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Comparison of stochastic search optimization algorithms for the laminated composites under mechanical and hygrothermal loadings

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Abstract

The aim of the present study is to design the stacking sequence of the laminated composites that have low coefficient of thermal expansion and high elastic moduli. In design process, multi-objective genetic algorithm optimization of the carbon fiber laminated composite plates is verified by single objective optimization approach using three different stochastic optimization methods: genetic algorithm, generalized pattern search, and simulated annealing. However, both the multi- and single-objective approaches to laminate optimization have been used by considerably few authors. Simplified micromechanics equations, classical lamination theory, and MATLAB Symbolic Math toolbox are used to obtain the fitness functions of the optimization problems. Stress distributions of the optimized composites are presented through the thickness of the laminates subjected to mechanical, thermal, and hygral loadings.

Keywords

stochastic optimization, laminated composites, dimensional stability, hygrothermal loading

Introduction

Laminated composites are widely used in aerospace, mechanics, and other branches of engineering applications due to their inherent tailorability. The materials used in aerospace structures like antenna, satellites, and missiles should have such features as low density, high stiffness and low coefficients of thermal and moisture expansions.[1] Carbon fiber reinforced polymer composite materials can satisfy these requirements with an appropriate stacking sequence.[2] In order to optimally design laminated composite materials with such a stacking sequence, it is necessary to perform some of the optimization methods. Design and optimization of the laminated composite materials are one of the most interesting subject of engineering because of that traditional optimization techniques may not be applied to composites or may be used only in limited cases. A detailed discussion of various optimization methods and algorithms can be found in[3] for general application and in[4] for composite design problems. Due to the complexity of the composite design and optimization problems, the use of stochastic optimization methods such as genetic algorithm (GA) and simulated annealing (SA) algorithm are appropriate. There are a few papers considering comparison of stochastic search algorithms in structural mechanics[5,6] and review of optimization methods in composites.[7,8] Optimization of laminated composite materials for only some specific problems have been studied by many researchers by using multi-objective[9–12] or single objective approaches.[13–21] However, Costa et al.[22] have considered both of multi-objective and single-objective approaches to laminate optimization problems.

Genetic algorithm is the most frequently used optimization method for composite design problems when

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compared to other stochastic search techniques.[9–16] A methodology for the multi-objective optimization of laminated composite materials has been proposed by Pelletier and Vel.[10] A multi-objective genetic algorithm has been used to obtain Pareto-optimal designs for the model problems. They have found that non-linearities in the shape of the Pareto-optimal front enable to perform trade-off studies when choosing a particular design. Aydin and Artem[12] have considered multi-objective optimal design of the laminated composites using genetic algorithms. MATLAB Genetic Algorithm and Direct Search Toolbox is used to obtain Pareto-optimal design for three different model problems. The objectives of the problems are to maximize the Young's moduli and minimize the coefficient of thermal expansion (CTE) simultaneously for 8- and 16-layered carbon/epoxy composites. Apalak et al.[13] have studied layer optimization for maximum fundamental frequency of laminated composite plates under any combination of the three classical edge conditions by means of genetic algorithm. They used an artificial neural network model in order to reduce the time searching for the optimal lay-up sequence. The problem for adjustment of residual stresses in unsymmetric composites has been studied by Hufenbach et al.[14] The new laminate design method has been verified by exemplary experiments and numerical calculations on unsymmetric glass and carbon fiber-reinforced plastics. The method has been applied to the design of multi layered curved hybrid structures.

Another stochastic optimization method used in composite design is simulated annealing. A constant thickness optimization of laminated composite has been presented by Deng et al.[17] The edging stress of a composite plate is the objective function for the SA algorithms and an efficient use of SA in the optimum stacking sequence of a composite laminate plate has been accomplished. The optimization of laminated and sandwich plates with respect to buckling load and thickness has been performed by Di Sciuva et al.[18] Genetic algorithm and simulated annealing methods have been employed and algorithms have provided almost the same results. In the study carried out by Erdal and Sonmez,[19] they have attempted to develop a procedure that can locate global optimum designs of composite laminates with minimum liability to buckling even in quite a large design space. They have adopted an improved version of SA for buckling optimization of composite. Reliability of the algorithm has been investigated in different load ratios.

Generalized pattern search algorithm (GPSA) is a mostly local search algorithm[16] and the use of the method in composite optimization is considerably few. GPSA has been used for optimal stacking sequence of a 64-layer composite plate made of graphite epoxy by Karakaya and Soykasap.[16] The optimization implementation has been done using MATLAB Genetic Algorithm and Direct Search Toolbox. They have concluded that the genetic algorithm is expensive but more effective in finding distinct global optima than generalized pattern search algorithm.

Since moisture and temperature cause some changes on mechanical properties of the polymer matrix composites, dimensional change induced by moisture and temperature is a significant feature in design of the composites.[23] Therefore, some researchers[2,12,20,21] have considered investigation of moisture and temperature effects on composite materials. For example, Le Rich and Gaudin[2] have investigated design of dimensionally stable composite laminates as space materials accounting for thermal, hygral, and mechanical constraints. In the context of composite materials, dimensional stability can be defined as the ability of a material to retain its dimensions when subjected to environmental effects such as temperature/moisture changes. Thermal expansion of composites is of primary importance to the dimensional stability of many structures. The ply orientation of laminated composite influences the linear CTE of a resin matrix composite. Therefore, using fiber orientation angles as design variables are appropriate.[24]

The purpose of the present study is to design dimensionally stable $(\pm\theta_1/\pm\theta_2)_s$ eight-layered laminated composite plate. In-plane design of symmetric and balanced carbon/epoxy composite satisfying the conditions of low CTE in longitudinal and high elastic moduli in longitudinal and/or transverse directions have been considered. In this work, the design has two main parts: mechanical analysis and optimization.

In the first part, simplified micromechanics expressions are used to predict the stiffness and thermal expansion coefficients of a lamina using constituent material properties. The classical lamination theory is utilized to determine the effective elastic modulus and the effective thermal expansion coefficients.

In optimization part, the problems (see Table 6) are formulated as multi-objective optimization problems and solved using GA. Then, an alternative single objective formulations with the non-linear constraints, obtained from multi-objective optimization, are utilized for verification of the multi-objective approach. The stochastic search methods GA, GPSA, and SA have been used to solve single objective optimization problems. The effective elastic moduli and the thermal expansion coefficient have been defined as fitness functions of the optimization problems. MATLAB Optimization Toolbox and Symbolic Math Toolbox[25–27] are used to obtain Pareto-optimal and best designs for 12 different model problems.

After completing the design process, stress distributions through the thickness have been investigated for the optimized composites subjected to the hygral, thermal, and mechanical loadings.

Comparison papers for optimization of laminated composites are quite few in literature and therefore, the present study fills a gap in composite design. We have considered both multi-objective and single-objective approaches to verify our study for the laminated composite optimization problems. However, in literature, either single-objective or multi-objective approach have been used for composite design and optimization.

Mechanical analysis

Macromechanical analysis

Classical lamination theory is used to analyze the infinitesimal deformation of thin laminated structures. In this theory, it is assumed that laminate is thin and wide, perfect bounding exists between laminas, there exists a linear strain distribution through the thickness, all laminas are macroscopically homogeneous, and they behave in a linearly elastic manner.[28] Thin laminated composite structure subjected to in plane loadings N_x, N_y is shown in Figure 1. Cartesian coordinate system x, y, and z define global coordinates of the layered material. A layer-wise principal material coordinate system is denoted by 1, 2, and 3, and fiber direction is oriented at angle θ to the x axis. Representation of laminate convention for the n-layered structure with total thickness h is given in Figure 2. In order to generalize the total strains including mechanical, thermal, and hygral effects, the following strain expression can be used:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{bmatrix} + \begin{bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{bmatrix} + \begin{bmatrix} \varepsilon_x^H \\ \varepsilon_y^H \\ \gamma_{xy}^H \end{bmatrix} \quad (1)$$

where $[\varepsilon]$, $[\varepsilon^M]$, $[\varepsilon^T]$, $[\varepsilon^H]$ are total, mechanical, thermal, and hygral strains, respectively.

Based on the classical lamination theory, the stress–strain relation for the k-th layer of a composite plate can be written in the following form

$$\begin{bmatrix} \sigma_x^M \\ \sigma_y^M \\ \sigma_{xy}^M \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \varepsilon_{xy}^o \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} - \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T - \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}_k \Delta C \right) \quad (2)$$

where $[\overline{Q}_{ij}]_k$ are the elements of the transformed reduced stiffness matrix, $[\varepsilon^o]$ is the mid-plane strains, $[\kappa]$ is curvatures, $\Delta T, \Delta C$ are temperature and moisture changes, and $[\alpha]_k$, $[\beta]_k$ are the coefficients of thermal and moisture expansions.

