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# On "A Flexible Window Function for Spectral Analysis"

In March 2010, a windowing function that is based on the well-known Butterworth filters was proposed [1] with the claim that it can solve "the tradeoff problem between the frequency resolution and spectral leakage in the estimated power spectral density, unavoidable with the conventional windows." That claim is a consequence of not considering the available time duration in the window design for which we provide an illustration in this comment correspondence.

The Butterworth window is defined as the truncated impulse response of a Butterworth filter constituted by the initial positive part of the impulse response from the beginning until the first zero crossing [1]. The shortcoming in the claim is that the proposed window does not maintain a constant time duration; however this constraint on the window duration is crucial in window design, and it is the reason for the mentioned classical tradeoff between spectral leakage and frequency resolution. After realizing this constant duration constraint, one can easily deduce that the proposed benefit of the Butterworth window in controlling the "3-dB bandwidth and sidelobe attenuation independently" disappears, and it becomes only a regular window design choice as long as the mainlobe-width versus sidelobe attenuation tradeoff allows.

As the article suggests, for some windows, such as Hanning or Hamming, the sidelobe attenuation is fixed. If the constraint on the window duration, or length, is removed, it will be possible to adjust the mainlobe width, or the frequency resolvability, of these windows by increasing the window duration. This means that the independently adjustable 3 dB-bandwidth of the Butterworth window proposed in [1] is nothing special. On the contrary, this property is shared by any conventional window function once they are granted a freely adjustable window length.

Considering a more common case, if we assume that the window duration is fixed, which is the case for power spectrum estimation of some time sequence, the ultimate limit of frequency resolution of these discrete Fourier transform-based estimation methods is set by the rectangular window and it corresponds to the smallest 3 dB-bandwidth of the mainlobe. This limit is approximately $1/NT$, where $N$ and $T$ denote the number of samples used to define the window and the sampling time, respectively. The resulting minimum sidelobe attenuation is 13.3 dB. Any attempt to increase sidelobe attenuation by a smoother window will increase the mainlobe width [2]. That is why the tabulated classical window functions specify the width of the mainlobe as a function of the window duration given by $N \times T$.

A modified Butterworth filter definition that corresponds to a fair comparison with other classical windows ought to have a constraint on the window duration. Since for a certain data the sampling frequency is also a noncontrollable parameter, the cutoff frequency of the Butterworth filter and the filter order must be determined together to have the target window length. A good strategy might be

- choosing the filter order N, which sets the sidelobe attenuation
- adjusting the cutoff frequency accordingly to make the positive initial part of the impulse response attain the desired window length.

In Table 1, we compared the frequency characteristics of this modified Butterworth window with some classical windows. Two of these windows, particularly Kaiser and Chebyshev, have free parameters that allow a tradeoff between sidelobe attenuation and frequency resolution just like the Butterworth window [3]–[5].

The sampling frequency, $f_s = 2{,}048$ Hz and the length of the windows are chosen to be comparable with the fourth-order Butterworth filter in Table 3 of [1]. The rectangular window has the smallest 3 dB-bandwidth that is in agreement with the normalized value in [3], 0.89, and can be found as

$$0.89 \times \frac{1}{NT} = 0.89 \times \frac{2{,}048\ \text{Hz}}{4{,}157} = 0.4385\ \text{Hz}.$$

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[TABLE 1] COMPARISON OF FREQUENCY CHARACTERISTICS ($f_s$ = 2,048 Hz).

