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A Self Tuning RISE Controller Formulation

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Abstract— **In recent years, controller formulations using robust integral of sign of error (RISE) type feedback have been successfully applied to a variety of nonlinear dynamical systems. The drawback of these type of controllers however, are *(i)* the need of prior knowledge of the upper bounds of the system uncertainties and *(ii)* the absence of a proper gain tuning methodology. To tackle the aforementioned weaknesses, in our previous work [1] we have presented a RISE formulation with a time-varying compensation gain to cope for the need of upper bound of the uncertain system. In this study, we have extended our previous design to obtain a fully self tuning RISE feedback formulation. Lyapunov based arguments are applied to prove overall system stability and extensive numerical simulation studies are presented to illustrate the performance of the proposed method.**

I. Introduction

When dealing with unstructured nonlinear systems with uncertain dynamical parameters, use of robust controller formulations are among the most preferred methods. However, due to the use of the *signum* function in their design most robust controllers are discontinuous, like the variable structure and the sliding mode controllers. Also with most robust controller designs, convergence of the error signal to an ultimate bound can be guaranteed, and over–shrinking this ultimate bound causes chattering, which is undesirable. To our best knowledge, the first continuous and asymptotically stable robust controller was presented in [2] and [3]. In [2], motivated by the work of [4], authors designed a continuous robust controller for a class of nonlinear systems. In this methodology the integral of the sign of the error was utilized instead of the sign of error used in standard sliding mode controllers. This method was then referred as RISE (short for Robust Integral of Sign of Error) feedback [5] and have been successfully applied to a variety of nonlinear dynamical systems including autonomous flight control [6], underwater vehicle control [7], control of special classes of multiple input multiple output (MIMO) nonlinear systems [8], [9], and even time delay compensation [10]. Similar to that of most robust–type controllers, the RISE feedback makes use of a constant high gain to compensate the overall uncertainties in the continuously differentiable system dynamics. To adjust this high controller gain, the knowledge of the upper bounds of the overall system uncertainties is required. Specifically, the knowledge of the upper bounds of vectors (functions of the desired system trajectories) containing system uncertainties are necessary in classical RISE feedback formulations where the uncertainty compensation gain is constant. In cases where this information is not available simply applying extra high gains to compensate for the system uncertainties is not a preferred approach. Researchers applied adaptive [5] and neural network (NN) based [11], [12] feedforward compensation techniques in conjunction with RISE feedback in order to decrease the heavy control effort enforced to the system by this high gain.

Recently, in [13], Jagannathan *et. al* proposed a controller formulation that utilized RISE feedback having an adaptive uncertainty compensation gain fused with NN feedforward term. However, this formulation didn't guarantee that the proposed time–varying adaptive gain would remain bounded under the closed-loop operation due to lack of proof of $\mathcal{L}_1$ boundedness of the error term. In [1], the need of prior knowledge of upper–bounds of the vector containing the desired system dynamics plus uncertainties (and their derivatives) for the control gain selection was removed via the use of an adaptive compensation gain formulation. The use of an adaptive compensation gain reduces the heavy control effort and therefore eliminates the need of extra feedforward compensation methods. The analysis given in [1] also provided the $\mathcal{L}_1$ boundedness of the error term utilized in the design of the time–varying gain. In this work, we have extended the result in [1] to obtain a fully self tuning RISE feedback formulation. On top of a time–varying uncertainty compensation gain the proposed methodology also provides a time–varying feedback gain which eases the overall tuning process for RISE feedback type robust controllers. According to the authors' best knowledge, this work is the first attempt to design a self tuning process for RISE controllers.

The rest of the paper is organized in the following manner: The error system development and controller design are presented in Section II. The stability analysis and the main result are given in Section III. Simulation studies performed on two different systems are presented in Section IV, and concluding remarks are given in Section V.

