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A New Robust Controller Formulation for the Full–State Feedback Position Tracking of a Small–Scaled Unmanned Model Helicopter

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Abstract: This work focuses on the robust attitude tracking control problem for a small–scaled unmanned helicopter where the actual system inputs, namely the elevator servo input, the aileron servo input and the rudder servo input, are used in the controller formulation. The design process is divided into two parts. Initially the problem is transformed into a second order system with an uncertain non–symmetric input gain matrix by utilizing some reasonable simplifications for the rotor model under the hovering flight conditions. Then a novel robust control methodology is utilized via a matrix decomposition method. The stability of the overall system is ensured by Lyapunov type analysis where asymptotic position tracking is ensured. Numerical simulation results are presented to demonstrate the efficiency of the proposed method.

Keywords: Robust control, Nonlinear systems, Small–scaled Unmanned Model Helicopter.

1. INTRODUCTION

Helicopters are versatile aerial vehicles that can perform hover and vertical take–off and landing maneuvers. Due to the aforementioned versatility, helicopters are useful for both military and civilian applications, however the corresponding flight dynamics are highly nonlinear and contains uncertainties associated with the dynamical terms. In addition to these, strong coupling effects and natural instability of their system dynamics make the controller design problem a challenging task. In general, the control system of the unmanned helicopters can be divided into two parts; namely as inner–loop level control and the outer–loop level control. These parts are related with attitude and position control, respectively. Since the position tracking can be ensured via inner–loop control, designing a controller for the attitude control of helicopters is considered as the main control objective in this study.

Some examples from the previous work on attitude control of helicopter systems in literature are given as follows; in (Sakamato et al., 2006), Sakamato et al. designed a PID controller for the linearized dynamics of the attitude control of the helicopter while the other similar approaches were realized with LQR control, output regulation and feedback linearization in (Liu et al., 2013), (Nao et al., 2003) and (Kagawa et al., 2005), respectively. Moreover, H_∞ control (Gadewadikar et al., 2008), (Kato et al., 2003) and sliding mode control (Xian et al., 2015) are other approaches for attitude control designs of linearized dynamics. In (Suzuki et al., 2011), an adaptive attitude controller for a small unmanned helicopter is designed by using quartenion feedback provided by the backstepping control method. In (Tee et al., 2008), a robust adaptive neural network controller was presented for helicopters in vertical flight, with dynamics in single–input–single–output nonlinear nonaffine form. Because of highly uncertain and nonlinear dynamics of helicopters robust control design is another popular approach for the attitude control of helicopters. In (Shin et al., 2010), a position tracking control system was developed for a rotorcraft–based unmanned aerial vehicle using robust integral of the signum of the error feedback and neural network feedforward terms. In (Liu et al., 2014), a nonlinear robust attitude tracking control scheme was developed for a small–scaled unmanned helicopter under input constraints. In addition to these, neural network based control (Shin et al., 2012) and fuzzy control (Kadmiry and Driankov, 2004) are other approaches that were used for attitude control of helicopters.

In this work, the design and the corresponding stability analysis for a novel robust controller for the attitude tracking control scheme for a small–scaled unmanned helicopter has been presented. Our design is based on the actual inputs namely the elevator servo input, the aileron servo input and the rudder servo input. To express the effects of these inputs on the system dynamics, rigid body and the rotor dynamics are combined in the mathematical model of the helicopter (Fantoni and Lozano, 2002). In the men-

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tioned mathematical model, input torque is expressed as a function of actual inputs and the vector of actual inputs is premultiplied with a non–symmetric input gain matrix to obtain this expression. Designing a controller that is based on actual inputs is a realistic approach and it can be seen as a necessity for the applicability of the designed controller. However the symmetric nature of the input gain matrix is a critical case for the Lyapunov–based control designs. Realizing a robust control design that can provide the attitude control of a small–scaled unmanned helicopter by taking into account the non–symmetric nature of the input gain matrix is one of the most important aspects of this study. The stability of the closed–loop error dynamics of the designed controller is proven via Lyapunov–based stability analysis. The performance of the designed controller is then demonstrated via numerical simulations.

2. HELICOPTER MODEL

The dynamic model of a helicopter has the following form(Fantoni and Lozano, 2002)

$$M_h(\eta)\ddot{\eta} + C_h(\eta,\dot{\eta})\dot{\eta} + G_h(\eta) + f_d(t) = \tau \tag{1}$$

where $\eta = [\,\phi\ \theta\ \psi\,]^T$ with $\phi(t)$, $\theta(t)$ and $\psi(t) \in \mathbb{R}$ being the yaw, roll and pitch angles, is the position vector and $\dot{\eta}(t)$, $\ddot{\eta}(t) \in \mathbb{R}^3$ are the first and second order time derivatives of the position vector, respectively. $M_h(\eta) \in \mathbb{R}^{3\times3}$ is the inertia matrix, $C_h(\eta,\dot{\eta}) \in \mathbb{R}^{3\times3}$ is the matrix containing the Coriolis–centrifugal forces and $G_h(\eta) \in \mathbb{R}^3$ is the vector of conservative forces. To represent the unknown external disturbances that may be effective on the helicopter, the term $f_d(t) \in \mathbb{R}^3$ is added to the dynamic model. The torque input vector is represented by $\tau(t) \in \mathbb{R}^3$. Also note that the inertia matrix denoted by $M_h(\eta)$ is symmetric and positive definite, and satisfies the following inequalities (Liu et al., 2014)

