STUDIES ON MODIFIED NEWTONIAN DYNAMICS AND DARK MATTER

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ABSTRACT

STUDIES ON MODIFIED NEWTONIAN DYNAMICS AND DARK MATTER

The flat rotation curves of the galaxies are considered to be anomalous observations according to Newtonian dynamics. There are two different approaches to explain this challenge - Dark Matter (DM) and Modified Newtonian Dynamics (MOND). Both of them possess some failures as well as many successes. Beyond these failures, they have much more fundamental difficulties such as the lack of any direct or indirect detection of proposed dark matter candidates and the lack of a full-fledged relativistic version of MOND theory. In this thesis, focus will be on these fundamental problems.

First, the relativistic MOND theory will be studied. The first successful relativistic version is Tensor-Vector-Scalar (TeVeS) theory based on bimetric gravity. However, the addition of vector and scalar fields into the theory by hand is not much different than the addition of dark matter. In this study, TeVeS theory will be constructed in a more natural way. To do this, at first standard Einstein-Hilbert action will be extended by using non-Riemannian structures (torsion, non-metricity, etc.) from metric-affine formalism. It will then be shown that obtained extended theory of gravity could turn into a Tensor-Vector-Scalar theory via decomposition of affine connection as Levi-Civita connection and rank(1,2) tensorial structure composed of lower rank fundamental and composite fields such as vector and scalar fields.

Subsequently, it will be continued with a study on the relativistic MOND theory, without requiring an action principle. In this study, energy momentum tensor part will be modified rather than the geometrical part of the Einstein field equations. This could be considered as the first dynamical approach to relativistic MOND in the literature. It will be shown that the modified field equations obtained via this dynamical approach can be reduced to true MONDian force in the non-relativistic limit in some astrophysical domains. This study can be also considered as an extension of Milgrom's modified inertia approach to relativistic domain.

Finally, a new phenomenological model- Higgsed Stueckelberg scenario - involving a hidden vector field with an accompanying scalar field ensuring the gauge invariance will be proposed. It will be shown that the contributions from the hidden fields could stabilize the Higgs boson mass at one-loop, where the set up can accommodate naturally a viable DM candidate.

ÖZET

MODİFİYE NEWTON DİNAMİĞİ VE KARANLIK MADDE ÜZERİNE ÇALIŞMALAR

Galaksilerin düz dönme eğrileri Newton dinamiğine göre beklenenden farklı bir gözlemdir. Bu aykırı gözlemi açıklamak için iki farklı yaklaşım bulunmaktadır: karanlık madde ve Modifiye Newton Dinamiği (MOND). İki yaklaşım da birçok başarılarının yanısıra bazı başarısızlıklara sahiptir. Bu başarısızlıkların ötesinde önerilen karanlık madde adaylarının doğrudan veya dolaylı olarak algılamanın olmaması ve MOND teorisinin tam teşekküllü bir relativistik versiyonunun yokluğu gibi daha temel sorunları vardır. Bu tezde, karanlık madde ve MOND teorisinin bu temel problemleri üzerine odaklanılmıştır.

İlk olarak MOND teorisinin relativistik genellemesi çalışılmıştır. MOND teorisinin ilk öne sürülen başarılı relativistik versiyonu bimetrik gravitasyonuna dayanan Tensör-Vektör-Skaler (TeVeS) teorisidir. Fakat vektör ve skaler alanların teoriye el ile eklenmesi, karanlık madde eklemekten çok farklı değildir. Bu çalışmada, TeVeS teorisi daha doğal bir yol ile elde edilmiştir. Metrik-afin formalizasyonundan gelen Riemann uzayında bulunmayan yapılar vasıtasıyla Einstein-Hilbert eylemi genişletilmiştir. Bu genişletilmiş gravitasyon teorisinin afin bağlantısının Levi-Civita bağlantısı ve (vektör ve skaler gibi daha düşük ranklı temel ve bileşik alanlardan oluşan) rank(1,2) tensor yapısı şeklinde ayrıştırılması yoluyla bir tensör-vektör-skaler teorisine dönüştüğü gösterilmiştir.

Daha sonra aksiyon prensibi olmaksızın bir relativistik MOND teorisi çalışmasıyla devam edilmiştir. Bu çalışmada, Einstein alan denklemlerinin geometrik kısmının modifikasyonundan ziyade enerji momentum tensörü kısmı modifiye edilmiştir. Bu, literatürde relativistik MOND için ilk dinamik yaklaşım olarak düşünülebilir. Böyle bir dinamik yaklaşım ile elde edilmiş modifiye alan denklemlerinin bazı astrofiziksel alanlarda relativistik olmayan limitte gerçek MONDian kuvvetine indirgenebileceği gösterilmiştir. Bu çalışma Milgrom'un modifiye eylemsizlik yaklaşımının relativistik alana genişletilmesi olarak da düşünülebilinir.

Son olarak, ayar değişmezliği bir skaler alan tarafından sağlanan saklı bir vektör alan içeren yeni bir fenomolojik model-Higgsed Stueckelberg senaryosu öne sürülmüştür. Bu saklı alanlardan gelen katkıların tek halka düzeyinde Higgs bozonu kütlesinin stabilizasyonunu sağladığı, ve ayrıca bu senaryonun doğal olarak oluşan geçerli karanlık madde adaylarını içinde barındırdığı gösterilmiştir.

To the memory of my aunt Nurhan LEUE

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CHAPTER 1

INTRODUCTION

The motion is the action of changing position with respect to time and a reference frame. It might be seen as the simplest branch of physics to study, but it is not. Actually, it is the oldest concept and is more complex than estimated in elementary physics. From past to present, physicists have been trying to describe the motion. Aristotales, Copernicus, Kepler, Galileo and Newton are the major physicists and astronomers studying this notion. Eventually, Newton was able to come up with a way to interpret it with three fundamental laws called as Newton's laws of motion. These laws are briefly summarized as follows:

1. Newton's first law of motion: Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied on it.

2. Newton's second law of motion: The relationship between an object's mass m, its acceleration $\mathbf{a}$, and the applied force $\mathbf{F}$ is

$$\mathbf{F} = m\mathbf{a} \tag{1.1}$$

3. Newton's third law of motion: For every action there is an equal and opposite reaction.

The motion of bodies is explained very well via these three laws. The most powerful one among them is the second one without any doubt since it describes the dynamics of systems. In addition to these three laws of motion, Newton put forward a gravitational force law. According to this law, every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them (Newton, 1687).

$$\mathbf{F} = \frac{G_N m_1 m_2}{r^2}\hat{r} \tag{1.2}$$

where G_N is an empirical gravitational constant (Newton constant). Its value in SI units is $G_N \simeq 6.67384 \times 10^{-11} m^3 kg^{-1} s^{-2}$ (Mohr et al., 2016).

Newton's law of gravitation and Newton's laws of motion are the cornerstones of the classical mechanics. However, at this point the right question to ask is this: Are these laws valid for all scales? The first answer to this question comes from the microscopic world where Newtonian dynamics seem to fail. To describe the motion of atoms and the subatomic particles, there is a new branch of physics, called "Quantum Mechanics". The second answer to the question on the validity of Newton's laws of motion comes from the galactic systems. Up to galactic scales there is no problem. However, for larger scales, some challenges arise in describing the motion of bodies via Newtonian mechanics.

In 1970's Vera Rubin observed that the rotation curve of spiral galaxies was nearly flat (Rubin and Ford Jr, 1970; Rubin et al., 1980). Some of the observation results of Vera Rubin are given in the Figure1.1 below:

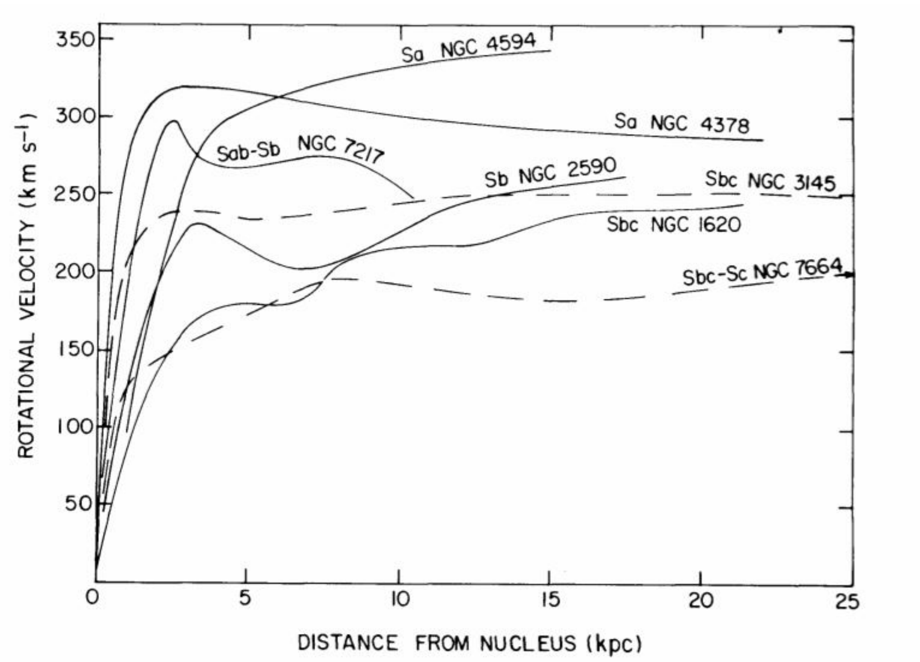

Figure 1.1. Rotational velocities for seven galaxies Source: (Rubin et al., 1978)

This was a surprising result. Because, according to Newtonian dynamics, velocities of stars in a galaxy should decrease as we go further away from the center of the galaxy ($v(r) \propto 1/\sqrt{r}$). However, the observations show that after particular distances from the center of the galaxy, the velocities remain approximately constant. This controversy is called as "Flat Rotation Curve of galaxies". This problem can be easily seen in Fig.1.2. As it may clearly be understood, the observed velocities of stars at the outer orbits do not match the expected ones from Newtonian Dynamics. The rotational velocities of bodies at the outer parts of the galaxy are far faster than the predicted values. Therefore, one may easily state that Newton's laws of motion fail in explaining the motion of the bodies at the galactic scales. There are two approaches to explain this challenge.

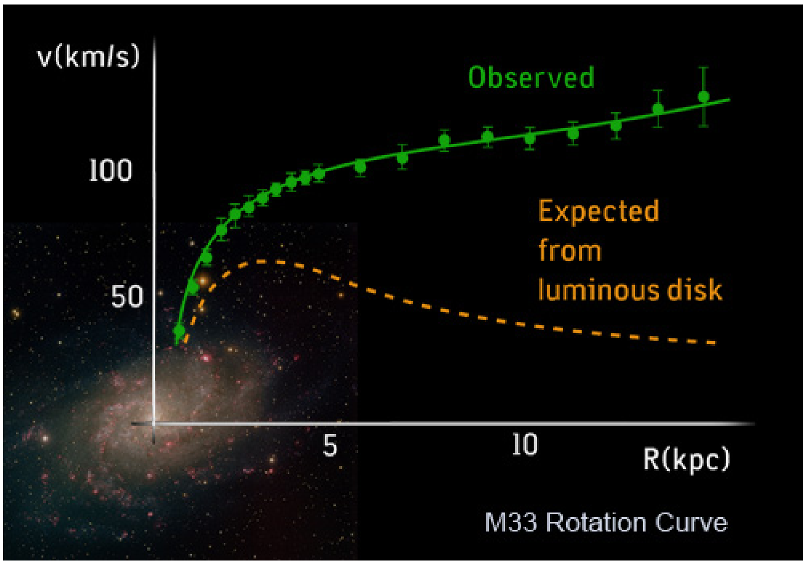

Figure 1.2. Flat rotation curve of the galaxy M33, comparatively. Green points: observed velocities of the bodies orbiting the galaxy, The lower dashed lines: predicted velocities by Newtonian mechanics with luminous matter in the galaxy, Source: NOAO, AURA, NSF, T.A.Rector.

Newtonian dynamics is either not valid at galactic scales and thus should be modified or not sufficient to explain the motion of the bodies with the amount of luminous matter. In the literature, there are two most leading explanations on this anomalous observation: **Dark Matter (DM)** and **Modified Newtonian Dynamics (MOND)**. Considering these two approaches, physics underlying the motion of the bodies at any scale with respect to the sizes and the velocities may be summarized such as in the Figure 1.3:

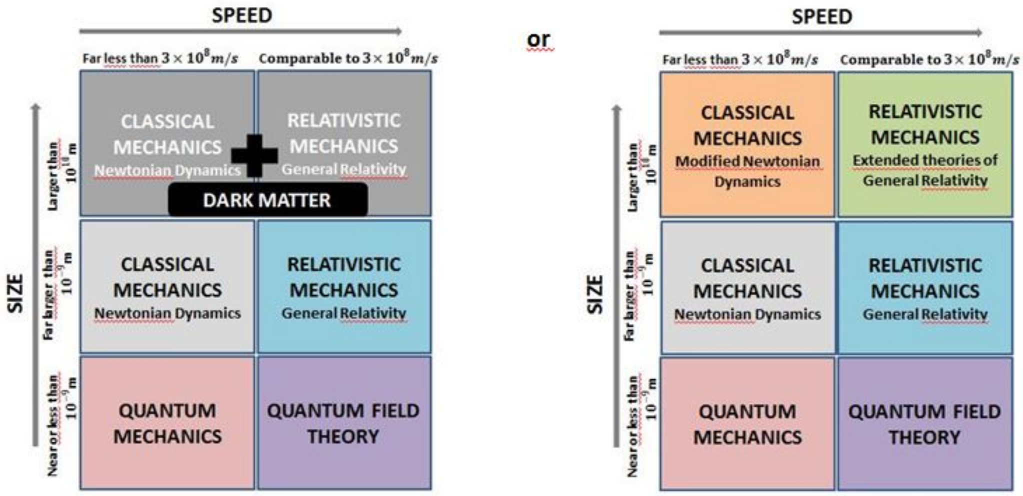

Figure 1.3. Physics underlying the motion of the bodies with respect to the sizes and the velocities. Left: DM approach, Right: MOND approach

To explain the flat rotation curve of galaxies, Vera Rubin used the statement that the observed mass is much less than the predicted one since there is a non-shining mass - Dark Matter distributed there. Prior to the observation of flat rotation curve, this statement had been used by Zwicky to explain the anomalous motions realized in Coma clusters in the 1930s (Zwicky, 1933). This new type of non-baryonic matter is called **Cold Dark Matter** (CDM) and the corresponding model of universe is called ΛCDM. The term Λ stands for the cosmological constant which is responsible for the late-time accelerated expansion of the universe. ΛCDM model is accepted widely as the Standard Cosmological Model. According to this model, Newtonian mechanics together with its corresponding relativistic generalization General Relativity (GR) are the right theories to describe the motion of bodies at any scales (except at quantum scales). However, as we have mentioned above, to be able to explain the aforementioned controversial observations, there must exist a different kind of matter which does not interact with electromagnetic field. It only interacts with the baryonic matter via gravitational interaction, such that it can be detected from its gravitational effect.

From past to present, by using the concept of DM the flat rotation curves of the galaxies have been well explained. In Fig.1.4 there is an example of rotation curve which is explained by DM scenario. According to this scenario,at the outer parts of the galaxies there should be dark halos whose mass changes directly proportional to the distance:

$M(r) \propto r$ (mass density $\rho(r) \propto 1/r^2$). This could be a true statement for missing mass

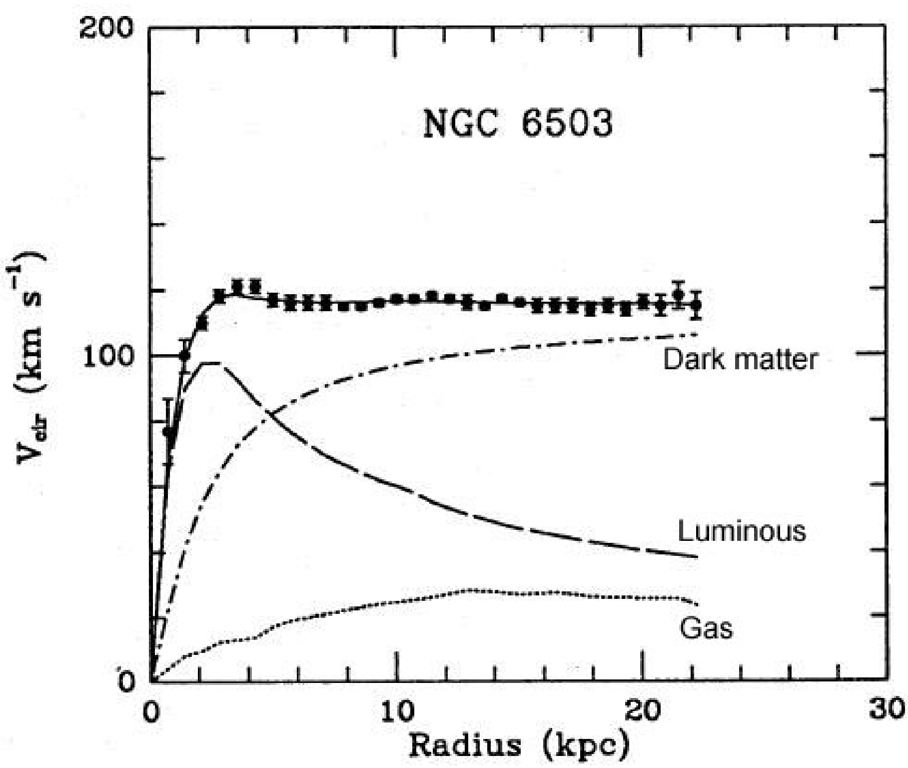

Figure 1.4. Rotation curve of the galaxy NGC6503 with DM Source: (Broeils et al., 1991)

problem in the galaxies and at larger scales, but the lack of any direct or indirect detection of dark matter particle is still a problem. Despite there is no detection of DM, it has been widely accepted by scientific community. Moreover, it is still continued to be tested by the increasing numbers of modern astronomical observations which are also progressively upgrading their precession limits. The latest observational data from Planck is consistent with the estimations of ΛCDM model (Ade et al., 2016).

An alternative explanation to the flat rotation curves of galaxies is **Modified Newtonian Dynamics** proposed by Milgrom in 1983. According to Milgrom, to explain the flat rotation curves, there is no need to propose a non-shining matter - dark matter - unless there is an obligation to adhere to Newtonian dynamics. The modification of Newtonian dynamics may be conceived as another alternative for describing this confounding observations (Milgrom, 1983a,b,c). This choice is not a new way in explaining the anomalous observations in physics. There are many examples in the past, such as special relativity, Quantum mechanics, etc. Although MOND shows many successes at galactic scales (Sanders and McGaugh, 2002; Begeman et al., 1991; Famaey and McGaugh, 2012), at larger scales it is not as successful as DM. Moreover, despite there are many attempts (Bruneton and Esposito-Farese, 2007; Skordis, 2009; Famaey and McGaugh, 2012; Sanders, 1997; Bekenstein, 2004b; Sanders, 2005; Zlosnik et al., 2006, 2007; Bertolami et al., 2007, 2008; Stabile and Scelza, 2011; Bernal et al., 2011) the lack of full fledge relativistic version of MOND may be confer as one of the biggest problems of the model.

As understood from the statements above, when the whole universe is considered, there is still need for dark matter and dark energy in explaining various anomalous observations (especially at large scales). There has been a big debate among scientists studying MOND theory and those studying dark matter for years. Outside this debate there have been some studies on the combination of these three phenomena (dark matter, dark energy and MOND) to explain the whole universe (Khoury, 2015). According to us, It is not time to be fanatic on neither MOND nor dark matter. It is the best to work on both until one of them is completely confirmed.

This thesis involves three different studies: two of which are related to relativistic version of MOND theory and one is related to DM. Therefore, before going on with these studies it would be appropriate to give the basics of DM and MOND.

- **Dark Matter**

  The concept of DM has started to be spread with the studies of Fritz Zwicky in the 1930s on the motions of galaxies in the Coma Clusters. Zwicky proposed that a large amount of invisible matter - dark matter - should exist to keep these galaxies bound together (Zwicky, 1933). More than 40 years later, Vera Rubin used the same concept for explaining the flat rotation curves of the galaxies (Rubin and Ford Jr, 1970; Rubin et al., 1980). After that time, many observational evidences at a wider range of scales have confirmed the existence of DM. The latest results of the Planck satellite indicate that DM accounts for 26.8% (ordinary matter 4.9% and dark energy 68.3%) of the cosmological matter density (Ade et al., 2016), and nowadays there is broad unanimity that the DM is most likely to be made up of an entirely new elementary particle. However, the identity of the DM particle is still one of the phenomenal mysteries in modern particle physics, astrophysics, and cosmology. To understand the dark matter concept basically, introductory level information about its strong observational evidences, candidates and detection methods is given below.

– **Observational evidences of DM from astrophysics and cosmology**

The most powerful evidence for the existence of DM, without any doubt, is the flat rotation curves of galaxies (Rubin and Ford Jr, 1970; Bosma, 1978; Rubin et al., 1980). In addition, there are many other gravitational effects explained very well by DM paradigm at galactic and larger scales. Velocity dispersions of galaxies (Faber and Jackson, 1976; Minchin et al., 2005), gravitational lensing (Wu et al., 1998; Refregier, 2003; Massey et al., 2007), CMB fluctuations (Hinshaw et al., 2009; Ade et al., 2016),structure formation (Springel et al., 2005) and bullet cluster indicates the strong evidences for DM.

– **Constituents of DM**

As mentioned above, dark matter explains many cosmological observations in a simple way. However, the constituents of dark matter are still unknown. The most plausible scenarios are based on non-baryonic new particles such as Weakly Interacting Massive Particles (WIMPs) (Barger et al., 2008) and axions (Weinberg, 1978; Preskill et al., 1983). Additionaly, the large objects like Massive Compact Halo Objects (MACHOs) such as black holes (Hawkins, 2011; Alcock et al., 2000) are considered as baryonic dark matter candidates.

MACHOs are detected by searching for the gravitational microlensing amplification of light. The results of the searches indicates that the maximum 20% of the dark halo can be in the form of MACHOs with $0.5M_\odot$ (Becker et al., 1999). This shows that there is not enough MACHOs to fill the dark halo in a galaxy. On the other hand, since WIMPs are electrically neutral massive particles which do not interact very strongly with other matter, they are strong candidates for DM. This is why, in recent years the studies on Dark matter generally include WIMPs as DM candidates.

According to Cosmic Microwave Background measurement of Planck satellite, the latest density of non-baryonic dark matter and the baryonic matter (Ade et al., 2016)

$$\Omega_{nbm}h^2 = 0.1186 \pm 0.0020$$
$$\Omega_b h^2 = 0.02226 \pm 0.00023 \tag{1.3}$$

where h is the Hubble constant (in units of $100kms^{-1}Mpc^{-1}$) whose present-day value is given by $H_0 = 100h = 67.8 \pm 0.9km/(s.Mpc)$. As understood, the amount of non-baryonic matter density (DM density) is approximately five times bigger than the amount of ordinary matter density. This is not a small fraction, therefore the detection of DM is one of very important steps to explain universe.

– **Detection of DM**

Many experiments are presently searching for a signal of DM (especially WIMP-like ones). There are three ways to detect DM: direct detection, indirect detection and particle collider. Direct detection experiments rely on measuring the energy deposited via the interaction of WIMPs with nuclei in a detector on Earth. To avoid the background from cosmic radiation, this kind of experiments are performed in underground laboratories. The latest experiment PandaX-II indicates that there is no dark matter particle up to the cross section $2.5 \times 10^{-46} cm^2$ for a range of mass between 5 and $1000 GeV/c^2$ (Tan et al., 2016). On the other hand, WIMP-WIMP annihilation is a potential signal of DM which is considered as indirect detection. The last method to detect a DM particle is searching for them at particle collider experiments. Producing and detecting DM particles in an accelerator is a good way toward confirming the existence of dark matter. Moreover, this detection method is ideal as it allows a control on initial states and the experiments conducted in particle accelerators are reproducible. Since DM candidate particles are not supposed to interact with electromagnetic field, they escape detection as neutrinos do in particle collider experiments. They appear as a characteristic signal of missing energy. This missing energy might be determined from the observed jets, photons, heavy quarks and leptons. In spite of many proposed dark matter model, there has been no detection of dark matter particles at colliders, yet.

The Figure 1.5 summarizes the detection methods with a simple schematic representation.