II. Error System Development

In this section[1], for the compactness of the presentation the following single input single output (SISO) nonlinear system

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[1]As the proposed work aims to extend the results in [2], the notation in [2] is borrowed for a better comparison with the results in this paper.

is considered [2]

$$mx^{(n)} + f = u \tag{1}$$

where $x^{(i)}(t) \in \mathbb{R}$ $i = 0, ..., n$ are the system states, $m\left(x, \dot{x}, \cdots, x^{(n-1)}\right)$, $f\left(x, \dot{x}, \cdots, x^{(n-1)}\right) \in \mathbb{R}$ are uncertain nonlinear functions, and $u(t) \in \mathbb{R}$ is the control input. The standard assumption that the uncertain function $m(\cdot)$ being positive (*i.e.*, $m(\cdot) > 0$) is utilized in the subsequent development. Therefore, following bounds are assumed

$$\underline{m} \le m(x) \le \overline{m}\left(|x|, |\dot{x}|, \cdots, \left|x^{(n-1)}\right|\right) \tag{2}$$

where $\underline{m} \in \mathbb{R}$ is a positive constant and $\overline{m}(\cdot)$ is some positive non–decreasing function of its arguments. The uncertain functions $m(\cdot)$ and $f(\cdot)$ are assumed to be continuously differentiable up to their second order time derivatives. It is highlighted that while the development in this paper is for the SISO system model in (1), extension to MIMO systems is straightforward[2].

To quantify the tracking control objective, the output tracking error, denoted by $e_1(t) \in \mathbb{R}$, is defined as

$$e_1 \triangleq x_r - x \tag{3}$$

where $x_r(t) \in \mathbb{R}$ represents the reference trajectory which is assumed to be bounded with bounded continuous time derivatives (*i.e.*, $x_r^{(i)}(t) \in \mathcal{L}_\infty$ for $i = 0, \cdots, (n+2)$). The main control objective is to ensure that the output tracking error in (3) converge asymptotically to zero, that is $|e_1(t)| \to 0$ as $t \to \infty$ by designing a continuous robust control law under full–state feedback (*i.e.*, $x^{(i)}$, $i = 0, \cdots, (n-1)$ are measurable).

To facilitate the control design, auxiliary error signals, denoted by $e_i(t) \in \mathbb{R}$, $i = 2, \cdots, n$, are defined in the following manner

$$e_2 \triangleq \dot{e}_1 + e_1 \tag{4}$$
$$\vdots$$
$$e_n \triangleq \dot{e}_{n-1} + e_{n-1} + e_{n-2}. \tag{5}$$

It is noted that a general expression for $e_i(t)$ $i = 2, \cdots, n$ in terms of $e_1(t)$ and its time derivatives can be obtained as

$$e_i = \sum_{j=0}^{i-1} a_{i,j} e_1^{(j)} \tag{6}$$

where $a_{i,j} \in \mathbb{R}$ are known positive constant coefficients with $a_{n,(n-1)} = 1$. To ease the presentation of the subsequent stability analysis, another auxiliary error, denoted by $r(t) \in \mathbb{R}$, is defined to have the following form

$$r \triangleq \dot{e}_n + \alpha e_n \tag{7}$$

where $\alpha \in \mathbb{R}$ is a positive constant gain. It is noted that, the definition of $r(t)$ has $\dot{e}_n(t)$ which requires unmeasurable $x^{(n)}(t)$ then it is clear that $r(t)$ is not measurable and thus cannot be utilized in the control design.

[2]A numerical simulation study is conducted to demonstrate performance of application to a second order MIMO system.

After multiplying both sides of the time derivative of (7) with $m(\cdot)$, substituting the second time derivative of (6) for $i = n$, and the time derivative of (1), the following open–loop dynamics for $r(t)$ can be obtained

$$m\dot{r} = -\frac{1}{2}\dot{m}r - e_n - \dot{u} + N \tag{8}$$

where $N\left(x, \cdots, x^{(n)}, e_1, \cdots, e_n, r, x_r^{(n+1)}\right) \in \mathbb{R}$ is an auxiliary function defined as

$$\begin{aligned} N \triangleq\ & m\left[x_r^{(n+1)} + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + \alpha \dot{e}_n\right] \\ &+ \dot{m}\left(\frac{1}{2}r + x^{(n)}\right) + \dot{f} + e_n. \end{aligned} \tag{9}$$

