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Comment on: "Application of Exp-function method for (3 + 1)-dimensional nonlinear evolution equations" [Comput. Math. Appl. 56 (2008) 1451–1456]

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ABSTRACT

We show that Boz and Bekir [A. Boz, A. Bekir, Application of Exp-function method for (3+1)-dimensional nonlinear evolution equations, Comput. Math. Appl. 56 (2008) 1451–1456] obtained some incorrect solutions for the equations studied by means of the Exp-function method. We verify our assertion by direct substitution and pole order analysis. In addition, we provide the correct results using the same approach.

1. Introduction

Kudryashov [1] states that "Nowadays there are a lot of computer software programs like MATHEMATICA and MAPLE. Using these codes it is possible to have complicated analytical calculations to search for different forms of solutions for nonlinear evolution equations and many authors use computer codes to look for exact solutions. However using computer programs many investigators do not take into account some important properties of differential equations. Therefore some authors obtain "new" cumbersome exact solutions of nonlinear differential equations with some errors and mistakes". Accordingly, Parkes [2–4] and Kudryashov [5–9] have been constantly warning the research community for disguised, incorrect, and equivalent results.

Recently, the Exp-function method [10], among the others, has attracted more attention for solving nonlinear evolution equations (NEEs). Naturally, it has been adapted, generalized and extended for different kinds of problems. However, there have been some precedents when solutions obtained by the Exp-function method are misleading, for example, see [11–14]. The procedure of the Exp-function method is tedious and complicated. It is almost impossible to handle without a computer algebra system. The method assumes an ansatz, which is based on trying rational combinations of exponential functions, involving many unknown parameters to be specified at the stage of solving the problem. One of the pitfalls of the method is that it sometimes leads to inconsistent systems of algebraic equations for the unknown parameters. Thus, we believe that one must be in eagle-eyed solving mode when using the Exp-function method.

2. The Kadomstev–Petviashvili equation

In Section 3 of [15], the authors analyzed the (3 + 1)-KP (Kadomstev–Petviashvili) equation in the form

$$u_{xt} - 6(u_x)^2 + 6uu_{xx} - u_{xxxx} - u_{yy} - u_{zz} = 0. \qquad (1)$$

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Using the transformation $\xi = kx + ly + mz + wt$, where k, l, m, and w are arbitrary constants, they reduced Eq. (1) to an ODE of the form

$$(kw - l^2 - m^2)u'' - 6k^2(u')^2 + 6k^2uu'' - k^4u'''' = 0, \tag{2}$$

where a prime denotes differentiation with respect to ξ. Then, applying the ansatz (the case $p = c = 1$ and $q = d = 1$ of the Exp-function method)

$$u(\xi) = \frac{a_1\exp(\xi) + a_0 + a_{-1}\exp(-\xi)}{\exp(\xi) + b_0 + b_{-1}\exp(-\xi)}, \tag{3}$$

to Eq. (2) and solving the resultant system of algebraic equations (involving the arbitrary constants a_1, a_0, a_{-1}, b_0, b_{-1}, k, l, m and w), they derived the relation (the formula (3.11) in [15])

$$a_{-1} = \frac{b_{-1}(8m^2 + 2l^2 + m + 2)}{12}, \qquad a_1 = \frac{8m^2 + 2l^2 + m - 22}{12}, \qquad b_0 = 0, \qquad a_0 = 0, \qquad k = -\frac{1}{2}, \qquad w = \frac{m}{4} - \frac{3l^2}{2}, \tag{4}$$

which corresponded to a solution of Eq. (1) in the form (the formula (3.12) in [15])

$$u_1(x, y, z, t) = \frac{\frac{1}{12}(8m^2 + 2l^2 + m - 22)\exp(\xi) + \frac{1}{12}b_{-1}(8m^2 + 2l^2 + m + 2)\exp(-\xi)}{\exp(\xi) + b_{-1}\exp(-\xi)}, \tag{5}$$

where $\xi = -\frac{1}{2}x + ly + mz + (\frac{1}{4}m - \frac{3}{2}l^2)t$, while b_{-1}, l, and m remain arbitrary.

