Bianchi I model: an alternative way to model the present-day Universe

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ABSTRACT

Although the new era of high-precision cosmology of the cosmic microwave background (CMB) radiation improves our knowledge to understand the infant as well as the present-day Universe, it also leads us to question the main assumption of the exact isotropy of the CMB. There are two pieces of observational evidence that hint towards there being no exact isotropy. These are: first, the existence of small anisotropy deviations from isotropy of the CMB radiation and secondly, the presence of large angle anomalies, although the existence of these anomalies is currently a huge matter of debate. These hints are particularly important since isotropy is one of the two main postulates of the Copernican principle on which the Friedmann Robertson Walker (FRW) models are built. This almost-isotropic CMB radiation implies that the universe is almost an FRW universe, as is proved by previous studies. Assuming that the matter component forms the deviations from isotropy in the CMB density fluctuations when matter and radiation decouples, we here attempt to find possible constraints on the FRW-type scale and Hubble parameter by using the Bianchi type I (BI) anisotropic model which is asymptotically equivalent to the standard FRW. To obtain constraints on such an anisotropic model, we derive average and late-time shear values that come from the anisotropy upper limits of the recent *Planck* data based on a model independent shear parameter of Maartens, Ellis & Stoeger and from the theoretical consistency relation. These constraints lead us to obtain a BI model which becomes an almost-FRW model in time, and which is consistent with the latest observational data of the CMB.

Key words: methods: analytical – cosmology: theory – early Universe – large-scale structure of Universe.

1 INTRODUCTION

The accepted model of the present-day Universe is homogenous and isotropic on large scales and is defined by the Robertson Walker (RW) metric in the de Sitter space–time. On the other hand, the new high-precision cosmology era brings some new insights to our understanding of the infant universe via two important hints of observational evidences of broken isotropy of the cosmic microwave background (CMB) radiation. The first hint for broken isotropy is small temperature anisotropies with approximately 10^{-5} amplitude, while the second hint is the family of some large angle anomalies (Bennett et al. 2003) that are the alignment of quadrupole and octopole moments (Copi, Huterer & Starkman 2004; de Oliveira-Costa et al. 2004; Ralston & Jain 2004; Land & Magueijo 2005), large-scale asymmetry (Eriksen et al. 2004; Hansen, Banday & Górski 2004), the unusually coldspot (Vielva

et al. 2004) and the low quadrupole moment. Although there are various studies to explain the cosmological origin of the large angle anomalies such as anisotropic inflation (Berera, Buniy & Kephart 2004; Gordon et al. 2005; Gümrükçüoglu & Peloso 2007; Pereira, Pitrou & Uzan 2007; Koivisto & Mota 2008; Yokoyama & Soda 2008), inhomogeneous spaces (Jaffe et al. 2006; Land & Magueijo 2006; Bridges et al. 2007; Pontzen & Challinor 2007), local spherical voids (Inoue & Silk 2006), an initial phase of inflation (Contaldi et al. 2003; Donoghue, Dutta & Ross 2009) and a non-trivial spherical topology (Luminet et al. 2003), it was not certain that the anomalies were observational artefacts. Recent data from the *Planck* satellite have fuelled suggestions that these anomalies could represent real features of the CMB map of the Universe (Planck Collaboration 2013a); however, this is currently a topic of debate. On the other hand, this possible observational evidence has a key importance since the small temperature anisotropies and large angle anomalies may be caused by some unknown mechanisms or an anisotropic phase during the evolution of the Universe. This statement is particularly interesting since it aids us to modify the

current model or to construct an alternative model to decode the effects of the early universe on the present-day large-scale structure without affecting the processes in the nucleosynthesis.

The answer of how to construct a new universe model or modify the standard model may be found in the early theories. According to the theories proposed by Misner (1968) and Gibbons & Hawking (1977), anisotropy of the early stage of the universe may turn into an isotropic present universe and initial anisotropies die away. As is known from the present-day observational data that there are anisotropies in the CMB; therefore, it is possible that the anisotropies or anomalies are imprints of this early anisotropic phase on the CMB. Apart from this, Stoeger, Maartens & Ellis (1995) and Maartens, Ellis & Sroeger (1996) show if all fundamental observers measure the CMB radiation to be almost isotropic in an expanding universe, the universe is locally almost spatially homogeneous and isotropic in a region. This result formalizes the way of an almost Friedmann Robertson Walker (FRW) space–time since the time of the decoupling of matter and radiation based on the evidence almost isotropy of the CMB (Hawking & Ellis 1973; Stoeger et al. 1995; Maartens et al. 1996; Cea 2014). One may question the effects of an almost-FRW model on the primordial nucleosynthesis. Barrow (1997) shows that it is possible for anisotropic fluids to create a measurable temperature anisotropy in the CMB without having any significant effects upon the primordial nucleosynthesis of He^4. In fact, Campanelli, Cea & Tedesco (2007) discuss this matter and confirm Barrow's result.

Bianchi models can be alternatives to the standard FRW models with small deviations from the exact isotropy in order to explain the anisotropies and anomalies in the CMB. In this framework, Jaffe et al. (2005) propose that the large angle anomalies can be mimicked by using a specific solution of the Bianchi type VII_h ($BVII_h$) universe based on models developed by Collins & Hawking (1973) and Barrow, Juszkiewicz & Sonoda (1985). In addition, Pontzen & Challinor (2007) point out that the polarization signal in the $BVII_h$ universe can mimic the several large angle anomalous features observed in the CMB. However, Planck Collaboration (2013b) show that the $BVII_h$ model is not consistent with the observational data from the *Planck* satellite. Apart from this, one of the large angle anomalies of the CMB; the low quadrupole moment is particularly important since it may indicate a presence of Bianchi type I (BI) anisotropic evolution in the early universe. How this happens? The low quadrupole moment shows a great amount of power suppression at large scales. This suppression cannot be satisfied by the standard dark energy dominated cold dark matter model as in indicated by the recent *Planck* and the *Wilkinson Microwave Anisotropy Probe* (Hinshaw et al. 2013) results (Planck Collaboration 2013c). Particularly, Planck Collaboration (2013c) underline that the standard model is incomplete. Earlier, Martinez-Gonzalez & Sanz (1995) have proved that the small quadrupole component of the CMB temperature found by *Cosmic Background Explorer* (*COBE*; Smoot et al. 1992) implies that if the universe is homogeneous but anisotropic BI then it necessarily must be a small departure from the flat Friedmann model. Later on, Cea (2014), Campanelli et al. (2007) and Campanelli, Cea & Tedesco (2006) show that the low quadrupole moment can be reduced in a plane symmetric BI universe. Recently, Aluri et al. (2013) analyse the state space of a BI universe with anisotropic sources. In their study, assuming the universe contains anisotropic model including matter and dark energy components since decoupling, they find that this type of BI model contributes dominantly to the CMB quadrupole. Given its importance for studying the possible effects of an anisotropy in the early universe on present-day observations, many researchers have

investigated the BI model from different perspectives. Examples of these studies are string theory (Alexeyev, Toporensky & Ustiansky 2001; Rao, Vinutha & Sireesha 2008; Rathore & Mandawat 2009; Bali & Gupta 2010; Rikhvitsky, Saha & Visinescu 2012), dynamical properties (Salucci & Fabbri 1983; Chimento & Forte 2006; Ellis 2006; Akarsu & Kılınç 2010a,b; Adhav, Gadodia & Bansod 2011; Appleby & Linder 2013; Singh & Chaubey 2012; Ali & Rahaman 2013; Kohli & Haslam 2013; Mostafapoor & Grøn 2013; Pradhan, Jaiswal & Khare 2013; Singh & Chaubey 2013), the singularity problem (Belinskij, Khalatnikov & Lifshits 1970; Khalatnikov et al. 2003; Bronnikov, Chudayeva & Shikin 2004), the spinor/scalar field (Saha 2001, 2005, 2006, 2013; Saha & Boyadjiev 2004; Fay 2005; Kucukakca, Camci & Semiz 2012; Pradhan, Singh & Amirhashchi 2012; Carloni, Vignolo & Fabbri 2013) and perturbations in the early phase of inflation (Gümrükçüoglu & Peloso 2007; Pereira et al. 2007; Dong 2010; Bali 2011; Kofman, Uzan & Pitrou 2011; Aluri & Jain 2012). Separately, Maartens et al. (1995a) obtain evolution equations by which matter imposes anisotropies on freely propagating CMB radiation, leading to a new model independent way of using anisotropy measurements to limit the deviations of the Universe from an FRW geometry. Following this, Maartens et al. (1996) show how to place the quadrupole and octopole direct and explicit limits on the shear, vorticity, Weyl tensor and density gradients. Later, Stoeger, Araujo & Gebbie (1997) point out that it is possible to find limits on all anisotropy parameters such as shear, viscosity, Weyl tensor and density gradients, etc. if one can determine limits on the anisotropy components from the CMB measurements.