The above auxiliary function is partitioned as sum of two auxiliary signals which are denoted by $N_r\left(x_r, \cdots, x_r^{(n)}\right)$, $\tilde{N}\left(x, \cdots, x^{(n)}, e_1, \cdots, e_n, r, x_r^{(n+1)}\right) \in \mathbb{R}$ and are defined as

$$N_r \triangleq N|_{x=x_r, \cdots, x^{(n)}=x_r^{(n)}} \tag{10}$$
$$\tilde{N} \triangleq N - N_r. \tag{11}$$

It should be noted that since both $N_r(t)$ and $\dot{N}_r(t)$ are functions of the desired trajectory and its time derivatives, they are bounded functions of time (*i.e.*, $N_r(t)$, $\dot{N}_r(t) \in \mathcal{L}_\infty$).

Remark 1: Since the auxiliary signal $N(\cdot)$ defined in (9) is continuously differentiable, Mean Value Theorem [14] can be utilized to show that $\tilde{N}(\cdot)$ can be upper bounded as

$$\left|\tilde{N}(\cdot)\right| \le \rho(\|z\|)\|z\| \tag{12}$$

where $\|\cdot\|$ denotes the standard Euclidean norm, $\rho : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is some globally invertible, non–decreasing function of its argument and $z(t) \in \mathbb{R}^{(n+1)}$ is the combined error signal defined as

$$z \triangleq [e_1, \cdots, e_n, r]^T. \tag{13}$$

Based on the subsequent stability analysis, the following continuous robust controller is proposed

$$\begin{aligned} u(t) =\ & k(t)e_n(t) - k(t_0)e_n(t_0) \\ &+ \alpha\int_{t_0}^{t} k(\sigma)e_n(\sigma)d\sigma \\ &+ \int_{t_0}^{t} \hat{\beta}(\sigma)\operatorname{sgn}(e_n(\sigma))d\sigma \end{aligned} \tag{14}$$

where $k(t) \in \mathbb{R}$ is a time–varying control gain which is updated according to

$$k(t) = k_c + \frac{1}{2}e_n^2(t) + \alpha\int_{t_0}^{t} e_n^2(\sigma)d\sigma \tag{15}$$

where $k_c \in \mathbb{R}$ is the positive constant part of $k(t)$, $\hat{\beta}(t) \in \mathbb{R}$ is a subsequently designed time–varying (uncertainty compensation) control gain, α was introduced in (7) and $\operatorname{sgn}(\cdot)$ is the standard signum function. The constant term

$k(t_0)\, e_n(t_0)$ is added to the controller to ensure $u(t_0) = 0$. The time–varying control gain $\hat{\beta}(t)$ is decomposed as

$$\hat{\beta}(t) = \hat{\beta}_1(t) + \beta_2 \tag{16}$$

where $\hat{\beta}_1(t) \in \mathbb{R}$ is its time–varying part and $\beta_2 \in \mathbb{R}$ is its positive constant part (*i.e.*, $\beta_2 > 0$). The time–varying part of the control gain is designed as

$$\hat{\beta}_1 = \begin{cases} e_n(t) - |e_n(t_0)| + \alpha \int_{t_0}^{t} |e_n(\sigma)|\, d\sigma & \text{if } e_n > 0 \\ -|e_n(t_0)| + \alpha \int_{t_0}^{t} |e_n(\sigma)|\, d\sigma & \text{if } e_n = 0 \\ -e_n(t) - |e_n(t_0)| + \alpha \int_{t_0}^{t} |e_n(\sigma)|\, d\sigma & \text{if } e_n < 0 \end{cases} \tag{17}$$

and taking its time derivative results in

$$\dot{\hat{\beta}}_1 = \begin{cases} \dot{e}_n(t) + \alpha e_n \mathrm{sgn}(e_n) & \text{if } e_n > 0 \\ \alpha e_n \mathrm{sgn}(e_n) & \text{if } e_n = 0 \\ -\dot{e}_n(t) + \alpha e_n \mathrm{sgn}(e_n) & \text{if } e_n < 0. \end{cases} \tag{18}$$

