# JOINT RECONSTRUCTION OF SURFACE GEOMETRY AND REFLECTION PROPERTIES BY USING IMAGE BASED METHODS 

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#### Abstract

\section*{JOINT RECONSTRUCTION OF SURFACE GEOMETRY AND REFLECTION PROPERTIES BY USING IMAGE BASED METHODS}


In this thesis, we aim to capture realistic geometrical descriptions of real world scenes and objects with a special effort to characterize reflection properties.

After a brief review of the stereo imaging literature, we show our contributions to enhance stereo matching performance by identifying and eliminating specular surface reflections. The identification of specular reflection can be done both passively and actively. We use dichromatic-based methods to identify and eliminate specular reflections passively. We utilize polarization imaging methods to do the same job actively.

In this work we also study structured light based methods that can give better reconstruction results compared to stereo imaging methods. We propose three laser scanners equipped with a pair of line lasers and a method to calibrate these systems. Another convenient way to obtain good surface reconstruction results using structured light is to use projectors that can be used as a light source that project complicated patterns. We show our results from a digital camera-projector-based scanning system as well. This system can robustly generate a very dense reconstruction of surfaces.

We also use the projector based scanning system to determine the surface reflection properties. Using high dynamic range imaging (HDRI) techniques makes it possible for us to estimate scene radiance values. Since we can determine the incoming and outgoing light directions, we are able to measure bidirectional reflectance distribution function (BRDF) values from reconstructed surface points for corresponding directions. If the sample surface have not only diffuse reflection components but also a sufficient amount of specular highlights, it is possible to approximate BRDF corresponding to a surface by fitting an analytical BRDF model to the measured data. In our work we preferred to use Phong BRDF model.

Finally, we present results with rendered synthetic images where the parameter values of the Phong model were estimated using scans of real objects.

## ÖZET

## YÜZEY GEOMETRİSİ VE YANSIMA ÖZELLİKLERİNİN GÖRÜNTÜ TABANLI YÖNTEMLER KULLANILARAK BİRLEŞİK GERİÇATIMI

Bu tez çalışmasında gerçek sahnelerin ve nesnelerin gerçekçi geometrik tanımlarını, aynı zamanda yansıma özelliklerini de karakterize etmeye çalışarak elde etmeye çalışıyoruz.

Stereo imgeleme literatürü ile ilgili genel bir özet yaptıktan sonra stereo imgeleme yöntemlerindeki stereo eşleşme başarımının aynasal yansımaların tanımlanması ve yokedilmesi ile arttırılabilmesine yönelik kendi katkılarımızı da sunuyoruz. Aynasal yansıma özelliklerinin tanımlanması pasif ve aktif olarak gerçekleştirilebilmektedir. Dikromatik tabanlı yöntemler bu işlemin pasif olarak gerçekleştirilebilmesini sağlamaktadır. Polarizasyon imgeleme yöntemlerini kullanarak aynı işi aktif olarak da gerçekleştiriyoruz.

Bu çalışmada aynı zamanda stereo imgeleme yöntemlerine nazaran çok daha iyi geri çatım sonuçları verebilen yapısal aydınlatma tabanlı yöntemleri de ele aldık. Öncelikle çift çizgisel lazer kullanan üç ayrı tarayıcı sistem ve bunlara ilişkin bir kalibrasyon yöntemi öneriyoruz. Yapısal aydınlatma kullanarak iyi bir yüzey geri çatımı elde etmek için kullanılabilecek olan bir diğer yöntem ise karmaşık örüntüler yansıtabilen projektörleri kullanmaktır. Burada aynı zamanda dijital fotoğraf makinesi ve projektör ile oluşturulan bir tarama sitemi ile elde etitğimiz sonuçları da gösteriyoruz. Bu sistem yüzeylere ait çok yoğun geri çatımların elde edilebilmesini sağlamaktadır.

Projektör tabanlı sistemi aynı zamanda yüzeylerin yansıtıcılık özelliklerinin geri çatımı için de kullanıyoruz. Yüksek dinamik erimli imgeleme (YDEI) teknikleri sahnedeki ışınırlık değerlerini tahmin edebilmemizi mümkün kılmaktadır. Yüzeye gelen ve yüzeyden yansıyan ışık yönlerini de belirleyebildiğimiz için geri çatılan yüzey noktalarındaki çift yönlü yansıma dağılımı fonksiyonu (ÇYDF) değerlerini ilgi yönler için ölçebiliyoruz. Şayet örnek yüzey yayınık yansıma bileşeninin yanısıra yeterli müktarda aynasal yansıma bileşenine de sahip ise ölçülen veriye analitik bir ÇYDF fonsiyonunun oturtulması ile yüzeyin ÇYDF'sine yaklaşmak mümkün olabilmektedir. Çalışmamızda Phong ÇYDF modelini kullanmayı tercih ettik.

Son olarak elde ettiğimiz sonuçları gerçek nesnelerin taranması ile kestirilen Phong ÇYDF modeli parametre değerlerinin kullanılması ile gerçeklenmiş sentetik imgelerle sunuyoruz.

To my family ...

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## LIST OF ABBREVIATIONS

CN Color Normalization
SR Specularity RemovalPIPolarization Images
DS3DLS Double Stripe 3D Laser Scanner
LR Linear Regression
LM Levenberg-Marquardt
UPC Unorganized Point Cloud
3D Three Dimensional
SFS Shape from Shading
HDRI High Dynamic Range Imaging
BRDF Bi-directional Reflectance Distribution FunctionRMSRoot Mean Square
HD High Definition
ICA Independent Component Analysis
TRS Transmitted Radiance Sinusoidal
SVD Singular Value Decomposition
NLSA Nonlinear Least Squares Algorithm
RHT Randomized Hough Transform
NMA Nelder Mead Algorithm
SMP Stereo Matching Problem, Stereo Matching Performance
DRM Dichromatic Reflection Model
NIR Neutral Interface ReflectionRGBRed Green Blue
IICS Inverse Intensity Chromaticity Space
HSI .Hue, Saturation, Intensity Color Space
NCCBSM Normalized Cross Correlation Based Stereo Matching
PSNRPeak Signal to Noise Ratio
PBRT .Physically Based Ray Tracer

## CHAPTER 1

## INTRODUCTION

Recent advances in computers and digital imaging technologies have made it possible for researchers to deploy complex methods and algorithms to capture geometrical descriptions of the surrounding physical environment. Real-time systems and applications where the 3D scenes are captured instantly have become widely popular in gaming industry (e.g. X-Box Kinect). Such systems can sense the 3D environment and they are able to understand the players' body gestures by roughly identifying them in 3D space. If the aim is not only to sense but also to correctly describe the surrounding 3D space in detail, it is still a difficult task to perform with conventional sensory devices and computers.

In this thesis we aimed to extract a description the 3D environment by using different image based methods. Following a survey of various methods, we show our contributions to the existing literature. Firstly, we focused on the effect of specular surface reflections on stereo imaging causing false matching results. The identification of specular reflection can be done both passively and actively. We prefer to use dichromatic-based methods to identify and eliminate specular reflections passively. We also use polarization imaging method to do it actively. Consequently, we show that eliminating specular reflections in scenes can significantly improve stereo matching performance.

Conventional stereo imaging methods are usually not accurate in acquiring scene depth information. If more detailed reconstruction results are needed, using structured light systems is a better choice. In this thesis we also study structured light based methods and propose three laser scanners with a pair of line lasers that utilizes a novel method to calibrate these systems. Another convenient way to obtain accurate surface reconstruction is to use projector based scanning systems. In our work we also propose a projector based scanning system. This system can generate dense reconstruction of a scene. Since the system, both the projector and camera, is calibrated the reconstruction results are metrically consistent and can be used for measurement if needed. Moreover, the projector based system helps us to characterize surface reflection properties by using some basic assumptions.

Arguably, the most common and comprehensive way to define surface reflection is using bidirectional reflectance distribution functions (BRDFs). Analytical BRDF models are used for realistic rendering in computer graphics. These functions can be fitted to
measured data and mimic a materials surface reflection characteristics. In our work we show that even with limited surface reflection information, it is possible to generate a limited BRDF measurement data set for objects in the scene and estimate parameters of analytical BRDF models which can be used to capture and represent sample surfaces' physical reflection properties to some extend.

### 1.1. Reconstruction of Surface Geometry

Reconstruction of 3D surface geometry is an active research area in both computer vision and computer graphics. Due to physical and geometrical varieties of object surfaces, many different application specific scanning systems, methods and algorithms have been developed in order to gather 3D surface geometry.

Stereo imaging methods, especially two-frame stereo methods are widely studied in computer vision. Solving the stereo correspondence problem is the fundamental task in these methods. In (Scharstein and Szeliski, 2002) a detailed taxonomy and performance analysis of these methods can be found. In traditional stereo imaging, the sensor position is changed where the scene illumination is preferably kept constant. On the contrary, in photometric stereo techniques ((Horn, 1986), (Woodham, 1989)), the sensor position is fixed and the scene illuminant position is changed accordingly. Photometric stereo is also known as shape from shading (SFS). Photometric stereo techniques originally assumes the surfaces are pure Lambertian, i.e. there is no specular reflection from the surface which is a very strict generalization which makes it difficult to solve SFS problem since Lambertian model is an idealized model and real material surfaces are not always pure Lambertian. In (Prados and Faugeras, 2005) the corresponding problem is modeled more precisely by rewriting adding an additional term and solve the problem as a partial differential equation, which gives better results compared to conventional SFS solving methods. Also in (Ahmed and Farag, 2006) it is proposed to define the surface reflection with OrenNayar model (Oren and Nayar, 1993). The model is more robust in representing diffuse surface reflection of real materials.

Methods for determining depth from focus (Ens and Lawrence, 1993) require multiple images of the scene taken with different focal parameters, where the depth from defocus methods ((Subbarao and Surya, 1994) and (Watanabe and Nayar, 1998)) requires fewer images. In both methods, the reflections in the scene are considered to be Lambertian. For semiglossy dielectric materials, shape from polarization can be used. In (Atkinson and Hancock, 2007) and (Rahmann and Canterakis, 2001) polarization imag-
ing methods are used to estimate local surface normals and to roughly recover the surface shape.

Another common method to capture 3D information is to use structured light. Using projection of a coded light pattern, it is possible to capture a depth image from a single image frame. Different coding strategies are used for generating light patterns (Pages et al., 2003) (Salvi et al., 2004) (Salvi et al., 2010). Structured light patterns are also shown to be effective when used in conjunction with stereo techniques (Scharstein and Szeliski, 2003).

In laser scanner systems, emitters that project light stripes are used. In these systems, the main problem is the calibration of the light source (Niola et al., 2011) (Zhou and Zhang, 2005) (Wei et al., 2005). But laser stripe based scanner systems suffer from occlusion problem. A shadow is produced when the light stripe is blocked from the viewpoint by a surface. In (Ozan and Gümüştekin, 2014a) we implemented three laser scanner systems with two line lasers which minimizes scene occlusions. This setup is preferable to a single laser - multiple camera scenario since it significantly reduces cost and gives more flexibility in different types of scanning problems.

The only constraint imposed in (Ozan and Gümüştekin, 2014a) is about positioning the two light sources. The stripes should be visible in different halves of the image frame. We also propose a method to calibrate lasers with respect to the camera. The method uses geometrical constraints like coplanarities and orthogonalities imposed in the scene. In (Furukawa and Kawasaki, 2009), calibration techniques using similar geometrical constraints are reported. However their method needs user interaction to identify planar surfaces. In our study the detection of planar surfaces are performed automatically during calibration. Once the calibration is performed, a 3D model can be reconstructed from the data acquired by simultaneously using the two laser stripes. In (Park et al., 2001) a least squares solution that optimizes linear transformations corresponding to light sources are searched using a complex calibration object that produces 18 distinct point positions. Although their test setup is similar to one of the scanners studied here, the main difference in our approach is that it attempts to generalize double stripe scanning problem of different possible scanning scenarios using planarities and orthogonalities occurring in the scene.

There are also different projector based structured light systems using different light patterns. In (Young et al., 2007), a method using frequency coded structured light patterns is proposed. In frequency coded structured light methods, monochromatic light planes with different frequencies are projected to the scene. In color-coded structured
light methods, (Caspi et al., 1998) and (Chen et al., 2008), the color information of the light planes is used to encode the distribution of the light stripes over sample subjects. In another group of structured light methods, pseudo random coded structured light patterns are used. In (Lavoie et al., 2004), random coded structured light with a grid pattern and two camera views are used to recover 3D object model by solving stereo correspondence problem.

### 1.2. Reconstruction of Surface Reflection Properties

Physically based rendering is an active research area in computer graphics field. The most important aspect of realistic rendering is to describe the appearance of a material by its interaction with light. The BRDFs are used for that purpose. Also BRDF is the most convenient data structure to store reflection properties of a material. Since it contains every possible incoming and outgoing light directions, it is a hard and time consuming process to measure real material BRDFs. In (Hünerhoff et al., 2006) a BRDF measurement technique with a gonioreflectometer installed to a robot arm is proposed. It is also possible to measure BRDF by using digital imaging techniques as well (Marschner et al., 2000), (Matusik, 2003). The public database of Matusik's measurements (Matusik, 2003) is yet the most comprehensive measured BRDF database of 100 materials. This database is used as ground truth for testing existing or new analytical BRDF models.

