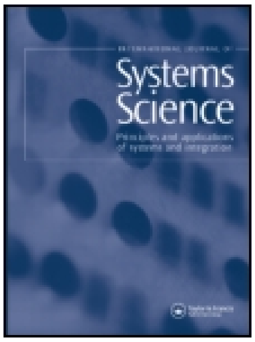

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A novel online adaptive time delay identification technique

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Time delay is a phenomenon which is common in signal processing, communication, control applications, etc. The special feature of time delay that makes it attractive is that it is a commonly faced problem in many systems. A literature search on time-delay identification highlights the fact that most studies focused on numerical solutions. In this study, a novel online adaptive time-delay identification technique is proposed. This technique is based on an adaptive update law through a minimum–maximum strategy which is firstly applied to time-delay identification. In the design of the adaptive identification law, Lyapunov-based stability analysis techniques are utilised. Several numerical simulations were conducted with Matlab/Simulink to evaluate the performance of the proposed technique. It is numerically demonstrated that the proposed technique works efficiently in identifying both constant and disturbed time delays, and is also robust to measurement noise.

Keywords: adaptive identification; time delay; signal processing

1. Introduction

Time delay is a widely used phenomenon in dynamical systems in a wide variety of disciplines such as chemistry, biology, communications, mechanics, control, and signal processing applications. In dynamical systems, time delay may have negative effects such as instability and/or reduction in performance. On the other hand, its accurate identification is crucial for several signal processing applications such as distance measurement and localisation systems (Brennan, Gao, & Joseph, 2007; Gao, Brennan, & Joseph, 2009; Giraudet & Glotin, 2006; Lui, Chan, & So, 2009; Martin et al., 2002). Due to its implementation in several disciplines and to overcome its negative effects, a significant amount of research was devoted to the time-delay phenomenon, its effects on systems, and identification and control methods. A broad overview on time delay, and its effects on systems and open problems may be found in Richard (2003).

Identification of time delay in systems is an important research area and several techniques and algorithms are available. The proposed approaches in the literature usually use least-squares algorithms (Bai & Chyung, 1993; Tuch, Feuer, & Palmor, 1994), gradient algorithms, correlation analysis, filter-based techniques, or stochastic-approximation techniques (Banyasz & Keviczky, 1994; Zhou & Frank, 2000). Tugnait (1996) proposed an adaptive frequency domain filter based on high-order statistics for a class of error-in-variable models and applied this method to time-delay identification. Zhou and Frank (2000) developed an approach based on a modified tracking filter for time-delay identification for a class of nonlinear autoregressive processes with exogenous inputs. Tuch et al. (1994) also considered least squares for time-delay identification. However, the stability analysis led to some strict conditions that must be satisfied. In Diop, Kolmanovsky, Moraal, and van Nieuwstadt (2001), a system with time-delayed input was considered and the least-squares method presented in Tuch et al. (1994) was utilised to identify the time delay. However, the identification method necessitated the consideration of a strict assumption which obstruct zero crossing of the derivative of the input signal. Furthermore, while the stability analysis yielded exponential stability, however, as indicated in the paper, in implementation, ultimate convergence can be provided as opposed to exponential convergence.

So (2002) presented an unbiased impulse response estimation approach for time-delay identification between signals received at two spatially separated sensors. Zhang and Li (2006) analysed the time-varying communication delay and proposed a time-delay identification method based on the steepest descent algorithm. But, the stability analysis of the method relies on the system being linear, and when the system is nonlinear, the stability of the method should be investigated for concave and convex cases separately, and in convex case, the method fails to ensure convergence. Wen et al. proposed an adaptive structure to address time-delay identification in noisy environment and developed a stochastic-gradient algorithm to calculate the optimum solution (Wen, Li, & Wen, 2007). Shaltaf presented a neuro-fuzzy technique for identification of time delay embedded within a received noisy and delayed replica of a

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known reference signal (Shaltaf, 2007). Sharma and Joshi (2007) proposed a number of estimators-based fractional Fourier transform for time-delay identification. Harva and Raychaudhury (2008) proposed a Bayesian approach for identifying time delay between signals that are irregularly sampled. Bhardwaj and Nath (2010) proposed a maximum likelihood identifier for time-delay identification. Bastard et al. presented a modified version of the estimation of signal parameters via rotational invariance techniques algorithm which takes both transmitted pulse shape and any noise into account, to identify time delay in backscattered radar signals (Bastard, Baltazart, & Wang, 2010). Another commonly utilised tool is the cross-correlation method. In this method, basically, the measured delayed signal is correlated with an array of signals which have different delays and their regressions are compared. Unfortunately, such methods cannot be run online. A good categorisation and comparison of time-delay identification methods may be found in Bjorklund and Ljung (2003).

Literature review highlighted the fact that most of the available time-delay estimation methods are for linear or linearised system models and they are either inadequate for nonlinear systems or they impose strict constraints. On the other hand, some methods cannot be used online due to their nature. Review of the relevant literature yielded the fact that time delay is a nonlinear parameter affecting the systems (nonlinearly) and thus nonlinear parameter identification techniques should be proposed for time-delay identification. In this study,[1] a novel online adaptive time-delay identification technique is presented to identify constant time delays for the systems of the form in (1). The technique is novel in the sense that a nonlinear-adaptive identification technique is firstly adopted as a time-delay identification method. The proposed time-delay identification algorithm is based on a minimum–maximum optimisation algorithm. It is mathematically proven that the developed estimator identifies unknown time delays upon satisfaction of a nonlinear persistent excitation condition. In the design of the adaptive identification law, Lyapunov-based stability analysis techniques were utilised. As a consequence of Lyapunov-based methods, the identification algorithm is robust to noise, variations, and time delay, and also compensates for some unmodelled nonlinearities (Dydek, Annaswamy, & Lavretsky, 2010). The robustness of the technique to noise and disturbed delay was demonstrated by extensive numerical simulation results.

2. Plant model

The general model considered in this paper is of the following form:

$$\begin{aligned} q(\tau,\Pi) = {} & a_{10}\Psi_1(t) + a_{11}\Psi_1(t-\tau_1) + \cdots + a_{1n}\Psi_1(t-\tau_n) \\ & + \cdots + a_{m0}\Psi_m(t) + a_{m1}\Psi_m(t-\tau_1) \\ & + \cdots + a_{mn}\Psi_m(t-\tau_n) \end{aligned} \tag{1}$$

which can be rewritten as

$$q(\tau,\Pi) = \sum_{i=1}^{m} a_{i0}\Psi_i(t) + \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\Psi_i(t-\tau_j), \tag{2}$$

where $q(\cdot) \in \mathbb{R}$ is a measurable signal, $\Pi(\cdot)$ is a measurable function including known and measurable parameters, $\Psi_i(t) \in \mathbb{R}$, $i = 1, \ldots, m$, are arbitrary chosen functions, a's are attenuation factors and $\tau = [\tau_1 \ldots \tau_n]^T \in \mathbb{R}^n$, where τ_j, $j = 1, 2, \ldots, n$, denote time delays. We assume that only the time delays are unknown and all the remaining parameters are known.

The signal $q(\cdot)$ in (1) consists of a sum of different signals and their time-delayed forms as a general description of delayed systems. A delayed signal received by a sensor can be given as a simplest example to this kind of systems. This kind of systems are usually used as localisation or distance measurement applications. Meanwhile, the received signal can be an emitted signal itself and its delayed forms.

The estimation of leak location in water distribution pipes can be given as an interesting real-world example to such kind of systems (Wen, Wen, & Li, 2008). In this system, the purpose is to estimate the location of the leak in a water distribution pipe by estimating the time delay in the acoustic signals travelling inside the pipe measured by receivers at points A and B, which is depicted in Figure 1. The system can be modelled as (Wen et al., 2008)

$$\begin{aligned} x_A(t) &= s(t), \\ x_B(t) &= \delta s(t-\tau), \end{aligned} \tag{3}$$

where x_A and x_B are measured signals at points A and B, respectively, $s(t)$ is the emitted signal, δ is the attenuation factor, and τ is the time delay. Some other applications in which time-delay information is utilised can be found in Giraudet and Glotin (2006), Yu, Qi, and Fanrong (2012), Chen, Liu, Kong, and He (2011), Quazi (1981), Qin, Huang, and Zhang (2003), Aghasi, Hashemi, and Khalaj (2011), Wei, Wang, and Wan (2006), and Azimi-Sadjadi, Charleston, Wilbur, and Dobeck (1998).