In-plane normal force resultants (force per unit width) N_x^M, N_y^M and shear force resultant N_{xy}^M on a laminate have the following relations:

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} - \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} - \begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix} \quad (3)$$

The matrices $[A]$ and $[B]$ appearing in Equation (3) can be defined as

$$A_{ij} = \sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_k - h_{k-1}) \quad (4)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \quad (i,j = 1,2,6) \quad (5)$$

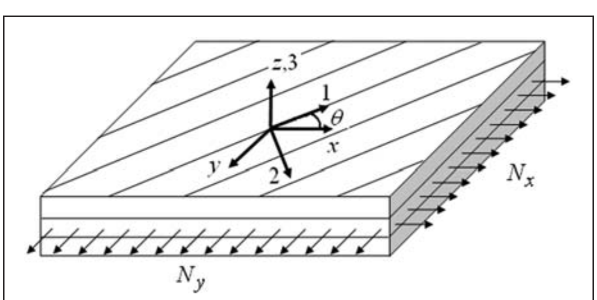

Figure 1. A thin fiber reinforced laminated composite subjected to in plane loading.

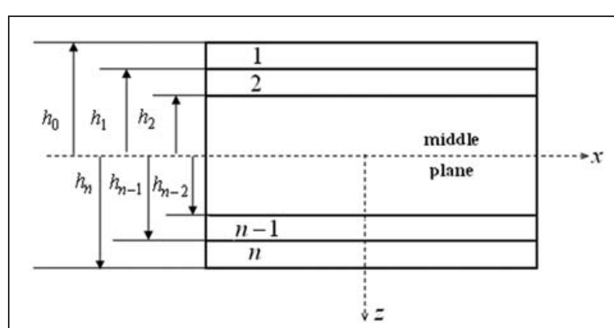

Figure 2. Laminate convention.

and $[N^T], [N^C]$ are the resultant thermal and hygral forces, respectively:

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (h_k - h_{k-1}) \tag{6}$$

$$\begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix} = \Delta C \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix}_k (h_k - h_{k-1}) \tag{7}$$

In macromechanical analysis, it is also convenient to introduce the effective elastic properties of symmetric balanced or symmetric cross-ply laminated plates subjected to in-plane loading. The effective thermal and moisture expansion coefficients and elastic moduli can be calculated using the following relations[28]:

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{\substack{\Delta C=0 \\ \Delta T=1}} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix}, \tag{8}$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}_{\substack{\Delta C=1 \\ \Delta T=0}} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \begin{bmatrix} N_x^C \\ N_y^C \\ N_{xy}^C \end{bmatrix}, \tag{9}$$

$$E_x = \frac{1}{hA_{11}^*}, \tag{10}$$

$$E_y = \frac{1}{hA_{22}^*} \tag{11}$$

where $[A^*] = [A]^{-1}$, h is total thickness of the composite.

Micromechanical analysis

Simplified micromechanical expressions[2] used to predict the stiffness and coefficient of thermal expansion of a lamina using constituent material properties (given in Table 1) are as follows:

$$E_1 = V_f E_{1f} + (1 - V_f) E_m \tag{12}$$

$$E_2 = \frac{E_m}{1 - \sqrt{V_f}(1 - E_m/E_{2f})} \tag{13}$$

Table 1. Constituent material properties

Fiber properties	Matrix properties
$E_{1f} = 550.2\,\text{GPa}$	$E_{1m} = 4.34\,\text{GPa}$
$E_{2f} = 9.52\,GPa$	$E_{2m} = 4.34\,\text{GPa}$
$G_{12f} = 6.9\,GPa$	$G_{12m} = 1.59\,\text{GPa}$
$\nu_{12f} = 0.2$	$\nu_{12m} = 0.37$
$\alpha_{1f} = -1.35.\ 10^{-6}/°C$	$\alpha_{1m} = 43.92.\ 10^{-6}/°C$
$\alpha_{2f} = 6.84.\ 10^{-6}/°C$	$\alpha_{2m} = 43.92.\ 10^{-6}/°C$
$\beta_{1f} = —$	$\beta_{1m} = 2000.10^{-6}/\%M$
$\beta_{2f} = —$	$\beta_{2m} = 2000.10^{-6}/\%M$

$$G_{12} = \frac{G_m}{1 - \sqrt{V_f}(1 - G_m/G_{12f})} \tag{14}$$

$$\nu_{12} = V_f \nu_{12f} + (1 - V_f)\nu_m \tag{15}$$

$$\alpha_1 = \frac{V_f \alpha_{1f} E_{1f} + (1 - V_f)\alpha_m E_m}{E_1} \tag{16}$$

$$\alpha_2 = \alpha_{2f}\sqrt{V_f} + (1 - \sqrt{V_f})(1 - V_f \nu_m E_{1f}/E_1)\alpha_m \tag{17}$$

$$\beta_1 = \frac{(1 - V_f)\beta_m E_m}{E_1} \tag{18}$$

$$\beta_2 = (1 - \sqrt{V_f})\beta_m \left(1 + \sqrt{V_f} \frac{(1 - \sqrt{V_f})E_m}{\sqrt{V_f}E_2 + (1 - \sqrt{V_f})E_m}\right) \tag{19}$$

where subscripts 1 and 2, f and m appearing in above equations denote the longitudinal and transverse directions, fiber and matrix properties, respectively. V_f represents the fiber volume fraction of the lamina, G is the shear modulus. In these formulations, fibers are assumed to be transversely isotropic.

Optimization

Essentially, optimization of a structure can be defined as finding the best design or elite designs by minimizing the specified single or multi-objectives that satisfy all the constraints. Single- and multi-objective optimizations are the main approaches used in structural design problems. In single objective approach, an optimization problem consists of a single objective function, constraints, and bounds. However, the design of practical composite structures often requires the maximization or minimization of multiple, often conflicting, two or more objectives, simultaneously. In such a case, multi-objective formulation is used and a set of solutions are obtained with different trade-off called Pareto optimal. Only one solution is to be chosen from the set of solutions for practical engineering usage. There is no such thing as the best solution with respect to all objectives in multi-objective optimization.[10]

Optimization techniques can be classified as traditional and non-traditional. Traditional optimization techniques, such as constrained variation and lagrange multipliers, are analytical and find the optimum solution of only continuous and differentiable functions.[3] Since composite design problems generally have discrete search spaces, the traditional optimization techniques cannot be used. In these cases, the use of stochastic optimization methods such as genetic algorithms (GA), generalized pattern search algorithm (GPSA), and simulated annealing (SA) are appropriate.

Optimization Toolbox of MATLAB has been used to solve the design optimization problems. MATLAB Optimization Toolbox includes a few routines for solving optimization problems using Direct Search (DS), GA, and SA methods. All of these methods have been used in the design of composite materials by many researchers.[9–19] DS functions include two DS algorithms called the Generalized Pattern Search Algorithm and the Mesh Adaptive Search (MADS) algorithm. The Toolbox has optimization solvers *ga*, s*imulannealbnd,* and *patternsearch* for single objectives and *gamultiobj* for multi objectives. The multi-objective GA function *gamultiobj* uses a variant of NSGA-II. Parameters for the solvers are given in Tables 2–5.

Single-objective optimization

A standard mathematical formulation of the single-objective optimization consists of an objective function, equality and/or inequality constraints, and design

Table 2. Genetic algorithm parameters for multi-objective approach used in the model problems 1a, 2a, and 3a

Population type	Double vector
Population size	40
Initial range	[−90 −90 ; 90 90]
Selection function	Tournament
Cross-over fraction	0.8
Mutation function	Adaptive feasible
Cross-over function	Intermediate; Ratio = 1.0
Migration direction	Both; Fraction = 0.2, Interval = 20
Initial penalty	10
Penalty factor	100
Hybrid function	None
Stopping criteria	Generation = 100; stall generation = 50; function tolerance = 10^{-6}

Table 3. Genetic algorithm parameters for single-objective approach used in model problems 1b, 2b, and 3b

Population type	Double vector
Population size	20
Creation function	Use constraint dependent
Initial population	[ ]
Initial scores	[ ]
Initial range	[−300; −100]
Scaling function	Top, quantity = 12
Selection function	Tournament; Tournament size = 7
Elite count	2
Cross-over fraction	0.6
Mutation function	Use constraint Dependent
Cross-over function	Scattered
Migration direction	Both; fraction = 0.2, interval = 20
Initial penalty	10
Penalty factor	100
Hybrid function	None
Stopping criteria	Generation = 100; stall generation = 50; function tolerance = 10^{-6}

Table 4. Generalized pattern search algorithm (GPSA) parameters for single-objective approach used in model problems 1c, 2c, and 3c

Poll method	GPS positive basis 2N
Complete poll	Off
Polling order	Consecutive
Complete search	Off
Search method	None
Mesh initial size	1.0
Mesh max size	Infinity
Mesh accelerator	Off
Mesh rotate	On
Mesh scale	On
Mesh expansion factor	2.0
Contraction factor	0.5
Initial penalty	1.0
Penalty factor	100
Bind tolerance	10^-3
Stopping criteria	Mesh tolerance = 10^{-6}; Max iterations = 200 * number of variables; Max function evaluations = 2000 * number of variables; Time limit = infinity × tolerance = 10^{-6}; Function tolerance = 10^{-6}; Non-linear constraint tolerance = 10^{-6}.