Moreover, the authors took one further step by taking $b_{-1} = 1$ in (5) and obtained two other solutions to Eq. (1) as follows (the formulas (3.13) and (3.14) in [15] respectively):

$$u_2(x, y, z, t) = 8m^2 + 2l^2 + m - 10 - 12\tanh\left(-\frac{1}{2}x + ly + mz + \left(\frac{1}{4}m - \frac{3}{2}l^2\right)t\right), \tag{6}$$

$$u_3(x, y, z, t) = 8m^2 + 2l^2 + m - 10 - 12\mathrm{i}\tan\left(-\frac{1}{2}x + ly + mz + \left(\frac{1}{4}m - \frac{3}{2}l^2\right)t\right), \tag{7}$$

where i is the imaginary unit, while l and m remain arbitrary.

However, the direct substitution of the function (5) into Eq. (1) gives

$$u_{xt} - 6(u_x)^2 + 6uu_{xx} - u_{xxxx} - u_{yy} - u_{zz} = \frac{-24b_{-1}(e^{mt+4ly+4mz} + b_{-1}^2e^{6l^2t+2x})}{(e^{\frac{mt}{2}+2ly+2mz} + b_{-1}e^{3l^2t+x})^5}e^{3l^2t+mt+x+4ly+4mz}, \tag{8}$$

which is not zero in the general case. Thus, the functions (5)–(7) cannot be solutions of Eq. (1).

Remark 1. In fact, it can be shown by inspection that the function (5) cannot be a solution of Eq. (1). Using pole order analysis [6], we observe that a solution of Eq. (1) must have a pole of second order. But, the function (5) has a pole of first order. They do not match.

Remark 2. It is worth pointing out here that $(3 + 1)$-KP equation is a three spatial dimensional analog of the KdV equation. In other words, it is of the form

$$(u_t + auu_x + bu_{xxx})_x + cu_{yy} + du_{zz} = 0, \tag{9}$$

where the constants a, b, c, and d are chosen appropriately. For example, with $a = 6$ and $b = c = d = -1$, we obtain

$$u_{xt} + 6(u_x)^2 + 6uu_{xx} - u_{xxxx} - u_{yy} - u_{zz} = 0. \tag{10}$$

The key point here is that the coefficients of the second and third terms of Eq. (10) are the same. However, the first "+" sign in Eq. (10) is replaced by a "−" sign in Eq. (1). Let us, for a while, assume that Eq. (1) was misprinted in [15]. In this case, according to pole order analysis again, it is obvious that the function (5) cannot be a solution of the correct form of the $(3 + 1)$-KP equation (10).

Remark 3. We do not attempt to solve Eq. (1) by the Exp-function method because it is not the correct form of the $(3+1)$-KP equation as mentioned above. On the other hand, using the ansatz (3), we conclude that the $(3+1)$-KP equation (10) admits the bounded solution

$$u(x, y, z, t) = a_1 - \frac{4k^2b_0\exp(\xi)}{(b_0 + 2\exp(\xi))^2}, \qquad \xi = kx + ly + mz + \left(k^3 + \frac{l^2}{k} + \frac{m^2}{k} - 6ka_1\right)t, \tag{11}$$

where k, l, m, a_1, and b_0 remain arbitrary.

It is clear that the solution function (11) has a pole of second order as expected.

3. The potential-YTSF equation

In Section 4 of [15], the authors considered the (3 + 1)-dimensional so-called potential-Yu–Toda–Sassa–Fukuyama equation (or simply the potential-YTSF equation) which reads

$$-4u_{xt}+u_{xxxz}+4u_xu_{xz}+2u_{xx}u_z+3u_{yy}=0. \tag{12}$$

Using the transformation $\xi=kx+ly+mz+wt$, where k, l, m, and w are arbitrary constants, and omitting the integration constant, they reduced Eq. (12) to an ODE of the form

$$k^3mu'''+3k^2m(u')^2+(3l^2-4kw)u'=0, \tag{13}$$

where a prime denotes differentiation with respect to ξ. Then, applying the ansatz (the case $p=c=2$ and $q=d=1$ of the Exp-function method)