In this study, assuming that the CMB anisotropies show themselves as slight deviations from isotropy in the density fluctuations since decoupling of matter and radiation, we formulate possible constraints on the separations from isotropy by modelling the evolution of the BI model that is asymptotically the FRW one. Then, our main goals here are to construct an anisotropic BI model that leads us to obtain functional forms of deviations from the standard FRW one, and to limit the most up to date constraints via the two distortion/shear parameters of Maartens et al. (1995a,b) by using the upper bounds of the dipole, quadrupole and octopole moments of the recent *Planck* data.

The structure of this paper is the following. First, we give the general framework of the BI model including the necessary theoretical background and the isotropization criteria of the BI model to the FRW one at decoupling. After providing the well-known exact solutions of the field equations of the BI model, the criteria of deviations with two different dynamical behaviours are obtained which are directly related with the sign of deviation/anisotropy parameters. Then, we calculate the upper limits of the deviations satisfying the average and late-time distortion/shear values obtained from the *Planck* anisotropy limits. Finally, the discussion and conclusions are summarized.

2 BI MODELS

The BI model admits the metric element that has different scalefactors in each direction,

$$dl^2 = c^2 dt^2 - a_1^2(t) dx^2 - a_2^2(t) dy^2 - a_3^2(t) dz^2, \tag{1}$$

where a_1, a_2 and a_3 represent three different scalefactors which are a function of time t. If we admit the energy–momentum tensor of a perfect fluid, then the field equations of the BI universe

are found as

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = 8\pi G \rho, \tag{2a}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} = -\frac{8\pi G}{c^2} p, \tag{2b}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_2 \dot{a}_1}{a_2 a_1} = -\frac{8\pi G}{c^2} p, \tag{2c}$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_3 \dot{a}_2}{a_3 a_2} = -\frac{8\pi G}{c^2} p. \tag{2d}$$

Here, the dot represents the derivatives in terms of time t. To solve the system of equations (2), we define the following new variables which are simply the directional Hubble parameters,

$$H_1 \equiv \frac{\dot{a}_1}{a_1}, \quad H_2 \equiv \frac{\dot{a}_2}{a_3}, \quad H_3 \equiv \frac{\dot{a}_3}{a_3}. \tag{3}$$

Their first derivatives are

$$\dot{H}_1 = \frac{\ddot{a}_1}{a_1} - \left(\frac{\dot{a}_1}{a_1}\right)^2, \quad \dot{H}_2 = \frac{\ddot{a}_2}{a_2} - \left(\frac{\dot{a}_2}{a_2}\right)^2, \quad \dot{H}_3 = \frac{\ddot{a}_3}{a_3} - \left(\frac{\dot{a}_3}{a_3}\right)^2. \tag{4}$$

Inserting variables (3) and their derivatives (4) into the Einstein field equations (2), we reformulate the field equations in terms of the directional Hubble parameters,

$$H_1 H_2 + H_1 H_3 + H_2 H_3 = 8\pi G \rho, \tag{5a}$$

$$\dot{H}_3 + H_3^2 + \dot{H}_1 + H_1^2 + H_3 H_1 = -\frac{8\pi G}{c^2} p, \tag{5b}$$

$$\dot{H}_1 + H_1^2 + \dot{H}_2 + H_2^2 + H_1 H_2 = -\frac{8\pi G}{c^2} p, \tag{5c}$$

$$\dot{H}_3 + H_3^2 + \dot{H}_2 + H_2^2 + H_3 H_2 = -\frac{8\pi G}{c^2} p. \tag{5d}$$

In addition to this, the energy conservation equation $T_{\nu;\mu}^{\mu} = 0$ yields

$$\dot{\rho} = -\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\left(\rho + \frac{p}{c^2}\right) = -3H\left(\rho + \frac{p}{c^2}\right). \tag{6}$$

As is known, the BI universe has a flat metric with $k = 0$ which implies that its total density is equal to the critical density. The critical density is given by

$$\rho = \rho_c = \frac{1}{8\pi G}(H_1 H_2 + H_1 H_3 + H_2 H_3). \tag{7}$$

2.1 General solution

In this subsection, we derive the analytical solutions of the field equations of the BI models in terms of the directional Hubble parameters. To do this, first we add the last three equations of system (5), which yields

$$2\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=1}^{3} H_i\right) + 2\left(H_1^2 + H_2^2 + H_3^2\right) + \left(H_3 H_2 + H_1 H_2 + H_3 H_1\right)$$

$$= \frac{-24\pi G}{c^2} p. \tag{8}$$

After substituting the following term,

$$\sum_{i=1}^{3} H_i^2 = \left(\sum_{i=1}^{3} H_i\right)^2 - 2(H_3 H_2 + H_1 H_2 + H_3 H_1), \tag{9}$$

and equation (5a) of system (5) into equation (8), we then obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=1}^{3} H_i\right) + \left(\sum_{i=1}^{3} H_i\right)^2 = 12\pi G\left(\rho - \frac{p}{c^2}\right). \tag{10}$$

The mean of the three directional Hubble parameters in the BI universe is given by

$$H \equiv \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right). \tag{11}$$

Substituting the mean (11) into equation (10), a non-linear first-order differential equation is obtained,

$$\dot{H} + 3H^2 = 4\pi G\left(\rho - \frac{p}{c^2}\right). \tag{12}$$

Here, this dynamical equation shows evolution of the Hubble parameter of the related BI cosmology.

In addition to this, it is possible to write equation (12) in terms of volume element V by using the following relation between volume and the mean Hubble parameter of the BI,

$$H = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}t}\ln(a_1 a_2 a_3) = \frac{1}{3}\frac{\dot{V}}{V}. \tag{13}$$

As is seen, the multiplication of the scalefactors in different directions is defined as the volume element of the BI universe $V \equiv a_1 a_2 a_3$. Using this relation between volume and the mean Hubble parameter in equation (12), the volume evolution equation of the BI models is obtained,

$$\ddot{V} - 3\left[4\pi G\left(\rho - \frac{p}{c^2}\right)\right]V = 0. \tag{14}$$

On the basis of the above, we find the following alternative form for system (5):

$$\dot{H}_1 + 3H_1 H = 4\pi G\left(\rho - \frac{p}{c^2}\right), \tag{15a}$$

$$\dot{H}_2 + 3H_2 H = 4\pi G\left(\rho - \frac{p}{c^2}\right), \tag{15b}$$

$$\dot{H}_3 + 3H_3 H = 4\pi G\left(\rho - \frac{p}{c^2}\right). \tag{15c}$$

These expressions allow us to write down the generic solution of the directional Hubble parameters,

$$H_i(t) = \frac{1}{\mu(t)}\left[K_i + \int \mu(t) 4\pi G\left(\rho(t) - \frac{p(t)}{c^2}\right)\mathrm{d}t\right], i = 1, 2, 3, \tag{16}$$

where K_i are the integration constants. The integration factor μ is defined as

$$\mu(t) = e^{\int^t 3H(s)\,\mathrm{d}s}. \tag{17}$$

The integration factor μ in the solutions (16) is derived from the system (15) by the particular solution of the system itself.