The general model in (1) satisfies the conditions given in Assumptions 2.1, 2.2, 2.4, and 2.5.

Assumption 2.1: *It is assumed that τ, the unknown time-delay vector, is bounded and in a known hypercube $\Omega \subset \mathbb{R}^n$.*

Assumption 2.2: *It is assumed that the function $q(\cdot)$ is either concave or convex on a simplex[2] Ω_s in $\mathbb{R}^n$, and also $\Omega_s \supset \Omega$.*

Definition 2.3: A function W is convex on Ω if it satisfies the following inequality:

$$W(\kappa\nu_1 + (1-\kappa)\nu_2) \le \kappa W(\nu_1) + (1-\kappa)W(\nu_2), \quad \forall \nu_1, \nu_2 \in \Omega, \tag{4}$$

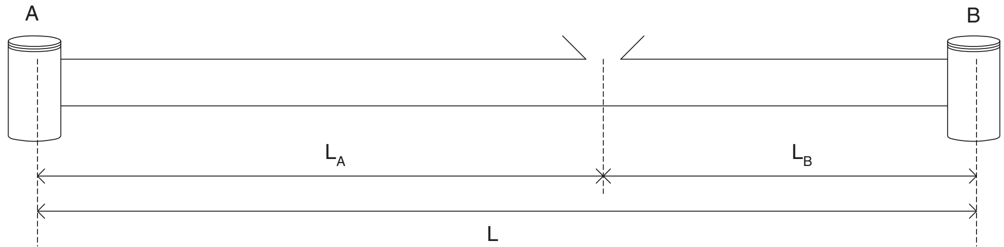

Figure 1. A segment of a water distribution pipeline system.

and concave if it satisfies the following inequality:

$$W(\kappa \nu_1 + (1-\kappa)\nu_2) \geq \kappa W(\nu_1) + (1-\kappa)W(\nu_2), \quad \forall \nu_1, \nu_2 \in \Omega, \tag{5}$$

where $0 \leq \kappa \leq 1$.

Assumption 2.4: *It is assumed that the function $\Pi(t)$ is a continuous function of time, bounded, and Lipschitz in t as follows:*

$$\|\Pi(t_1) - \Pi(t_2)\| \leq L_1 |t_1 - t_2|, \qquad \forall t_1, t_2 \in \mathbb{R}^+, \tag{6}$$

where $L_1 \in \mathbb{R}^+$ is the Lipschitz constant.

Assumption 2.5: *It is assumed that $q(\tau, \Pi)$ is Lipschitz with respect to its arguments as*

$$|q(\tau + \Delta\tau, \Pi + \Delta\Pi) - q(\tau, \Pi)| \leq L_2(\|\Delta\Pi\| + \|\Delta\tau\|), \tag{7}$$

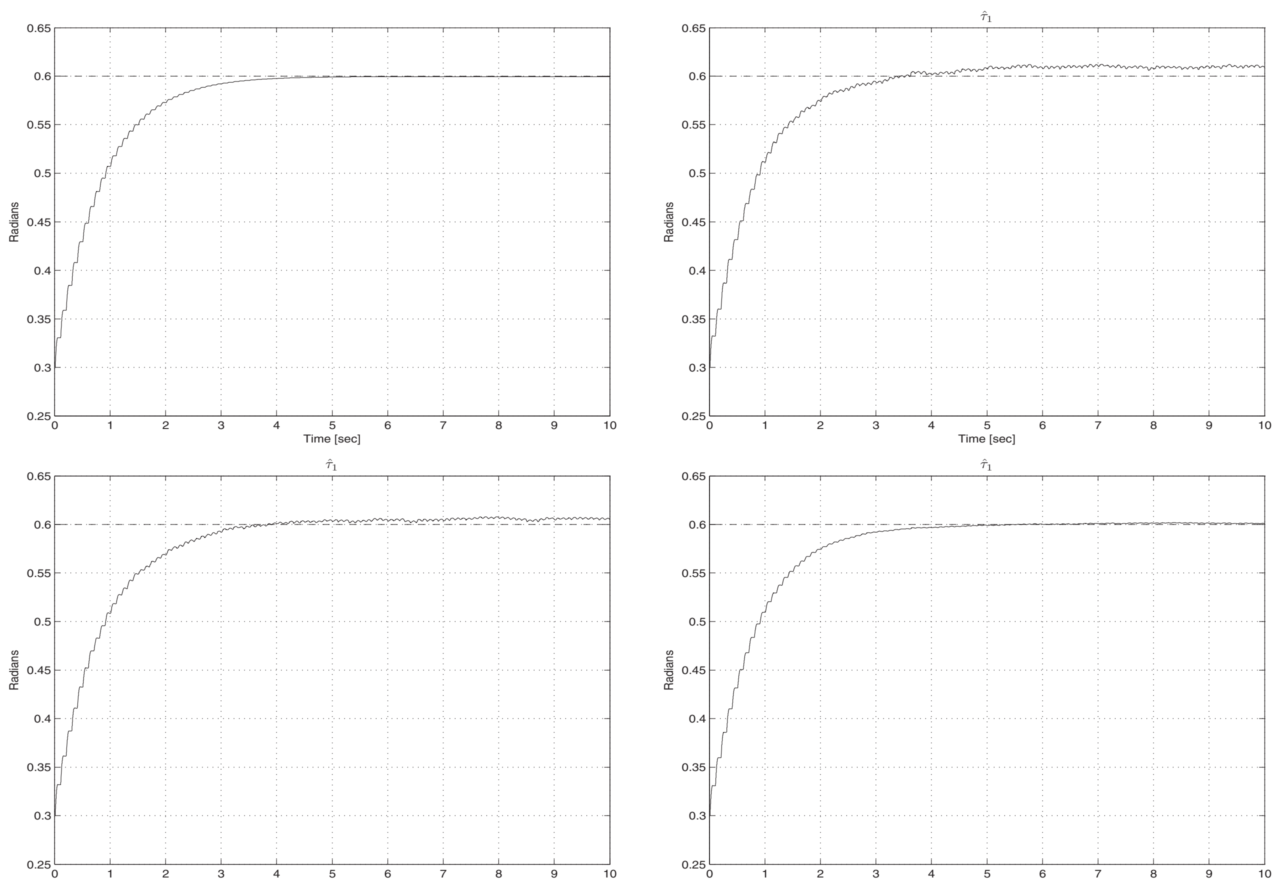

Figure 2. $\hat{\tau}_1(t)$ for constant delay for case I without noise (left top) and in the presence of additive noise with an SNR of 10 (right top), 20 (left bottom), and 30 dB (right bottom).