Table 5. Simulated annealing solver parameters for single-objective approach used in model problems 1d, 2d, and 3d

Annealing function	Fast annealing
Reannealing interval	100
Temperature update function	Exponential temperature update
Initial temperature	100
Acceptance probability function	Simulated annealing acceptance
Data type	Double
Stopping criteria	Max iterations = infinity; Max function evaluations = 3000 * number of variables; Stall iterations = 500 * number of variables; Function tolerance = 10^{-6}

variables. In our study, the single-objective optimization problem with fiber orientation angles $\theta_1, \theta_2, \ldots, ., \theta_n$ can be stated as follows:

$$\begin{aligned} &\textbf{minimize} \;\; f(\theta_1, \theta_2, \ldots, \theta_n) \\ &\textbf{such that} \;\; g_i(\theta_1, \theta_2, \ldots, \theta_n) \geq 0 \quad i = 1, 2, \ldots, r \\ &\qquad\qquad\quad p_j(\theta_1, \theta_2, \ldots, \theta_n) = 0 \quad j = 1, 2, \ldots, m \end{aligned}$$

where f is objective function, $\theta_1, \theta_2, \ldots, \theta_n$ are the design variables, and g, p are the constraints of the problem. In composite design and optimization problems mass, stiffness, displacements, residual stresses, thickness, vibration frequencies, buckling loads, and cost are used as objective functions.[4] In this study, elastic modulus E_x is considered as objective function of the single-objective optimization problems.

Multi-objective optimization

A multi-objective optimization problem can be stated as follows:

$$\begin{aligned} &\textbf{minimize} \;\; f_1(\theta_1, \theta_2, \ldots, \theta_n), f_2(\theta_1, \theta_2, \ldots, \theta_n), \ldots, \\ &\qquad\qquad\; f_t(\theta_1, \theta_2, \ldots, \theta_n) \\ &\textbf{such that} \;\; g_i(\theta_1, \theta_2, \ldots, \theta_n) \geq 0 \quad i = 1, 2, \ldots, r \\ &\qquad\qquad\quad p_j(\theta_1, \theta_2, \ldots, \theta_n) = 0 \quad j = 1, 2, \ldots, m \end{aligned}$$

where $f_1, f_2, \ldots, f_t$ represent the objective functions to be minimized simultaneously.[3]

The main difficulties in multi-objective optimization problems are to minimize the distance of the generated solutions to the Pareto set and to maximize the diversity of the developed Pareto set. Detailed analysis of multi-objective optimization can be found in.[29]

Stochastic optimization algorithms

Stochastic optimization methods are optimization algorithms based on probabilistic elements, either in the objective function with the constraints or in the algorithm itself or both of them.[30] Genetic algorithm, particle swarm optimization, ant colony optimization, simulated annealing, tabu search, harmony search, and generalized pattern search algorithm are examples of the stochastic search techniques used in engineering applications. In composite laminate design problems, derivative calculations or their approximations are impossible to obtain or is often costly. Therefore, stochastic search methods have the advantage of requiring no gradient information of the objective functions and the constraints. In this paper, only GA, GPSA, and SA have been considered and used without any modification for design of the laminated composites. In the following subsections, steps of the algorithms are briefly overviewed.

Genetic algorithm

The GA is a stochastic optimization and search technique that allows obtain alternative solutions for some of the complex engineering problems such as increasing composite strength, developing dimensionally stable and light-weight structures, etc. GA method utilizes the principles of genetics and natural selection. This method is simple to understand and uses three simple operators: selection, crossover, and mutation. GA always considers a population of solutions instead of a single solution at each iteration. It has some advantages in parallelism and robustness of genetic algorithms. It also improves the chance of finding the global optimum point and helps to avoid local stationary point. However, GA is not guaranteed to find the global optimum solution to a problem. GA has been applied to the design of a variety of composite structures ranging from simple rectangular plates to complex geometries.

Generalized pattern search algorithm

GPSA has been defined for derivative-free unconstrained optimization of functions by Torczon[31] and later extended to take non-linear constrained optimization problems into account. GPSA is a direct search method which finds a sequence of points that approach the optimal point. Each iteration is divided into two phases: the search phase and the poll phase. In the search phase, the objective function is evaluated at a finite number of points on a mesh. The main task of the search phase is to find a new point that has a lower objective function value than the best current solution which is called the incumbent. In the poll phase, the objective function is evaluated at the neighboring mesh points, so as to see whether a lower objective function value can be obtained.[32] GPSA has some collection of vectors that form the pattern and has two commonly used positive basis sets; the maximal basis with $2N$ vectors and the minimal basis with $N + 1$ vectors.

In order to clarify the algorithm, a laminated composite plate optimization problem including two independent variables θ_1 and θ_2 in the objective function has been considered. In this case, pattern consists of the vectors $v_1 = [1 \quad 0]$, $v_2 = [0 \quad 1]$, $v_3 = [-1 \quad 0]$, $v_4 = [0 \quad -1]$ for the positive basis $2N$ or $v_1 = [1 \quad 0]$, $v_2 = [0 \quad 1]$, $v_3 = [-1 \quad -1]$ for the positive basis $N + 1$. The pattern search begins at a provided initial point vector θ_0. In this example problem, $\theta_0 = [10 \quad 50]$, the mesh size $\Delta^m = 5$ and positive basis $2N$ are taken into account. At the first iteration, the following mesh points can be calculated as

$$[1 \quad 0] \times 5 + [10 \quad 50] = [15 \quad 50]$$
$$[0 \quad 1] \times 5 + [10 \quad 50] = [10 \quad 55]$$
$$[-1 \quad 0] \times 5 + [10 \quad 50] = [5 \quad 50]$$
$$[0 \quad -1] \times 5 + [10 \quad 50] = [10 \quad 45]$$

and the algorithm computes the objective function at these mesh points before polls.[16,30,33] If the algorithm finds an objective function value which is smaller than the value at $\theta_0 = [10 \quad 50]$, the poll at corresponding iteration is called as 'successful.' Supposing the vector $[10 \quad 55]$ satisfies the condition, the algorithm sets the next point in the sequence equal to $\theta_1 = [10 \quad 55]$. After obtaining a successful poll, the algorithm multiplies the current mesh size by *expansion factor*. For example, if the *expansion factor* is taken as 2, the *mesh size* for the second iteration becomes $5 \times 2 = 10$ and the mesh at the second iteration is to be:

$$[1 \quad 0] \times 10 + [10 \quad 55] = [20 \quad 55]$$
$$[0 \quad 1] \times 10 + [10 \quad 55] = [10 \quad 65]$$
$$[-1 \quad 0] \times 10 + [10 \quad 55] = [0 \quad 55]$$
$$[0 \quad -1] \times 10 + [10 \quad 55] = [10 \quad 45]$$

Now, suppose that $\theta_2 = [0 \quad 55]$ produce smaller objective function value than the value at $\theta_1 = [10 \quad 55]$. This procedure repeats until none of the mesh points has a smaller objective function value than the value at last (say n) successful poll iteration. This poll is called 'unsuccessful' in the corresponding iteration. In this case, the algorithm does not change the current point at the next iteration as:

$$\theta_{n+1} = \theta_n \tag{20}$$

In such a case, the algorithm multiplies the current mesh size by given *contraction factor* and the algorithm then polls with a smaller mesh size. The algorithm stops when any of the stopping criteria conditions occurs.

Simulated annealing algorithm

SA is a random-search technique and it is based on the simulation of thermal annealing of heated solids to achieve the minimum function value in a minimization problem.[3] It is possible to solve mixed-integer, discrete, or continuous optimization problems by using SA. In this algorithm, a new point is randomly generated at each iteration and the algorithm stops when any of the stopping criteria are satisfied. The distance of the new point from the current point or the extent of the search is based on Boltzmann's probability distribution. The distribution implies the energy of a system in thermal equilibrium at temperature T. Boltzmann's probability distribution can be written in the following form[3,33]:

$$P(E) = e^{-E/kT} \tag{21}$$

where P(E) represents the probability of achieving the energy level E, k is the Boltzmann's constant, and T is temperature.[3]

SA algorithm has the following steps:

1. Start with an initial vector x_1 and assign a high temperature value to the function.
2. Generate a new design point randomly and find the difference in function values.
3. Determine whether the new point is better or worse than the current point.
4. If the value of a randomly generated number is larger than $e^{-\Delta E/kT}$, accept the point x_{i+1}.
5. If the point x_{i+1} is rejected, then the algorithm generates a new design point x_{i+1} randomly.

