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Mathematical models of the Bandpass problem and OrderMatic computer game

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ABSTRACT

A new combinatorial optimization problem, the Bandpass problem, was defined in Bell and Babayev (2004) [4]. Recently, this problem was investigated in detail in Babayev et al. (2009) [5]. In this paper, we first present some new mathematical models of the Bandpass problem. Then related to this problem, we introduce a software called OrderMatic which is very useful for teaching permutations.

1. Introduction

Due to the rapid growth in Internet usage, more bandwidth is needed. A technology named dense wavelength division multiplexing (DWDM) provides a platform to exploit the huge capacity of optical fiber. DWDM increases the number of communication channels within a fiber-optic cable, thereby letting service providers obtain much more bandwidth without installing a new cable [1]. DWDM works by combining and transmitting multiple signals simultaneously at different wavelengths on the same fiber. One fiber is transformed into multiple virtual fibers. Theoretically, more than 1000 channels may be multiplexed in a fiber. DWDM with more than 200 wavelengths has already been demonstrated [2]. A DWDM system with 200 signals can expand a basic 10 Gbit/s fiber system to a total capacity of over 2 Tbit/s over a strand of fiber. A simple architecture of a WDM system is presented in Fig. 1.

An optical add–drop multiplexer (OADM) is one of the most important elements in a fiber optic network. An OADM is a device that can add, block, pass or redirect various wavelengths in a fiber optic network. OADM consists of two logical input ports, namely In and Add, as well as two logical output ports, namely Out and Drop [3]. A typical OADM is shown in Fig. 2.

Each OADM facilitates flows on some wavelengths to exit the cable according to their paths. In each OADM, special cards control each wavelength; they may either pass through the OADM or may be dropped at their destination. DWDM is efficient in long-haul (600 km or less) communication. But it is expensive in short distances, because the communication requires a lot of add/drop processes. Many producers and researchers try to find out how to reduce cost of metropolitan (80 km or less) DWDM systems. The Bandpass problem was proposed to serve this purpose. If the wavelengths are consecutive such as $\lambda_m, \lambda_{m+1}, \ldots, \lambda_{m+k}$, then it is possible to pack wavelengths, and this reduces the cost of optic communication networks. In Section 2, we give the definition of the Bandpass problem. In Section 3, we give new mathematical models for the Bandpass

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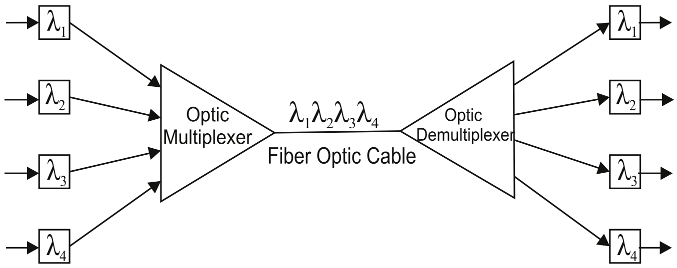

Fig. 1. A simple WDM system with 4 wavelengths.

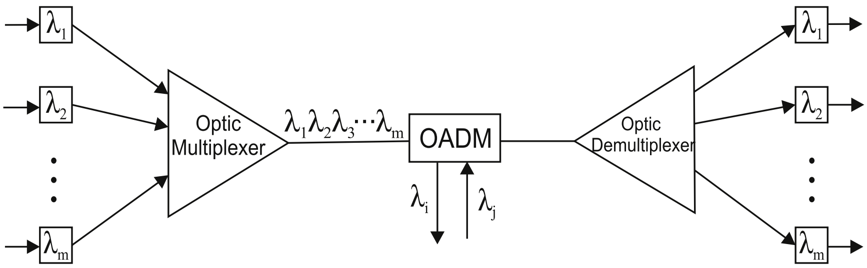

Fig. 2. OADM in a DWDM system.

a

i \ j	1	2	3	4
1	1	0	0	1
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1
5	0	0	1	1
6	0	1	1	1
7	1	0	0	1
8	0	1	0	0

b

i \ j	1	2	3	4
1	1	0	0	1
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1
5	0	0	1	1
6	0	1	1	1
7	1	0	0	1
8	0	1	0	0

c

i \ j	1	2	3	4
1	1	0	0	1
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1
5	0	0	1	1
6	0	1	1	1
7	1	0	0	1
8	0	1	0	0

d

i \ j	1	2	3	4
1	1	0	0	1
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1
5	0	0	1	1
6	0	1	1	1
7	1	0	0	1
8	0	1	0	0

Fig. 3. Overlapping bandpasses.

problem, In Section 4, we present a game related to the Bandpass problem and we end the paper with a list of open problems in Section 5.

