Motion Synthesis of a Planar Watt II Type Six-Bar Mechanism with Two End-Effectors

Gökhan Kiper

Abstract The study deals with motion generation with closed-loop mechanisms with several end-effectors. As a case study a single degree-of-freedom planar Watt II type six-bar mechanism with two end-effectors is worked on. Dyad formulation with complex numbers is made use of for the mathematical model. It is found that the motion synthesis is possible for at most three poses of the two end-effectors. The formulations are illustrated with numerical examples.

Keywords Watt II type six-bar mechanism · Motion generation · Dyad formulation

1 Introduction

The formulations for the dimensional synthesis of mechanisms with single end-effector are well-known [1–3]. These methods may as-well be used for mechanisms with several end-effectors. By this way, mechanisms with less actuators may be utilized in applications which require multiple simultaneous operations. Some examples for such a system are a gripper with several fingers which have non-symmetric motions, a pick-and-place manipulator for relocation of several different objects and a surgery robot with several end-effectors for different operations. Recently Simo-Serra et al. devised methodologies for designing mechanisms with multiple end-effectors [4], specifically applied to multi-fingered robotic hands with several serial chains and fingers meeting in a common palm [5]. Shen et al. [6, 7] worked on the design of Watt I type planar six-bar mechanisms to be used as a mechanical finger, where prescribed motions of the middle and distal sections of the

G. Kiper (✉)
Izmir Institute of Technology, Izmir, Turkey
e-mail: gokhankiper@iyte.edu.tr

finger. Also, Wobrecht et al. [8] designed a planar 8-link single degrees-of-freedom (dof) exoskeleton finger mechanism for which path and motion generation problems are solved for two of the links. In [8], the authors use numerical optimization techniques for the synthesis.

In [6, 7], the motion of one of the links is described with respect to the other link with prescribed motion. That is, the relative motion of the dependent link is used for the synthesis. However, in an application with multiple end-effectors, the end-effector motions with respect to the base may be independently described. This study is an initial attempt for working out dimensional synthesis of such mechanisms with several end-effectors. As the first step, analytical motion synthesis formulation is developed for a single dof Watt II type planar six-bar mechanism with two end-effectors for two and three poses of the end-effectors. The dyad formulation with complex numbers is presented in Sect. 2. Two and three pose synthesis is the subject of Sects. 3 and 4, respectively. Examples are provided for both two and three pose synthesis. Further studies are discussed in Sect. 5.

2 Dyad Formulation

The Watt II type planar six-bar mechanism shown in Fig. 1 has end-effectors P and Q attached to the coupler links of the two-four bar loops. The task is to relocate the end-effectors from poses P_1, Q_1 to P_j, Q_j for j = 2, 3, … etc. For the mathematical model of this problem we make use dyad formulation following the notation in [2]. Accordingly, one of the two four bar loops consists of the dyads ($\mathbf{W}_1$, $\mathbf{Z}_1$) and ($\mathbf{W}_3$, $\mathbf{Z}_3$), and the other loop consists of the dyads ($\mathbf{W}_2$, $\mathbf{Z}_2$) and ($\mathbf{W}_4$, $\mathbf{Z}_4$). We represent

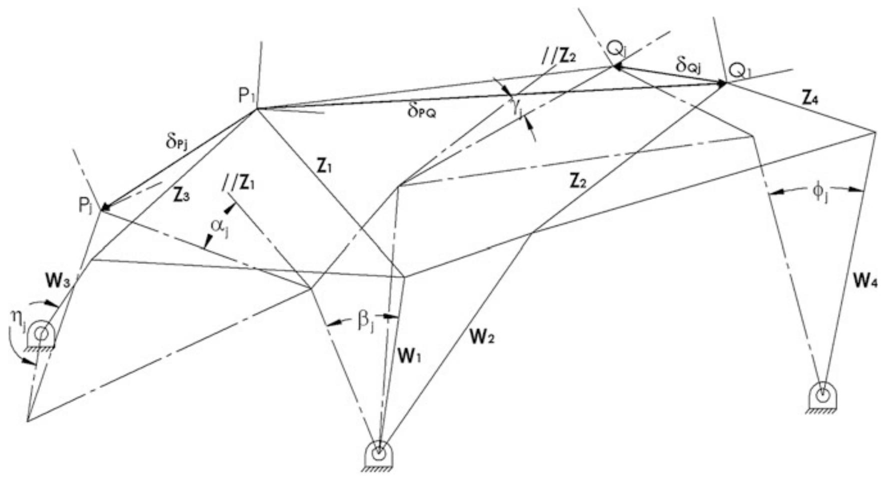

Fig. 1 Planar six-bar mechanism

Table 1 Number of scalar unknowns and free selections for two and three pose synthesis