However, it should be noted that the algorithm accepts a worse point based on an acceptance probability.[3]

Model problems

Stacking sequences optimization for eight-layered symmetric-balanced laminated composites made of carbon/epoxy have been considered in this study. Main objective of the study is to design $(\pm\theta_1/\pm\theta_2)_s$ laminates satisfies the conditions: low CTE in longitudinal and high elastic moduli in longitudinal and/or transverse directions. The fiber orientation angles θ_1 and θ_2 are selected as design variables and the limiting values are $-90 \leq \theta_1, \theta_2 \leq 90$ in the continuous domain. The fiber volume fraction and thickness of each layer are assumed as 0.50 and 150.10^{-6}m, respectively.

In this study, we have considered three main problems (1a, 2a, 3a) defining in-plane designs and optimization of thin plate composites. The optimization problems are firstly formulated based on multi-objective approach. An alternative single-objective formulation including the non-linear constraints is utilized for verification of the multi-objective optimization. Multi-objective optimization problems (1a, 2a, 3a) aim to minimize the CTE while simultaneously maximizing the elastic moduli. In the single-objective representation of the problems (1b-d, 2b-d, 3b-d), CTE obtained from the multi-objective formulations are used to define the non-linear constraints of the single optimization problems. This proposed approach is quite recommended since it is possible to obtain relatively small feasible solution space and it clarifies definition of the front. Details of these model problems with different loading cases,[2] optimization types, objective functions, constraints, and bound are presented in Table 6.

The present study consists of three main parts: (i) Micro and macro mechanical analyses, (ii) Optimization, and (iii) Stress analysis. In the first part, simplified micromechanics expressions are used to predict the elastic modulus and thermal and moisture expansion coefficients of a lamina using constituent material properties. The classical lamination theory is utilized to determine the effective elastic moduli, the effective coefficient of thermal and moisture expansions for composite laminates.[4] In the second part, the effective elastic properties, which define the fitness functions, have been used to obtain the optimum fiber orientation angles of each layer by stochastic search techniques GA, GPSA, and SA. MATLAB *Optimization Toolbox* solvers *ga, patternsearch, simulannealbnd* and *gamultiobj* are used to obtain best designs and Pareto-optimal designs for model problems. In the last part, optimized composites obtained from multi-objective design have been considered for stress analysis. Stress distributions through the thickness of the composites subjected to the mechanical, thermal, and hygral loadings have been calculated and shown graphically in Figures 14–17.

Results and discussion

In this section, results of laminated composite optimal design studies based on the multi- and single-objective optimizations are presented for coefficient of thermal expansion and elastic moduli. If the fiber orientation angles are selected as 0° for all lamina, as expected, E_x becomes maximum ($E_x = 277.3\,\text{GPa}$). However, this design is not suitable for minimum coefficient of thermal expansion ($\alpha_x = -1.0.10^{-6}/^{o}\text{C}$). Similarly, if all the fibers have 32° orientation, the CTE becomes minimum ($\alpha_x = -5.24.10^{-6}/^{o}\text{C}$), but this is again not an appropriate design for E_x of the composite ($E_x = 40.8\,\text{GPa}$). Regarding this fact, the model problems have been optimized for the purpose of

Table 6. Model problems

Problems	Loadings	Optimization types	Objective functions	Constraints/bound
1a	LC1	Multi-objective GA	E_x, α_x(for multi-obj.)	B1
1b		Single-objective GA	E_x(for single-obj.)	C1
1c		Single-objective GPSA		
1d		Single-objective SA		
2a	LC1	Multi-objective GA	E_x, E_y, α_x(for multi-obj.)	B1
2b		Single-objective GA	E_x(for single-obj.)	C2
2c		Single-objective GPSA		
2d		Single-objective SA		
3a	LC2	Multi-objective GA	E_x, α_x(for multi-obj.)	B1
3b		Single-objective GA	E_x(for single-obj.)	C3
3c		Single-objective GPSA		
3d		Single-objective SA		

where

Loading Cases
LC1: $F_x = 20kN,\ F_y = 20kN,\ F_{xy} = 0,\ \Delta T = -150^{\circ}C, \Delta C = 0\ \%$
LC2: $F_x = 50kN,\ F_y = 1kN,\ F_{xy} = 0,\ \Delta T = -150^{\circ}C,\ \Delta C = 2\ \%$

Constraints
C1: $-90^{\circ} \leq \theta_1 \leq 90^{\circ},\ -90^{\circ} \leq \theta_2 \leq 90^{\circ},\ \alpha_x \leq -2.63.10^{-6}/^{\circ}C$
C2: $-90^{\circ} \leq \theta_1 \leq 90^{\circ},\ -90^{\circ} \leq \theta_2 \leq 90^{\circ},\ \alpha_x \leq -2.31.10^{-6}/^{\circ}C,\ E_y \geq 9.7\,GPa$

Bound
C3: $-90^{\circ} \leq \theta_1 \leq 90^{\circ},\ -90^{\circ} \leq \theta_2 \leq 90^{\circ},\ \alpha_x \leq -3.21.10^{-6}/^{\circ}C$
B1: $-90^{\circ} \leq \theta_1 \leq 90^{\circ},\ -90^{\circ} \leq \theta_2 \leq 90^{\circ}$

Table 7. Pareto-optimal designs for the model problems 1a and 3a and the corresponding CMEs[12]

Design	E_x (GPa)	α_x (10^{-6}/°C)	β_x (10^{-6}/%M)	θ_1 (Deg)	θ_2 (Deg)
1	115.9	−3.66	−58.8	25.9	−19
2	127.6	−3.49	−54.1	24.6	−18
3	152.7	−3.21	−46.1	20	−18
4	169.4	−2.89	−37.3	14.9	−20.5
5	188.0	−2.63	−29.9	13.8	−18.5
6	194.2	−2.44	−24.7	20.6	−10.2
7	206.9	−2.16	−17.0	4.7	−21.6
8	222.1	−1.99	−12.2	19.5	−2.6
9	239.9	−1.80	−6.8	8.1	−13.7

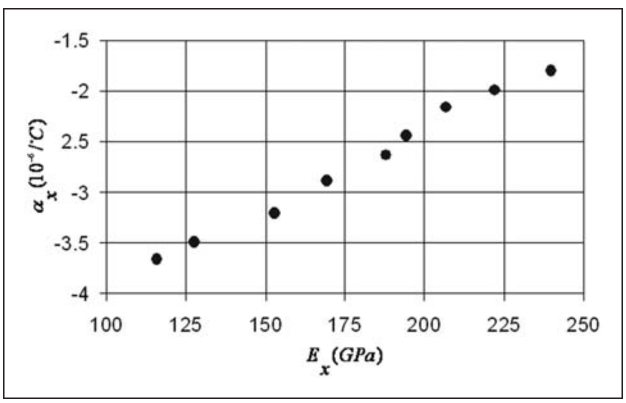

Figure 3. Pareto-optimal designs for maximum E_x and minimum α_x for model problems 1a and 3a.

