$\{T\} = [c]\{S\} - [e]\{E\} \quad (2.35)$$

$$\{D\} = [e]^T \{S\} + [\varepsilon]\{E\} \quad (2.36)$$

where  $\{T\}$  is stress,  $\{S\}$  is mechanical strain,  $\{E\}$  is electric field,  $\{D\}$  is electric flux density vectors. Also,  $[c]$ ,  $[e]$ , and  $[\varepsilon]$  denotes the elastic stiffness matrix evaluated at constant electric field, the piezoelectric matrix and the dielectric matrix evaluated at constant mechanical strain, respectively.

The finite element discretization is accomplished by expressing the field variables in terms of nodal freedoms vector and element shape functions. Therefore, if the nodal displacements vector is denoted by  $\{u\}$ , the displacements vector  $\{u_c\}$  which has the displacements within element in the x, y, z directions is written as (ANSYS, 2004)

$$\{u_c\} = [N^u]^T \{u\} \quad (2.37)$$

where  $[N^u]$  is shape function matrix for displacements and given by

$$[N^u] = \begin{bmatrix} N_1 & 0 & 0 & \Lambda & N_n & 0 & 0 \\ 0 & N_1 & 0 & \Lambda & 0 & N_n & 0 \\ 0 & 0 & N_1 & \Lambda & 0 & 0 & N_n \end{bmatrix} \quad (2.38)$$

where the subscript  $n$  represents the node number in the finite element. It is useful to express the nodal displacements vector  $\{u\}$  in open form

$$\{u\} = \{UX_1 \quad UY_1 \quad UZ_1 \quad \Lambda \quad UX_n \quad UY_n \quad UZ_n\}^T \quad (2.39)$$

Similarly, if the nodal electrical potential vector is denoted by  $\{V\}$ , the electrical potential  $V_c$  within element is written as (ANSYS, 2004)

$$V_c = \{N^V\}^T \{V\} \quad (2.40)$$

where  $\{N^V\}$  is shape function vector of electrical potential shape function and given by

$$\{N^V\}^T = (N_1 \quad N_2 \quad \Lambda \quad N_n) \quad (2.41)$$

in which  $N_i$  is the shape function for node  $i$ . Again, it is useful to express the nodal electrical potential vector  $\{V\}$  in open form as

$$\{V\} = \{V_1 \quad V_2 \quad \Lambda \quad V_n\}^T \quad (2.42)$$

The strain vector  $\{S\}$  is related to the displacement vector  $\{u\}$  as

$$\{S\} = [B_u] \{u\} \quad (2.43)$$

where  $[B_u]$  is written as

$$[B_u] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \quad (2.44)$$

On the other hand, the electric field vector  $\{E\}$  is related to the electrical potential vector  $\{V\}$  as

$$\{E\} = -[B_V]\{V\} \quad (2.45)$$

where  $[B_V]$  is written as

$$[B_V] = \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{Bmatrix} \{N^V\}^T \quad (2.46)$$

The coupled finite element matrix equation is obtained using the Equations (2.37) to (2.46) in the variational principle available in the paper of Allik and Hughes (1970) and expressed for static problem by using the notation in ANSYS (2004) as

$$\begin{bmatrix} [K] & [K^z] \\ [K^z]^T & [K^d] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{V\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{L\} \end{Bmatrix} \quad (2.47)$$

where  $[K]$ ,  $[K^d]$ , and  $[K^z]$  are structural stiffness, dielectric conductivity, and piezoelectric coupling matrices, respectively.  $\{F\}$  and  $\{L\}$  are nodal force and nodal charge vectors, respectively. They are given in ANSYS (2004) as

$$[K] = \int_{vol} [B_u]^T [c] [B_u] d(vol) \quad (2.48)$$

$$[K^d] = \int_{vol} [B_v]^T [\varepsilon] [B_v] d(vol) \quad (2.49)$$

$$[K^z] = \int_{vol} [B_u]^T [e] [B_v] d(vol) \quad (2.50)$$

After finding the nodal displacement and electric potential for an element, stresses and electric flux density at any point within the finite element can be obtained by substituting Equations (2.43) and (2.45) into Equations (2.35) and (2.36) as

$$\{T\} = [c][B_u]\{u\} + [e][B_v]\{V\} \quad (2.51)$$

$$\{D\} = [e]^T [B_u]\{u\} - [\varepsilon][B_v]\{V\} \quad (2.52)$$

The finite element model of the smart curved beam with variable curvature is generated in ANSYS by using SOLID45 and SOLID5 for beam and piezoelectric patch, respectively. A computer code in ANSYS is developed by using APDL (ANSYS Parametric Design Language).

## CHAPTER 3

### NUMERICAL RESULTS AND DISCUSSION

#### 3.1. Validation of Model

The axis of the curved beam with the equation  $r=4\theta$ , ( $0^\circ < \theta < 90^\circ$ ) is generated in ANSYS by using a spline that fits to a series of 50 keypoints. The length of the axis is  $s=476.5$  mm. Cross-section of the beam with  $b=20$  mm and  $h=1.5$  mm is dragged along the spline in order to form the volume. Fixed-free boundary conditions are used. Several numbers of elements to mesh the curved beam are used to have the theoretical tip displacement due to the load acting on tip and then element size in axial, tangential and radial directions are decided as 5 mm, 4.9635 mm, and 1.5 mm, respectively. Details are presented in this section. The meshed curved beam is shown in Figure 3.1.

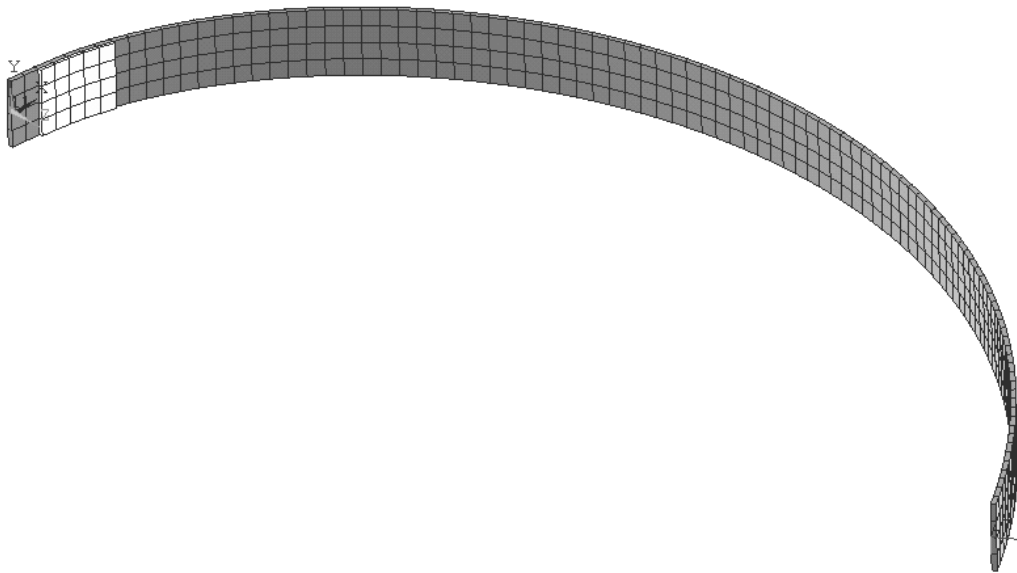


Figure 3.1 The curved beam and smart patch as meshed

The piezoelectric layer with  $b_p=20$  mm,  $s_p=25$  mm, and  $t_p=1$  mm is bonded to concave surface of the curved beam at the distance  $d_p=10$  mm from the root. Its mesh is shown in Figure 3.1 by light grey color.