In practice rendering a scene by using measured BRDF data is not the optimal solution to obtain fast results. Hence analytical BRDF models which mimics real material reflections are preferred to obtain realistic rendering results faster. The speed is especially important for realtime hardware rendering which is widely used in today's computer game industry. In (Ngan et al., 2005) various analytical BRDF models are surveyed and their fitting performances are analyzed. Also a novel method to measure anisotropic materials, such as velvet, BRDFs is also proposed. A small database of four anisotropic BRDF measurements can be publicly reached from the authors website.

In our work we use the projector based 3D scanning system not only for reconstructing surface geometry but also for surface reflection properties. The system also gives us the flexibility to make radiance measurements from within the scene. Furthermore, it is possible to convert gathered geometrical and radiometric information to BRDF data which can be later used to estimate parameters of an appropriate analytical BRDF model. The radiometric measurements can be done by performing high dynamic range imaging (HDRI) method proposed in (Debevec and Malik, 2008).

### 1.3. Contributions to Existing Literature

In this work we are reconstructing both surface geometry and surface reflection properties. Furthermore while reconstructing one feature we make the use of the other, i.e. we use surface reflection properties to improve reconstruction results of surface geometry and we use the surface geometry to improve reconstruction results of surface reflection properties.

Firstly, we want to enhance stereo matching performance of stereo imaging methods by eliminating specular reflections in the scene. While performing specularity elimination we propose a single-image-based method which is mainly used to identify illuminant chromaticity. Compared to existing single-image-based illuminant chromaticity estimation methods, we propose a simple and easily implementable method. Moreover, we use the illuminant chromaticty to eliminate specular reflections in the image and use the specular free stereo image pairs to generate a better depth map of a scene. We similarly use polarization images to eliminate specular reflections in the scene to enhance stereo matching performance. Hence we use surface reflection properties, in this case specular reflection, to enhance reconstructed surface geometry.

Secondly, we want to reconstruct surface reflection properties by using surface geometry. We introduce a surface reconstruction system which makes the use of a camera and a projector. Furthermore, we propose a method to reconstruct surface reflection properties by using the reconstructed surface geometry and radiometric measurements performed by high dynamic range imaging. The proposed method makes use of the definition of bidirectional reflectance distribution function. Hence we use surface geometry to reconstruct surface reflection properties.

### 1.4. Thesis Outline

We divide the thesis into two main chapters. In Chapter 2 we focus on reconstruction of surface geometry. In Section 2.2, we analyze stereo matching methods. We also show that by analyzing surface reflection we can eliminate the artifact called specular reflection. Eliminating specular reflection in the scene can be performed by using both active or passive methods and it significantly enhances the performance of stereo imaging methods. We show that reconstructed surface reflection properties, i.e. specularity, can jointly be used to enhance depth maps generated using stereo methods.

In Section 2.3, we also introduce two types of structured light systems and their corresponding calibration methods. First system is constructed with two line lasers aligned parallel. We give three versions of this double stripe 3D laser scanner. The second scanning system is a projector and camera based scanning system. The structured light based scanning system have the ability to generate a dense reconstruction of a scene which is to be scanned.

In Chapter 2, the main focus is the reconstruction of surface reflection properties. We claim that the projector based structured light system introduced in Section 2.3 can also be used to reconstruct the surface reflection properties. We start by reviewing some basic radiometric quantities. Then we introduce the BRDF concept and some basic analytical BRDF models. We measure reflection data from the scene and obtain our own BRDF measurement data. We use an analytical BRDF model to fit the corresponding measurements and show the results of reconstructed reflection data. Hence in Chapter 2 we show that reconstructed surface geometry can be used to identify surface reflection and can jointly be used to reconstruct surface reflection properties.

Finally in Chapter 4 we summarize and conclude our work and give ideas about our possible future works.

## CHAPTER 2

## RECONSTRUCTION OF SURFACE GEOMETRY

### 2.1. Background and Motivation

In this chapter we present our contributions to surface reconstruction methods. Stereo imaging is the basic method which mainly inherits the 3D perception of human visual system. Two images taken by two side by side cameras can give a rough information about scene depth. Stereo Matching Problem (SMP) is the main problem to be solved. A detailed analysis of SMP solution methods can be seen in (Scharstein and Szeliski, 2002). Stereo imaging methods tend to fail if there exist specular reflections in the scene. Here, we propose two methods to eliminate the specular reflection by identifying it. Eliminating specular component of surface reflections significantly improves the quality of scene depth images.

Moreover, in this chapter we propose structured light scanning systems. First we introduce three generic laser scanners which can be used for different purposes. We also propose a novel calibration method for our these scanners.

Recent advances in projector technology made it possible to find affordable HD projectors for daily use. Hence it became a convenient way to use projector as a structured light source. In this chapter we also propose a projector-camera system which can give dense reconstruction results as well. Corresponding system also makes it possible to reconstruct surface reflection properties which is the subject of the next chapter. We start by identifying and eliminating specular reflections.

### 2.2. Stereo Imaging Methods

Stereo imaging is the most common and intuitive method for recovering depth information in a scene. In this section we will consider stereo imaging methods in detail. We will also present our contributions to the existing literature by means of enhancing stereo matching performance. Our proposed methods aim to eliminate specular reflections.

### 2.2.1. Enhancing Stereo Matching Performance by Color Normalization and Specularity Removal

The main goal of stereo methods is to generate a disparity map from image pairs. Specular reflections, especially in cases where the position of the specular reflection changes drastically, cause unwanted results. This position change is mostly due to a change in the relative orientation of the object surface with respect to the camera. The light source position may also change independently.

In (Lin et al., 2002), a stereo-enhanced color processing method which deals with the unwanted effect of specular variations on surfaces is given by Lin et al. This method uses an image sequence to remove specularity with a strict assumption that all scene points present diffuse reflection in at least one of the images in the image sequence. Specularities are also a problem in 3D object tracking applications and treated as noise. (Lagger, 2009) addresses this problem and solves it by using the position change of specularities on a surface in a set of images to find the number and direction of multiple lights sources. In (Bhat and Nayar, 1998), Bhat and Nayar embedded a physically based model of reflected light into a stereo matching method to obtain a better stereo matching result in the specular regions of images, but their analysis required controlled lighting conditions and fully calibrated camera positions.

Specular reflections also make shape from shading algorithms fail. In (Mallick et al., 2005) it is proposed to pre-process source images to separate diffuse and specular reflections and obtain specular-free images which hold not the color but the shading information of original images. This makes it possible to use generated specular free images to solve the corresponding SFS problem.

In (Ozan and Gümüştekin, 2011a), we propose a method which does not have restrictive assumptions. Furthermore, it does not require a complicated experimental setup. The only assumption in the proposed method is that the pixel values in the images are not saturated. This assumption holds in most cases where exposure settings avoid saturation and objects do not contain mirror-like highly reflective parts. Our method is based on identifying surface reflections, i.e. specular and diffuse reflections by using dichromatic reflection model. Dichromatic reflection model is a linearized model of surface reflection for dielectric material surfaces.

### 2.2.1.1. Dichromatic Reflection Model (DRM)

Most of the dielectric materials have semi glossy surfaces which can cause specular highlights. The nature of the illuminant is also important such that the same object may not have specular reflections once illuminated by different kind of illuminants. In Figure 2.1a a ceramic frog figurine which is illuminated by a directional light source can be seen. One can easily observe the specular reflections following the surface geometry. Once the same object is illuminated by a diffuse light source the specularity is not evident (see Figure 2.1b).


Figure 2.1. Frog figurine illuminated with different lighting conditions.

In (Shafer, 1985) it is proposed to represent the surface reflection as a linear combination of both specular and diffuse components. This simplified model is popular and widely used in computer graphics and geometric optics literature to solve various problems. If light source chromaticity is to be estimated, this model can be used. Even with a single image light source chromaticity can be estimated.

The chromaticity of scene illuminant can be approximately estimated by analyzing the specular component of surface reflections in a scene. In (Tan et al., 2003) a color constancy method is proposed to estimate illuminant chromaticity from a single image. The method uses the dichromatic reflection model as a basis.

Definition Dichromatic reflection model (DRM) states that surface reflection is a linear combination of diffuse and specular reflections for dielectric materials. For a single wavelength $(\lambda)$ of the spectrum this can be given as Equation 2.1 for a single point in the space.

$$
\begin{equation*}
I(\lambda, \overline{\mathbf{x}})=w_{d}(\overline{\mathbf{x}}) S_{d}(\lambda, \overline{\mathbf{x}}) E(\lambda, \overline{\mathbf{x}})+w_{s}(\overline{\mathbf{x}}) S_{s}(\lambda, \overline{\mathbf{x}}) E(\lambda, \overline{\mathbf{x}}) \tag{2.1}
\end{equation*}
$$

In this equation, $\overline{\mathbf{x}}$ represents 3D world coordinate of the point. $w_{d}(\overline{\mathbf{x}})$ and $w_{s}(\overline{\mathbf{x}})$ are weights for diffuse and specular reflections where $S_{d}(\lambda, \overline{\mathbf{x}})$ and $S_{s}(\lambda, \overline{\mathbf{x}})$ are the diffuse and specular spectral reflectance functions. $E(\lambda, \overline{\mathbf{x}})$ is the spectral energy distribution function of the illumination (Tan et al., 2003)

For most dielectric objects, the spectral reflectance distribution of the specular reflection component is similar to the spectral energy distribution of the incident light. This phenomenon is called as the neutral interface reflection (NIR) (Lee et al., 1990). Hence it can be assumed that $S_{s}(\lambda, \overline{\mathbf{x}})$ is wavelength independent and can be written as a constant $k_{s}(\overline{\mathbf{x}})$ which yields us to Equation 2.2.

$$
\begin{equation*}
I(\lambda, \overline{\mathbf{x}})=w_{d}(\overline{\mathbf{x}}) S_{d}(\lambda, \overline{\mathbf{x}}) E(\lambda, \overline{\mathbf{x}})+\tilde{w}_{s}(\overline{\mathbf{x}}) E(\lambda, \overline{\mathbf{x}}), \text { where } \tilde{w}_{s}(\overline{\mathbf{x}})=w_{s}(\overline{\mathbf{x}}) k(\overline{\mathbf{x}}) . \tag{2.2}
\end{equation*}
$$

A digital camera behaves as an integrator for this relation in the visible spectrum. Conventional digital cameras have three sensors for red green and blue (RGB) color of the visible spectrum. For an image taken by a digital camera Equation 2.2 can be written as Equation 2.3.

$$
\begin{equation*}
I_{c}(\mathbf{x})=w_{d}(\mathbf{x}) \int_{\Omega} S_{d}(\lambda, \mathbf{x}) E(\lambda) q_{c}(\lambda) d \lambda+\tilde{w}_{s}(\mathbf{x}) \int_{\Omega} E(\lambda) q_{c}(\lambda) d \lambda . \tag{2.3}
\end{equation*}
$$

In this equation x represents the two dimensional (2D) image coordinates, $q_{c}(\lambda)$ digital cameras sensors response function where $c$ represents sensor type (R,G and B). The integrals are defined over the visible spectrum ( $\Omega$ ). Since the color of illumination is uniform $E$ becomes independent of image coordinate. The equation can further be simplified and written as Equation 2.4.

$$
\begin{equation*}
I_{c}(\mathbf{x})=m_{d}(\mathbf{x}) \Lambda_{c}(\mathbf{x})+m_{s}(\mathbf{x}) \Gamma_{c} . \tag{2.4}
\end{equation*}
$$

In this equation $m_{d}(\mathbf{x})=w_{d}(\mathbf{x}) L(\mathbf{x}) k_{d}$ where $L(\mathbf{x})$ is the spectral magnitude of the surface irradiance and $k_{d}$ is the ratio between surface irradiance and scene radiance. Similarly $m_{s}(\mathbf{x})=\tilde{w}_{s}(\mathbf{x}) L(\mathbf{x})$. Moreover $\Lambda_{c}(\mathbf{x})$ and $\Gamma_{c}$ can be written as Equation 2.5 in which $s(\lambda, \mathbf{x})$ and $e(\lambda)$ are the normalized surface reflectance and illumination spectral energy distribution functions respectively.

$$
\begin{align*}
\Lambda_{c}(\mathbf{x}) & =\int_{\Omega} s(\lambda, \mathbf{x}) e(\lambda) q_{c}(\lambda) d \lambda, \\
\Gamma_{c} & =\int_{\Omega} e(\lambda) q_{c}(\lambda) d \lambda . \tag{2.5}
\end{align*}
$$

Definition Chromaticity $c(\mathbf{x})$ can be defined as Equation 2.6. By considering the above derivations three types of chromaticity can be considered: image chromaticity $(c(\mathbf{x}))$,
surface chromaticity $\left(\Lambda_{c}(\mathbf{x})\right)$ and illumination chromaticity $\left(\Gamma_{c}\right)$. Where $\sum \Lambda_{i}=\sum \Gamma_{i}=$ 1.

$$
\begin{align*}
c(\mathbf{x}) & =\frac{I_{c}(\mathbf{x})}{\sum I_{i}(\mathbf{x})} \\
\text { where } \sum I_{i}(\mathbf{x}) & =I_{r}(\mathbf{x})+I_{g}(\mathbf{x})+I_{b}(\mathbf{x}) \tag{2.6}
\end{align*}
$$

By plugging Equation 2.4 into Equation 2.6, image chromaticity can be written as Equation 2.7. For simplicity image location vector x can be removed and $m_{s}$ and $m_{d}$ can be related as in Equation 2.8

$$
\begin{gather*}
c(\mathbf{x})=\frac{m_{d}(\mathbf{x}) \Lambda_{c}(\mathbf{x})+m_{s}(\mathbf{x}) \Gamma_{c}}{m_{d}(\mathbf{x}) \sum \Lambda_{i}(\mathbf{x})+m_{s}(\mathbf{x}) \sum \Gamma_{i}} .  \tag{2.7}\\
m_{s}=\frac{m_{d}\left(\Lambda_{c}-c\right)}{c-\Gamma_{c}} \tag{2.8}
\end{gather*}
$$

From Equation 2.4 and Equation 2.8, Equation 2.9 can be written.

$$
\begin{equation*}
I_{c}=m_{d}\left(\Lambda_{c}-\Gamma_{c}\right)\left(\frac{c}{c-\Gamma_{c}}\right) . \tag{2.9}
\end{equation*}
$$

Definition In Equation 2.9, a parameter $p=m_{d}\left(\Lambda_{c}-\Gamma_{c}\right)$ can be used which makes it possible to write Equation 2.10. This representation is called to as inverse intensity chromaticity space (IICS). Inverse intensity ( $1 / \sum I_{i}$ ) chromaticity (c) space makes it possible to estimate illumination chromaticity $\left(\Gamma_{c}\right)$.

$$
\begin{equation*}
c=p \frac{1}{\sum I_{i}}+\Gamma_{c} . \tag{2.10}
\end{equation*}
$$

In (Tan et al., 2003) it is shown that the points in IICS can be shown as lines in $p-c$ space by using Hough Transform. The line intersections are concentrated at a single point which can be used as an estimate for $\Gamma_{c}$. This requires significant amount of processing time since the domain is interchanged and the line intersections for each line pair combinations are calculated.