Table 8. Pareto optimal designs of the model problem 2a and the corresponding CMEs[12]

Design	E_x (GPa)	E_y (GPa)	α_x (10^{-6}/°C)	β_x (10^{-6}/%M)	θ_1 (Deg)	θ_2 (Deg)
1	149. 5	9.1	−2. 90	−36. 3	13	26.5
2	159. 5	16.9	−1. 90	−9.7	1.2	34.7
3	159. 8	13.0	−2. 21	−18. 3	6.5	31.2
4	161. 3	7.4	−3. 00	−40. 5	15.5	21.2
5	170. 2	8.4	−2. 69	−31. 8	11.6	23.7
6	172. 4	11.8	−2. 20	−17. 9	3.9	29.2
7	181. 9	8. 0	−2. 57	−28. 4	10.9	22.1
8	184. 1	9. 7	−2. 31	−21. 0	5.7	25.5
9	190. 2	7. 2	−2. 60	−29. 2	13.7	18.3
10	193. 2	8. 5	−2. 34	−21. 8	7.4	22.7
11	202. 6	9. 0	−2. 15	−16. 5	1.2	23.3
12	217. 8	8. 1	−2. 04	−13. 4	2.5	20.3
13	234. 0	7. 6	−1. 85	−8. 1	2.1	17.4
14	245. 9	7. 4	−1. 67	−3. 3	0.3	15.2

minimizing the CTE while maximizing the elastic moduli, simultaneously. More reliable solutions and the corresponding CMEs for multi-objective optimization of the model problems (1a and 3a) are given in

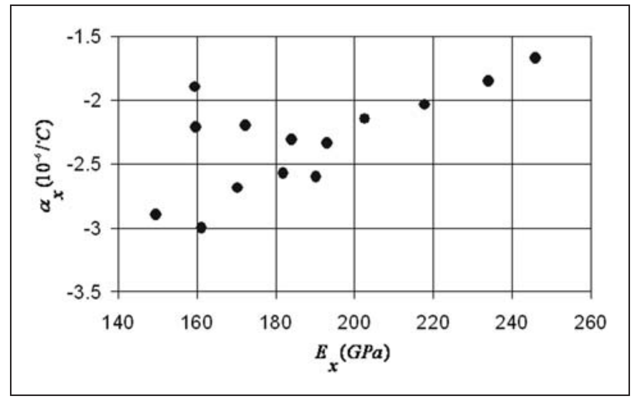

Figure 4. Pareto-optimal designs for maximum E_x and minimum α_x for model problem 2a.

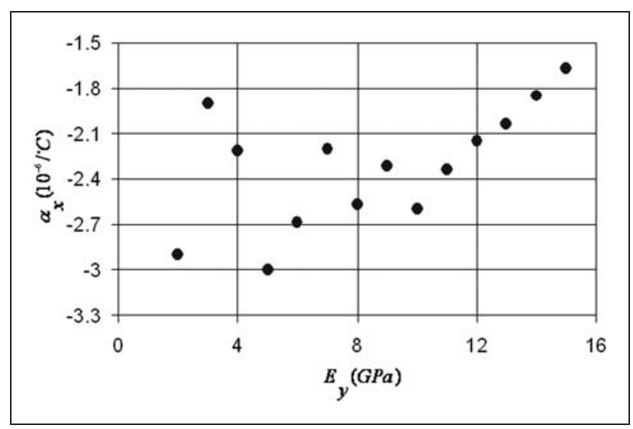

Figure 5. Pareto-optimal designs for maximum E_y and minimum α_x for model problem 2a.

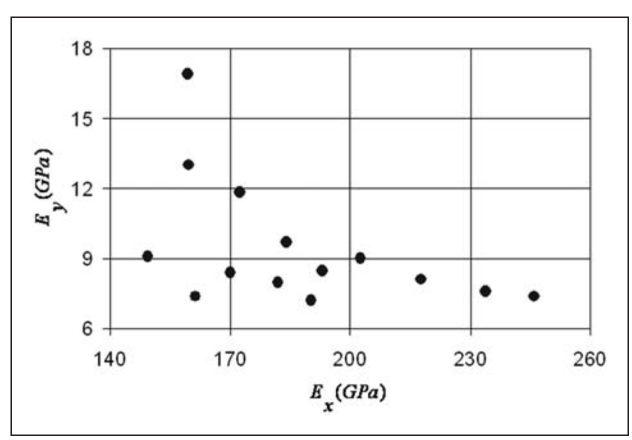

Figure 6. Pareto-optimal designs for maximum E_x and maximum E_y for model problem 2a.

Table 7. The set of (Pareto-optimal) solutions have been obtained when the maximum number of generations has reached to 51. For practical engineering use, only one of these solutions is to be chosen. For example, if one assumes $E_x \geq 188\,\text{GPa}$ and $\alpha_x \leq -2.63.10^{-6}/^{\circ}\text{C}$, design 5 is to be an appropriate solution and therefore the stacking sequence becomes $[\pm 13.8/\mp 18.5]_s$. Distribution of set of solutions are also given in Figure 3. Pareto-optimal design for model problem 2a is listed in Table 8. In model problem 2a, design 8 is to be chosen with the assumptions $E_x \geq 180\,GPa$, $E_y \geq 9.5\,GPa$, $\alpha_x \leq -2.30.10^{-6}/^{\circ}C$

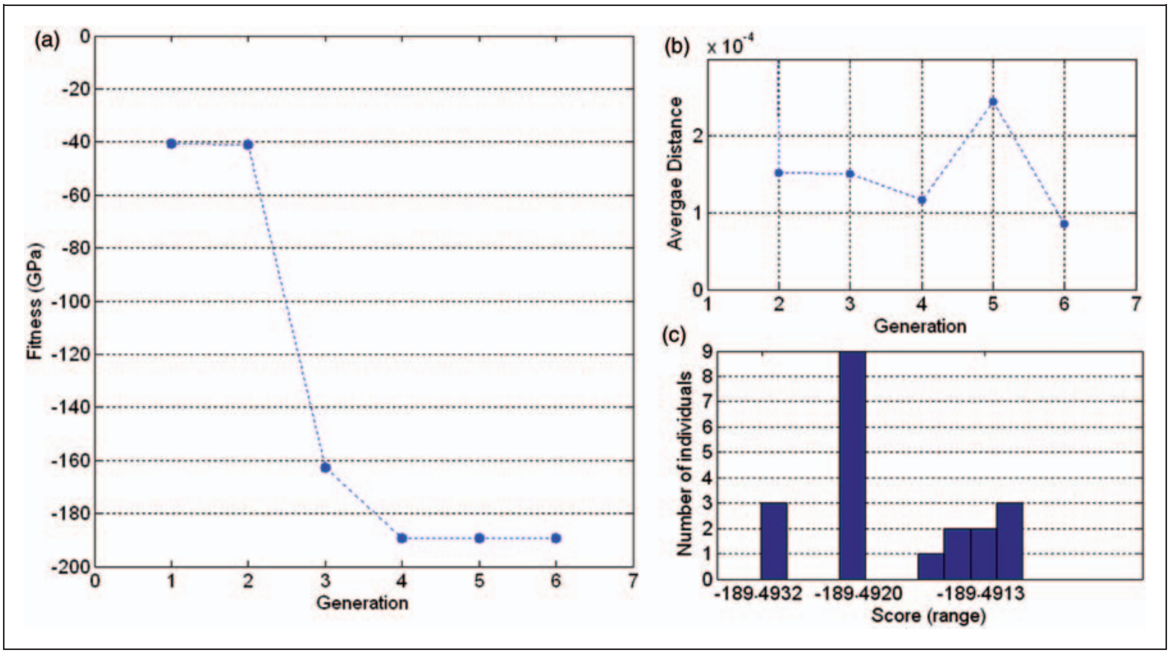

Figure 7. Single-objective genetic algorithm (GA) results for problem 1b; (a) evolution of the fitness function, (b) average distance between individuals, and (c) histogram for individuals.

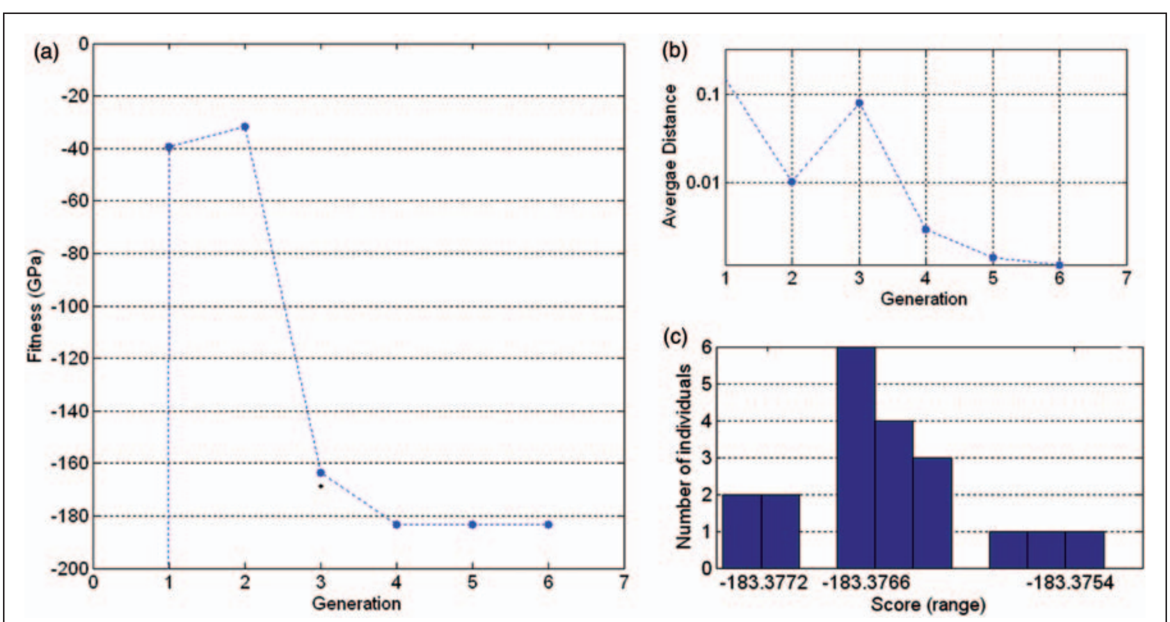

Figure 8. Single-objective genetic algorithm (GA) results for problem 2b; (a) evolution of the fitness function, (b) average distance between individuals, and (c) histogram for individuals.