	Equations (1–3)			Equations (4–5)		
	Unknowns	Equations	Free selections	Unknowns	Equations	Free selections
2 poses	9	6	3	10	4	6
3 poses	10	10	–	12	8	4

all vectors with complex numbers, i.e. $\mathbf{W}_1 = W_{1x} + W_{1y}\hat{\imath}$, etc. We can write the following vector equations from Fig. 1:

$$\mathbf{W}_2 + \mathbf{Z}_2 - \mathbf{W}_1 - \mathbf{Z}_1 = \delta_{PQ} \tag{1}$$

$$\left(e^{\hat{\imath}\beta_j} - 1\right)\mathbf{W}_1 + \left(e^{\hat{\imath}\alpha_j} - 1\right)\mathbf{Z}_1 = \delta_{Pj} \tag{2}$$

$$\left(e^{\hat{\imath}\beta_j} - 1\right)\mathbf{W}_2 + \left(e^{\hat{\imath}\gamma_j} - 1\right)\mathbf{Z}_2 = \delta_{Qj} \tag{3}$$

$$\left(e^{\hat{\imath}\eta_j} - 1\right)\mathbf{W}_3 + \left(e^{\hat{\imath}\alpha_j} - 1\right)\mathbf{Z}_3 = \delta_{Pj} \tag{4}$$

$$\left(e^{\hat{\imath}\phi_j} - 1\right)\mathbf{W}_4 + \left(e^{\hat{\imath}\gamma_j} - 1\right)\mathbf{Z}_4 = \delta_{Qj} \tag{5}$$

where $\delta_{PQ} = \overrightarrow{P_1Q_1}$, $\delta_{Pj} = \overrightarrow{P_1P_j}$, $\delta_{Qj} = \overrightarrow{Q_1Q_j}$; β_j, η_j, ϕ_j are crank link rotations and α_j, γ_j are coupler link rotations as shown in Fig. 1. The angles are directed such that counterclockwise is positive. Actually Eqs. (2), (4) and (3), (5) represent two different solutions for the dyads of a four bar. The difference of the solution from the classical four-bar design comes from Eq. (1).

δ_{PQ}, δ_{Pj}, δ_{Qj}, α_j and γ_j are given in the motion generation problem. The unknowns are $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{W}_2$, $\mathbf{Z}_2$, $\mathbf{W}_3$, $\mathbf{Z}_3$, $\mathbf{W}_4$, $\mathbf{Z}_4$, β_j, η_j and ϕ_j. The number of scalar unknowns and free selections for two and three pose synthesis are listed in Table 1. Exact motion synthesis for more than three poses of the end-effectors is not possible. In Sects. 3 and 4 we formulate the solution for two and three pose problems.

3 Two Pose Synthesis

For the two pose synthesis problem, three scalar parameters out of $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{W}_2$, $\mathbf{Z}_2$, and β_2 can be selected freely. It is wise to select β_2 as one of the specified parameter values, because it appears in trigonometric functions, which causes nonlinearity. Two more scalars from $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{W}_2$ and $\mathbf{Z}_2$ should be selected. In general, there is no superiority of one over other possible selections. However, from numerical examples we found out that two parameters among the nine cannot be selected arbitrarily. For example selecting W_{1x} and W_{1y} does not work. Depending on the application, the designer needs to decide on which parameters to be assumed. For illustration of the formulation let's select W_{1x} and Z_{1x}. From real and imaginary parts of Eq. (2)

$$\left(\cos\beta_j - 1\right)W_{1x} - \sin\beta_j W_{1y} + \left(\cos\alpha_j - 1\right)Z_{1x} - \sin\alpha_j Z_{1y} = \delta_{Pjx} \tag{6}$$

$$\sin\beta_j W_{1x} + \left(\cos\beta_j - 1\right)W_{1y} + \sin\alpha_j Z_{1x} + \left(\cos\alpha_j - 1\right)Z_{1y} = \delta_{Pjy} \tag{7}$$