As can be seen from solutions (15), the initial values/integration constants determine the solution of each directional Hubble parameter. These values are the origin of the anisotropy. Note that the generic solution of the directional Hubble parameters (16) is incomplete. To obtain exact solutions of the Hubble parameters and therefore the Einstein equations, we need one more equation which is known as the equation of state,

$$p = \gamma \rho c^2. \tag{18}$$

Here, the adiabatic parameter γ is characterized by a component of the universe dominating its expansion,

(i) $\gamma = 1/3$, radiation-dominated universe,
(ii) $\gamma = 0$, matter-dominated universe.

Before giving the analytical formalisms of the different epochs of the BI models, it is useful to define a general isotropy criterion of BI-type Universe models that is essential to obtain asymptotically FRW ones.

3 ISOTROPIZATION OF BI MODELS INTO FRW UNIVERSE

The isotropic and homogeneous nature of the large-scale structure may be an asymptotic situation emerging from an anisotropic nature of the universe is formed by the matter component during decoupling. That is why it is essential to define an isotropizaton criterion which should explain how the anisotropy parameters disappear or become negligible when the Universe evolves into the present epoch, $t \to t_0$.

Bronnikov et al. (2004) and Saha (2006, 2009) define isotropization as expansion factors of the BI universe that grow at the same rate at late stages of the evolution. It is assumed that a BI model becomes isotropic if the ratio of each directional expansion factor $a_i(t)$ ($i = 1, 2, 3$) and the expansion factor of the total volume $a(t)$ tends to be a constant value,

$$\frac{a_i}{a} \to \text{constant} > 0 \quad \text{when} \quad t \to \infty. \tag{19}$$

Note that the total expansion factor $a(t)$ has the contribution from each directional expansion factor,

$$a = (a_1 a_2 a_3)^{1/3} = V^{1/3}. \tag{20}$$

The anisotropic models satisfying condition (19) become isotropic. As a particular case of condition (19), one may choose the constant as unity. This indicates that when the universe evolves into the present day $t = t_0$, its dynamics become equivalent to the FRW. As a consequence of this choice, even highly anisotropic the BI models become isotropic in time. Since our goal is to obtain a model that becomes an FRW one in the late-time limit, then it should be specifically indicated that the anisotropic parts of each scalefactor of the BI models should tend to be identical and equivalent to unity in the limit of present day $t = t_0$. Under this condition, the critical density of the BI models is reduced to

$$\frac{\rho}{\rho_c} = \Omega = 1, \quad \rho_c = \frac{3H_0^2}{8\pi G}, \tag{21}$$

where the total mean density Ω of the Bianchi models is unity due to the flat geometry of the BI metric (1). Apart from this general isotropization criterion (19), we consider the following two widely used anisotropy criteria in the literature (Jacobs 1968; Bronnikov et al. 2004; Saha 2006, 2009):

$$A = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_i^2 - H^2}{H^2} \right) \to 0, \tag{22}$$

$$\sigma^2 = \frac{3}{2} A H^2 \to 0, \tag{23}$$

where A is the mean anisotropy parameter while σ^2 is the shear scalar and it is defined as

$$\sigma^2 \equiv \sigma_{ij} \sigma^{ij}, \tag{24}$$

where σ_{ij} is the shear tensor. The shear tensor indicates any tendency of distortion into an ellipsoidal shape of the initially spherical region. Therefore, the shear scalar σ^2 represents the distortion rate of the region. The mean anisotropy parameter A in equation (22) is correlated with the expansion divergence Θ also known as expansion scalar which is related to the expansion rate/Hubble parameter as

$$\Theta = \nabla . \boldsymbol{v} = 3H, \tag{25}$$

this leads to

$$\Theta \equiv \frac{1}{3} \frac{\nabla . \boldsymbol{v}}{3H} = \sum_{i=1}^{3} \left(\frac{H_i - 1}{H} \right), \tag{26}$$

in which v represents the velocity field. Note that the isotropy of every point of the Universe implies that the vorticity ω and shear σ of the matter are zero (Collins & Hawking 1973). The vorticity ω is the rate of rotation of a set of axes fixed in the matter, relative to a set of inertial axes defined by gyroscopes. In the BI universe, the vorticity parameter is zero since there is no rotation by definition. The distortion rate is the difference between the Hubble parameters of the matter in each orthogonal direction, and its value is non-zero for the BI models. These parameters, A and σ^2, have two constraints; one of them is theoretical and it directly comes from the consistency relation of the analytical solution of the field equations of the BI metric (2) via the integration constants (Jacobs 1968; Bronnikov et al. 2004; Saha 2006, 2009, 2014; Pereira et al. 2007; Gupt & Singh 2012; Amirhashchi 2014),

$$\sum_{i=1}^{3} K_i = 0. \tag{27}$$

The second constraint is the observational value of the shear parameter (23). Bunn, Ferreira & Silk (1996) analysed 4-yr data of the *COBE* satellite to constrain the allowed parameters of the $BVII_h$ model. This model evolves into an FRW Universe and the definitive upper limits on the amount of shear, $[\sigma/H]_0$ and vorticity, $[\omega/H]_0$ are

$$\left[\frac{\sigma}{H} \right]_0 < 3 \times 10^{-9}, \tag{28}$$

$$\left[\frac{\omega}{H} \right]_0 < 10^{-6}, \quad \Omega_0 = 1. \tag{29}$$

Apart from the above limits, Stoeger et al. (1995) show how to relate the CMB anisotropies to growing density in homogeneities in an almost-FRW expanding universe. Later on, Maartens et al. (1995a,b) relate CMB anisotropies with anisotropies and inhomogeneities in the large-scale structure of the universe and show the way of placing limits on those anisotropies and inhomogeneities simply by using CMB quadrupole and octopole limits. Note that these limits are upper bounds on the multipoles of the CMB temperature anisotropy and are independent from any models for the source of perturbations including inflationary models (Maartens et al. 1995a,b; Stoeger et al. 1999). Hence, the distortion/shear is limited as follows (Maartens et al. 1995a,b):

$$\frac{|\sigma_{ij}|}{\Theta} < \frac{5}{3}\epsilon_1 + 3\epsilon_2 + \frac{3}{7}\epsilon_3, \tag{30}$$

in which ϵ stands for limits on the dipole, the quadrupole and the octopole components, respectively. Therefore, if one can obtain ϵ in equation (30) from the CMB measurements, then one can determine the upper bond of the distortion/shear parameter by relating the squares of the rotationally invariant rms multipole ΔT_l^2 coefficients

in the usual Legendre polynomial expansion as

$$\langle \epsilon_l \rangle^2 = 3^l \frac{(2l)!}{2^l \, (l!)^2} \Delta T_l^2, \tag{31}$$

in which ΔT_l^2 are directly related with the anisotropy rms such as $\Delta T_2^2 = Q_{rms}^2$ and $\Delta T_3^2 = O_{rms}^2$ in which Q_{rms} and O_{rms} are the rms quadrupole and octopole amplitudes that are obtained from the CMB observations as is shown by Bennett et al. (1994) that $(\Delta T_l)^2$ are compatible with the *COBE* data. Using (31), the average quadrupole $\langle \epsilon_2 \rangle$ and octopole $\langle \epsilon_3 \rangle$ limits are found in terms of rms, which are

$$\langle \epsilon_2 \rangle = \sqrt{13.5} \frac{Q_{rms}}{T_0}, \tag{32}$$

$$\langle \epsilon_3 \rangle = \sqrt{67.5} \frac{O_{rms}}{T_0}, \tag{33}$$

where $T_0 = 2.7255$ K is the average temperature of the CMB since Stoeger et al. (1997, 1999) obtain the multipoles for $\Delta T/T$, $Q_{rms} = 10.7 \pm 7$ μK (Bennett et al. 1994; Kogut et al. 1996) and $O_{rms} = 16 \pm 8$ μK are the best-fitting value, rms quadrupole and octopole amplitudes of *COBE* in which the dipole rms is set as zero $\epsilon_1 = 0$ (Stoeger et al. 1997, 1999). Then, Stoeger et al. (1997, 1999) find average distortion by using calculated anisotropy limits in (32) and (33) in the shear equation (30) based on the *COBE* data,

$$\left\langle \frac{|\sigma_{ij}|}{\Theta} \right\rangle < 4.4 \times 10^{-5}. \tag{34}$$