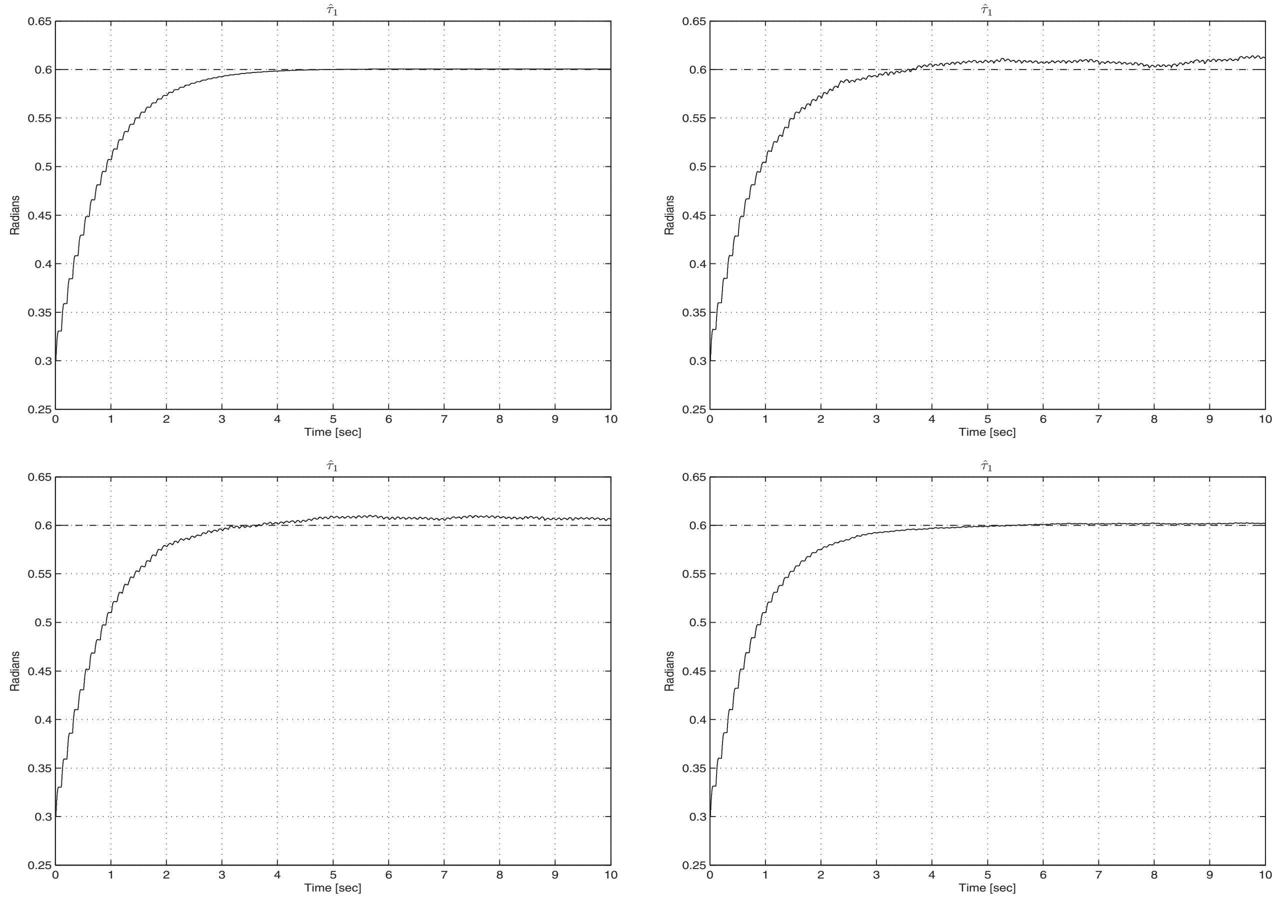

Figure 3. $\hat{\tau}_1(t)$ for disturbed time delay for case I without noise (left top) and in the presence of additive noise with an SNR of 10 (right top), 20 (left bottom), and 30 dB (right bottom).

where $\Delta\Pi \triangleq \Pi(t_1) - \Pi(t_2)$, $\Delta\tau \triangleq \tau(t_1) - \tau(t_2)$, *and* $L_2 \in \mathbb{R}^+$ *is the Lipschitz constant.*

3. Delay estimation

The estimate form of (2) is defined as

$$\hat{q} = \sum_{i=1}^{m} \Psi_i(t) + \sum_{i=1}^{m}\sum_{j=1}^{n} \Psi_i(t - \hat{\tau}_j), \tag{8}$$

where $\hat{q} \triangleq q(\hat{\tau}, \Pi) \in \mathbb{R}$. An auxiliary filter signal, denoted by $q_f(t) \in \mathbb{R}$, is designed as follows:

$$\dot{q}_f = -\alpha q_f + q, \quad q_f(t_0) = 0, \tag{9}$$

where $\alpha \in \mathbb{R}$ is a positive constant. The estimate form of (9) is designed as

$$\dot{\hat{q}}_f = -\alpha(\hat{q}_f - \varepsilon \text{sat}(r)) + \hat{q} - a^* \text{sat}(r), \tag{10}$$

where $\hat{q}_f(t)$ and $\dot{\hat{q}}_f(t) \in \mathbb{R}$ are the estimates of $q_f(t)$ and $\dot{q}_f(t)$, respectively, $\varepsilon \in \mathbb{R}$ is the desired precision, $a^*(t) \in \mathbb{R}$ is the tuning function and $r(t) \in \mathbb{R}$ is defined as follows:

$$r \triangleq \frac{\tilde{q}_f}{\varepsilon}, \tag{11}$$

where $\tilde{q}_f(t) \in \mathbb{R}$ is an error signal defined as follows:

$$\tilde{q}_f \triangleq \hat{q}_f - q_f. \tag{12}$$

In (10), sat$(\cdot)$ is a saturation function and is defined as

$$\text{sat}(z) = \begin{cases} 1, & z \geqslant 1 \\ z, & |z| < 1 \\ -1, & z \leqslant -1 \end{cases}. \tag{13}$$

After taking the time derivative of (12), the below expression may be obtained:

$$\dot{\tilde{q}}_f = -\alpha \tilde{q}_{f\varepsilon} + \hat{q} - q - a^* \text{sat}(r), \tag{14}$$

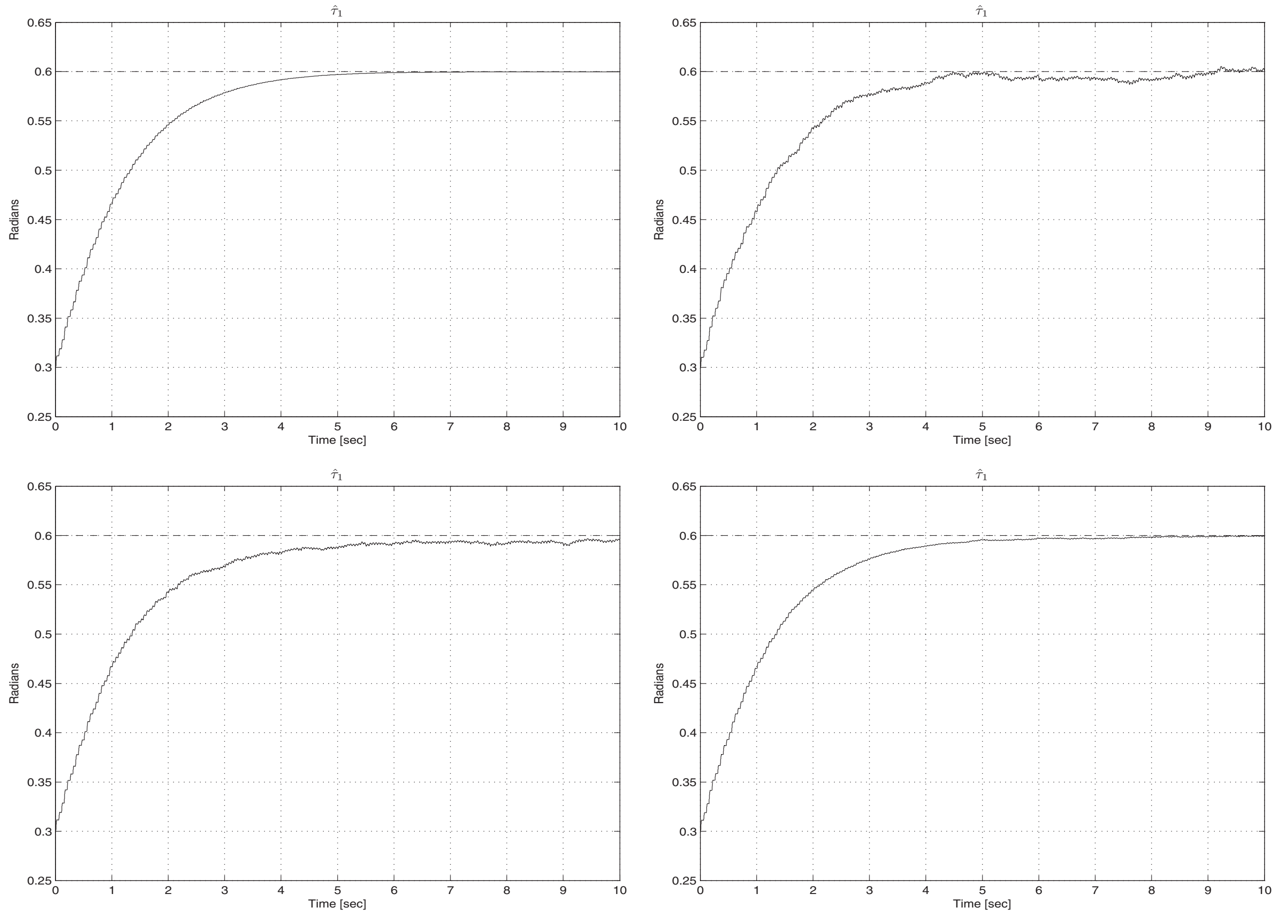

Figure 4. $\hat{\tau}_1(t)$ for constant delay for case II without noise (left top) and in the presence of additive noise with an SNR of 10 (right top), 20 (left bottom), and 30 dB (right bottom).