| WINDOW | FREE PARAMETER | LENGTH (SAMPLES) | 3 dB BANDWIDTH | SIDELOBE ATTENUATION |
|---|---|---|---|---|
| RECTANGULAR | | 4,157 | 0.436 Hz | –13.3 dB |
| TRIANGULAR | | 4,157 | 0.627 Hz | –26.5 dB |
| HANNING | | 4,157 | 0.709 Hz | –31.5 dB |
| KAISER | $\alpha = 2$ | 4,157 | 0.488 Hz | –18.4 dB |
| | $\alpha = 4$ | 4,157 | 0.590 Hz | –30.0 dB |
| CHEBYSHEV | $\beta = 1$ | 4,157 | 0.440 Hz | –20.0 dB |
| | $\beta = 2$ | 4,157 | 0.590 Hz | –40.0 dB |
| BUTTERWORTH (ORDER=4) | $f_c = 0.439$ Hz | 4,157 | 0.685 Hz | –28.8 dB |
| BUTTERWORTH (ORDER=3) | $f_c = 0.439$ Hz | 3,655 | 0.742 Hz | –24.2 dB |
| | $f_c = 0.386$ Hz | 4,157 | 0.652 Hz | –24.2 dB |
| BUTTERWORTH (ORDER=2) | $f_c = 0.439$ Hz | 3,299 | 0.817 Hz | –18.2 dB |
| | $f_c = 0.348$ Hz | 4,157 | 0.649 Hz | –18.2 dB |

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been enabled by faster wireless networks, more capable hardware, and the appetite of consumers to search for information on the go. This special issue includes two articles on this topic. The first article by Feng et al. describes all of the technologies required for mobile voice search, including speech recognition, query understanding, search, and multimodal interfaces. The authors address the challenges involved, including high background noise, very large vocabularies in open Web search, error correction, and the integration of touch and speech. The second article by Seltzer et al. presents complete in-car multimedia interactive (infotainment) systems, which have the potential to significantly reduce driver distraction while enabling users to perform their desired tasks. The described systems exploit and integrate advanced spoken language-based media search and information retrieval. In this overview, the authors discuss the algorithms and methods used to perform speech recognition, spoken query information retrieval, and multimodal interaction.

There have been numerous advances in mobile media search in the areas of image and video processing. The article by Girod et al. reviews the state of the art in content-based image retrieval using photos taken by camera phones to initiate visual search queries about objects found in the mobile environment. Such applications can be used for identifying commercial products and finding information about movies, CDs, or products of the visual arts. In addition to the description of the overall system architecture, special attention was given to minimizing the size of the data sent over the network to reduce latency and improve user experience, and compressing the inverted index on the server to reduce the memory usage and processing latency.

Finally, the article by Schroth et al. extends the content-based image retrieval to the important application of location recognition and mapping. Video recordings on a mobile device are used as a visual fingerprint of the environment, which is matched to a georeferenced database to determine the location and pose of the mobile user. Such an estimation is used to supplement or refine existing global positioning system information and further facilitate location-based services. Video processing techniques are used to extract robust features from complex dynamic visual scenes. Methods for preloading relevant and informative database image features to the mobile client according to the user location and context are also presented.

## THANK YOU

We thank all the authors who submitted proposals to this special issue, particularly those with accepted articles for the hard work they put in that made this special issue possible. We also thank all the reviewers for providing valuable feedback and recommendations that have vastly improved the articles. Finally, we are also grateful to Li Deng and Dan Schonfeld for their support and guidance throughout the process of preparing and finalizing the articles. We hope you will enjoy this special issue.

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## [letter to EDITOR] continued from page 7

In Table 3 of [1], four of the listed Butterworth windows have smaller 3 dB-bandwidths compared to rectangular window. This is not possible if the window length is kept constant. Additionally, Kaiser window with $\alpha = 4$ and the Chebyshev window with $\beta = 2$ have both better sidelobe attenuation and smaller 3 dB-bandwidth when compared to fourth-order Butterworth filter of the same length. If one considers all of the Butterworth filters having the same window length, a tradeoff between 3 dB-bandwidth and sidelobe attenuation can be seen, just like the case with some conventional windows. In the table, we included other Butterworth filters of order two and three with the same cutoff frequency of 0.439 Hz. But the window lengths of these filters differ from the one of the fourth-order filter.

## CONCLUSION

A window derived from the Butterworth filter does allow a tradeoff between frequency resolution and sidelobe attenuation, but it has no special property to eliminate this tradeoff. Furthermore, some classical windows, such as the Kaiser and Chebyshev windows, can tackle that tradeoff problem more successfully.

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