The material properties of curved beam and piezoelectric layer given in Tables 3.1 and 3.2, respectively, are taken from Malgaca (2007).

Table 3.1. Material Properties of curved beam

Material	Aluminum
Young modulus (N/m <sup>2</sup> )	62 10 <sup>9</sup>
Density (kg/m <sup>3</sup> )	2676
Poisson's ratio	0.32

Table 3.2. Material Properties of piezoelectric layer

Material	BM532 (PZT-5H)
$c_{11}$ (N/m <sup>2</sup> )	12.6 10 <sup>10</sup>
$c_{12}$ (N/m <sup>2</sup> )	7.95 10 <sup>10</sup>
$c_{13}$ (N/m <sup>2</sup> )	8.41 10 <sup>10</sup>
$c_{33}$ (N/m <sup>2</sup> )	11.7 10 <sup>10</sup>
$c_{44}$ (N/m <sup>2</sup> )	2.33 10 <sup>10</sup>
$e_{31}$ (C/m <sup>2</sup> )	-6.5
$e_{33}$ (C/m <sup>2</sup> )	23.3
$e_{25}$ (C/m <sup>2</sup> )	17
$\epsilon_{11}$ (F/m)	1.503 10 <sup>-8</sup>
$\epsilon_{22}$ (F/m)	1.503 10 <sup>-8</sup>
$\epsilon_{33}$ (F/m)	1.3 10 <sup>-8</sup>

The material data given in Table 3.1 and 3.2 are coded in APDL by using the following lines:

```

mp,ex,1,62e9           ! Young modulus of aluminum
mp,dens,1,2676        ! Density of aluminum
mp,nuxy,1,0.32       ! Poisson's ratio of aluminum

```

```

mp,dens,2,7350          ! Density of PZT
mp,perx,2,15.03e-9     ! epsilon11
mp,pery,2,15.03e-9     ! epsilon22
mp,perz,2,13e-9        ! epsilon33
tb,piez,2              ! Table of e for PZT
tbdata,3,-6.5          ! e31
tbdata,6,-6.5          ! e32
tbdata,9,23.3          ! e33
tbdata,14,17           ! e25
tbdata,16,17           ! e16
tb,anel,2              ! Table of c for PZT
tbdata,1,126e9,79.5e9,84.1e9 ! c11, c12, c13
tbdata,7,126e9,84.1e9 ! c22, c23
tbdata,12,117e9        ! c33
tbdata,16,23.25e9      ! c44
tbdata,19,23e9         ! c55
tbdata,21,23e9         ! c66

```

In order to present the correctness and numerical accuracy of the finite element model of the beam without piezoelectric layer due to mesh size selection, an equivalent straight cantilever beam having the same length and cross-section is loaded from its tip by 0.5 N. Then, the tip displacement of the cantilever beam is found from analytical method and finite element model. The results are given in Table 3.3.

Table 3.3. Tip displacements of cantilever beam (m)

Analytical	0.0517
Numerical by FEM	0.0514

The difference between analytical and numeric result is 0.580271%. Thus, it can be concluded from Table 3.3 that the present finite element model gives satisfactory result based on the mesh distribution used to model the beam.

The analytical solution for the curved beam with piezoelectric layer in the selected geometry is not available in the existing literature. Therefore, the piezoelectric layer in the form of thin rectangular prism with  $l_{xp}=25$  mm,  $l_{yp}=20$  mm, and  $t_p=1$  mm is fixed left hand and loaded from the right hand as shown in Figure 3.2 in order to validate finite element model of the laminar form of piezoelectric layer. Although, the piezoelectric layer is bonded concave side of curved beam by adopting its curvature, the curvature of the piezoelectric layer is neglected in this validation to compare the numerical results with analytical results obtained from Equation (2.22).

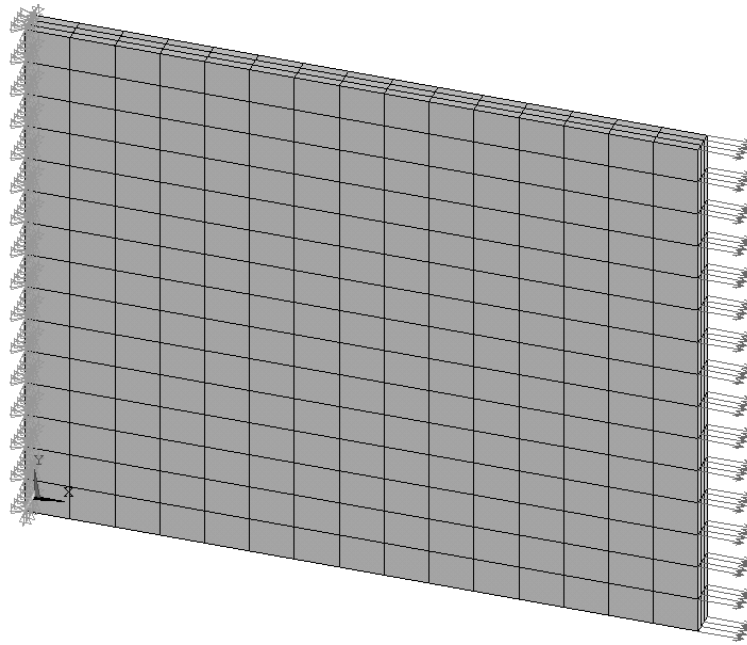


Figure 3.2. The finite element model of piezoelectric layer

Analytical results are calculated by using  $E_3=-2 \cdot 10^5$  Volt/m and  $T_1=25 \cdot 10^5$  N/m<sup>2</sup>. Other data for PZT-5H are  $s_{11}^E=16.5 \cdot 10^{-12}$  m<sup>2</sup>/N and  $d_{13}=-274 \cdot 10^{-12}$  C/N. Thus,

- Strain in  $x$  direction due to stress acting in  $x$  direction is equal to  $s_{11}^E T_1=0.413 \cdot 10^{-4}$ .
- Strain in  $x$  direction due to electric field in  $z$  direction is equal to  $d_{11}^E E_3=0.548 \cdot 10^{-4}$ .
- Strain in  $x$  direction due to stress acting in  $x$  direction and electric field applied in  $z$  direction is equal to  $S_1=s_{11}^E T_1+ d_{11}^E E_3=0.961 \cdot 10^{-4}$ .

Finite element model results are given in Figures 3.3-3.5 in the same order of the analytical results presented above.