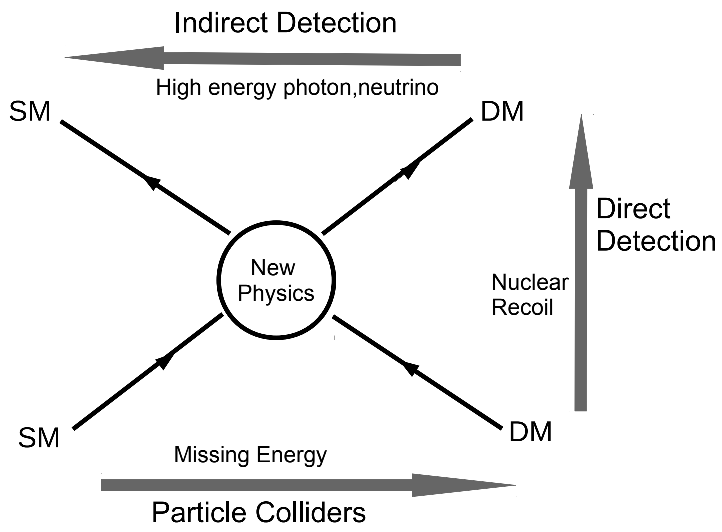

Figure 1.5. Schematic representation of three detection methods

In this thesis, we will propose an extended SM called as Higgsed Stueckelberg scenario which involves a hidden vector field and a scalar field ensuring the gauge invariance. Via detailed collider and dark matter analyses of the model, we show that these hidden fields serve as perfect dark matter candidates.

- **MOND Theory**

  One another most leading explanation for the flat rotation curves is MOND theory proposed by Milgrom in 1983 (Milgrom, 1983a,b,c). Contrary to dark matter paradigm, its main claim is based on the modification of Newton's second law in a such a way to obtain constant velocities at the outer parts of galaxies where the acceleration is very small due to the weak gravitational force. This modification may be also considered as a classical generalization of Newton's second law via an interpolation function behaving asymptotically with respect to the the acceleration. The form of this generalized MONDian force is given as follows:

$$\mathbf{F} = m\mu\left(\frac{|\mathbf{a}|}{a_0}\right)\mathbf{a} \tag{1.4}$$

where μ is the interpolation function and $a_0 = 1.2 \times 10^{-8} cm/s^2$ is an emprical universal constant in the units of acceleration. Here,the most important point is the behaviour of the interpolation function μ in the larger and smaller limits of acceleration with respect to the universal acceleration constant a_0. Analytically, μ is given as a piecewise function

$$\mu\left(\frac{a}{a_0}\right) = \begin{cases} 1 & \text{if } a \gg a_0 \\ \frac{a}{a_0}, & \text{if } a \ll a_0 \end{cases}$$

such that in the high acceleration limit $a \gg a_0$ (at the distances close to the center of the galaxy) the function μ takes the value 1, then the Eq. (1.4) reduces to Newton's second law given in Eq. (1.1). On the other hand, at the low acceleration limit $a \ll a_0$ the function μ goes to a/a_0, then the MONDian force takes the quadratic form in acceleration as

$$F = m\frac{a^2}{a_0} \tag{1.5}$$

This form of force provides the expected constant velocity as

$$v = \sqrt[4]{GMa_0} \tag{1.6}$$

It is seen from the Eq. (1.6) that the velocity is independent of the distance r. This result is consistent with the flat rotation curve obtained by observations. MOND is a promising theory due to its achivements at galactic scales.

The cornerstone of MOND theory is the existence of a new emperical constant a_0 with the dimension of acceleration and hence interpolation function is the heart of the theory. This constant puts a boundary between Newtonian and MONDian dynamics. As $a_0 \to 0$, acceleration $|\mathbf{a}|$ of any system may be viewed as bigger than a_0, and then MOND force law is reduced to Newton's second law. Otherwise, MONDian force, which possesses a quadratic form in acceleration, holds for low accelerated systems ($|\mathbf{a}| < a_0$) which should not be the case. A similar constant also exists in Quantum Mechanics: $\hbar$. As $\hbar \rightarrow 0$, the equations of motion take the classical forms. This similarity makes one think that the disclosure of the motions of the bodies with small accelerations via a new force law may be the right track to follow.

MOND was originally proposed to explain flat rotation curves of galaxies. However, to make it an alternative theory to DM at all scales, there exist ongoing studies on MOND at galactic and also larger scales. The most leading one of these studies is about the relativistic version of MOND theory. Although GR has been the most successful relativistic theory of gravity from past to present, like Newtonian mechanics it fails to account for some dynamics at galactic and larger scales. As mentioned above, to explain these dynamics, there are two different approaches also in the relativistic limit. The first one is adding dark matter and dark energy to the systems and the other one is the modification of the standard theory of general relativity to extend MOND to the relativistic domain. In literature, there are many extended theories of general relativity reducing to the MONDian force for low accelerations in non-relativistic limits (Bekenstein and Sanders, 2006; Bekenstein, 2009, 2004b; Zlosnik et al., 2006; Mavromatos and Sakellariadou, 2007; Bernal et al., 2011). The first and most successful extended version with action principle is Tensor-Vector-Scalar theory proposed by Bekenstein (Bekenstein, 2004b). Before going on the theoretical formulation of TeVeS theory, it would be appropriate to give some basics of GR. General Relativity (GR) is the relativistic generalization of Newtonian mechanics. It was first proposed by A. Einstein in 1915 as the theory of space-time and matter. According to GR, matter curve the space-time and this curvature tells the matter how to move.
GR is a purely metric theory of gravity. In other words, only dynamical variable of the system is metric which is responsible for measuring the distances in spacetime. The full action of the theory is given by

$$S_{EH}[g,\psi] = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}R(g) + \int d^4x\,\sqrt{-g}L_m(g,\psi) \tag{1.7}$$

where the first term in Eq.(1.7) is the gravitational part and the second term is the matter part. This action is called Einstein-Hilbert action. g is the determinant of the metric tensor $g_{\mu\nu}$, R is Ricci scalar defining the curvature of the spacetime and L_m is Lagrangian of any kind of ordinary matter. The metric convention is (-,+,+,+) and the speed of light is set to one.

Gravitational equation of motion is obtained by taking the variation with respect to the metric tensor (according to least action principle) as

$$G_{\mu\nu} = R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = 8\pi G_N T_{\mu\nu} \tag{1.8}$$

Here, $T_{\mu\nu}$ is the energy-momentum tensor of matter describing the density and flux of energy and momentum in spacetime, $G_{\mu\nu}$ is called Einstein tensor. Left Hand Side (LHS) of this equation is gravitation part and Right Hand Side (RHS) is matter part. The conservation of energy directly comes from the LHS (geometrical side). It can be seen easily by taking the divergence of it.

As it mentioned, GR is not sufficient to explain some dynamics at astrophysical bodies such as galaxies and large scale structures. TeVeS may be considered as an alternative theory of gravitational interaction. Unlike GR, it is a bi-metrical theory. This means that there are two different metric in the system. One of them is standard gravitational metric $g_{\mu\nu}$ known from GR, the other one is called as physical metric whose form is;

$$\tilde{g}_{\mu\nu} = e^{2\phi}g_{\mu\nu} - 2\sinh(2\phi)u_\mu u_\nu \tag{1.9}$$

where ϕ is a scalar field and u_μ and u_ν are unit vector fields. In this relativistic theory, there is no need to postulate a different kind of unshining matter-CDM. Scalar and vector fields undertake the task of it. The TeVeS action can be written as:

$$S = \int d^4x \left(L_g + L_s + L_\upsilon\right) \tag{1.10}$$

The first term is standard Einstein-Hilbert Lagrangian

$$L_g = \frac{1}{16\pi G_N} R\sqrt{-g} \tag{1.11}$$

and the second term corresponds to the scalar field Lagrangian in the form as:

$$L_s = \frac{1}{2}\left(\sigma^2 h^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{1}{2}\frac{G^2}{\ell}\sigma^4 F(kG\sigma^2)\right)\sqrt{-g} \tag{1.12}$$

where $h^{\alpha\beta} = g^{\alpha\beta} - u^\alpha u^\beta$, ℓ is a constant length, k is the dimensionless scalar coupling constant and F is an unspecified dimensionless function. σ is non-dynamical scalar field while ϕ is dynamical one. The last term in Eq.(1.10) is the vector field Lagrangian:

$$L_\upsilon = \frac{K}{32\pi G_N}\left[g^{\alpha\beta}g^{\mu\nu}B_{\alpha\mu}B_{\beta\nu} + 2\frac{\lambda}{K}g^{\mu\nu}u_\mu u_\nu - 1\right]\sqrt{-g} \tag{1.13}$$

where $B_{\alpha\beta} = \partial_\alpha u_\beta - \partial_\beta u_\alpha$, K is dimensionless vector coupling constant of the theory. The physical metric mentioned above takes the role in matter part. The action of matter can be written by using physical metric as in the following closed form:

$$S = \int d^4x L_m\left(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha_{;\mu}...\right)\sqrt{-\tilde{g}} \tag{1.14}$$

The equation of motion obtained from Eq.(1.10) reduces to MOND in the non-relativistic limit. Moreover, it is consistent with gravitational lensing and some other cosmological observations.

In spite of these achievements, TeVeS theory can not explain some astrophysical observations such as recent observational evidence of dark matter - bullet clusters. It is not a full-fledge relativistic version of MOND theory. Furthermore, the addition of scalar and vector fields by hand gives rise to the question "What is the difference TeVeS theory from DM hypothesis?".

As mentioned above, there are two alternative explanations to the flat rotation curves of the galaxies. Although DM has much more achievements at galactic scales and at larger ones, it has not been observed by experimentally in direct and indirect searches. This is the most relevant challenge of DM hypothesis. On the other hand, MOND theory is not a complete theory and the lack of a full-fledge relativistic version of MOND is one of its biggest problems.

These problems of DM and MOND are our main motivations.The thesis is divided broadly into three parts. In total there will be five chapters and three appendices. Firstly, we start with the relativistic generalization of MOND theory. In Chapter 2 we focus on the first successful relativistic version of MOND - TeVeS theory proposed by Bekenstein (Bekenstein, 2004b). TeVeS theory possesses extra degrees of freedom such as vector and scalar fields added by hand. In this study, we obtain TeVeS theory via metric-affine gravity in a more natural way. In Chapter 3, we propose a relativistic version of MOND without action principle (at the equation of motion level). We modify the energy-momentum tensor part of Einstein Field Equations. The main importance of this modification is that we modify the matter part rather than the geometrical part (Mishra and Singh, 2012; Bernal et al., 2011). In Chapter 4, we propose some possible dark matter candidates and the theory that lies behind them. We also give the detailed collider analyses of these candidates. Then, we conclude in Chapter 5.

CHAPTER 2

SCALARS, VECTORS AND TENSORS FROM METRIC-AFFINE GRAVITY

In this chapter, we obtain TeVeS theory with a different formalism: metric-affine formalism. The metric-affine gravity provides a useful framework for analyzing gravitational dynamics since it treats metric tensor and affine connection as fundamentally independent variables. We show that, a metric-affine gravity theory composed of the invariants formed from non-metricity, torsion and curvature tensors can be decomposed into a theory of scalar, vector and tensor fields. These fields are natural candidates for the ones needed by various cosmological and other phenomena. Indeed, we show that the model accommodates TeVeS gravity (relativistic modified gravity theory), vector inflation, and aether-like models. Detailed analyses of these and other phenomena can lead to a standard metric-affine gravity model encoding scalars, vectors and tensors.

2.1. Introduction

Spacetime is a smooth manifold $M(g; \between)$ endowed with a metric g and connection $\between$. Metric is responsible for measuring the distances while affine connection governs the straightness of curves and twirling of the manifold. These two geometrical structures, the metric and connection, are fundamentally independent geometrical variables, and they play completely different roles in spacetime dynamics. If they are to exhibit any relationship it derives from dynamical equations a *posteriori*. This fact gives rise to an alternative approach to Einstein's standard theory of general relativity: Metric-Affine Gravity.

The standard theory of general relativity is a purely metric theory of gravity since connection is completely determined by the metric and its partial derivatives, *a priori*. This determination is encoded in the Levi-Civita connection,

$$\Gamma^{\lambda}_{\alpha\beta} = \frac{1}{2} g^{\lambda\rho} \left(\partial_{\alpha} g_{\beta\rho} + \partial_{\beta} g_{\rho\alpha} - \partial_{\rho} g_{\alpha\beta} \right) \tag{2.1}$$

which defines a metric-compatible covariant derivative (Carroll, 2004).

The metric-affine theory of gravity (similar to the Palatini formalism (Palatini, 1919; Einstein, 1925) in philosophy), which treats an metric tensor and connection as independent variables (Carroll, 2004; Peldan, 1994; Magnano, 1995), encodes a more general approach to gravitation by breaking up the a priori relation (2.1). This breaking inherently reveals the new dynamic structures torsion, nonmetricity in addition to curvature.

In this work we shall study metric-affine gravity in regard to decomposing the affine connection into independent vectors, tensors and scalars. We shall, in particular, be able to derive certain interactions using solely the *geometrical* sector with no reference to the matter sector that contains the known forces and species. Our starting point will be the fundamental independence of connection and metric, and the field content of the connection in the most general case.

The outline of this study is as follows. In Sec. II below we first construct the most general 'connection' involving physically 'distinct and independent' structures, and then form a general action containing vector and tensor fields. In Sec. III we give specific applications of the derived action to vector inflation and TeVeS theory. Here we also discuss the relation of the model to the ones in the literature. In Sec. IV we conclude.

2.2. Tensor-Vector Theories from Non-Riemannian Geometry

An affine connection, whose components to be symbolized by $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$, governs parallel transport of tensor fields along a given curve in spacetime, and parallel transport around a closed curve, after one complete cycle, results in a finite mismatch if the spacetime is curved. Curving is uniquely determined by the Riemann curvature tensor

$$\mathbb{R}^{\mu}_{\alpha\nu\beta}(\mathbb{\Gamma}) = \partial_{\nu}\,\mathbb{\Gamma}^{\mu}_{\beta\alpha} - \partial_{\beta}\,\mathbb{\Gamma}^{\mu}_{\nu\alpha} + \mathbb{\Gamma}^{\mu}_{\nu\lambda}\mathbb{\Gamma}^{\lambda}_{\beta\alpha} - \mathbb{\Gamma}^{\mu}_{\beta\lambda}\mathbb{\Gamma}^{\lambda}_{\nu\alpha} \tag{2.2}$$

which is a tensor field made up solely of the non-tensorial objects $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$ and their partial derivatives. Notably, higher rank tensors involving $(n+1)$ partial derivatives of $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$ are given by n-th covariant derivatives of $\mathbb{R}^{\mu}_{\alpha\nu\beta}$, and hence, $\mathbb{R}^{\mu}_{\alpha\nu\beta}$ acts as the seed tensor field for a complete determination of the spacetime curvature.

Affine connection determines not only the curving but also the twirling of the spacetime. This effect is encoded in the torsion tensor

$$\mathbb{S}^{\lambda}_{\alpha\beta}(\mathbb{\Gamma}) = \mathbb{\Gamma}^{\lambda}_{\alpha\beta} - \mathbb{\Gamma}^{\lambda}_{\beta\alpha} \tag{2.3}$$

which participates in structuring of the spacetime together with curvature tensor. Torsion vanishes in geometries with symmetric connection coefficients, $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}=\mathbb{\Gamma}^{\lambda}_{\beta\alpha}$.

The spacetime gets further structured by the notions of distance and angle if it is endowed with a metric tensor $g_{\alpha\beta}$ comprising clocks and rulers needed to make measurements. The connection coefficients and metric tensor are fundamentally independent quantities. They exhibit no *a priori* known relationship, and if they are to have any it must derive from some additional constraints. This property is best expressed by the non-metricity tensor

$$\mathbb{Q}^{\alpha\beta}_{\lambda}(g, \mathbb{\Gamma}) = \nabla^{\mathbb{\Gamma}}_{\lambda} g^{\alpha\beta} \tag{2.4}$$

which is non-vanishing for a general connection $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$. This rank (2,1) tensor would identically vanish if the connection were compatible with the metric. Indeed, in GR, for instance, the constraint to relate $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$ to $g_{\alpha\beta}$ is realized by imposing $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}= \Gamma^{\lambda}_{\alpha\beta}$ from the scratch, where $\Gamma^{\lambda}_{\alpha\beta}$ is the Levi-Civita connection (2.1) which respect to which metric stays covariantly constant, $\nabla^{\Gamma}_{\lambda} g_{\alpha\beta} = 0$, and hence, non-metricity vanishes identically. Furthermore, for this particular connection, the torsion also vanishes identically since $\Gamma^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha}$ by definition.

The curving and twirling of the spacetime are governed by the connection $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$. The metric tensor has nothing to do with them, and the Riemann curvature tensor (2.2) contracts, with no involvement of the metric tensor, in three different ways to generate the associated Ricci tensors of $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$:

- $\mathbb{R}_{\alpha\beta}(\mathbb{\Gamma}) \equiv \mathbb{R}^{\mu}_{\alpha\mu\beta}(\mathbb{\Gamma})$,
- $\widehat{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma}) \equiv \mathbb{R}^{\mu}_{\alpha\beta\mu}(\mathbb{\Gamma}) = -\mathbb{R}_{\alpha\beta}(\mathbb{\Gamma})$,
- $\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma}) \equiv \mathbb{R}^{\mu}_{\mu\alpha\beta}(\mathbb{\Gamma}) = \partial_{\alpha}\, \mathbb{\Gamma}^{\mu}_{\beta\mu} - \partial_{\beta}\, \mathbb{\Gamma}^{\mu}_{\alpha\mu}$.

The reason for having more than one Ricci tensor is that the Riemann tensor (2.2) possesses only a single symmetry $\mathbb{R}^{\mu}_{\alpha\nu\beta}(\mathbb{\Gamma}) = -\mathbb{R}^{\mu}_{\alpha\beta\nu}(\mathbb{\Gamma})$. It is this symmetry property that gives the relation $\widehat{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma}) = -\mathbb{R}_{\alpha\beta}(\mathbb{\Gamma})$ between the first two Ricci tensors above. The third Ricci tensor $\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma})$ does not exist in the General Relativity (GR) since symmetries of the Riemann tensor, $\mathbb{R}_{\mu\alpha\nu\beta}(\Gamma) \equiv g_{\mu\mu'}\mathbb{R}^{\mu'}_{\alpha\nu\beta}(\Gamma) = -\mathbb{R}_{\mu\alpha\beta\nu}(\Gamma) = -\mathbb{R}_{\alpha\mu\nu\beta}(\Gamma) = \mathbb{R}_{\nu\beta\mu\alpha}(\Gamma)$, admits only one single independent Ricci tensor, the $\mathbb{R}_{\alpha\beta}(\Gamma)$ defined above.

Unlike the Riemann and Ricci tensors, the curvature scalar is obtained only by contraction with the inverse metric. Therefore, one finds the curvature scalar

$$\mathbb{R}(g, \mathbb{\Gamma}) \equiv g^{\alpha\beta}\mathbb{R}_{\alpha\beta}(\mathbb{\Gamma}) = -g^{\alpha\beta}\widehat{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma}) \equiv -\widehat{\mathbb{R}}(g, \mathbb{\Gamma}) \tag{2.5}$$

from the first two Ricci tensors listed above. Likewise, the third Ricci tensor contracts to

$$\overline{\mathbb{R}}(g, \mathbb{\Gamma}) = g^{\alpha\beta}\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma}) = 0 \tag{2.6}$$

as dictated by the anti-symmetric nature of $\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma})$. As a result, the theory possesses two distinct Ricci tensors but a single Ricci scalar.

The action density describing matter and gravity is formed by invariants generated by the tensor fields above plus the matter Lagrangian. A partial list includes

$$\begin{aligned}
&\mathbb{R},\ \mathbb{S}\bullet\mathbb{S},\ \mathbb{Q}\bullet\mathbb{Q},\ \mathbb{Q}\bullet\mathbb{S},\\
&\mathbb{R}^2,\ \mathbb{R}\bullet\mathbb{R},\ \overline{\mathbb{R}}\bullet\overline{\mathbb{R}},\\
&\mathbb{R}\bullet\mathbb{S}\bullet\mathbb{S},\ \mathbb{R}\bullet\mathbb{Q}\bullet\mathbb{Q},\ \mathbb{R}\bullet\mathbb{Q}\bullet\mathbb{S},\\
&\overline{\mathbb{R}}\bullet\mathbb{S}\bullet\mathbb{S},\ \overline{\mathbb{R}}\bullet\mathbb{Q}\bullet\mathbb{Q},\ \overline{\mathbb{R}}\bullet\mathbb{Q}\bullet\mathbb{S},\\
&\mathbb{S}\bullet\mathbb{S}\bullet\mathbb{S}\bullet\mathbb{S},\ \mathbb{Q}\bullet\mathbb{Q}\bullet\mathbb{Q}\bullet\mathbb{Q},\\
&\mathbb{S}\bullet\mathbb{Q}\bullet\mathbb{Q}\bullet\mathbb{Q},\ \mathbb{S}\bullet\mathbb{S}\bullet\mathbb{Q}\bullet\mathbb{Q},\\
&\mathbb{S}\bullet\mathbb{S}\bullet\mathbb{S}\bullet\mathbb{Q},\ \mathcal{L}_{\texttt{matter}}(g, \mathbb{\Gamma}, \psi)
\end{aligned} \tag{2.7}$$

where $\mathcal{L}_m(g, \mathbb{\Gamma}, \psi)$ is the matter Lagrangian which explicitly involves the matter and radiation fields ψ, the metric g and the connection $\mathbb{\Gamma}$. The first line of the list consists of mass dimension-2 invariants while the rest involve mass dimension-4 ones. Those structures having mass dimension-5 or higher are not shown. Also not shown are the invariants involving the covariant derivatives of the tensors. The bullet ($\bullet$) stands for contraction of the tensors in all possible ways by using the metric tensor, in case needed.

The scalars in (2.7), most of which do not exist at all in the GR, contain novel degrees of freedom reflecting the non-Riemannian nature of the underlying geometry. These degrees of freedom can be explicated via the decomposition of the connection

$$\mathbb{\Gamma}^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\alpha\beta} + \Delta^{\lambda}_{\alpha\beta} \tag{2.8}$$

with respect to the Levi-Civita connection (2.1), which is the most natural connection one would consider in the presence of the metric tensor. In this decomposition, $\Delta^{\lambda}_{\alpha\beta}$, being the difference between two connections, is a rank (1,2) tensor field, and it is the source of various non-Riemannian invariants listed in (2.7). To this end, in response to (2.8), the Ricci curvature tensor $\mathbb{R}_{\alpha\beta}(\mathbb{\Gamma})$ splits as

$$\mathbb{R}_{\alpha\beta}(\mathbb{\Gamma}) = R_{\alpha\beta}(\Gamma) + \mathcal{R}_{\alpha\beta}(\Delta) \tag{2.9}$$

where $R_{\alpha\beta}(\Gamma) \equiv \mathbb{R}_{\alpha\beta}(\Gamma)$ is the Ricci curvature tensor of the Levi-Civita connection, and

$$\mathcal{R}_{\alpha\beta} = \nabla_{\mu}\Delta^{\mu}_{\beta\alpha} - \nabla_{\beta}\Delta^{\mu}_{\mu\alpha} + \Delta^{\mu}_{\mu\nu}\Delta^{\nu}_{\beta\alpha} - \Delta^{\mu}_{\beta\nu}\Delta^{\nu}_{\mu\alpha} \tag{2.10}$$

where $\nabla_{\alpha} \equiv \nabla^{\Gamma}_{\alpha}$ is the covariant derivative of the Levi-Civita connection $\Gamma^{\lambda}_{\alpha\beta}$. This tensor is a rank (0,2) tensor field generated by the tensorial connection $\Delta^{\lambda}_{\alpha\beta}$ alone. It is actually not a true curvature tensor as it is generated by none of the covariant derivatives $\nabla^{\mathbb{\Gamma}}$ or ∇^{Γ}. It is a 'quasi' curvature tensor.

In response to (2.8), the purely non-Riemannian Ricci tensor $\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma})$ takes the form

$$\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma}) \quad = \quad \partial_{\alpha}\mathtt{V}_{\beta} - \partial_{\beta}\mathtt{V}_{\alpha} = \nabla^{\Gamma}_{\alpha}\mathtt{V}_{\beta} - \nabla^{\Gamma}_{\beta}\mathtt{V}_{\alpha} \tag{2.11}$$

wherein the second equality, which ensures that $\mathbb{R}_{\alpha\beta}(\mathbb{\Gamma})$ is a rank (0,2) anti-symmetric tensor field, follows from the symmetric nature of the Levi-Civita connection, $\Gamma^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha}$. It is obvious that $\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma})$, in the form (2.11), is nothing but the field strength tensor

$$\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma}) \equiv \mathtt{V}^{(-)}_{\alpha\beta} \equiv \partial_{\alpha}\mathtt{V}_{\beta} - \partial_{\beta}\mathtt{V}_{\alpha} \tag{2.12}$$

of the Abelian vector

$$\mathtt{V}_{\alpha} = \Delta^{\mu}_{\alpha\mu} \tag{2.13}$$

which is of purely geometrical origin. Consequently, purely non-Riemannian curvature tensor $\overline{\mathbb{R}}_{\alpha\beta}(\Gamma)$ plays a strikingly different role compared to $\mathbb{R}_{\alpha\beta}(\Gamma)$ in that it directly extracts a vector field out of the underlying geometry.