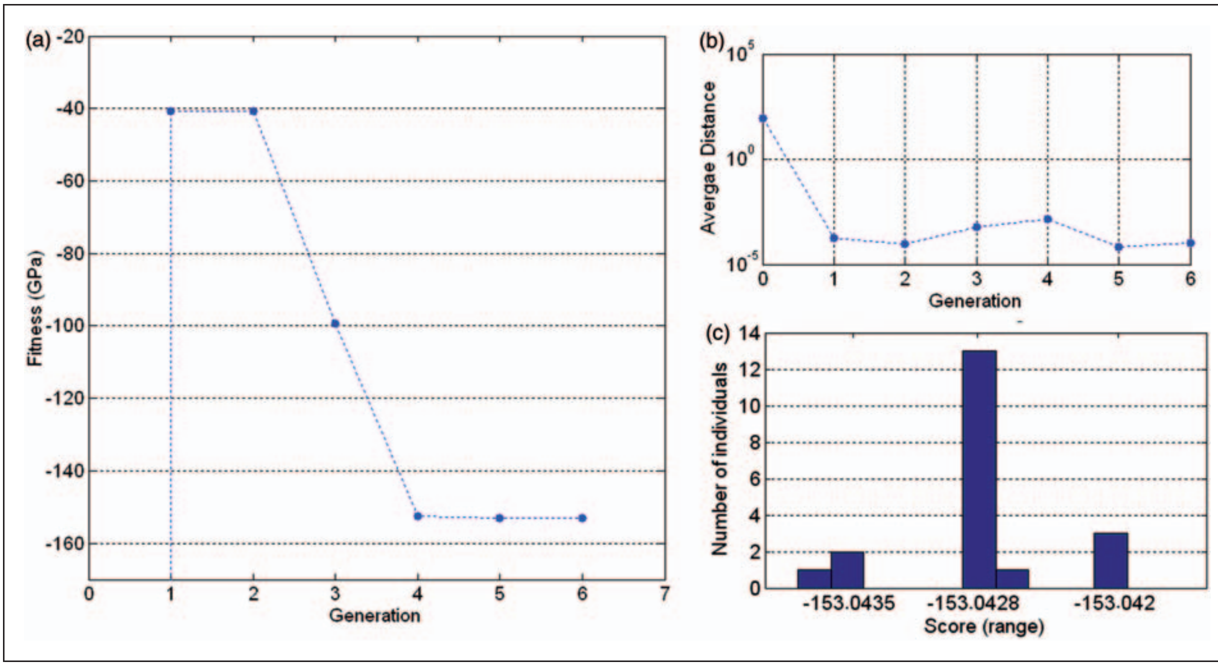

Figure 9. Single-objective genetic algorithm (GA) results for problem 3b; (a) evolution of the fitness function, (b) average distance between individuals, and (c) histogram for individuals.

and therefore, the corresponding stacking sequence for the composite becomes $[\pm 5.7/\pm 25.5]_s$.

Distributions of Pareto-optimal solution of problem 2a are illustrated in Figures 4–6 in the objective functions spaces, $E_x - \alpha_x, E_y - \alpha_x, E_x - E_y$, respectively. The Pareto-optimal curves enable us to perform trade-off studies. It is noted that design problems 1a–3a have been previously solved by Aydin and Artem.[12]

After obtaining the multi-objective GA results, we have performed single-objective optimization methods such as GA, GPSA, and SA. Single-objective GA results are given in Figures 7, 8, and 9 for problems 1b, 2b, and 3b, respectively. Evolutions of the fitness function value for those problems are illustrated in Figures 7a–9a. It is observed from the figures, this value converges after 4 generations as a result of relatively small feasible solution space obtained from the multi-objective solutions. Average distances between individuals for each generation are given in Figures 7b–9b. The fitness functions final values for problems 1b–3b has been supported by 9, 6, and 13 individuals, respectively, and given in Figures 7c–9c.

GPSA iteration steps for problems 1c–3c are illustrated in Figures 10–12, respectively. As it is seen from the figures, the optimum results are obtained after 4 iterations and decreasing mesh sizes are observed. In SA method (Figure 13), relatively much more iteration is obtained for the solution compared with GA and GPSA. Table 9 gives a comparison of the results obtained from multi-objective GA, single objective GA, GPSA, and SA. In model problems 1a-d, maximum E_x value is obtained from single objective GPSA and maximum corresponding E_y from multi-objective

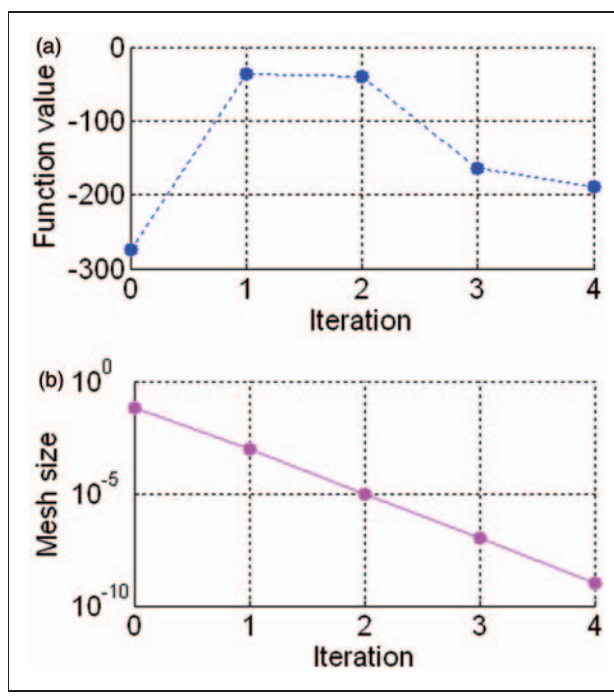

Figure 10. Single-objective generalized pattern search algorithm (GPSA) results for problem 1c; (a) iteration steps for fitness function value and (b) variation of mesh size.

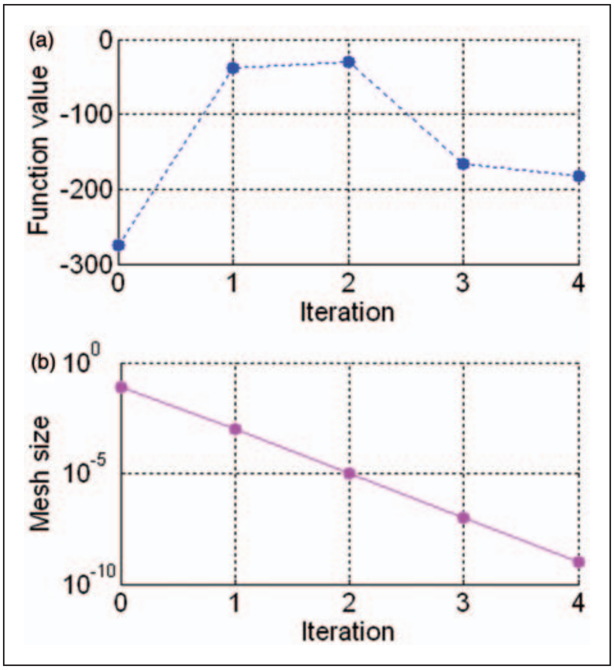

Figure 11. Single-objective generalized pattern search algorithm (GPSA) results for problem 2c; (a) iteration steps for fitness function value and (b) variation of mesh size.

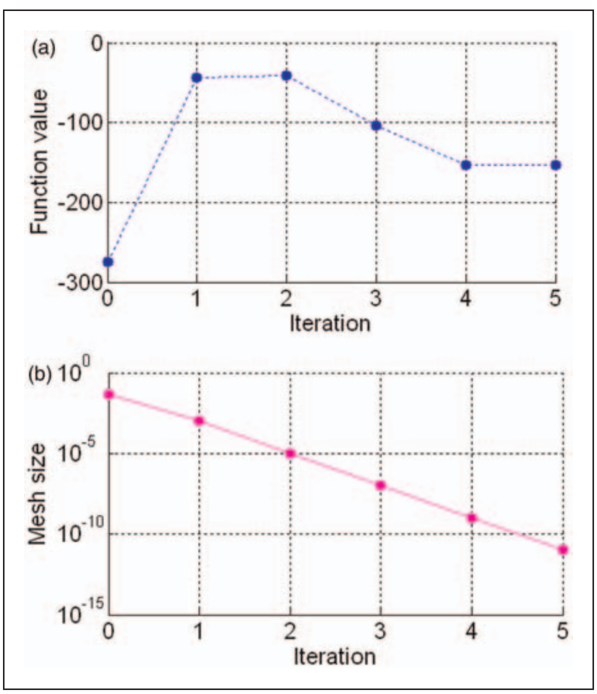

Figure 12. Single-objective generalized pattern search algorithm (GPSA) results for problem 3c; (a) iteration steps for fitness function value and (b) variation of mesh size.