Apart from the distortion equation (30) which is a model-independent parameter, Maartens et al. (1995a) show that if the dipole, quadrupole and octopole limits are homogeneous to first order, then distortion can be found directly without any further assumption,

$$\left(\frac{|\sigma_{ij}|}{\Theta} \right)_0 = \left(\frac{16}{15} \frac{\Omega_r}{\Omega_m} \right)_0 \epsilon_2, \tag{35}$$

which is also defined as the late-time limit of the shear. As is seen above, the distortion only depends on quadrupole upper bound ϵ_2 rather than octopole anisotropy limit. In distortion, equation (35) Ω_r and Ω_m are the radiation and matter density parameters. Using this distortion equation (35), Maartens et al. (1996) obtain the distortion to characterize the deviations from isotropy of a space–time which has nearly BI symmetry for the *COBE* data in which the quadrupole is $\epsilon \approx 10^{-5}$ and $(\Omega_r/\Omega_m)_0 = 2.5 \, h^{-2} \times 10^{-5}$ (Kolb & Turner 1990), that is,

$$\left(\frac{\sigma_{ij}}{\Theta} \right)_0 = 2.7 \, h^{-2} \times 10^{-10}, \tag{36}$$

where the parameter h is estimated as $0.4 < h < 1.0$.

4 EVOLUTION OF ANISOTROPIC DEVIATIONS FROM FRW IN DECOUPLING

In this section, we show that the given BI model provides a solution of expansion factors and Hubble parameters of the FRW model with extra parameters that decrease in time. These extra terms are defined as deviations from isotropy of the FRW model. Here, we obtain functional forms of deviations from the FRW isotropic model by using anisotropic and homogeneous BI metric in order to investigate the dynamical characteristics of separations from isotropy during the decoupling. These deviations from isotropy are possibly detected as tiny anisotropy fluctuations in the CMB resulting in

observed anisotropy and inhomogeneities such as distortion even though anisotropies are very small.

As is expected, radiation and matter decouples during decoupling, that happens right after radiation–matter equality at $z_{eq} \approx 3300$ ($t \approx 10^4$ yr) with a temperature of approximately 3×10^5 K. After this equality the temperature of the universe drops to 3000 K, and the plasma turns into neutral gas around $z < 3300$ ($t \approx 10^4$ yr).

Since we here investigate deviations from isotropy by using the BI metric, our first starting point is to explain the dynamical behaviour of the BI model in which matter and radiation components are decoupled, and later on, matter starts to rule the evolution. In such a period, the energy conservation equation has contributions from both matter and radiation components in the BI universe as the FRW one. As a result, the radiation and matter state equation decouples, in which the pressure term of the matter component vanishes due to its adiabatic parameter $\gamma = 0$ while the radiation pressure becomes proportional to one third of the radiation density in equation of state (18),

$$p_m = 0, \quad p_r = \frac{1}{3}\rho_r. \tag{37}$$

Hence, the energy conservation equation (6) in the radiation–matter period decouples as well,

$$\dot{\rho}_r = -4H_{rm}\rho_r, \quad \dot{\rho}_m = -3H_{rm}\rho_m, \tag{38}$$

which leads to

$$\rho_r = \rho_{r,0} \frac{V_{rm,0}}{V_{rm}}, \quad \rho_m = \rho_{m,0} \frac{V_{rm,0}}{V_{rm}}. \tag{39}$$

Here, the radiation ρ_r and matter ρ_m densities as well as the volume element in the decoupling era V_{rm} are normalized to their present-day values, $\rho_{r,0}$, $\rho_{m,0}$, $V_{rm,0}$ in which the present-day values of the volume element are equal to unity. The normalization of the densities (39) will help us to compare the dynamical parameters to the recent observational results. Then, the normalized densities for two components are written by using the definition of the critical density (21), which are

$$\rho_{r,0} = \rho_{c,0}\Omega_{r,0} = \frac{3H_0^2}{8\pi G}\Omega_{r,0} \text{ and } \rho_{m,0} = \rho_{c,0}\Omega_{m,0} = \frac{3H_0^2}{8\pi G}\Omega_{m,0}. \tag{40}$$

Our goal is to show deviations from isotropy by dynamical parameters such as Hubble parameter and expansion factor by using the BI model, which has the FRW model embedded. Therefore, first the form of the mean Hubble parameter H_{rm} which has the contributions from three directional Hubble parameters is obtained by using the dynamical evolution equation (12) and equation of state (37) in order to obtain the exact solution of the directional Hubble parameters in the radiation+matter-dominated BI model from equations (16),

$$\dot{H}_{rm} + 3H_{rm}^2 = 4\pi G \left(\frac{2}{3}\rho_r + \rho_m \right). \tag{41}$$

This non-linear equation gives the dynamical evolution of the mean Hubble parameter at decoupling. equation (41) can be transformed into an equation that satisfies the total volume evolution of the model by substituting relation (13) and the normalized radiation and matter densities (39), which is

$$\ddot{V}_{rm} - 4\pi G \left(\rho_{m,0} + \frac{2}{3} \frac{\rho_{r,0}}{V_{rm}^{1/3}} \right) = 0. \tag{42}$$

Multiplying this equation with $\dot{V}_{rm}$, integrating it in terms of time, and substituting the normalized densities (40) in, we then obtain,

$$\dot{V}^2 - 9H_0^2 \Omega_{m,0} V - 9H_0^2 \Omega_{r,0} V^{2/3} = 0. \qquad (43)$$

Here, we obtain a relation between the mean Hubble parameter and the volume element of the related epoch. This equation is rearranged as follows:

$$\left(\frac{\dot{V}_{rm}}{V_{rm}} \right)^2 = 9H_0^2 \left(\frac{\Omega_{m,0}}{V_{rm}} + \frac{\Omega_{r,0}}{V_{rm}^{4/3}} \right). \qquad (44)$$

The integration of the above equation allows us to derive a relation for the time component as a function of volume during decoupling,

$$H_0 t = \frac{4}{3} \frac{V_{rme}^{2/3}}{\sqrt{1 - \Omega_{m,0}}} \left(\sqrt{1 + \left(\frac{V_{rm}}{V_{rme}} \right)^{1/3}} \left(\frac{1}{2} \left(\frac{V_{rm}}{V_{rme}} \right)^{1/3} - 1 \right) \right), \qquad (45)$$

where $V_{rme} \equiv \left(\frac{\Omega_{r,0}}{\Omega_{m,0}} \right)^3$. Hence, the volume element can be found,

$$V_{rm} \approx \frac{9}{4} H_0^2 \Omega_{m,0} t^2 + 5 \left(\frac{\Omega_{rad,0}}{\Omega_{m,0}} \right)^3 = V_m + 5V_{rme}. \qquad (46)$$