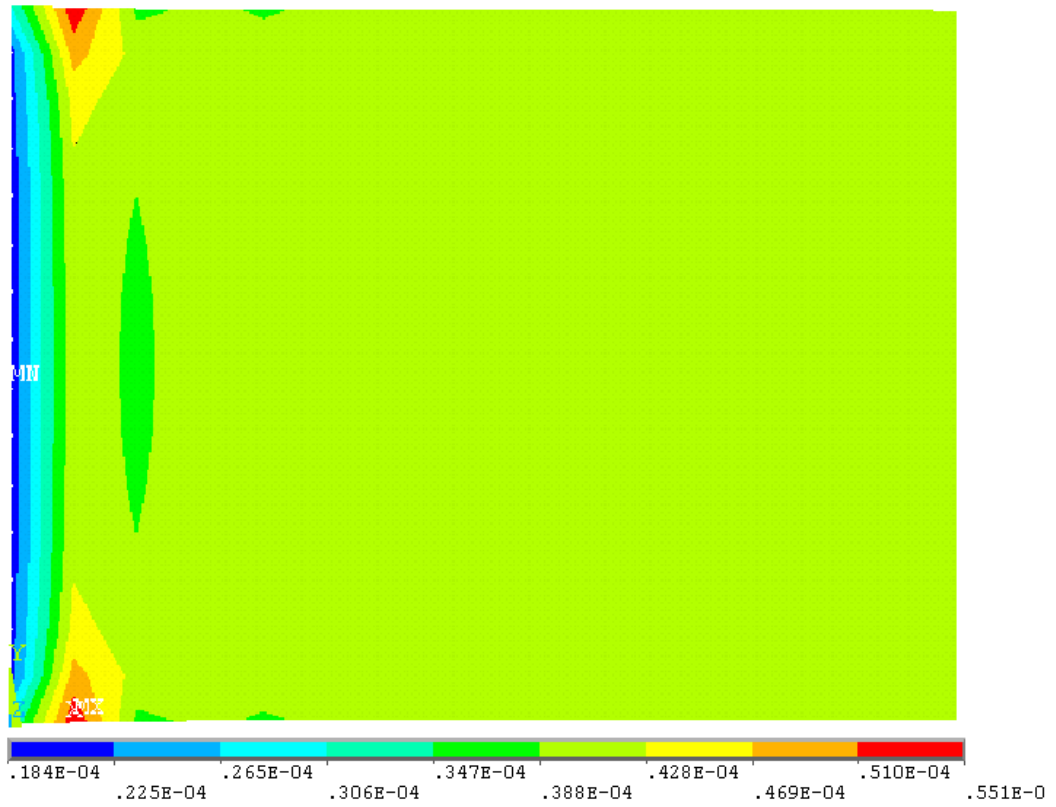


Figure 3.3. Strain in x direction due to force applied in x direction

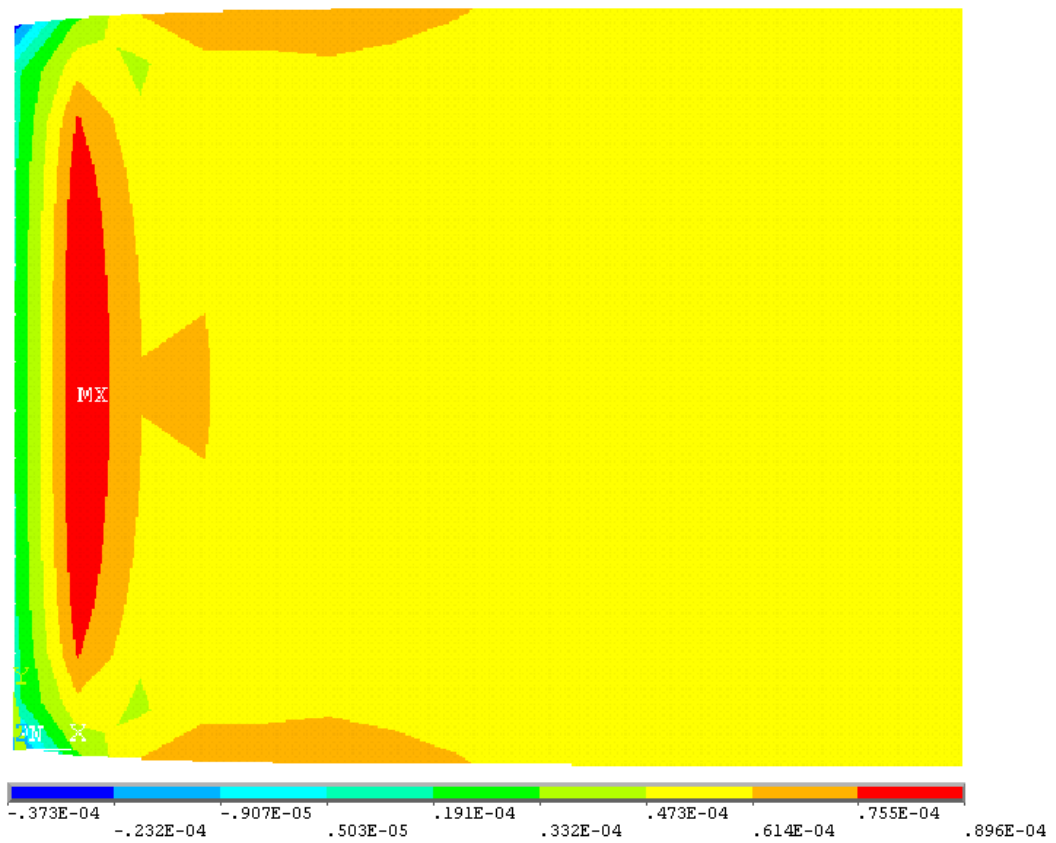


Figure 3.4. Strain in x direction due to voltage applied in z direction

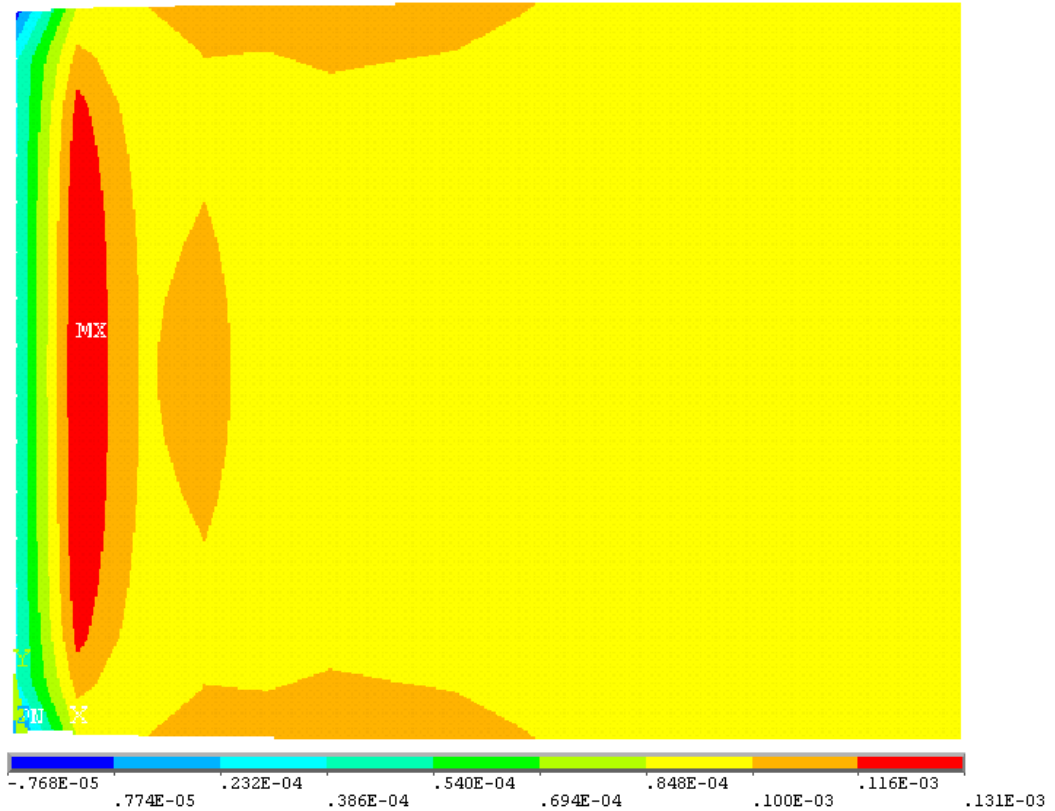


Figure 3.5. Strain in  $x$  direction due force and voltage

When the Figures 3.3-3.5 are examined, the following validations can be said:

- Strain in  $x$  direction due to stress acting in  $x$  direction is generally shown by the color of which scale is the fourth part from the right. It has an interval  $[0.388E-04, 0.428E-04]$ . The analytical result  $0.413 \cdot 10^{-4}$  is in this interval.
- Strain in  $x$  direction due to electric field in  $z$  direction is generally shown by the color of which scale is the third part from the right. It has an interval  $[0.473E-04, 0.614E-04]$ . The analytical result  $0.548 \cdot 10^{-4}$  is in this interval.
- Strain in  $x$  direction due to stress acting in  $x$  direction and electric field applied in  $z$  direction is generally shown by the color of which scale is the third part from the right. It has an interval  $[0.848E-04, 1.000E-03]$ . The analytical result  $0.961 \cdot 10^{-4}$  is in this interval.

Therefore, the finite element model of piezoelectric layer is proven.

### 3.2. Analysis with Present Model

The curved beam model described Section 3.1 illustrates an electromechanical model of curved beam with piezoelectric layer bonded to concave side of the curved beam as a patch. The voltage generation of the bonded piezoelectric layer due to the load acting on tip of the curved beam is the main topic in this study. The tip load is selected as 0.1 N and 0.5 N in the analysis.