$$\underline{m}\,\|\xi\|^2 \le \xi^T M_h(\eta)\,\xi \le \bar{m}\,\|\xi\|^2\ \forall \xi \in \mathbb{R}^3 \tag{2}$$

where $\underline{m}$ and $\bar{m} \in \mathbb{R}$ are positive bounding constants. The torque input $\tau(t)$ is expressed as (Mettler, 2003), (Cai et al., 2011)

$$\tau = S_h^{-T}(A\upsilon_c + B) \tag{3}$$

where $S_h(\eta) \in \mathbb{R}^{3\times3}$ denotes the velocity transformation matrix from the body frame to the inertia frame and defined as

$$S_h \triangleq \begin{bmatrix} 1 & s_\phi s_\theta/c_\theta & c_\phi s_\theta/c_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi/c_\theta & c_\phi/c_\theta \end{bmatrix} \tag{4}$$

where s_ϕ, s_θ, c_ϕ and c_θ denote $\sin(\phi)$, $\sin(\theta)$, $\cos(\phi)$ and $\cos(\theta)$, respectively. At this point it should be noted that $\cos(\theta)$ term used as denominator for some terms of this matrix does not cause indefiniteness of these terms because of the possible interval of θ. More detailed explanations about the possible intervals of yaw, roll and pitch angles an other model parameters can be found in modeling studies (Fantoni and Lozano, 2002), (Mettler, 2003) and (Cai et al., 2011). In (3), $\upsilon_c(t) \in \mathbb{R}^3$ is a vector that is expressed as $\upsilon_c = [\,a\ b\ T_T\,]^T$ where $a(t)$, $b(t) \in \mathbb{R}$ are the flapping angles and $T_T(t) \in \mathbb{R}$ is the tail rotor thrust. In addition to these, $A \in \mathbb{R}^{3\times3}$ and $B \in \mathbb{R}^3$ are a constant invertible matrix and a constant vector, respectively. A simplified model for flapping angles and tail rotor thrust at hovering flight condition can be expressed as in (Mettler, 2003)

$$\begin{aligned} a &= A_b b - A_{lon}\delta_{lon} \\ b &= -B_a a + B_{lat}\delta_{lat} \\ T_T &= K_{ped0}\delta_{ped} \end{aligned} \tag{5}$$

where A_b, A_{lon}, B_a, B_{lat} and $K_{ped0} \in \mathbb{R}$ are constant parameters that are related with the helicopter dynamics and $\delta = [\,\delta_{lon}\ \delta_{lat}\ \delta_{ped}\,]^T$ denotes the actual control input that contains the elevator servo input $\delta_{lon}(t)$, the aileron servo input $\delta_{lat}(t)$ and the rudder servo input $\delta_{ped}(t)$. As a result, a simplified rotor model can be expressed as

$$\tau = S_h^{-T}(AC_\delta\delta + B) \tag{6}$$

where the constant matrix $C_\delta \in \mathbb{R}^{3\times3}$ is defined as

$$C_\delta = \begin{bmatrix} -\dfrac{A_{lon}}{A_bB_a+1} & \dfrac{A_bB_{lat}}{A_bB_a+1} & 0 \\ \dfrac{B_{lat}}{A_bB_a+1} & \dfrac{B_aA_{lon}}{A_bB_a+1} & 0 \\ 0 & 0 & K_{ped0} \end{bmatrix} \tag{7}$$

In view of (6), the dynamic model given in (1) can be re–arranged as follows

$$\ddot{\eta} = h + g\delta \tag{8}$$

where $h(\eta,\dot{\eta}) \in \mathbb{R}^3$ and $g(\eta) \in \mathbb{R}^{3\times3}$ are defined as

$$h \triangleq M_h^{-1}\left(S_h^{-T}B - C_h\dot{\eta} - G_h - f_d\right) \tag{9}$$

$$g \triangleq M_h^{-1}S_h^{-T}AC_\delta \tag{10}$$

We also would like to note that the uncertain functions h and g are assumed to be at least second–order differentiable (*i.e.*, h, $g \in \mathcal{C}^2$) in the rest of analysis.

3. MATHEMATICAL DESCRIPTION OF THE PROBLEM

Based on the assumption that g being a real valued matrix with non–zero leading principal minors, the following matrix decomposition is utilized (Costa et al., 2003), (Morse, 1993)

$$g = S(\eta)DU(\eta) \tag{11}$$

where $S(\eta) \in \mathbb{R}^{3\times3}$ is a symmetric positive definite matrix, $D \in \mathbb{R}^{3\times3}$ is a diagonal matrix with entries being ±1, and $U(\eta) \in \mathbb{R}^{3\times3}$ is a unity upper triangular matrix. Similar to (Costa et al., 2003) and (Chen et al., 2008), it is assumed that D is available for control design.