In (Ozan and Gümüştekin, 2011b) we propose a much simpler method which is easier to implement and runs faster than the method in (Tan et al., 2003). The method in (Ozan and Gümüştekin, 2011b) starts with selecting specular pixel candidates. This selection is done by using thresholds in hue-saturation-intensity color space (HSI) as proposed in (Lehmann and Palm, 2001) which can be given as in Equation 2.11.

$$
\begin{align*}
& I=\frac{I_{R}+I_{G}+I_{B}}{3}>T_{1} I_{\max }, \\
& S=1-\frac{\min \left(I_{R}, I_{G}, I_{B}\right)}{I}<T_{2} S_{\max } . \tag{2.11}
\end{align*}
$$



Figure 2.2. Pixels in the specular region of image can be selected by using thresholds shown in Equation 2.11. The Figure shows the corresponding pixes in IICS

In this equation $I_{\max }$ and $S_{\max }$ represents the maximum intensity and saturation values in the whole image respectively. $T_{1}$ and $T_{2}$ are user defined threshold parameters. Threshold parameters are not necessarily to be selected precisely since the aim is to roughly select the specular pixels in the image.

The method in (Ozan and Gümüştekin, 2011b) proposes to make the statistical analysis in IICS without Hough Transform. In Figure 2.2 an experimental result where specular candidate pixels are shown in IICS. The aim is to find an estimate of $\Gamma_{c}\left(\Gamma^{\prime}\right)$ which resides on $y$ axis of the plot.

The method says that any point selected on $y$ axis can be an estimate. If the normalized distribution of the slopes of lines ( $p$ ) connecting points on IICS to the selected candidate $\Gamma^{\prime}$ are plotted for two different values of $\Gamma^{\prime}$, Figure 2.3 is obtained. Normalized distributions are normalized histogram of slope distributions. If the standard deviation of these plots are separately plotted Figure 2.4 is obtained. Our observations show that $\Gamma^{\prime}$ value which corresponds to the global maximum of this plot is an optimal estimate for $\Gamma_{c}$.

In Figure 2.5 experimental measurements can be seen. The images are taken by placing different color filters in front of the cameras flash. Figures $2.5 \mathrm{a}, 2.5 \mathrm{~b}$ and 2.5 c are


Figure 2.3. Normalized distribution of the slopes in Figure 2.2 for different $\Gamma_{c}$ candidates $\Gamma^{\prime}$.


Figure 2.4. Standard deviations of slope distributions for $\Gamma^{\prime}$ values are plotted. The global maximum point of this plot gives the optimal estimate $\Gamma^{\prime}$ for $\Gamma_{c}$.


Figure 2.5. Frog figurine illuminated by light sources with different chromaticity values. Light source chromaticities are measured by using a white balance card and given below each image in RGB format. (a), (b) and (c) illuminated by green dominant light sources where (d), (e) and (f) are illuminated by red dominant light sources.
taken with different green filters where figures $2.5 \mathrm{~d}, 2.5 \mathrm{e}$ and 2.5 f are taken with different red filters.

In order to generate ground truth for estimations a white balance card is captured with the same illuminant setting for each of the measurements. In Table 2.1 estimates for each of the three color channel results are shown together with the ground truth values.

Another metric is to use the slope distributions directly. It also gives the best estimate for illuminant chromaticity values. To show the corresponding method Figure 2.6 can be considered. The corresponding scene is illuminated by a light source which has a blue dominant wavelength spectrum. It is expected to find a significant blue chromaticity value once the proposed analysis is performed. If the specular regions in the image are roughly filtered by using the above thresholding method in Equation 2.11 and corresponding pixels are shown in IICS Figure 2.7 is obtained for three color channels.

Similar to the above method for any point on chromaticity axis, slopes of lines connecting that point with the pixels distributed in IICS can be calculated and plotted. It is expected to observe that the diversity of slopes declines. To highlight this better two extreme values, namely 0.0001 and 0.9999 , are considered for $\Gamma^{\prime}$. The calculated slopes for each color channels are given in Figure 2.8. If the standard deviation of these

Table 2.1. Ground Truth/Estimated illuminant chromaticity values of different illuminants in experiments shown in Figure 2.5.

| Experiment | $\Gamma_{R} / \Gamma^{\prime}$ | $\Gamma_{G} / \Gamma^{\prime}$ | $\Gamma_{B} / \Gamma^{\prime}$ |
| :--- | :--- | :--- | :--- |
| Figure 2.5a | $0.3211 / \mathbf{0 . 3 2 4 9}$ | $0.4329 / \mathbf{0 . 4 1 5 7}$ | $0.2458 / \mathbf{0 . 2 5 6 8}$ |
| Figure 2.5b | $0.3340 / \mathbf{0 . 3 3 2 3}$ | $0.3735 / \mathbf{0 . 3 6 6 3}$ | $0.2924 / \mathbf{0 . 3 0 4 4}$ |
| Figure 2.5c | $0.3418 / \mathbf{0 . 3 3 2 7}$ | $0.3474 / \mathbf{0 . 3 5 4 9}$ | $0.3107 / \mathbf{0 . 3 1 7 8}$ |
| Figure 2.5d | $0.5684 / \mathbf{0 . 5 6 6 7}$ | $0.2960 / \mathbf{0 . 3 1 5 1}$ | $0.1354 / \mathbf{0 . 1 1 7 7}$ |
| Figure 2.5e | $0.4633 / \mathbf{0 . 4 3 2 6}$ | $0.3330 / \mathbf{0 . 3 4 3 2}$ | $0.2035 / \mathbf{0 . 2 1 9 0}$ |
| Figure 2.5f | $0.3834 / \mathbf{0 . 3 6 2 6}$ | $0.3417 / \mathbf{0 . 3 4 1 5}$ | $0.2745 / \mathbf{0 . 3 0 7 1}$ |



Figure 2.6. A test scene illuminated by a blue dominant light source with ground truth value [0.1824, 0.3323, 0.4852]
distributions are calculated for different $\Gamma^{\prime}$ values we obtain plots in Figures 2.9b, 2.9d and 2.9f.

The minimum of these plots points the approximate estimate for illuminant chromaticity for the corresponding color channel. Hence the estimated chormaticities for the light source illuminating the test scene in Figure 2.6 are found as [0.1881, 0.3425, 0.4695] which are close to the ground truth values $[0.1824,0.3323,0.4852]$. Moreover if the slope distributions are calculated for best $\Gamma$ estimates for the color channels, Figures 2.9a, 2.9c and 2.9 e are obtained respectively. It can be observed that slopes are much more concentrated.


Figure 2.7. Roughly selected specular pixels in Figure 2.6 can be shown in IICS for each of the color channels.


Figure 2.8. 2.8a, 2.8c, 2.8e The calculated slope values where $\Gamma^{\prime}=0.0001$ for red, green and blue color channels respectively. 2.8b, 2.8d, 2.8 f The calculated slope values where $\Gamma^{\prime}=0.9999$ for red, green and blue color channels respectively.


Figure 2.9. 2.9a, 2.9c, 2.9e Standard deviations of calculated slope values for different values of $\Gamma^{\prime} \in[0,1] .2 .9 \mathrm{~b}, 2.9 \mathrm{~d}, 2.9 \mathrm{f}$ the slope distributions for best $\Gamma$ estimates for red, green, blue color channels respectively, It can be observed that slopes are much more concentrated.

### 2.2.1.2. Specularity Removal by Color Normalization

In Section 2.2.1.1 we show the estimation of scene illuminant chromaticity by using single image. The method we propose in (Ozan and Gümüştekin, 2011a) is an extension to (Ozan and Gümüştekin, 2011b). The estimated chromaticity value $\Gamma^{\prime}$ can be used to normalize the image by considering Equation 2.4 and using Equation 2.12.

$$
\begin{equation*}
\mathbf{I}^{n}=\frac{\mathbf{I}}{\boldsymbol{\Gamma}^{\prime}}=m_{d}^{n} \boldsymbol{\Lambda}^{n}+m_{s}^{n} \boldsymbol{\Gamma}^{n} . \tag{2.12}
\end{equation*}
$$

In this equation $\Gamma^{\prime}$ is the estimated illuminant chromaticity and $\mathbf{I}^{n}$ represents the normalized image. $\Gamma^{n}$ is the illuminant chromaticity for the normalize image and $\Gamma^{n}=$ $[1 / 3,1 / 3,1 / 3]^{\prime}$ which implies that the illuminant is pure white.

In (Shen et al., 2008) it is suggested that for a normalized image, $I^{n}$, the minimum of the RGB value of a pixel can be extracted from each channel and the resultant image $\tilde{\mathbf{I}}_{c}$ becomes specular-free (see Equation 2.13). In (Ozan and Gümüştekin, 2011a) we claim that using specular-free images instead of original images increases the stereo matching performance significantly.

$$
\begin{equation*}
\tilde{\mathbf{I}}_{c}=\mathbf{I}_{c}^{n}-\min \left(I_{R}^{n}, I_{G}^{n}, I_{B}^{n}\right) . \tag{2.13}
\end{equation*}
$$

In order to present the enhancement in stereo matching performance, a reference stereo image pair Figure 2.10a and 2.10b is used. These images are captured in a homogeneously illuminated soft box. There is no significant specular reflection from the object surface. When this image pair is used in normalized cross correlation based stereo matching (NCCBSM) algorithm a depth image in Figure 2.10c is obtained. This image is considered as ground truth that can be used to measure the success of the proposed algorithm. NCCBSM finds correlations between selected window areas from image pairs by using Equation 2.14.

$$
\begin{equation*}
C O R=\frac{\sum_{i=0}^{N}\left(X_{i}-\bar{X}\right) *\left(Y_{i}-\bar{Y}\right)}{\sqrt{\sum_{i=0}^{N}\left(X_{i}-\bar{X}\right)^{2} *\left(Y_{i}-\bar{Y}\right)^{2}}} \tag{2.14}
\end{equation*}
$$

In this equation $X$ and $Y$ represents selected windows from the image pair. The windows have the same size and $N$ is the number of pixels in the window area. $\bar{X}$ and $\bar{Y}$ are the mean value of pixel intensities for the windows. The disparity between pixels which have the maximum correlation value gives the depth information.

Another image pair is given in Figures 2.10d and 2.10e, where specular reflections are evident. Moreover the dominant wavelength of the light source used while taking
these images are different than each other. This causes an apparent shift in hue. Once NCCMSM algorithm is directly applied to this image pair the depth image in Figure 2.10f is obtained. As it can be seen from the figure estimated depth values are significantly distorted.


Figure 2.10. (a)-(b) is the stereo image pair captured in homogeneously illuminated soft box where (c) is the ideal case depth image generated using this image pair. This image is used as ground truth for the successive experimental analysis. (d)-(e) the stereo image pair with apparent specular reflections and different illuminant colors where (f) is the depth image generated using this image pair. PSNR wrt. Figure (c) $=19.24 d B$. (g)-(h) is the specular free image pair calculated by using Figures (d) and (e) where (i) is the enhanced depth image generated by using this pair. PSNR wrt. Figure (c) $=24.65 \mathrm{~dB}$.

If the color normalization method given above is applied to Figures 2.10d and 2.10 e , specular-free images in Figures 2.10 g and 2.10 h are obtained respectively. The stereo matching algorithm gives the resultant depth image in Figure 2.10i. The improvement can be seen and also can be measured by calculating peak signal to noise ratio (PSNR) between pairs 2.10c -2.10 f and $2.10 \mathrm{c}-2.10 \mathrm{i}$. Our measurements show that PSNR increases from $19.24 d B$ to $24.65 d B$ for this specific scenario.