W_{1y} and Z_{1y} can be solved linearly from Eqs. (6–7) to obtain:

$$W_{1y} = \frac{\begin{bmatrix} \left(\cos\alpha_j - 1\right)\delta_{Pjx} + \sin\alpha_j\delta_{Pjy} \\ -\left(\cos\left(\alpha_j - \beta_j\right) - \cos\alpha_j - \cos\beta_j + 1\right)W_{1x} - 2\left(2 - \cos\alpha_j\right)Z_{1x} \end{bmatrix}}{\sin\left(\alpha_j - \beta_j\right) - \sin\alpha_j + \sin\beta_j} \tag{8}$$

$$Z_{1y} = -\frac{\begin{bmatrix} \left(\cos\beta_j - 1\right)\delta_{Pjx} + \sin\beta_j\delta_{Pjy} \\ -\left(2 - \cos\beta_j\right)W_{1x} - \left(\cos\left(\alpha_j - \beta_j\right) - \cos\alpha_j - \cos\beta_j + 1\right)Z_{1x} \end{bmatrix}}{\sin\left(\alpha_j - \beta_j\right) - \sin\alpha_j + \sin\beta_j} \tag{9}$$

Once $\mathbf{W}_1$ and $\mathbf{Z}_1$ are set, $\mathbf{W}_2$ and $\mathbf{Z}_2$ can be linearly solved from Eqs. (1) and (3) to obtain:

$$\mathbf{W}_2 = \frac{\left(e^{\hat{i}\gamma_2} - 1\right)\left(\mathbf{W}_1 - \mathbf{Z}_1 + \delta_{PQ}\right) - \delta_{Q_2}}{e^{\hat{i}\gamma_2} - e^{\hat{i}\beta_2}} \tag{10}$$

$$\mathbf{Z}_2 = \frac{\delta_{Q_2} - \left(e^{\hat{i}\beta_2} - 1\right)\left(\mathbf{W}_1 - \mathbf{Z}_1 + \delta_{PQ}\right)}{e^{\hat{i}\gamma_2} - e^{\hat{i}\beta_2}} \tag{11}$$

For the other two dyads, it is necessary to select three parameters per dyad. Let's select η_2 and $\mathbf{W}_3$ for one dyad and ϕ_2 and $\mathbf{W}_4$ for the other dyad. Then $\mathbf{Z}_3$ and $\mathbf{Z}_4$ can be easily solved from Eqs. (4–5) as:

$$\mathbf{Z}_3 = \frac{\delta_{Pj} - \left(e^{\hat{i}\eta_j} - 1\right)\mathbf{W}_3}{e^{\hat{i}\alpha_j} - 1} \tag{12}$$

$$\mathbf{Z}_4 = \frac{\delta_{Qj} - \left(e^{\hat{i}\phi_j} - 1\right)\mathbf{W}_4}{e^{\hat{i}\gamma_j} - 1} \tag{13}$$

Since the formulation is analytical, the unique result is obtained almost instantly once the assumed parameter values are specified. By changing the assumed parameter values, the designer can converge to viable solutions and obtain a satisfactory mechanism.

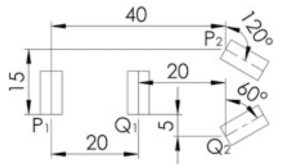

Fig. 2 Two pose pick-and-place application for two objects

As an example consider two poses of the two rectangular objects shown in Fig. 2. From the figure, $\delta_{PQ} = 20$, $\delta_{Pj} = 40 + 15\hat{i}$, $\delta_{Qj} = 20 - 5\hat{i}$; $\alpha_2 = -120°$, $\gamma_j = -60°$. The computations are done using Maple® software. After several trials, considering link length ratios and link collision avoidance, a proper selection for the assumed parameters is done as $\beta_2 = -90°$, $W_{1x} = -35$, $Z_{1x} = 6$, $\eta_2 = -60°$, $\mathbf{W}_3 = -30 + 5\hat{i}$, $\phi_2 = -100°$ and $\mathbf{W}_4 = -5 + 10\hat{i}$. Using Eqs. (8–13), the remaining parameters are computed as $W_{1y} = 12.902$, $Z_{1y} = 1.268$, $\mathbf{W}_2 = -3.170 + 12.268\hat{i}$, $\mathbf{Z}_2 = -5.830 + 1.902\hat{i}$, $\mathbf{Z}_3 = -7.887 + 10.207\hat{i}$ and $\mathbf{Z}_4 = -3.711 + 2.804\hat{i}$. Hence resulting link lengths are $|\mathbf{W}_1| = 37.302$, $|\mathbf{Z}_1| = 6.133$, $|\mathbf{W}_2| = 12.671$, $|\mathbf{Z}_2| = 6.133$, $|\mathbf{W}_3| = 30.414$, $|\mathbf{Z}_3| = 12.899$, $|\mathbf{W}_4| = 11.180$, $|\mathbf{Z}_4| = 4.651$, $|\mathbf{Z}_1 + \mathbf{Z}_3| = 16.515$, $|\mathbf{Z}_2 + \mathbf{Z}_4| = 16.515$ and fixed joint coordinates with respect to P_1: $(29, -14.170)$, $(37.887, -15.207)$, $(28.711, -12.804)$ in order of dyads 1, 3 and 4. The two poses of the resulting mechanism is illustrated in Fig. 3.

