

Filamentary structures of the cosmic web and the nonlinear Schrödinger type equation

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Abstract. We show that the filamentary type structures of the cosmic web can be modeled as solitonic waves by solving the reaction diffusion system which is the hydrodynamical analogous of the nonlinear Schrödinger type equation. We find the analytical solution of this system by applying the Hirota direct method which produces the dissipative soliton solutions to formulate the dynamical evolution of the nonlinear structure formation.

1. Introduction

The large scale structure of the Universe is marked by prominent filamentary features embedded within a weblike network, the cosmic web. Extensive N-body simulations are used to model this complex and intricate dynamical structure. However we need to find a full analytical formalism to understand this complex structure. Analytical methods for studying the evolution of cosmological density perturbations which cause this intricate structure formation are classified into two broad classes: the Eulerian and the Zel'dovich approximations. While the Eulerian approximation provides an accurate description of the gravitational instability in the linear regime, the Zel'dovich approximation is an exact solution of the fluid equations as long as particle trajectories do not cross. When the trajectories cross, the velocity field becomes multi valued and this causes non-singularities in the density field. The adhesion theory has been proposed to avoid this velocity singularity problem in the Zel'dovich approximation. In the adhesion approximation, when shell crossing occurs, the particles are assumed to stick to each other by introducing an artificial viscosity term in the Burger's equation. In the special case, when the viscosity term tends to zero, the geometrical interpretation of the solution of the Burger's equation can be used to determine the skeleton of the large scale structure. In this limit structures formed in the adhesion model are infinitely thin and the adhesion approximation reduces to the Zel'dovich approximation outside of mass concentration.

An alternative method to these analytical approaches is based on the idea suggested by Spiegel. He showed the correspondence between the fluid structure equations and the nonlinear wave equations [1] and in 1993 Widrow and Kaiser were the first to apply the Schrödinger representation to the problem of the cosmological structure formation for cold dark matter (CDM) [2]. They developed an advanced nonlinear numerical model known as the Schrödinger Method to follow the nonlinear evolution of the dark matter field by offering an alternative particle mesh code. This new numerical model described the matter as a Schrödinger field

obeying the coupled classical Schrödinger and Poisson equations. This code has been modified by Davis and Widrow [3]. Another extension of the Schrödinger Method has been done by Coles [4] in which he suggested that the nonlinear Schrödinger equation is a good candidate to model CDM but, in the same paper, he pointed out that this nonlinear equation is not easy to solve analytically. Coles and Spencer [5] demonstrated a wave mechanical approach to solve the caustic problem in the Zel'dovich approximation, when trajectories cross and this approach is similar to the adhesion approach except the pressure term. The pressure term in this new approach has the same effect as the viscosity term in the adhesion theory to avoid the singularities of the density field. By following this work, Short and Coles proposed a different approach to study the large scale structure formation based on a wave mechanical description of self gravitating CDM called free particle approximation [6]. They transformed the usual hydrodynamical equations of motion into the linear Schrödinger equation and they showed that the free particle approximation is useful into the mildly nonlinear regime and it has the same result as the adhesion approximation.

In this study, we derive fully analytical solution of the nonlinear wave equations (nonlinear Schrödinger type equation and the reaction diffusion equation) which are the fluid dynamical analogous of each other [7]. To do this, first we show that the cubic nonlinear Schrödinger type equation satisfies the cosmological fluid dynamical equations and by following the idea suggested by Coles and Spencer [5], we assume this fluid has the gas pressure that is related to the density via $p = K\chi^2$, where K is a positive constant. Then we derive a complete analytical solution of this nonlinear fluid by applying the Hirota direct method which produces the special soliton waves called dissipatons. To do this, we obtain the nonlinear Schrödinger equation from the fluid dynamical equation by applying the Madelung transformation and then we transform the nonlinear Schrödinger type equation into the reaction diffusion system which produces the soliton solutions by applying the Hirota direct method [8] to find the exact solution of the nonlinear wave equations. It means that the filamentary type structures of the cosmic web in the Einstein de Sitter (EdS) Universe can be modeled in a complete analytical way.