### 2.2.2. Enhancing Stereo Matching Performance by using Polarization Images

When initially unpolarized light is reflected from a surface, it becomes partially polarized. This fact applies to both specular and diffuse component of the reflected light. The polarization of light results from the directionality of the molecular electron charge density interacting with the electromagnetic field of the incident light (Atkinson and Hancock, 2005). The specular reflection component is usually strong in its intensity and polarized significantly compared to the diffuse component. On the other hand, the intensity of the diffuse component is weak and it tends to be unpolarized except near occluding contours.

The polarization state of light can be detected by placing a linear polarizing filter in front of the camera. The difference in the electrical characteristics between metals and dielectrics causes differences in the light which is reflected. Hence, degree of polarization varies for dielectric and metal objects. This variation can be used to discern between metal and dielectric surfaces (Wolff, 1990). Dielectric surfaces partially polarize light upon specular reflection. Polarization caused by surface reflection is also used in order to provide surface geometry constraints. In (Rahmann and Canterakis, 2001), reconstruction of surfaces has been performed by using specular polarization. A diffusion polarization based method for surface reconstruction is proposed in (Atkinson and Hancock, 2006).


Figure 2.11. Effect of polarization can be seen in the images. Images are gathered by installing a linear polarizer in front of a camera. (a) Specular reflections are evident. (b) Polarizer is oriented such that specular reflections are mostly filtered.

The effect of polarization is illustrated in Figure 2.11. A simple setup has been constructed by simply placing a linear polarizer in front of a camera attached to a tripod. Two consecutive images are taken without changing the camera position but changing the linear polarizer angle. As it can be seen from the figure, the light reflected by the table is polarized and it can be effectively blocked by using a linear polarizer filter.

Polarization can also be used to separate diffuse and specular components. In (Umeyama and Godin, 2004), by using a linear polarizer and independent component analysis (ICA), specular and diffuse components of surface reflection are separated.

### 2.2.2.1. Estimating TRS Parameters from Images

The observations show that if a linear polarizer is placed in front of a camera and rotated, the measured pixel brightness changes according to transmitted radiance sinusoid (TRS) (Atkinson and Hancock, 2006). A sample experimental result is given in Figure 2.12. The change of pixel brightness especially in specular component of the reflection can easily be observed. TRS can be represented as Figure 2.13 and formulated as Equation 2.15.


Figure 2.12. Pixel brightness change in successive measurements of a scene with varying linear polarizer orientations (a) Polarizer angle $\theta_{\text {pol }}=0^{0}$ (b) Polarizer angle $\theta_{\text {pol }}=30^{\circ}$ (c) Polarizer angle $\theta_{\text {pol }}=60^{\circ}$ (d) Polarizer angle $\theta_{\text {pol }}=90^{\circ}$.

$$
\begin{equation*}
I\left(\theta_{p o l}, \phi\right)=\frac{I_{\max }+I_{\min }}{2}+\frac{I_{\max }-I_{\min }}{2} \cos \left(2 \theta_{\text {pol }}-2 \phi\right) . \tag{2.15}
\end{equation*}
$$

Each pixel in the images has its own TRS (Equation 2.15) in polarimetric image sequences. The equation can be organized by separating constant and variable values as in Equation 2.16. This form constitutes an over determined linear system (Equation 2.17) and the solution of this linear system requires $N \geq 3$ measurements (Huynh et al., 2010).

| Transmitted Radiance |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Figure 2.13. Transmitted radiance sinusoidal can be represented in accordance with Equation 2.15 .

$$
\begin{align*}
& I_{i}\left(\theta_{f}, \phi\right)=\left[\begin{array}{c}
1 \\
\cos \left(2 \theta_{f}\right) \\
\sin \left(2 \theta_{f}\right)
\end{array}\right]^{T}\left[\begin{array}{c}
\frac{I_{\max }+I_{\min }}{2} \\
\frac{I_{\text {max }}-I_{\min }}{2} \cos (2 \phi) \\
\frac{I_{\text {max }}-I_{\text {min }}}{2} \sin (2 \phi)
\end{array}\right] \\
&= a_{i}^{T} \mathbf{x} .  \tag{2.16}\\
& \text { Such that, } \mathbf{I}=\left[\begin{array}{c}
I\left(\theta_{f}^{1}, \phi\right) \\
I\left(\theta_{f}^{2}, \phi\right) \\
\vdots \\
I\left(\theta_{f}^{N}, \phi\right)
\end{array}\right] \quad \text {, and, } \mathbf{A}=\left[\begin{array}{c}
a_{1}^{T} \\
a_{2}^{T} \\
\vdots \\
a_{N}^{T}
\end{array}\right] .
\end{align*}
$$

The desired parameter values can be rewritten as Equation 2.18 by using $x$ which is the solution of Equation 2.17. In order to find $x$ two numerical analysis techniques, Linear Regression and Levenberg-Marquardt, can be used. A brief information about these numerical methods can be found in Appendix A.

$$
\begin{align*}
I_{\max } & =x_{1}+\sqrt{x_{2}^{2}+x_{3}^{2}}, \\
I_{\min } & =x_{1}-\sqrt{x_{2}^{2}+x_{3}^{2}}, \\
\phi & =\frac{1}{2} \arctan \frac{x_{3}}{x_{2}} . \tag{2.18}
\end{align*}
$$

### 2.2.2.2. Specularity Removal by using TRS

By using Linear Regression and Levenberg-Marquardt TRS parameters especially $I_{\text {min }}$ can be estimated and specular free images can be generated by using $I_{\text {min }}$ values for each pixel. In (Ozan and Gümüştekin, 2013) we claim that using specular-free images obtained by using $I_{\text {min }}$ values significantly increases stereo matching performance in the scenes where there exist specular reflections.

In order to present the enhancement in stereo matching performance a reference stereo image pair taken under diffuse illumination (Figure 2.14a and 2.14b) is used. We use the resultant depth image in Figure 2.14c as ground truth.

In our experiments we use 3,4 and 36 images to estimate TRS parameters by using Linear Regression and Levenberg Marquardt methods. By using obtained specular-free image pairs depth images can be generated. In Figure 2.15 the resultant depth images and their PSNR values with respect to the ground truth data (Figure 2.14) are given below each image and also tabulated in Table 2.2. As it is expected, Levenberg-Marquardt gives


Figure 2.14. (a)-(b) Stereo image pair taken by using a diffuse lighting condition. (c) The depth image generated by using (a) and (b). This image is used as ground truth in further analysis.
better results since it can handle the nonlinear nature of the system given in Equation 2.16 better than Linear Regression solution. The numerical results shows that using three or four images makes it possible to estimate TRS in polarization imaging analysis. This makes analysis more practical and in the specific case in (Ozan and Gümüştekin, 2013) it can be used to enhance stereo matching performance.


Figure 2.15. Depth images which are generated using numerical analysis techniques, linear regression (LR) and Levenberg-Marquardt (LM). PSNR values wrt. ground truth image in Figure 2.14 are given below each image. (a) LR with 3 images (b) LR with 4 images (c) LR with 36 images(d) LM with 3 images (e) LM with 4 images (f) LM with 36 images.

### 2.2.2.3. A Multi-view Application of Proposed Method

In (Ozan and Gümüştekin, 2014b) we applied the method in (Ozan and Gümüştekin, 2013) to a multi-view imaging system. We used a catadioptric system with planar mirrors. Similar setups can be also seen in (Gluckman and Nayar, 2001) and (Wu et al., 2009). Four planar mirrors and a camera can capture five different views of an object with a single take. A sample take of a checkerboard pattern can be seen in Figure 2.16a.

In order to calibrate the system a checker board pattern is captured (Figure 2.16a). By using one real and four virtual views captured from mirror reflections can be used to find both intrinsic and extrinsic calibration parameters of the camera. In order to find camera calibration parameters we use Zhang's Homography approach (Zhang, 2000) (Also see Appendix B).

We use checkerboard corner information to rectify different views, this process aligns the views and makes it easier to find correspondences in image pairs. NCCBSM is used to find corresponding pixels in image pairs. 3D space coordinates of a point can

Table 2.2. PSNR values wrt. numerical analysis method and number of images used.

|  | 3 images | 4 images | 36 images |
| :--- | :--- | :--- | :--- |
| Linear Regression | 16.74 | 18.61 | 18.82 |
| Levenberg-Marquardt | 19.38 | 19.39 | $\mathbf{1 9 . 6 0}$ |



Figure 2.16. (a) The checkerboard pattern viewed in our multi-view system. (b) Detected checkerboard corners in a sample view. The corners are detected with sub-pixel accuracy.
be deduced from two corresponding pixel locations in a stereo image pair. The problem can be defined as a linear equation and be solved by using singular value decomposition (SVD) (see Appendix C).

In (Ozan and Gümüştekin, 2014b) we say that multi-view 3D reconstruction performance can be enhanced by using polarization imaging. By following same procedure given in (Ozan and Gümüştekin, 2013), the TRS parameters can be estimated and the negative effect of specular reflections can be alleviated.

Four images are taken for different polarization angles. In order to perform corresponding polarimetric measurements a lighting system is also proposed. Instead of changing polarizing filter in front of the camera the polarization state of the light source is changed. To build a homogeneous light source power leds are arrayed (see Figure 2.17a). The leds are selected such that they are as directional as possible. Moreover, tube channels with a diameter close to led width is placed in front of leds. This makes the lighting con-
dition much more collimated. Polarization filters with four different polarization angles are placed at the end of the tubes. By enabling adjacent pixels, four polarized lighting conditions with four polarization angles are obtained. In Figure 2.17b an enabled led group can be seen. By enabling four groups separately polarization imaging is performed for polarizer angles $0^{0}, 45^{0}, 90^{\circ}$ and $135^{\circ}$. The whole imaging system can be seen in Figure 2.17c.


Figure 2.17. (a) The led array used to light the scene. (b) Single led group activated. There are four led groups emitting light with four different polarization angles. (c) The whole system.

The effect of specularity is minimized in images by finding $I_{\text {min }}$ values in TRS (Equation 2.16). In (Ozan and Gümüştekin, 2014b) a Maya calendar object which has semi-glossy surface properties are analyzed. Figure 2.18a shows a single frame from one of the views without any polarization analysis. The effect of specularity can be observed. Figure 2.18 b shows the same view generated by $I_{\text {min }}$ values of each pixels TRS. It can be observed that the effect of specularity is significantly reduced.

Closeup image of the reconstruction result without a polarization analysis can be seen in Figure 2.19a. Closeup image of the reconstruction result after making the polarization analysis and generating specular-free images can be seen in Figure 2.19b. The enhancement in the reconstruction can be observed by comparing two results.

Both polarization based methods and color normalization methods can be used to eliminate specularity in the scene. As it is given above, specularity removal can further enhance stereo matching performance. Color normalization methods can be applied by using single image. But inorder to apply polarization based methods we need at least three images inorder to correctly identify TRS parameters for a surface point. Both methods


Figure 2.18. (a) A view where the effect of specular reflections are apparent. (b) The same view where the effect of specular reflections are removed.
are works best for dielectric surfaces where the surface can be described as semi-glossy. If the surface has mirror-like reflections both color normalization and polarization based methods tend to fail in removing the whole specular component. Morover, if the image takes are overly exposed, i.e. there exist saturated pixels, it is not possible to correctly remove specular component.

In Figure 2.20, we apply the color normalization method to an overly exposed scene. The source image is Figure 2.20a. The illuminant chromaticity can be estimated by analysing this image. Moreover, the illuminant chromaticity information can be used to obtain normalized image in Figure 2.20b. But once we calculate specular-free image by using 2.13, we get Figure 2.20c. Since the specularity is so high in the source image, the corresponding pixels have saturated color information and the obtained specular-free image has some defects in the originally specular area.

### 2.3. Structured Light Based Methods

In this section we will deal with structured light based scanning systems. We will propose double stripe 3D scanners and a novel method for their calibration. Also we will introduce a projector based structured light system in detail.


Figure 2.19. (a) The closeup view of reconstructed surface of Maya Calendar figure. The reconstruction is performed by using the stereo image pair where the specular reflections are evident. Hence reconstructed result lacks some regions. (b) The same reconstruction which is performed by using the stereo image pair where the specular reflections are removed by using polarization imaging. The reconstructed result is better than the former one. (c) Front view of the reconstruction which is performed after removing specular reflection.


Figure 2.20. (a) An over-exposed scene where the same frog figurine is used (b) The normalized image. Illuminant chromaticity can be estimated and used for image normalization. (c) Since the specularity is so high in the source image, the obtained specular-free image has some defects in the originally specular area.

### 2.3.1. Laser Scanner Systems

In (Ozan and Gümüştekin, 2014a), we propose 3D scanning systems that utilize a pair of laser stripes. Three types of scanning systems are implemented to scan environments, rough surfaces of near planar objects and small 3D objects. These scanners make use of double laser stripes to minimize the undesired effect of occlusions. Calibration of these scanning systems is important. In (Ozan and Gümüştekin, 2014a) and (Ozan and Gümüştekin, 2012), the main focus is on the calibration problem. The 3D points corresponding to laser stripes are used in an optimization procedure that imposes geometrical constraints such as coplanarities and orthogonalities. It is shown that, calibration procedure proposed here, significantly improves the alignment of 3D points scanned by using two laser stripes.