where (9) and (10) were utilised and $\tilde{q}_{f\varepsilon}(t) \in \mathbb{R}$ is the tuning error defined as

$$\tilde{q}_{f\varepsilon} \triangleq \tilde{q}_f - \varepsilon \text{sat}(r). \tag{15}$$

The tuning function $\tilde{q}_{f\varepsilon}(t)$ and the saturation function sat(r) ensure that the estimator is continuous even if a discontinuous solution of the minimum–maximum algorithm is obtained (Annaswamy, Skantze, & Loh, 1998). The update law is developed with a projection as follows:

$$\dot{\hat{\tau}} = \text{Proj}\{-\Gamma \tilde{q}_{f\varepsilon} \phi^*\}, \tag{16}$$

where $\phi^*(t) \in \mathbb{R}^n$ is the sensitivity function, $\Gamma \in \mathbb{R}^{n\times n}$ is a positive-definite diagonal gain matrix, and the projection algorithm ensures that $\hat{\tau}(t)$ always belongs to the hypercube Ω. The projection algorithm is defined as

$$\hat{\tau}_j = \begin{cases} \hat{\tau}_j, & \text{if } \hat{\tau}_j \in [\tau_{j,\min}, \tau_{j,\max}] \\ \tau_{j,\min}, & \text{if } \hat{\tau}_j < \tau_{j,\min} \\ \tau_{j,\max}, & \text{if } \hat{\tau}_j > \tau_{j,\max} \end{cases}, \tag{17}$$

where the subscript j denotes the jth element of the corresponding vector $\forall j = 1, 2, \ldots, n$, and $\tau_{j,\min}$ and $\tau_{j,\max} \in \mathbb{R}$ are the minimum and maximum values of the jth component of τ, respectively. The projection strategy in (17) is utilised to guarantee the boundedness of $\hat{\tau}(t)$; thus, $\phi^*(t)$ can be upper bounded as follows:

$$\|\phi^*(t)\| \leqslant L_\phi \qquad \forall t \geqslant t_0, \tag{18}$$

where $L_\phi \in \mathbb{R}$ is a positive constant. The terms $\phi^*(t)$ and $a^*(t)$ are defined from the following minimum–maximum optimisation problem[3]:

$$a^* = \min_{\phi\in\mathbb{R}^n} \max_{\tau\in\Omega_s} J(\phi, \tau), \tag{19}$$

$$\phi^* = \arg \min_{\phi\in\mathbb{R}^n} \max_{\tau\in\Omega_s} J(\phi, \tau), \tag{20}$$

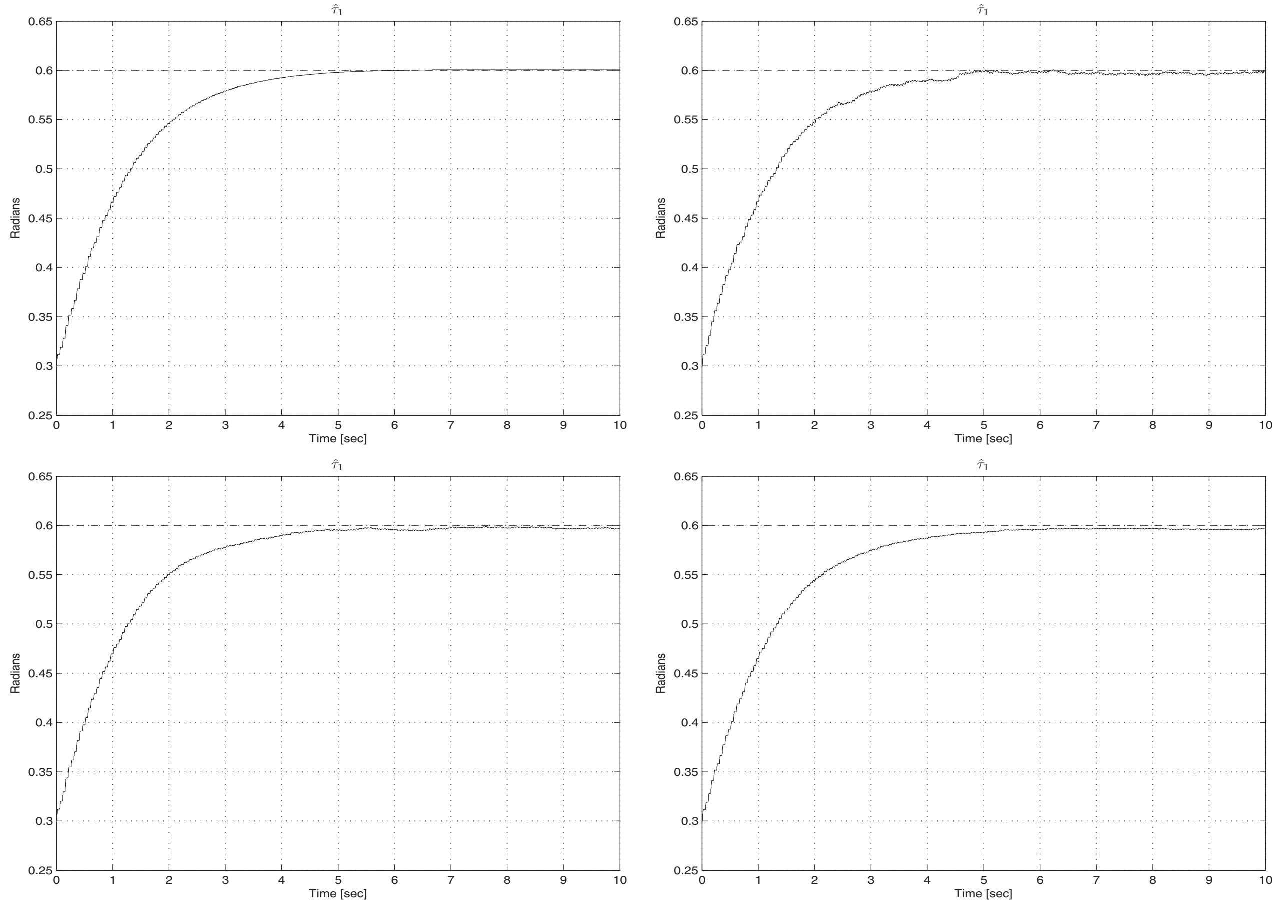

Figure 5. $\hat{\tau}_1(t)$ for disturbed time delay for case II without noise (left top) and in the presence of additive noise with an SNR of 10 (right top), 20 (left bottom), and 30 dB (right bottom).

where $J(r, q, \hat{q}, \tilde{\tau}, \phi) \in \mathbb{R}$ is a performance index and is given as follows:

$$J(\cdot) = \text{sat}(r)[\hat{q} - q - (\Gamma\tilde{\tau})^T\phi], \tag{21}$$

where $\tilde{\tau}(t) \in \mathbb{R}^n$ is the parameter estimation error defined as follows:

$$\tilde{\tau} \triangleq \hat{\tau} - \tau. \tag{22}$$

The solutions of $\phi^*(t)$ and $a^*(t)$ can be obtained as follows:

(1) when $\tilde{q}_f(t) < 0$

$$a^* = \begin{cases} 0 & \text{if } q \text{ is concave on } \Omega_s \\ A_1 & \text{if } q \text{ is convex on } \Omega_s \end{cases} \tag{23}$$

$$\phi^* = \begin{cases} \nabla q(\hat{\tau}) & \text{if } q \text{ is concave on } \Omega_s \\ A_2 & \text{if } q \text{ is convex on } \Omega_s \end{cases} \tag{24}$$

(2) when $\tilde{q}_f(t) \geqslant 0$

$$a^* = \begin{cases} A_1 & \text{if } q \text{ is concave on } \Omega_s \\ 0 & \text{if } q \text{ is convex on } \Omega_s \end{cases} \tag{25}$$

$$\phi^* = \begin{cases} A_2 & \text{if } q \text{ is concave on } \Omega_s \\ \nabla q(\hat{\tau}) & \text{if } q \text{ is convex on } \Omega_s \end{cases} \tag{26}$$

where $A(t) \in \mathbb{R}^{n+1}$ is given as follows:

$$A = [A_1\ A_2^T]^T = G^{-1}b, \tag{27}$$

where $A_1(t) \in \mathbb{R}$ and $A_2(t) \in \mathbb{R}^n$, and $G(t) \in \mathbb{R}^{(n+1)\times(n+1)}$ and $b(t) \in \mathbb{R}^{n+1}$ are defined as

$$G = \begin{bmatrix} -1 & \beta\Gamma(\hat{\tau} - \tau_{s1})^T \\ \vdots & \vdots \\ -1 & \beta\Gamma(\hat{\tau} - \tau_{s(n+1)})^T \end{bmatrix}, \tag{28}$$