Remark 1. As it is explained in a detailed manner in (Fantoni and Lozano, 2002), g becomes a non–symmetric input gain matrix for the helicopter model studied in this paper. The significance of this non–symmetric nature may cause reduction in performance and even instability for the Lyapunov–based control designs when not appropriately dealt with. At this point it should be noted that by only using the availability of D in it, an independent structure from the uncertain function g is obtained for the control design and the mentioned non–symmetry is appropriately dealt with. This situation can be seen as an another advantageous aspect of this control design.

Taking the time derivative of the system model in (8) yields

$$\dddot{\eta} = \dot{h} + \dot{g}g^{-1}(\ddot{\eta} - h) + SDU\dot{\delta} \tag{12}$$

where (11) was also utilized. Multiplying both sides of (12) with $M(\eta) \triangleq S^{-1} \in \mathbb{R}^{3\times3}$ results

$$M\dddot{\eta} = f + DU\dot{\delta}. \tag{13}$$

with $f(\eta,\dot{\eta},\ddot{\eta}) \triangleq M\left(\dot{h} + \dot{g}g^{-1}(\ddot{\eta} - h)\right) \in \mathbb{R}^3$.

4. ERROR SYSTEM DEVELOPMENT

The main control objective is to ensure that the system output $\eta(t)$ tracks a sufficiently smooth reference trajectory while ensuring all signals within the closed–loop system remain bounded. In order to quantify the tracking control objective, the tracking error signal, $e_1(t) \in \mathbb{R}^3$, is defined to have the following form

$$e_1 \triangleq \eta_r - \eta \tag{14}$$

where $\eta_r(t) \in \mathbb{R}^3$ is the reference trajectory satisfying

$$\eta_r(t) \in \mathcal{C}^3 \text{ and } \eta_r(t), \dot{\eta}_r(t), \ddot{\eta}_r(t) \in \mathcal{L}_\infty. \tag{15}$$

In the controller development, it will be assumed that the system states $\eta(t)$ and $\dot{\eta}(t)$ are available.

To facilitate the control design, an auxiliary error, denoted by $e_2(t) \in \mathbb{R}^3$ is defined in the following form

$$e_2 \triangleq \dot{e}_1 + e_1. \tag{16}$$

Also a filtered error like term $r(t) \in \mathbb{R}^3$ is defined to have the following form

$$r \triangleq \dot{e}_2 + \alpha e_2 \tag{17}$$

where $\alpha \in \mathbb{R}^{3\times3}$ is a constant positive definite, diagonal, gain matrix. Simplifying the presentation of the stability analysis by eliminating higher order time derivatives from it is the main purpose of the definition of filtered error like term. It should be noted that, since $\dot{e}_2(t)$ is unavailable, then $r(t)$ is also unavailable for control design. It should further be noted that the auxiliary error signals in (16) and (17) are introduced to obtain a stability analysis where only first order time derivatives are utilized. After differentiating (17) and pre–multiplying the resulting equation with $M(\eta)$, the following expression can be derived

$$M\dot{r} = M(\dddot{\eta}_r + \ddot{e}_1 + \alpha\dot{e}_2) - f - DU\dot{\delta} \tag{18}$$

where (13), (14) and (16) were utilized. After defining an auxiliary function, $N(\eta,\dot{\eta},\ddot{\eta},t) \in \mathbb{R}^3$, as

$$N \triangleq M(\dddot{\eta}_r + \ddot{e}_1 + \alpha\dot{e}_2) - f + e_2 + \frac{1}{2}\dot{M}r \tag{19}$$

the expression in (18) can be reformulated to have the following form

$$M\dot{r} = -\frac{1}{2}\dot{M}r - e_2 - DU\dot{\delta} + N. \tag{20}$$

Furthermore, the filtered error dynamics in (20) can be rearranged as

$$M\dot{r} = -\frac{1}{2}\dot{M}r - e_2 - D(U - I_3)\dot{\delta} - D\dot{\delta} + \widetilde{N} + \bar{N} \tag{21}$$

where $D\dot{\delta}(t)$ were added and subtracted to the right–hand side, $I_3 \in \mathbb{R}^{3\times3}$ is the standard identity matrix, and $\bar{N}(t)$, $\widetilde{N}(t) \in \mathbb{R}^3$ are auxiliary functions defined as follows

$$\bar{N} \triangleq N|_{\eta=\eta_r,\dot{\eta}=\dot{\eta}_r,\ddot{\eta}=\ddot{\eta}_r} \tag{22}$$

$$\widetilde{N} \triangleq N - \bar{N}. \tag{23}$$

The main idea behind adding and subtracting $D\dot{\delta}(t)$ term to the right–hand side of (21) is to make use of the fact that $U(\eta)$ is unity upper triangular, and thus $(U - I_3)$ is strictly upper triangular.