GA. Another observation from the table is that, for problems 1b-d, 2b-d, and 3b-d, by considering maximization of E_x based on single-objective algorithms, the

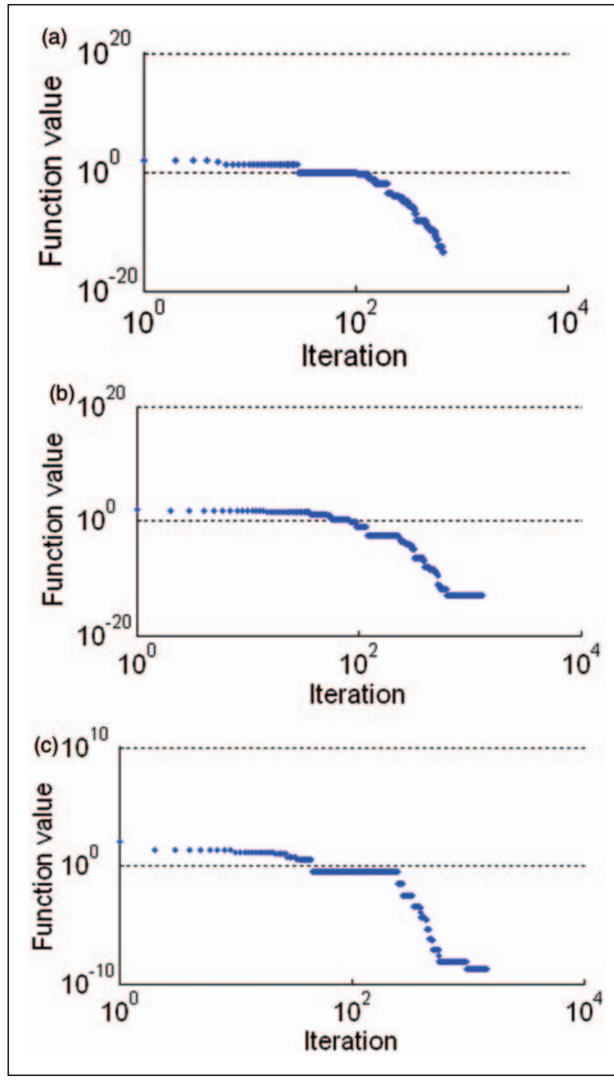

Figure 13. Single-objective simulated annealing (SA) algorithm iteration steps for (a) problem 1d, (b) problem 2d, and (c) problem 3d.

corresponding CTE values have been calculated using the stacking sequences and minimum values of them are obtained in case of SA algorithm.

Stress analysis

Investigation of stresses for optimized problems under combined loading gives some additional information about composite design. This type of information provides production of safer structures.[34] For this purpose, the through the thickness normal and shear stress distributions of the optimized (using multi-objective approach) composite plate under the mechanical, thermal and hygral loads are presented in Figures 14–17 for model problems 1a-3a. In Figure 14, it can be observed that maximum normal stresses occur in ply numbers

Table 9. Comparison of the results obtained from multi-objective genetic algorithm (GA), single-objective GA, generalized pattern search algorithm (GPSA), and simulated annealing (SA)

Problem	Optimization type	E_x (GPa)	E_y (GPa)	α_x (10^{-6}/°C)	β_x (10^{-6}/%M)	Stacking sequence
1a	Multi-objective GA[12]	187.90	7.3	−2.63	−29.88	$(\pm13.8/\pm18.5)_s$
1b	Single-objective GA	188.93	7.1	−2.64	−30.25	$(\pm16.1)_{2s}$
1c	Single-objective GPSA	189.54	7.1	−2.63	−29.99	$(\pm16/\pm16.1)_s$
1d	Single-objective SA	188.26	7.1	−2.65	−30.43	$(\pm17/\mp15.3)_s$
2a	Multi-objective GA[12]	184.22	9.6	−2.31	−20.96	$(\pm5.7/\pm25.5)_s$
2b	Single-objective GA	183.48	9.7	−2.31	−21.10	$(\pm5.8/\pm25.6)_s$
2c	Single-objective GPSA	183.48	9.7	−2.31	−21.20	$(\pm25.6/\pm5.8)_s$
2d	Single-objective SA	182.00	9.8	−2.32	−21.38	$(\pm6/\pm25.8)_s$
3a	Multi-objective GA[12]	152.77	7.2	−3.21	−46.08	$(\pm20/\pm18)_s$
3b	Single-objective GA	152.66	7.1	−3.22	−46.38	$(\pm19.3/\pm18.7)_s$
3c	Single-objective GPSA	152.65	7.1	−3.22	−46.41	$(\pm18.9/\pm19.1)_s$
3d	Single-objective SA	151.01	7.2	−3.23	−46.66	$(\pm17.8/\pm20.5)_s$

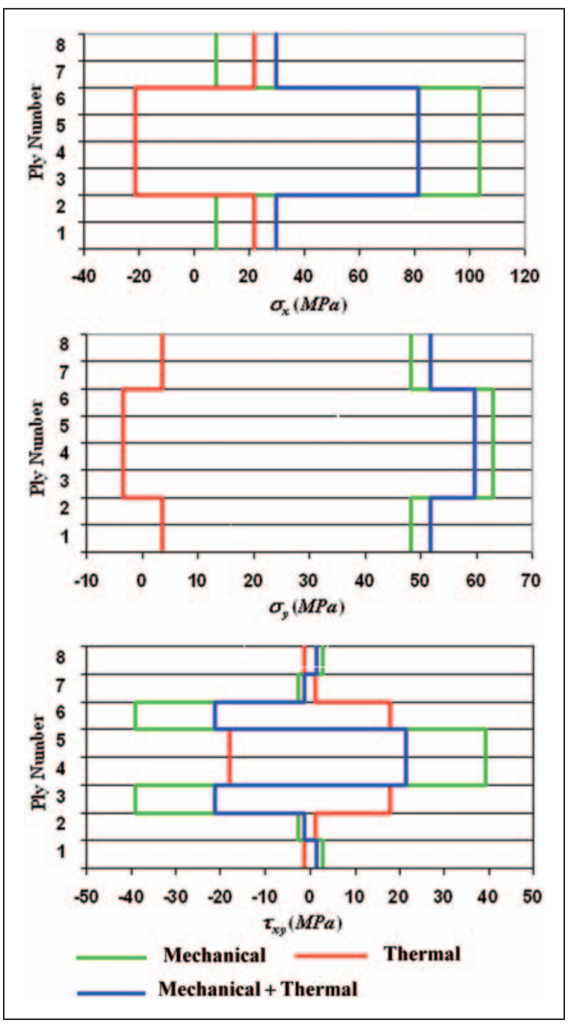

Figure 14. Stress distributions of the composite subjected to combination of mechanical and thermal loads for model problem 1a ($N_x = 20\,kN/m$, $N_y = 20\,kN/m$, $N_{xy} = 0\,kN/m$ $\Delta T = -150°C$).