Alternatively, in a more compact form, the time–varying gain $\hat{\beta}_1(t)$ in (17) can be rewritten as

$$\hat{\beta}_1(t) = |e_n(t)| - |e_n(t_0)| + \alpha \int_{t_0}^{t} |e_n(\sigma)|\, d\sigma \tag{19}$$

from which its time derivative is obtained as

$$\begin{aligned} \dot{\hat{\beta}}_1 &= \dot{e}_n \mathrm{sgn}(e_n) + \alpha |e_n| \\ &= r \mathrm{sgn}(e_n) \end{aligned} \tag{20}$$

where the definition of $r(t)$ in (7) was utilized. Notice from (19) that $\hat{\beta}_1(t_0) = 0$. The definitions (19) and (20) will be preferred in the subsequent stability analysis.

Remark 2: At this point, we would like to compare our controller in (14) with the controller in [2]. To do that, recall the controller formulation in [2]

$$\begin{aligned} u(t) &= k\left[e_n(t) - e_n(t_0) + \alpha \int_{t_0}^{t} e_n(\sigma)\, d\sigma\right] \\ &+ \beta \int_{t_0}^{t} \mathrm{sgn}(e_n(\sigma))\, d\sigma \end{aligned} \tag{21}$$

where k and β are constant control gains. It is clear that, the only difference between these two controllers is that the controller gains in our design are time-varying where they were constant in [2]. While this is the only difference in the control design, the stability analysis in [2] requires the constant control gain β to be greater than the sum of the upper bound of the **uncertain** function N_r with the upper bound of its time derivative scaled by $\frac{1}{\alpha}$. However, in our controller design, a time-varying control gain, namely $\hat{\beta}(t)$ is utilized instead. Similarly, the control gain k was required to be chosen large enough when compared to the initial conditions of the system. However, in our controller design, a time-varying control gain, namely $k(t)$ is utilized instead.

At this stage, to substitute into (8), the time derivative of the control input in (14) is calculated

$$\dot{u} = \dot{k} e_n + k r + \left(\hat{\beta}_1 + \beta_2\right) \mathrm{sgn}(e_n) \tag{22}$$

where (7) and (16) were utilized, and thus the closed–loop error system for $r(t)$ is obtained as

$$m\dot{r} = -\frac{1}{2}\dot{m}r - e_n - kr - \dot{k}e_n - \left(\hat{\beta}_1 + \beta_2\right) \mathrm{sgn}(e_n) + N_r + \tilde{N}. \tag{23}$$

III. Stability Analysis

Before presenting the main result of this section, two lemmas are stated where both of which will later be utilized in the proof of the theorem.

Lemma 1: The auxiliary function, denoted by $L_1(t) \in \mathbb{R}$, is defined as

$$L_1 \triangleq r\left(N_r - \beta_1 \mathrm{sgn}(e_n)\right) \tag{24}$$

where $\beta_1 \in \mathbb{R}$ is a positive constant. Provided that β_1 satisfy

$$\beta_1 \geq \|N_r(t)\|_{L_\infty} + \frac{1}{\alpha}\left\|\dot{N}_r(t)\right\|_{L_\infty} \tag{25}$$

where $\|\cdot\|_{L_\infty}$ denotes infinity norm, then

$$\int_{t_0}^{t} L_1(\sigma)\, d\sigma \leq \zeta_{b_1} \tag{26}$$

where $\zeta_{b_1} \in \mathbb{R}$ is a positive constant.

Proof: The proof is available in [1]. ■

Lemma 2: The auxiliary function, denoted by $L_2(t) \in \mathbb{R}$, is defined as

$$L_2 \triangleq -\beta_2 \dot{e}_n \mathrm{sgn}(e_n). \tag{27}$$

Provided that $\beta_2 > 0$ then

$$\int_{t_0}^{t} L_2(\sigma)\, d\sigma \leq \zeta_{b_2} \tag{28}$$

where $\zeta_{b_2} \in \mathbb{R}$ is a positive constant.