5. CONTROLLER DESIGN

Based on the open–loop error system in (21) and the subsequent stability analysis the control input $\delta(t)$ is designed as

$$\delta = DK\left[e_2(t) - e_2(t_0) + \alpha\int_{t_0}^{t} e_2(\sigma)\,d\sigma\right] + D\Pi \tag{24}$$

where the auxiliary signal $\Pi(t) \in \mathbb{R}^3$ is generated according to the following equation

$$\dot{\Pi} = \beta\mathrm{Sgn}(e_2), \Pi(t_0) = 0_3. \tag{25}$$

In (24) and (25), K, $\beta \in \mathbb{R}^{3\times3}$ are constant, diagonal, positive definite, gain matrices, $0_3 \in \mathbb{R}^3$ is a vector of zeros and $\mathrm{Sgn}(\cdot) \in \mathbb{R}^3$ is the vector signum function. Notice that $\delta(t)$ depend on $\eta(t)$, $\dot{\eta}(t)$ and not $\ddot{\eta}(t)$. The control gain matrix K is chosen as $K = I_3 + k_p I_3 + \mathrm{diag}\{k_{d,1}, k_{d,2}, 0\}$ where k_p, $k_{d,1}$ and $k_{d,2} \in \mathbb{R}$ are constant, positive, control gains, and $\mathrm{diag}\{\cdot\}$ is used to represent the entries of a diagonal matrix. Finally, the closed–loop error system for $r(t)$ is obtained as follows by substituting the time derivative of (24) into (21)

$$\begin{aligned} M\dot{r} = &-\frac{1}{2}\dot{M}r - e_2 - Kr + \widetilde{N} + \bar{N} \\ &- D(U - I_3)DKr - DUD\beta\mathrm{Sgn}(e_2) \end{aligned} \tag{26}$$

where (17), (25) and the fact that $DD = I_3$ was utilized.

Before proceeding with the stability analysis, we would like to draw attention to the last two terms of (26) which will be investigated separately.

Note that, after utilizing the fact that $(U - I_3)$ being strictly upper triangular, the term $D(U - I_3)DKr$ is rewritten as

$$D(U - I_3)DKr = \begin{bmatrix} \Lambda + \Phi \\ 0 \end{bmatrix} \tag{27}$$

where $\Lambda(t)$, $\Phi(t) \in \mathbb{R}^2$ are auxiliary functions with their entries $\Lambda_i(t)$, $\Phi_i(t) \in \mathbb{R}$, $i = 1, 2$, being upper bounded as

$$|\Lambda_i| \leq \sum_{j=i+1}^{3} k_j\rho_{i,j}(\|z\|)\|z\||r_j| \leq \rho_{\Lambda_i}(\|z\|)\|z\| \tag{28}$$

$$|\Phi_i| \leq \sum_{j=i+1}^{3} k_j\zeta_{\bar{U}_{i,j}}|r_j| \leq \zeta_{\Phi_i}\|z\|. \tag{29}$$

The $DUD\beta\mathrm{Sgn}(e_2)$ term is rewritten as

$$DUD\beta\mathrm{Sgn}(e_2) = \begin{bmatrix} \Psi \\ 0 \end{bmatrix} + \Theta \tag{30}$$

where $\Psi(t) \in \mathbb{R}^2$ and $\Theta(t) \in \mathbb{R}^3$ are auxiliary functions defined as

$$\begin{bmatrix} \Psi \\ 0 \end{bmatrix} = D\left(U - \bar{U}\right)D\beta\mathrm{Sgn}(e_2) \tag{31}$$

$$\Theta = D\bar{U}D\beta\mathrm{Sgn}(e_2) \tag{32}$$

where $\bar{U}(\eta_r) \triangleq U|_{\eta=\eta_r} \in \mathbb{R}^{3\times3}$ is a function of reference trajectory. The terms $\Psi_i(t) \in \mathbb{R}$, $i = 1, 2$ and $\Theta_i(t) \in \mathbb{R}$, $i = 1, 2, 3$, are upper bounded as

$$|\Psi_i| \leq \sum_{j=i+1}^{3} \beta_j \rho_{i,j} (\|z\|) \|z\| \leq \rho_{\Psi_i} (\|z\|) \|z\| \quad (33)$$

$$|\Theta_i| \leq \sum_{j=i}^{3} \beta_j \zeta_{\bar{U}_{i,j}} \leq \zeta_{\Theta_i} \quad (34)$$