2. Fluid dynamical equations in the Einstein de Sitter Universe $\Omega(t) = 1$

To find the analytical solution of the nonlinear wave equation, we take our starting point as the scaled expressions of the Newtonian equations for a self gravitating perfect fluid in the comoving coordinate system. The continuity, the Bernoulli and the Poisson equations

$$\frac{\partial \chi}{\partial D} + \vec{\nabla}_x \cdot \chi \vec{\nabla}_x \phi_v = 0 \quad (1)$$

$$\frac{\partial v'}{\partial D} + v' \vec{\nabla}_x v' = -\vec{\nabla}_x \mathcal{V} - \frac{1}{\chi} \vec{\nabla}_x p \quad (2)$$

$$\nabla_x^2 \phi(x, t) = 4\pi G a^2 \rho_u \chi \quad (3)$$

where G is the gravitational constant and $a(t)$ is the scale factor. We express the equations of motion in terms of the density excess δ or density contrast $\chi(\vec{x}, t)$, the peculiar velocity $\vec{v}(\vec{x}, t)$ and the comoving gravitational potential $\phi(\vec{x}, t)$. The density contrast χ is defined by

$$\chi(\vec{x}, t) = 1 + \delta(\vec{x}, t) = \frac{\rho(\vec{x}, t)}{\rho_u(t)} \quad (4)$$

where $\rho_u(t)$ is the universal cosmic background density and D is the growth factor. In order to describe the evolution against the background of an expanding background, it is sensible

to describe the evolution in terms of the expansion factor $a(t)$ or, even more convenient and appropriate, in terms of the linear density growth factor $D(t)$ (See [5]). In the case of an Einstein-de Sitter Universe, we have

$$D(t) = a(t) \quad (5)$$

The other important term in the Bernoulli equation (1) the effective potential \mathcal{V} includes contributions from the velocity potential ϕ_v and the gravity potential ϕ ,

$$\mathcal{V} = \frac{3\Omega}{2f^2D} \left(\phi_v + \theta \right) \quad (6)$$

with the scaled gravity potential θ defined as

$$\theta \equiv \frac{2\phi}{3\Omega a^2 D H^2}. \quad (7)$$

We also introduce the comoving velocity potential ϕ_v for the scaled peculiar velocity \vec{v}'

$$\vec{v}' = \vec{\nabla}_x \phi_v \quad (8)$$

3. Relation between the nonlinear Schrödinger and fluid dynamical equations

The fluid equations (1) and (2) can be obtained by decomposing the nonlinear Schrödinger type equation into its real and imaginary parts by using a special transformation called Madelung transformation proposed by [9] in which the dark matter can be presented as complex scalar field. The form of this transformation is given by

$$\psi(\vec{x}, t) = \sqrt{\chi} \exp \left\{ i \frac{\phi_v}{\nu} \right\} \quad (9)$$

where ν is an adjustable parameter with dimension equal to that of the comoving velocity potential ϕ_v . The scaled density function χ in (9) satisfies the relation

$$\chi = \psi\psi^* = |\psi|^2, \quad |\psi|^2 \geq 0 \quad (10)$$

Note that this equality is the analogous of the probability density of the quantum mechanics. Since we use the nonlinear theory, $|\psi|^2$ gives the mass density of its constituent microscopic particles at that point [10] and it is positive definite $|\psi|^2 \geq 0$ but not essentially unity. This gives us the right to name the nonlinear wave equation as the nonlinear Schrödinger type equation and it is given by the following equation

$$i\nu \frac{\partial \psi}{\partial a} = -\frac{1}{2} \nu^2 \nabla_x^2 \psi + \mathcal{V} \psi + \kappa^2 |\psi|^2 \psi + \mathcal{P} \psi \quad (11)$$

where \mathcal{P} is defined as the regulation term given as

$$P = \frac{\nu^2 \nabla_r^2 |\psi|}{2 |\psi|} \quad (12)$$

The reason for substituting the regulation term P into the nonlinear Schrödinger type equation is to avoid the extra term called the quantum potential in the Euler equation. Equation (11) originally explains the dynamical evolution of the Bose-Einstein condensate in which large fraction of the bosons occupy the lowest quantum state of the external potential and all wave functions overlap each other where the quantum effect becomes apparent on a macroscopic scale. Here we will use this fact to model the macroscopic wave function of bosons ψ obeys the nonlinear Schrödinger equation. The nonlinear term $\kappa^2 |\psi|^2$ in (11) shows that the pressure term resulting from the inter particle interactions and κ is interpreted as the nonlinearity parameter which is a constant value. This nonlinear parameter and the constant parameter of the equation of state satisfies $\kappa = \sqrt{K}$.