In order to see the in-plane displacements of the curved beam under the tip load acting in plane of the neutral axis of the curved beam, deformed and undeformed states of the respective system are illustrated in Figure 3.6 and Figure 3.11 for tip load 0.1 N and 0.5 N, respectively. Displacements of the tip of the curved beam in  $x$  direction can be read from aforementioned figures as 0.00562 mm and 0.028101 mm for tip load 0.1 N and 0.5 N, respectively. Results indicate that displacements are linearly proportional to tip load as expected from the linear elasticity.

Electrical fields generated by strains in piezoelectric layer due to the tip load 0.1 N are shown for entire curved beam in Figure 3.7 and for PZT layer in Figure 3.8. Also, Figure 3.12 and Figure 3.13 are counterparts of Figures 3.7 and Figure 3.8 for tip load 0.5 N. Maximum values of electrical field in Figure 3.8 and Figure 3.9 are 10354 V/m and 51771 V/m, respectively. They show that electrical field linearly proportional to tip load again as expected from the theory.

Strains of PZT layer in  $x$  direction due to tip load 0.1 N and 0.5 N are shown in Figure 3.9 and Figure 3.14, respectively. The reason for selecting direction of  $x$  only is the 31 operating mode of piezo patch. Maximum strain values in PZT layer in  $x$  direction which is related with operating mode 31 from related figures are found as follows: 0.223E-04 for 0.1 N, 0.111E-03 for 0.5 N. The numerical values are proportional to each other.

Moreover, in order to check the stress levels of the curved beam due to the tip load, stress contours in root region of curved beam and PZT layer are presented in Figure 3.10 and Figure 3.15 for tip load 0.1 N and 0.5 N, respectively. Maximum stress values in PZT layer in  $x$  direction as follows: 0.169E+07 Pa for 0.1 N, 0.845E+07 Pa for 0.5 N. The numerical values are proportional to each other.

The mentioned values can be seen from the figures below, where the numbers below the figures increases from left to right.