As a result of (2.8), the torsion and non-metricity tensors

$$\mathbb{S}^{\lambda}_{\alpha\beta}(\Gamma) = \Delta^{\lambda}_{\alpha\beta} - \Delta^{\lambda}_{\beta\alpha} \tag{2.14}$$

$$\mathbb{Q}^{\alpha\beta}_{\lambda}(g,\Gamma) = \Delta^{\alpha}_{\lambda\mu} g^{\mu\beta} + \Delta^{\beta}_{\lambda\mu} g^{\alpha\mu} \tag{2.15}$$

reduce to plain algebraic expressions in terms of $\Delta^{\lambda}_{\alpha\beta}$.

Having explicated the $\Delta^{\lambda}_{\alpha\beta}$ dependencies of the fundamental tensor fields, it is time to ask what the tensorial connection actually is and what information about the geometry can be extracted from it. In other words, $\Delta^{\lambda}_{\alpha\beta}$, which embodies non-Riemannian ingredients of the underlying geometry, must be refined in order to extract the novel geometrical degrees of freedom it contains. As the first option to think of, it is possible that there exist a fundamental rank (1,2) tensor field $\delta^{\lambda}_{\alpha\beta}$, and the connection $\Delta^{\lambda}_{\alpha\beta}$ equals just this fundamental tensor field. Though this is possible, at present there is no indication for such higher spin fields, and thus, it is convenient to leave this possibility aside. The other option to think of is that $\Delta^{\lambda}_{\alpha\beta}$ could be made up of lower spin fields, *i. e.* vectors, spinors and scalars. To this end, given its rank (1,2) nature, it is obvious that the tensorial connection must be decomposable into vector fields, which might be fundamental fields or composites formed out of spinors or scalars. In general, $\Delta^{\lambda}_{\alpha\beta}$ possesses 64 independent elements, and hence, it should be fully parameterizable by 3 independent vector fields, whose nature will be further analyzed in the sequel. One of the vectors is already defined by the contraction $\mathtt{V}_{\alpha}$ in (2.13). The other two

$$\mathtt{U}_{\alpha} = \Delta^{\mu}_{\mu\alpha} \tag{2.16}$$

and

$$\mathtt{W}^{\alpha} = g^{\mu\nu}\Delta^{\alpha}_{\mu\nu} \tag{2.17}$$

are conveniently defined through the remaining two distinct contractions of $\Delta^{\lambda}_{\alpha\beta}$. These two vectors, unlike $\mathtt{V}_{\alpha}$, do not possess an immediate kinetic term, and if they are to have any, it must come from the invariants involving the gradients of the fundamental tensors in (2.7).

Given the metric tensor $g_{\alpha\beta}$, $\mathtt{V}_\alpha$ in (2.13), $\mathtt{U}_\alpha$ in (2.16), and $\mathtt{W}_\alpha$ in (2.17), the tensorial connection $\Delta^\lambda_{\alpha\beta}$ can be algebraically decomposed as

$$\begin{aligned}\Delta^\lambda_{\alpha\beta} &= \delta^\lambda_{\alpha\beta} + a_v \mathtt{V}^\lambda g_{\alpha\beta} + b_v \mathtt{V}_\alpha \delta^\lambda_\beta + c_v \delta^\lambda_\alpha \mathtt{V}_\beta \\ &+ a_u \mathtt{U}^\lambda g_{\alpha\beta} + b_u \mathtt{U}_\alpha \delta^\lambda_\beta + c_u \delta^\lambda_\alpha \mathtt{U}_\beta \\ &+ a_w \mathtt{W}^\lambda g_{\alpha\beta} + b_w \mathtt{W}_\alpha \delta^\lambda_\beta + c_w \delta^\lambda_\alpha \mathtt{W}_\beta \\ &+ \frac{1}{M^2} \sum \left(\nu_{xy} \mathtt{V}^\lambda + \upsilon_{xy} \mathtt{U}^\lambda + \omega_{xy} \mathtt{W}^\lambda \right) \mathtt{X}_\alpha \mathtt{Y}_\beta \end{aligned} \tag{2.18}$$

because of its higher spin assuming that a fundamental rank (1,2) tensor field $\delta^\lambda_{\alpha\beta}$ does not exist at all. The sum in the last line runs over $\mathtt{X}, \mathtt{Y} = \mathtt{V}, \mathtt{U}, \mathtt{W}$, and M is a mass scale expected to be around the fundamental scale of gravity, M_{Pl}. The decomposition necessarily involves linear and trilinear combinations of the vectors. There cannot exist any other acceptable combinations of the vectors. The expansion is unique in structure. However, one notices that all three defining relations (2.13), (2.16), (2.17) are algebraic in nature, and thus, the dimensionless coefficients a's, …, ω's cannot be prohibited to involve dressing factors of the form $\mathtt{I}^\delta/M^\delta$ where $\delta \geq 0$ and $\mathtt{I}$ is an invariant generated by bilinear contractions of the vectors $\mathtt{V}$, $\mathtt{U}$, $\mathtt{W}$. These dressing factors introduce invariants with higher and higher mass dimension. The defining relations (2.13), (2.16) and (2.17) are too few to determine all the expansion coefficients in (2.18). Therefore, all one can do is to express nine of the coefficients in terms of the rest. For instance, the coefficients in the linear sector can be expressed in terms of those in the trilinear sector, leaving ν's, υ's and ω's undetermined, and accordingly, all the invariants in (2.7) can be expanded via (2.18) to determine the dynamics of $\mathtt{V}_\alpha$, $\mathtt{U}_\alpha$, and $\mathtt{W}_\alpha$. Nevertheless, as clearly suggested by (2.18), the main effect of trilinear terms is to generate quartic and higher order interactions of vectors. Putting emphasis on quadratic interactions, the trilinear terms can thus be left aside though they can be straightforwardly included in the formulae below by processing the complete $\Delta^\lambda_{\alpha\beta}$ in (2.18). Proceeding thus with linear terms in (2.18), one finds

$$\begin{aligned} a_v &= c_v = a_u = b_u = b_w = c_w = -\frac{1}{18} \\ b_v &= c_u = a_w = \frac{5}{18} \end{aligned} \tag{2.19}$$

for which $\Delta^{\lambda}_{\alpha\beta}$ gets decomposed linearly in terms of $\mathtt{V}_\alpha$, $\mathtt{U}_\alpha$ and $\mathtt{W}_\alpha$.

Given the decomposition in (2.18) of the tensorial connection, all the invariants in (2.7) can be expressed in terms of $\mathtt{V}_\alpha$, $\mathtt{U}_\alpha$ and $\mathtt{W}_\alpha$ to determine their dynamics as vector fields hidden in the non-Riemannian geometry under consideration. To start with, the curvature scalar $\mathbb{R}(g, \mathbb{\Gamma})$, as follows from (2.9), is composed of the GR part $R(g, \Gamma)$ and the quasi curvature scalar $g^{\alpha\beta}\mathcal{R}_{\alpha\beta}(\Delta) \equiv \mathcal{R}(g, \Delta)$. In response to the linear part of the decomposition of $\Delta^{\lambda}_{\alpha\beta}$ in (2.18), the latter takes the form

$$\mathcal{R}(g, \Delta) = \nabla \cdot (\mathtt{W} - \mathtt{U}) + \frac{1}{18}\Big(\mathtt{V}\cdot\mathtt{V} + \mathtt{U}\cdot\mathtt{U} + \mathtt{W}\cdot\mathtt{W} - 4\mathtt{V}\cdot\mathtt{U} - 4\mathtt{V}\cdot\mathtt{W} + 14\mathtt{U}\cdot\mathtt{W}\Big) \quad (2.20)$$

which shows that a term linear in $\mathbb{R}(g, \mathbb{\Gamma})$ in the gravitational Lagrangian yields the Einstein-Hilbert term $R(g, \Gamma)$ in GR plus a theory of three vector fields in which each vector develops a 'mass term' and mixes with the others quadratically. The vectors do not acquire a kinetic term from $\mathbb{R}(g, \mathbb{\Gamma})$ since the first term at the right-hand side of (2.20), the divergence of $\mathtt{W}_\alpha - \mathtt{U}_\alpha$, does not contribute to dynamics as it can be integrated out of the action by using $\sqrt{-g}\nabla \cdot (\mathtt{W} - \mathtt{U}) = \partial_\alpha\left(\sqrt{-g}\,(\mathtt{W}^\alpha - \mathtt{U}^\alpha)\right)$. One, however, notices that this term becomes important in higher curvature terms like $\mathbb{R}^2(g, \mathbb{\Gamma})$.

From (2.11) it is already known that $\overline{\mathbb{R}}_{\alpha\beta}(\mathbb{\Gamma})$ is the field strength tensor of the vector field $\mathtt{V}_\alpha$. Then the associated invariant in (2.7) becomes

$$\overline{\mathbb{R}} \bullet \overline{\mathbb{R}} = \mathtt{V}^{(-)\alpha\beta}\mathtt{V}^{(-)}_{\alpha\beta} \quad (2.21)$$

which is nothing but the kinetic term of the Abelian vector $\mathtt{V}_\alpha$.

Corresponding to the decomposition in (2.18), the torsion and non-metricity tensors take the explicit form

$$\mathbb{S}^{\lambda}_{\alpha\beta} = \frac{1}{3}\left(\mathtt{V}_\alpha\delta^{\lambda}_{\beta} - \delta^{\lambda}_{\alpha}\mathtt{V}_\beta\right) - \frac{1}{3}\left(\mathtt{U}_\alpha\delta^{\lambda}_{\beta} - \delta^{\lambda}_{\alpha}\mathtt{U}_\beta\right), \quad (2.22)$$

$$\mathbb{Q}^{\alpha\beta}_{\lambda} = \frac{1}{9}\Big(5\mathtt{V}_\lambda g^{\alpha\beta} - \mathtt{V}^\alpha\delta^{\beta}_{\lambda} - \delta^{\alpha}_{\lambda}\mathtt{V}^\beta - \mathtt{U}_\lambda g^{\alpha\beta} + 2\mathtt{U}^\alpha\delta^{\beta}_{\lambda} + 2\delta^{\alpha}_{\lambda}\mathtt{U}^\beta - \mathtt{W}_\lambda g^{\alpha\beta} + 2\mathtt{W}^\alpha\delta^{\beta}_{\lambda} + 2\delta^{\alpha}_{\lambda}\mathtt{W}^\beta\Big), \quad (2.23)$$

and thus, the related invariants in (2.7) read as

$$\mathbb{S}\bullet\mathbb{S} = 2\left(\mathtt{V}\cdot\mathtt{V}+\mathtt{U}\cdot\mathtt{U}-2\mathtt{V}\cdot\mathtt{U}\right), \tag{2.24}$$

$$\mathbb{Q}\bullet\mathbb{Q} = \frac{2}{9}\Big(22\mathtt{V}\cdot\mathtt{V}+7\mathtt{U}\cdot\mathtt{U}+7\mathtt{W}\cdot\mathtt{W}+20\mathtt{V}\cdot\mathtt{U}+20\mathtt{V}\cdot\mathtt{W}+14\mathtt{U}\cdot\mathtt{W}\Big), \tag{2.25}$$

$$\mathbb{Q}\bullet\mathbb{S} = \frac{4}{3}\Big(2\mathtt{V}\cdot\mathtt{V}+\mathtt{U}\cdot\mathtt{U}-3\mathtt{V}\cdot\mathtt{U}-\mathtt{V}\cdot\mathtt{W}+\mathtt{U}\cdot\mathtt{W}\Big). \tag{2.26}$$

This completes the decomposition of the quadratic invariants of the vector fields as generated by the curvature, torsion and non-metricity tensors. It is clear that these invariants provide a kinetic term only for $\mathtt{V}_\alpha$; the other two vectors, $\mathtt{U}_\alpha$ and $\mathtt{W}_\alpha$, acquire no kinetic term from any of the invariants in (2.7). Nevertheless, a short glance at (2.22) and (2.23) immediately reveals that the invariants formed by the gradients of curvature, torsion and non-metricity tensors can generate the requisite kinetic terms. Specifically, from (2.22) it is found that

$$\mathbb{D}_{\alpha\beta} = \nabla^{\emptyset}_{\lambda}\mathbb{S}^{\lambda}_{\alpha\beta} \supset -\frac{1}{3}\mathtt{V}^{(-)}_{\alpha\beta} + \frac{1}{3}\mathtt{U}^{(-)}_{\alpha\beta} \tag{2.27}$$

where the terms $O\left(\Delta^2\right)$ are suppressed on the basis of unnecessity. The first term at the right-hand side is the field strength tensor of $\mathtt{V}_\alpha$ as mentioned in (2.11) and (2.12). The second term is new in that it is the field strength tensor of the $\mathtt{U}_\alpha$ field. Therefore, divergence of torsion tensor generates the requisite kinetic term for $\mathtt{U}_\alpha$, and the associated invariant

$$\mathbb{D}\bullet\mathbb{D} \supset \frac{1}{9}\Big(\mathtt{V}^{(-)\alpha\beta}\mathtt{V}^{(-)}_{\alpha\beta} + \mathtt{U}^{(-)\alpha\beta}\mathtt{U}^{(-)}_{\alpha\beta} - 2\mathtt{V}^{(-)\alpha\beta}\mathtt{U}^{(-)}_{\alpha\beta}\Big) \tag{2.28}$$

encodes the kinetic terms of $\mathtt{V}_\alpha$ and $\mathtt{U}_\alpha$ as well as their kinetic mixing. One notices that, not only the divergence operation (2.27) but also

$$g^{\rho\alpha}\nabla^{\emptyset}_{\rho}\mathbb{S}^{\lambda}_{\alpha\beta} = -g^{\rho\alpha}\nabla^{\emptyset}_{\rho}\mathbb{S}^{\lambda}_{\beta\alpha} \tag{2.29}$$

give contributions to the kinetic terms of vectors with similar structures as (2.28).

The candidate kinetic terms of $\mathtt{V}_\alpha$ in (2.21), and the kinetic term of $\mathtt{U}_\alpha$ in (2.28) are of the form expected of an $U(1)$ invariance. Of course, such an invariance is explicitly broken by the 'mass terms' generated by curvature, torsion and non-metricity tensors. This is not the whole story, however. The kinetic terms generated by the derivatives of the non-metricity tensor in (2.23) also violate possible $U(1)$ invariance suggested by (2.21) and (2.28). To see this, one notes that

$$\begin{aligned}\mathbb{N}^{\alpha\beta} = g^{\rho\lambda}\nabla^{\Diamond}_{\rho}\mathbb{Q}^{\alpha\beta}_{\lambda} \;&\supset\; \frac{1}{9}\Big(5\nabla\cdot\mathtt{V}g^{\alpha\beta} - \mathtt{V}^{(+)\alpha\beta} - \nabla\cdot\mathtt{U}g^{\alpha\beta} + 2\mathtt{U}^{(+)\alpha\beta} \\ &- \;\nabla\cdot\mathtt{W}g^{\alpha\beta} + 2\mathtt{W}^{(+)\alpha\beta}\Big)\end{aligned} \tag{2.30}$$

where

$$\mathtt{V}^{(+)}_{\alpha\beta} \equiv \nabla_{\alpha}\mathtt{V}_{\beta} + \nabla_{\beta}\mathtt{V}_{\alpha} \tag{2.31}$$

is the symmetric counterpart of the anti-symmetric field strength tensor $\mathtt{V}^{(-)}_{\alpha\beta}$ in (2.12). This definition holds also for the other vectors. Then the invariant generated by (2.30) reads as

$$\mathbb{N}\bullet\mathbb{N} \supset \frac{1}{162}\sum_{i,j=1}^{3}\mathtt{A}^{(+)}_{i\alpha\beta}\mathtt{K}^{\alpha\beta\mu\nu}_{ij}\mathtt{A}^{(+)}_{j\mu\nu} \tag{2.32}$$

where $\mathtt{A}_i \in (\mathtt{V},\mathtt{U},\mathtt{W})$, and $\mathtt{K}^{\alpha\beta\mu\nu}_{ij}$ is the (i, j)-th entry of the matrix-valued tensor

$$\mathtt{K}^{\alpha\beta\mu\nu} = \begin{pmatrix} \mathtt{K}^{\alpha\beta\mu\nu}_{11} & \mathtt{K}^{\alpha\beta\mu\nu}_{12} & \mathtt{K}^{\alpha\beta\mu\nu}_{13} \\ \mathtt{K}^{\alpha\beta\mu\nu}_{21} & \mathtt{K}^{\alpha\beta\mu\nu}_{22} & \mathtt{K}^{\alpha\beta\mu\nu}_{23} \\ \mathtt{K}^{\alpha\beta\mu\nu}_{31} & \mathtt{K}^{\alpha\beta\mu\nu}_{32} & \mathtt{K}^{\alpha\beta\mu\nu}_{33} \end{pmatrix} \tag{2.33}$$

where

$$\begin{aligned}\mathtt{K}^{\alpha\beta\mu\nu}_{11} &= 202g^{\alpha\beta}g^{\mu\nu} + g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu} \\ \mathtt{K}^{\alpha\beta\mu\nu}_{12} &= \mathtt{K}^{\alpha\beta\mu\nu}_{21} = \mathtt{K}^{\alpha\beta\mu\nu}_{13} = g^{\alpha\beta}g^{\mu\nu} - 2g^{\alpha\mu}g^{\beta\nu} - 2g^{\alpha\nu}g^{\beta\mu} \\ \mathtt{K}^{\alpha\beta\mu\nu}_{22} &= \mathtt{K}^{\alpha\beta\mu\nu}_{23} = \mathtt{K}^{\alpha\beta\mu\nu}_{32} = \mathtt{K}^{\alpha\beta\mu\nu}_{33} = -2g^{\alpha\beta}g^{\mu\nu} + 4g^{\alpha\mu}g^{\beta\nu} + 4g^{\alpha\nu}g^{\beta\mu}\end{aligned}$$

which describes the kinetic mixing among the three vector fields. As for the divergence of torsion in (2.28), one notices that, not only the divergence operation (2.30) but also

$$\nabla^{\between}_{\alpha}\mathbb{Q}^{\alpha\beta}_{\lambda} = \nabla^{\between}_{\alpha}\mathbb{Q}^{\beta\alpha}_{\lambda} \tag{2.34}$$

give contributions similar to that in (2.32). In addition to these, contraction of

$$\nabla^{\between}\mathbb{Q} \bullet \nabla^{\between}\mathbb{S} \quad = \quad 0 \tag{2.35}$$

due to symmetry conditions.

Having done with the decomposition of various invariants in terms of the vector fields $\mathtt{V}$, $\mathtt{U}$ and $\mathtt{W}$, we now turn to analysis of interactions in such a non-Riemannian setup. The most general action functional describing ‘gravity’ and ‘matter’ is of the form

$$I = \int d^4x \, \sqrt{-g} \left\{ \mathfrak{L}\left(\mathbb{R}, \overline{\mathbb{R}}, \mathbb{S}, \mathbb{Q}\right) + L_m\left(g, \between, \psi\right) - V_0 \right\} \tag{2.36}$$

which contains action densities for geometric and material parts, respectively. V_0 stands for the vacuum energy (containing the bare cosmological term fed by the geometrical sector), and ψ stands for matter and radiation fields, collectively. Neither the geometrical $\mathfrak{L}$ nor the matter Lagrangian L_m contains any constant energy density; all such energy components are collected in V_0. The geometrical part reads explicitly as

$$\begin{aligned} \mathfrak{L} \quad = \quad & \frac{1}{2}M^2_{Pl}\left(\mathbb{R} + c_S\mathbb{S} \bullet \mathbb{S} + c_Q\mathbb{Q} \bullet \mathbb{Q} + c_{QS}\mathbb{Q} \bullet \mathbb{S}\right) \\ + \quad & c'_S\nabla^{\between}\mathbb{S} \bullet \nabla^{\between}\mathbb{S} + c'_Q\nabla^{\between}\mathbb{Q} \bullet \nabla^{\between}\mathbb{Q} + c'_{QS}\nabla^{\between}\mathbb{Q} \bullet \nabla^{\between}\mathbb{S} \\ + \quad & c_{R^2}\mathbb{R}^2 + c_{RR}\mathbb{R} \bullet \mathbb{R} + c_{\overline{RR}}\overline{\mathbb{R}} \bullet \overline{\mathbb{R}} + O\left(\frac{1}{M^2_{Pl}}\right) \end{aligned} \tag{2.37}$$

where we have discarded terms $O\left(1/M^2_{Pl}\right)$. Moreover, we have discarded higher-derivative terms $\Box^{\between}\mathbb{R}$ and the like. c’s are all dimensionless couplings. The mass dimension-2 terms are naturally scaled by the fundamental scale of gravity, M_{Pl}. One notices that $\mathbb{R}^2$ and $\mathbb{R} \bullet \mathbb{R}$ contain higher-curvature terms $R(g,\Gamma)^2$ and $R_{\alpha\beta}(\Gamma)R^{\alpha\beta}(\Gamma)$, respectively. Indeed, leaving aside the non-dynamical terms, one can show that

$$\mathbb{R}^2\left(\mathbb{\Gamma}\right) \supset R(g,\Gamma)^2 + \Big((\nabla.\mathtt{W})^2 - 2(\nabla.\mathtt{W})(\nabla.\mathtt{U}) + (\nabla.\mathtt{U})^2 \Big) \tag{2.38}$$

and

$$\begin{aligned}
\mathbb{R}\left(\mathbb{\Gamma}\right) \bullet \mathbb{R}\left(\mathbb{\Gamma}\right) \;\; \supset \;\; & R^2(g,\Gamma) + R_{\mu\nu}(g,\Gamma) R^{\mu\nu}(g,\Gamma) + \frac{1}{648}\Big(-4(\nabla.\mathtt{V})^2 + 162(\nabla.\mathtt{U})^2 + 167(\nabla.\mathtt{W})^2 \\
- \;\; & 330(\nabla.\mathtt{U})(\nabla.\mathtt{W}) - 6(\nabla.\mathtt{V})(\nabla.\mathtt{U}) + 4(\nabla.\mathtt{V})(\nabla.\mathtt{W}) + 16\nabla_\mu \mathtt{V}_\nu \nabla^\nu \mathtt{V}^\mu \\
+ \;\; & 24\nabla_\mu \mathtt{V}_\nu \nabla^\nu \mathtt{U}^\mu + 9\nabla_\mu \mathtt{U}_\nu \nabla^\mu \mathtt{U}^\nu + 10\nabla_\mu \mathtt{V}_\nu \mathtt{V}^{(-)\mu\nu} - 18\nabla_\mu \mathtt{V}_\nu \mathtt{U}^{(-)\mu\nu} \\
- \;\; & 8\nabla_\mu \mathtt{V}_\nu \mathtt{W}^{(+)\mu\nu} + 8\nabla_\mu \mathtt{U}_\nu \mathtt{U}^{(-)\mu\nu} - 6\nabla_\mu \mathtt{U}_\nu \mathtt{W}^{(+)\mu\nu} + 2\nabla_\mu \mathtt{W}_\nu \mathtt{W}^{(+)\mu\nu} \Big)
\end{aligned} \tag{2.39}$$

wherein the GR-related parts are seen to involve higher-derivative interactions. In this sense, the GR-part (the terms $R^2(g,\Gamma)$ and $R_{\mu\nu}(g,\Gamma)R^{\mu\nu}(g,\Gamma)$) brings forth ghosts. Clearly, these terms must be absent (c_{R^2} and c_{RR} must vanish) if such ghosty contributions in GR are to be avoided. The remaining terms, after using their decompositions in terms of the vector fields $\mathtt{V}$, $\mathtt{U}$ and $\mathtt{W}$, give rise to the action

$$\begin{aligned}
I \;\; = \;\; & \int d^4x \, \sqrt{-g} \left\{ \frac{1}{2} M_{Pl}^2 R + L_m\left(g, \mathbb{\Gamma}, \psi\right) - V_0 \right\} \\
+ \;\; & \int d^4x \, \sqrt{-g} \Big\{ c_{VV} \mathtt{V}^{(-)\alpha\beta} \mathtt{V}^{(-)}_{\alpha\beta} + c_{UU} \mathtt{U}^{(-)\alpha\beta} \mathtt{U}^{(-)}_{\alpha\beta} + c_{VU} \mathtt{V}^{(-)\alpha\beta} \mathtt{U}^{(-)}_{\alpha\beta} \\
+ \;\; & \mathtt{V}^{(+)}_{\alpha\beta} \mathbf{k}_{VV}^{\alpha\beta\mu\nu} \mathtt{V}^{(+)}_{\mu\nu} + \mathtt{U}^{(+)}_{\alpha\beta} \mathbf{k}_{UU}^{\alpha\beta\mu\nu} \mathtt{U}^{(+)}_{\mu\nu} + \mathtt{W}^{(+)}_{\alpha\beta} \mathbf{k}_{WW}^{\alpha\beta\mu\nu} \mathtt{W}^{(+)}_{\mu\nu} + \mathtt{V}^{(+)}_{\alpha\beta} \mathbf{k}_{VU}^{\alpha\beta\mu\nu} \mathtt{U}^{(+)}_{\mu\nu} + \mathtt{V}^{(+)}_{\alpha\beta} \mathbf{k}_{VW}^{\alpha\beta\mu\nu} \mathtt{W}^{(+)}_{\mu\nu} \\
+ \;\; & \mathtt{U}^{(+)}_{\alpha\beta} \mathbf{k}_{UW}^{\alpha\beta\mu\nu} \mathtt{W}^{(+)}_{\mu\nu} + M_{Pl}^2 \Big(\frac{1}{2} a_{VV} \mathtt{V}^\alpha \mathtt{V}_\alpha + \frac{1}{2} a_{UU} \mathtt{U}^\alpha \mathtt{U}_\alpha + \frac{1}{2} a_{WW} \mathtt{W}^\alpha \mathtt{W}_\alpha + a_{VU} \mathtt{V}^\alpha \mathtt{U}_\alpha + a_{VW} \mathtt{V}^\alpha \mathtt{W}_\alpha \\
+ \;\; & a_{UW} \mathtt{U}^\alpha \mathtt{W}_\alpha \Big) \Big\}
\end{aligned} \tag{2.40}$$

where the first integral at the right-hand side is precisely the Einstein-Hilbert action in GR (plus the contribution of matter and radiation), and the second integral pertains to a theory of three vector fields in a spacetime with metric $g_{\alpha\beta}$. The Einstein-Hilbert action above would receive contributions from higher-curvature (and thus typically ghosty) terms had we kept $\mathbb{R}^2$ and $\mathbb{R} \bullet \mathbb{R}$ terms in (2.37).