4. Reaction diffusion system in the EdS Universe

We transform the nonlinear Schrödinger type wave equation into an equivalent reaction diffusion wave equation which is the fluid analogous of equation (11) [7]. To obtain the reaction diffusion analogous of equation (11) we define two real functions

$$Q^+(x, a) \equiv \sqrt{\chi} e^{\frac{\phi v}{\nu}}, \quad Q^-(x, a) \equiv \sqrt{\chi} e^{-\frac{\phi v}{\nu}} \quad (13)$$

These real functions and the wave function of equation (11) satisfy the following relations

$$-Q^+Q^- = \chi = \psi\psi^* = |\psi|^2 \quad (14)$$

so fluid equations (1) and (2) generated by (11) can be written as the reaction diffusion system which is the analogous of (11)

$$\begin{aligned} \nu \frac{\partial Q^{(+)}}{\partial a} + \frac{\nu^2}{2} \nabla_x^2 Q^{(+)} - \kappa^2 Q^{(+)} Q^{(-)} Q^{(+)} - \mathcal{V} Q^{(+)} &= 0 \\ -\nu \frac{\partial Q^{(-)}}{\partial a} + \frac{\nu^2}{2} \nabla_x^2 Q^{(-)} - \kappa^2 Q^{(-)} Q^{(+)} Q^{(-)} - \mathcal{V} Q^{(-)} &= 0 \end{aligned} \quad (15)$$

Here we represent the decoupling reaction diffusion system. The second equation in system (15) is the time reversible of the first reaction diffusion equation and it is crucial for the existence of the Hamiltonian structure and integrable system [7, 11]. In this study we do not take into account the time reversible reaction diffusion equation as we do not use the negative time values in order to have physical consistency in the large scale structure. We consider the full nonlinear Schrödinger Poisson system in which the evolution of CDM can be fully described for the gravitational field in terms of the cosmological model. Jones suggested an analytical model for nonlinear clustering in which he showed that the baryonic matter driven by CDM potential can be modeled in terms of the heat equation [12] and we know that the heat equation is the particular case of the reaction diffusion equation. It means that we can model the evolution of baryonic matter by solving (15) whose solution automatically gives the solution of (11) thanks to relation (14). It is interesting to note that in the linear regime the effective potential \mathcal{V} is equal to zero. In fact, the conclusion of $\mathcal{V} = 0$ stretches out much further into the quasi-linear regime, for as long the Zel'dovich formalism still describes the motion of matter elements in the Universe. Hence the first equation in system (15) is reduced to

$$\nu \frac{\partial Q^{(+)}}{\partial a} + \frac{\nu^2}{2} \nabla_x^2 Q^{(+)} - \kappa^2 Q^{(+)} Q^{(-)} Q^{(+)} = 0 \quad (16)$$

5. Hirota bilinear form

Our approach is to find the exact solution of (16) by following the method developed by Hirota [8]. When we apply the Hirota direct method to the reaction diffusion equation, the soliton solutions are obtained. The crucial step of the Hirota method is a change of variables which converts the equation of motion to an equation of Hirota bilinear type. The appropriate change of variables is not obvious; fortunately we are guided by the change of variables which is used for the nonlinear Schrödinger equation.