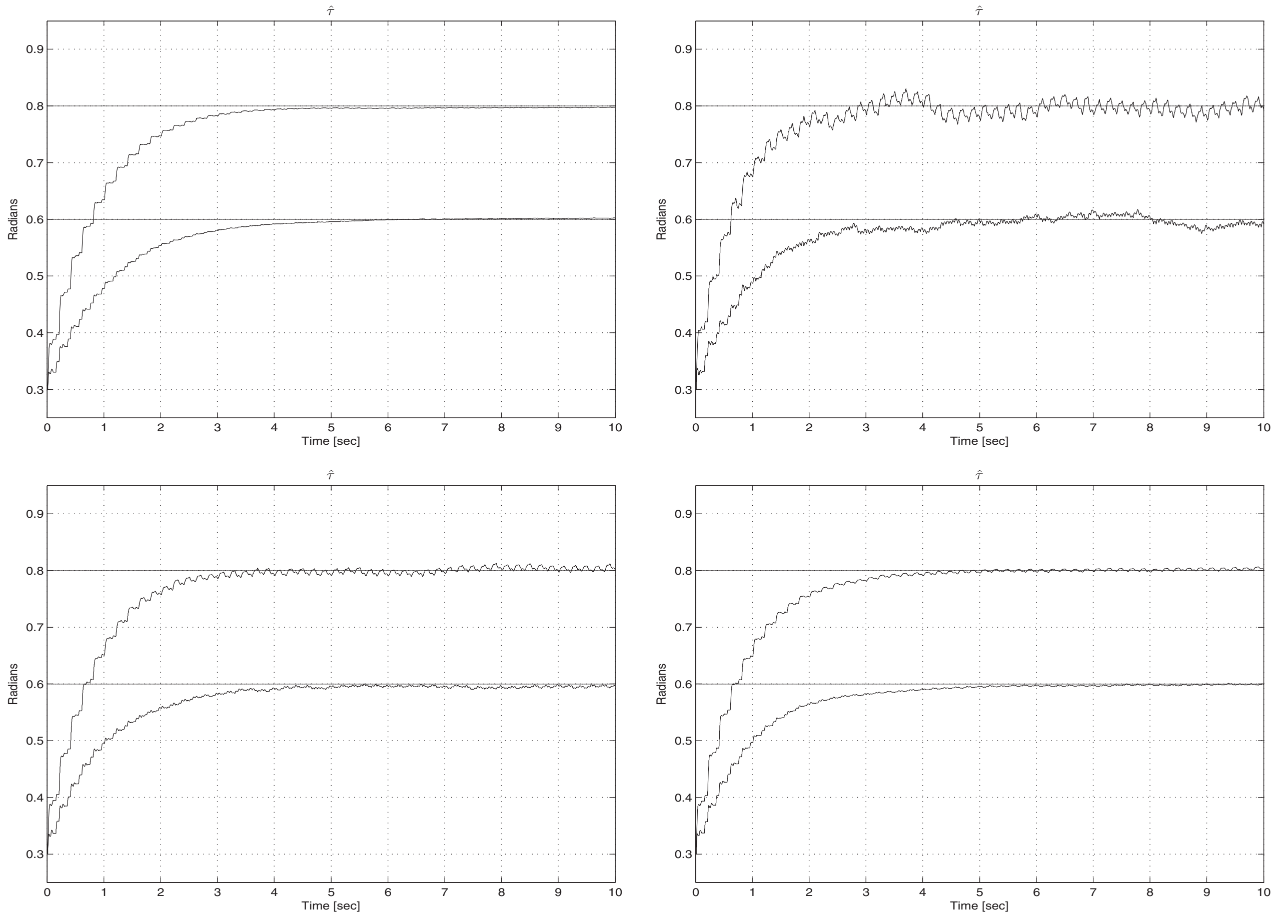

Figure 6. $\hat{\tau}(t)$ for constant delay for case III without noise (left top) and in the presence of additive noise with an SNR of 10 (right top), 20 (left bottom), and 30 dB (right bottom).

$$b = \begin{bmatrix} \beta(\hat{q} - q_{s1}) \\ \vdots \\ \beta(\hat{q} - q_{s(n+1)}) \end{bmatrix}, \tag{29}$$

where $\beta \in \mathbb{R}$ is defined as follows:

$$\beta = \begin{cases} 1 & \text{if } q \text{ is convex on } \Omega_s \\ -1 & \text{if } q \text{ is concave on } \Omega_s. \end{cases} \tag{30}$$

In (29), $q_{sh} \triangleq q(\tau_{sh}, \Pi)\ \forall h = 1, 2, \ldots, n+1$, where $\tau_{sh} \in \mathbb{R}^n$ are the vertices of the simplex Ω_s. In (24) and (26), $\nabla q(\hat{\tau}) \in \mathbb{R}^n$ is the gradient function given as follows:

$$\nabla q(\hat{\tau}) = \frac{\partial q}{\partial \tau}\ |_{\tau=\hat{\tau}}\,. \tag{31}$$

The hypercube Ω may be obtained by using minimum and maximum values of τ. The vertices of the simplex Ω_s may be obtained by first inscribing Ω in a n-dimensional sphere and then inscribing this sphere inside an $(n+1)$-dimensional polytope.

4. Stability analysis

Theorem 4.1: *The adaptive update law in* (16) *ensures that* $\tilde{q}_{f\varepsilon}(t) \in L_2 \cap L_\infty$*; hence, the stability of the identifier and the global boundedness of the overall adaptive system are guaranteed. The estimator ensures that* $\|\tilde{\tau}(t)\| \leqslant \gamma$ *as* $t \to \infty$ *provided the following nonlinear persistent excitation condition holds:*

$$\beta(\Pi(t_2))(q(\hat{\tau}(t_1), \Pi(t_2)) - q(\tau, \Pi(t_2))) \geqslant \varepsilon_u \|\hat{\tau}(t_1) - \tau\|, \tag{32}$$

where $\gamma \in \mathbb{R}$ *is a constant defined as*

$$\gamma \triangleq \frac{8\varepsilon c_1}{\varepsilon_u^2}, \tag{33}$$

where $c_1 \in \mathbb{R}$ *is a constant defined as* $c_1 \triangleq 4L_1L_2 + 2\nu L_2 L_\phi + \nu L_\phi^2$*, where* $\nu \in \mathbb{R}$ *is the maximum eigenvalue*