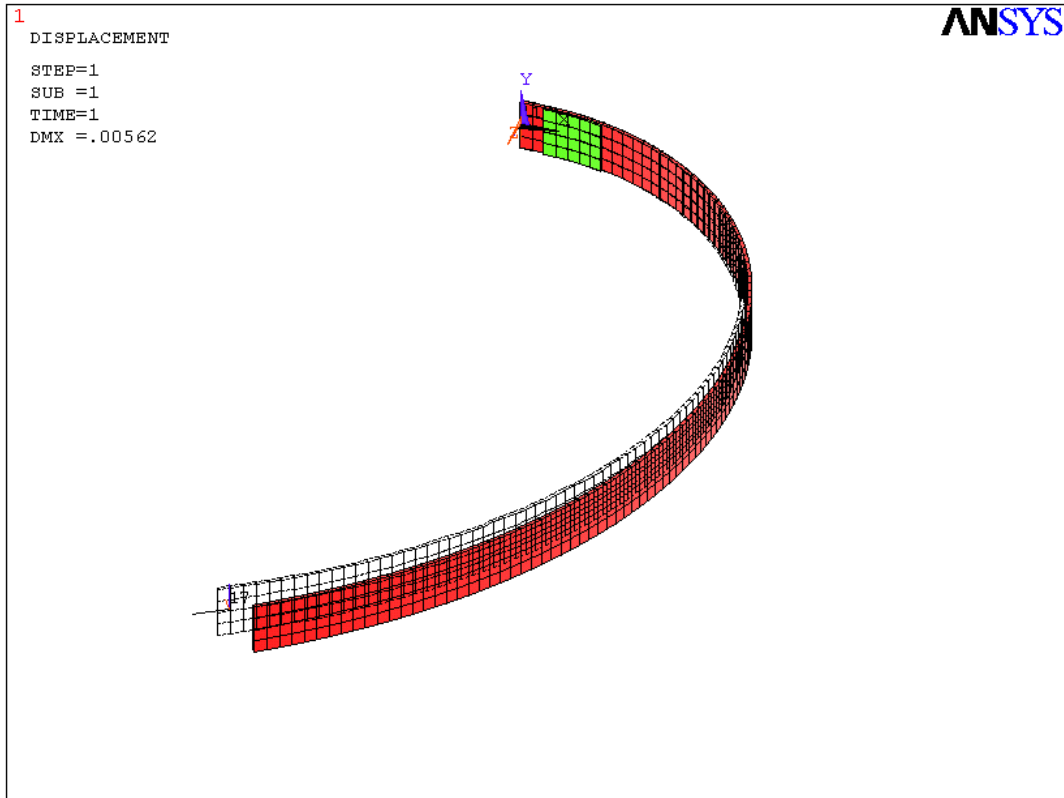


Figure 3.6. Deformed and undeformed states of curved beam under tip load 0.1 N

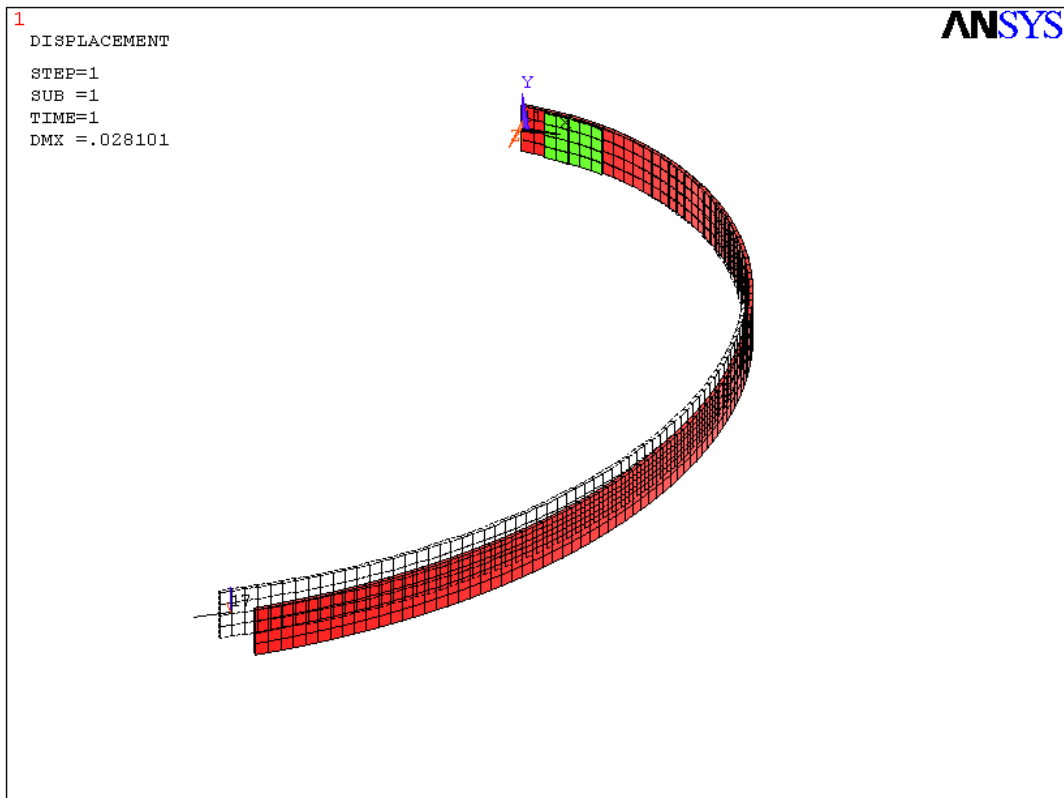


Figure 3.7. Deformed and undeformed states of curved beam under tip load 0.5 N

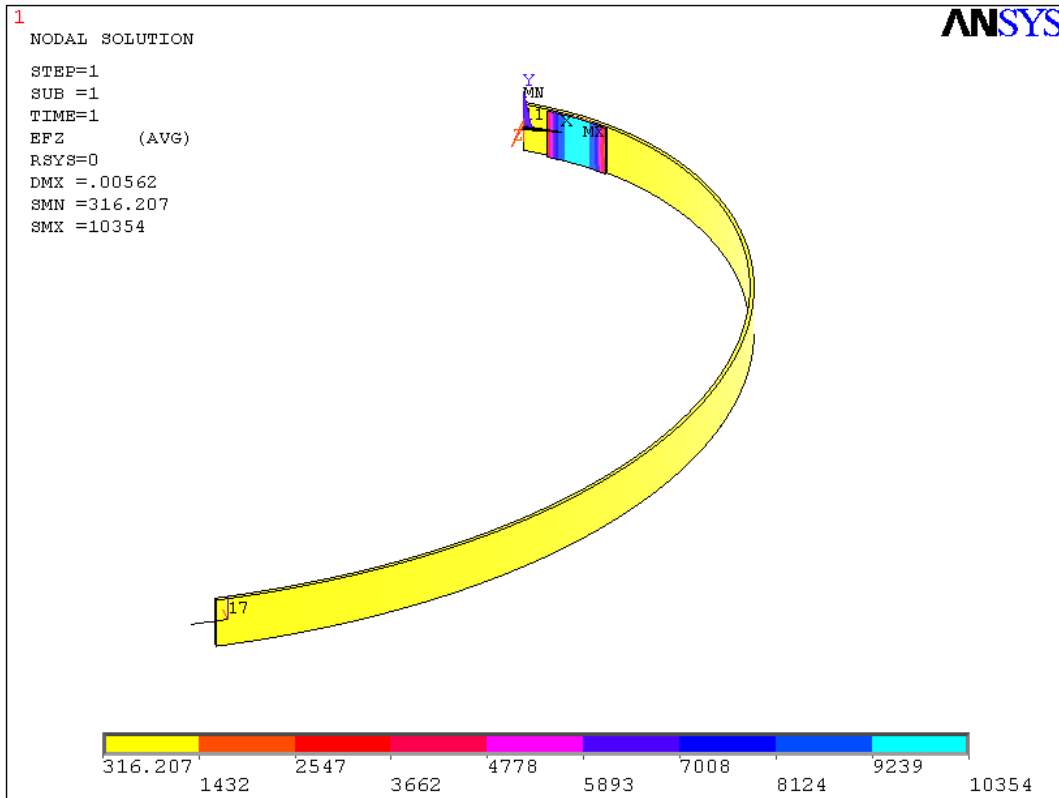


Figure 3.8. Electric field of smart curved beam in z direction due to tip load 0.1 N