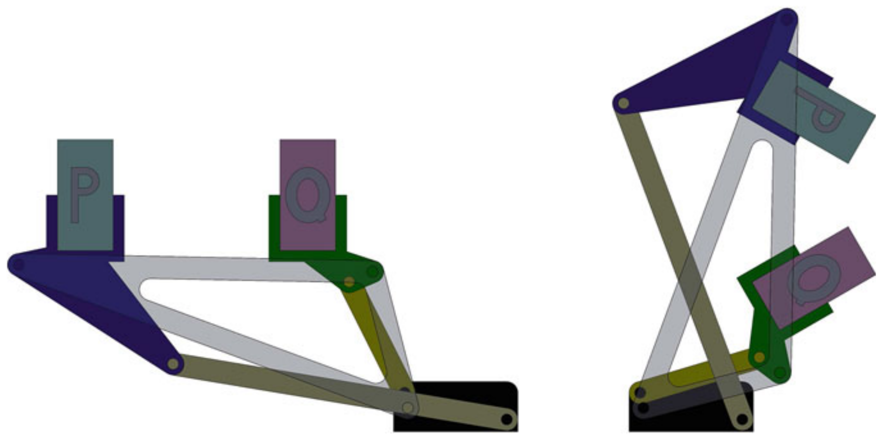

Fig. 3 Two poses of the designed mechanism

4 Three Pose Synthesis

For j = 2, 3, Eqs. (1–3) constitute a nonlinear set of equations. However, Eqs. (2–3) are linear in $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{W}_2$ and $\mathbf{Z}_2$. Solving $\mathbf{W}_1$ and $\mathbf{Z}_1$, from Eq. (2):

$$\mathbf{W}_1 = \frac{\left(e^{\hat{i}\alpha_3} - 1\right)\delta_{P_3} - \left(e^{\hat{i}\alpha_2} - 1\right)\delta_{P_2}}{\left(e^{\hat{i}\beta_2} - 1\right)\left(e^{\hat{i}\alpha_3} - 1\right) - \left(e^{\hat{i}\alpha_2} - 1\right)\left(e^{\hat{i}\beta_3} - 1\right)} \tag{14}$$

$$\mathbf{Z}_1 = \frac{\left(e^{\hat{i}\beta_2} - 1\right)\delta_{P_3} - \left(e^{\hat{i}\beta_3} - 1\right)\delta_{P_2}}{\left(e^{\hat{i}\beta_2} - 1\right)\left(e^{\hat{i}\alpha_3} - 1\right) - \left(e^{\hat{i}\alpha_2} - 1\right)\left(e^{\hat{i}\beta_3} - 1\right)} \tag{15}$$

$$\mathbf{W}_2 = \frac{\left(e^{\hat{i}\gamma_3} - 1\right)\delta_{Q_3} - \left(e^{\hat{i}\gamma_2} - 1\right)\delta_{Q_2}}{\left(e^{\hat{i}\beta_2} - 1\right)\left(e^{\hat{i}\gamma_3} - 1\right) - \left(e^{\hat{i}\gamma_2} - 1\right)\left(e^{\hat{i}\beta_3} - 1\right)} \tag{16}$$

$$\mathbf{Z}_2 = \frac{\left(e^{\hat{i}\beta_2} - 1\right)\delta_{Q_3} - \left(e^{\hat{i}\beta_3} - 1\right)\delta_{Q_2}}{\left(e^{\hat{i}\beta_2} - 1\right)\left(e^{\hat{i}\gamma_3} - 1\right) - \left(e^{\hat{i}\gamma_2} - 1\right)\left(e^{\hat{i}\beta_3} - 1\right)} \tag{17}$$