In essence, under the decomposition in (2.18), the non-Riemannian gravitational theory in (2.37) reduces to a tensor-vector theory of the type in (2.40) (leaving aside the matter sector $L_m\left(g, \mathbb{\Gamma}, \psi\right)$). One notices that the general connection $\mathbb{\Gamma}^\lambda_{\alpha\beta}$ can directly couple to matter fields as encoded in the matter Lagrangian. According to types of the matter fields, these couplings give rise to additional structures (like hyper-momentum) which involve torsion and non-metricity. In (Vitagliano et al., 2011; Sotiriou and Liberati, 2007), various effects of the general connection on the matter sector are analysed in detail. The vector part of the action is written in a rather generic form by admitting that various terms listed above plus similar ones coming, for example, from (2.29) and (2.34) give rise to, at the quadratic level, the structures in (2.40) with dimensionless coefficients $c_{VV}, \ldots, a_{UW}$. These coefficients can be expressed as linear combinations of the coefficients weighing individual contributions.

The tensor-vector theory in (2.40) has been obtained for a general setup involving curvature, torsion and non-metricity tensors exhaustively. The theory is GR plus a theory of three vectors $\mathtt{V}$, $\mathtt{U}$ and $\mathtt{W}$. Any constraint or selection rule imposed on the non-Riemannian geometry results in a more restricted theory. It could thus be useful to discuss certain aspects of (2.40) here:

- Theory consists of three vector fields $\mathtt{V}$, $\mathtt{U}$ and $\mathtt{W}$. The vector action contains two types of kinetic terms: ones with $\mathtt{X}^{(-)}_{\alpha\beta}$ and those with $\mathtt{X}^{(+)}_{\alpha\beta}$. The $\mathtt{V}$ and $\mathtt{U}$ possess both types of kinetic terms while $\mathtt{W}$ possesses only the second type *i. e.* $\mathtt{W}^{(+)}_{\alpha\beta}$. The $\mathtt{X}^{(-)}_{\alpha\beta}$ and hence the corresponding kinetic terms obviously possess an Abelian invariance. However, there is no such invariance for the kinetic terms involving $\mathtt{X}^{(+)}_{\alpha\beta}$. Therefore, the vector fields contained in (2.40) are not associated with a gauge theory; they are not vectors originating from need to realize a local U(1) invariance.

  The coefficients $c_{VV}, \ldots, k_{UV}^{\alpha\beta\mu\nu}$, which seem being left arbitrary, can actually be fixed in terms of the coefficients of individual terms in (2.37) which contribute to that particular structure. The kinetic terms, both $\mathtt{X}^{(-)}_{\alpha\beta}$ and $\mathtt{X}^{(+)}_{\alpha\beta}$ type, receive contributions from various structures, as addressed before. In particular, contributions of the alternative structures given in (2.29) and (2.34) must also be included in forming the vector action in (2.40).

- A highly crucial aspect concerns the signs of the coefficients $c_{VV}, \ldots, k_{UV}^{\alpha\beta\mu\nu}$ in the kinetic part of the vector action. The kinetic terms of $\mathtt{V}$, $\mathtt{U}$ and $\mathtt{W}$ must have the correct sign required of a ghost-free theory. Indeed, any sign-flip in the kinetic terms causes vector ghosts to show up in the spectrum. The various coefficients in (2.37) must comply with this requirement.

- The vectors exhibit not only the kinetic mixings $\mathtt{X}^{(-)}_{\alpha\beta}\mathtt{Y}^{(-)\alpha\beta}$ and $\mathtt{X}^{(+)}_{\alpha\beta}\mathtt{Y}^{(+)\alpha\beta}$ but also mass mixings of the form $\mathtt{X}^{\alpha}\mathtt{Y}_{\alpha}$, as shown in the last line of the vector action. Their masses and mixings are proportional to M_{Pl} with respective coefficients $a_{VV}, \ldots, a_{UW}$.

In $\{\mathtt{V}, \mathtt{U}, \mathtt{W}\}$ basis their mass-squared matrix reads as

$$\frac{1}{2} M_{Pl}^2 \begin{pmatrix} a_{VV} & a_{VU} & a_{VW} \\ a_{VU} & a_{UU} & a_{UW} \\ a_{VW} & a_{UW} & a_{WW} \end{pmatrix} \tag{2.41}$$

each entry of which can be extracted from (2.37) as

$$\begin{aligned} a_{VV} &= \frac{1}{18} + 2c_S + \frac{44}{9} c_Q + \frac{8}{3} c_{QS} \,, \\ a_{UU} &= c_{WW} + 2c_S + \frac{4}{3} c_{QS} \,, \\ a_{WW} &= \frac{1}{18} + \frac{14}{9} c_Q \,, \\ a_{VU} &= -\frac{1}{9} - 2c_S + \frac{20}{9} c_Q - 2c_{QS} \,, \\ a_{VW} &= -\frac{1}{9} + \frac{20}{9} c_Q - \frac{2}{3} c_{QS} \,, \\ a_{UW} &= \frac{7}{18} + \frac{14}{9} c_Q + \frac{2}{3} c_{QS} \,. \end{aligned} \tag{2.42}$$

It is the eigenvalues of (2.41) that determine the light and heavy vector spectrum in the theory. For having a stable theory free from tachyons, the eigenvalues of (2.41) must each be positive semi-definite. This puts stringent constraints on the elements $a_{VV}, \cdots, a_{UW}$ (See Appendix B for further details.). If off-diagonal entries are small *i. e.* if c_S, c_Q and c_{QS} are chosen appropriately then all three vector bosons weigh $M_{Pl}/3\sqrt{2}$. Alternatively, if the mixings are sizeable, or equivalently, if all entries of (2.41) are of similar size then there will exist two light and one heavy vectors in the spectrum. Depending on the hierarchy of the couplings, there could exist just one light state instead of two (Demir, 2004). In any case, it is with the hierarchy of the couplings that the vector boson spectrum can exhibit different hierarchies. Needless to say, the intra-hierarchy of the mass matrix entries $a_{VV}, \ldots, a_{UW}$ is determined by the couplings c_S, c_Q and c_{QS} via the relations (2.42).

Actually, having the vector fields with masses around M_{Pl} should come by no surprise; the underlying theory (2.37) is a pure gravity of non-Riemannian structure, and the mass scale in the theory is automatically fixed by the fundamental scale of gravity M_{Pl}. However, the statement ‘a Planckian-mass vector field’ depends crucially on what we mean by the vector field: Is it fundamental or is it a composite structure? We will discuss answers and consequences of these questions in the sequel.

- As is obvious from the general procedure, reduction of the non-Riemannian gravity gives rise to GR plus extra degrees of freedom represented by the vector fields in (2.40). These extra degrees of freedom can have astrophysical and cosmological implications, and can give rise to observable phenomena at high-energy particle colliders. These fields may form an invisible sector which couples to known matter via Higgs or vector boson portals. We shall discuss some of their cosmological effects in the next section.

- The framework we have reached in (2.40) is a rather general one in that we have imposed no condition on metric, connection and any other geometro-dynamical quantity. Imposition of certain selection rules, though seems to cause loss of generality, does actually prove highly useful for extracting information about behavior of the system in certain reasonable situations. Here we shall discuss two such limiting cases:

  - **Symmetric Connection:** We first discuss the possibility of symmetric connection *i. e.* $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}=\mathbb{\Gamma}^{\lambda}_{\beta\alpha}$. The prime implication of this selection rule is that the torsion tensor identically vanishes, $\mathbb{S}^{\lambda}_{\alpha\beta} = 0$. This statement is equivalent to imposing

$$\mathtt{V}_{\alpha} = \mathtt{U}_{\alpha}, \tag{2.43}$$

    as is manifestly suggested by the decomposition of $\Delta^{\lambda}_{\alpha\beta}$ in (2.18). This constraint is seen to nullify the invariants $\mathbb{S}\cdot\mathbb{S}$ and $\mathbb{S}\cdot\mathbb{Q}$, in agreement with vanishing torsion. This particular relation between $\mathtt{V}$ and $\mathtt{U}$ reduces the vector action in (2.40) into a theory of two vectors: the $\mathtt{V}$ and $\mathtt{W}$. The structure remains similar to that in (2.40) yet various terms containing $\mathtt{V}$ and $\mathtt{U}$ merge together to give more compact relations.

  - **Antisymmetric Tensorial Connection:** This time we consider the relation $\Delta^{\lambda}_{\alpha\beta} = -\Delta^{\lambda}_{\beta\alpha}$ for the tensorial connection not for $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$. Actually, since $\Gamma^{\lambda}_{\alpha\beta}$ is symmetric the connection $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}$ possesses no obvious symmetry under the exchange of α and β. The prime implication of the anti-symmetric Δ is that the geodesics of test bodies remain as in the GR. This, however, does not mean that one can eliminate the non-Riemannian effects. The reason is that the geodesic deviation, which involves the Riemann tensor $\mathbb{R}^{\alpha}_{\mu\beta\nu}$, directly feels the non-GR components of the curvature tensor. In the language of the expansion (2.18), anti-symmetric $\Delta^{\lambda}_{\alpha\beta}$ gives

$$\mathtt{V}_{\alpha} = -\mathtt{U}_{\alpha} \quad \text{and} \quad \mathtt{W}_{\alpha} = 0 \tag{2.44}$$

which reduces thus the vector action in (2.40) to theory of a single vector field $\mathtt{V}$.

Here we have highlighted certain salient features of the Tensor-Vector theory of (2.40) in regard to various structures and limiting cases the vector part can take.

2.3. Applications to Cosmology

Up to now, we have constructed a general action which consists of all possible vector and tensor fields. In addition to this, we have given two limiting cases as symmetric and antisymmetric tensorial connection. In next two subsections, by using antisymmetric tensorial connection limit and some constraints, we obtain two well-known actions which are defined in modified gravity theories These are TeVeS gravity and Vector Inflation.

2.3.1. TeVeS Gravity

In spite of its great success in describing the solar system, General Relativity (GR) fails to account for dynamics at galactic scales without postulating a large amount of cold dark matter (CDM) – non-baryonic, non-relativistic, electrically neutral, weakly interacting particles of weak-scale masses (Bertone et al., 2005). The asymptotic flatness of the galaxy rotation curves, which occurs towards galaxy outskirts involving extremely small accelerations, manifestly disagrees with predictions of the GR unless the galactic region is populated by non-shining, and hence, astrophysically unobservable CDM.

Apart from this, there are problems with structure formation: with the baryonic matter alone, the large-scale structure as we observe it would not have been formed yet if gravity is described by GR. Indeed, GR demands large amounts of 'dark components' (23% 'dark matter' for structure formation and 73% 'dark energy' for late-time inflation) to be able to account for the mounting cosmo-physical precision data (coming from observations on microwave background (Komatsu et al., 2009), large scale structure (Tegmark et al., 2006), and supernovae (Astier et al., 2006)). However, the way these dark components enter into gravitational field equations does not involve their origins and nature; they are treated as 'fluids' with right density and equation of state. Nevertheless, the positron excess reported by recent observations (Adriani et al., 2009; Abdo et al., 2009) on cosmic rays, if interpreted to come from decays or annihilations of dark matter, can be taken as indirect signals of dark matter (though there are alternative arguments in favor of astrophysical sources (Hooper et al., 2009; Yüksel et al., 2009) of positron excess).

This 'dark paradigm' necessitated by GR can in fact be evaded if an alternative description of Nature takes over at extremely small accelerations and curvatures. This is what has been postulated by Milgrom (Milgrom, 1983a; Bekenstein and Milgrom, 1984), who replaced Newton's second law of motion with

$$\mu\left(\frac{\left|\vec{a}\right|}{a_0}\right)\vec{a} = -\vec{\nabla}\Phi_N \tag{2.45}$$

where Φ_N is the gravitational potential, $\mu(x) \rightsquigarrow 1(x)$ for $x \gg 1(x \ll 1)$, and $a_0 \simeq 10^{-10}\ m/s^2$ is an acceleration scale appropriate for galaxy outskirts (Bernal et al., 2011). This proposal, despite its empirical success, had to wait for the relativistic generalizations of (Sanders, 1997, 2005; Bekenstein, 2004b,a) to become a complete, alternative theory of gravitational interactions (see also the review (Skordis, 2009)). The relativistic generalization, dubbed as tensor-vector-scalar (TeVeS) theory of gravity, involves the *geometrical* fields V_μ and ϕ in addition to the metric tensor $g_{\mu\nu}$ such that, while the matter sector involves $g_{\mu\nu}$ only, the gravitational sector involves

$$\widetilde{g}_{\mu\nu} = e^{2\phi}g_{\mu\nu} - 2\sinh(2\phi)A_\mu A_\nu \tag{2.46}$$

whose action can be generalized to incorporate aether effects (Zlosnik et al., 2006, 2007; Skordis, 2008), too. Various astrophysical and cosmological phenomena exhibit observable signatures of TeVeS (Giannios, 2005; Chiu et al., 2006; Skordis, 2006; Diaz-Rivera et al., 2006; Chen and Zhao, 2006; Sagi and Bekenstein, 2008; Chen, 2008; Contaldi et al., 2008; Bekenstein and Sagi, 2008; Tamaki, 2008; Ferreira et al., 2008; Mavromatos et al., 2009; Ferreras et al., 2009; Lasky, 2009; Sagi, 2010; Reyes et al., 2010).

TeVeS is essentially a bi-metrical gravitational theory where matter and gravity are distinguished by the metric fields they operate with. It is thus natural to expect a reformulation in bi-metrical language (Banados et al., 2009; Bañados et al., 2009) wherein certain interactions and properties follow deductively.

The material produced in the last section is general and detailed enough to have a re-look at the TeVeS gravity. In this section we will argue that TeVeS type extended gravity theories do naturally follow from the non-Riemannian theories of the form (2.37) under the decomposition (2.18).

- To establish contact with TeVeS gravity, it is necessary to discuss first the function μ defined in (2.45). In relativistic formulation, μ is a non-dynamical field in the action. Variation of the action with respect to μ fixes 'gradient' of its potential $dV(\mu)/d\mu$ in terms of the remaining terms in which μ appears at least linearly. Basically, μ must multiply the kinetic term of (scalars or vectors) so that its force $dV(\mu)/d\mu$ is fixed in terms of the field gradient-squareds (actually the kinetic terms of the fields) in accord with the requirements of the MOND. In summary, the MOND relation (2.45) for μ arises from the equation of motion for μ (to be solved via $dV(\mu)/d\mu$ in terms of the kinetic terms of the fields in the spectrum). The relativistic theory of (Bekenstein, 2004b,a) requires that μ should be non-dynamical, that is, it should have no kinetic term. Therefore, the Lagrangian of μ can be directly constructed from couplings in the action (2.40). We do this as follows:
  - First, we postulate that the vacuum energy density V_0 in (2.36) and (2.40) can actually be decomposed as

$$V_0 = V(\mu) + \Delta V \tag{2.47}$$

    where ΔV is a constant additive energy density while V varies with μ. At this stage μ is a hypothetical parameter having no solid physical basis.
  - We further postulate that the coefficients $c_{VV}, \dots, k_{UV}^{\alpha\beta\mu\nu}$ weighing the individual kinetic terms in the vector part of the action (2.40) do actually depend on the parameter μ at least in a linear fashion. In fact, it is not necessary to make all these constants vary with μ; the μ dependence of one single parameter suffices.
- Under these instructed changes for 'creating' or 'explicating' the non-dynamical

field μ, the action (2.40) becomes essentially the Tensor-Vector Theory of (Zlosnik et al., 2006, 2007; Skordis, 2008). This theory is obtained by eliminating the scalar field through the constraints on the bimetrical theory of (Bekenstein, 2004b,a), and is shown to be a viable replacement for cold dark matter. As an aether theory, it works as good as the model in (Bekenstein, 2004b,a) as far as the MOND-change of gravity is concerned. The main distinction between the theory obtained here and that of (Zlosnik et al., 2006, 2007; Skordis, 2008) is that the model here consists of three vectors in the most general case. If one specializes to cases like (2.43) or (2.44), however, the model obtained here gets closer to the aether theory of (Zlosnik et al., 2006, 2007; Skordis, 2008), which is shown therein to be an alternative to the cold dark matter.

Consequently, the Tensor-Vector theory in (2.40) provides a general enough framework (in terms of parameters and number of vector fields) for generating the TeVeS gravity of (Milgrom, 1983a; Bekenstein and Milgrom, 1984; Sanders, 1997, 2005; Bekenstein, 2004b,a; Skordis, 2009) through the analyses in (Zlosnik et al., 2006, 2007; Skordis, 2008). It should be kept in mind that, the TeVeS gravity of (Sanders, 1997, 2005; Bekenstein, 2004b,a) is based on a bimetrical theory where the geometrical sector proceeds with metric involving a scalar field, vector field and the metric field used by the matter Lagrangian. The theory in the present work, however, provides a compact approach to TeVeS gravity via the decomposition of the tensorial connection in (2.18).

- At this point, one may wonder why we are dealing with the Tensor-Vector theory of (Zlosnik et al., 2006, 2007; Skordis, 2008) instead of the true TeVeS gravity of (Sanders, 1997, 2005; Bekenstein, 2004b,a; Skordis, 2009). Actually, as we will shown below, the action (2.40) naturally contains the true TeVeS gravity. To this end, the right question to ask concerns the vector fields themselves: Are they fundamental vector fields or composites of some other fields? They each could be of either nature. Whatever their structure, however, they must be true vectors on the spacetime manifold such that their vector property must not depend on the connections $\between^{\lambda}_{\alpha\beta}$ or $\Gamma^{\lambda}_{\alpha\beta}$ or $\Delta^{\lambda}_{\alpha\beta}$. The reason is that the vectors themselves are just parameterizing the connection via (2.18), and hence, their independence from the connection is required by the logical consistency of the construction. This constraint prohibits all structures but

$$\mathtt{V}_\alpha = a_1 V_\alpha + \frac{a_0}{M_{Pl}} \partial_\alpha \phi \tag{2.48}$$

where V_α is a fundamental vector, and ϕ is a fundamental scalar field. The vector property of V_α is obvious. Why the ϕ-dependent part is a vector is guaranteed by the fact that $\nabla_\alpha^{(any\ connection)} \phi = \partial_\alpha \phi$, and hence, it is a vector on the manifold independent of the connection; may it be $\lozenge^\lambda_{\alpha\beta}$ or $\Gamma^\lambda_{\alpha\beta}$ or some other structure. Obviously, if ϕ is to be a new degree of freedom (not a scalar formed form V_α itself) then it is necessary to reduce the degrees of freedom contained in V_α by one unit. Any 'gauge constraint' such as $\nabla \cdot V = 0$ proves sufficient for this purpose. Under these conditions, the expansion (2.48) operates on each of the vectors $\mathtt{V}$, $\mathtt{U}$ and $\mathtt{W}$ with their respective scalar fields.

It is obvious that replacement of (2.48) and similar relations for $\mathtt{U}_\alpha$ and $\mathtt{W}_\alpha$ into the vector action in (2.40) will yield a general tensor-vector-scalar theory of gravity. The main difference from (Sanders, 1997, 2005; Bekenstein, 2004b,a; Skordis, 2009) will be the number of vectors and scalars in the theory. The difference will be the dependence of the action on the scalars: Only the gradients of scalars are involved. The scalars themselves do not enter the action. Nevertheless, one arrives at a tensor-vector-scalar theory of gravity, and the theory is parametrically and dynamically wide enough to cover the standard TeVeS gravity.

- As a concrete case study, here we shall discuss the reduced theory after imposing the condition (2.44). The action (2.40) reduces to

$$\begin{aligned} I &= \int d^4x \sqrt{-g} \left\{ \frac{1}{2} M_{Pl}^2 R + L_m(g, \lozenge, \psi) - V_0 \right. \\ &+ \overline{c}_{VV} \mathtt{V}^{(-)\alpha\beta} \mathtt{V}^{(-)}_{\alpha\beta} + \mathtt{V}^{(+)}_{\alpha\beta} \overline{\mathtt{k}}_{VV}^{\alpha\beta\mu\nu} \mathtt{V}^{(+)}_{\mu\nu} \\ &+ \left. \frac{1}{2} M_{Pl}^2 \overline{a}_{VV} \mathtt{V}^\alpha \mathtt{V}_\alpha \right\} \end{aligned} \tag{2.49}$$

where the terms involving $\mathtt{V}$ and $\mathtt{U}$ in (2.40) combine to form the over-lined coefficients in here. The terms involving $\mathtt{W}$ in (2.40) are all nullified in accord with (2.44). For instance, one directly finds

$$\overline{a}_{VV} = \frac{1}{3} + 8c_S + 2c_Q + 8c_{QS} \tag{2.50}$$

form (2.42). The reduced theory in (2.49) is precisely the one in (Zlosnik et al., 2006, 2007; Skordis, 2008) except for the absence of quartic-in-$\mathtt{V}$ terms. The couplings in and dynamics of the two theories can be matched via the terms involved in two cases. This situation becomes especially clear after using $\mathtt{V}^{\alpha}\mathtt{V}_{\alpha} = -1$ in the tensor-vector theory of (Zlosnik et al., 2006, 2007; Skordis, 2008).

Now, it is time to analyze (2.49) under the decomposition (2.48). One finds

$$\begin{aligned} I &= \int d^4x \sqrt{-g}\Bigg\{\frac{1}{2}M_{Pl}^2 R + L_m(g, \Diamond, \psi) - V_0 + a_1^2 \overline{c}_{VV} V^{(-)\alpha\beta} V^{(-)}_{\alpha\beta} + a_1^2 V^{(+)}_{\alpha\beta} \overline{\mathtt{k}}_{VV}^{\alpha\beta\mu\nu} V^{(+)}_{\mu\nu} \\ &+ \frac{1}{2} M_{Pl}^2 a_1^2 \overline{a}_{VV} V^{\alpha} V_{\alpha} + M_{Pl} a_1 a_0 \overline{a}_{VV} V^{\alpha} \partial_{\alpha}\phi + a_0^2 \overline{a}_{VV} \partial^{\alpha}\phi \partial_{\alpha}\phi + O\left(\frac{1}{M_{Pl}}\right)\Bigg\} \qquad (2.51) \end{aligned}$$

from which it is seen that setting $V_0 \equiv V(\mu) + \Delta V$ and $a_0 = \bar{a}_0\mu$ essentially suffices to reproduce the results of TeVeS gravity (Bekenstein, 2004b,a; Skordis, 2009). Setting $V^{\alpha}V_{\alpha} = -1$ as a constraint on the vector field, the mass term of V^{α} in (2.51) just adds up to the vacuum energy V_0.

Before closing this section we comment on MOND. The MOND theory (or its relativistic realization TeVeS) has been put forth as an alternative to the Dark Matter paradigm. As for any model, there are phenomena for which TeVeS cannot give a satisfactory explanation. Indeed, while it can explain flat rotation curves with no need to Dark Matter, it has phenomenological shortcomings related to explanations of the other DM evidences such as Bullet Cluster. Nevertheless, like the Dark Matter paradigm all these models are under theoretical and experimental investigation, and one can find better realizations in terms of various constraints. The non-Riemannian origin we discuss is not special to TeVeS or any other specific modeling; it holds in general and its parameter space can be constrained by astrophysical observations or collider experiments.