Proof: The proof is available in [1]. ■

The tracking result will now be proven via the following theorem.

Theorem 1: The controller in (14) with the time–varying gain in (16) and (19) ensures semi–global asymptotic convergence of the tracking error and its time derivatives in the sense that $\left|e_1^{(i)}(t)\right| \to 0$ as $t \to \infty$ provided that α of (7) is selected to satisfy $\alpha > \frac{1}{2}$, and β_2 is chosen to be positive.

Proof: Following Lyapunov function candidate, denoted by $V(y, t) \in \mathbb{R}$, is defined as

$$V \triangleq \frac{1}{2}\sum_{j=1}^{n} e_j^2 + \frac{1}{2} m r^2 + \frac{1}{2}\tilde{\beta}_1^2 + P_1 + P_2 \tag{29}$$

where $P_1(t)$, $P_2(t) \in \mathbb{R}$ are defined as

$$P_1 \triangleq \zeta_{b_1} - \int_{t_0}^{t} L_1(\sigma)\, d\sigma \tag{30}$$

$$P_2 \triangleq \zeta_{b_2} - \int_{t_0}^{t} L_2(\sigma)\, d\sigma \tag{31}$$

and $\tilde{\beta}_1(t) \in \mathbb{R}$ is defined as

$$\tilde{\beta}_1 \triangleq \beta_1 - \hat{\beta}_1 \tag{32}$$

and $y(t) \in \mathbb{R}^{(n+4)\times 1}$ is defined as

$$y \triangleq \left[z^T, \tilde{\beta}_1, \sqrt{P_1}, \sqrt{P_2}\right]^T \quad (33)$$

where $z(t)$ was defined in (13).

From the proofs of Lemmas 1 and 2, it is clear that $P_1(t)$ and $P_2(t)$ are non–negative and thus $V(y,t)$ is also non–negative. The Lyapunov function in (29) can be bounded as

$$\frac{1}{2}\min\{1,\underline{m}\}\|y\|^2 \leq V \leq \max\left\{\frac{1}{2}\overline{m}(\|y\|), 1\right\}\|y\|^2 \quad (34)$$

where (2) was utilized.

After taking the time derivative of (29) and substituting (5), (7) and (23), following expression can be obtained

$$\begin{aligned}\dot{V} = & -\sum_{j=1}^{n-1} e_j^2 - \alpha e_n^2 + e_{n-1}e_n - r^2 - kr^2 \\ & + r\tilde{N} - \alpha\beta_2|e_n| - \dot{k}re_n \end{aligned} \quad (35)$$

where (24) and (27) were also utilized. By using the fact that $e_{n-1}e_n \leq \frac{1}{2}\left(e_{n-1}^2 + e_n^2\right)$, an upper bound on (35) can be obtained as

$$\begin{aligned}\dot{V} \leq & -\min\left\{\frac{1}{2}, \alpha - \frac{1}{2}\right\}\|z\|^2 + \frac{\rho^2(\|z\|)}{4k}\|z\|^2 \\ & - \alpha\beta_2|e_n| - \dot{k}re_n \end{aligned} \quad (36)$$

where (12) was utilized. Provided that α is selected to satisfy $\alpha > \frac{1}{2}$ and the time–varying control gain $k(t)$ is updated according to update rule given in (15). From (36), following expression is stated

$$\dot{V} \leq -\gamma\|z\|^2 - \alpha\beta_2|e_n| - r^2e_n^2 \leq -\gamma\|z\|^2 - \alpha\beta_2|e_n| \quad (37)$$