The only constraint imposed in (Ozan and Gümüştekin, 2014a) about positioning the two light sources is that parallel stripes should be visible in different halves of the image frame. Geometrical constraints like coplanarities and orthogonalities are imposed in the scene. In (Furukawa and Kawasaki, 2009), calibration techniques using similar geometrical constraints are reported. However their method needs user interaction to identify planar surfaces. In our work the detection of planar surfaces are performed automatically during calibration. Once the calibration is performed, a 3D model can be reconstructed from the point data acquired simultaneously using the two laser stripes. Also in (Park et al., 2001) a least squares solution that optimizes linear transformations corresponding to light sources are searched using a complex calibration object that produces 18 distinct point positions.

In this study, the perspective camera model is used. For convenience the axis naming convention in (Trucco and Verri, 1998) is adopted. As seen in Figure 2.21, the image plane is considered to be parallel to the $X Y$ plane where $f$ is the focal distance measured $+Z$ direction.

The scanners are built with a standard CCD camera and two low cost line lasers. They are designed to handle different scanning scenarios. In this work we also propose a calibration method which does not necessarily require a calibration object. It uses the planarities and orthogonalities in the scene. If it is not possible to find any planarity and/or orthogonality information, a simple calibration object is used.

Scanner-1: The scanning head is placed on a platform which can be controlled by computer and makes pan movement. Schematic and actualization of the scanner can be seen in Figures 2.22 a and 2.22 b respectively. The radial motion of the platform enables


Figure 2.21. Perspective Camera Model as given in (Trucco and Verri, 1998).
the scanner to scan a surrounding environment. The laser emitters are placed such that they are approximately parallel to each other.

Scanner-2: It is designed to scan surfaces of objects placed on floor. Schematic and actualization of the scanner can be seen in Figures 2.22c and 2.22d respectively. Camera and lasers are attached to a platform which is capable of moving in $X$ axis. This linear motion enables us to scan a surface continuously along $X$ direction.

Scanner-3: It is designed to scan 3D objects. Scanning head is fixed. The corresponding object is placed on a turntable. Schematic and actualization of the scanner can be seen in Figures 2.22e and 2.22 f respectively. By rotating the object for $360^{\circ}$, object geometry is gathered.

In all three cases, usage of two line lasers alleviates the effect of scene occlusion. This can be seen in a reconstruction result performed by Scanner-3 in Figure 2.23. The scanning results in Figures 2.23a and 2.23b are individual scans of left and right lasers respectively. Once these results are merged together (see Figure 2.23c), they complement each other and occlusion effects are significantly eliminated.

### 2.3.1.1. Calibration of Double Stripe 3D Laser Scanner Systems using Planarity and Orthogonality Constraints

The scanning systems used in this work produce two sets of scanning data which are due to two laser beams. Using two line lasers minimizes the effect of occlusions


Figure 2.22. (a), (c) and (e) are the graphical representations of the proposed laser scanners. (b), (d) and (f) are the implemented scanners which are used in our experiments.


Figure 2.23. Usage of two laser sources alleviates the effect of scene occlusions.
in the scanning results as shown previously, but it also complicates the calibration of the system. We need accurate information about position and orientation of the laser sources to achieve coherent scanning results. In this work, by using known geometrical constraints in the scene, an objective function is defined and Nelder-Mead minimization procedure (Nelder and Mead, 1965) is used to find the scanner parameters by minimizing the objective function.

In Figure 2.24, scanning results of a person sitting in front of a wall can be seen. This scan data is obtained by using Scanner-1. Figure 2.24a is the point cloud data which is generated by using the laser source on the left $\left(\operatorname{laser}_{L}\right)$. Similarly Figure 2.24 b is generated by the right source $\left(\operatorname{laser}_{R}\right)$. When these scanning results are brought to the same coordinate space it can be seen that the merged sets do not correctly overlap (e.g. Figures 2.24 c and 2.24 d ). This example shows that using the best-effort manual measurements for the system parameters may provide realistic results for individual laser stripes but the combined data severely suffer from the calibration errors.

The camera calibration is performed by using Matlab Toolbox for Camera Calibration (Bouguet, 2004) which is a MATLAB ${ }^{\mathrm{TM}}$ implementation based on (Zhang, 2000).

After calibrating the camera, the image acquisition process is started. The ideal environment is a dark room with surrounding objects. In this scenario the sensor noise becomes significant. To eliminate the sensor noise, median temporal filtering is used. For each stop of the actuators, five consecutive frames are captured and median-filtered. A temporal-filtered frame is captured as in Figure 2.25a and the undistorted (i.e. corrected from radial distortion using lens calibration results) version can be seen in Figure 2.25b.

The ambiguities caused by the varying strength of reflected laser light, multiple


Figure 2.24. Sample scan with Scanner-1. The scene consists of a person sitting in front of a wall. (a) The point data which is generated by the data from the right laser. (b) The point data which is generated by the data from the left laser. (c) Side view of merged data. (d) Top view of merged data.
reflections and independent light sources can be reduced by detecting dominant paths that are most likely due to the reflection of projected stripes. In this work, this is done by using dynamic programming (Dijkstra, 1959). The shortest path algorithm is modified to extend the horizontal neighborhood of each pixel using a margin of pixels on each side. The jumps in the stripe reflections due to surface and depth variations are handled by robustly detecting optimal paths in multiple segments. Figure 2.25.c illustrates detected segments in the presence of discontinuities. Besides finding the optimal paths, this algorithm reduces the width of the stripe reflection to a single pixel.

At this stage, the experimental results show that the precision of 3D points calculated by using the detected path are limited since the calculations are performed in pixel accuracy. The unwanted aliasing effects are clearly visible when smooth planar surfaces are observed. (Figure 2.26a). To hadle this problem the paths are regenerated in sub-pixel accuracy using Nonlinear Least Squares Algorithm (NLSA). As proposed in (Izquierdo et al., 1999) the width profile of the reflected stripe can be considered as a Gaussian dis-


Figure 2.25. (a) A sample captured frame with lens distortion. (b) Undistorted version of (a). Straightness of the right laser stripe is a visible cue on the presence of lens distortion. (c) Initial estimation of stripe positions by multi-segment shortest path algorithm. The actual grayscale images are inverted. Boxed regions are shown in detail in the upper-right corner of each figure.
tribution. By fitting a Gaussian function to the laser profile, the approximate peak value can be detected with sub-pixel accuracy. The optimal paths found by NLSA significantly improve the accuracy of the reconstructed 3D data (e.g. Figure 2.26b). The details of NLSA implementation are given in Appendix D.


Figure 2.26. (a) Reconstructed point cloud with pixel accuracy. (b) Reconstructed point cloud with sub-pixel accuracy. Boxed regions of both figures are shown in detail in the upper-right corner.

### 2.3.1.2. Obtaining 3D Point Cloud

Considering the assumed perspective camera model, coordinate axes (Figure 2.21) and the proposed scanner geometry, the laser triangulation can be depicted as Figure 2.27. For simplicity, the 2D geometry is shown in $X Z$ plane since the extension to $Y$ axis is straightforward. In this representation, subscript $L$ and $R$ represents "left" and "right" respectively.

Light plane $\Pi_{L}$ intersects with the objects in the surrounding environment. A point $\mathbf{p}_{L}$ is the intersection point of the plane $\Pi_{L}$ and line $l_{L}$ which means that $\mathbf{p}_{L}$ solves Equations 2.19 and 2.20 simultaneously. A plane can be defined with one point and one normal vector as in Equation 2.19. Similarly, a line can be defined as in Equation 2.20.

$$
\begin{gather*}
\left(\Pi_{L}\right):\left(\mathbf{p}-\mathbf{D}_{L}\right) \cdot \boldsymbol{n}_{L}=0 . \\
\left(l_{L}\right): \mathbf{p}=k \boldsymbol{v}_{L} . \tag{2.20}
\end{gather*}
$$

Substituting Equation 2.20 in Equation 2.19 to find $k$ and substituting this expression back in Equation 2.20 the vector $\mathbf{p}_{L}$ pointing to the object surface can be found as:

$$
\begin{equation*}
\mathbf{p}_{L}=\left(\frac{\mathbf{D}_{L} \cdot \boldsymbol{n}_{L}}{\boldsymbol{v}_{L} \cdot \boldsymbol{n}_{L}}\right) \boldsymbol{v}_{L} . \tag{2.21}
\end{equation*}
$$

In the above derivation, since the distance $d$ (See Figure 2.27) is a design parameter by construction, $\mathbf{D}_{L}$ is approximately known. Initially, $n_{L}$ can be considered as the unit vector pointing $+X$ direction. Moreover, $\boldsymbol{v}_{L}$ can be written explicitly as Equation 2.22 with ( $x_{i m}, y_{i m}$ ) pixel coordinates of the point $\mathbf{p}_{L}$ on the image plane $\pi,\left(o_{x}, o_{y}\right)$ the pixel coordinates of the image center, $\left(s_{x}, s_{y}\right)$ the effective size of the pixel in the horizontal and vertical directions and $f$ the focal length of the camera. By writing $\boldsymbol{v}_{L}$ explicitly, the intrinsic parameters of the camera are embedded into the solution (Equation 2.23).

$$
\begin{gather*}
\boldsymbol{v}_{L}=\left[\begin{array}{c}
-\left(x_{i m}-o_{x}\right) s_{x} \\
-\left(y_{i m}-o_{y}\right) s_{y} \\
f
\end{array}\right]  \tag{2.22}\\
\boldsymbol{p}_{L}=\left[\begin{array}{c}
-\left(\frac{\mathbf{D}_{L} \cdot \boldsymbol{n}_{L}}{\boldsymbol{v}_{L} \cdot \boldsymbol{n}_{L}}\right)\left(x_{i m}-o_{x}\right) s_{x} \\
-\left(\frac{\mathbf{D}_{L} \cdot{ }_{L}}{\boldsymbol{v}_{L} \cdot \boldsymbol{n}_{L}}\right)\left(y_{i m}-o_{y}\right) s_{y} \\
\left(\frac{\mathbf{D}_{L} \cdot \boldsymbol{n}_{L}}{\boldsymbol{v}_{L} \cdot \boldsymbol{n}_{L}}\right) f
\end{array}\right] \tag{2.23}
\end{gather*}
$$

Similarly $\mathbf{p}_{R}$ can found as in Equation 2.24:

$$
\boldsymbol{p}_{R}=\left[\begin{array}{c}
-\left(\frac{\mathbf{D}_{R} \cdot \boldsymbol{n}_{R}}{\boldsymbol{v}_{R} \cdot \boldsymbol{n}_{R}}\right)\left(x_{i m}-o_{x}\right) s_{x}  \tag{2.24}\\
-\left(\frac{\mathbf{D}_{R} \cdot \boldsymbol{n}_{R}}{\boldsymbol{v}_{R} \cdot \boldsymbol{n}_{R}}\right)\left(y_{i m}-o_{y}\right) s_{y} \\
\left(\frac{\mathbf{D}_{R} \cdot \boldsymbol{i}_{R}}{\boldsymbol{v}_{R} \cdot \boldsymbol{n}_{R}}\right) f
\end{array}\right]
$$

Using Equations 2.23 and 2.24, and precomputed intrinsic camera parameters (i.e. $o_{x}, o_{y}$, $s_{x}, s_{y}$ and $f$ ) for projecting image points corresponding to light stripes back to 3D space, point clouds can be created as in Figure 2.26.

At each step of the mechanical actuators, an offset (which is angular for Scanners$1 \& 3$ and translational for Scanner-2) is included in the corresponding terms.


Figure 2.27. Laser triangulation geometry for two independent line lasers .

### 2.3.1.3. Geometrical Constraints in the Scenes

Details of three scanner configurations are given. They are designed for different scanning scenarios but they share the same calibration problem involving the usage of second line laser. The first scanner, Scanner-1, is designed to scan enclosed environments. A sample scene where a person is sitting in front of a wall can be seen in Figure 2.28a. The most significant geometrical constraints are the wall and the floor data where they represent orthogonal planes. Also it is known that when the best calibration is achieved, corresponding data points from either of the lasers become coplanar on these planes.

The second scanner, Scanner-2, is designed to scan near planar objects. A sample scene in which a Sphinx Mask is laid on the floor can be seen in Figure 2.28b. In this case, there are no apparent geometrical constraints like orthogonal planes in the scene. Also for Scanner-3, if an arbitrary object (e.g. Frog figurine in Figure 2.28c) is scanned, finding an apparent geometrical constraint is not possible as well. Hence for Scanners 2 and 3 an L-shaped calibration object (see Figure 2.29) is utilized to create necessary geometrical constraints in the scene. Once this calibration object is scanned, desired geometrical constraints are obtained. An L-shaped object can also be used for Scanner-1 if two orthogonal planes are not available in the scanned angular interval.


Figure 2.28. Scanned samples used in the scanning experiments by using 2.28a Scanner-1, 2.28b Scanner-2 and 2.28c Scanner-3 respectively.


Figure 2.29. L-Shaped calibration figure.