We analyze equation (16) and construct one- and two- soliton solutions. To obtain the solution, we applied the following steps: First we make the bilinearization by applying the suitable transformations of (16) which is

$$Q^{(+)} = \frac{1}{\sqrt{2}} \frac{\nu g^+}{\kappa f}, \quad Q^{(-)} = \frac{1}{\sqrt{2}} \frac{\nu g^-}{\kappa f} \quad (17)$$

where $g^\pm(x, a)$ and $f(x, a)$ are the new differentiable functions which help us to write the reaction diffusion equation as a combination of bilinear equation. After substituting the derivatives of the new functions $Q^{(+)}(x, a)$ in terms of the independent variables into equation (16)

$$\begin{aligned} \frac{\partial Q^{(+)}}{\partial a} &= \frac{1}{\sqrt{2}} \frac{\nu}{\kappa} \left[\frac{D_a(g^+ \cdot f)}{f^2} \right] \\ \nabla_x^2 Q^{(+)} &= \frac{1}{\sqrt{2}} \frac{\nu}{\kappa} \left[\frac{D_x^2(g^+ \cdot f)}{f^2} - \frac{g^+ D_x^2(f \cdot f)}{f^2} \right] \end{aligned}$$

where D is called the Hirota D-operator or the Hirota derivative which is defined as

$$\begin{aligned} D_x^n(g^\pm \cdot f) &= \sum_{l=0}^n (-1)^l \binom{n}{l} g_{(n-l)x}^\pm f_l \\ &= g_{nx}^\pm f - n g_{(n-1)x}^\pm f_x + \dots + (-1)^n g f_{nx} \end{aligned} \quad (18)$$

where the subscripts indicate the partial differentiation and dot (.) is multiplication symbol. Hence equation (16) can be expressed as the following pair of the Hirota bilinear equations

$$\begin{aligned} \left(\nu D_a + \frac{\nu}{2} D_x^2 \right) (g^+ \cdot f) &= 0 \\ D_x^2 (f \cdot f) &= -g^+ g^- \end{aligned} \quad (19)$$

6. Perturbation analysis of the soliton solutions

To find the solution of equation (16) in terms of one-, two- ...etc soliton waves the Hirota perturbation method has been applied. To do this, we use two different differential functions defined in transformation (17) g and f . Let us define them as

$$g^+ = \epsilon^0 g_0^+ + \epsilon^1 g_1^+ + \epsilon^2 g_2^+ + \dots \quad (20)$$

and

$$f = \epsilon^0 f_0 + \epsilon^1 f_1 + \epsilon^2 f_2 + \dots \quad (21)$$

where g_0^+ and f_0 are non zero constants and $f_i, g_i^+, i = 1, 2, \dots, N$ are exponential functions. We insert f and g into the Hirota bilinear form (19) of equation (16).

6.1. One soliton solution

First we can take $f = 1 + \epsilon^2 f_1$ and $g = \epsilon g_1^\pm$ where $f_i = g_i = 0$ for all $i \geq 2$ and they are exponential functions. For a nontrivial solution, f_0 and g_0^\pm should not vanish at the same time. At first let us examine the Hirota perturbation on the bilinear system and make the coefficients identically zero. The coefficient of ϵ 's gives us

$$\epsilon^0 : D_x^2(1 \cdot 1) = 0 \quad (22)$$

$$\epsilon^1 : \left(\nu D_a + \frac{\nu^2}{2} D_x^2 \right) (g_1^+ \cdot 1) = 0 \quad (23)$$

$$\epsilon^2 : 2D_x^2(1 \cdot f_1) = -g_1^+ g_1^- \quad (24)$$

$$\epsilon^3 : \left(\nu D_a + \frac{\nu^2}{2} D_x^2 \right) (g_1^+ \cdot f_1) = 0 \quad (25)$$

$$\epsilon^4 : D_x^2(f_1 \cdot f_1) = 0 \quad (26)$$

and then by the coefficient of ϵ^1 , we can easily show that $g_1^\pm = \pm e^{\eta_1^\pm}$ satisfies the differential form and here we choose the form of the function η_1 as $\eta_1^\pm = p_1^\pm x \pm \Omega_1^\pm t + \eta_0^\pm$. Hence again using the same form we find the following relations between the parameters of the η_1^\pm which is

$$\Omega_1^\pm = -\frac{\nu}{2} p_1^{\pm 2} \quad (27)$$

then $\eta_1^\pm = p_1^\pm x \mp \frac{\nu}{2} p_1^{\pm 2} t + \eta_{10}^\pm$ so we get the solution as