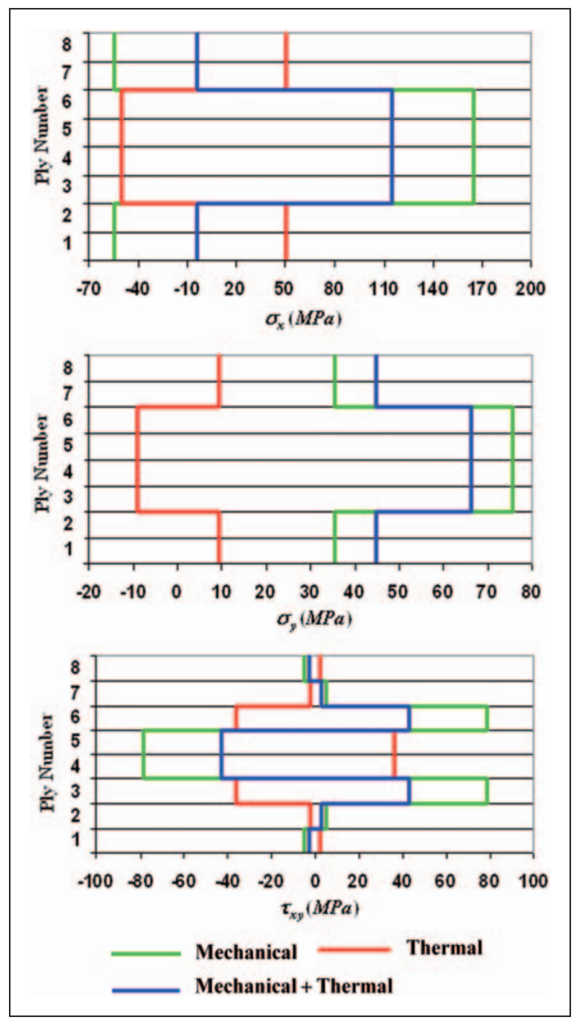

Figure 15. Stress distributions of the composite subjected to combination of mechanical and thermal loads for model problem 2a ($N_x = 20\,kN/m$, $N_y = 20\,kN/m$, $N_{xy} = 0\,kN/m$ $\Delta T = -150°C$).

3–6 and shear stress in 4 and 5 when the composite subjected to mechanical load. Applying only thermal load, relatively lower stress values are obtained for both normal and shear stresses. Combination of thermal and mechanical loading leads to decrease the effect of mechanical load and therefore this produces lower values of normal and shear stresses at the plies where the maximum stresses occur. As it is seen from Figure 15, stress distributions for model problem 2a show similar behavior to 1a. Effects of loadings on stress distributions are also presented for model problem 3a in Figures 16 and 17. It can be seen from Figure 16, mechanical load is significantly effective compared to hygral and thermal loads for normal stress σ_x. However, for σ_y, thermal load dominates the others. It would be so due to relatively low mechanical load along y direction. It can be concluded from Figure 16 that it is advantageous to absorb moisture. Unfortunately moisture degrades the strength of laminates.[34] Therefore, it is not really an advantage to use the stress-relieving tendencies of moisture absorption. The maximum values of normal stresses occur in the interior plies of the composite subjected to both mechanical and thermal loads (Figure 17). However,

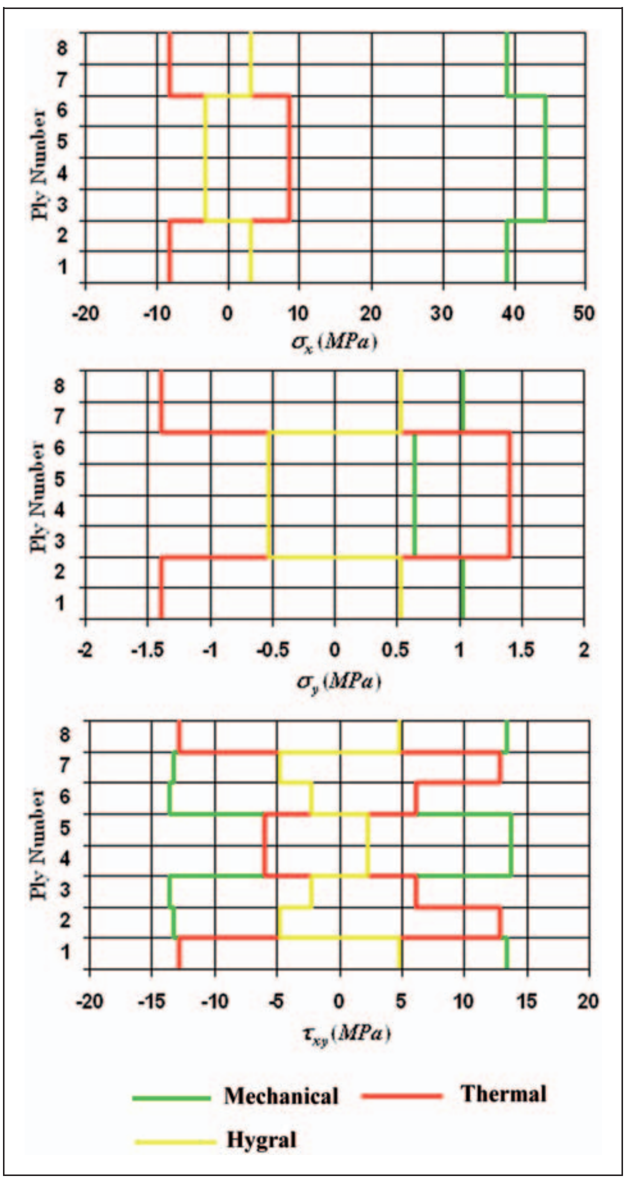

Figure 16. Stress distributions of the composite subjected to mechanical, thermal and hygral loads for problem 3a ($N_x = 50\ kN/m$, $N_y = 1kN/m$, $N_{xy} = 0$, $\Delta T = -150°C$, $\Delta M = 2\%$).

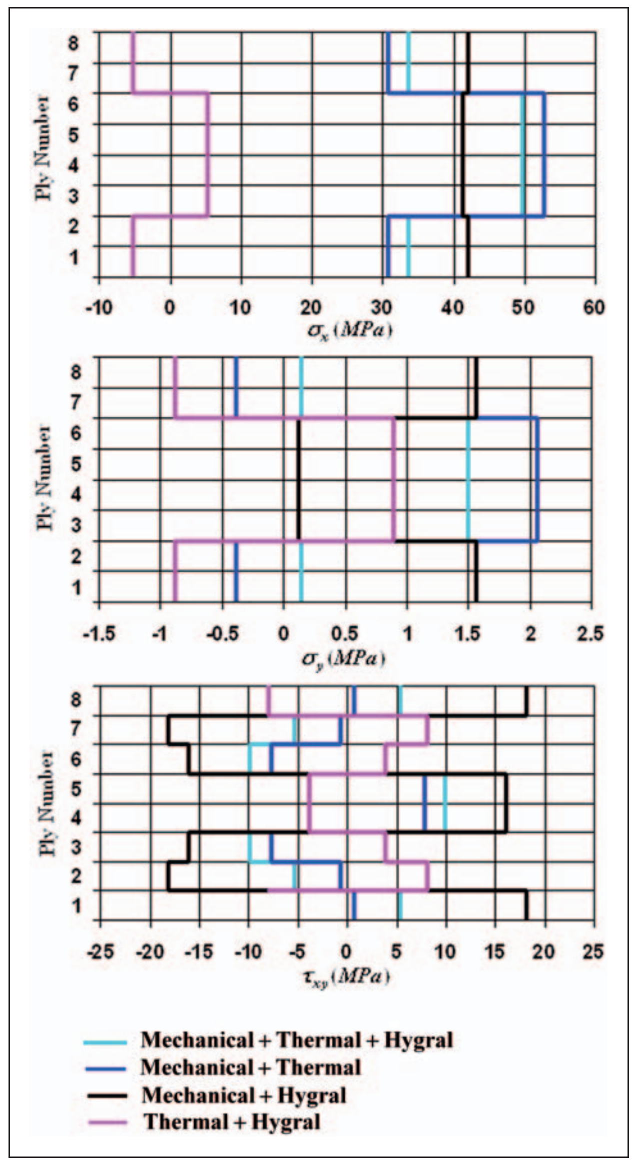

Figure 17. Stress distributions of the composite subjected to combination of mechanical, thermal and hygral loads for problem 3a ($N_x = 50\ kN/m$, $N_y = 1kN/m$, $N_{xy} = 0$, $\Delta T = -150°C$, $\Delta M = 2\%$).

negative values for thermal stresses leads to relatively low stresses in the exterior plies. Another observation is that for Figure 17, shear stress distribution is more complicated compared to the normal stress distributions.

Conclusion

In the present study, we have compared stochastic search optimization algorithms GA, GPSA, and SA for the laminated composites subjected to mechanical and hygrothermal loadings. Single- and multi-objective optimization approaches have been proposed to design the carbon fiber-reinforced epoxy-laminated composites. The fiber orientations of the composites are chosen as design variables. Simplified micromechanical equations, classical lamination theory, and MATLAB Symbolic Math Toolbox have been utilized in order to obtain objective functions and constraints. MATLAB Optimization Toolbox is used to solve the model problems. We have concluded with the following observations:

1. Even if the number of iterations of the algorithms GA, GPSA, and SA are quite different, the CPU times are approximately the same,
2. Since the constraints used in single objective approach significantly narrows the search space, we get the results with fewer number of iteration,
3. All the methods carried out in the present study have produced almost the same results with different stacking sequences,
4. Regarding mechanical analysis, shear and normal stress distributions of the optimized composite plates have been presented and results showed that effect of mechanical loads dominate to hygral and thermal loads.

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