Therefore, the mean Hubble parameter is obtained as

$$H_{rm} \approx \frac{2}{3} \frac{1}{t} \frac{1}{\left[1 + 5 \frac{V_{rme}}{V_m} \right]}, \qquad (47)$$

by using relation (13). Here, in the limit of $V_{rm} \gg V_{rme}$ ($t \to \infty$), the solution of equation (44) approaches the mean Hubble parameter of the matter dominated BI universe,

$$H_{rm} \to H_m = \frac{2}{3} \frac{1}{t}. \qquad (48)$$

Taking into account that the radiation and matter become equivalent when $V_{rme}^{1/3} = 2.963 \times 10^{-4}$ (Ryden 2003), the redshift z_{eq} when decoupling starts, can be obtained,

$$\frac{\Omega_m}{\Omega_r} = V_{rme}^{-1/3} \frac{a}{a_0} = \frac{\Omega_{m,0}}{\Omega_{r,0}} \frac{1}{1+z}, \quad 1 + z_{eq} \approx 3.375 \times 10^3. \qquad (49)$$

This result leads to the redshift value $z_{eq} \approx 3300$. In the FRW Universe, radiation–matter equality took place at a scalefactor $a_{rm} \equiv \Omega_{r,0} / \Omega_{m,0} \approx 2.8 \times 10^{-4}$. Here, we assume that the particles that are non-relativistic today were also non-relativistic at z_{eq}; this should be a safe assumption, with the possible exception of massive neutrinos, which make a minority contribution to the total density (Trodden & Carroll 2004). It follows that the integration factor of the epoch in terms of volume element (46) from equation (17), is obtained,

$$\mu_{rm} = 4\Omega_{m,0}^3 V_{rm}. \qquad (50)$$

Substituting the integration factor (50) and using the equation of state (18) for the decoupling case in the solution of the directional Hubble parameters (16), we obtain

$$H_i t_0 = \underbrace{\frac{\alpha_i}{4\Omega_{m,0}^3} \left(\frac{t_0}{t} \right)^2 b}_{\text{Deviations}} + \underbrace{\frac{2}{3} \left(\frac{t_0}{t} \right) b}_{\text{FRW: Hubble parameter}}, \qquad (51)$$

where the parameter b and the normalized deviation/anisotropy coefficients α_i are defined as

$$b \equiv \frac{V_m}{V_{rm}}, \quad \alpha_i \equiv \frac{K_i}{t_0}. \qquad (52)$$

As is seen in the directional Hubble parameters (H_1, H_2, H_3, inequations 51), the last term on the right-hand side is the same in each

directional Hubble parameter which is the standard Hubble parameter of the FRW model at decoupling. The first terms on left-hand side with the anisotropy coefficients α_i are only dependent on the initial values K_i (see equation 16). As is seen, even small differences in the expansion rates of the given epoch may cause deviations from the standard model. The normalized scalefactors are derived from the directional Hubble parameters (51) with a direct integration in terms of cosmic time in transformation (3), which are obtained as

$$a_{n,i} = \underbrace{e^{\left[\frac{\alpha_i}{4\sqrt{5}\beta} \tan^{-1} \left(\beta \frac{\left(1 - \frac{t_0}{t} \right)}{1 + 5 \frac{V_{rme}}{V_m} \frac{t}{t_0}} \right) \right]}}_{\delta_{n,i}: \text{ Deviations}} \underbrace{\left[1 + \left(\frac{\left(\frac{t}{t_0} \right)^2 - 1}{1 + 5 \frac{V_{rme}}{V_m}} \right) \right]^{1/3}}_{\text{FRW}}, \qquad (53)$$

where the density-dependent parameter β is defined as

$$\beta \equiv \left(\Omega_{r,0} \Omega_{m,0} \right)^{3/2}. \qquad (54)$$

Note that anisotropic parts of the scalefactors or deviations from isotropy $\delta_{n,i}$ should satisfy (Saha 2014; Amirhashchi 2014)

$$\prod_{i=1}^3 \delta_{n,i} = 1, \qquad (55)$$

It is crucial to note that even though the BI model shows deviations from isotropic FRW in each direction, the overall volume of the universe behaves as the standard FRW model. As a result, the total volume element is not affected by the directional deviations (expansion/contraction(s)) in the decoupling. One easily can prove that via the multiplication of scalefactors (53), which is related to volume via definition (19) and taking into account the consistency relation of integration constants (27), the sum of the normalized anisotropy coefficients α_i disappears,

$$K_1 + K_2 + K_3 = 0 \implies \alpha_1 + \alpha_2 + \alpha_3 = 0. \qquad (56)$$

As a result, the total volume element is not affected by the directional deviations in the decoupling. Therefore, the total volume becomes the volume of the universe given by the FRW in the related epoch. Apart from this, the critical anisotropy coefficients are obtained from the first derivative test of the normalized scalefactors (53), at which points the directional Hubble parameters become zero. These critical coefficients are given by

$$\alpha_i = -\frac{8}{3} \Omega_{m,0}^3 \left(\frac{t}{t_0} \right), \qquad (57)$$

in which $\Omega_{m,0} = 0.3175$ (Planck Collaboration 2014). Assuming deviations from isotropy starts in decoupling, the critical anisotropy value is calculated as $-4.5 \ 10^{-7}$ at $t/t_0 = 5.2 \ 10^{-6}$ which is approximately the radiation–matter equality $z_{eq} = 3300$. Then, we investigate the dynamical behaviours of the directional normalized scalefactors around these critical points. As a result, we obtain expansion and contraction criteria of the scalefactors in terms of the critical anisotropy coefficients as follows:

$$\text{Expansion } \alpha_i = \begin{cases} \text{If } \alpha_i < 0, & \text{then } |\alpha_i| < \frac{8}{3} \Omega_{m,0}^3 \left(\frac{t}{t_0} \right), \\ \text{If } \alpha_i > 0, & \text{then } \alpha_i > -\frac{8}{3} \Omega_{m,0}^3 \left(\frac{t}{t_0} \right), \end{cases} \qquad (58a)$$

$$\text{Contraction } \alpha_i = \left\{ \text{If } \alpha_i < 0, \quad \text{then } |\alpha_i| > \frac{8}{3} \Omega_{m,0}^3 \left(\frac{t}{t_0} \right). \right. \qquad (58b)$$