### 2.3.1.4. Plane Detection in 3D Point Cloud Data

In this work the planar features in the scene are found automatically. First, scanning and preliminary 3D reconstruction with initial (manually measured) parameters are performed. Since the number of scanned points is usually extremely high, a brute force method to find dominant planes in 3D data takes an excessive amount of time with conventional computers. Hence an efficient method is needed. In (Okada et al., 2001), the parameterization of a plane is described as Equation 2.25 where $(\rho, \theta, \beta)$ are plane parameters and $(x, y, z)$ triplet is a point on the plane (see Figure 2.30).


Figure 2.30. Parameterization of a plane in 3D.

$$
\begin{equation*}
\rho=(x \cos (\beta)+y \sin (\beta)) \cos (\theta)+z \sin (\theta) . \tag{2.25}
\end{equation*}
$$

As proposed in (Okada et al., 2001), plane detection in a point cloud data can be performed by using Hough Transform once the plane is represented with angular parameters (i.e. $(\rho, \theta, \beta)$ ). Conventionally, Hough Transform is used to find lines or circles in 2 D images. Plane finding in 3D point cloud data is a higher order problem which requires a higher dimensional search. In (Borrmann et al., 2011), plane detection methods using Hough Transform are surveyed. Among those methods, Randomized Hough Transform (RHT) is a proper choice for our problem. With a slight modification to the representation in (Borrmann et al., 2011), pseudo code of the RHT method we used in our approach can be seen in Appendix E. When the RHT plane detection algorithm is applied to the point cloud generated by using left laser (Figure 2.24b), the wall data and floor data can be decomposed as Figures 2.31a and 2.31b respectively.

Once the co-planar points representing a plane is found for both left and right stripes, the corresponding analytical equation can be found by Linear Least Squares Al-
gorithm (LLSA). LLSA based 3D plane fitting allows us to test validity of system parameters so that planes acquired using two stripes are aligned correctly. To solve 3D plane fitting problem we considered the plane defined in Equation 2.26.

$$
\begin{equation*}
A x+B y+C=z \tag{2.26}
\end{equation*}
$$

In order to find $(A, B, C)$ triplets from 3D point data $(x, y, z)$ triplets, we focus on the minimization of $\|H u-b\|^{2}$. Here $u=\left[\begin{array}{lll}A & B & C\end{array}\right]^{T}, H$ and $b$ can be constructed from observations. This problem can be solved as in Equation 2.27 (Strang, 2007).

$$
\begin{equation*}
H^{T} H \hat{u}=H^{T} b . \tag{2.27}
\end{equation*}
$$

We can reorganize Equation 2.27 as Equation 2.28:

$$
\left[\begin{array}{ccc}
\sum_{i=0}^{m} x_{i}^{2} & \sum_{i=0}^{m} x_{i} y_{i} & \sum_{i=0}^{m} x_{i}  \tag{2.28}\\
\sum_{i=0}^{m} x_{i} y_{i} & \sum_{i=0}^{m} y_{i}^{2} & \sum_{i=0}^{m} y_{i} \\
\sum_{i=0}^{m} x_{i} & \sum_{i=0}^{m} y_{i} & \sum_{i=0}^{m} 1
\end{array}\right]\left[\begin{array}{c}
A \\
B \\
C
\end{array}\right]=\left[\begin{array}{c}
\sum_{i=0}^{m} x_{i} z_{i} \\
\sum_{i=0}^{m} y_{i} z_{i} \\
\sum_{i=0}^{m} z_{i}
\end{array}\right]
$$

The solution to Equation 2.28 gives us the estimated plane parameters $(A, B, C)$. In our implementation the pixel locations corresponding to dominant planes are recorded for each image. These image pixels are used in the optimization steps. This makes it possible to complete calibration without imposing extensive memory and computation time requirements.


Figure 2.31. (a) Wall data (b) Floor data, detected from point cloud in Figure 2.26b.

### 2.3.1.5. Nelder-Mead Algorithm

In order to estimate the parameters that represent the projection planes of laser stripes (i.e. $\Pi_{L}$ and $\Pi_{R}$ in Figure 2.27), an optimization procedure is required. In our study the problem is defined as a multidimensional minimization problem. Nelder Mead Algorithm (NMA) (Nelder and Mead, 1965) is an appropriate approach to solve such problems. The problem is solved by calculating objective function $f(\bullet)$ at simplex vertices and performing some procedural geometrical operations to update these vertices. At each step $f(\bullet)$ is calculated for new vertex values. Each vertex corresponds to parameters representing a feasible solution. The details of NMA implementation can be found in Appendix F.

The reconstruction problem for Scanner- 1 and 2 is very similar. The only difference is that; in Scanner-1 the movement of the scanner is a rotation around $Y$ whereas the movement of Scanner-2 is a translation along $X$. The laser projection planes can be identified with $\mathbf{D}_{i}, \boldsymbol{n}_{i}$ (where $i=L, R$ ). Surface normals $\boldsymbol{n}_{i}$ can be represented by two angles (i.e. azimuth and zenith angles in spherical coordinates) $\theta_{i}$ and $\beta_{i}$.


Figure 2.32. Axis representations of Scanner-3.

The configuration for Scanner-3 (see Figure 2.32) is slightly different. In addition to the configuration parameters given for Scanners 1 and 2, the rotation around turntable's $Y_{T}$ axis should also be embedded in the solution. Coordinates of the turntable's rotation center (i.e. $\boldsymbol{O}_{T}$ ) should be expressed with respect to cameras optical center (i.e. $\boldsymbol{O}$ ). The position of the turntable is included in the optimization procedure with three parameters $D_{x}, D_{y}$ and $D_{z}$. Initial distance values can be approximately found by measuring the distances manually. In order to correctly identify the rotation around turntable axis, the


Figure 2.33. Scanning non-convex objects using Scanner-3.
necessary rotation angles $\Theta_{x}, \Theta_{y}$ and $\Theta_{z}$, which aligns corresponding coordinate axes, are also included in the optimization procedure. The laser emitters can be positioned such that the stripes intersect behind the rotation center. This offset distance is dependent on the size and shape of the object that needs to be scanned. It is possible to scan nonconvex objects if a significant offset which should be lower than approximate radius of the object is used (e.g. Figure 2.33). In Figure 2.33a the laser stripes are intersected at the rotation axis of the turntable. Since some part of the non-convex sample is located beyond the intersection point, an ambiguity is observed while generating point clouds by using Equations 2.23 and 2.24. This ambiguity is resolved by intersecting lasers beyond the rotation axis (Figure 2.33b).

### 2.3.1.6. Objective Function

Since our aim is to make 3D point measurements from left and right stripes perfectly overlap, we have to find the best parameter values which correctly identify the laser sources' positions and orientations. These parameters are listed in Equation 2.29. $\boldsymbol{S}_{1}, \boldsymbol{S}_{2}$ and $\boldsymbol{S}_{3}$ are the parameter sets used for Scanners 1,2 and 3 respectively.

Once the optimal parameter values are found, the best overlap is obtained. This makes the two corresponding planes (e.g. wall data from each of the right and left measurements in Figures 2.24a and 2.24b ) co-planar. The objective function given in Equation 2.30 measures the coplanarities and the orthogonalities together.


Figure 2.34. Representation of the geometry used to define Objective Function in NMA.

The geometrical constraints of the calibration problem can be illustrated as in Figure 2.34. The planar point clouds are named as $\pi_{i}^{j}$ where subscript $i$ represents laser source (i.e. either left or right) and superscript $j$ represents the corresponding surface (i.e. either $a$ or $b$ ) of the scanned environment or the calibration pattern. After optimizing the scanner parameters, $\pi_{L}^{a}$ and $\pi_{R}^{a}$ become coplanar in $\Pi^{a}$. Similarly $\pi_{L}^{b}$ and $\pi_{R}^{b}$ become co-planar in $\Pi^{b}$.

To measure the coplanarity we utilized linear-least squares method to find the best fitting plane equations for $\Pi^{j}$ s which represent the merged plane pairs. The fitting error (i.e $\operatorname{Err}\left(\Pi^{j}\right)$ ) serves as a measure that quantifies coplanarity of two planes (i.e $\pi_{L}^{j}$ and $\pi_{R}^{j}$ ). The orthogonality constraint is included in the objective function as the dot product of normals $n^{a}, n^{b}$ (of surfaces $\Pi^{a}$ and $\Pi^{b}$ ).

$$
\begin{align*}
\mathbb{S}_{1} & =\left\{\theta_{L}, \beta_{L}, \theta_{R}, \beta_{R}, D_{L}, D_{R}\right\}, \\
\mathbb{S}_{2} & =\left\{\theta_{L}, \beta_{L}, \theta_{R}, \beta_{R}, D_{L}, D_{R}\right\},  \tag{2.29}\\
\mathbb{S}_{3} & =\left\{\theta_{L}, \beta_{L}, \theta_{R}, \beta_{R}, D_{L}, D_{R}, D_{x}, D_{y}, D_{z}, \Theta_{x}, \Theta_{y}, \Theta_{z}\right\} . \\
f\left(\mathbb{S}_{i}\right) & =M A X\left\{\operatorname{Err}\left(\Pi^{a}\right), \operatorname{Err}\left(\Pi^{b}\right)\right\} *\left(1+\left(n^{a} \cdot n^{b}\right)\right),  \tag{2.30}\\
i & =1,2,3 .
\end{align*}
$$

### 2.3.1.7. Calibration Results

In this study three different laser scanners are studied. Each scanner has two line lasers and a CCD camera (see Figure 2.22). Using two laser stripes brings up the problem
of aligning two scanning results gathered from each one of the laser sources. To correctly align scanning results, both laser light planes should be correctly identified with respect to a reference coordinate frame which is the camera coordinate system described in Figure 2.21.

In the previous sections, the problem is defined and geometrically parameterized. It is shown that parameters can be optimized by a minimization procedure. NMA is chosen for this multidimensional minimization problem. Sample optimization results can be seen in Table 2.3. For calibration of Scanner-1, wall and floor data from the room scan is used. For Scanners 2 and 3, scanning result of the calibration object (see Figure 2.29) is used. Angles are given in degrees and distances are given in millimeters (mm).

To visualize the performance of the optimization procedure, parameter variations during NMA iterations are given in Figure 2.36. Since the parameters are in different metrics they are normalized with respect to the maximum value achieved during iterations.

Variations in objective function value during NMA iterations are given in Figure 2.35. The algorithm is stopped at a point where the parameter and objective function variations are negligibly small.


Figure 2.35. Value of objective function (see Equation 2.30) during NMA iterations.

The total computation time on a set of 250 1280x 1024 images using a Core i5 2.53 GHz computer is typically less than 5 minutes. The most time consuming operations are RHT and NMA. Detection of four dominant planes using RHT for Scanner-3 setup is completed in 62 seconds. NMA (whose performance is highly dependent on initial parameter assignments) for the same setup is completed in 192 seconds. Scanner-1 and Scanner-2 computations are completed in a shorter period due to less number of parameters, even though twice as many (500) image frames are used.


Figure 2.36. Parameter variations during NMA iterations are given. Since the parameters are in different metrics they are normalized with respect to the maximum value achieved during iterations. The original initial and final values are given in Table 2.3. (a) (b) and (c) show parameter optimization for scanners 1, 2 and 3 respectively.

Table 2.3. Sample Optimization Results for Parameters. Angle values are given in degrees, distances are given in millimeters (mm).

| Parameters | Scanner-1 | Scanner-2 | Scanner-3 |
| :---: | :---: | :---: | :---: |
| $\theta_{L}$ | 0.0000 .211 | 36.00037 .130 | 15.00014 .456 |
| $\beta_{L}$ | 0.000 -0.003 | 0.0000 .332 | 0.000-4.012 |
| $\theta_{R}$ | 180.000179 .787 | 145.000143 .633 | 160.000160 .898 |
| $\beta_{R}$ | 0.0000 .841 | 0.000-0.295 | 0.0-5.531 |
| $D_{L}$ | 500.000502 .950 | 250.000251 .913 | 137.700134 .489 |
| $D_{R}$ | -500.000-502.574 | -250.000-247.474 | -137.700-138.033 |
| $D_{x}$ | N/A | N/A | 0.00019 .692 |
| $D_{y}$ | N/A | N/A | -92.100-96.645 |
| $D_{z}$ | N/A | N/A | 448.400439 .066 |
| $\Theta_{x}$ | N/A | N/A | 22.60017 .553 |
| $\Theta_{y}$ | N/A | N/A | 0.000-24.126 |
| $\Theta_{z}$ | N/A | N/A | 0.000 -1.883 |
| $f(\bullet)$ | 22.1660 .074 | 3.3320 .482 | $1.241 \mathbf{0 . 1 4 7}$ |

Reconstructed scanning results can be seen in Figures 2.37-2.39. Non-overlapping results before optimization can be seen in Figures 2.37a, 2.38a, 2.39a and improved overlapping results can be seen in Figures 2.37b, 2.38b and 2.39b.

A final test is done to test the validity of the procedure using an object with known geometrical properties. In order to generate worst case scenario, Scanner-3 is used in this experiment, since it involves additional parameters compared to other scanners. A sphere with a diameter of 150 mm was chosen as the test object. The point clouds are shown before and after optimization in parts $\mathrm{a} \& \mathrm{~b}$ of Figure 2.40. Parts $\mathrm{c} \& \mathrm{~d}$ of the same figure show the histograms of distances from detected sphere center to 3D points. Same number of data points are randomly selected for before \& after optimization cases. Numerical results from this experiment is given in Table 2.4. It can be seen that, refinement in the estimated parameter set makes it possible to generate 3D points preserving the shape of the object. In this experiment the size of the object is correctly identified, but it should be noted that the distance measurements are dependent mostly on the parameters $D_{L}$ and $D_{R}$ which are the distances from camera to laser sources (See Figure 2.27). The alignment of point sets does not guarantee convergence to physically correct values. To determine the correct scales of $D_{L}$ and $D_{R}$, precise physical measurements of these terms can be used as fixed values or a calibration object with known geometry can be utilized.