2. The Bandpass problem

The Bandpass problem was first presented in Annual INFORMS meeting, in October 2004 by Bell and Babayev [4]. Before introducing models of the Bandpass problem, we give the definition of the bandpass and the Bandpass problem.

Definition 2.1. Let $B > 1$ be a positive integer. B consecutive non-zero entries of an $m \times n$ binary matrix A in the same column form a Bandpass. B is called the bandpass number.

Every non-zero entry of a column can be included in only one bandpass. The definition implies that several bandpasses in the same column cannot have any common rows. Consider as an example, the matrix A with $m = 8$ and $n = 4$ presented in Fig. 3(a), with bandpass number $B = 3$. Note that there is a single bandpass number which is the same for all columns. In Fig. 3(a) the matrix A, columns 1 and 2 contain no bandpasses because they have no $B = 3$ consecutive non-zero elements. Column 3 contains 5 consecutive non-zero elements from row 2 to row 6. Therefore, rows 2–4 (Fig. 3(b)) or rows 3–5 (Fig. 3(c)) or rows 4–6 (Fig. 3(d)) in the third column form a bandpass. But according to the definition of the bandpass, only one of these three groups of non-zero elements can be taken as a bandpass for this column. Because they have overlapping elements.

In order to have more than one bandpass, the column should have more than one group of non-overlapping consecutive nonzero elements. Each group should not have less than B elements. For instance, column 4 may have two bandpasses, one of the following three pairs of groups of rows: rows 1–3 and 4–6 (Fig. 4(a)), rows 1–3 and 5–7 (Fig. 4(b)), rows 2–4 and 5–7 (Fig. 4(c)).

i \ j	1	2	3	4
1	1	0	0	1
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1
5	0	0	1	1
6	0	1	1	1
7	1	0	0	1
8	0	1	0	0

a

i \ j	1	2	3	4
1	1	0	0	1
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1
5	0	0	1	1
6	0	1	1	1
7	1	0	0	1
8	0	1	0	0

b

i \ j	1	2	3	4
1	1	0	0	1
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1
5	0	0	1	1
6	0	1	1	1
7	1	0	0	1
8	0	1	0	0

c

Fig. 4. Non-overlapping bandpasses.

Let A be an $m \times n$ matrix and n_j be the number of bandpasses in column j for some $1 \le j \le n$. Then, $n_j \le \left\lfloor \sum_{i=1}^{m} a_{ij}/B \right\rfloor$.

If a column of the matrix has no bandpass but the total number of non-zero elements in the column is equal to or more than the bandpass number B, then the rows of the matrix can be relocated in order to form a bandpass in this column. For example, column 1 in the matrix (Fig. 3(a)) has no bandpass, but relocating row 1 to position 2, or 5, creates a bandpass in this column. Notice that many other relocations of rows also create a bandpass in column 1. However, this may affect the existing bandpasses in other columns.

The Bandpass problem is to find an optimal permutation of rows of a binary matrix which produces the maximum total number of bandpasses of given bandpass number B in all columns. It is very easy to find an optimal permutation of rows for a matrix which includes one or two columns. However, the Bandpass problem is NP-hard for a matrix with more than two columns [5].

3. Mathematical models of the Bandpass problem

In this section, we give four new mathematical models of the Bandpass problem.

3.1. The Boolean model of the Bandpass problem

This model is given in detail by Babayev et al. in [5]. Let $A = (a_{ij})$ be an $m \times n$ matrix and a_{ij} be an element of the matrix A. Let B be a bandpass number of A. Let us define decision variables as follows:

$$x_{ik} = \begin{cases} 1, & \text{if row } i \text{ is relocated to position } k; \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{kj} = \begin{cases} 1, & \text{if row } k \text{ is the first row of a bandpass in column } j; \\ 0, & \text{otherwise.} \end{cases}$$