These criteria of the critical values are obtained by applying the second derivative test to the scalefactors (53). Therefore, the
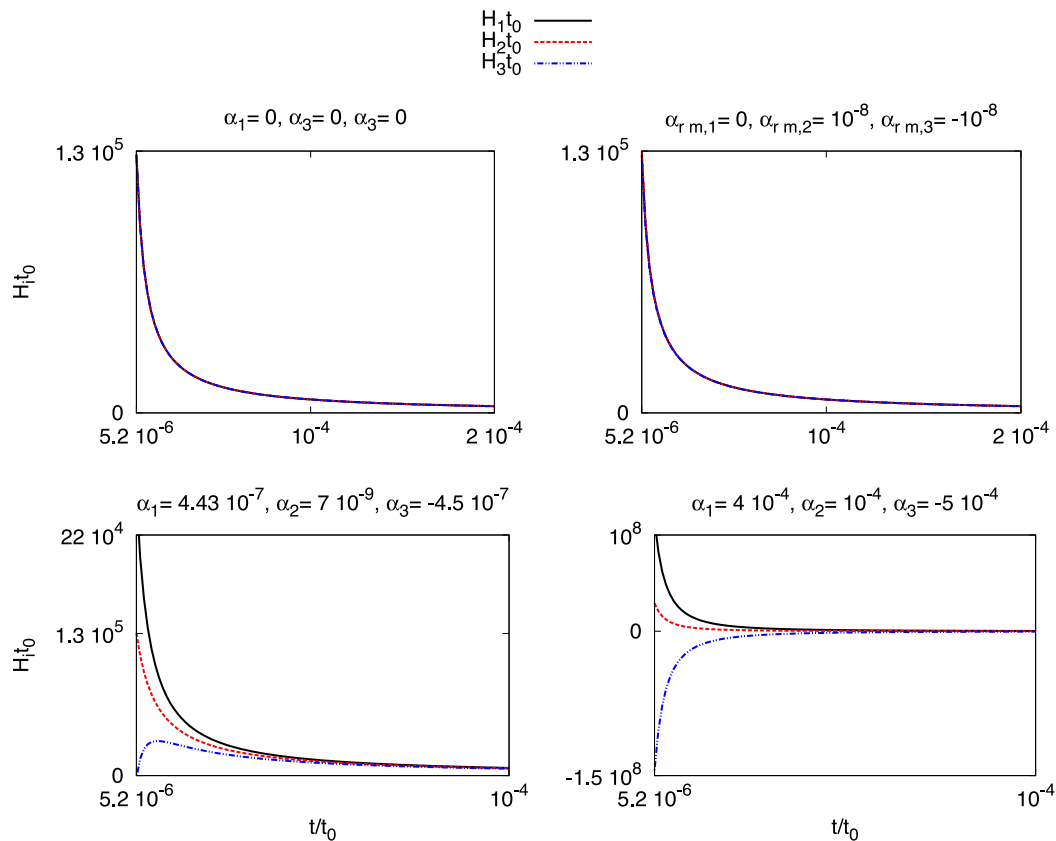
Figure 1. Emerging of the negative anisotropy part $H_3 < 0$ of the directional Hubble parameters in the decoupling for the matter density $\Omega_{\mathrm{m},0} = 0.3175$.

criteria (58) show the dynamical characteristics of the scalefactors via a large set of anisotropy coefficients admitting positive and negative numbers. The physical interpretation of negative anisotropy coefficients is equivalent to contraction of the related scalefactor(s) while positive anisotropy coefficients demonstrate expansion characteristics of the scalefactor(s).

These initial expansions and/or contractions of the scalefactors also affect the dynamical behaviours of the directional Hubble parameters due to their dynamical relation. A possible contraction of one of the directional scalefactors causes slowing down in the rate of expansion leading to zero or even negative expansion rates depending on how strong slowing down is in the related directions during the emergence of anisotropies. On the other hand, anisotropic expansions of the directional scalefactors cause increase in the expansion rates. Therefore, criteria (58) indicate another criteria on the directional Hubble parameters by presenting emerging out of positive and negative branches (adopting Gümrükçüoglu & Peloso 2007) of the expansion rates which are the same as expansion criterion (58a) and contraction criterion (58b) of the directional scalefactors. Although anisotropies may form negative and/or positive branches of the directional Hubble parameters (or expansion and/or contraction of the directional scalefactors) at the very early stages of the evolution; later on, these negative/positive directional Hubble parameters (or contracting/expanding directional scalefactors) tend to merge and become the Hubble parameter (the scalefactor) of the FRW model in time (see Figs 1 and 2).

Figs 1 and 2 show how the directional Hubble and scale parameters change in terms of time t/t_0 around and at the critical anisotropy coefficient relatively for the same set of coefficients. In Fig. 1, the left-upper panel with zero anisotropy coefficients presents the evo-

lution of standard FRW Hubble parameter. Therefore, the rest of the panels indicate separations from the evolution of the standard FRW Hubble parameter with non-zero anisotropy coefficients. Particularly, we aim to show the emerging of negative Hubble parameters in Fig. 1. To show this evolution, the anisotropy coefficients of the two axes of expansion are kept positive ($\alpha_1 > 0$ and $\alpha_2 > 0$) from the upper-right to the lower-right panels. Note that positive anisotropy coefficients ($\alpha_1 > 0$ and $\alpha_2 > 0$) represent positive separations from the standard FRW Hubble parameter indicating initial speeding up process in the related directions at decoupling. On the other hand, in each panel (from upper right to lower right) the anisotropy coefficients of the third axis H_3 accept negative values (note that the sum of a set coefficients are zero satisfying the consistency relation 27). These negative valued directional Hubble parameters indicate different dynamical behaviours depending on comparison of the coefficients with the critical value based on the criteria (58). The critical value is obtained as -4.5×10^{-7} by using equation (57) at initial time $t/t_0 = 5.2 \times 10^{-6}$. Therefore, the third directional Hubble parameter (H_3) with the anisotropy coefficient that is equal to the exact critical value -4.5×10^{-7} shows a halt in expansion or zero expansion rate at the beginning of decoupling $t/t_0 = 5.2 \times 10^{-6}$ (see lower-left panel). As is seen in the lower-right panel, the initially negative third directional Hubble parameter emerges since the anisotropy coefficients accepts $|\alpha_i| > 4.5 \times 10^{-7}$. Note that the negativity of directional Hubble parameters (contraction rate) gets stronger for the values of anisotropy coefficient higher than the critical anisotropy coefficient (see criterion 58b). Here, after we call the positive and negative separations from standard FRW Hubble parameter as positive and negative branches by following Gümrükçüoglu & Peloso (2007).
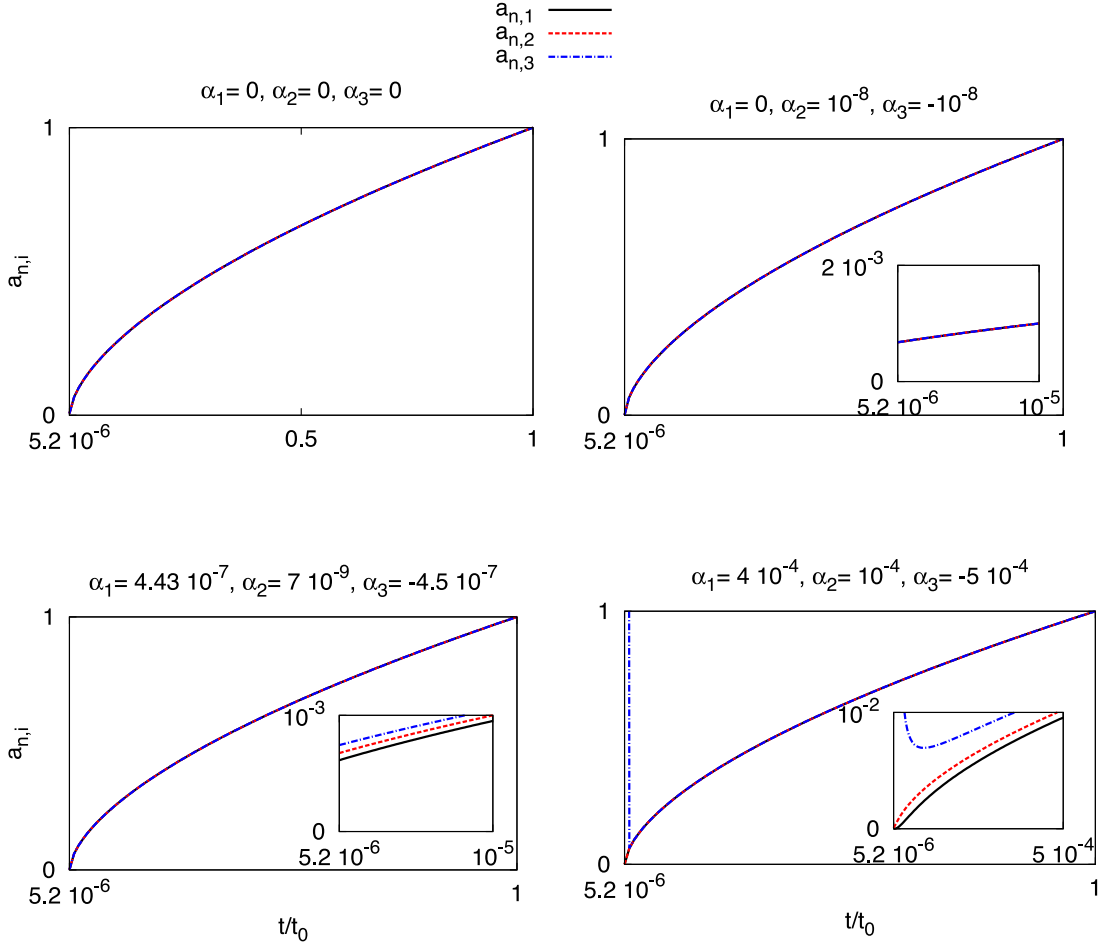
Figure 2. The time evolution of the normalized scalefactors starting from the decoupling with different sets of anisotropy coefficients α_i.