2.3.2. Vector Inflation

According to the standart big bang cosmology, which is defined by using Friedmann-Robertson-Walker (FRW) metric, universe is homogeneous and isotropic on large scales (Liddle and Lyth, 2000; Tsujikawa, 2003). In addition to this, observations of Hubble in redshifts of galaxies shows that universe is expanding. To understand dynamical properties of expansion, the solutions of Einstein equation for FRW metric are required. Combination of these solutions is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M_{pl}^2}(\rho + 3p) \tag{2.52}$$

as ρ implies energy density and p is pressure and a is scale factor. In the ligth of equation (2.52) one can think that universe expands by decelerating in case of $(\rho + 3p) > 0$. However, this deceleration doesn't solve some problem of standart big bang cosmology such as flatness, horizon and so on. To solve these problems, accelerated expansion of universe in early stage is treated instead of decelerated one i.e $(\rho + 3p) < 0$ and this type of expansion is called "inflation". Inflation is generally driven by scalar fields to prevent anisotropy occured in higher spin fields (Chiba, 2008). However, scalar inflation models have fine-tuning problem and also scalar bosons which is base of these models aren't observed by experiments(Maki et al., 2010). Therefore, vector inflation model is condsidered instead of scalar inflation model. (Ford, 1989; Golovnev et al., 2008) Also p-forms inflation model is also considered in literature(Germani and Kehagias, 2009).

Vector inflation was firstly proposed in (Ford, 1989) by using spacelike vector fields. In Ford's paper vector fields gave anisotropic solution of inflation. So, instead of spacelike vector fields, it was shown that timelike vector fields under some constraints of vector field potential give rise to desired inflationary expansion (Koh, 2011; Jacobson and Mattingly, 2001; Carroll and Lim, 2004). The other problems vector fields have can be solved by using a triplet of mutually orthogonal vector fields and non- minimally coupling.

In this section, we show that after a regularization ,the action (2.40) obtained by using the anti-symmetric connection constraint give the same action in (Koh, 2011; Jacobson and Mattingly, 2001; Carroll and Lim, 2004) which is most general action of vector inflation theory.

Combining the abelian and non-abelian part of vector field and defining new dimensionless coefficients lead to the action (leaving aside the matter sector):

$$I = \int d^4x \, \sqrt{-g}\left\{\frac{1}{2}M_{Pl}^2 R + \frac{1}{2}\kappa^{\alpha\beta\mu\nu}\nabla_\alpha V_\beta \nabla_\mu V_\nu + V(\xi)\right\} \tag{2.53}$$

where

$$V(\xi) = \frac{1}{2} M_{Pl}^2 \overline{a}_{VV} V^\alpha V_\alpha \tag{2.54}$$

and

$$\kappa^{\alpha\beta\mu\nu} = \kappa_1 g^{\alpha\beta} g^{\mu\nu} + \kappa_2 g^{\alpha\mu} g^{\beta\nu} + \kappa_3 g^{\alpha\nu} g^{\beta\mu} \tag{2.55}$$

$\xi = V^\alpha V_\alpha$, and $\kappa_1, \kappa_2, \kappa_3$ are random coefficients coming from general action.

$$\begin{aligned} \kappa_1 &= \frac{44}{18} c_Q^{'}, \\ \kappa_2 &= \frac{8c_s^{'} + 2c_Q^{'}}{18}, \\ \kappa_3 &= \frac{2c_Q^{'} - 8c_s^{'}}{18} \end{aligned} \tag{2.56}$$

Assigning suitable values (by excluding ones leading to linear instabilities or negative-energy ghosts) to these coefficients reproduce the same results with the action of general vector inflation in (Koh, 2011; Jacobson and Mattingly, 2001; Carroll and Lim, 2004).

2.4. Conclusion

Metric-affine gravity generalizes the GR by accommodating an affine connection that extends the Levi-Civita connection. The tensorial part of the connection, under general conditions, can be decomposed into three independent vector fields (and a fundamental rank (1,2) tensor field, if any) which can be fundamental fields or gradients of some scalar fields. By this way the vector, scalar and tensor fields come into play when the metric-affine action is decomposed accordingly. The resulting theory is rather general. By imposing judicious constraints, theory can be reduced to more familiar ones like TeVeS gravity, vector inflation or aether-like models, in general. In the text we have given a detailed discussion of the TeVeS gravity and vector inflation.

From this work, one concludes that metric-affine gravity is rich enough to supply various vector and scalar fields needed in cosmological phenomena. Analyses of various effects may lead to a standard model of metric-affine gravity.

CHAPTER 3

RELATIVISTIC MOND FROM MODIFIED ENERGETICS

In this part, we obtain a relativistic version of MOND theory without action principle. We begin to investigate the question of what modifications in energy-momentum tensor can yield correct MOND regime. As a starting study, we refrain from insisting on an action principle and focus exclusively on the equations of motion. The present work, despite the absence of an explicit action functional, can be regarded to extend Milgrom's modified inertia approach to relativistic domain. Our results show that a proper MOND limit arises if energy-momentum tensor is modified to involve determinant of the metric tensor in reference to the flat metric, where the latter is dynamically generated as in gravitational Higgs mechanism. This modified energy-momentum tensor is conserved in both Newtonian and MONDian regimes.

3.1. Introduction

Observations of several decades, ranging from the initial measurements by Oort (see the discussion in (Kuijken and Gilmore, 1989)) to the primal ones by Rubin (Rubin and Ford Jr, 1970; Rubin et al., 1980), have shown that galaxies exhibit flat rotation curves, manifestly violating the Keplerian dynamics. This universal anomalous dynamics has been interpreted in two distinct ways. The first, first proposed by Zwicky (Zwicky, 1933) in 1933, refers to Dark Matter (DM) hypothesis. According to the DM paradigm, there must be a distribution of non-shining matter at the outer skirts of galaxies to measure approximately constant velocities after particular distances from the centre of galaxies. The DM hypothesis provides viable explanations not only for flat rotation curves but also for various cosmological and astronomical observations describing different phases of the evolution of Universe. Several experimental groups have been searching for DM particle by utilizing various detection methods (see the recent review volumes (Bertone, 2010; Fornengo, 2008)). So far, no signal of DM has been observed.

The second interpretation, first proposed by Milgrom (Milgrom, 1983a,b,c) in

1983, postulates that the observed flat rotation curves result from modifications in the Newtonian laws of motion. In this approach, instead of adding unknown ingredients to galactic matter, one exercises modifications in motion equations which dominate at the skirts of the galaxies. To this end, Newton's law of motion $\vec{F} = m\vec{a}$ changes to

$$\vec{F} = m\mu\left(\frac{a}{a_0}\right)\vec{a} \tag{3.1}$$

where $\vec{F}$ is the net force acting on the material point which has inertia m and acceleration $\vec{a}$ (with $a^2 = \vec{a}\cdot\vec{a}$). This dynamical equation, structuring Milgrom's MOND theory (Milgrom, 1983a,b,c), is characterized by the empirical function $\mu(a/a_0)$ where $a_0 \simeq 1.2\times10^{-10}\mathrm{ms}^{-2}$ is a constant acceleration scale for all galaxies (Giné, 2009). It appears in (3.1) as a critical acceleration scale set galactically by the mass M and radius R of the galaxy as $(G_N M)/R^2 \simeq a_0$ and cosmologically by the present-day value H_0 of the Hubble parameter as $(cH_0)/2\pi \simeq a_0$ (Easson et al., 2011).

The heart of the MOND theory is the empirical function $\mu(a/a_0)$. There is yet no dynamical theory for it; however, its asymptotic behavior is not difficult to guess

$$\mu(x) \asymp \begin{cases} 1 & \text{if } x > 1 \\ x & \text{if } x < 1 \end{cases} \tag{3.2}$$

if all the successes of the Newtonian theory are to be maintained. Here x does not need to be very large or small compared to unity because $\mu(x)$ can attain its asymptotics even when x is close to unity. For instance, the empirical form

$$\mu(x) = \frac{x\left(\frac{3}{2}\right)^{\frac{2}{n^2}}}{\left(\frac{1}{1+x^n} + x^n\right)^{\frac{1}{n}}} \tag{3.3}$$

facilitates the asymptotics in (3.2) almost independently of x provided that n is large. Indeed, taking $n = 50$ one finds $\mu(x) = 0.700, 0.900, 0.986, 0.996, 1.000, 1.000$ for $x = 0.7, 0.9, 0.99, 1.01, 1.1, 2.0$, respectively.

The behaviour in (3.2) ensures that matter in the galaxy exhibits flat rotation curves far away from the galactic center. Indeed, in the limit of small accelerations the equation of motion (3.1) takes the form

$$\vec{F} = m\frac{a\vec{a}}{a_0} \tag{3.4}$$

so that at large radii R corresponding to outer skirts of the galaxy one finds not the Keplerian law $|\vec{F}| = (mv^2)/R$ but $|\vec{F}| = (mv^4)/(a_0R^2)$ which yields the constant speed

$$v^4 = G_N M a_0 \tag{3.5}$$

for $|\vec{F}| = (G_N mM)/R^2$. This relation accounts for the observed flat rotation curves (Rubin and Ford Jr, 1970; Rubin et al., 1980; Sanders and Verheijen, 1998; Sanders and Noordermeer, 2007; Brownstein and Moffat, 2006). The constant speed (3.5) is the reason for and result from the whole idea of MOND. It depends crucially on the behaviour of the empirical function (3.2) at low accelerations.

The empirical MOND relation in (3.1), supported by (3.2) and (3.3), needs be formulated at a more fundamental level. In this regard, there arises two different interpretations. In the first, after setting $\vec{a} = -\vec{\nabla}\phi_g$ with ϕ_g being the gravitational potential, one formulates MOND as a modification in gravitational laws (see the reviews (Bruneton and Esposito-Farese, 2007; Skordis, 2009; Famaey and McGaugh, 2012)). In this case, one is necessarily led to modified Newtonian gravity (Bekenstein and Milgrom, 1984; Milgrom, 2010b) or General Relativity (GR) extended by geometrical scalar and vector fields (Sanders, 1997; Bekenstein, 2004b; Sanders, 2005; Zlosnik et al., 2006, 2007; Karahan et al., 2013). Besides, there are alternative approaches based on $f(R)$ gravity (Bertolami et al., 2007, 2008; Stabile and Scelza, 2011; Bernal et al., 2011), bimetric gravity (Milgrom, 2009, 2010a), time foliation (Blanchet and Tiec, 2008; Blanchet and Marsat, 2011), nonlocal metric theories (Soussa and Woodard, 2003; Deffayet et al., 2011), Galileons (Babichev et al., 2011), and Horava-Lifshitz gravity (Romero et al., 2010). In general, modified gravity theories introduced to replace the DM necessarily lead to MONDian structure.

In the second interpretation, one conceives the equation of motion (3.1) as defining an acceleration-dependent inertia $m(a) = m\mu(a/a_0)$. This approach, the modified inertia approach proposed in (Milgrom, 1994, 1999), in the non-relativistic limit, keeps gravitational laws unchanged yet lets in nonlinear kinetic terms. In this framework, it is found that the kinetic term of the point mass involves all derivatives of acceleration (Milgrom, 1994, 1999; Romero and Zamora, 2006; McCulloch, 2007) yet it is stable and respects causality (Bruneton and Esposito-Farese, 2007; Skordis, 2009; Famaey and McGaugh, 2012; Soussa and Woodard, 2003; Deffayet et al., 2011). In the present work, we pursue this modified inertia viewpoint to generalize it to general-relativistic domain. The experience from non-relativistic study (Milgrom, 1994, 1999) ensures that forming an action functional must be difficult, if not impossible, in the relativistic domain. We thus focus exclusively on the equations of motion without specifying an action principle to derive them.

3.2. Modified Energetics

As the beginning phase of a study programme aiming at finding dynamical alternatives to modified gravity models of relativistic MOND (Bruneton and Esposito-Farese, 2007; Skordis, 2009; Famaey and McGaugh, 2012), in this section we study gravitational field equations where MOND phase is understood as changes in matter energy-momentum tensor. This approach, aiming at carrying Milgrom's modified inertia approach (Milgrom, 1994, 1999) into relativistic domain at the level of equations of motion, is based on the matter energy-momentum tensor $T^{(N)}_{\mu\nu}$ in Newtonian domain and exploits its expected non-conservation in the MOND regime to derive MONDian dynamics in an empirical way. Having a complete knowledge of the interactions of matter, its energy-momentum tensor $T^{(N)}_{\mu\nu}$ (with energy density T^N_{00}, pressure T^N_{ii}, momentum density T^N_{0i} and shear stress T^N_{ij}) is strictly conserved in the Newtonian regime. However, the same $T^{(N)}_{\mu\nu}$ is not conserved in the MONDian regime because matter develops extra interactions even if one is not able to know them explicitly. Those extra interactions generalize $T^{(N)}_{\mu\nu}$ to a conserved energy-momentum tensor $T_{\mu\nu}$ which can be approached only empirically in the absence of a complete dynamical model (see (Mishra and Singh, 2012, 2014) for a similar approach to modified gravity framework for MOND). We now give an empirical implementation of this dynamical picture starting with Einstein field equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu} \tag{3.6}$$

in which $T_{\mu\nu}$ is the conserved energy-momentum tensor of matter at all acceleration scales ranging from $a = 0$ to $a = \infty$. In general, $T_{\mu\nu}$ is conserved on the equations of motion, and these equations necessarily encode the novel interactions of matter responsible for the

MOND. However, those new interactions are not known and our knowledge of $T_{\mu\nu}$ is incomplete; we are able to know it only when $a > a_0$ for which it equals $T_{\mu\nu}^{(N)}$. Consequently, on an empirical basis we write for $T_{\mu\nu}$

$$T_{\mu\nu} = \mu\left(\mathfrak{a}\right)\left[T_{\mu\nu}^{(N)} - Qg_{\mu\nu}\right] + Qg_{\mu\nu} \tag{3.7}$$

where $\mu(x)$ is the MOND function in (3.2), Q is a scalar, and $\mathfrak{a}$ is yet another scalar which is to be judiciously constructed to have the empirical limit

$$\mathfrak{a} \xrightarrow{v \ll c} \mathfrak{a}_{NR} = \frac{a}{a_0} \tag{3.8}$$

at non-relativistic energies. This correspondence between the relativistic ($\mathfrak{a}$) and non-relativistic (a) regimes is crucial for the empirical structure in (3.7) to give a consistent framework.

Physically, the grand energy-momentum tensor $T_{\mu\nu}$ must correctly reproduce the Newtonian and MONDian regimes. This is analyzed case by case in Table 3.1 as a function of the divergence of $T_{\mu\nu}^{(N)}$. As suggested by the table, underlying dynamics can be revealed after a proper understanding of $T_{\mu\nu}$ and this requires $T_{\mu\nu}^{(N)}$, $\mathfrak{a}(T)$ and $Q(T)$ to be constructed in detail. We detail these physical variables in the three consecutive subsections that follow.

3.2.1. Physical Properties of $T_{\mu\nu}^{(N)}$

It has been emphasized previously, specifically in Table 3.1, that $T_{\mu\nu}^{(N)}$ has the same form as the energy-momentum tensor of matter in Newtonian regime yet it does not qualify as true energy-momentum tensor in the MOND regime simply because its conservation is spoiled by novel interactions of matter that arise at accelerations below a_0. The higher-derivative self interactions studied in (Milgrom, 1994, 1999; Romero and Zamora, 2006; McCulloch, 2007) form a concrete example of such effects. Let us consider, as an illustrative example, dust (pressureless matter having only energy density in the comoving

Acceleration	MOND Function	Energy-Momentum Tensor	Matter Dynamics
$\mathfrak{a} \gtrsim 1$	$\mu(\mathfrak{a}) \simeq 1$	$T_{\mu\nu} \simeq T^{(N)}_{\mu\nu}$ $\left(\nabla^\mu T_{\mu\nu} = 0\right.$ hence $\left.\nabla^\mu T^{(N)}_{\mu\nu} = 0\right)$	This is 'Newtonian regime'. Acceleration of matter is above a_0 and $\mu(\mathfrak{a})$ ensures $T_{\mu\nu} \simeq T^{(N)}_{\mu\nu}$ so that $T^{(N)}_{\mu\nu}$ is symmetric and divergence-free ($\nabla^\mu T^{(N)}_{\mu\nu} = 0$) in agreement with (3.6). In Newtonian regime thus $T^{(N)}_{\mu\nu}$ qualifies as the known conserved energy-momentum tensor of matter.
$\mathfrak{a} \lesssim 1$	$\mu(\mathfrak{a}) \simeq \mathfrak{a}$	$T_{\mu\nu} \not\simeq T^{(N)}_{\mu\nu}$ $\left(\nabla^\mu T_{\mu\nu} = 0\right.$ yet $\left.\nabla^\mu T^{(N)}_{\mu\nu} \neq 0\right)$	This is 'MONDian regime'. Acceleration of matter is below a_0 and $\mu(\mathfrak{a})$ leads to $T_{\mu\nu} \not\simeq T^{(N)}_{\mu\nu}$ so that $T^{(N)}_{\mu\nu}$ is symmetric yet not divergence-free ($\nabla^\mu T^{(N)}_{\mu\nu} \neq 0$). In MOND regime thus it is $T_{\mu\nu}$ not $T^{(N)}_{\mu\nu}$ which qualifies as the conserved energy-momentum tensor of matter. In this small acceleration regime, matter develops novel interactions that make $\nabla^\mu T^{(N)}_{\mu\nu} \neq 0$ yet the scalars $\mathfrak{a}$ and Q help $T_{\mu\nu}$ be conserved and give the observed flat rotation curves.

Table 3.1. The acceleration dependence of the energy-momentum tensor $T_{\mu\nu}$ of matter. In general, $\mathfrak{a} = \mathfrak{a}(T^{(N)})$ and $Q = Q(T^{(N)})$ are functions of the energy-momentum tensor $T^{(N)}_{\mu\nu}$. These scalars take appropriate values for Newtonian ($T^{(N)}_{\mu\nu}$ is conserved) and MONDian ($T^{(N)}_{\mu\nu}$ is not conserved) regimes. Namely, matter develops novel interactions (such as the higher-derivative kinetic terms, determined in (Milgrom, 1994, 1999) in the non-relativistic regime) at small accelerations and its known energy-momentum tensor $T^{(N)}_{\mu\nu}$ starts exhibiting non-conservation properties.

frame) for which

$$T^{(N)}_{\mu\nu} = \rho u_\mu u_\nu \tag{3.9}$$

where ρ and u_μ are energy density and velocity, respectively. (One recalls that $T^{(N)}_{\mu\nu} = \int d\tau \rho u_\mu u_\nu$ for a relativistic particle with trajectory $y_\mu(\tau)$ and energy density $\rho = mc^2\delta^4(x - y(\tau))$.) It is divergence-free, $\nabla^\mu T^{(N)}_{\mu\nu} = 0$, because densities and flows of dust are all conserved. However, this conservation property holds only in normal circumstances where Newtonian laws of motion are valid. In MONDian regime, where dust develops higher-derivative kinetic interactions for instance, conservation breaks down, $\nabla^\mu T^{(N)}_{\mu\nu} \neq 0$. On dimensional grounds, it is likely to have structures of the form

$$\nabla^\mu T^{(N)}_{\mu\nu} \sim \rho a_0 u_\nu \tag{3.10}$$

in addition to terms involving derivatives of acceleration. In the absence of an invariant action (like the non-relativistic model in (Milgrom, 1994, 1999)), this non-conservation can be understood neither in origin nor in structure ($\rho a_0 u_\nu$ in (3.10) is just an example). Therefore, our goal is not to construct a model of the non-conservation of $T^{(N)}_{\mu\nu}$ but to determine its consequences for structures and dynamics of $\mathfrak{a}$ and Q.

3.2.2. Physical Properties of the Acceleration Scalar $\mathfrak{a}$

The acceleration scalar $\mathfrak{a}$, which must have the non-relativistic limit $\mathfrak{a}_{NR}$ given in (3.8), must be constructed judiciously to correctly cover the Newtonian and MONDian regimes. Hence, besides the crucial relation (3.8), it must have the following properties.

1. By our construction shown in Table 3.1, $\mathfrak{a}$ must vary with the divergence of $T^{(N)}_{\mu\nu}$ as

$$\begin{aligned} &\mathfrak{a} > 1 \ \text{ if } \ \nabla^\mu T^{(N)}_{\mu\nu} = 0 \\ &\mathfrak{a} < 1 \ \text{ if } \ \nabla^\mu T^{(N)}_{\mu\nu} \neq 0 \end{aligned} \tag{3.11}$$

while $\nabla^\mu T_{\mu\nu} = 0$ in both cases.

2. Being a scalar field, $\mathfrak{a}$ involves contractions of the divergences of $T^{(N)}_{\mu\nu}$. This necessarily brings in the gravitational acceleration $\vec{\nabla}\phi_g$ through the gravitational potential $\phi_g = -1 - g_{00}$ arising in the Newtonian limit of the metric tensor $g_{\mu\nu}$. However, presence of $\vec{\nabla}\phi_g$ must be prohibited for $\mathfrak{a}$ to yield the kinetic acceleration in (3.8). It is easy to see that this cannot be accomplished without using an independent source of ϕ_g and the most natural source as such is the determinant $g = \text{Det}(g_{\mu\nu})$ of the metric tensor. However, being a scalar density rather than a scalar, g cannot appear in $\mathfrak{a}$ by itself; it must be divided by another scalar density to achieve covariance. This other scalar density necessitates a new metric $\overline{g}_{\mu\nu}$, and naturally leads to a bi-metrical picture (whose relevance for MOND has been discussed in (Milgrom, 2009, 2010a)). Then, acceleration scalar possess the functional form

$$\mathfrak{a} = \mathfrak{a}\left(a_0, \nabla^\mu T^{(N)}_{\mu\nu}, T^{(N)}_{\mu\nu}, g_{\mu\nu}, g_{\mu\nu}\overline{g}^{\mu\nu}, g/\overline{g}\right) \tag{3.12}$$

where $\overline{g} = \text{Det}(\overline{g}_{\mu\nu})$ arises as an additional variable to be dynamically determined.

These two points plus (3.8) must be taken into account in formulating $\mathfrak{a}$. However, the formulation process becomes utterly incomplete unless the additional metric $\overline{g}_{\mu\nu}$ is demystified. In the two subsections that follow, we first study $\overline{g}_{\mu\nu}$ and then construct a model of $\mathfrak{a}$.

3.2.2.1. Construction of $\overline{g}_{\mu\nu}$

The second metric tensor $\overline{g}_{\mu\nu}$, required to eliminate the gravitational acceleration $\vec{\nabla}\phi_g$ from the acceleration scalar $\mathfrak{a}$, can be ascribed different structures depending on the underlying dynamics. For instance, one may consider identifying it with $T^{(N)}_{\mu\nu}$ itself but this attempt fails because its determinant vanishes in the case of dust (see equation (3.9) above). Alternatively, one may take $\overline{g}_{\mu\nu}$ as a second metric tensor with its own curvature and dynamics but this setup, as was already elaborated by Milgrom in ((Milgrom, 2009, 2010a)) (see also (Soussa and Woodard, 2004)), gives a modified gravity theory for MOND. This and other possible modified gravity models fall outside the scope of the present work because the goal here is to develop a dynamical approach to relativistic MOND similar in philosophy to Milgrom's modified inertia approach (Milgrom, 1994, 1999).

Our approach to $\overline{g}_{\mu\nu}$ is dynamical rather than geometrical. In other words, the dynamics underlying the asymptotics in Table 3.1 and structures in (3.12) proceed with not only $T^{(N)}_{\mu\nu}$ but also $\overline{g}_{\mu\nu}$. Thus, $\overline{g}_{\mu\nu}$ is a low-acceleration dynamical field, maybe one of many as such, which facilitates the MOND regime. In modeling the dynamics, we interpret the coupling $g^{\mu\nu}\overline{g}_{\mu\nu}$ between the two metrics as the kinetic term of four real scalars ϕ^m ($m = 0, \ldots, 3$), and construct the defining relation

$$\overline{g}_{\mu\nu} = \frac{1}{M^4}\eta_{mn}\partial_\mu\phi^m\partial_\nu\phi^n \tag{3.13}$$

where η_{mn} is the flat Minkowski metric, and hence, scalar spectrum contains a a ghosty (negative kinetic term) mode. We assume that ϕ^m develop the nontrivial backgrounds

$$\langle\overline{g}_{\mu\nu}\rangle = \begin{cases} 0 & \text{if} \quad \langle\phi^m\rangle = 0 \\ \eta_{\mu\nu} & \text{if} \quad \langle\phi^m\rangle = M^2 x^m \end{cases} \tag{3.14}$$

depending on whether the diffeomorphism invariance is exact ($\langle\phi^m\rangle = 0$) or spontaneously broken ($\langle\phi^m\rangle = M^2 x^a$) in the vacuum state governed by the vacuum expectation value $\langle\phi^a\rangle$ of the scalars. Here, the scale M is around a_0. The dynamics leading to (3.14) can be known only in a setting where all interactions of matter and extra fields like ϕ^a are specified. The diffeomorphism-breaking vacuum here sets the flat Minkowski metric $\eta_{\mu\nu}$ as the background metric about which $g_{\mu\nu}$ can be expanded in a perturbation series.