The Mean Value Theorem in (Khalil, 2002) can be utilized to develop the following upper bounds

$$\left\| \widetilde{N}(t) \right\| \leq \rho_{\widetilde{N}} (\|z\|) \|z\| \quad (35)$$

$$\left\| \widetilde{U}_{i,j}(t) \right\| \leq \rho_{i,j} (\|z\|) \|z\| \quad (36)$$

where $\rho_{\widetilde{N}}(\cdot)$, $\rho_{i,j}(\cdot) \in \mathbb{R}$ are non–negative, globally invertible, non–decreasing functions of their arguments, and $z(t) \in \mathbb{R}^9$ is defined by

$$z \triangleq \begin{bmatrix} e_1^T & e_2^T & r^T \end{bmatrix}^T. \quad (37)$$

It can be seen from (15), (19), (22) that $\bar{N}(t)$ and $\bar{U}_{i,j}(t)$ are bounded in the sense that

$$\left| \bar{N}_i(t) \right| \leq \zeta_{\bar{N}_i} \quad (38)$$

$$\left| \bar{U}_{i,j}(t) \right| \leq \zeta_{\bar{U}_{i,j}} \quad (39)$$

$\forall t$ where $\zeta_{\bar{N}_i}$, $\zeta_{\bar{U}_{i,j}} \in \mathbb{R}$ are positive bounding constants. From (34), it is easy to see that $\|\Theta(t)\| \leq \zeta_\Theta \ \forall t$ is satisfied for some positive bounding constant $\zeta_\Theta \in \mathbb{R}$, and from (28), (29) and (33), the following inequality can be obtained

$$|\Lambda_i| + |\Phi_i| + |\Psi_i| \leq \rho_i (\|z\|) \|z\| \quad (40)$$

where $\rho_i(\|z\|) \in \mathbb{R}$ $i = 0, 1, 2$, are non–negative, globally invertible, non–decreasing functions satisfying

$$\rho_{\Lambda_i} + \rho_{\Psi_i} + \zeta_{\Phi_i} \leq \rho_i. \quad (41)$$

As a result of the fact that $\bar{U}(t)$ being unity upper triangular, $\Theta(t)$ in (32) can be rewritten as

$$\Theta = (I_3 + \Omega) \beta \mathrm{Sgn}(e_2) \quad (42)$$

where $\Omega(t) \triangleq D(\bar{U} - I_3) D \in \mathbb{R}^{3\times3}$ is a strictly upper triangular matrix. Since it is a function of the reference trajectory and its time derivatives, its entries, denoted by $\Omega_{i,j}(t) \in \mathbb{R}$, are bounded in the sense that

$$|\Omega_{i,j}(t)| \leq \zeta_{\Omega_{i,j}} \ \forall t \quad (43)$$

where $\zeta_{\Omega_{i,j}} \in \mathbb{R}$ are positive bounding constants.

At this point, we are now ready to continue with the stability analysis of the proposed robust controller.

6. STABILITY ANALYSIS

In this section, first the boundedness of the error signals will be proven under the closed–loop operation by utilizing an initial Lyapunov based analysis. Then a lemma will be presented and an upper bound for the integral of the absolute values of the entries of $\dot{e}_2(t)$ will be obtained by using this result. This upper bound will later be utilized in another lemma to prove the non–negativity of a Lyapunov–like function that will be used in the final analysis which proves asymptotic stability of the tracking error.

Theorem 1. For the dynamic model given in (8), the controller in (24) and (25) guarantees the boundedness of all the closed–loop signals including the error signals in (14), (16) and (17) provided that the control gains $k_{d,i}$ and k_p are chosen large enough compared to the initial conditions of the system and the following condition is satisfied

$$\lambda_{\min}(\alpha) \geq \frac{1}{2} \quad (44)$$

where the notation $\lambda_{\min}(\alpha)$ denotes the minimum eigenvalue of the gain matrix α, previously introduced in (17).

Proof 1. The non–negative function $V_1(z) \in \mathbb{R}$ is defined as

$$V_1 \triangleq \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2} r^T M r. \quad (45)$$

By utilizing (2), (45) can be bounded in the following manner

$$\frac{1}{2} \min\{1, \underline{m}\} \|z\|^2 \leq V_1(z) \leq \frac{1}{2} \max\{1, \bar{m}\} \|z\|^2, \quad (46)$$

where $z(t)$ was defined in (37), and the terms $\underline{m}$, $\bar{m}$ were defined in (2). The time derivative of (45) can be upper bounded as

$$\dot{V}_1 \leq -\left(\lambda_1 - \frac{\rho_{\widetilde{N}}^2(\|z\|)}{4k_p} - \sum_{i=1}^{2} \frac{\rho_i^2(\|z\|)}{4k_{d,i}} \right) \|z\|^2 + \delta_h \varepsilon^2 \quad (47)$$

where $\lambda_1 \triangleq \min\left\{ \frac{1}{2}, \lambda_{\min}(\alpha) - \frac{1}{2}, 1 - \frac{1}{4\delta_h} \right\}$, $\delta_h \in \mathbb{R}$ is a positive bounding constant, $\varepsilon \triangleq \zeta_{\bar{N}} + \zeta_\Theta$, and $\varepsilon \|r\| \leq \frac{1}{4\delta_h} \|r\|^2 + \delta_h \varepsilon^2$ was utilized. Provided that the controller gains $k_{d,i}$ and k_p are selected sufficiently large [larger than functions of the initial values of the norm of $z(t)$], it can be ensured that the terms presented in parenthesis in (47) are always positive, and utilizing (46), the following inequality can be obtained

$$\dot{V}_1 \leq -\beta_{g_1} V_1 + \delta_h \varepsilon^2 \quad (48)$$

where $\beta_{g_1} \in \mathbb{R}$ is a positive constant. From (45), and (48), it can be concluded that $V_1(t) \in \mathcal{L}_\infty$, therefore $e_1(t)$, $e_2(t)$, and $r(t)$ are uniformly ultimately bounded. Standard signal chasing arguments can then be utilized to prove that all the signals remain bounded under the closed–loop operation.