Substituting Eqs. (14–17) in Eq. (1) results in a complex equation in terms of the unknowns β_2 and β_3, only:

$$\mathbf{A}e^{\hat{i}2\beta_2} + \mathbf{B}e^{\hat{i}2\beta_3} + \mathbf{C}e^{\hat{i}\beta_2}e^{\hat{i}\beta_3} + \mathbf{D}e^{\hat{i}\beta_2} + \mathbf{E}e^{\hat{i}\beta_3} + \mathbf{F} = \mathbf{0} \tag{18}$$

where

$$\begin{aligned}
\mathbf{A} &= \left(e^{\hat{i}\gamma_3} - 1\right)\delta_{P_3} - \left(e^{\hat{i}\alpha_3} - 1\right)\delta_{Q_3} + \left(e^{\hat{i}(\alpha_3+\gamma_3)} - e^{\hat{i}\alpha_3} - e^{\hat{i}\gamma_3} + 1\right)\delta_{PQ} \\
\mathbf{B} &= \left(e^{\hat{i}\gamma_2} - 1\right)\delta_{P_2} - \left(e^{\hat{i}\alpha_2} - 1\right)\delta_{Q_2} + \left(e^{\hat{i}(\alpha_2+\gamma_2)} - e^{\hat{i}\alpha_2} - e^{\hat{i}\gamma_2} + 1\right)\delta_{PQ} \\
\mathbf{C} &= -\left(e^{\hat{i}\gamma_3} - 1\right)\delta_{P_2} - \left(e^{\hat{i}\gamma_2} - 1\right)\delta_{P_3} + \left(e^{\hat{i}\alpha_3} - 1\right)\delta_{Q_2} - \left(e^{\hat{i}\alpha_2} - 1\right)\delta_{Q_3} \\
&\quad - \left(e^{\hat{i}(\alpha_2+\gamma_3)} + e^{\hat{i}(\alpha_3+\gamma_2)} - e^{\hat{i}\alpha_2} - e^{\hat{i}\alpha_3} - e^{\hat{i}\gamma_2} - e^{\hat{i}\gamma_3} - 2\right)\delta_{PQ} \\
\mathbf{D} &= \left(e^{\hat{i}\gamma_3} - 1\right)e^{\hat{i}\alpha_3}\delta_{P_2} - \left(e^{\hat{i}(\alpha_2+\gamma_3)} - e^{\hat{i}\alpha_2} - e^{\hat{i}\gamma_2} + e^{\hat{i}\gamma_3}\right)\delta_{P_3} \\
&\quad - \left(e^{\hat{i}\alpha_3} - 1\right)e^{\hat{i}\gamma_3}\delta_{Q_2} - \left(e^{\hat{i}(\alpha_3+\gamma_2)} + e^{\hat{i}\alpha_2} - e^{\hat{i}\alpha_3} - e^{\hat{i}\gamma_2}\right)\delta_{Q_3} \\
&\quad + \left(e^{\hat{i}(\alpha_2+\gamma_3)} + e^{\hat{i}(\alpha_3+\gamma_2)} - 2e^{\hat{i}(\alpha_3+\gamma_3)} - e^{\hat{i}\alpha_2} + e^{\hat{i}\alpha_3} - e^{\hat{i}\gamma_2} + e^{\hat{i}\gamma_3}\right)\delta_{PQ}
\end{aligned}$$