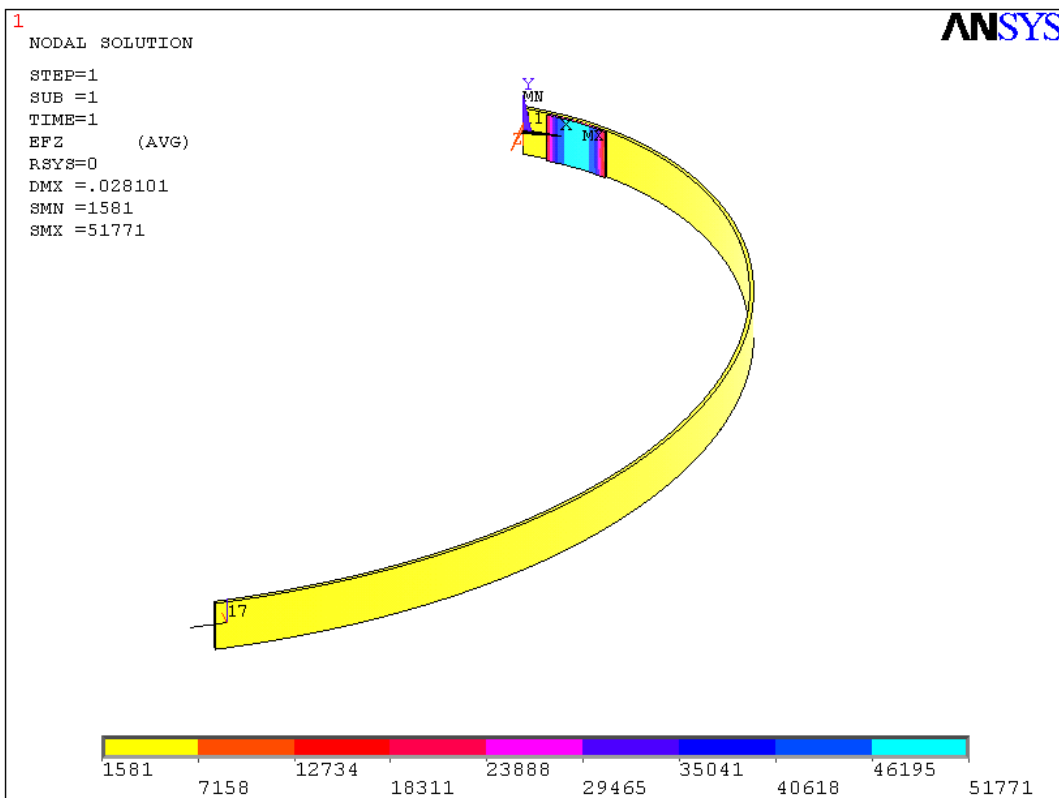


Figure 3.9. Electric field of smart curved beam in z direction due to tip load 0.5 N