Lemma 1. Provided that $e_2(t)$ and $\dot{e}_2(t)$ are bounded, the integral of the absolute value of the i^{th} entry of $\dot{e}_2(t)$, $i = 1, 2, 3$, can be upper bounded by using a term expressed in terms of the i^{th} entry of $e_2(t)$, $i = 1, 2, 3$.

Proof 2. The proof is similar to that of the one given in (Stepanyan and Kurdila, 2009).

Lemma 2. Consider the term

$$L \triangleq r^T \left(\bar{N} - (I_3 + \Omega) \beta \mathrm{Sgn}(e_2) \right) \quad (49)$$

where $\Omega(t)$ introduced in (42) is a strictly upper triangular matrix that is a function of reference trajectory and its time derivatives. The integral of (49) can be upper bounded as follows by utilizing the appropriate selection of the entries of the control gain β

$$\int_{t_0}^{t} L(\sigma) d\sigma \leq \zeta_L \quad (50)$$

where $\zeta_L \in \mathbb{R}$ is a positive bounding constant.

Proof 3. It can be obtained from Appendix 2 of (Bidikli et al., 2016) by putting 2 and 3 instead of n and m, respectively.

Theorem 2. Given the dynamic model of the helicopter of the form (8), the controller of (24) and (25) ensures that the tracking error and its time derivatives converge to zero asymptotically in the sense that

$$\left\| e_1^{(i)}(t) \right\| \to 0 \text{ as } t \to +\infty \ , \ \forall i = 0, 1, 2$$

provided that α is chosen to satisfy (44), the entries of β are chosen appropriately, and $k_{d,i}$ and k_p are chosen large enough compared to the initial conditions of the system.

Proof 4. It can be obtained from the proof of Theorem 4.1 of (Bidikli et al., 2016) by putting 2 and 3 instead of n and m, respectively.

7. NUMERICAL SIMULATION RESULTS

We utilized a numerical simulation to substantiate the performance of the designed robust controller. All system parameters are obtained from the results at (Fantoni and Lozano, 2002), (Mettler, 2003) and (Cai et al., 2011). The mathematical model of the helicopter in (1) was utilized with the inertia matrix that have following form

$$M = \begin{bmatrix} c_0 & 0 & 0 \\ 0 & c_1 + c_2 \cos(c_3 \psi) & c_4 \\ 0 & c_4 & c_5 \end{bmatrix}$$

The Coriolis–centrifugal forces matrix and vector of conservative forces have the following forms

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_6 \sin(2c_3\psi)\dot{\psi} & c_6 \sin(2c_3\psi)\dot{\theta} \\ 0 & -c_6 \sin(2c_3\psi)\dot{\theta} & 0 \end{bmatrix},$$

$$G = \left[\, c_7 \cos(\phi) \ \ 0 \ \ 0 \,\right]^T.$$

The constant parameters were

$$c_0 = 7.5,\ c_1 = 0.4305,\ c_2 = 3 \times 10^{-4},\ c_3 = -4.143,$$
$$c_4 = 0.108,\ c_5 = 0.4993,\ c_6 = 6.214 \times 10^{-4},\ c_7 = -73.58.$$

The simplified rotor dynamics in (6) are given as

$$A = \begin{bmatrix} c_8\dot{\psi}^2 & 0 & 0 \\ 0 & c_{11}\dot{\psi}^2 & 0 \\ c_{12}\dot{\psi} + c_{13} & 0 & c_{15}\dot{\psi}^2 \end{bmatrix},\ B = \begin{bmatrix} c_9\dot{\psi} + c_{10} \\ 0 \\ c_{14}\dot{\psi}^2 + c_{15} \end{bmatrix}$$

with the constant parameters that are given as

$$c_8 = 3.411,\ c_9 = 0.6004,\ c_{10} = 3.679,\ c_{11} = -0.1525,$$
$$c_{12} = 12.01,\ c_{13} = 10^5,\ c_{14} = 1.204 \times 10^{-4},\ c_{15} = -2.642.$$

The other parameters were used with their following numerical values

$$A_{lon} = -0.1,\ A_{lat} = 0.0313,\ A_b = -0.189$$
$$B_{lon} = 0.0138,\ B_{lat} = 0.14,\ B_a = 0.368$$
$$K_{ped} = 2.16.$$