where $\gamma \in \mathbb{R}$ is some positive constant. From (29), (34) and (37), it is clear that $V(y,t) \in \mathcal{L}_\infty$ and thus $e_1(t), \cdots, e_n(t)$, $r(t)$, $\tilde{\beta}_1(t)$, $P_1(t)$, $P_2(t) \in \mathcal{L}_\infty$. Boundedness of $e_n(t)$ and $r(t)$ can be utilized along with (7) to show that $\dot{e}_n(t) \in \mathcal{L}_\infty$. These boundedness statements can be utilized along with (4)–(6) to prove that $\dot{e}_1(t), \cdots, \dot{e}_{n-1}(t) \in \mathcal{L}_\infty$. From (22), it can easily be concluded that $\dot{u}(t) \in \mathcal{L}_\infty$. The boundedness of the auxiliary error signals and their time derivatives can be utilized along with (6) to conclude that $e_1^{(i)}(t) \in \mathcal{L}_\infty$ $i = 1, \cdots, n$, which can then be utilized along with (3) and its time derivatives to prove that $x^{(i)}(t) \in \mathcal{L}_\infty$ $i = 1, \cdots, n$. The above boundedness statements can be utilized along with $m(\cdot)$, $f(\cdot) \in \mathcal{C}_2$, to prove that $m(\cdot)$, $f(\cdot)$, $\dot{m}(\cdot)$, $\dot{f}(\cdot) \in \mathcal{L}_\infty$. From (23), it is concluded that $\dot{r}(t) \in \mathcal{L}_\infty$.

After integrating (37) in time, following expression can be obtained

$$\gamma\int_{t_0}^{\infty}\|z(\sigma)\|^2 d\sigma + \alpha\beta_2\int_{t_0}^{\infty}|e_n(\sigma)| d\sigma \leq V(t_0) - V(\infty) \quad (38)$$

and since $V(\infty) \geq 0$ following expressions are obtained

$$\int_{t_0}^{\infty}\|z(\sigma)\|^2 d\sigma \leq \frac{V(t_0)}{\gamma} \quad (39)$$

$$\int_{t_0}^{\infty}|e_n(\sigma)| d\sigma \leq \frac{V(t_0)}{\alpha\beta_2}. \quad (40)$$

From (39) and (40), it is clear that $z(t) \in \mathcal{L}_2$ and $e_n(t) \in \mathcal{L}_1$. Since $e_n(t) \in \mathcal{L}_1 \cap \mathcal{L}_\infty$, from (19), it is concluded that $\hat{\beta}_1(t) \in \mathcal{L}_\infty$, and since $r(t) \in \mathcal{L}_\infty$, then from (20), it is clear that $\dot{\hat{\beta}}_1(t) \in \mathcal{L}_\infty$. Since $e_n(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, from (15), it is clear that $k(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can be utilized to prove that all the remaining signals remain bounded under the closed–loop operation. Since $z(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{z}(t) \in \mathcal{L}_\infty$, Barbalat's Lemma [15] can be utilized to prove that $\|z(t)\| \to 0$ as $t \to \infty$, and from its definition in (13), it is clear that the tracking error and its time derivatives asymptotically converge to zero. ■

IV. Example Systems and Simulation Results

In order to substantiate the theoretical results, the proposed nonlinear controller has been tested on two different systems. The first system is a generalized first order system that contains scalar variables only and the second system is a two link, direct–drive, planar robot manipulator system that contains vectoral variables. The main purpose of the usage of two different type systems is show that the controller can be performed efficiently for different variable types.

A. First Order Generalized Scalar System

The equations of motions are given as [16]

$$\dot{x} = -x + \delta_0(t) + u \quad (41)$$

where unknown time–varying parameter is set to be

$$\delta_0(t) = \sin(t) + \cos(\pi t) \quad (42)$$

$x \in \mathbb{R}$ denotes the state variable and $u \in \mathbb{R}$ denotes the control input. The system initial position has been set to $x(0) = 2$. The control objective is to make $x(t)$ follow a sinusoidal desired trajectory given as

$$x_r(t) = \sin(t). \quad (43)$$

Satisfactory tracking result was obtained when the control gains and the constant part of time–varying control gain $k(t)$ are selected as

$$\alpha = 2 \quad k_c = 10 \quad \beta_2 = 10. \quad (44)$$

The results are shown in Figures 1–4. The tracking result is depicted in Figure 1, while the tracking error, control input and the adaptive term are shown in Figures 2, 3 and 4, respectively. From Figures 1 and 2, it is clear that the tracking control objective was met.