In Fig. 2, the upper-left panel shows the evolution of the standard FRW scalefactor from decoupling to present day. Panels from upper right to lower right demonstrate the evolution of the possible separations from the standard FRW. Here, the lower-right panel shows a contraction until a certain time, then this contraction turns into expansion in third direction. Note that directional scalefactors especially with highly negative values satisfying the criteria (58b) show initial contraction but, later on, this behaviour starts turning into an expansion. This transformation of initial contraction to expansion is known as bouncing behaviour of the directional scalefactors in the BI models. When we choose any of the anisotropy coefficients with negative value $\alpha_i < 0$ satisfying the condition $|\alpha_i| > 4.5 \times 10^{-7}$, we can see this bouncing behaviour of the related directional scalefactor.

4.1 Isotropization criteria of the radiation and matter dominated BI model

Here, the isotropization criteria of the BI model at the decoupling are given by equations (22) and (23). Hence, the isotropization of deviations from the exact isotropy is obtained by the anisotropy,

$$A = \frac{3}{64\Omega_{m,0}^6}\left(\frac{t_0}{t}\right)^2 \sum_i^3 \alpha_i^2 \to 0 \text{ if } t \to t_0, \tag{59}$$

and the shear parameters,

$$\left[\frac{\sigma}{\Theta}\right]^2 = \frac{1}{128\Omega_{m,0}^6}\left(\frac{t_0}{t}\right)^4 \sum_i^3 \alpha_i^2 \to 0 \text{ if } t \to t_0. \tag{60}$$

Also, the ratio of anisotropy A and the shear $\left[\frac{\sigma}{\Theta}\right]^2$ can be obtained for the same sum of squared anisotropy coefficients as

$$\left(\frac{A}{\sigma^2}\right)\Theta^2 = 6\left(\frac{t}{t_0}\right)^2. \tag{61}$$

As is seen, at the present day $t = t_0$, the ratio of anisotropy and distortion becomes 6. This simple formulation indicates that the expansion of the universe Θ increases during the evolution which is consistent with observations (Riess et al. 1998; Schmidt et al. 1998; Perlmutter et al. 1999) while the ratio of pure anisotropy parameters (A/σ^2) should die away in time because of the expansion of the universe. Considering the present-day Universe is isotropic on large scales, this is an expected result. Fig. 3 shows the time evolution of the ratio of anisotropy and distortion parameters (61) for the same sum of squared anisotropy coefficients. As is seen in Fig. 3, the anisotropy (A/σ^2) is more dominant than expansion during the decoupling $t/t_0 \approx 5.3 \times 10^{-6}$ when matter creates the deviations from isotropy. However, these anisotropy parameters die away in time and the expansion of the universe takes over the evolution. Therefore, the anisotropic BI behaviour of the universe due to the matter component at decoupling turns into an isotropic FRW one at the present day.
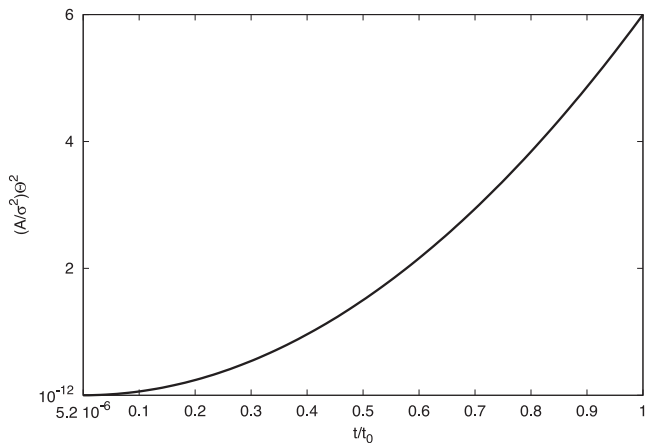
Figure 3. Evolution of the ratio of the anisotropy–shear parameters starting from decoupling to the present day $t = t_0$.

Here, we obtain an upper bound of the distortion based on anisotropy limits directly obtained from the recent *Planck* foreground-subtracted temperature power spectrum $\mathcal{D}_l$ (Planck Collaboration: Ade et al. 2013d) by adopting the same formalism of Maartens et al. (1995a,b). Note that the squares of rotationally invariant rms multipole moments $(\Delta T_l)^2$ should relate to each other in order to obtain correct multipole upper limits,

$$(\Delta T_l)^2 = \mathcal{D}_l \frac{2l+1}{2l\,(l+1)}, \tag{62}$$

in which $\mathcal{D}_l$ is defined as

$$\mathcal{D}_l = \frac{l\,(l+1)}{2\pi} C_l, \quad C_l = \frac{1}{2l+1} \sum_{m=-l}^{+l} |a_{lm}|^2, \tag{63}$$

and C_l is equivalent to the sum of the expansion coefficients a_{lm} of the temperature anisotropy in spherical harmonics. The quadrupole $(l = 2)$ and octopole $(l = 3)$ anisotropy are given by Planck Collaboration (2014) as

$$\mathcal{D}_2 \approx 299.5 \ [\mu K^2], \quad \mathcal{D}_3 \approx 1000 \ [\mu K^2]. \tag{64}$$

According to this, the average quadrupole and octopole limits are found from the method of Maartens et al. (1995a,b) by using the quadrupole and octopole anisotropies of the *Planck* data (64) in the average anisotropy definitions (32) and (33), which are

$$\langle \epsilon_2 \rangle \approx 1.506 \times 10^{-5}, \quad \langle \epsilon_3 \rangle \approx 3.640 \times 10^{-5}. \tag{65}$$

Substituting these values into distortion equation (30), we obtain distortion based on the *Planck* anisotropy from the power spectrum of the temperature fluctuations, which is

$$\left\langle \frac{|\sigma_{ij}|}{\Theta} \right\rangle \quad < \quad 6.078 \times 10^{-5}. \tag{66}$$

Here, we neglect the effect of the dipole component depending on Planck Collaboration (2014) in which a bulk flow has been significantly constrained by *Planck* studies of the kinetic Sunyaev–Zeldovich effect in which two different methods used to detect dipole as a consequence; in all cases, the measured dipoles are compatible with zero. On the other hand, Planck Collaboration (2013d) report that our motion modulates and aberrates the CMB temperature fluctuations which is an order 10^{-3} effect applied to fluctuations which are already one part in roughly 10^{-5}, so it is quite small. Nevertheless, it becomes detectable with the all-sky coverage, high angular resolution and low noise levels of the *Planck*
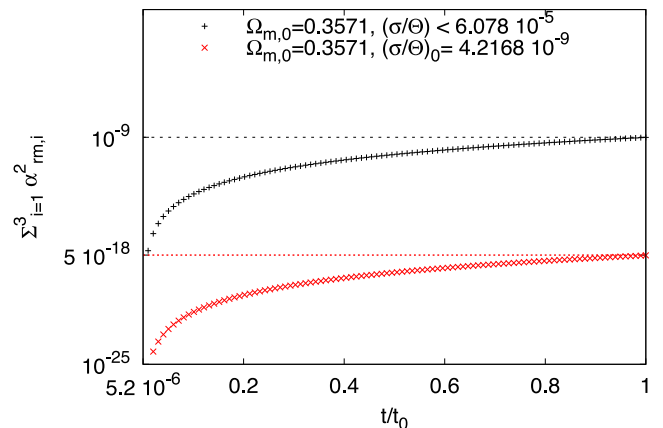