The unknown external disturbance term f_d was modeled as

$$f_d(t) = \begin{bmatrix} f_{d_c11} + f_{d_c12}\sin(10t) \\ f_{d_c21} + f_{d_c22}\sin(10t) \\ f_{d_c31} + f_{d_c32}\sin(10t) \end{bmatrix} (\text{deg}) \qquad (51)$$

where the constant coefficients f_{d_c11}, f_{d_c12}, f_{d_c21}, f_{d_c22}, f_{d_c31} and f_{d_c32} were randomly selected from the interval $(0, 1]$. Moreover, the additive zero mean white noise with 50 dB signal–to–noise ratio was added to position and velocity sensors to obtain a more realistic approach by modeling noise of sensors. The reference position $\eta_r(t)$ was selected as

$$\eta_r(t) = \left[\, 10\sin(0.1t) \ \ 15\sin(0.1t) \ \ 20\sin(0.1t) \,\right]^T \ (\text{deg}).$$

To ease the gain tuning by getting rid of all bounding conditions about control gains and system uncertainties, the self–tuning algorithm in (Bidikli et al., 2013) and (Bidikli et al., 2014) was utilized after choosing $\alpha = I_3$ that yielded $K = \text{diag}\{2.8, 1.3, 4.75\}$ and $\beta = \text{diag}\{3.6, 1.5, 4.2\}$.

The actual and reference positions are shown in Figure 1, while the position tracking errors and the control inputs are shown in Figures 2 and 3, respectively. Simulation results confirm that the proposed controller meets the tracking objective.

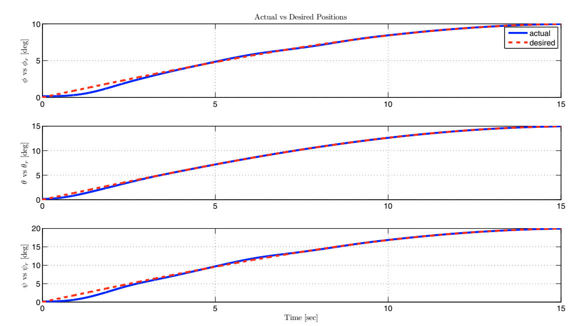

Fig. 1. Tracking results

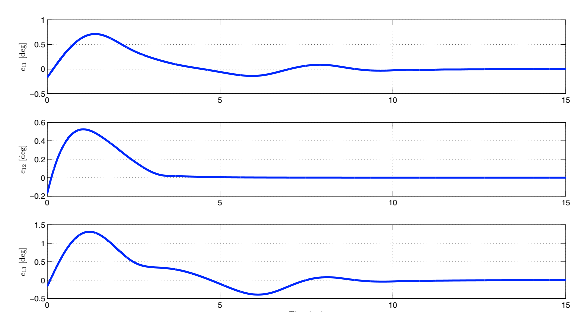

Fig. 2. Tracking error $e_1(t)$

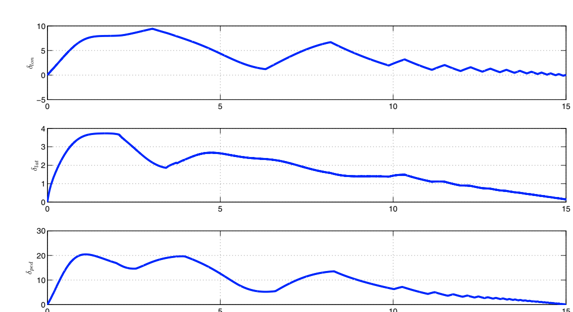

Fig. 3. Actual control input $\delta(t)$

8. CONCLUSIONS

In this paper, attitude tracking control of a small–scaled unmanned helicopter is considered. To realize this purpose, the overall problem is transformed into a second order system by utilizing some reasonable simplifications for the rotor model under the hovering flight conditions. A continuous nonlinear robust controller, that compensates the dynamical uncertainties and the asymmetry in the input gain matrix have been proposed. Provided that the input gain matrix has non–zero leading principle minors the proposed controller ensures semi–global asymptotic tracking. The overall analysis is supported by Lyapunov based arguments. The performance of the designed controller is demonstrated via simulation studies.

The main advantages of the designed controller can be summarized as:

- Highly uncertain flight dynamics, strong coupling effects and the natural instability of the small–scaled unmanned model helicopter are coped with.
- The attitude control of the small–scaled unmanned helicopter is provided by taking the highly nonlinear dynamics of it into account.
- The non–symmetric structure of the input gain matrix of the small–scaled unmanned model helicopter is compensated.
- Different from the past works that designed control input torques, actual control inputs (i.e., δ_{lon}, δ_{lat}, δ_{ped}) are designed.

All of these aspects show the realisticity and applicability of the designed controller for the real time applications.

Future work will concentrate on output feedback versions of the proposed method in order to remove the need of velocity measurement in the controller implementation.