$$g_1^\pm = \pm e^{\eta_1^\pm}, \quad f = 1 + e^{\eta_1^+ + \eta_1^- + \gamma} \quad (28)$$

where γ is defined in the following form

$$\gamma \equiv \ln \frac{1}{2} \frac{1}{(p_1^+ + p_1^-)^2}$$

Then the solution of the RD system has the form

$$Q^+(x, a) = \frac{1}{\sqrt{2}} \frac{\nu}{\kappa} \frac{e^{\eta_1^+}}{1 + e^{\eta_1^+ + \eta_1^- + \gamma}}, \quad Q^-(x, a) = -\frac{1}{\sqrt{2}} \frac{\nu}{\kappa} \frac{e^{\eta_1^-}}{1 + e^{\eta_1^- + \eta_1^+ + \gamma}} \quad (29)$$

This solution admits the exponentially growing and decaying components known as dissipation. It is useful to note that adiabatic parameter κ defines two different families of solutions of system (15) depending on the sign of κ . For $\kappa > 0$, the solution of equation (16) is given by a bright soliton, corresponding to a local maximum of the scaled density distribution $\chi = |\psi|^2$. For $\kappa < 0$, the solution of reaction diffusion (16) is given by a dark soliton, corresponding to a local minimum in the scaled density distribution. Here we investigate the bright soliton solution to obtain the density function of the local maxima represented as filaments. Hence the scaled density function can be written as

$$-Q^+(x, a)Q^-(x, a) = \psi\psi^* = |\psi|^2 = \frac{1}{2} \frac{\nu^2}{\kappa^2} \frac{e^{\eta_1^+ + \eta_1^-}}{(1 + e^{\eta_1^+ + \eta_1^- + \gamma})^2} \quad (30)$$

which is equal to the density contrast

$$\chi = \frac{\nu^2}{\kappa^2} \frac{k^2}{\cosh^2 \left[kx + \frac{\nu}{2} vkt + \frac{\eta_0^+ + \eta_0^-}{2} + \frac{1}{2} \gamma \right]} \quad (31)$$

where k is represented as wave number and v is the velocity of the dissipative soliton

$$k \equiv \frac{p_1^+ + p_1^-}{2}, \quad v \equiv p_1^- - p_1^+$$

The scaled density function shows the perfect soliton wave shape presented as filamentary type structure. When we change the parameters p_1^+ and p_1^- we can see the different type of solitonic wave type structures which bear a striking geometric resemblance to the filaments of the cosmic web. One soliton solution provides the information of the distribution of the scaled density in terms of the expanding scale factor $a(t)$, in other words, it shows the evolution of the scaled density function in the EdS Universe in Figure 6.1.

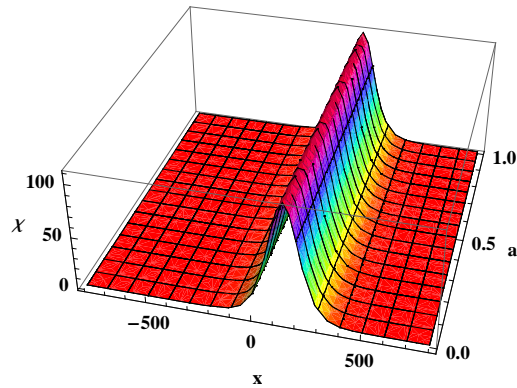


Figure 1. 3D plot of the scaled density function χ represented by one soliton solution of the reaction diffusion equation in terms of $\kappa = 1/\sqrt{2}$ with parameters $p_1^- = -0.1$ and $p_1^+ = 0.121$.