This induction mechanism is similar to what happens in gravitational Higgs mechanism (Percacci, 1991; Hooft, 2007; Kakushadze, 2008a,b; Demir and Pak, 2009) in which a second metric tensor $\overline{g}_{\mu\nu}$ is needed for writing a sensible graviton mass term through the kinetic term $g^{\mu\nu}\overline{g}_{\mu\nu}$ of scalars and through the ratio of the determinants $g/\overline{g}$. Nevertheless, as was throughly analyzed in (Demir and Pak, 2009), these two contributions, instead of adding, can cancel each other to keep graviton massless, or equivalently, gravity unmodified. This does not mean that the metric tensors in (3.14) do not participate in other physical processes. Indeed, they can well generate our targeted structures involving the gravitational acceleration $\vec{\nabla}\phi$. Consequently, we associate the metric tensors in (3.14) with the two phases of motion as

$$\begin{aligned} \langle\overline{g}_{\mu\nu}\rangle = 0 \quad &\Longrightarrow \quad \text{Newtonian regime} \\ \langle\overline{g}_{\mu\nu}\rangle = \eta_{\mu\nu} \quad &\Longrightarrow \quad \text{MONDian regime} \end{aligned} \tag{3.15}$$

keeping in mind that gravity is not necessarily massive. Indeed, the model of (Demir and Pak, 2009) offers a wide parameter space to set $V_1'(4) = 0$ in equation (26) and $\zeta V_1'(4) = 0$ in equation (27). Moreover, potential terms in equation (11) give enough freedom to realize massless and massive gravity phases. Therefore, as will be proven below, the MOND regime can be realized by using the metrics in (3.14) without the necessity of modifying gravity.

3.2.2.2. Construction of $\mathfrak{a}$

Having fixed all the variables in (3.12), we now start formulating the acceleration scalar $\mathfrak{a}$. The kinetic term $g^{\mu\nu}\overline{g}_{\mu\nu}$ of scalars do not contribute to $\vec{\nabla}\phi_g$, and hence, the argument of $\mathfrak{a}$ in (3.12) represent the optimal list of dynamical variables. Out of various possibilities, we consider for $\mathfrak{a}$ a simple structure

$$\mathfrak{a}^2 a_0^2 \left(T^{(N)}\right)^2 = \nabla^\alpha T^{(N)\beta}_{\ \ \ \ \alpha} \nabla^\theta T^{(N)}_{\theta\beta} + c_1 \left(T^{(N)}\right)^2 \nabla_\alpha \left(\frac{g}{\overline{g}}\right) \nabla^\alpha \left(\frac{g}{\overline{g}}\right) + c_2 T^{(N)} \nabla^\alpha \left(\frac{g}{\overline{g}}\right) \nabla^\theta T^{(N)}_{\theta\alpha} \quad (3.16)$$

where all indices are raised and lowered with $g_{\alpha\beta}$ so that $T^{(N)} = g^{\alpha\beta}T^{(N)}_{\alpha\beta}$ is the trace of the matter energy-momentum tensor in Newtonian domain. Here, the dimensionless constants $c_{1,2}$ will be fixed in the weak field limit by imposing (3.8). The presence of the metric determinants in (3.16) is crucially important for MOND because gravitational acceleration $\vec{\nabla}\phi_g$ is generated by derivatives of $g/\overline{g}$ (not $g^{\mu\nu}\overline{g}_{\mu\nu}$, for instance).

Having fixed its functional form in (3.16), we now start checking if $\mathfrak{a}$ satisfies its defining asymptotics in (3.2) and Table 3.1. This requires its evaluation in the two vacua in (3.14) since they correspond to the Newtonian and MONDian regimes as indicated in (3.15).

1. $\langle\overline{g}_{\mu\nu}\rangle = 0$ *and* $\nabla^\mu T^{(N)}_{\mu\nu} = 0$. In this vacuum, $\langle\overline{g}\rangle$ vanishes identically and, as follows from (3.16), $\mathfrak{a}$ becomes infinitely large thanks to the fact that $c_{1,2} > 0$, as will be proven below. Now, having found $\mathfrak{a} > 1$, one gets $\mu(\mathfrak{a}) \simeq 1$ and this gives $T_{\mu\nu} \simeq T^{(N)}_{\mu\nu}$ from (3.7). Thus, the Einstein field equations (3.6) reduce to

$$G_{\mu\nu} = 8\pi G_N T^{(N)}_{\mu\nu} \quad (3.17)$$

in which consistency of the Bianchi identity on $G_{\mu\nu}$ is maintained by the conservation of $T^{(N)}_{\mu\nu}$. This conservation, $\nabla^\mu T^{(N)}_{\mu\nu} = 0$, gives the usual Newtonian equations for free-fall

$$\vec{a} = -\vec{\nabla}\phi_g \tag{3.18}$$

for dust distribution characterized by the energy-momentum tensor in (3.9). Clearly, this equation holds if the metric tensor takes the form

$$g_{\mu\nu} = \text{Diag.}\left(-(1+2\phi_g), 1, 1, 1\right)_{\mu\nu} \tag{3.19}$$

as appropriate for the non-relativistic limit.

In conclusion, as conjectured in equation (3.15), the minimum energy configuration $\langle \overline{g} \rangle$ gives rise to the Newtonian regime for motion. Small perturbations about this vacuum makes $\overline{g} \neq 0$ but this determinant is expected to be sufficiently small to secure the Newtonian regime $\mathfrak{a} > 1$.

2. $\langle \overline{g}_{\mu\nu} \rangle = \eta_{\mu\nu}$ *and* $\nabla^\mu T^{(N)}_{\mu\nu} \neq 0$. In this vacuum, in the non-relativistic limit in which metric tensor is given by (3.19), the acceleration scalar defined in (3.16) becomes

$$\mathfrak{a}^2_{NR} = \frac{\vec{a}\cdot\vec{a}}{a_0^2} + (2-c_2)\frac{\vec{a}\cdot\vec{\nabla}\phi_g}{a_0^2} + (1-c_2+c_1)\frac{\vec{\nabla}\phi_g\cdot\vec{\nabla}\phi_g}{a_0^2} \tag{3.20}$$

for dust whose energy-momentum tensor is given partly by (3.9) and partly by extra interactions occurring in low-acceleration regime. It is due to this alleged extra piece that $T^{(N)}_{\mu\nu}$ in (3.9) satisfies $\nabla^\mu T^{(N)}_{\mu\nu} \neq 0$.

It is clear that, the acceleration scalar exhibits correct non-relativistic limit if

$$c_1 = 1\ ,\quad c_2 = 2 \tag{3.21}$$

because then the last two terms of (3.20) drop out to enable the required limit in (3.8). Thus, the construct in (3.16) for $\mathfrak{a}$ does indeed reduce to the acceleration of the point mass rather than the gravitational acceleration $-\vec{\nabla}\phi_g$. The non-relativistic result in (3.20), which holds for $\mathfrak{a} < 1$ or equivalently $a < a_0$, entails $\mu(\mathfrak{a}) \simeq \mathfrak{a}$ so that Einstein field equations (3.6) takes the form

$$G_{\mu\nu} = 8\pi G_N \left\{\mathfrak{a}\left[T_{N\,\mu\nu} - Qg_{\mu\nu}\right] + Qg_{\mu\nu}\right\} \tag{3.22}$$

where the scalar field Q is to be chosen judiciously to make the right-hand side to have vanishing divergence. This constraint, ensuring conservation of $T_{\mu\nu}$, can be difficult to satisfy if Q does not involve $T^{(N)}_{\mu\nu}$ and $g/\overline{g}$. As a plausible structure, we set

$$Q = \frac{g\,T^{(N)}}{\overline{g}} \tag{3.23}$$

where one can of course consider alternative structures giving similar results in the non-relativistic limit. In $\langle\overline{g}_{\mu\nu}\rangle = \eta_{\mu\nu}$ vacuum, in the non-relativistic limit, conservation of $T_{\mu\nu}$ gives

$$\begin{aligned}\nabla_\mu\left(\frac{a}{a_0}\right)\left\{\rho u^\mu u^j - \rho\left(\frac{g}{\overline{g}}\right)g^{\mu j}\right\} &= -\frac{a}{a_0}\left\{\rho\left(a^j + \nabla^j\phi_g\right) - \rho\nabla^j\phi_g - (1+2\phi_g)\nabla_\mu\rho g^{\mu j}\right\} \\ &- \rho\nabla^j\phi_g - (1+2\phi_g)\nabla_\mu\rho g^{\mu j}\end{aligned} \tag{3.24}$$

where the metric tensor is given by (3.19). This differential equation is too involved to suggest the MOND dynamics. Nevertheless, a closer look reveals that, if *(i)* energy density ρ varies slowly in space ($|\vec{\nabla}\rho| \ll \rho|\vec{\nabla}\phi_g|$) and if *(ii)* acceleration $\vec{a}$ varies slowly both in space and time ($|\nabla_\mu\rho| \ll \rho|\nabla_\mu\phi_g|$) then one gets from (3.24)

$$\frac{a\vec{a}}{a_0} = -\vec{\nabla}\phi_g \tag{3.25}$$

which is the desired MOND relation given in equation (3.4).

3. *Non-Conservation of $T^{\mu\nu}_N$*. Having obtained motion equations in the two regimes of $\overline{g}_{\mu\nu}$, we now turn to a discussion of the non-conservation of $T^{\mu\nu}_N$. In view of

the discussions summarized in Table 3.1, the energy-momentum tensor $T_{\mu\nu}$, introduced in (3.6) and defined in (3.7), is always conserved. This is necessary for the consistency of the gravitational field equations (3.6). The $T_N^{\mu\nu}$ tensor, however, is conserved only in the Newtonian regime. To see how these conservation features hold, it proves useful to examine the divergence of $T_N^{\mu\nu}$

$$\nabla_\mu T_N^{\mu\nu} = f_N^\nu \tag{3.26}$$

where

$$f_N^\nu = -[\nabla_\alpha \ln \mu(\mathfrak{a})]\,(T_N^{\alpha\nu} - Q g^{\alpha\nu}) + \left(1 - \frac{1}{\mu(\mathfrak{a})}\right)\nabla^\nu Q \tag{3.27}$$

as follows from (3.6) with (3.7). It is obvious that, in the Newtonian regime, $\mu(\mathfrak{a}) \rightarrow 1$ and f_N^ν vanishes identically to ensure conservation of $T_N^{\mu\nu}$. In MONDian regime, however, $\mu(\mathfrak{a}) \rightarrow \mathfrak{a} \neq 1$, and f_N^ν stays non-vanishing. This prohibits conservation of $T_N^{\mu\nu}$. These features are precisely the ones listed in Table 3.1. The MONDian force is consistent with (3.22). Since $\mathfrak{a}$ is related to $\overline{g}_{\mu\nu}$ as in (3.16), the second metric $\overline{g}_{\mu\nu}$ turns out to be a fundamental ingredient of the entire formalism. Not surprisingly, effective forces similar to f_N^ν also arise in modified gravity theories which couple curvature and energy-momentum tensor $T_N^{\mu\nu}$ directly (Haghani et al., 2013; Sharif and Zubair, 2013; Odintsov and Sáez-Gómez, 2013).

In this section, we have succeeded to get the MONDian dynamics starting from (3.6) by defining the acceleration scalar $\mathfrak{a}$ as in (3.16), the Q scalar as in (3.23), and the second metric tensor as in (3.13). Moreover, we have explicitly ensured conservation of the total energy-momentum tensor $T^{\mu\nu}$ while determining effective MOND force associated with the non-conservation of $T_N^{\mu\nu}$. The analysis here provides an existence proof.

3.3. Conclusion and Future Prospects

In the present paper, we reported our results on relativistic MOND as derived from modified dynamics rather than modified gravity. Our approach is an empirical one and gives the beginning stage of a general investigation of relativistic MOND. The formalism developed, though lacks an action principle, can be regarded as generalizing Milgrom's modified inertia approach (Milgrom, 1994, 1999; Romero and Zamora, 2006; McCulloch, 2007) to relativistic domain. It is based on the energy-momentum tensor of matter. The reason for this is that, the energy-momentum tensor of matter in Newtonian regime, which necessarily looses its conservation property due to extra interactions occurring at sub-Hubble accelerations, seems to provide correct path way to quadratic acceleration in MOND regime. In fact, this dynamical structure cannot follow from other sources such as potentials, metric tensor and curvature tensor. The main observation behind our approach is that, matter possesses its usual energy-momentum tensor under the usual circumstances where Newtonian laws hold. However, the same matter, at exceedingly small accelerations below the Hubble scale, develops novel interactions causing non-conservation of its energy-momentum tensor, and it is with these interactions that MONDian dynamics arises. Our empirical relativistic model is essentially a bi-metric theory. However, our approach to the second metric tensor mimics models of gravitational Higgs mechanism in which the vacuum expectation value of the second metric tensor equals the flat Minkowski metric, and it provides the requisite terms clearing the gravitational acceleration contributions to enable the quadratic acceleration piece needed for MOND.

The present study can be extended in various aspects for rectifying and improving the present model.

- In the present work we have taken matter at the skirts of galaxies as dust. For an accurate analysis of matter distribution, however, one may need to extend it to perfect fluid and other forms of matter.
- In obtaining the MOND equation of motion (3.25) we have neglected contributions from spatial variation of ρ. The situation can be improved by incorporating such terms from (3.24). The effect can be pronounced especially at the arms of spirals where dust density changes sharply.

Last but not least, the present model would be grossly improved if an invariant action could be written. The alleged action, which must directly generalize Milgrom's modified inertia approach in (Milgrom, 1994, 1999) to relativistic velocities could be too complicated to construct due mainly to the presence of the fixed acceleration scale a_0. It might necessitate a_0 to be included in relativistic transformations.

CHAPTER 4

HIGGSED STUECKELBERG VECTOR AND HIGGS QUADRATIC DIVERGENCE

In this chapter we show that, a hidden vector field whose gauge invariance is ensured by a Stueckelberg scalar and whose mass is spontaneously generated by the Standard Model Higgs field contributes to quadratic divergences in the Higgs boson mass squared, and even leads to its cancellation at one-loop when Higgs coupling to gauge field is fine-tuned. In contrast to mechanisms based on hidden scalars where a complete cancellation cannot be achieved, stabilization here is complete in that the hidden vector and the accompanying Stueckelberg scalar are both free from quadratic divergences at one-loop. This stability, deriving from hidden exact gauge invariance, can have important implications for modelling dark phenomena like dark matter, dark energy, dark photon and neutrino masses. The hidden fields can be produced at the LHC.The detailed analysis on the fingerprints of this scenario at the Large Hadron Collider and at the Future Circular Collider are also presented. We find that the model is within the reach of current and future colliders and the results are not in conflict with existing data. Last but not least, this scenario accommodates naturally occurring viable dark matter (DM) candidates.

4.1. Introduction

With the discovery of a new resonance at the Large Hadron Collider (LHC), having a mass $m_h = 125.9 \pm 0.4$ GeV (Aad et al., 2012; Collaboration et al., 2012) and couplings well consistent with the Standard Model (SM) predictions (Ellis and You, 2013; Djouadi and Moreau, 2013), the Higgs naturalness problem (Weisskopf, 1939; Wilson, 1971; Susskind, 1979) has become the foremost problem to be tackled. The resolution, if any, brings its own new physics structure. The squared-masses of fundamental scalars, contrary to chiral fermions and gauge bosons whose masses are protected by chiral and gauge invariances, receive additive quantum corrections proportional to Λ^2 – the UV boundary of the SM. In explicit terms, one-loop quantum correction to Higgs squared-mass, originally computed by Veltman (Veltman, 1981), reads as

$$(\delta m_H^2)_{\text{quad}} = \frac{\Lambda^2}{16\pi^2}\left(6\lambda_H + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6g_t^2\right) \tag{4.1}$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings of the SM, respectively, and $g_t = m_t/\upsilon_H$ ($\upsilon_H = 246$ GeV is the VEV of the Higgs field) is the top quark Yukawa coupling. The top quark, being the most strongly coupled SM particle to the Higgs field, induces the biggest contribution and ensures a nonvanishing, unremovable coefficient before Λ^2. The Higgs boson mass is stabilized to electroweak scale if $|\delta m_H^2| < m_H^2 < \Lambda^2$. This is the Veltman condition (VC). The parameters in it have all been measured, and it violates the LHC results for $\Lambda > 500$ GeV (Feng, 2013; Wells, 2015; Giudice, 2013; Altarelli, 2013).

Having no symmetry to prevent the Higgs boson mass from sliding to the higher scales via (4.1), frequently a cancellation mechanism is implemented via fine-tuning of counter terms in which low and high energy degrees of freedom are mixed. This renders the whole procedure unnatural. It would be more natural, if the cancellation occurs by means of a symmetry principle at higher scales, or if it arises by accidental cancellations of certain terms. In fact, models of new physics constructed to complete the SM beyond Fermi energies have all been motivated by Higgs naturalness problem (Feng, 2013; Wells, 2015; Giudice, 2013; Altarelli, 2013) (see also (Liu and Nath, 2013; Masina and Quiros, 2013; Lu et al., 2014; Fowlie, 2014) for studies within supersymmetry). So far, however, in the 7 TeV and 8 TeV LHC searches reaching out beyond the TeV domain, no compelling sign of evidence for new physics has been found (Flechl, 2013; Feng et al., 2010).

In consequence, having no TeV scale new physics for achieving naturalness, one is forced to understand the electroweak unnaturalness within the SM plus general relativity, albeit with some imperative extensions required by specifics of the approach taken. In 1995, conformal symmetry (Bardeen, 1995) was proposed as a mechanism for solving the Higgs mass hierarchy problem (the latest studies on the conformal symmetry as a solution to the fine-tuning problem may be found in (Demir, 2012; Heikinheimo et al., 2014; Tavares et al., 2014; Kawamura, 2013; Holthausen et al., 2013; Antipin et al., 2014; Guo and Kang, 2015)). Recently, the Higgs coupling to spacetime curvature has been found to stabilize the electroweak scale by a harmless, soft fine-tuning (Demir, 2014). Furthermore, anti-gravity effects have been claimed to improve Higgs naturalness (Salvio and Strumia, 2014). Alternatively, one may view the parameters chosen by nature as the necessity of existence, and this leads to anthropic considerations (Agrawal et al., 1997). In variance with all these approaches, a fine-tuning method based on singlet scalars (Chivukula and Golden, 1991; Chivukula et al., 1992; Bjorken, 1992) has also been employed. In this approach, main idea is to cancel the quadratic divergences in Higgs boson mass with the loops of the singlet scalars that couple to Higgs field (Ruiz-Altaba et al., 1991; Peyranere et al., 1991; Andrianov et al., 1995; Kundu and Raychaudhuri, 1996; Bazzocchi et al., 2007). This method, though a fine-tuning operation by itself, nullifies the quadratic divergences and accommodates viable dark matter candidates (Chakraborty and Kundu, 2013; McDonald, 1994; Demir, 1999; Burgess et al., 2001; Barger et al., 2008, 2009; Guo and Wu, 2010; Djouadi et al., 2012; Gonderinger et al., 2012; Batell et al., 2012; Baek et al., 2012; Biswas and Majumdar, 2013; Bélanger et al., 2013; Cline and Kainulainen, 2013; Demir et al., 2014; Haba et al., 2014). Nevertheless, for real singlet scalars with vacuum expectation value (VEV), it is not possible to kill the quadratic divergences consistently because there is a mixing between the CP-even component of the Higgs field and the real singlet scalar, and it does not allow for simultaneous cancellation of the quadratic divergences in Higgs boson and singlet scalar masses (Karahan and Korutlu, 2014). There are also studies on two-Higgs doublet models without flavor changing neutral currents, demonstrating that, although the cancellation in the coefficient of the one-loop quadratically divergent terms is possible, the parameter space is severely constrained (Chakraborty and Kundu, 2014b). An additional complex scalar triplet extension of the SM has also been studied and proven to be a solution to the fine-tuning problem (Chakraborty and Kundu, 2014a).

In the present work, as a completely new approach never explored before, we study protection of the Higgs boson mass by a SM-singlet gauge field (not a scalar field as in (Ruiz-Altaba et al., 1991; Peyranere et al., 1991; Andrianov et al., 1995; Kundu and Raychaudhuri, 1996; Bazzocchi et al., 2007)). In contrast to the attempts based on hidden scalars (Ruiz-Altaba et al., 1991; Peyranere et al., 1991; Andrianov et al., 1995; Kundu and Raychaudhuri, 1996; Bazzocchi et al., 2007; Chakraborty and Kundu, 2013, 2014b,a), which are now known to be unable to simultaneously protect the masses of the Higgs boson and the singlet scalar (Karahan and Korutlu, 2014), in the present work, we consider a hidden $U(1)$ gauge field V_μ whose invariance is ensured by a Stueckelberg scalar S and whose mass is spontaneously induced by the SM Higgs field. We show that V_μ and S enable cancellation of the quadratic divergence in Higgs boson mass with no quadratic divergence arising in their own masses. It is important that the SM Higgs boson is stabilized at one-loop along with already-stable hidden gauge and Stueckelberg scalar. This phenomenological advantage has important implications not only for stabilizing the Higgs boson mass but also for correlating the SM Higgs field with hidden sectors.

The study is organized as follows. In Section 2 below, we construct the model starting from the basic Stueckelberg setup. Section 3 is devoted to computation of the quadratic divergences and vanishing of the Higgs mass divergence by fine-tuning. We conclude in Section 4.

4.2. The Model

In this section, we consider a massive Abelian gauge field V_μ accompanied by a real scalar field $S(x)$, introduced to preserve the gauge invariance of the theory. Originally proposed by Stueckelberg (Stueckelberg, 1938a,b) and noted afterwards by Pauli (Pauli, 1941) that, V_μ satisfies a restricted $U(1)$ gauge invariance, with the gauge function $\Theta(x)$ obeying a massive Klein-Gordon equation. The mechanism provides an alternative to the Higgs mechanism, where the vector boson acquires its mass with the breakdown of the gauge invariance of not the Lagrangian but of the vacuum. These features are encoded in the Stueckelberg model (Körs and Nath, 2004, 2005)

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}^2 + \frac{1}{2}m^2\left(V^\mu - \frac{1}{m}\partial^\mu S\right)^2 - \frac{1}{2}\left(\partial_\mu V^\mu + mS\right)^2 \tag{4.2}$$

where m is the common mass for V_μ and S. Despite its massive spectrum, this model enjoys a $U(1)_m$ invariance

$$\begin{aligned} V_\mu(x) &\to V'_\mu(x) = V_\mu(x) + \partial_\mu\Theta(x), \\ S(x) &\to S'(x) = S(x) + m\Theta(x) \end{aligned} \tag{4.3}$$

provided that $\left(\Box + m^2\right)\Theta(x) = 0$. Consequently, in spite of its nonvanishig hard mass, V_μ enjoys exact gauge invariance, albeit with a restricted gauge transformation function $\Theta(x)$ (Körs and Nath, 2004, 2005). In the massless limit, $m \to 0$, the Stueckelberg Lagrangian (4.2) reduces to $\mathcal{L}_{m=0} = -\frac{1}{4}V_{\mu\nu}^2 + \frac{1}{2}\partial_\mu S\,\partial^\mu S$, which is obviously $U(1)_m$ invariant in Lorentz gauge ($\partial_\mu V^\mu = 0$) with an unrestricted $\Theta(x)$. Interestingly, the Stueckelberg scalar S, transforming like the gauge field V_μ in massive case, turns into a gauge-singlet scalar in massless limit.

Inspired from the Stueckelberg model (4.2), we propose the Higgsed Stueckelberg model

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}^2 + \lambda_1 H^\dagger H\left(V^\mu - \frac{1}{\sqrt{\lambda_1}a_H}\partial^\mu S\right)^2 - \frac{1}{2}\left(\partial_\mu V^\mu + \sqrt{\lambda_1}a_H S\right)^2 \tag{4.4}$$

where λ_1 is a positive dimensionless constant and a_H is a mass parameter. This model is manifestly gauge-invariant under both the hidden $U(1)_m$ invariance with $m \to \sqrt{\lambda_1}a_H$, and the electroweak gauge group $SU(2)_L \otimes U(1)_Y$. The Higgs potential $V(H) = m_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2$ and hence the total energy is minimized at the Higgs field configuration

$$\langle H^\dagger H\rangle = \begin{cases} \frac{v_H^2}{2} & \text{if } m_H^2 < 0, \\ 0 & \text{if } m_H^2 > 0, \end{cases} \tag{4.5}$$

where $v_H = \sqrt{-\frac{m_H^2}{\lambda_H}}$ is the Higgs VEV in the broken phase ($m_H^2 < 0$), to which masses of the SM particles are all proportional. In this phase electroweak gauge group $SU(2)_L \otimes U(1)_Y$ is spontaneously broken down to electromagnetism. In unbroken phase ($m_H^2 > 0$) electroweak group stays exact and all the SM particles but Higgs boson are massless.