We can formulate the Boolean model of the Bandpass problem as follows:

$$\max \sum_{j=1}^{n} \sum_{k=1}^{m-B+1} y_{kj} \tag{3.1a}$$

subject to

$$\sum_{k=1}^{m} x_{ik} = 1, \quad i = 1, \ldots, m \tag{3.1b}$$

$$\sum_{i=1}^{m} x_{ik} = 1, \quad k = 1, \ldots, m \tag{3.1c}$$

$$\sum_{i=k}^{k+B-1} y_{ij} \le 1, \quad j = 1, \ldots, n, \; k = 1, \ldots, m - B + 1 \tag{3.1d}$$

$$B \cdot y_{kj} \le \sum_{i=k}^{k+B-1} \sum_{r=1}^{m} a_{rj} x_{ri}, \quad j = 1, \ldots, n, \; k = 1, \ldots, m - B + 1 \tag{3.1e}$$

$$x_{ik}, \; y_{kj} \in \{0, 1\}, \quad i = 1, \ldots, m, \; j = 1, \ldots, n, \; k = 1, \ldots, m. \tag{3.1f}$$

The constraints in (3.1b) express the fact that row i must be relocated into one new position k only and the constraints in (3.1c) express that only one row i must be relocated to each new position k. The constraints in (3.1d) guarantee that no two bandpasses may have a common element. The constraints in (3.1e) guarantee to find the coordinates of bandpasses.

In this model, there are $2m + 2n(m - B + 1)$ constraints, i.e., $O(mn)$.

3.2. Integer programming model of the Bandpass problem

In this model, each x_i is a positive integer such that

$$x_i := \begin{cases} p, & \text{if row } i \text{ is relocated to position } p; \\ i, & \text{otherwise.} \end{cases}$$

Therefore, the model is called the integer programming model [6] of the Bandpass problem.

The other variables are as defined previously in (3.1) We can formulate this model as follows:

$$\max \sum_{j=1}^{n} \sum_{k=1}^{m-B+1} y_{kj} \tag{3.2a}$$

subject to

$$\sum_{i=k}^{k+B-1} y_{ij} \le 1, \quad j = 1, \ldots, n,\ k = 1, \ldots, m - B + 1 \tag{3.2b}$$

$$B \cdot y_{kj} \le \sum_{i=k}^{k+B-1} \sum_{r=1}^{m} a_{rj} x_{ri}, \quad j = 1, \ldots, n,\ k = 1, \ldots, m - B + 1 \tag{3.2c}$$

$$y_{kj} \in \{0, 1\}, \quad i = 1, \ldots, m,\ j = 1, \ldots, n,\ k = 1, \ldots, m \tag{3.2d}$$

$$x_i \in \{1, \ldots, m\}, \quad i = 1, \ldots, m \tag{3.2e}$$

$$x_i \ne x_j \quad \text{for } i \ne j,\ i = 1, \ldots, m,\ j = 1, \ldots, n. \tag{3.2f}$$

In this model, there are $2n(m - B + 1)$ constraints, i.e., $O(mn)$.

3.3. Combinatorial model of the Bandpass problem

This model represents all possible arrangements of rows (permutations). The model deals with which permutation π of the rows maximizes the total number of bandpasses in all columns. π is a permutation which is an arrangement, or an ordering of $(1, \ldots, m)$. That is, $\pi = \begin{pmatrix} 1 & 2 & 3 & \ldots & m \\ \pi(1) & \pi(2) & \pi(3) & \ldots & \pi(m) \end{pmatrix}$. For example, for $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 5 & 2 & 3 \end{pmatrix}$, $\pi(1) = 4$, $\pi(2) = 1, \pi(3) = 5, \pi(4) = 2, \pi(5) = 3$. Let us define decision variables as follows:

$$y_{\pi(k)j} := \begin{cases} 1, & \text{if row } k \text{ is the first row of a bandpass in column } j; \\ 0, & \text{otherwise.} \end{cases}$$

We can formulate the model as follows:

$$\max \sum_{j=1}^{n} \sum_{k=1}^{m-B+1} y_{\pi(k)j} \tag{3.3a}$$

subject to

$$\sum_{i=k}^{k+B-1} y_{\pi(i)j} \le 1, \quad j = 1, \ldots, n,\ k = 1, \ldots, m - B + 1 \tag{3.3b}$$

$$B \cdot y_{\pi(k)j} \le \sum_{i=k}^{k+B-1} a_{\pi(i)j}, \quad j = 1, \ldots, n,\ k = 1, \ldots, m - B + 1 \tag{3.3c}$$

$$y_{\pi(k)j} \in \{0, 1\}, \quad j = 1, \ldots, n,\ k = 1, \ldots, m. \tag{3.3d}$$