6.2. Two soliton solution

We give the analytical expression of two soliton solution of the RD system. Hence to find two-soliton solution, we take $f = 1 + \epsilon^2 f_1 + \epsilon^4 f_2$ and $g = \epsilon g_1 + \epsilon^3 g_2$ where $\eta_j^\pm = p_j^\pm x \mp \frac{\nu}{2} p_j^{\pm 2} t + \eta_{j0}^\pm$, $j = 1, 2$. Note that here $f_j = g_j = 0$ for all $j \geq 3$. At first we substitute f and g into the equation (19) and we try to make the coefficients of ϵ^m , $m = 0, \dots, 8$ to vanish. By following the Hirota method we obtain the two soliton solution

$$\epsilon^0 : D_x^2(1.1) = 0 \quad (32)$$

$$\epsilon^1 : \left(\nu D_a + \frac{\nu^2}{2} D_x^2 \right) (g_1^+ . 1) = 0 \quad (33)$$

$$\epsilon^2 : 2D_x^2(1.f_1) = -g_1^+ g_1^- \quad (34)$$

$$\epsilon^3 : \left(\nu D_a + \frac{\nu^2}{2} D_x^2 \right) (g_1^+ . f_1 + g_2^+) = 0 \quad (35)$$

$$\epsilon^4 : D_x^2(2f_2 + f_1.f_1) = -(g_1^+ g_2^- + g_2^+ g_1^-) \quad (36)$$

$$\epsilon^5 : \left(\nu D_a + \frac{\nu^2}{2} D_x^2 \right) (g_1^+ . f_2 + g_2^+ . f_1) = 0 \quad (37)$$

$$\epsilon^6 : D_x^2(f_1 f_2 + f_2 . f_1) = -g_2^+ g_2^- \quad (38)$$

$$\epsilon^7 : \left(\nu D_a + \frac{\nu^2}{2} D_x^2 \right) (g_2^+ . f_2) = 0 \quad (39)$$

$$\epsilon^8 : D_x^2(f_2 f_2) = 0 \quad (40)$$

By following the same way as in the one soliton solution, we obtain g

$$g^\pm = \pm [e^{\eta_1^\pm} + e^{\eta_2^\pm} + e^{\eta_1^+ + \eta_1^- + \eta_2^\pm + \alpha_1} + e^{\eta_1^\pm + \eta_2^+ + \eta_2^- + \alpha_2}] \quad (41)$$

where parameters α_1 and α_2 are defined as

$$\alpha_1 \equiv \ln \frac{1}{2} \frac{(p_2^+ - p_1^+)^2}{(p_1^+ + p_1^-)^2 (p_2^+ + p_1^-)^2}$$

$$\alpha_2 \equiv \ln \frac{1}{2} \frac{(p_2^+ - p_1^+)^2}{(p_1^+ + p_2^+)^2 (p_2^+ + p_2^-)^2} \quad (42)$$

and the general form of function f is

$$f = 1 + e^{\eta_1^+ + \eta_1^- + \gamma_{11}} + e^{\eta_2^+ + \eta_1^- + \gamma_{21}} + e^{\eta_1^+ + \eta_2^- + \gamma_{12}} + e^{\eta_2^+ + \eta_2^- + \gamma_{22}} + e^{\eta_1^+ + \eta_1^- + \eta_2^+ + \eta_2^- + \xi} \quad (43)$$

where the parameters are defined as

$$\begin{aligned} \gamma_{11} &\equiv \ln \frac{1}{2} \frac{1}{(p_1^+ + p_1^-)^2} \\ \gamma_{12} &\equiv \ln \frac{1}{2} \frac{1}{(p_1^+ + p_2^-)^2} \\ \gamma_{21} &\equiv \ln \frac{1}{2} \frac{1}{(p_2^+ + p_1^-)^2} \\ \gamma_{22} &\equiv \ln \frac{1}{2} \frac{1}{(p_2^+ + p_2^-)^2} \\ \xi &\equiv \ln \frac{1}{4} \frac{|p_2^+ - p_1^+|^4}{(p_2^- + p_2^+)^2 (p_1^+ + p_1^-)^2 |p_1^+ + p_2^-|^4} \end{aligned}$$