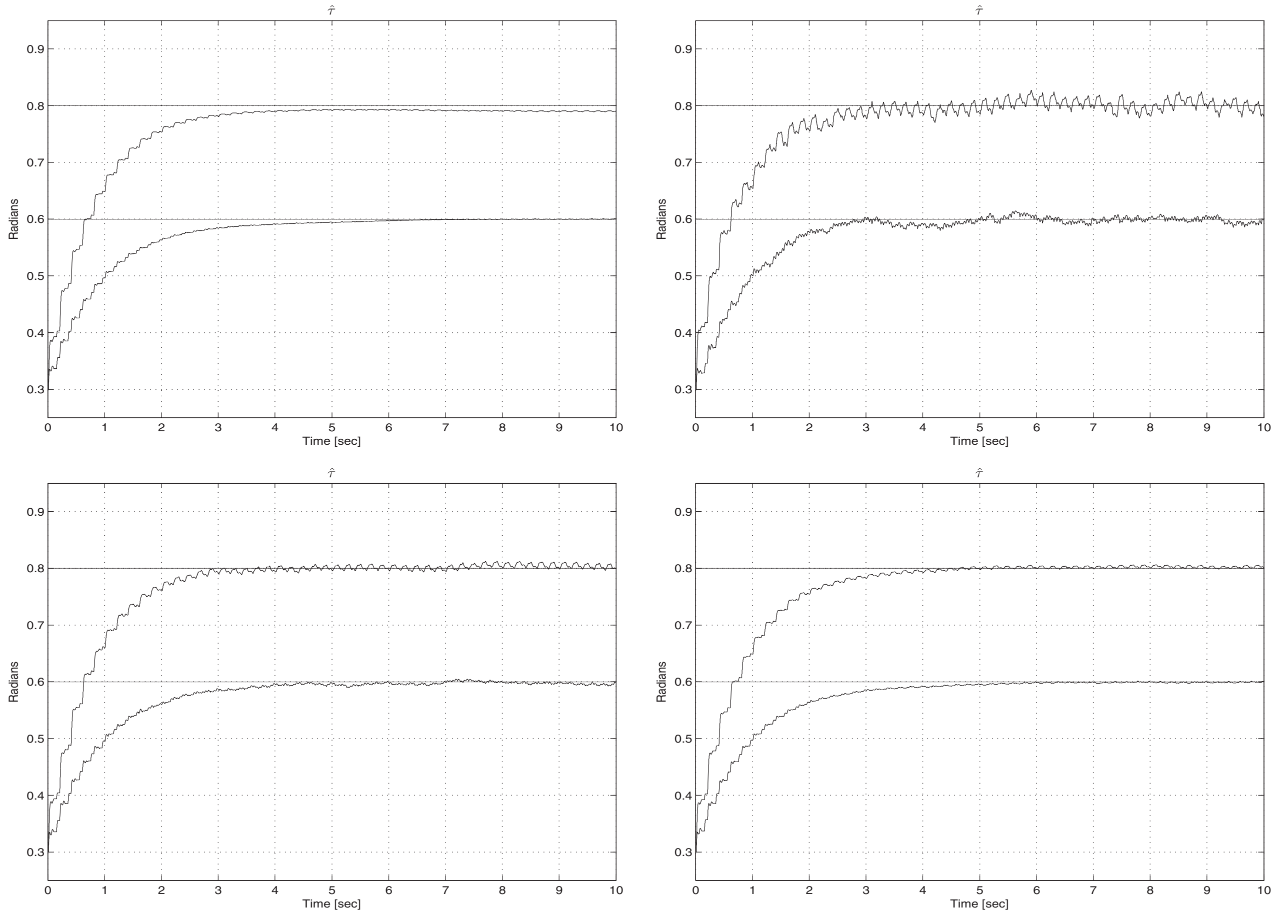

Figure 7. $\hat{\tau}(t)$ for disturbed time delay for case III without noise (left top) and in the presence of additive noise with an SNR of 10 (right top), 20 (left bottom), and 30 dB (right bottom).

of Γ, $t_2 \in [t_1, t_1 + T_0]$, $t_1 > t_0$, *and* $T_0, \varepsilon_u \in \mathbb{R}$ *are positive constants.*

Proof: The reader is referred to Bayrak (2013) for the full proof and Nath, Tatlicioglu, and Dawson (2010) for a similar proof. □

Remark 1: From its definition in (33), it is clear that γ can be made smaller by choosing a smaller precision ε. It is also clear that as $\varepsilon \to 0$, $\gamma \to 0$; thus, the time-delay identification error also goes to zero in the sense that $\|\tilde{\tau}(t)\| \to 0$.

5. Numerical simulation results

The performance of the proposed technique was evaluated by conducting several numerical simulations using Matlab/Simulink. Numerical simulation performance was investigated for various cases. The below parameters and initial conditions were used in all cases, without being changed, to demonstrate a better comparison of results. The performance of the proposed technique was evaluated with additive noise where white Gaussian noise with 10, 20, and 30 dB signal to noise ratio (SNR) was, separately, injected to $q(t)$ to demonstrate robustness against noise. During the simulation, the update law in (16) was utilised with gains $\alpha = 600$, $\Gamma = 3000$, and the desired precision was chosen as $\varepsilon = 0.00001$. The control gains were adjusted via trial and error. The desired precision is required to be chosen very close to zero and this eased the choice of ε. For a better comparison, in all the sub-cases, same gains were used. As a result, the gains α and Γ were required to be chosen big for the algorithm to be robust to additive noise, additive disturbances, and jumps in the delays. At this point, we would like to highlight the fact that if we were to choose control gains differently for all sub-cases, then smaller gains would work for both noise-free and disturbance-free cases. In addition, we would also like to note that genetic algorithms or similar methods can be utilised to adjust the gains. The lower and upper bounds of phase shifts[4] were 0.1 and 1.1 radians, respectively. The initial values of q_f and $\hat{q}_f$ were set to 0 and the initial value of $\hat{\tau}$ was 0.3 radians.

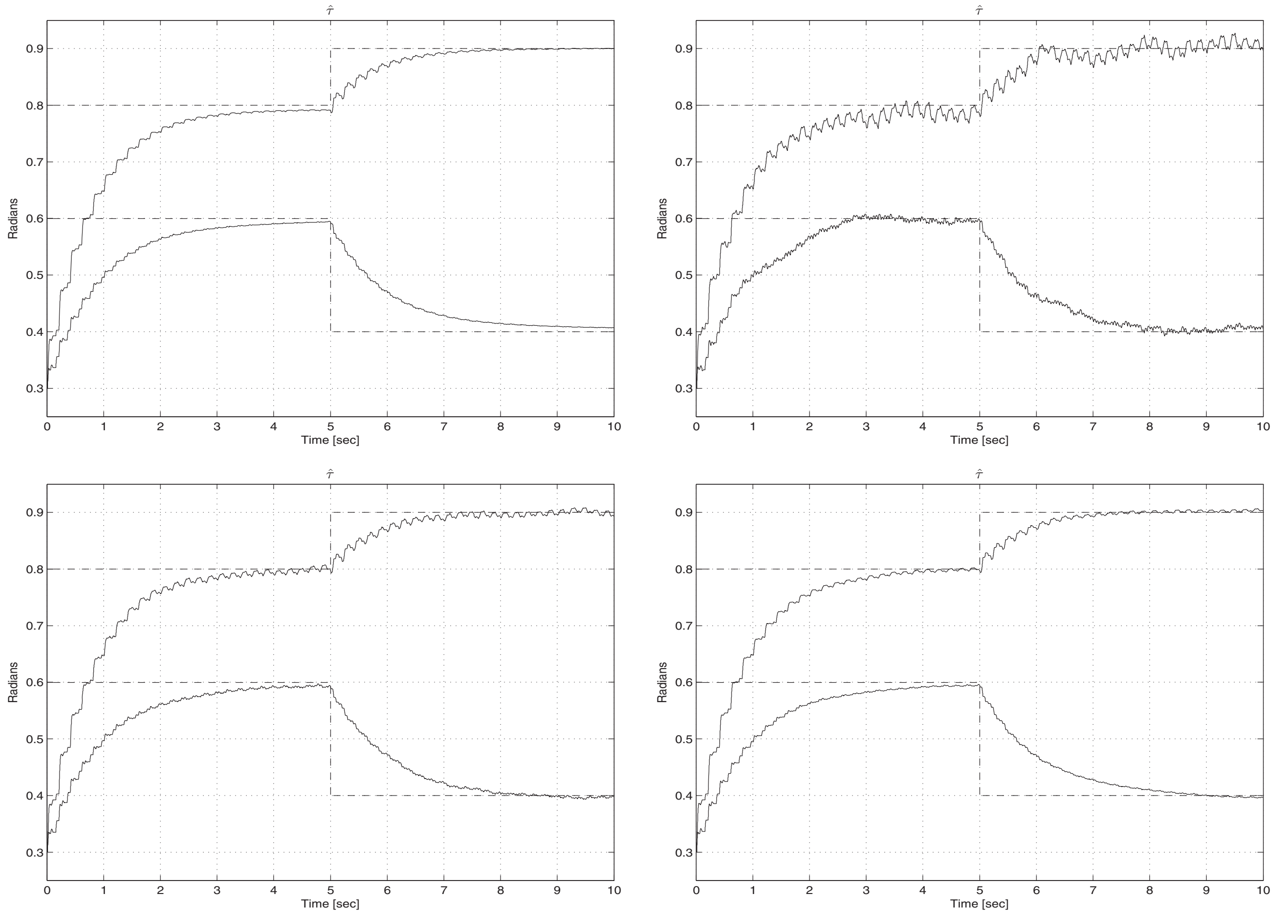

Figure 8. $\hat{\tau}(t)$ for constant delay for case IV without noise (left top) and in the presence of additive noise with an SNR of 10 (right top), 20 (left bottom), and 30 dB (right bottom).