From (4.4) it is clear that, the two phases of the SM directly leave distinguishable effects on the mass of V_μ and kinetic term of S. And the Stueckelberg structure in (4.2) is achieved properly if the mass parameter a_H can keep track of the two electroweak phases. This feature is implemented into the Higgsed Stueckelberg model (4.4) by setting

$$a_H = \text{Re}\left(\sqrt{-\frac{m_H^2}{\lambda_H}}\right) = \begin{cases} v_H & \text{if } m_H^2 < 0, \\ 0 & \text{if } m_H^2 > 0, \end{cases} \tag{4.6}$$

which obviously dogs the Higgs VEV in (4.5). It turns out that $\langle H^\dagger H\rangle = a_H^2/2$ in both broken and exact electroweak phases, and $v_H = a_H$ specifically in the broken phase. This switching ability of a_H ensures that, in the broken phase of electroweak group, there arises, in addition to the massive SM spectrum, a massive vector V_μ with mass $M_V^2 = \lambda_1 v_H^2$ and a massive scalar $m_S^2 = \lambda_1 a_H^2$. In the unbroken phase, however, the Higgs field stands as the only massive field. The rest, inclusing V_μ and S, are all massless. In what follows, we will work in the physical vacuum of the broken electroweak phase and necessarily set $a_H = \upsilon_H$ everywhere.

It is instructive to study the transcription of the Stueckelberg $U(1)_m$ symmetry in (4.3) into the Higgsed Stueckelberg case. To this end, one notes that the Stueckelberg scalar $S(x)$ facilitates $U(1)_m$ gauge invariance of the hidden sector, and also, helps keep the Hamiltonian positive definite [1] (Stueckelberg, 1938a,b). In this formalism, Lorentz subsidiary condition does not follow from equation of motion. Imposing an operator equation of the form $\partial^\mu V_\mu^{(-)}(x)|\mathbf{phys}\rangle = 0$, where $V_\mu^{(-)}(x)$ involves the free field annihilation operators, however, gives rise to conflict between the operator equation and the canonical commutation relations. This puzzle is solved via the introduction of an additional scalar field $S(x)$, replacing the operator equation with $\Phi(x)|\mathbf{phys}\rangle \equiv [\partial^\mu V_\mu^{(-)}(x) + mS^{(-)}(x)]|\mathbf{phys}\rangle = 0$, where $S^{(-)}(x)$ also involves free field annihilation operators. The operator equation decreases the number of degrees of freedom of the Lagrangian to four. The required constraint to decrease it to three for a massive vector field comes into play with the gauge transformation

$$\begin{aligned} V_\mu(x) &\rightarrow V'_\mu(x) = V_\mu(x) + \partial_\mu \Theta(x), \\ S(x) &\rightarrow S'(x) = S(x) + \sqrt{\lambda_1}\upsilon_H \Theta(x) \end{aligned} \tag{4.7}$$

which closely follows the Stueckelberg transformation (4.3). The $U(1)_m$ invariance is ensured if $(\partial^2 + \lambda_1 \upsilon_H^2)\Theta(x) = 0$. This restricted gauge invariance changes to an unrestricted, standard gauge invariance in the unbroken ($m_H^2 > 0$) electroweak phase in which V_μ and S are massless and non-interacting. Moreover, S is a gauge singlet in this phase. The V_μ and its Stueckelberg companion S do possess identical masses in broken and unbroken phases of the electroweak symmetry. In broken phase, Stueckelberg-Feynman gauge,

[1] Note that the last term in (4.4) can also be written as $\mathcal{L}_{\rm gf} = -\frac{1}{2\alpha}(\partial_\mu V^\mu + \alpha\sqrt{\lambda_1}\upsilon_H S)^2$, where α is a real parameter, similar to t'Hooft's parametrization for Abelian Higgs model. The choice of $\alpha = 1$ corresponds to the Stueckelberg-Feynman gauge. When $\alpha \neq 1$, the restriction on the gauge function changes to $(\Box + \alpha\lambda_1\upsilon_H^2)\Lambda(x) = 0$. It is also possible to choose two different parameters α_1 and α_2, to check the gauge independence of the parameters. However, there is the disadvantage that the terms of the form $V^\mu\partial_\mu B$ survives for this choice. In the present work, we will work in Stueckelberg-Feynman gauge.

their propagators read as

$$\Delta_{\mu\nu} = -\frac{i\, g_{\mu\nu}}{q^2 - m^2}, \qquad \Delta = \frac{i}{q^2 - m^2} \tag{4.8}$$

where $m^2 = \lambda_1 v_H^2$ is the common mass for V_μ and S.

4.3. Phenomenology

In this section we study quantum corrections to masses of the Higgs boson h and Stueckelberg fields S and V_μ. The main constraint on the model is that Higgs boson must weigh $m_h = 125.9$ GeV (Aad et al., 2012; Collaboration et al., 2012). As follows from (4.4), there are three-point and four-point interactions among the vector boson V_μ, the Stueckelberg field S, and the Higgs field h. The vertex factors are summarized in the Appendix C. The Higgsed Stueckelberg hidden sector then modifies the Veltman condition (4.1) as

$$(\delta m_H^2)_{\text{quad}} = \frac{\Lambda^2}{16\pi^2}\left(\frac{11}{3}\lambda_H + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6g_t^2 + \lambda_1\right) \tag{4.9}$$

wherein λ_1 shows up as a new degree of freedom. In the philoshopy of the original attempts in (Ruiz-Altaba et al., 1991; Peyranere et al., 1991; Andrianov et al., 1995; Kundu and Raychaudhuri, 1996; Bazzocchi et al., 2007), one can suppress $(\delta m_H^2)_{\text{quad}}$ by choosing λ_1 appropriately. In particular, $(\delta m_H^2)_{\text{quad}}$ vanishes for $\lambda_1 = 4.41$. The V_μ and S are degenerate in mass, and for this specific value of λ_1 they weigh $m = \sqrt{\lambda_1} v_H = 517$ GeV. It is possible to decrease the value of λ_1 by simply introducing N such fields, which in turn lowers the masses of the new fields while increasing their number. In Figure 4.1, a schematic representation of the one-loop quantum corrections to Higgs mass is shown in our extended scenario. As it is apparent from this figure, a hidden Abelian gauge sector splendidly cancels the quadratically divergent contributions to Higgs mass from the SM fields.s

It is clear that suppressing $(\delta m_H^2)_{\text{quad}}$ requires λ_1 to be finely tuned. The fine-tuning here is of the same size as the fine-tunings required for hidden scalar sectors (Ruiz-Altaba et al., 1991; Peyranere et al., 1991; Andrianov et al., 1995; Kundu and Raychaudhuri, 1996; Bazzocchi et al., 2007; Chakraborty and Kundu, 2013; Karahan and Korutlu, 2014;

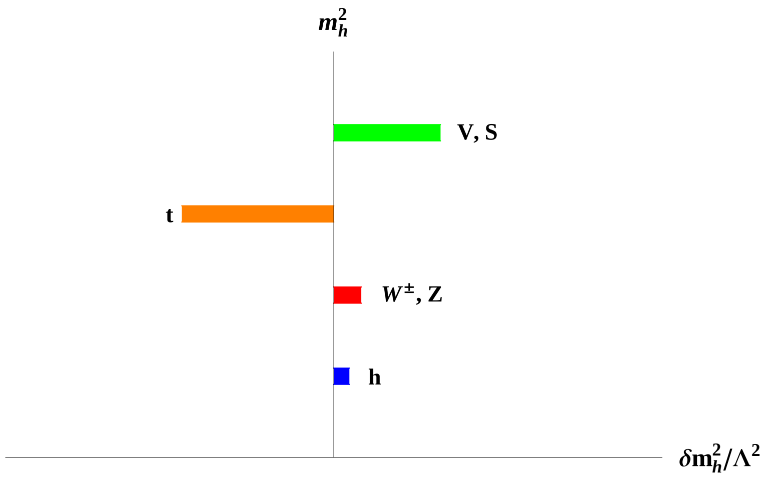

Figure 4.1. The schematic representation of the quadratically divergent contributions to Higgs boson mass at one-loop level. Here, h denotes the Higgs boson, $W^{\pm}$, Z the electroweak bosons, t the top quark, V,S the hidden gauge boson V_μ and the Stueckelberg scalar S, respectively. Higgs mass is protected from destabilizing quantum effects when the hidden gauge sector is included.

Chakraborty and Kundu, 2014b,a). There is one big difference, however. Indeed, these models based on hidden scalars suffer from the fact that masses of the hidden scalars and of the SM Higgs boson cannot be protected simultaneously (Karahan and Korutlu, 2014). The hidden scalar continues to have a mass $\mathcal{O}(\Lambda)$ after suppressing the radiative contribution to the Higgs boson mass. In the Higgs-Stueckelberg model this impasse is overcome. To see this, one notes that mass of the Stueckelberg field does actually receive quadratically divergent radiative corrections from two self energy diagrams (one with Higgs boson in the loop and another with both Higgs and the Stueckelberg field S in the loop). The self energy diagram with a Higgs boson and vector boson V_μ in the loop diverges logarithmically. The spruceness of this scenario emerges at this point in that the quadratically-divergent contributions to the mass of the Stueckelberg field from the two loop diagrams cancel out to give

$$(\delta m_S^2)_{\text{quad}} = 0 \tag{4.10}$$

In the same manner, the mass of V_μ is protected against quadratically-divergent quantum corrections

$$(\delta m_V^2)_{\text{quad}} = 0 \tag{4.11}$$

Leaving aside the logarithmic corrections, masses of $V\mu$ and S are found to be UV-insensitive. This is actually expected by gauge invariance because there exists an unbroken $U(1)_m$ invariance in both broken and unbroken electroweak phases. The invariance protects the mass of V_μ. Interestingly, it also protects the mass of S because S by itself acts like a gauge field when V_μ is massive and becomes a non-interacting $U(1)_m$ singlet when V_μ is massless. Clearly, the radiative stability of the hidden sector can have important implications for modelling 'dark phenomena' like Dark Matter, Dark Energy, Dark Photon and neutrino masses.

4.4. Collider Analysis

In the Higgsed Stueckelberg scenario, the V_μ and S states appear as Higgs portal fields. By virtue of the fact that they do not interact with any other SM particles but the Higgs boson, the HS fields remain cosmologically stable and might serve as perfect DM candidates. Accordingly, even though they can be pair produced in the decay of Higgs Boson, of which the dominant production mode is the gluon fusion at both LHC and FCC, they remain undetectable and would appear only as missing energy. The Feynman diagram of the process is shown in Fig. 4.2. The virtual Higgs boson, when produced in association with the gluon fusion, may decay into either two S particles, two V_μ states or one S and one V_μ. Additional production channels are not taken into account, as they contribute to the forenamed process at least ten times less than the gluon fusion mode for the whole energy range (See the recent analysis on the prospects for Higgs physics at energies up to 100 TeV (Baglio et al., 2015)). For the numerical analysis of our model, we modified the SM in LanHEP-3.2.0 (Semenov, 2016), and exported the new model to FeynArts-3.9 (Hahn, 2000) and FormCalc-9.0 (Hahn and Perez-Victoria, 1999). LHAPDF-6.1.5 (Buckley et al., 2015) is used for evaluating parton density functions (PDFs). We have computed the number of events generated via gluon fusion channel at LHC with center of mass energies of $\sqrt{s} = 13$ TeV at 18 fb^{-1}, and at FCC with $\sqrt{s} = 100$ TeV at 100 fb^{-1} as a function of λ_1. Although, for a natural scenario at one loop one needs $\lambda_1 = 4.41$, at higher loops, this result is subject to changes and this is why we have plotted our results with

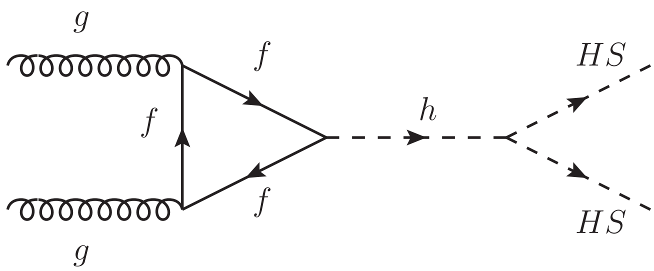

Figure 4.2. The Feynman diagram for the pair production of S and V_μ states via the dominant production mode of gluon fusion. g are the gluon fields, f are the fermions of the SM, h is the Higgs boson and HS stand for the hidden sector fields V_μ and S.

respect to the λ_1 parameter. The results are summarized in the following three figures.

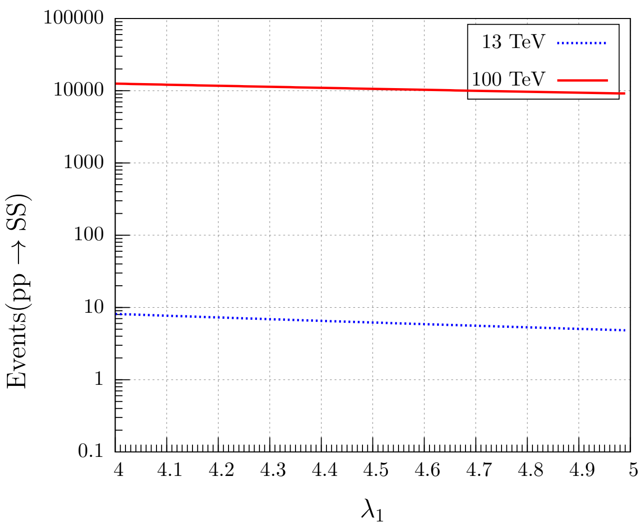

Figure 4.3. The number of events generated per year for the process $pp \to SS$ via gluon fusion channel at LHC with $\sqrt{s} = 13$ TeV and luminosity 18 fb^{-1}, and at FCC with c.m. energy $\sqrt{s} = 100$ TeV and luminosity 100 fb^{-1}. The MMHT2014nnlo68cl PDF set has been used.

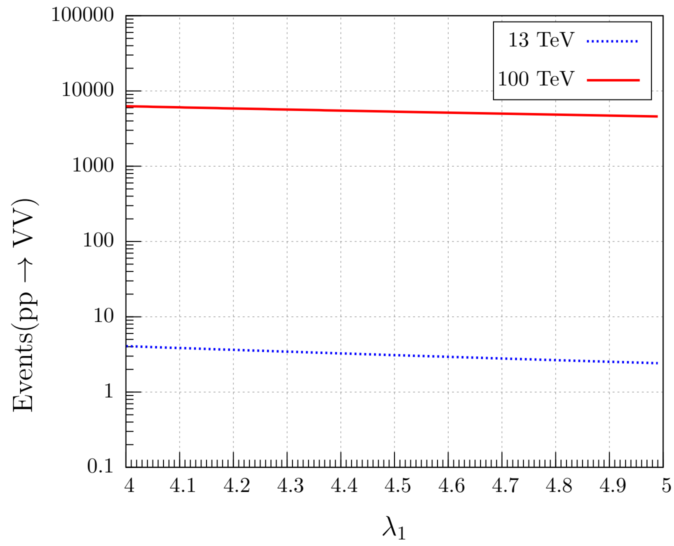

Figure 4.4. The number of events generated per year for the process $pp \to VV$ via gluon fusion channel at LHC with $\sqrt{s}$ = 13 TeV and luminosity 18 fb^{-1}, and at FCC with c.m. energy $\sqrt{s}$ = 100 TeV and luminosity 100 fb^{-1}. The MMHT2014nnlo68cl PDF set has been used.

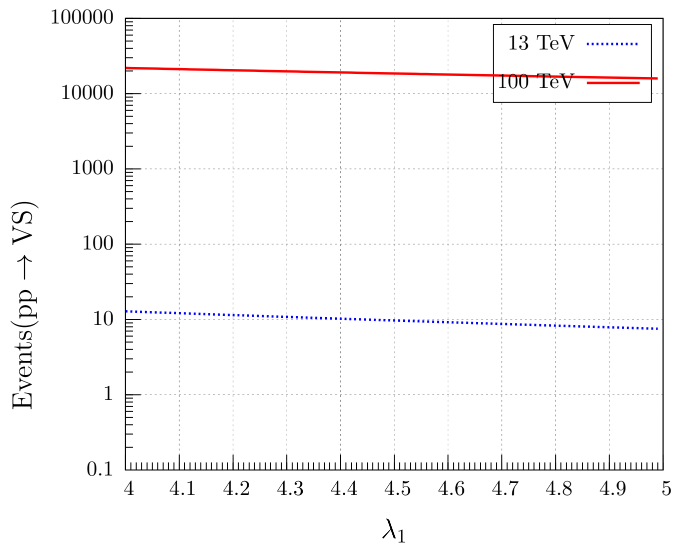

Figure 4.5. The number of events generated per year for the process $pp \to VS$ via gluon fusion channel at LHC with $\sqrt{s}$ = 13 TeV and luminosity 18 fb^{-1}, and at FCC with c.m. energy $\sqrt{s}$ = 100 TeV and luminosity 100 fb^{-1}. The MMHT2014nnlo68cl PDF set has been used.

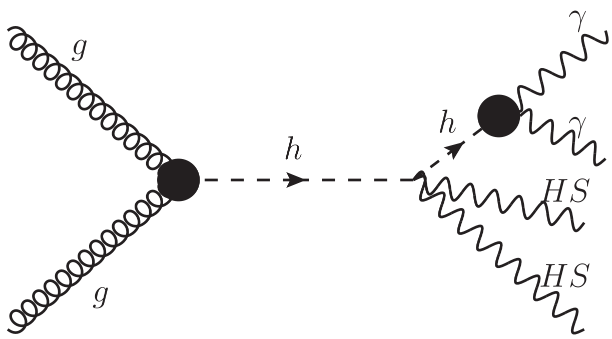

Figure 4.6. The Feynman diagram for the pair production of photons together with the HS states.g are the gluon fields, f are the fermions of the SM, h is the Higgs boson and HS stand for the hidden sector fields V_μ and S and γ is the photon.

All three give similar results in the sense that the number of events generated decreases as the λ_1 increases. Given that the masses of HS particles are found by $m_{HS} = \sqrt{\lambda_1}\upsilon_H$ where $\upsilon_H = 246$ GeV (Demir et al., 2015), in the parameter space with $4.0 \leq \lambda_1 \leq 5.0$, we obtain 492 GeV $\leq m_{HS} \leq$ 550 GeV. Hence, clearly, the Higgesd Stueckelberg scenario reopens the door for new physics at TeV scale and the results obtained are within the reach of both high-luminosity LHC (HL-LHC) and FCC. As both S and V_μ states are stable, they contribute to the invisible decays of the virtual Higgs boson. The most recent ATLAS searches set the upper limit for the invisible branching ratio of Higgs boson to 0.23 (Aad et al., 2015). Then, the ratio

$$R(h \to \mathrm{inv}) = \frac{\sigma(gg \to HS\ HS)}{\sigma(gg \to h)\mathrm{BR}(h \to \mathrm{inv})} \tag{4.12}$$

for the highest production rate of hidden sector in the Higgsed Stueckelberg scenario reads 0.00015 for 13 TeV center of mass energy, and 0.0026 for 100 TeV. The production cross sections for the Higgs boson might be found in the recent paper (Baglio et al., 2015). The smallness of the ratio indicates that, the Hidden sector does not bring any deviation in the invisible decays of Higgs boson and therefore our model is well consistent with the existing data. This result steer us into the elegant solution for the DM problem of the SM.

One possible observation channel of this scenario at LHC might be through the diphoton production. The Feynman diagram is given in Fig 4.6. We compute number of events generated at the LHC at $\sqrt{s} = 13$ TeV with integrated luminosity 18 fb^{-1} both

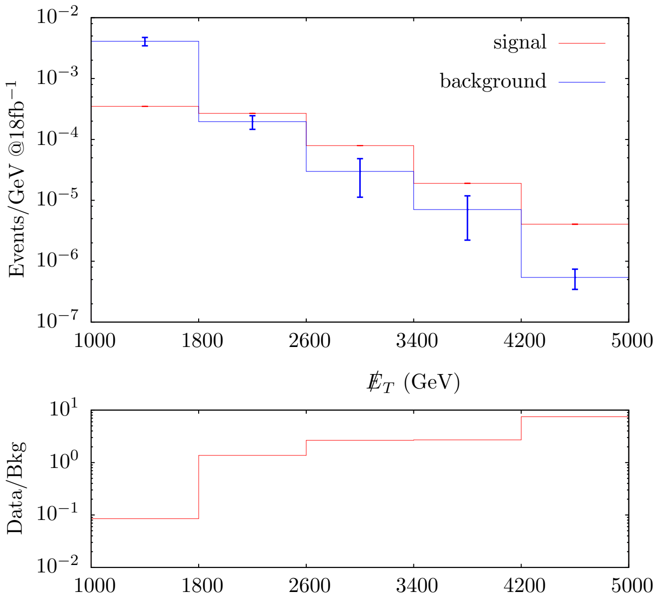

Figure 4.7. The number of events generated per year for the process $pp \to HS\gamma\gamma$ via gluon fusion channel at LHC with $\sqrt{s}$ = 13 TeV and luminosity 18 fb $^{-1}$ with respect to the missing transverse energy. The MMHT2014nnlo68cl PDF set has been used. The error bars are statitistical.

with respect to photon transverse momentum and mising energy. The simulation programs used in the analysis of this events are FeynRules (Christensen and Duhr, 2009; Christensen et al., 2011; Alloul et al., 2014) for implementing our model and CalcHEP (Belyaev et al., 2013) for the calculation of Feynman diagrams, integration over multi-particle phase space and parton level event simulation. The interface between FeynArts and Calchep is validated by (Christensen et al., 2011). Fig. 4.7 and Fig. 4.8 summarizes our results. The analysis indicate that it is possible to detect the fingerprint of this scenario through diphoton production channel in the missing energy higher than 1.8 TeV.

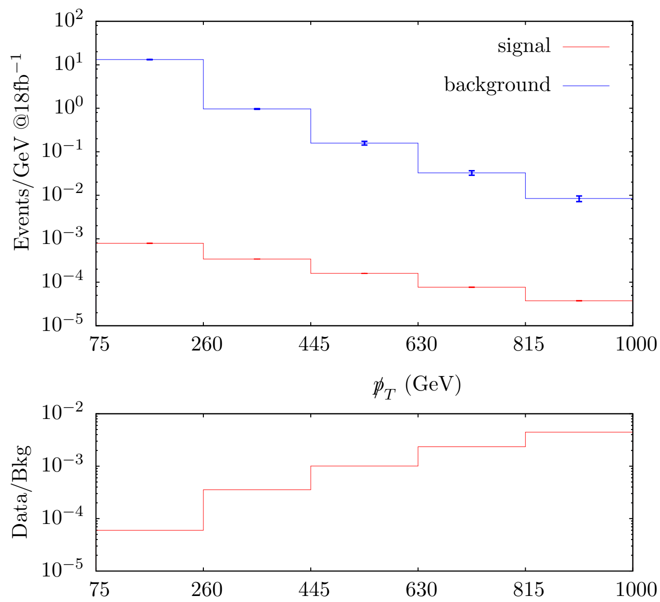

Figure 4.8. The number of events generated per year for the process $pp \rightarrow HS\gamma\gamma$ via gluon fusion channel at LHC with $\sqrt{s}$ = 13 TeV and luminosity 18 fb^{-1} with respect to the photon transverse momentum. The MMHT2014nnlo68cl PDF set has been used. The error bars are statitistical.

4.5. Dark Matter Analysis

Assuming a replica of the Lagrangian given in Eq. (4.4) with $V_\mu \to V'_\mu$, $S \to S'$ and $\lambda_1 \to \lambda_2$ one can obtain a natural Higgs scenario together with the DM candidates V'_μ and S' satisfying the 2013 PLANCK measurement of the relic density (Ade et al., 2014)

$$\Omega_{\mathrm{CDM}} h^2 = 0.1199 \pm 0.0027 \tag{4.13}$$

at 68% CL. While the naturalness of the Higgs boson is achieved via the unprimed fields in Lagrangian given in Eq. (4.4), the relic density result of Planck Space Telescope is satisfied with the primed fields. The DM analysis is completed via the routines of Micromegas software (Bélanger et al., 2015). The relic density constraint fixes the coupling of primed HS fields with the Higgs field as $\lambda_2 = 0.0465$.

The condition in Eq. (4.9) with the additional terms from the replica HS reads

$$(\delta m_H^2)_{\mathrm{quad}} = \frac{\Lambda^2}{16\pi^2}\left(\frac{4}{3}\lambda_H + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6g_t^2 + \lambda_1 + \lambda_2\right) \tag{4.14}$$

resulting an effective naturalness for Higgs at the TeV scale.