Then the scaled density function in terms of the two soliton solution (6.2) can be written as by applying (41) and (43) into the relation (14)

$$\chi = \frac{1}{2} \frac{\nu^2 g^+ g^-}{\kappa^2 f^2} \quad (44)$$

7. Conclusion

In this study we show a methodology for dealing with the nonlinear Schrödinger type equation and the reaction diffusion equation which are derived from the cosmological fluid equations. This special methodology is called the Hirota direct method and by applying this method to the nonlinear Schrödinger or its hydrodynamical analogous the reaction diffusion equation, we obtain the analytical solution of these nonlinear systems. Due to the nature of the Hirota method which is based on the perturbation approximation, the solution of the reaction diffusion equation and the nonlinear Schrödinger type equation produce the dissipative soliton type waves called dissipatons. When we increase the order of the perturbations in the Hirota method from one-soliton solution to N- soliton solutions, these waves show striking similarity to the intricate structure of filamentary type features of the cosmic web in 2 + 1 dimension. Apart from the filamentary structures represented as local maxima of the density field called bright solitons depending on the positive nonlinear term κ , we can construct the structures of the local minima called dark solitons and these dark solitons can be represented as empty negative density regions called voids around the filaments.

It is necessary to note that in this study we have focused on the detailed calculations of one soliton solution and added the graphics for two-soliton solutions because of the complexity of the calculations. We will give the technical details of the higher order solutions in the future paper.

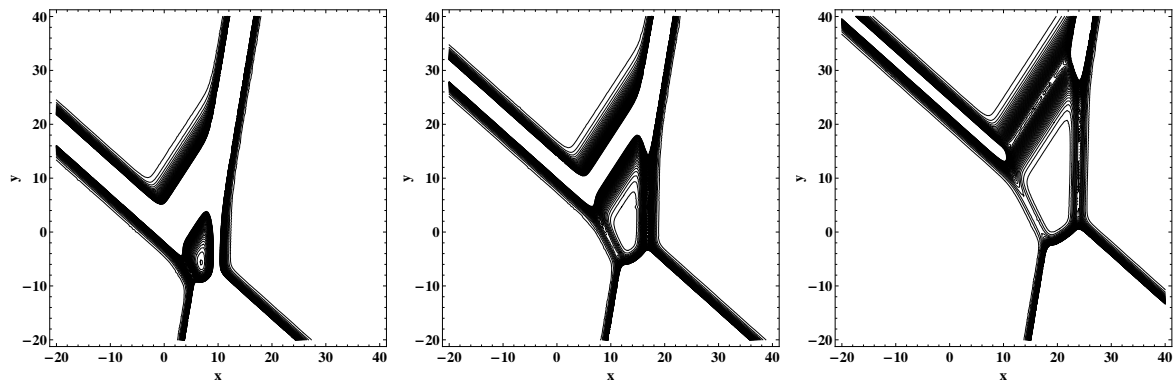


Figure 2. Contour plots of the scaled density or the density contrast function for two soliton solution of the reaction diffusion equation at the red shift values $z = 0.1, 0.05, 0$ in the phase space (from left to right). We can easily see that the scaled density of the filaments increase with respect to time evolution which means that the matter merges through the bridges into the filaments. In the late time steps, these matter bridges disappear because of the merging of the matter into the high density regions/lumps.

References

- [1] Spiegel E A 1980 *Physica* **1D** 236
- [2] Widrow L M and Kaiser N 1993 *Astrophys. J.* **416** L71
- [3] Davis G and Widrow L M 1997 *Astrophys. J.* **485** 484
- [4] Coles P 2002 *Mon. Not. R. Astron. Soc.* **330** 421
- [5] Coles P and Spencer K 2003 *Mon. Not. R. Astron. Soc.* **342** 176
- [6] Short C J and Coles P 2006 *Journal of Cosmological Astroparticle Physics* **12** 12
- [7] Pashaev O K and Lee J H 1998 *Journal of Mathematical Physics* **39** 102
- [8] Hirota R 1971 *Phys Rev. Letters* **27** 1192
- [9] Madelung E 1926 *Zeits. F. Phys.* **40** 3325
- [10] Johnston R, Lasenby A N and Hobson M P 2010 *Mon. Not. R. Astron. Soc.* **402** 2491
- [11] Lee J H and Pashaev O K 2002 *Modern Physics Letters A* **17** 1601
- [12] Jones B J T 1999 *Mon. Not. R. Astron. Soc.* **307** 376