$$\begin{aligned}
\mathbf{E} = & -\left(e^{\hat{i}(\alpha_3+\gamma_2)} - e^{\hat{i}\alpha_3} + e^{\hat{i}\gamma_2} - e^{\hat{i}\gamma_3}\right)\delta_{P_2} + \left(e^{\hat{i}\gamma_2} - 1\right)e^{\hat{i}\alpha_2}\delta_{P_3} \\
& + \left(e^{\hat{i}(\alpha_2+\gamma_3)} + e^{\hat{i}\alpha_2} - e^{\hat{i}\alpha_3} - e^{\hat{i}\gamma_2}\right)\delta_{Q_2} - \left(e^{\hat{i}\alpha_2} - 1\right)e^{\hat{i}\gamma_2}\delta_{Q_3} \\
& - \left(2e^{\hat{i}(\alpha_2+\gamma_2)} - e^{\hat{i}(\alpha_2+\gamma_3)} - e^{\hat{i}(\alpha_3+\gamma_2)} - e^{\hat{i}\alpha_2} + e^{\hat{i}\alpha_3} - e^{\hat{i}\gamma_2} + e^{\hat{i}\gamma_3}\right)\delta_{PQ} \\
\mathbf{F} = & \left(e^{\hat{i}\gamma_2} - e^{\hat{i}\gamma_3}\right)\left(e^{\hat{i}\alpha_3}\delta_{P_2} - e^{\hat{i}\alpha_2}\delta_{P_3}\right) - \left(e^{\hat{i}\alpha_2} - e^{\hat{i}\alpha_3}\right)\left(e^{\hat{i}\gamma_3}\delta_{Q_2} - e^{\hat{i}\gamma_2}\delta_{Q_3}\right) \\
& + \left(e^{\hat{i}(\alpha_2+\gamma_2)} - e^{\hat{i}(\alpha_2+\gamma_3)} - e^{\hat{i}(\alpha_3+\gamma_2)} + e^{\hat{i}(\alpha_3+\gamma_3)}\right)\delta_{PQ}
\end{aligned}$$

Multiplying Eq. (18) by $e^{-\hat{i}\beta_2}e^{-\hat{i}\beta_3}$ results in a simpler form:

$$\mathbf{A}e^{\hat{i}\beta_2}e^{-\hat{i}\beta_3} + \mathbf{B}e^{-\hat{i}\beta_2}e^{\hat{i}\beta_3} + \mathbf{F}e^{-\hat{i}\beta_2}e^{-\hat{i}\beta_3} + \mathbf{E}e^{-\hat{i}\beta_2} + \mathbf{D}e^{-\hat{i}\beta_3} + \mathbf{C} = \mathbf{0} \tag{19}$$

Writing real and imaginary parts of Eq. (19):

$$K_x c_2 c_3 + L_x s_2 s_3 + M_y s_2 c_3 + N_y c_2 s_3 + E_x c_2 + E_y s_2 + D_x c_3 + D_y s_3 + C_x = 0 \tag{20}$$

$$K_y c_2 c_3 + L_y s_2 s_3 - M_x s_2 c_3 + N_x c_2 s_3 + E_y c_2 - E_x s_2 + D_y c_3 - D_x s_3 + C_y = 0 \tag{21}$$

where c_2, s_2, c_3, s_3 stand for $\cos\beta_2$, $\sin\beta_2$, $\cos\beta_3$, $\sin\beta_3$, respectively, and $\mathbf{K} = K_x + K_y\hat{i} = \mathbf{A} + \mathbf{B} + \mathbf{F}$, $\mathbf{L} = L_x + L_y\hat{i} = \mathbf{A} + \mathbf{B} - \mathbf{F}$, $\mathbf{M} = M_x + M_y\hat{i} = -\mathbf{A} + \mathbf{B} + \mathbf{F}$ and $\mathbf{N} = N_x + N_y\hat{i} = \mathbf{A} - \mathbf{B} + \mathbf{F}$. Either of β_2 or β_3 can be eliminated from Eqs. (20–21). Let's eliminate β_3: First linearly solve for c_3 and s_3 from Eqs. (20–21). Then using $c_3^2 + s_3^2 = 1$, an equation in terms of only β_2 is obtained. This equation is of fourth order in terms of c_2 and s_2, which indicates that there are at most eight real solutions for β_2. After determining β_2, the corresponding $\beta_3 = \mathrm{atan2}(s_3, c_3)$ are found and $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{W}_2$ and $\mathbf{Z}_2$ are determined from Eqs. (14–17).

Table 1 indicates that for the three pose synthesis the 10 parameters are to be solved from Eqs. (1–3) without any free selected parameters. This implies that for given three poses of the two end-effectors, there are finitely many solutions. It is well-known from the five pose synthesis of a four-bar mechanism that in case of finitely many solutions, finding the solution is computationally problematic and usually it is hard to find a practically applicable solution. Also all poses may not be attained in the same assembly mode of the mechanism. These problems were also encountered in our computational studies as well.