5.1. Case I

The model in (34) was considered with $f_1 = 5$ Hz and $\tau_1 = 0.6$ radians. The estimate of τ_1 is presented in Figure 2 for both noiseless and noisy cases:

$$q(\tau, \Pi) = 0.9 \sin(2\pi f_1 t) + 0.8 \sin(2\pi f_1 t - \tau_1). \quad (34)$$

Another simulation was conducted for the same model where a sinusoidal perturbation was added to the time shift to demonstrate robustness to additive disturbances in the sense that

$$\tau_1 = 0.6 + 0.001\sin(2\pi 0.05 t). \quad (35)$$

In Figure 3, the estimate of $\tau_1(t)$ in (35) is presented for both noiseless and noisy cases. From Figures 2 and 3, it is clear that the identification of time delay is achieved even in the presence of noise and also disturbance.

5.2. Case II

The model in (36) was considered with $f_1 = 10$ Hz, $f_2 = 5$ Hz, and $\tau_1 = 0.6$ radians. The estimate of τ_1 is presented in Figure 4 for both noiseless and noisy cases:

$$q(\tau, \Pi) = 0.9 \sin(2\pi f_1 t - \tau_1) + 0.8 \sin(2\pi f_2 t). \quad (36)$$

Another simulation was conducted for the same model where a sinusoidal perturbation was added to the time delay to demonstrate robustness to additive disturbances as in (35). In Figure 5, the estimate of $\tau_1(t)$ is presented for both noiseless and noisy cases. From Figures 4 and 5, it is clear that the identification of time delay is achieved even in the presence of noise and also disturbance.

5.3. Case III

The model in (37) was considered with $f_1 = 10$ Hz, $f_2 = 5$ Hz, $\tau_1 = 0.6$ radians, and $\tau_2 = 0.8$ radians. The estimates of time delays are presented in Figure 6 for both noiseless

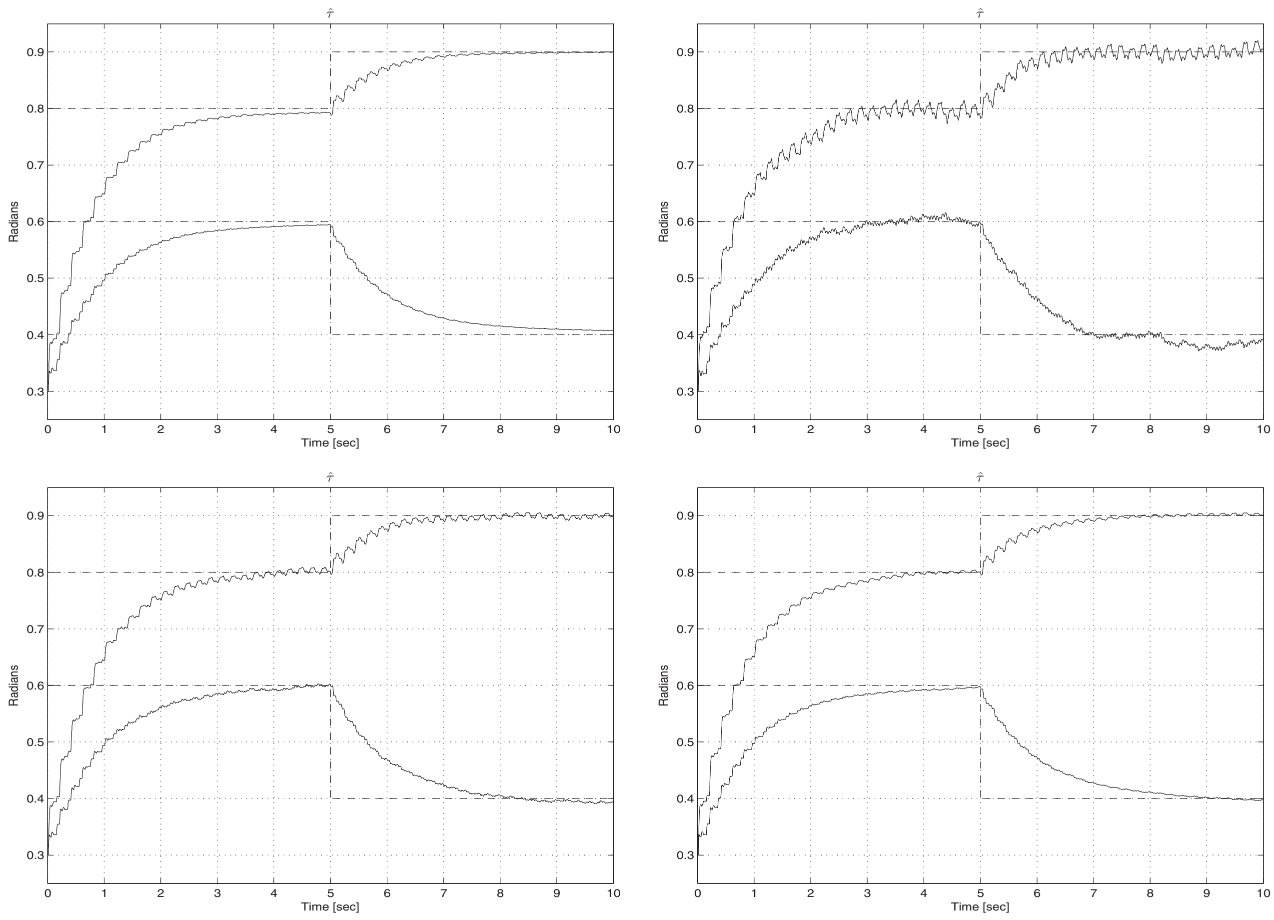

Figure 9. $\hat{\tau}(t)$ for disturbed time delay for case IV without noise (left top) and in the presence of additive noise with an SNR of 10 (right top), 20 (left bottom), and 30 dB (right bottom).

and noisy cases:

$$q(\tau, \Pi) = 0.9 \sin(2\pi f_1 t - \tau_1) + 0.8 \sin(2\pi f_2 t - \tau_2). \quad (37)$$

Another numerical simulation was conducted for the same model where a sinusoidal perturbation was added to the time delay to demonstrate robustness to additive disturbances in the sense that

$$\tau_1 = 0.6 + 0.001\sin(2\pi 0.05t), \quad (38)$$

$$\tau_2 = 0.8 + 0.001\cos(2\pi 0.05t). \quad (39)$$

In Figure 7, the estimates of $\tau_1(t)$ and $\tau_2(t)$ are presented for both noiseless and noisy cases. From Figures 6 and 7, it is clear that the identification of time delay is achieved even in the presence of noise and also disturbance.

5.4. *Case IV*

The model in (37) was considered with $f_1 = 10$ Hz and $f_2 = 5$ Hz. It is considered that the time delays are constant and jumping to different constant values after a while. Time delays were set to $\tau_1 = 0.6$ radians and $\tau_2 = 0.8$ radians, and after 5 seconds, time delays were changed as $\tau_1 = 0.4$ radians and $\tau_2 = 0.9$ radians. The estimates of time delays are presented in Figure 8 for both noiseless and noisy cases.

Another numerical simulation was conducted for the model in (37) where a sinusoidal perturbation was added to the time delay to demonstrate robustness to additive disturbances as in (38) and (39), and after 5 seconds, the time delays were changed to

$$\tau_1 = 0.4 + 0.001\sin(2\pi 0.05t), \quad (40)$$

$$\tau_2 = 0.9 + 0.001\cos(2\pi 0.05t). \quad (41)$$

In Figure 9, the estimates of $\tau_1(t)$ and $\tau_2(t)$ are presented for both noiseless and noisy cases. From Figures 8 and 9, it is clear that the identification of time delays is achieved when there is a sudden change in the time delay and even in the presence of noise and also disturbance.

6. Conclusion

In this work, a novel adaptive time-delay identification technique was presented. The technique is novel in the sense that a nonlinear parameter identification algorithm was utilised as the time-delay identification algorithm for the first time in the literature. In the design of the adaptive identification law, Lyapunov-based stability analysis techniques were utilised. As a consequence of Lyapunov-based methods, our identification algorithm is robust to parametric uncertainties and noise and also compensates for some unmodelled nonlinearities. Numerical simulation results were given to demonstrate the efficiency of the estimator for both constant and disturbed time delays and its robustness to additive noise. From the results, it is clear that the developed technique efficiently identifies constant and disturbed time delays and even when there is sudden change in the time delay.

The main results of this paper are as follows: (1) nonlinear parameter identification tools are utilised to identify time delays, (2) the identifier is continuous, (3) the developed time-delay identification algorithm can be applied online, (4) multiple time delays may be identified, (5) the developed identifier provides identification of time delay within a desired precision that can be adjusted to be very small, and (6) the identification algorithm is proven to be correct.