REFERENCES

B. Bidikli, E. Tatlicioglu, A. Bayrak, and E. Zergeroglu. A new robust 'integral of sign of error' feedback controller with adaptive compensation gain. In *Proc. IEEE Int. Conf. Decision and Control*, pages 3782–3787, Florence, Italy, 2013.

B. Bidikli, E. Tatlicioglu, and E. Zergeroglu. A self tuning RISE controller formulation. In *Proc. American Control Conf.*, pages 5608–5613, Portland, OR, USA, 2014.

B. Bidikli, E. Tatlicioglu, E. Zergeroglu, and A. Bayrak. An asymptotically stable robust controller formulation for a class of mimo nonlinear systems with uncertain dynamics. *International J. of Systems Science*, 47(12): 2913–2924, 2016.

G. Cai, B. M. Chen, and T. H. Lee. *Unmanned Rotorcraft Systems*. Springer–Verlag London Limited, London, England, 2011.

J. Chen, A. Behal, and D. M. Dawson. Robust feedback control for a class of uncertain MIMO nonlinear systems. *IEEE Trans. Autom. Control*, 53(2):591–596, 2008.

R. R. Costa, L. Hsu, A. K. Imai, and P. Kokotovic. Lyapunov-based adaptive control of MIMO systems. *Automatica*, 39(7):1251–1257, 2003.

I. Fantoni and R. Lozano. *Non–Linear Control for Underactuated Mechanical Systems*. Spring–Verlag London Limited, London, England, 2002.

J. Gadewadikar, F. L. Lewis, K. Subbarao, and B. M. Chen. Structured H_∞ command and control loop design for unmanned helicopters. *J. of Guidance, Control, and Dynamics*, 31(4):1093–1102, 2008.

B. Kadmiry and D. Driankov. A fuzzy gain–Scheduler for the attitude control of an unmanned helicopter. *IEEE Tr. on Fuzzy Systems*, 12(4):502–515, 2004.

M. Kagawa, H. Katayama, and A. Ichikawa. Attitude control of a helicopter model by feedback linearization. In *SICE Annual Conf.*, pages 1870–1875, Okayana, Japan, 2005.

A. Kato, H. Katayama, and A. Ichikawa. Attitude control of a helicopter model by nonlinear sampled–data H_∞ control. In *SICE Annual Conf.*, pages 138–143, Fukui, Japan, 2003.

H. K. Khalil. *Nonlinear Systems, 3rd Edition*. Prentice Hall, New York, NY, USA, 2002.

H. Liu, G. Lu, and Y. Zhong. Robust LQR attitude control of a 3–DOF laboratory helicopter for aggressive maneuvers. *IEEE Tr. on Industrial Electronics*, 60(10): 4627–4636, 2013.

X. Liu, B. Xian, Y. Zhang, and X. Zang. Nonlinear asymptotic attitude tracking control for an unmanned helicopter with input constraints. In *Proc. American Control Conf.*, pages 1402–1407, Portland, OR, USA, 2014.

B. Mettler. *Identification Modeling and Characteristics of Miniature Rotorcraft*. Kluwer Academic Publishers, Norwell, MA, USA, 2003.

A. S. Morse. A gain matrix decomposition and some of its applications. *Systems & Control Letters*, 21(1):1–10, 1993.

T. Nao, H. Katayama, and A. Ichikawa. Output regulation of a helicopter model. In *SICE Annual Conf.*, pages 126–131, Fukui, Japan, 2003.

T. Sakamato, H. Katayama, and A. Ichikawa. Attitude control of a helicopter model by robust PID controllers. In *Proc. of the Int. Symp. on Intelligent Control*, pages 1971–1976, Munich, Germany, 2006.

J. Shin, H. J. Kim, Y. Kim, and W. E. Dixon. Asymptotic attitude tracking of the rotorcraft-based UAV via RISE feedback and NN feedforward. In *Proc. IEEE Int. Conf. Decision and Control*, pages 3694–3699, Atlanta, GA, USA, 2010.

J. Shin, H. J. Kim, Y. Kim, and W. E. Dixon. Autonomous flight of the rotorcraft–based UAV using RISE feedback and NN feedforward terms. *IEEE Tr. on Control Systems Technology*, 20(5):1392–1399, 2012.

V. Stepanyan and A. Kurdila. Asymptotic tracking of uncertain systems with continuous control using adaptive bounding. *IEEE Trans. Neural Netw.*, 20(8):1320–1329, 2009.

S. Suzuki, D. Nakazawa, K. Nonami, and M. Tawara. Attitude control of small electric helicopter by using quaternion feedback. *J. of System, Design, and Dynamics,*, 5(2):231–247, 2011.

K. P. Tee, S. S. Ge, and F. E. H. Tay. Adaptive neural network control for helicopters in vertical flight. *IEEE Tr. on Control Systems Technology*, 16(4):753–762, 2008.

B. Xian, G. Jianchuan, Z. Yao, and Z. Bo. Sliding mode tracking control for miniature unmanned helicopters. *Chinese J. of Aeronautics*, 28(1):277–284, 2015.