B. Second Order Two Link Robot Manipulator System

The two link, direct–drive, planar robot manipulator having the following dynamic model [17]

$$\begin{aligned}\begin{bmatrix}\tau_1 \\ \tau_2\end{bmatrix} = & \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix}\begin{bmatrix}\ddot{q}_1 \\ \ddot{q}_2\end{bmatrix} \\ & + \begin{bmatrix} -p_3s_2\dot{q}_2 & -p_3s_2(\dot{q}_1 + \dot{q}_2) \\ p_3s_2\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix}\dot{q}_1 \\ \dot{q}_2\end{bmatrix} \\ & + \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix}\begin{bmatrix}\dot{q}_1 \\ \dot{q}_2\end{bmatrix} \end{aligned} \quad (45)$$

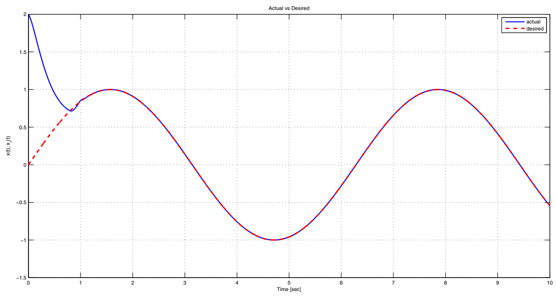

Fig. 1. Tracking Result

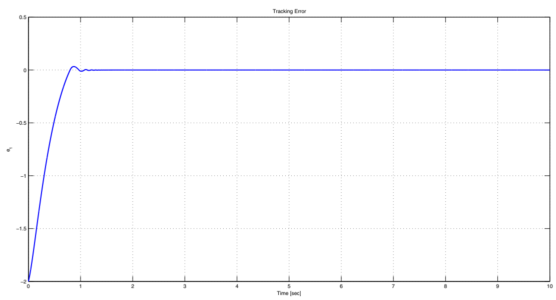

Fig. 2. Tracking Error

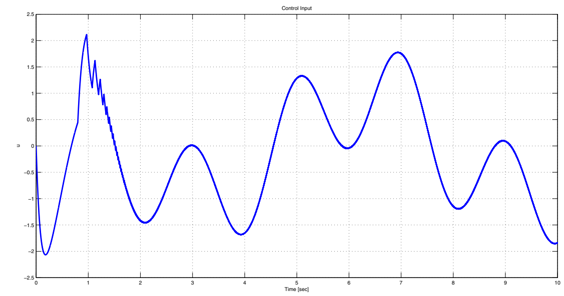

Fig. 3. Control Input

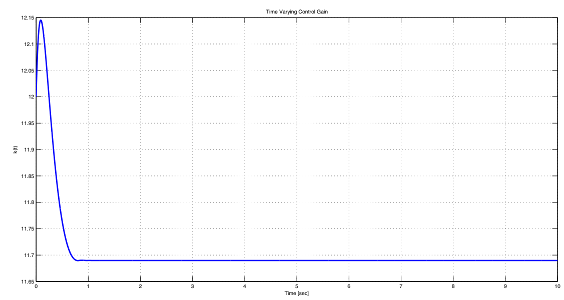

Fig. 4. Time–varying Control Gain $k(t)$

where $p_1 = 3.473\,[kg - m^2]$, $p_2 = 0.193\,[kg - m^2]$, $p_3 = 0.242\,[kg - m^2]$, $f_{d1} = 5.3\,[Nm - sec]$, $f_{d2} = 1.1\,[Nm - sec]$, $c_2 \triangleq \cos(q_2)$, and $s_2 \triangleq \sin(q_2)$. The control objective is to make $q_1(t)$ and $q_2(t)$ follow a sinusoidal desired trajectory given as

$$q_r(t) = \begin{bmatrix} 0.7\sin(t)\left(1 - \exp\left(-0.3t^3\right)\right) \\ 1.2\sin(t)\left(1 - \exp\left(-0.3t^3\right)\right) \end{bmatrix}. \quad (46)$$