There is much to be considered as future work. A possible extension is that the developed technique can be fused with other Lyapunov-based techniques since it is based on Lyapunov-type analysis synthesis tools. Specifically, a full system identification can be aimed by fusing the developed technique with other system identification tools (Yin, Ding, Haghani, Hao, & Zhang, 2012). It is also aimed to adopt this technique to time-delay identification for general classes of systems and to control applications by extending our previous results in Bayrak and Tatlicioglu (2012). In that sense, one possible research avenue is designing controllers for systems subject to unknown input time delay (Li, Jing, & Karimi, 2014).

Notes

1. Preliminary results of this work were published in Bayrak and Tatlicioglu (2011).
2. A simplex in $\mathbb{R}^n$ is a convex polytope with $n + 1$ vertices.
3. Although the derivations are very similar to that of Annaswamy et al. (1998), we presented them for the sake of completeness.
4. In examples, phase shifted form was used to illustrate the delayed sinusoidal signals. The phase shift in radians can be, easily, converted to time delay by a basic linear transformation.

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References

Aghasi, H., Hashemi, M., & Khalaj, B.H. (2011, May). Source localization through adaptive signal attenuation model and time delay estimation. In *Proceedings of the International Conference on Telecommunications* (pp. 151–156). Ayia Napa, Cyprus.

Annaswamy, A.M., Skantze, F.P., & Loh, A.P. (1998). Adaptive control of continuous time systems with convex/concave parametrization. *Automatica, 34*, 33–49.

Azimi-Sadjadi, M.R., Charleston, S., Wilbur, J., & Dobeck, G.J. (1998). A new time delay estimation in subbands for resolving multiple specular reflections. *IEEE Transactions on Signal Processing, 46*(12), 3398–3403.

Bai, E.-W., & Chyung, D.H. (1993). Improving delay estimates derived from least-square algorithms and Pade approximations. *International Journal of Systems Science, 24*(4), 745–756.

Banyasz, Cs., & Keviczky, L. (1994). Recursive time delay estimation method. *International Journal of Systems Science, 25*(11), 1857–1865.

Bastard, C.L., Baltazart, V., & Wang, Y. (2010). Modified esprit (m-esprit) algorithm for time delay estimation in both any noise and any radar pulse context by a GPR radar. *Signal Processing, 90*, 173–179.

Bayrak, A., & Tatlicioglu, E. (2011). A novel online adaptive time delay identification technique. In *Proceedings of the IEEE International Conference on Decision and Control* (pp. 3338–3343). Orlando, FL.

Bayrak, A. (2013). Online time delay identification and adaptive control for general classes of nonlinear systems (doctoral dissertation). Izmir Institute of Technology, Izmir, Turkey.

Bayrak, A., & Tatlicioglu, E. (2012). Online time delay identification and control for general classes of nonlinear systems. In *Proceedings of the IEEE International Conference on Decision and Control* (pp. 1591–1596). Maui, HI.

Bhardwaj, T.P., & Nath, R. (2010). Maximum likelihood estimation of time delays in multipath acoustic channel. *Signal Processing, 90*, 1750–1754.

Bjorklund, S., & Ljung, L. (2003). A review of time-delay estimation techniques. In *Proceedings of IEEE Conference on Decision and Control* (pp. 2502–2507). Maui, HA.

Brennan, M., Gao, Y., & Joseph, P. (2007). On the relationship between time and frequency domain methods in time delay estimation for leak detection in water distribution pipes. *Journal of Sound and Vibration, 304*, 213–223.

Chen, L., Liu, Y., Kong, F., & He, N. (2011). Acoustic source localization based on generalized cross-correlation time-delay estimation. *Procedia Engineering, 15*, 4912–4919.

Diop, S., Kolmanovsky, I., Moraal, P.E., & van Nieuwstadt, M. (2001). Preserving stability/performance when facing an unknown time-delay. *Control Engineering Practice, 9*, 1319–1325.

Dydek, Z., Annaswamy, A.M., & Lavretsky, E. (2010). Adaptive control and the NASA X–15–3 flight revisited. *IEEE Control Systems Magazine, 30*, 32–48.

Gao, Y., Brennan, M., & Joseph, P.F. (2009). On the effects of reflections on time delay estimation for leak detection in buried plastic water pipes. *Journal of Sound and Vibration, 325*, 649–663.

Giraudet, P., & Glotin, H. (2006). Real-time 3D tracking of whales by echo-robust precise TDOA estimates with a widely-spaced hydrophone array. *Applied Acoustics, 67*, 1106–1117.

Harva, M., & Raychaudhury, S. (2008). Bayesian estimation of time delays between unevenly sampled signals. *Neurocomputing, 72*, 32–38.

Li, H., Jing, X., & Karimi, H.R. (2014). Output-feedback-based H_∞ control for vehicle suspension systems with control delay. *IEEE Transactions on Industrial Electronics, 61*(1), 436–446.

Lui, K.W., Chan, F.K., & So, H. (2009). Accurate time delay estimation based passive localization. *Signal Processing, 89*, 1835–1838.

Martin, J.M., Jimenez, A.R., Seco, F., Calderon, L., Pons, J.L., & Ceres, R. (2002). Estimating the 3D-position from time delay data of US-waves: Experimental analysis and a new processing algorithm. *Sensors and Actuators A: Physical, 101*, 311–321.

Nath, N., Tatlicioglu, E., & Dawson, D.M. (2010). Range identification for nonlinear parameterizable paracatadioptric systems. *Automatica, 46*, 1129–1140.

Qin, H., Huang, J., & Zhang, Q. (2003). A novel joint estimator of direction-of-arrival and time-delay for multiple source localization. In *Proceedings of the International Conference on Neural Networks & Signal Processing* (pp. 1294–1297).

Quazi, A. (1981). An overview on the time delay estimate in active and passive systems for target localization. *IEEE Transactions on Acoustics, Speech & Signal Processing, 29*, 527–533.

Richard, J.P. (2003). Time-delay systems: An overview of some recent advances and open problems. *Automatica, 39*, 1667–1694.

Shaltaf, S. (2007). Neuro-fuzzy based time-delay estimation using DCT coefficients. *ISA Transactions, 46*, 21–30.

Sharma, K.K., & Joshi, S.D. (2007). Time delay estimation using fractional Fourier transform. *Signal Processing, 87*, 853–865.

So, H.C. (2002). Noisy input–output system identification approach for time delay estimation. *Signal Processing, 82*, 1471–1475.

Tuch, J., Feuer, A., & Palmor, Z.J. (1994). Time delay estimation in continuous linear time-invariant systems. *IEEE Transactions on Automatic Control, 39*(4), 823–827.

Tugnait, J.K. (1996). Frequency domain adaptive filters using higher-order statistics with application to adaptive time delay estimation. *International Journal of Adaptive Control & Signal Processing, 10*, 137–157.

Wei, X., Wang, L., & Wan, J. (2006, June). A new localization technique based on network TDOA information. In *Proceedings of International Conference on ITS Telecommunications* (pp. 127–130) Chengdu, China.

Wen, J., Li, P., & Wen, Y. (2007). A new method for unbiased time–delay estimation in noisy environments. *International Journal of Adaptive Control & Signal Processing, 21*, 623–634.

Wen, J., Wen, Y., & Li, P. (2008, June). Time delay estimation method for leak location of buried water distribution pipes. In *Proceedings of the World Congress on Intelligent Control and Automation* (pp. 3052–3056). Chongqing, China.

Yin, S., Ding, S.X., Haghani, A., Hao, H., & Zhang, P. (2012). A comparison study of basic data-driven fault diagnosis and process monitoring methods on the benchmark Tennessee Eastman process. *Journal of Process Control, 22*, 1567–1581.

Yu, B., Qi, W., & Fanrong, M. (2012, June). Application of time delay estimation in stress wave test of crude oil pipeline leakage. In *Proceedings of the World Automation Congress* (pp. 1–4). Puerto Vallarta, Mexico.

Zhang, T., & Li, Y. (2006). A control scheme for bilateral teleoperation systems based on time–varying communication delay identification. In *Proceedings of Systems and Control in Aerospace and Astronautics* (pp. 273–278). Harbin, China.

Zhou, D.H., & Frank, P.M. (2000). A real–time estimation approach to time–varying time delay and parameters of NARX processes. *Computers & Chemical Engineering, 23*, 1763–1772.