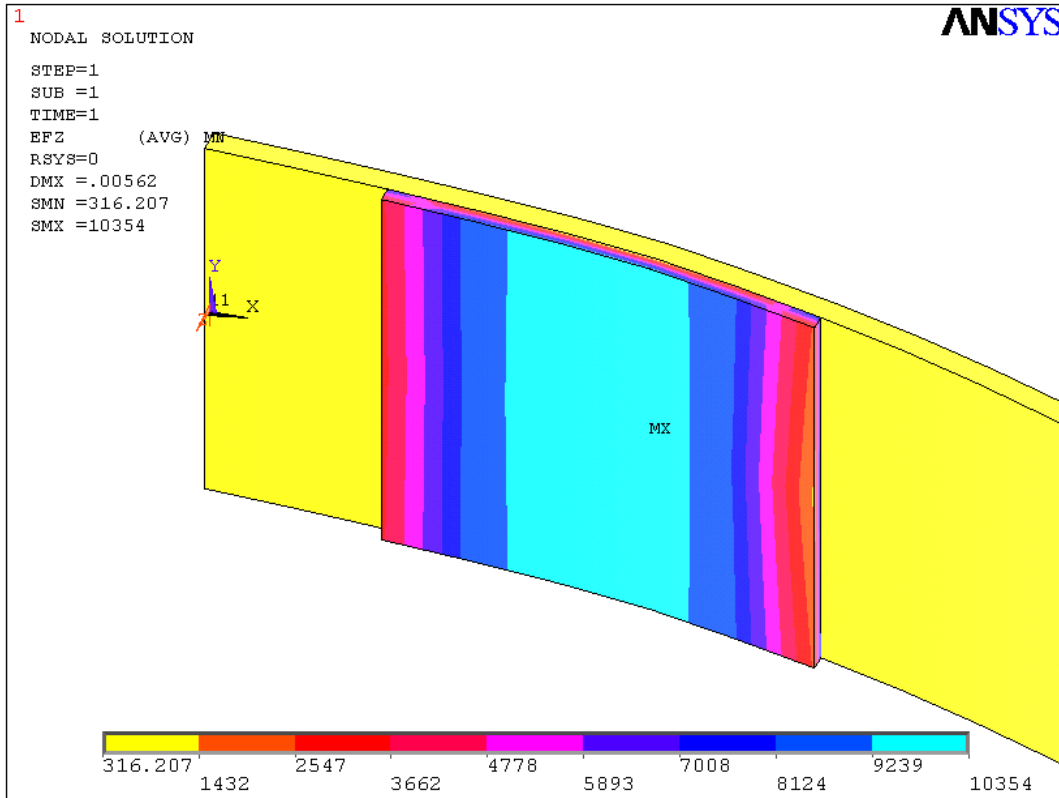


Figure 3.10. Electric field of PZT layer in z direction due to tip load 0.1 N

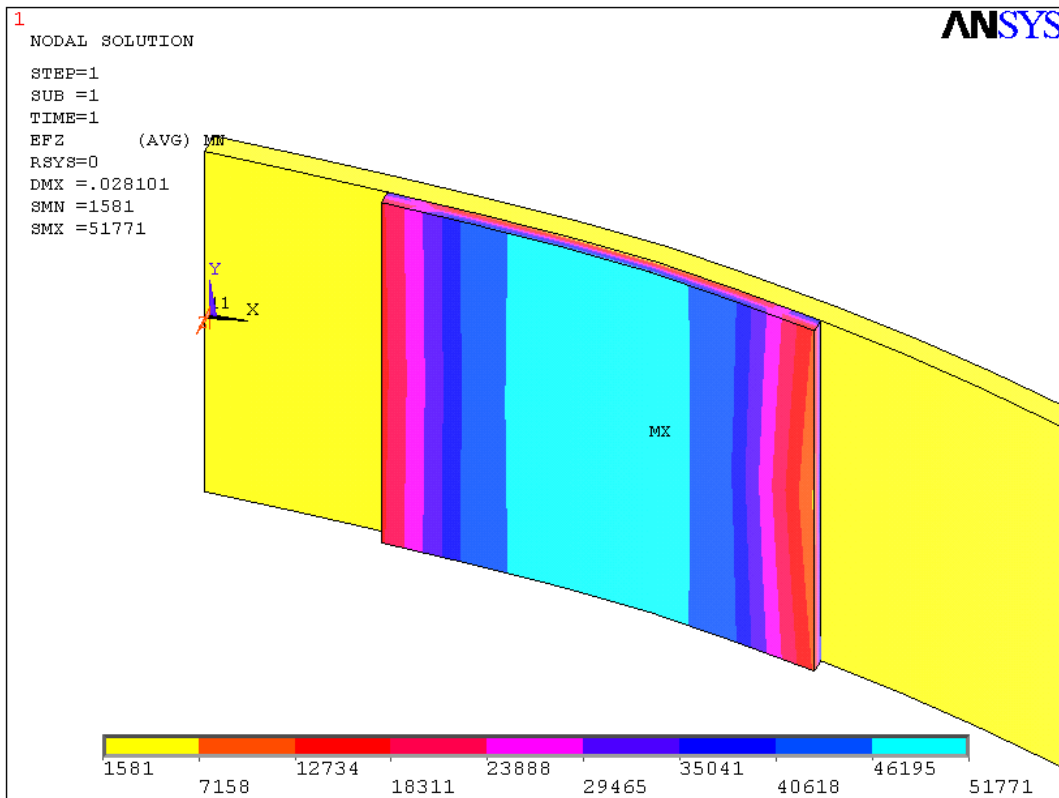


Figure 3.11. Electric field of PZT layer in z direction due to tip load 0.5 N

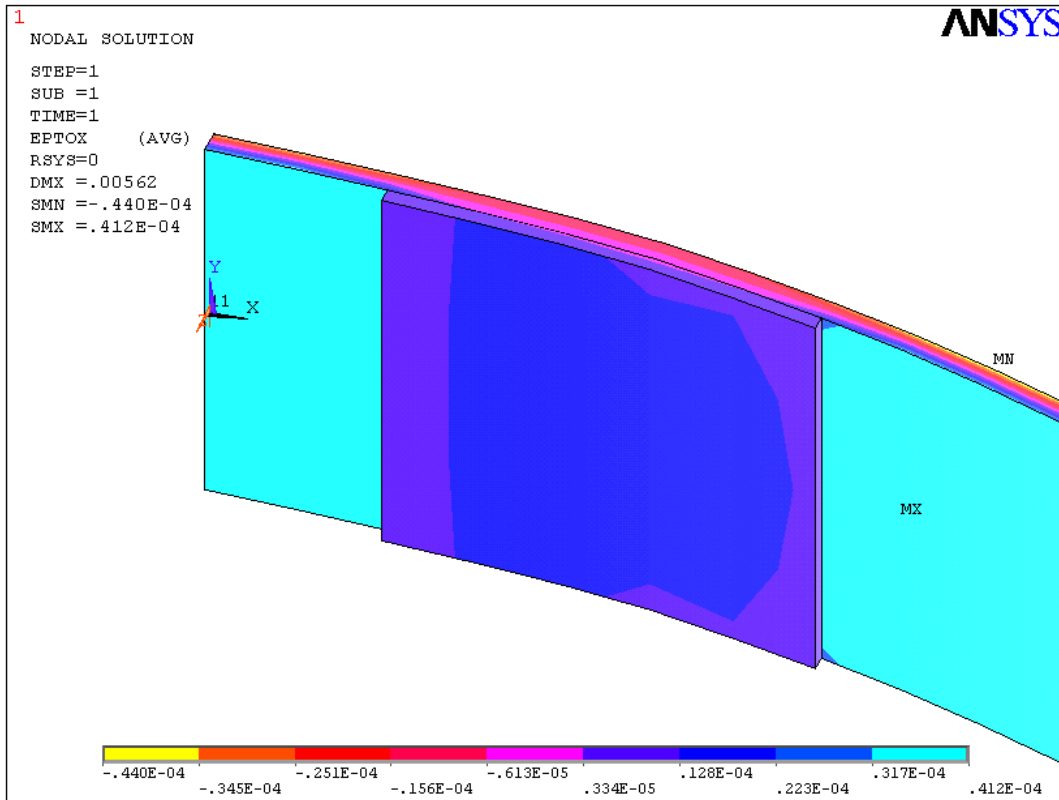


Figure 3.12. Strains of PZT layer in  $x$  direction due to tip load 0.1 N

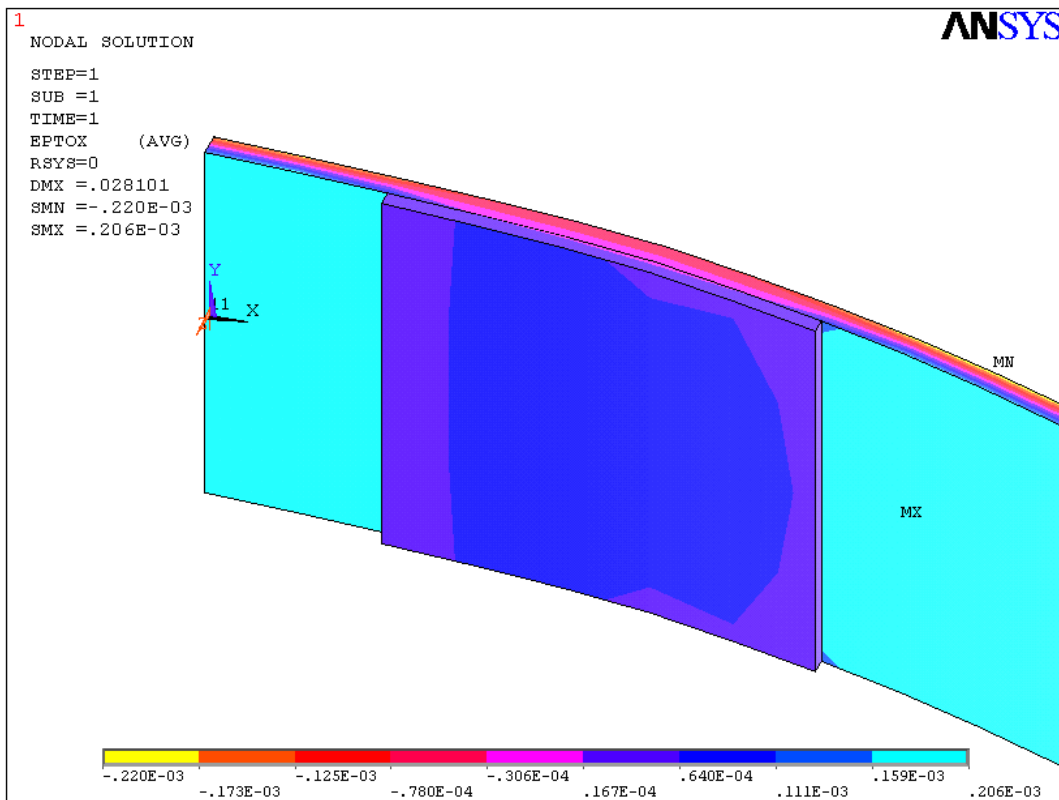


Figure 3.13. Strains of PZT layer in  $x$  direction due to tip load 0.5 N

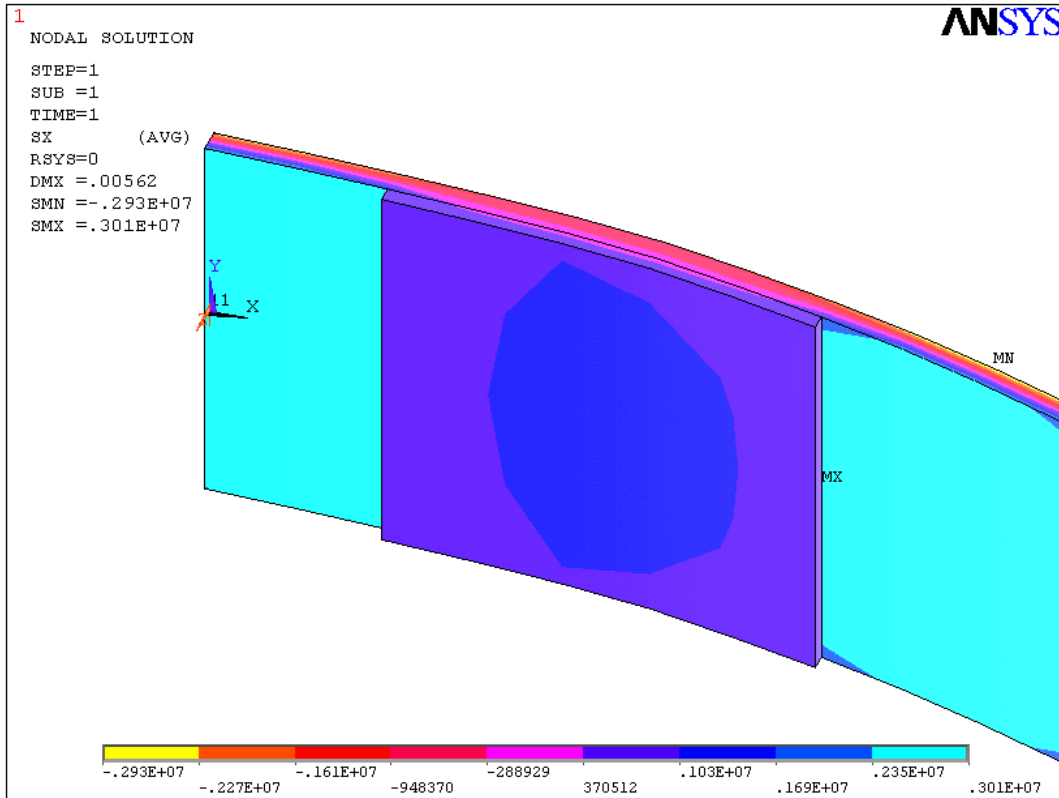


Figure 3.14. Stresses of PZT layer in x direction due to tip load 0.1 N

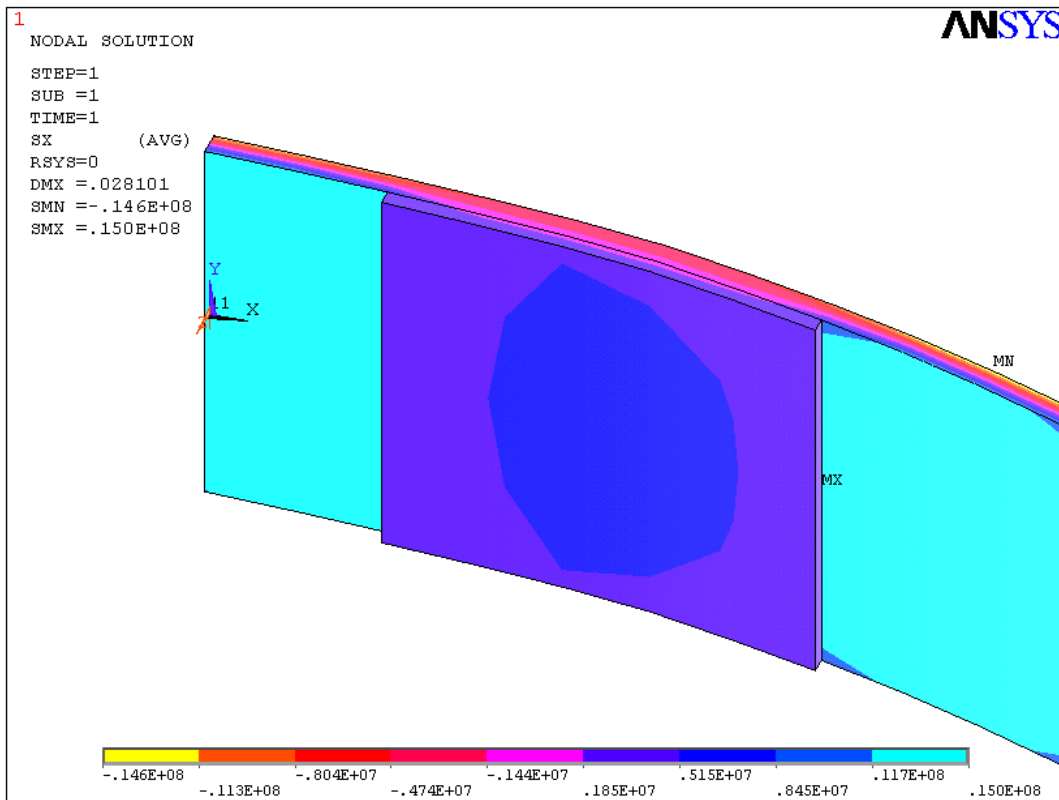


Figure 3.15. Stresses of PZT layer in x direction due to tip load 0.5 N



### 3.3. Discussion of Numerical Results

Electromechanical behavior of the curved beam with PZT layer is analyzed in Section 3.2. Results that were gathered with respect to the different tip load are given in Table 3.4.

Table 3.4. Results comparison for different loadings

	Tip load (N)	
	0.1	0.5
Tip displacement in $x$ direction (mm)	0.00562	0.028101
Maximum electric field in $z$ direction (V/m)	10354	51771
Maximum strain of PZT layer in $x$ direction	2.23E-05	1.11E-04
Maximum stress of PZT layer in $x$ direction (Pa)	1.69E+06	8.45E+06

The following statements can be said:

- The highest electric fields which are first part from the right, strains which are third part from the right and stresses which are third part from the right occurred at the middle of piezoelectric patches for both cases. Furthermore, the resultant patterns on the figures did not changed when the load was changed.
- The in-plane displacements of the curved beam under the tip load acting in plane of the neutral axis of the curved beam is obtained linearly proportional to tip load as expected from the linear elasticity.
- Electrical field generated by strains in piezoelectric layer due to the tip load is linearly proportional to tip load again as expected from the theory
- Strains of PZT layer in  $x$  direction due to tip load are proportional to loading.
- Stress levels in root region of curved beam and PZT layer are proportional to the loading.

## **CHAPTER 4**

### **CONCLUSIONS**

In this study, electromechanical behavior of piezoelectric smart curved beam with variable radius of curvature is investigated using the Finite Element Method.

The voltage generation due to the tip load applied to curved beam can be estimated by using strain values of piezoelectric patches calculated from piezoelectric constitutive equations and mechanics of materials. However, for a curved beam with variable curvature, estimation of the voltage generated by piezoelectric patch is not possible as a circular curved beam. Therefore, a numerical study is necessary to analyze the smart curved beam with variable curvature.

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