$$u(\xi)=\frac{a_2\exp(2\xi)+a_1\exp(\xi)+a_0+a_{-1}\exp(-\xi)}{\exp(2\xi)+b_1\exp(\xi)+b_0+b_{-1}\exp(-\xi)}, \tag{14}$$

to Eq. (13) and solving the resultant system of algebraic equations (involving the arbitrary constants a_2, a_1, a_0, a_{-1}, b_1, b_0, b_{-1}, k, l, m and w), they derived the relation (the formula (4.18) in [15])

$$a_0=\frac{a_1^2(2k-a_2)}{4k^2},\qquad b_0=\frac{-a_1^2}{4k^2},\qquad a_{-1}=0,\qquad b_{-1}=0,\qquad b_0=0,\qquad w=\frac{4k^3m+3l^2}{4k}, \tag{15}$$

which corresponded to a solution of Eq. (12) in the form (the formula (4.19) in [15])

$$u(x,y,z,t)=a_2+\frac{4k^2a_1\exp(\xi)+2ka_1^2}{4k^2\exp(2\xi)-a_1^2}, \tag{16}$$

where $\xi=kx+ly+mz+(\frac{4k^3m+3l^2}{4k})t$, while k, l, m, a_1, and a_2 remain arbitrary.

However, the direct substitution of the function (16) into Eq. (12) gives

$$-4u_{xt}+u_{xxxz}+4u_xu_{xz}+2u_{xx}u_z+3u_{yy}=\frac{-12k^5ma_1(2ke^{\frac{3l^2t}{4k}+k^2mt+kx+ly+mz}+a_1)}{(2ke^{\frac{3l^2t}{4k}+k^2mt+kx+ly+mz}-a_1)^3}e^{\frac{3l^2t}{4k}+k^2mt+kx+ly+mz}, \tag{17}$$

which is not zero in the general case. Thus, the function (17) cannot be a solution of Eq. (13).

On the other hand, using the ansatz (14), we conclude that the potential-YTSF equation (12) admits the solutions

$$u_1^{\pm}(x,y,z,t)=a_2-\frac{k(2b_0+(b_1\pm\sqrt{b_1^2-4b_0})\exp(\xi))}{\exp(2\xi)+b_0+b_1\exp(\xi)},\quad \xi=kx+ly+mz+\left(\frac{3l^2}{4k}+\frac{k^2m}{4}\right)t, \tag{18}$$

where k, l, m, b_0, b_1, and a_2 remain arbitrary;

$$u_2(x,y,z,t)=a_2-\frac{6kb_{-1}\exp(-\xi)}{\exp(2\xi)+b_{-1}\exp(-\xi)},\quad \xi=kx+ly+mz+\left(\frac{3l^2}{4k}+\frac{9k^2m}{4}\right)t, \tag{19}$$

where k, l, m, b_{-1}, and a_2 remain arbitrary;

$$u_3(x,y,z,t)=a_2-\frac{4kb_0(1+b_1\exp(-\xi))}{\exp(2\xi)+b_0+b_1\exp(\xi)+b_0b_1\exp(-\xi)},\quad \xi=kx+ly+mz+\left(k^2m+\frac{3l^2}{4k}\right)t, \tag{20}$$

where k, l, m, b_0, b_1, and a_2 remain arbitrary.

We can show by inspection that a solution of the potential-YTSF equation (12) must have a pole of first order. It is obvious that the solution functions (18)–(20) have poles of first order as well. They do match.

Remark 4. Of course, we added an integration constant C to the left side of the ODE (13) before applying the ansatz (14). But, the solution procedure of the Exp-function method leads to $C=0$. Otherwise, the resulting system of algebraic equations for the unknown parameters become inconsistent.

4. Conclusion

It seems that Boz and Bekir [15] were not able to solve the nonlinear algebraic systems for the unknown coefficients in a correct manner. They made one of the common errors from the list of Kudryashov [1], *sixth error: Some authors do not check the obtained solutions of nonlinear differential equations,* and presented some incorrect results**.**Working with MATHEMATICA interactively, we verified our results (11) and (18)–(20) by back-substitution; this provides an extra measure of confidence in the results.

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