Figure 4. Evolution of the sum of square anisotropy coefficients $\sum_i^3 \alpha_i^2$ for the average distortion 6.078×10^{-5} (black pluses) and the late-time distortion 4.2168×10^{-9} (red crosses) as upper limits in the shear equation (60) starting from the decoupling.

satellite. That is why we should be careful in order to construct an almost-FRW model by using the CMB observed anisotropy limits. Apart from the average general shear limit (66); here, we obtain the late-time limit of the shear which is also defined as the deviation from isotropy of a space–time that is nearly BI symmetry based on the *Planck* data as

$$\left(\frac{\sigma_{ij}}{\Theta} \right)_0 < 4.2168 \times 10^{-9}, \tag{67}$$

where we choose $(\Omega_r / \Omega_m)_0 = 2.8 \times 10^{-4}$ for the BI symmetry based on equation (49). Substituting the upper limits (67) and (66) into the shear equation (60), one can find upper limits on the sum of squared anisotropy coefficients as a time-dependent parameter. This leads us to obtain the two strong constraints on the deviations from isotropy. As a result, we can obtain a set of parameters at a given time and given observed shear parameter. In Fig. 4, the upper limits of the sum of the square of the anisotropy coefficients $\sum_{i=1}^3 \alpha_{rm,i}^2$ from equation (60) for the two upper distortion limits from *Planck* (Planck Collaboration 2014). According to this, the sum of the square anisotropy coefficients indicates a very small limit value 5×10^{-18} at the present day $t = t_0$ assuming the late-time upper limit of the shear while the sum reaches 10^{-9} for the average distortion.

5 SUMMARY AND DISCUSSION

Considering the observational evidence of the small anisotropic temperature fluctuations and the presence of the large angle anomalies in the CMB, we here aim to construct emerging of the deviations from isotropy in functional forms based on a BI model at the decoupling in which the matter component is believed to form these separations following (Hawking & Ellis 1973; Stoeger et al. 1995; Maartens et al. 1996; Cea 2014). Also, we attempt to put the most stringent upper limits on these functional separations by using the recent *Planck* anisotropy upper bounds (Planck Collaboration 2014) taking into account the two shear formalisms that are derived from evolution equations in decoupling based on a model independent method proposed by Maartens et al. (1995a,b).

To construct the separations starting from the decoupling, first we consider anisotropic, homogeneous BI space–time which is the most general case of the isotropic, homogeneous, flat FRW space–time. Following this, we introduce the isotropization criteria by using the

anisotropy (22) and shear (23) parameters from the previous studies (Jacobs 1968; Bronnikov et al. 2004; Saha 2006, 2009). These criteria are particularly important since they tell us how the separations from isotropy emerge out of the BI space–time, and later on, die out or become so small. We also introduce the two model-independent formalisms of the shear/distortion (Maartens et al. 1995a,b). These model-independent shears are called the average shear and the late-time shear that are given in equations (30) and (35).

Apart from the isotropization criteria, we obtain solutions of the three directional Hubble parameters and the scalefactors in the anisotropic space–time, in which normalized deviations from the FRW-type Hubble and scalefactors are explicitly found as in equations (51) and (53). Following this, the critical deviation/anisotropy coefficients (57) are obtained in the time-dependent functional forms. Depending on the sign of anisotropy coefficients α_i and their comparisons with the critical coefficient value at a given time, the directional scalefactors can present contraction and/or expansion-type separation(s) from the FRW scalefactor. To generalize these two different dynamical characteristics of the directional scalefactors, we formulate criteria of the expansion–contraction of the scalefactors (58).

According to criteria (58), it is found that the scalefactors with anisotropy coefficients satisfying expansion criterion (58a) show expansion-type separations from the FRW-type isotropy during the early phase of decoupling. On the other hand, anisotropy coefficients satisfying criterion (58b) demonstrate separations from the FRW scalefactor with a contraction behaviour in the related direction. However, these early expansion/contraction dynamical behaviour tends to die away or become so small in time. As a result, any contraction and expansion type of separations in the scalefactors may turn into FRW-type expansion. This tendency is shown in Fig. 2. Therefore, the scalefactor of the anisotropic direction experiences a bounce due to initial contraction and later expansion in Fig. 2. This behaviour is also mentioned in Gümrükçüoglu & Peloso (2007) and Gümrükçüğlu, Kofman & Peloso (2008) for a BI universe.

Moreover, criteria (58) of the scalefactors are extended to the directional Hubble parameters in order to investigate their initial dynamical characteristic at decoupling. Then, it is found that a possible contraction of one of the directional scalefactors causes the rate of expansion to stop or slow down, which leads to zero or even negative expansion rates depending on the strength of the slowing down in the related directions. As a result, criteria (58) turn into other criteria on emerging of initial positive and negative branches of the directional Hubble parameters (see Fig. 1). Therefore, the expansion criterion (58a) becomes an indicator of emerging of a positive Hubble parameter while criterion (58b) turns into an indicator of emerging of a negative Hubble parameter in the related direction at the given time. Here, it is crucial to note that we do not have any observational evidence for the contracting scalefactors or slowing down in expansion rates. Moreover, observational data support expansion of the universe. That is why we believe that one may exclude the contraction criterion, therefore the negative separation/anisotropy coefficients, in order to construct a model that is consistent with observations. On the other hand, as we pointed out before, the emerging of BI anisotropy tends to become an almost-FRW isotropy in time. As a result, we should not rule out a possible scenario of slight initial contractions at the beginning of decoupling which, later on, turn into the standard expansion characteristics of the FRW model.

Furthermore, the criteria (58) with the consistency relation (27) lead us to find that a set of coefficients form the initial conditions of the BI space–time at decoupling that turns into an isotropic FRW

model. Another constraint on the anisotropy coefficients comes from the observational data. Here, we calculate the average shear as 6.078×10^{-5} and the late-time shear as 4.2168×10^{-9} by using the upper anisotropy limits for the quadrupole and octopole components from the recent *Planck* temperature power spectrum by following Maartens et al. (1995a,b) and Stoeger et al. (1995). Then, we use these average and the late-time distortions in the distortion/shear equation (60) in order to obtain upper limits of the sum of the square of the anisotropy coefficients for a given normalized time (or redshift) for the two distortions found from the *Planck* data. As a result, the value of the sum of square of anisotropy parameters calculated at the present time $t = t_0$ is 5×10^{-18} assuming the late-time distortion upper limit while this value is 10^{-9} for the average distortion (Fig. 4). These upper shear values indicate very small anisotropy coefficients that represent small deviations from the standard FRW. This result is in agreement with Martinez-Gonzalez & Sanz (1995). They prove that if the universe is BI, then it necessarily must be a small departure from the flat Friedmann model. Extending this result to the *Planck* satellite and point out that a possible BI dynamical behaviour of the universe at decoupling leads to a small separations from isotropy of the FRW with the upper limits as 5×10^{-18} and 10^{-9} for the late-time shear and average shear, respectively.

In short, here we obtain the model based on the most up to date constraints for the average and the late-time shear parameters proposed by Maartens et al. (1995a,b) from the recent *Planck* data. These upper limits lead us to construct the deviations from isotropy in the dynamical form of the BI model turning into the FRW one in time starting from the time of decoupling. As a result, we construct the most stringent anisotropic model to date that is consistent with recent CMB observations.

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