(c) Front view after optimization

Figure 2.37. Reconstruction results for "person sitting in front of a wall" scan by Scanner-1.

Table 2.4. Mean and standard deviations of radius measurements for a sphere (shown in Figure 2.40) using left \& right lasers before and after optimization procedure.

| (Before Optim./After Optim.) | Left Laser | Right Laser | Left \& Right Laser |
| :--- | :--- | :--- | :--- |
| Mean | $99,528 / 75, \mathbf{3 9 9}$ | $122,61 / 74,989$ | $112,22 / 75,172$ |
| Standard Deviation | $6,2566 / \mathbf{0 , 5 0 7 5 8}$ | $4,9196 / \mathbf{0 , 5 3 7 7 6}$ | $12,7548 / \mathbf{0 , 5 6 2 2 3}$ |
| Number of Data Points | $90.000 / \mathbf{9 0 . 0 0 0}$ | $90.000 / 90.000$ | $90.000 / \mathbf{9 0 . 0 0 0}$ |



Figure 2.38. Reconstruction results for "sphinx mask" scan by Scanner-2.


Figure 2.39. Reconstruction results for "frog figurine" scan by Scanner-3.


Figure 2.40. A 150 mm diameter sphere scanned with Scanner-3. (a) Point cloud before optimization. (b) Point cloud after optimization. (c) Histogram of distances from detected center to 3D points before optimization. (d) Histogram of distances from detected center to 3D points after optimization.

### 2.3.2. Projector Based Structured Light Systems

If it is desired to perform a robust and dense surface reconstruction, projector based structured light imaging techniques are much appreciated. In this section we introduce a projector based scanning system to reconstruct surface geometry. We use a technique where both the camera and projector are treated as viewing devices. Hence they are calibrated in the same manner (Zhang and Huang, 2006) . Each visible point can be correctly located at each devices image plane without solving a correspondence problem, hence a dense reconstruction can be directly obtained. The assumption that a projector can be modeled similar to a camera enables us to calibrate the projector just same as the camera. The calibrated system can be considered as a stereo imaging system, the only difference is that SMP is implicitly solved and theoretically every image pixel can be projected to 3D coordinate system. There are several schemes for structured light
systems. Detailed analysis and comparison of these schemes can be found (Salvi et al., 2010) and (Geng, 2011). If the aim is to simply model surface geometry even a single image can be enough if color coded stripe patterns are used (Li and Zha, 2010). But color coded stripes are not easy to decode when the object chromaticity differs along the surface. Moreover most of the methods do not perform explicit calibration for projector side, hence the reconstructions are upto scale.

Binary coded patterns are much easier to decode since they are not effected by surface chromaticity. The main disadvantage of using binary patterns is that it needs more images for decoding, but since we do not aim to perform a real-time operation this is not of much a problem for our experiments.

For high definition projector and camera systems it is more convenient to use binary Gray code patterns since it is hard to differentiate black and white stripes for patterns representing lower bits (Xu and Aliaga, 2007). We have also followed the robust pixel classification algorithm given in (Xu and Aliaga, 2007). This method inherits the direct and global illumination component separation method proposed in (Nayar et al., 2006). Once the scene is illuminated with the binary Gray coded patterns, robust pixel classification can be used to find on-off pixels and hence projector pixel locations can be decoded.

We used Zhang's homography approach for both camera and projector calibration (Zhang, 2000) (see Appendix B). Moreover, we have adapted the local homography approach in (Moreno and Taubin, 2012) to calibrate projector.

### 2.3.2.1. Calibration of System Components

Since our main aim is to obtain a correct metric information from the scene, the system components, camera and projector, should be fully calibrated. In this section we describe the calibration in more detail.

Zhangs method is a convenient way to estimate camera calibration parameters from 2D views of a simple calibration pattern. In this work we use a 20 by 14 checkerboard pattern. The pattern can be simply obtained by a conventional digital printer. We also put the pattern between two rectangular Plexiglas frame to make it stable. The method uses homographies between the world coordinates and image coordinates of feature points on the pattern.

The projector has a front-end similar to a camera. The only difference is that projectors have light emitting CCDs where cameras have a sensor chip. In (Zhang and


Figure 2.41. Light pattern illuminated sample scene. These samples belong to the calibration pattern at its first position. Patterns encode the column and row coordinates of the pixels. At the top, pattern illuminated sample scene illuminated by column encoding least significant five patterns can be seen. At the bottom, pattern illuminated sample scene illuminated by row encoding least significant five patterns can be seen.

Huang, 2006) it is suggested that we can model the projector as a capturing device. And if we can find the projection of the feature points on the projectors CCD, we can calibrate the projector just like a camera.

The projector we use in this work is an Epson TW-5200 with HD resolution (1920 by 1080). To encode the pixels positions 11 binary bits are needed. We used the same calibration pattern used in camera calibration. The only difference is that we project 11 patterns ( 11 for column and 11 for row encoding). In Figure 2.41, pattern illuminated first calibration scene can be scene. Before calibrating the projector two major steps should be performed. Firstly the pattern illuminated scene images should be decoded. Secondly, the projection of the feature points on projector CCD should be found.

### 2.3.2.2. Robust Pixel Classification

Decoding light patterns is not a straight-forward task and should be handled carefully. The basic idea is to find ON-OFF state of the corresponding projector pixels by looking at the captured images by camera.

Simple thresholding does not work best to detect the pixel state. Robust pixel classification (Xu and Aliaga, 2007) uses a more detailed thresholding algorithm which is based on the method of separating direct and global components of light reflection in a scene (Nayar et al., 2006). This method shows that by using high frequency illumination patterns we can separate the direct and global reflection components.

In this work we use encoded light patterns where patterns representing least significant two bits can be considered as high frequency binary illumination patterns. We can show the direct and global illumination components of reflected light at a pixels position $p$ as $L_{d}(p)$ and $L_{g}(p)$ respectively by adopting the notation in (Moreno and Taubin, 2012).

In Nayar's original work they use shifting patterns. In our wok we use two images where the scene is illuminated by last two highest frequency patterns. Hence, If we represent our set of images as $\mathbb{S}=\left\{I_{1}, \cdots, I_{11}\right\}$, we can show the subset we use as $\mathbf{s}=\left\{I_{10}, I_{11}\right\}$.

We can define two values $L_{\max }^{p}$ and $L_{\min }^{p}$ for a pixel position $p$ as Equation 2.31. The method proposes that $L_{d}(p)$ and $L_{g}(p)$ can be approximated by using $L_{\text {max }}^{p}$ and $L_{\text {min }}^{p}$ as Equation 2.32.

$$
\begin{align*}
L_{\max }^{p} & =\max \left(I_{10}(p), I_{11}(p)\right), \\
L_{\min }^{p} & =\min \left(I_{10}(p), I_{11}(p)\right) .  \tag{2.31}\\
L_{d}(p) & =\frac{L_{\max }^{p}-L_{\min }^{p}}{1-b}, \\
L_{g}(p) & =2 \frac{L_{\min }^{p}-b L_{\max }^{p}}{1-b^{2}} . \tag{2.32}
\end{align*}
$$

Parameter $b$ represents the amount of light emitted by an OFF projector pixel. In practical experiments it is convenient to take $b=0.3$ (Moreno and Taubin, 2012). $L_{d}(p)$ and $L_{g}(p)$ are found by analyzing captured scene images, and used to determine ON-OFF state of the projector pixels by using the following set of rules.

## Robust Pixel Classification Algorithm

$$
\begin{array}{ll}
\text { 1: } & L_{d}<m \rightarrow \text { uncertain } \\
\text { 2: } & L_{d}>L_{b} \wedge p>\bar{p} \rightarrow \mathbf{O N} \\
\text { 3: } & L_{d}>L_{b} \wedge p<\bar{p} \rightarrow \mathbf{O F F} \\
\text { 4: } & p<L_{d} \wedge \bar{p}>L_{b} \rightarrow \mathbf{O F F} \\
\text { 5: } & p>L_{d} \wedge \bar{p}<L_{d} \rightarrow \mathbf{O N} \\
\text { 6: } & \text { ELSE } \rightarrow \text { uncertain }
\end{array}
$$

Here $m$ is a user defined value ( $m \in[0,255]$ ) for thresholding intensity. Low intensity may yield errors in decoding pixel states.

### 2.3.2.3. Finding Feature Points at The Projector Side

We first find the sub-pixel coordinates of the feature points of calibration pattern in the images captured by the camera. This information is used to calibrate the camera.


Figure 2.42. The reflection of the feature points on the calibration pattern can be calculated for both projector and camera. First sub-pixel corner locations found on the image taken by camera then these pixel positions are projected back to the projectors image plane. Local homography method is used for projection.

For projector calibration we have to find these points sub-pixel locations as seen by the projector (Figure 2.42).

Once the pixels in the captured images are analyzed and the decoding operation is performed, we can generate a virtual image which represents the image seen by projector. But this is a time consuming process for such a practical application. Hence we only calculate the values in a window around detected checkerboard corners locally. The size of the window may vary with respect to the image size. In this work we use 30 by 30 windows to find sub-pixel coordinates of feature points on the image plane of projector.

Locally decoded pixel values are used to find the projection of feature points on the image plane of the projector. As a result we can find sub-pixel feature point positions not only in the cameras image plane but also in the projectors image plane. This enables us to calibrate the intrinsic and extrinsic calibration parameters of both camera and the projector.

### 2.3.2.4. Reconstruction of Surface Geometry

Without solving an SMP, We can calculate the sub-pixel positions on both cameras and projectors image plane for any point if it is visible by camera and illuminated by the projector. Since we have two corresponding pixels positions in two different views, depth finding problem can be solved just like in a standard stereo or multi view imaging case. Here the depth value can be found by solving corresponding singular value decomposition (SVD) problem (see Appendix C).

By using MATLAB's GUI functions we are able to select region of interest (ROI) over the scene image (Figure 2.43). This helps us to eliminate redundant areas and speeds up the process significantly. In Figure 2.44 we show different experimental the scenes. The proposed scanning system gives dense reconstruction results as a point cloud. Once needed the corresponding surface meshes can be obtained by using the signed distance function method described in Appendix G.


Figure 2.43. MATLAB's GUI function makes it possible to select ROI.

(a) Experimental Scenes


Figure 2.44. (a) Image shows three different experiment scenes. (b) The reconstruction results of the corresponding scenes. The proposed scanner system generates dense point clouds of corresponding scenes.

## CHAPTER 3

## RECONSTRUCTION OF SURFACE REFLECTION PROPERTIES

### 3.1. Background and Motivation

In this chapter we will propose a method to reconstruct surface reflection properties. After recalling some radiometric quantities we will introduce BRDF concept. We will revisit the projector based structured light system introduced in Section 2.3.2 because the system is capable of giving dense reconstruction of corresponding scenes. The system calibration described in the same section also assures that the illumination and viewing directions can be identified for each reconstructed point.

Here we aim to use same setup to measure the radiance information from the scenes under our consideration as well. This is performed by using HDRI techniques. We use the point wise light directions together with the radiance measurements to construct a BRDF database for corresponding surface materials. Finally we show that overall reflection properties of materials can be approximated by fitting an analytical BRDF model to measurement data.

### 3.2. Fundamentals of Light

In thissection we will recall definitions of basic radiometric quantities. We will briefly discuss properties of light and how they relate to digital imaging. Corresponding quantities and their corresponding units can be seen in Table 3.1 and also in (Reinhard et al., 2010).

Definition The ability of materials to reflect light is called reflectance.
Definition Radiometry is the science concerned with measuring light.
Definition Light is a radiant energy $\left(Q_{e}\right)$ measured in joules.