Control gain parameters were selected as follows

$$\alpha = \{10\ 2\}\ \ k_c = diag\{5\ 25\}\ \ \ \beta_2 = diag\{10\ 2\}. \quad (47)$$

The results are shown in Figures 5–8. The tracking results are depicted in Figure 5, while the tracking errors, control inputs and the adaptive terms are shown in Figures 6, 7 and 8, respectively. From Figures 5 and 6, it is clear that the tracking control objective was met.

V. Conclusions

In this paper, we have presented a new self tuning RISE feedback type controller with time–varying feedback and an adaptive compensation gain. The proposed formulation does neither needs a tuning methodology nor require prior knowledge of upper–bounds of the vector containing the desired system dynamics plus functions containing uncertainties for the control gain selection. The controller formulation, achieved semi–global tracking and the stability result is backed up with a Lyapunov–type analysis. Extensive simulation studies are presented to illustrate the tracking performance of the proposed method.

When compared with the existing versions of the RISE feedback controllers, the results in this paper is the only design that addressed the self tuning of the controller gains. The time–varying controller gains designed in this paper can easily be applied to other RISE type controllers.

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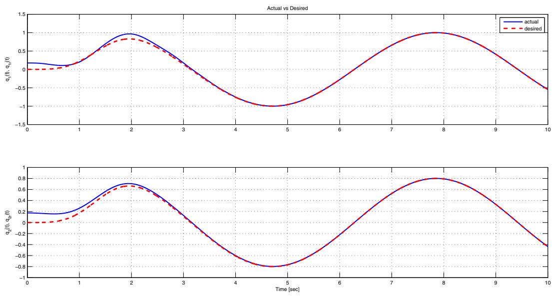

Fig. 5. Tracking Results

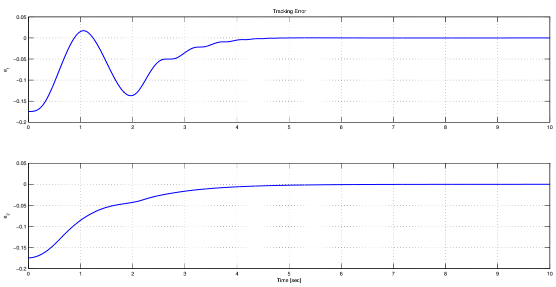

Fig. 6. Tracking Errors

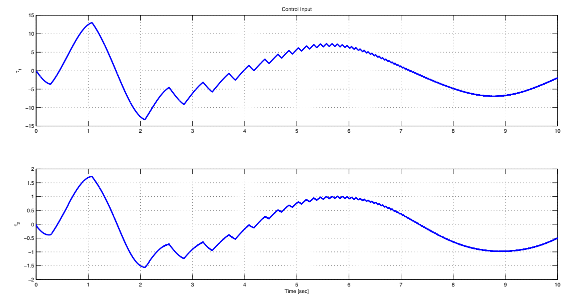

Fig. 7. Control Inputs

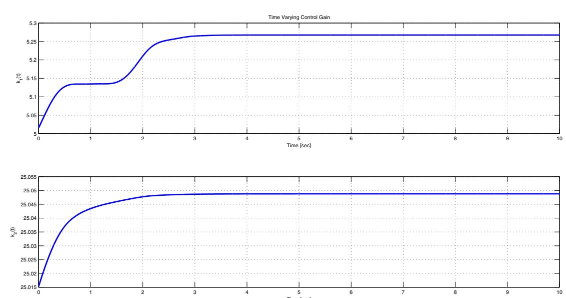

Fig. 8. Time–varying Control Gains $k_1(t)$,$k_2(t)$

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