3.4. Modelling of multiple Bandpass problem

In this model, it is supposed that the matrix $A_{m\times n}$ may have different bandpass numbers in its columns. For example; $B = 3$ in the first column, $B = 5$ in the second column etc. All the variables except $B_j \in \{B_1, \ldots, B_n\}$ are the same as the previous ones. Now, we can formulate the model as follows:

$$\max \sum_{j=1}^{n} \sum_{k_j=1}^{m-B_j+1} y_{k_j j} \tag{3.4a}$$

i \ j	1	2	3	4
1	0	1	1	1
2	1	1	0	1
3	1	0	0	0
4	0	1	0	1
5	1	1	1	1
6	1	0	0	0
7	1	0	0	1
8	1	1	1	0

Fig. 5. A_{8x4}, $B = 3$.

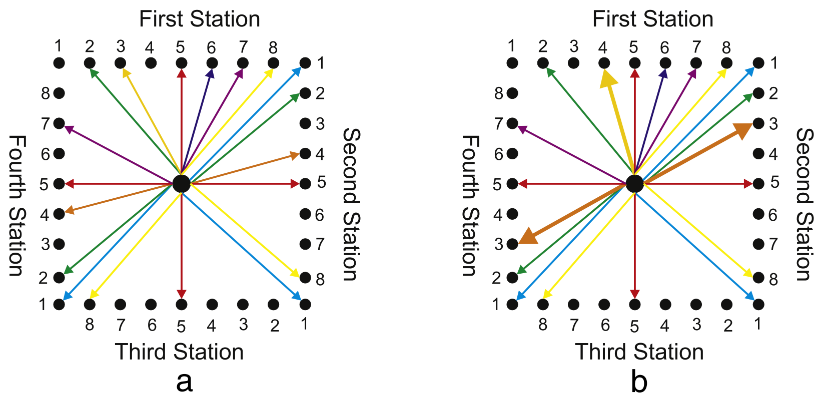

Fig. 6. Graph model of the Bandpass problem. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

subject to

$$\sum_{k=1}^{m} x_{ik} = 1, \quad i = 1, \dots, m \tag{3.4b}$$

$$\sum_{i=1}^{m} x_{ik} = 1, \quad k = 1, \dots, m \tag{3.4c}$$

$$\sum_{i=k_j}^{k_j+B_j-1} y_{ij} \le 1, \quad j = 1, \dots, n, \; k_j = 1, \dots, m - B_j + 1 \tag{3.4d}$$

$$B_j \cdot y_{k_j j} \le \sum_{i=k_j}^{k_j+B_j-1} \sum_{r=1}^{m} a_{rj} x_{ri}, \quad j = 1, \dots, n, \; k_j = 1, \dots, m - B_j + 1 \tag{3.4e}$$

$$x_{ik}, y_{k_j j} \in \{0, 1\}, \quad i = 1, \dots, m, \; j = 1, \dots, n, \; k = 1, \dots, m, \; k_j = 1, \dots, m - B_j + 1. \tag{3.4f}$$

3.5. Graph model of the Bandpass problem

This model is presented for better understanding of the Bandpass problem. We illustrate this model by an example. The matrix A in Fig. 5 says that there is a communication from a source to four different destinations (columns) on eight wavelengths (rows). We model this matrix as a star graph S_k (k is the number of 1s in the matrix A that indicates the number of edges in S). The internal node of the star graph is the source of communication. The stations (destinations) are indicated as sides of a polygon. In this example, four stations are indicated as sides of a square.

Wavelengths are stated as eight vertices at each station. Each wavelength which goes to the stations is colored differently as seen in Fig. 6. For example, the first wavelength is represented by the color blue. There is a communication to the second, third and fourth stations on this wavelength. The second wavelength is represented by the color green and the communication exists to the first, second and fourth stations on the second wavelength.