4.6. Conclusion and Outlook

The discovery of a new scalar (Aad et al., 2012; Collaboration et al., 2012) at the LHC, consistent with the SM Higgs boson, has accelerated studies on the UV-sensitivity of the Higgs boson. As opposed to the physical masses of chiral fermions and gauge bosons, which are protected by chiral and gauge symmetries, there is no symmetry principle to protect the Higgs boson mass against quadratically divergent quantum corrections. In the very absence of TeV-scale new physics, one is left with a finely-tuned Higgs sector where nature and degree of fine-tuning vary with the modeling details. In the presence of hidden scalars, despite the protection of the Higgs boson mass the hidden sector itself is UV-unstable. In case the hidden sector is formed by the spacetime curvature scalar, the fine-tuning is severe yet harmless because the SM fields and couplings are immune to its presence. The fine-tuning is as severe as hidden scalars in other field-theoretic approaches.

In this chapter of the thesis we have shown that a hidden sector spanned by an Abelian vector field whose mass is induced by electroweak breaking and whose gauge invariance is sustained by a Stueckelberg scalar can lead to stabilization of the Higgs boson mass by finely tuning its coupling to the SM Higgs field. In spite of this unavoidable fine-tuning, the Higgsed Stueckelberg model possesses the striking property that the hidden sector is insensitive to the UV scale. This stability, deriving from unbroken hidden gauge invariance, can have important collider, astrophysical and cosmological implications. Indeed, a stable hidden sector can be utilized in constructing viable models of Dark Matter, Dark Energy, Dark Photon and neutrino masses. The model can be tested at the LHC (and its successor FCC) via direct productions of V_μ and S fields.
We have also analysed the fingerprints of the V_μ and S fields from Higgsed Stueckelberg scenario at the HL-LHC and the FCC. Thanks to its simplistic formalism, the only new parameter in the model is λ_1, the coupling of the Higgs field with the HS fields. Although, at one loop the Higgs mass stability is realized via setting $\lambda_1 = 4.41$, considering the possible higher loop effects, the analysis is performed in the interval $4.0 \leq \lambda_1 \leq 5.0$. We compute the number of events generated for the HS fields against λ_1 at LHC with $\sqrt{s} = 13$ TeV and luminosity 18 fb^{-1}, and at FCC with c.m. energy $\sqrt{s} = 100$ TeV and luminosity 100 fb^{-1}, and find that the production cross sections $gg \to SS$, $gg \to V_\mu V_\nu$ and $gg \to SV_\mu$ decrease while λ_1 increases. The results are not in conflict with the existing data in the invisible Higgs branching ratio and are within the reach of both at HL-LHC and FCC. The possible observation channel of this scenario might be through diphoton production with missing energy which might serve as distinctive signal of this scenario.

CHAPTER 5

CONCLUSION

An anamolous observation -flat rotation curves of galaxies lead to two different hypothesis - Dark Matter and MOND. Dark Matter (DM) approach states that there is non-shining matter at the outer parts of galaxies. It is a well established hypothesis to explain anomalous observations not only at galactic scales but also at larger scales. In spite of many achievements, up to now the lack of direct and indirect detection of dark matter candidates proposed up to now yields the question whether there is really dark matter in the Universe or not. On the other hand, Modified Newtonian Dynamics (MOND) approach is based on the modification of well known laws of the motion - Newtonian Dynamics. However, the lack of full-fledge relativistic generalization of MOND is still a problem. Both of them possess some challenges as well as many achievements.In this thesis, we focus on the main problems of Dark Matter and MOND theory.

We began with the relativistic generalization of MOND theory. The first successful relativistic generalization is TeVeS theory based on bimetric gravity. TeVeS includes vector and scalar field actions in addition to the standard tensor field action. These extra degrees of freedom are added by hand in an unnatural way. In this study, we have shown that TeVeS-like theory can be obtained in a more natural way via metric-affine gravity wherein metric and connection is treated as two independent variables. We have obtained vector and scalar fields from the geometry, that is what we mean by a natural way. To do this, we first decomposed affine connection as Levi-Civita connection and rank (1,2) tensor field. By using the fact that a higher rank tensor field can be decomposed to the lower rank tensor fields such as vector fields, we have obtained a tensor-vector theory. Since there are three different ways for the contraction of rank(1,2) tensor field while getting vector fields, there arise also three different vector fields. To decrease the number of these fields, we used some plausible constraints on the tensorial structure- non-symmetric tensor field. Imposing this constraint makes the theory a tensor-vector theory. However, the lack of scalar field prevents us from obtainig a tensor-vector-scalar theory. At this point, we used the fact that a vector field may be composite field composed of a fundamental vector field and the gradient of a scalar field. Via this definition, we have obtained a true tensor-vector-scalar theory which is more familiar ones like TeVeS gravity

Secondly, we have studied the relativistic MOND theory without action principle.

The difference of our study from the ones in the literature is our dynamical approach to the relativistic MOND theory. We have focused on the modified dynamics rather than modified gravity. We began with the question that what modifications in energy-momentum tensor may yield the MONDian regime. Then, we modified the energy-momentum tensor by using a new scalar structure in relativistic domain called as acceleration scalar $\mathfrak{a}$ and another scalar quantity Q based on a second metric. The construction of these scalar quantities and the physics that lies under are very elegant. Acceleration scalar $\mathfrak{a}$ involves the divergence of energy-momentum tensor and the gradient of the other new scalar Q. The only relativistic structure related to the acceleration in weak field limit is the divergence of energy-momentum tensor. This is why we generalize the energy momentum tensor with a such that relativistic scalar quantity. The other scalar quantity is based on an induced metric defined by four real scalars developing non-trivial backgrounds depending on the whether the diffeomorphism invariance is exact or spontaneously broken. Therefore, the existence of second metric leads to arise two different regimes(Newtonian and MONDian regimes) depending on the whether the diffeomorphism invariance breaks or not. When diffeomorphism invariance is exact, equations of motion are reduced to the standard Einstein Field Equations, and then Newtonian dynamics in weak field limit. On the other hand, spontaneously broken of the diffeomorphism invariance yields to modified equations of motion which may be reduced to true MONDian force in the non-relativistic limit.

Finally, we have proposed an extended SM scenario called Higgsed Stueckelberg model. This scenario involves a hidden sector composed of an Abelian vector field whose gauge invariance is guaranteed by a Stueckelberg scalar. We have shown that the contributions from hidden sector to the quadratic divergences in the Higgs boson mass-squared may stabilize the Higgs boson mass at one-loop. Moreover, by adding the primed replicas of these fields with a new coupling constant, we have shown that primed Higgsed Stueckelberg fields can serve as perfect dark matter candidates which provides the current relic density of DM. In order to test the model at LHC and FCC, we have also analysed the fingerprints of the V_μ and S fields. As a result, the Higgsed Stueckelberg scenario is exquisite in the sense that it wins the battle against the destabilization of the Higgs mass and also does not push the new physics beyond TeV scale. As the quanta of HS fields are stable Higgs portal states, while they serve for the purposes of naturalness their primed replicas may satisfy the current relic density result without touching the stabilization of Higgs at the TeV scale.

The thesis is based on the following publications:

1. D. A. Demir, C. N. Karahan, B.Korutlu, O. Sargın, İ Turan, Fingerprinting the Higgsed Stueckelberg Scenario at Colliders,to be submitted,(2016).
2. D. A. Demir, C. N. Karahan, B.Korutlu, Higgsed Stueckelberg vector and Higgs quadratic divergence. Physics Letters B, 740, 46-50, (2015).
3. D. A. Demir, C. N. Karahan, Relativistic MOND from modified energetics. The European Physical Journal C, 74(12), 1-7, (2014)
4. C. N. Karahan, A. Altaş, D. A. Demir, Scalars, vectors and tensors from metric-affine gravity. General Relativity and Gravitation, 45(2), 319-343,(2013).

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APPENDIX A

CONTRACTION TENSORS

Contraction of tensors becomes a tedious operation as their rank becomes larger and larger. Already at the rank-3 level, there arise various possibilities in contracting the indices. Indeed, if one defines

$$\mathbb{A} \bullet \mathbb{B} \equiv \mathbb{A}^{\lambda}_{\alpha\beta} \Xi^{\alpha\beta\mu\nu}_{\lambda\rho} \mathbb{B}^{\rho}_{\mu\nu} \tag{A.1}$$

the contraction tensor $\Xi^{\alpha\beta\mu\nu}_{\lambda\rho}$ is found to have the most general form

$$\begin{aligned} \Xi^{\alpha\beta\mu\nu}_{\lambda\rho} &= g_{\lambda\rho}\left(g^{\alpha\beta}g^{\mu\nu} \oplus g^{\alpha\mu}g^{\beta\nu} \oplus g^{\alpha\nu}g^{\beta\mu}\right) \oplus \delta^{\alpha}_{\lambda}\left(g^{\beta\nu}\delta^{\mu}_{\rho} \oplus g^{\beta\mu}\delta^{\nu}_{\rho} \oplus g^{\mu\nu}\delta^{\beta}_{\rho}\right) \\ &\oplus \delta^{\beta}_{\lambda}\left(g^{\alpha\nu}\delta^{\mu}_{\rho} \oplus g^{\alpha\mu}\delta^{\nu}_{\rho} \oplus g^{\mu\nu}\delta^{\alpha}_{\rho}\right) \oplus \delta^{\mu}_{\lambda}\left(g^{\beta\nu}\delta^{\alpha}_{\rho} \oplus g^{\alpha\beta}\delta^{\nu}_{\rho} \oplus g^{\alpha\nu}\delta^{\beta}_{\rho}\right) \\ &\oplus \delta^{\nu}_{\lambda}\left(g^{\beta\mu}\delta^{\alpha}_{\rho} \oplus g^{\alpha\beta}\delta^{\mu}_{\rho} \oplus g^{\alpha\mu}\delta^{\beta}_{\rho}\right) \end{aligned} \tag{A.2}$$

where $\oplus$ implies $+$ or $-$ depending on whether symmetric or antisymmetric combinations of the indices are involved. Clearly, $\oplus$ also contains the appropriate symmetry factors.

As an example, let us take $\mathbb{B}^{\rho}_{\mu\nu} = \mathbb{S}^{\rho}_{\mu\nu}$ which is antisymmetric in (μ, ν). In this case, when contracting $\mathbb{S}^{\rho}_{\mu\nu}$ with $\Xi^{\alpha\beta\mu\nu}_{\lambda\rho}$ only the anti symmetric part of $\Xi^{\alpha\beta\mu\nu}_{\lambda\rho}$ in (μ, ν) matters. In other words, when $\mathbb{B}^{\rho}_{\mu\nu} = \mathbb{S}^{\rho}_{\mu\nu}$ we consider only

$$\begin{aligned} \Xi^{\alpha\beta[\mu\nu]}_{\lambda\rho} &= \frac{1}{2}\Big[g_{\lambda\rho}\left(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}\right) \oplus \delta^{\alpha}_{\lambda}\left(g^{\beta\nu}\delta^{\mu}_{\rho} - g^{\beta\mu}\delta^{\nu}_{\rho}\right) \oplus \delta^{\beta}_{\lambda}\left(g^{\alpha\nu}\delta^{\mu}_{\rho} - g^{\alpha\mu}\delta^{\nu}_{\rho}\right) \\ &\oplus \left[\delta^{\mu}_{\lambda}\left(g^{\beta\nu}\delta^{\alpha}_{\rho} \oplus g^{\alpha\beta}\delta^{\nu}_{\rho} \oplus g^{\alpha\nu}\delta^{\beta}_{\rho}\right) - \delta^{\nu}_{\lambda}\left(g^{\beta\mu}\delta^{\alpha}_{\rho} \oplus g^{\alpha\beta}\delta^{\mu}_{\rho} \oplus g^{\alpha\mu}\delta^{\beta}_{\rho}\right)\right]\Big] \end{aligned} \tag{A.3}$$

which is anti-symmetric in (μ, ν).

If $\mathbb{A}^{\lambda}_{\alpha\beta}$ in (A.1) is antisymmetric in (α, β) then we consider antisymmetric part of

(A.3).

$$
\begin{aligned}
\Xi^{[\alpha\beta][\mu\nu]}_{\lambda\rho} &= \frac{1}{4}\Big[\delta^{\alpha}_{\lambda}\left(g^{\beta\nu}\delta^{\mu}_{\rho} - g^{\beta\mu}\delta^{\nu}_{\rho}\right) - \delta^{\beta}_{\lambda}\left(g^{\alpha\nu}\delta^{\mu}_{\rho} - g^{\alpha\mu}\delta^{\nu}_{\rho}\right)\Big] \\
&+ \frac{1}{4}\Big[\delta^{\mu}_{\lambda}\left(g^{\beta\nu}\delta^{\alpha}_{\rho} - g^{\alpha\nu}\delta^{\beta}_{\rho}\right) - \delta^{\nu}_{\lambda}\left(g^{\beta\mu}\delta^{\alpha}_{\rho} - g^{\alpha\mu}\delta^{\beta}_{\rho}\right)\Big] \\
&+ \frac{1}{2}g_{\lambda\rho}\left(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}\right) \qquad \text{(A.4)}
\end{aligned}
$$

For instance, $\mathbb{S} \bullet \mathbb{S}$ will be computed by using this contraction tensor.

However, if $\mathbb{A}^{\lambda}_{\alpha\beta}$ in (A.1) is symmetric in (α, β) then we have to consider symmetric part of (A.3).

$$
\begin{aligned}
\Xi^{(\alpha\beta)[\mu\nu]}_{\lambda\rho} &= \frac{1}{4}\Big[\delta^{\alpha}_{\lambda}\left(g^{\beta\nu}\delta^{\mu}_{\rho} - g^{\beta\mu}\delta^{\nu}_{\rho}\right) + \delta^{\beta}_{\lambda}\left(g^{\alpha\nu}\delta^{\mu}_{\rho} - g^{\alpha\mu}\delta^{\nu}_{\rho}\right)\Big] \\
&+ \frac{1}{4}\Big[\delta^{\mu}_{\lambda}\left(g^{\beta\nu}\delta^{\alpha}_{\rho} + g^{\alpha\nu}\delta^{\beta}_{\rho}\right) - \delta^{\nu}_{\lambda}\left(g^{\beta\mu}\delta^{\alpha}_{\rho} + g^{\alpha\mu}\delta^{\beta}_{\rho}\right)\Big] \\
&+ \frac{1}{2}g^{\alpha\beta}\left(\delta^{\mu}_{\lambda}\delta^{\nu}_{\rho} - \delta^{\nu}_{\lambda}\delta^{\mu}_{\rho}\right) \qquad \text{(A.5)}
\end{aligned}
$$

For instance, $\mathbb{Q} \bullet \mathbb{S}$ should be computed by using this contraction tensor. In computing $\mathbb{Q} \bullet \mathbb{Q}$ we should symmetrize in both (μ, ν) and (α, β). Then contraction tensor of $\mathbb{Q} \bullet \mathbb{Q}$ is given

$$
\begin{aligned}
\Xi^{(\alpha\beta)(\mu\nu)}_{\lambda\rho} &= g^{\alpha\beta}g^{\mu\nu}g_{\lambda\rho} + \frac{1}{2}\Big[g_{\lambda\rho}\left(g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu}\right) \\
&+ g^{\mu\nu}\left(\delta^{\alpha}_{\lambda}\delta^{\beta}_{\rho} + \delta^{\beta}_{\lambda}\delta^{\alpha}_{\rho}\right) + g^{\alpha\beta}\left(\delta^{\mu}_{\lambda}\delta^{\nu}_{\rho} + \delta^{\nu}_{\lambda}\delta^{\mu}_{\rho}\right)\Big] \\
&+ \frac{1}{4}\Big[\delta^{\alpha}_{\lambda}\left(g^{\beta\nu}\delta^{\mu}_{\rho} + g^{\beta\mu}\delta^{\nu}_{\rho}\right) + \delta^{\beta}_{\lambda}\left(g^{\alpha\nu}\delta^{\mu}_{\rho} + g^{\alpha\mu}\delta^{\nu}_{\rho}\right)\Big] \\
&+ \frac{1}{4}\Big[\delta^{\mu}_{\lambda}\left(g^{\beta\nu}\delta^{\alpha}_{\rho} + g^{\alpha\nu}\delta^{\beta}_{\rho}\right) + \delta^{\nu}_{\lambda}\left(g^{\beta\mu}\delta^{\alpha}_{\rho} + g^{\alpha\mu}\delta^{\beta}_{\rho}\right)\Big] \\
&\qquad \text{(A.6)}
\end{aligned}
$$

In addition to these, one can compute contraction of divergence of tensors as

$$
\nabla^{\emptyset}A \bullet \nabla^{\emptyset}B = \nabla^{\emptyset}_{\lambda}A^{\lambda}_{\alpha\beta}\Xi^{\alpha\beta\mu\nu}\nabla^{\emptyset}_{\rho}B^{\rho}_{\mu\nu} \qquad \text{(A.7)}
$$

$\Xi^{\alpha\beta\mu\nu}$ is contraction tensor and defined in general form as

$$\Xi^{\alpha\beta\mu\nu} = g^{\alpha\beta}g^{\mu\nu} \oplus g^{\alpha\mu}g^{\beta\nu} \oplus g^{\alpha\nu}g^{\beta\mu} \tag{A.8}$$

If A is symmetric in (α,β) and B is symmetric in (μ,ν) contraction tensor takes the form

$$\Xi^{(\alpha\beta)(\mu\nu)} = g^{\alpha\beta}g^{\mu\nu} + \frac{1}{2}\left(g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu}\right) \tag{A.9}$$

this contraction tensor can be used to compute $\nabla\mathbb{Q} \bullet \nabla\mathbb{Q}$ because $\mathbb{Q}$ is symmetric in $(\alpha\beta)$. To compute $\nabla\mathbb{S} \bullet \nabla\mathbb{S}$, one needs contraction tensor which is antisymmetric both couple of indices.So,

$$\Xi^{[\alpha\beta][\mu\nu]} = \frac{1}{2}\left(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}\right) \tag{A.10}$$

If one writes contraction tensor of $\nabla\mathbb{Q} \bullet \nabla\mathbb{S}$, it is as;

$$\Xi^{(\alpha\beta)[\mu\nu]} = 0 \tag{A.11}$$

APPENDIX B

POSITIVE-DEFINITE MASS MATRIX

In the text, we mentioned that for a stable theory, each of the three eigenvalues must individually be positive. This leads to non-trivial constraints on the coefficients in (2.42). In this appendix we shall discuss certain related details. The eigenvalues of (2.41) follow from the cubic algebraic equation

$$-\lambda^3 + b\lambda^2 + c\lambda + d = 0 \tag{B.1}$$

where

$$\begin{aligned} b &= a_{VV} + a_{UU} + a_{WW}, \\ c &= -a_{VV}a_{UU} - a_{VV}a_{WW} - a_{UU}a_{WW} + a_{UW}^2 + a_{VU}^2 + a_{VW}^2, \\ d &= a_{VV}a_{UU}a_{WW} + 2a_{VU}a_{UW}a_{VW} - a_{UW}^2 a_{VV} - a_{VW}^2 a_{UU} - a_{VU}^2 a_{WW}\,. \end{aligned} \tag{B.2}$$

The roots of (B.1) must each be non-negative for guaranteeing absence of instabilities. The analytic expressions for roots are well-known. However, the constraint equations they lead to are too complicated to achieve specific statements about the elements of the mass matrix (2.41). Nevertheless, in a given specific problem, one can determine the allowed ranges for $a_{VV}, \cdots, a_{UW}$ at least numerically,

As an algebraically simpler case to exemplify, one can focus on the special case of vanishing discriminant, that is, one considers

$$\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2 \tag{B.3}$$

so that only two independent eigenvalues are left. Indeed, one has

$$\lambda_1 = -\frac{b}{3a} - \frac{2}{3a}\sqrt[3]{\frac{1}{2}[2b^3 - 9abc + 27a^2d]} \tag{B.4}$$

and

$$\lambda_2 = -\frac{b}{3a} + \frac{1}{3a}\sqrt[3]{\frac{1}{2}[2b^3 - 9abc + 27a^2d]} \tag{B.5}$$

For positive-definite mass matrix, λ_1 and λ_2 must each be positive:

$$\lambda_1 > 0 \Rightarrow -\frac{b}{2} < \sqrt[3]{\frac{1}{2}[2b^3 - 9abc + 27a^2d]} \tag{B.6}$$

and

$$\lambda_2 > 0 \Rightarrow b > \sqrt[3]{\frac{1}{2}[2b^3 - 9abc + 27a^2d]} \tag{B.7}$$

These two constraints lead one at once to the bound

$$-\frac{b}{2} < \sqrt[3]{\frac{1}{2}[2b^3 - 9abc + 27a^2d]} < b \tag{B.8}$$

Similar bounds can be derived for general as well as special cases (Demir, 2004). In general, constraints on various coefficients become more suggestive in some physically relevant special cases. We here thus exemplify two such cases: Symmetric and Antisymmetric connections.

1. Symmetric Connection : $\mathbb{\Gamma}^{\lambda}_{\alpha\beta}=\mathbb{\Gamma}^{\lambda}_{\beta\alpha}$

   As we have mentioned in the text, in this case, torsion tensor identically vanishes ($\mathbb{S}^{\lambda}_{\alpha\beta} = 0$), and consequently $\mathtt{V}_{\alpha} = \mathtt{U}_{\alpha}$. The theory then reduces to a two-vector theory of $\mathtt{V}$ and $\mathtt{W}$. From Eq. (2.40) the mass-squared matrix of vectors is found to be

$$\frac{1}{2}M_{Pl}^2\begin{pmatrix} a'_{VV} + a'_{UU} + 2a'_{VU} & a'_{VW} + a'_{UW} \\ a'_{VW} + a'_{UW} & a'_{WW} \end{pmatrix} \tag{B.9}$$

where various coefficients are given by

$$\begin{aligned}
a'_{VV} &= \frac{1}{18} + \frac{44}{9} c_Q \,, \\
a'_{UU} &= a_{WW} \,, \\
a'_{WW} &= \frac{1}{18} + \frac{14}{9} c_Q \,, \\
a'_{VU} &= -\frac{1}{9} + \frac{20}{9} c_Q \,, \\
a'_{VW} &= -\frac{1}{9} + \frac{20}{9} c_Q \,, \\
a'_{UW} &= \frac{7}{18} + \frac{14}{9} c_Q \,.
\end{aligned} \tag{B.10}$$

which follow from (2.40) for vanishing torsion. Clearly, c_Q is the only variable. The eigenvalues of (B.9) follow from the quadratic algebraic equation;

$$\lambda^2 + b'\lambda + c' = 0 \tag{B.11}$$

where

$$\begin{aligned}
b' &= -a'_{VV} - a'_{UU} - 2a'_{UU} - a'_{WW}, \\
c' &= (a'_{VV} + a'_{UU} + 2a'_{UU}) a'_{WW} - (a'_{VW} + a'_{UW})^2 \,.
\end{aligned} \tag{B.12}$$

From the equation (B.11), one directly determines the discriminant

$$\Delta = \frac{11680}{81} c_Q^2 + \frac{872}{162} c_Q + \frac{109}{324} \tag{B.13}$$

and eigenvalues

$$\begin{aligned}
\lambda_{1,2} &= \frac{a'_{VV} + a'_{UU} + 2a'_{UU} + a'_{WW} \pm \sqrt{\Delta}}{2} \\
&= \frac{-\frac{1}{18} + \frac{112}{9} c_Q \pm \sqrt{\Delta}}{2}
\end{aligned} \tag{B.14}$$

For a physically sensible theory, the eigenvalues must all be positive. By considering the constraint of positive discriminant and roots, one finds two appropriate intervals

$$c_Q < -0.046, \quad c_Q > 0.68 \tag{B.15}$$

This shows that except for the small interval containing origin, all values of c_Q lead to a stable massive two-vector theory.

2. Anti-symmetric tensorial connection: $V_\alpha = -U_\alpha$ and $W_\alpha = 0$

In this case we end up with a single-vector theory with mass-squared $\frac{1}{2}M_{Pl}^2\bar{a}_{VV}$ where $\bar{a}_{VV} = 1/3 + 8c_S + 2c_Q + 8c_{QS}$. This coefficient must be positive and hence

$$4c_S + c_Q + 4c_{QS} > -\frac{1}{6} \tag{B.16}$$

A much more special arises when non-metricity vanishes. In this special case, the coefficients c_Q and c_{QS} both vanishe, and one finds

$$c_S > -\frac{1}{24} \tag{B.17}$$

as a bound on c_S.

APPENDIX C

VERTEX FACTORS

Here we list the vertex factors:

$$\begin{aligned}
\lambda_{\mathrm{hhVV}} &= 2i\lambda_1 g^{\mu\nu}, \\
\lambda_{\mathrm{hhSS}} &= -\frac{2i}{\upsilon_H^2} k_\mu q_\nu g^{\mu\nu}, \\
\lambda_{\mathrm{hVV}} &= 2i\lambda_1 \upsilon_H g^{\mu\nu}, \\
\lambda_{\mathrm{hSS}} &= -\frac{2i}{\upsilon_H} k_\mu q_\nu g^{\mu\nu}, \\
\lambda_{\mathrm{hVS}} &= 2\sqrt{\lambda_1} k_\mu g^{\mu\nu}.
\end{aligned} \tag{C.1}$$

where k_μ is the momentum of S. We used $a_H = \upsilon_H$ in a_H dependent vertices.

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