For the solution for the remaining dyads we can choose two parameters freely per dyad. When we choose η_2, η_3, ϕ_2 and ϕ_3 the rest of the unknowns can be easily solved linearly. We may use Eqs. (14–17) by substituting η instead of β in Eqs. (14–15) for $\mathbf{W}_3$ and $\mathbf{Z}_3$ and ϕ instead of β in Eqs. (16–17) for $\mathbf{W}_4$ and $\mathbf{Z}_4$.

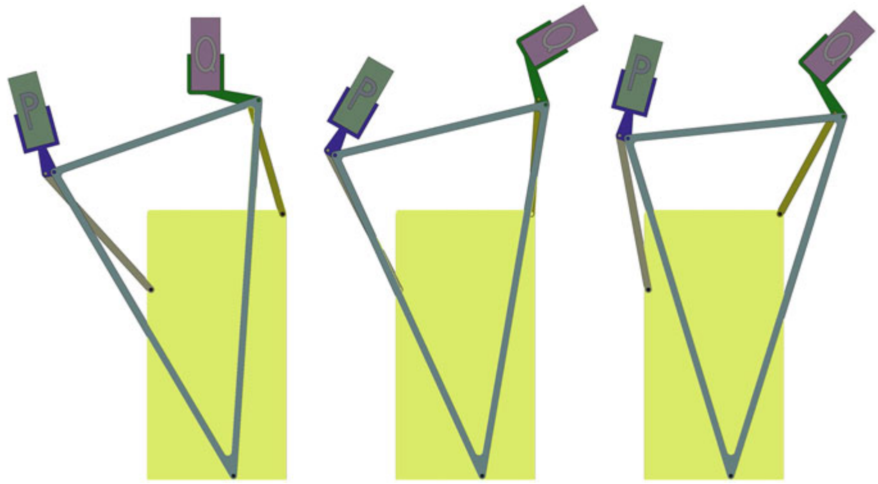

Fig. 4 Three poses of the designed mechanism

As an example consider the case where $\delta_{P_2} = 15 + 5\hat{i}$, $\delta_{P_3} = 25 + 10\hat{i}$, $\delta_{Q_2} = 20 + 10\hat{i}$, $\delta_{Q_3} = 30 + 5\hat{i}$, $\alpha_2 = -45°$, $\alpha_3 = -30°$, $\gamma_2 = -60°$ and $\gamma_3 = -45°$. Using Maple®, one of the solutions for β_2 is solved numerically resulting $\beta_2 = -5.908°$, $\beta_3 = -13.576°$, $\mathbf{W}_1 = -49.057 + 83.162\hat{i}$, $\mathbf{Z}_1 = -49.057 + 83.162\hat{i}$, $\mathbf{W}_2 = 6.848 + 102.869\hat{i}$ and $\mathbf{Z}_2 = -14.468 + 2.556\hat{i}$. For the other two dyads, assuming $\eta_2 = -15°$, $\eta_3 = -20°$, $\phi_2 = -20°$ and $\phi_3 = -45°$, the link vectors are solved as $\mathbf{W}_3 = -28.833 + 31.755\hat{i}$, $\mathbf{Z}_3 = -1.233 + 7.690\hat{i}$, $\mathbf{W}_4 = -8.850 + 30.196\hat{i}$ and $\mathbf{Z}_4 = -12.185 + 3.517\hat{i}$. The three poses of the resulting mechanism is illustrated in Fig. 4.

5 Conclusions

This study is a first attempt for kinematic synthesis of closed-loop mechanisms with several end-effectors, motions of which are independently described with respect to the base. Specifically, the motion synthesis problem for a Watt II mechanism with two end-effectors is issued. With dyad formulation it was shown that nine parameters may be selected freely for two pose synthesis and four parameters are free for three pose synthesis. Examples showed that three pose synthesis is prone to computational problems and the solutions are hardly practical. However, the two pose synthesis proved itself quite successful.

The methods discussed in this study can easily be adapted to other two loop planar mechanisms which comprise prismatic joints as well. Also similar formulation can be devised for spherical mechanisms. The dyad formulation may also be used for path generation problem for mechanisms with two end-effectors.

More than two end-effectors in a mechanism is quite rare, nevertheless it seems that at least two pose synthesis is possible for a mechanism with three end effectors. The main course of progress in our study will be focused on synthesis methods for multi-dof mechanisms with several end-effectors.

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