In this model, the objective is to maximize the number of bandpasses in matrix A as in all previous models defined in this paper. That is, for a given B the aim is to maximize total number of B consecutive edges to all stations. There are no

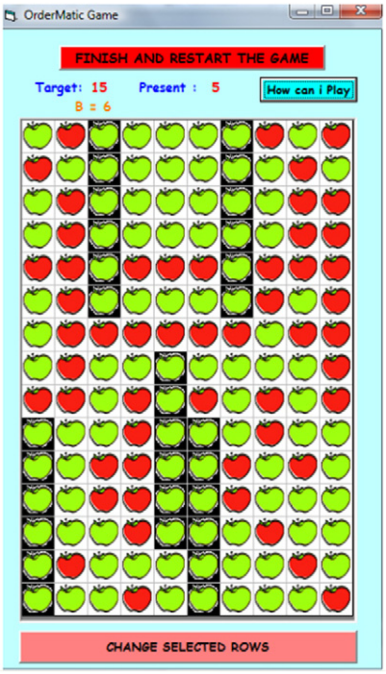

Fig. 7. The OrderMatic game. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

three consecutive edges to reach the second, third and fourth station in Fig. 6(a). If the fourth edge is swapped with the third edge in all stations then three consecutive edges are formed at the second and fourth station. New edges after swapping are indicated as thick arrows in Fig. 6(b). The consecutive edges at the first station are not destroyed while three consecutive edges are formed at the second and fourth stations. Consequently, this swapping increases the total number of bandpasses in the graph.

4. OrderMatic computer game

OrderMatic is designed for students who are interested in permutation. This game emphasizes the importance of permutation related to the Bandpass problem.

The exact solution of the Bandpass problem is found by choosing the correct permutation of rows which creates the maximum total number of bandpasses in all permutations. The auxiliary software (which allows to make calculations easier) transforms into a brain stimulating game.

In the OrderMatic game (Fig. 7), there are apples in colors red and green and three different levels; easy, normal and hard. There are different binary matrices and bandpass numbers related to hardness in each level. The aim of the game is to be able to order, as many as possible, consecutive green apples in a column of given bandpass number *B*. This ordering is shown on a black background on screen.

The player starts the game by choosing a level. Initially, a binary matrix and existing bandpasses are shown on the screen. The number of bandpasses in the current time is displayed on the "Present" tag. The maximum total number of bandpasses which would be found by arranging rows is displayed on the "Target" tag. The player chooses two rows by choosing two apples which are in different rows and ticks the button "Change Selected Rows". Then the chosen rows are relocated by the computer. If a new bandpass is formed or the number of bandpasses differs from previous one, the "Present" tag is updated. The game ends when the target number of bandpasses is reached.

The upcoming version of the game will include more levels and matrices. New matrices will be defined by players and the total number of bandpasses for these matrices of a given number *B* will be computed by a heuristic algorithm. Players can also share their permutations of rows which is a solution of the Bandpass problem in the library of Bandpass problems (BPLIB). This game is available on [7].

5. Conclusions

New models of a combinatorial optimization problem, called the Bandpass problem are introduced. The problem models a design problem in optical communications networks using wavelength division multiplexing technologies. In order to encourage research to meet the challenge of solving the problems, a library of Bandpass problems (BPLIB) is created in [7]. The library is open to the public and contains 90 problems.

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References

[1] N.N. Ravi, A. Pradyumnan, Special Edition Using Optical Networks, Que, 2001.
[2] Tutorial guide: dense wavelength division multiplexing, in: Circuits and Systems, The IEEE International Symposium ISCAS, 2001, pp. 9.3.1–9.3.10.
[3] J. Wagener, T. Strasser, Characterization of the economic impact of stranded bandwidth in fixed OADM relative to ROADM networks, in: Optical Fiber Communication Conference and Exposition and The National Fiber Optic Engineers Conference, Optical Society of America, 2006.
[4] G. Bell, D.A. Babayev, Bandpass problem, in: Annual INFORMS meeting, Denver, 2004.
[5] D.A. Babayev, G. Bell, U.G. Nuriyev, The Bandpass problem: combinatorial optimization and library of problems, Journal of Combinatorial Optimization (2009) 151–172.
[6] L.A. Woolsey, Integer Programming, Wiley, New York, 1998.
[7] D.A. Babayev, G. Bell, U.G. Nuriyev, M. Kurt, M.E. Berberler, A. Gursoy, Library of Bandpass problems, http://sci.ege.edu.tr/~math/BandpassProblemsLibrary.