Table 3.1. Radiometric symbols, names, and units.

| Quantity | Unit | Definition |
| :--- | :--- | :--- |
| Radiant Energy $\left(Q_{e}\right)$ | $J$ (joule) | $Q_{e}$ |
| Differential Solid Angle $(d \omega)$ | $s r$ (steradian) | $d \omega$ |
| Radiant Power $\left(P_{e}\right)$ | $J s^{-1}=W$ (Watt) | $P_{e}=\frac{d Q_{e}}{d e}$ |
| Radiant Exitance $\left(M_{e}\right)$ | $W m^{-2}$ | $M_{e}=\frac{d P_{e}}{d A_{e}}$ |
| Irradiance $\left(E_{e}\right)$ | $W m^{-2}$ | $E_{e}=\frac{d P_{e}}{d A_{e}}$ |
| Radiant intensity $\left(I_{e}\right)$ | $W r^{-1}$ | $I_{e}=\frac{d P_{e}}{d \omega}$ |
| Radiance $\left(L_{e}\right)$ | $W m^{-2} s r^{-1}$ | $L_{e}=\frac{d^{2} P_{e}}{d A \cos \theta d \omega}$ |

Definition The differential solid angle, $d \omega$, is the 2D angle in 3D space that an object subtends at a point (Dutre et al., 2006). It is a measure of how large the object appears to an observer looking from that point. Solid angle is expressed in a dimensionless unit called a steradian. The surface patch in Figure 3.1 can be given as Equation 3.1


Figure 3.1. The differential solid angle is the 2D angle in 3D space that an object subtends at a point.

$$
\begin{align*}
d A & =(r d \theta)(r \sin \theta d \phi),  \tag{3.1}\\
& =r^{2} \sin \theta d \theta d \phi .
\end{align*}
$$

Hence the solid angle subtended by the surface patch is Equation 3.2 from which the area of sphere, $S$, is found to be as $4 \pi s r$ (Equation 3.3).

$$
\begin{gather*}
d \omega=\frac{d A}{r^{2}}=\sin \theta d \theta d \phi  \tag{3.2}\\
S=\int_{0}^{\pi} \int_{0}^{2 \pi} \sin \theta d \theta d \phi,=4 \pi \tag{3.3}
\end{gather*}
$$

Solid angle is used in the description of light. It relates the flux to the intensity of light. The solid angle also represents the angular size of a light beam with its direction. In radiometry and illumination calculations, the integral over the incoming directions on the hemisphere $\Omega$ is often needed. This integral can be written in spherical coordinates and evaluated accordingly as Equation 3.4

$$
\begin{equation*}
\int_{\Omega} f(\theta, \phi) d \omega=\int_{0}^{2 \pi} \int_{0}^{\pi / 2} f(\theta, \phi) \sin \theta d \theta d \phi . \tag{3.4}
\end{equation*}
$$

Definition The flow of radiant energy can be measured. It is indicated with radiant power or radiant flux $\left(P_{e}\right)$ and is measured in joules per second $J s^{-1}$, or watts $W$. It is thus a measure of energy per unit of time.

Definition Radiant flux density is the radiant flux per unit area. It is known as irradiance $\left(E_{e}\right)$ for flux arriving from all possible directions at a point on a surface and as radiant exitance $\left(M_{e}\right)$ for flux leaving a point on a surface in all possible directions. Both irradiance and radiant exitance are measured in watts per square meter $W \mathrm{~m}^{-2}$. These are therefore measures of energy per unit of time as well as per unit of area.

Definition Radiance, $L$, is the radiant flux per unit solid angle $W m^{-2} s r^{-1}, d \vec{\omega}$ per unit projected area $d A$ (see Figure 3.2):

### 3.3. The Bidirectional Reflectance Distribution Function (BRDF)

Bidirectional reflectance distribution function identifies the light matter interaction for any given illumination and viewing directions. It is wavelength dependent but in the computer graphics literature the studies are performed in visible light spectrum with three color channels (RGB).

BRDF concept is used to improve realism in computer graphics. It is the most common and efficient way to model the light - matter interaction which has complicated


Figure 3.2. Radiance $L$
dynamics. The interaction depends both on physical characteristics of the matter and physical characteristics of the light. One can observe the different interactions by just looking at surrounding objects.

A BRDF describes how much light is reflected when light makes contact with a certain material. When analysing the light matter interaction, it can be observed that the intensity of reflected light varies for different illumination and viewing direction combinations. So it can be deduced that, BRDF should be defined as a function of incoming and outgoing light directions.

Different colors (hence different wavelengths) of light may be absorbed , reflected and transmitted to varying degrees depending on the physical properties of the material, which makes BRDF also wavelength dependent. Due to the inhomogeneity of materials, the light-matter interaction may also differ locally. This property is called as positional variance. There are two main types of BRDFs:

- Isotropic BRDFs: Isotropic BRDFs represent reflectance properties of surfaces for which the reflection properties are invariant with respect to rotation of surface around the normal vector.
- Anisotropic BRDFs: Anisotropy refers to BRDFs that describe reflectance properties that do exhibit change with respect to rotation of the surface around the normal vector. In practical applications materials are often assumed to be isotropic for simplicity.

Definition and formulation of BRDF was firstly introduced in (Nicodemus et al., 1992). Simple representation of BRDF can be seen in Figure 3.3. The BRDF, $f$, relates the reflected radiance and incoming irradiance as Equation 3.5

$$
\begin{align*}
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right) & =\frac{d L_{r}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right)}{d E_{i}\left(\boldsymbol{\omega}_{\boldsymbol{i}}\right)}  \tag{3.5}\\
& =\frac{d L_{r}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right)}{L_{i}\left(\boldsymbol{\omega}_{\boldsymbol{i}}\right)\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \boldsymbol{n}\right) d \boldsymbol{\omega}_{\boldsymbol{i}}},
\end{align*}
$$

Where $\mathbf{n}$ is the surface normal. If the incident radiance field at a surface location is known, the reflected radiance in all directions can be calculated. The relation between the reflected radiance and BRDF is given in Equation 3.6, $\Omega$ is the hemisphere of all incoming directions. $\omega_{i}$ and $\omega_{o}$ represents the normal vectors in incoming and outgoing directions respectively (Figure 3.4).


Figure 3.3. BRDF assumes that it is reflected at the same location at which it hits the surface.

$$
\begin{align*}
L_{r}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right) & =\int_{\Omega} f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right) d E\left(\boldsymbol{\omega}_{\boldsymbol{i}}\right), \\
& =\int_{\Omega} f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right) L_{i}\left(\boldsymbol{\omega}_{\boldsymbol{i}}\right)\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right) d \boldsymbol{\omega}_{\boldsymbol{i}} . \tag{3.6}
\end{align*}
$$

Where $\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right)=\cos \theta_{i}$.

The two most important properties of a BRDF are the reciprocity and energy conservation which can be given as follows:

Helmholtz's Law of Reciprocity: Helmholtz's Law of Reciprocity states that BRDF is independent of the direction in which the light flows. This can be shown as Equation 3.7

$$
\begin{equation*}
f\left(\omega_{i}, \omega_{o}\right)=f\left(\boldsymbol{\omega}_{o}, \omega_{i}\right) . \tag{3.7}
\end{equation*}
$$

Energy Conservation Law: Energy Conservation law states that a surface can not reflect more light than it receives. It can be shown in terms of BRDF as Equation 3.8

$$
\begin{equation*}
\int_{\Omega} f\left(\boldsymbol{\omega}_{i}, \boldsymbol{\omega}_{o}\right)\left(\boldsymbol{\omega}_{i} \cdot \mathbf{n}\right) d \boldsymbol{\omega}_{i} \leq 1, \forall \boldsymbol{\omega}_{o} \tag{3.8}
\end{equation*}
$$



Figure 3.4. $\omega_{i}, \omega_{o}$ and n vectors shown over the hemisphere over the surface point.

### 3.3.1. BRDF Measurement

Complete BRDF measurement is a detailed and time consuming process which requires to build a special device called gonioreflectometer. Gonioreflectometers are used to measure surface reflection of a sample surface for every incoming and outgoing light direction combinations.

In (Matusik, 2003) a gonioreflectometer system to measure BRDF over spherical samples is proposed. The measurements are performed for 100 different materials and an open database is generated. As proposed in (Matusik, 2003) a principle component analysis (PCA) is performed to reduce the measured data size. The measurements can be used in ray tracer programs such as physically based ray tracer (PBRT) (Pharr and Humphreys, 2010).


Figure 3.5. Spherical coordinate system

Generalized BRDF measurement of a planar surface by using a gonioreflectometer can be seen in Figure 3.6.In (Matusik, 2003) a setup similar to Figure 3.7 is used. Since it has a known and parameterized shape spherical samples can give multiple measurement information from a single frame. In (Ngan et al., 2005) a setup similar to Figure 3.8 is used. The cylindrical sample is used to make measurements for anisotropic materials such as velvet. The sample velvet stripes are cut with different angles and coated over the cylinder. As the cylinder is rotated around its vertical axis the system becomes capable of measuring anisotropic material BRDFs. Also some other possible BRDF measurement setups can be seen in Figures 3.9 and 3.10.

Real materials have complex BRDFs. In Figure 3.11, a ball object is rendered with six different measured BRDFs from (Matusik, 2003). Lighting is performed with a high dynamic range image from Debevec's Light Probe Image Gallery (Debevec, 1998). As it can be seen from the figure, materials have different characteristics with respect to light matter interaction. It is not possible to derive a unique parametric formula to capture the whole complexity of real material BRDFs. But yet, analytical models can mimic BRDF of real materials once the model and model parameters are chosen appropriately.

In order to measure BRDF we need the irradiance and the radiance values at a surface point. Scene radiance can be captured by cameras. If we consider the light prop-


Figure 3.6. Generalized BRDF measurement by using gonioreflectometer.


Figure 3.7. BRDF measurement can be done by using spherical samples with various light directions (Matusik, 2003).
agation in vacuum (see Figure 3.12) we can write the flux inside a hypothetical tube as Equation 3.9 which uses the fact that the flux coming $\left(d \Theta_{1}\right)$ is equal tote flux leaving $\left(d \Theta_{2}\right)$. The solid angles subtended by small surface patches at either side of the tube can be written as Equation 3.10. Hence it is easy to see that Equation 3.11 is true. From Equation 3.9 and 3.11 we can deduce Equation 3.12.


Figure 3.8. Cylindrical samples can be used for BRDF measurements. It is also possible to measure anisotropic BRDFs by using this setup (Ngan et al., 2005).


Figure 3.9. A possible BRDF measurement setup.

$$
\begin{gather*}
d \Phi_{1}=L_{1} d \omega_{1} d A_{1}=L_{2} d \omega_{2} d A_{2}=d \Phi_{2}  \tag{3.9}\\
d \omega_{1}=d A_{2} / r^{2}, d \omega_{1}=d A_{2} / r^{2}  \tag{3.10}\\
d \omega_{1} d A_{1}=\frac{d A_{1} d A_{2}}{r^{2}}=d \omega_{2} d A_{2}  \tag{3.11}\\
L_{1}=L_{2} \tag{3.12}
\end{gather*}
$$

A camera captures radiance reflected from a scene. But every capturing device, either digital or analog, have a response function which identifies the devices behaviour to the amount of irradiance incident to its sensor. Figure 3.13 briefly shows this process.

The relation between scene radiance and image irradiance can be derived by considering Figure 3.14. Subtended solid angles $d \boldsymbol{\omega}_{i}$ and $d \omega_{s}$ we can derive the relation


Figure 3.10. A possible BRDF measurement setup.
between $d A_{i}$ and $d A_{s}$ as in Equation 3.13. The solid angle subtended by the camera lens can be shown as Equation 3.14. We can assume that camera lens projects incoming flux from a surface patch onto the image plane (CCD in a digital camera case). This gives us the relation in Equation 3.15.

From the relations given in Equations 3.13-3.15 we can write the relation between scene radiance $L$ and image irradiance $E$ as Equation 3.16. Since $\alpha$ is a very small angle, effect of $\cos \alpha$ term is negligible $(\cos \alpha \approx 1)$.

$$
\begin{align*}
d \omega_{i} & =d \boldsymbol{\omega}_{s}, \\
\frac{d A_{i} \cos \alpha}{(f / \cos \alpha)^{2}} & =\frac{d A_{s} \cos \theta}{(z / \cos \alpha)^{2}}, \\
\frac{d A_{s}}{d A_{i}} & =\frac{\cos \alpha}{\cos \theta}\left(\frac{z}{f}\right)^{2}  \tag{3.13}\\
d \omega_{L} & =\frac{\pi d^{2}}{4} \frac{\cos \alpha}{(z / \cos \alpha)^{2}} .  \tag{3.14}\\
L\left(d A_{s} \cos \theta\right) d \omega_{L} & =E d A_{i} .  \tag{3.15}\\
E & =L \frac{\pi}{4}\left(\frac{d}{f}\right)^{2} \cos \alpha^{4} . \tag{3.16}
\end{align*}
$$

### 3.3.2. Camera Response Function and HDR Imaging

The cameras response function relates image irradiance at the image plane to measured pixel intensity values. It is possible to find a cameras response function by captur-


Figure 3.11. Rendered BRDFs of different materials. Material names are below each image.


Figure 3.12. Radiance is constant as it propagates along ray


Figure 3.13. Relationship between Scene and Measured Pixel Values.


Figure 3.14. Relation between scene radiance and image irradiance.
ing a scene with various exposure times. We implement the method used in high dynamic range imaging (HDRI) as described in (Debevec and Malik, 2008) to find out our cameras response functions. Scene radiance map can be calculated by using response functions (see Appendix H).

Method in (Debevec and Malik, 2008) can estimate the response function not only by using the color correction card but also by using such different scenes given in Figure 3.15. The recovered camera response functions for green channel with underlying data can be seen in Figure 3.16. In Figure 3.17 we can see that the recovered response function is very close even by using different scenes.

As it is stated in (Debevec and Malik, 2008) theoretically at least two images


Figure 3.15. (a) Color correction card used to recover camera response function. (b) Sample indoor and (c) Sample outdoor scenes are used to recover camera response function as well.


Figure 3.16. Green channel component of the response function of Nikon D60 recovered by the method in (Debevec and Malik, 2008). Top, recovered by using checker card. Bottom left, recovered by using the indoor scene where bottom right recovered by using the outdoor scene. Underlying data ( $E_{i} \Delta t_{j}$, $Z_{i j}$ ) shown as cyan circles. The logarithm is base $e$.


Figure 3.17. Recovered response functions in Figure 3.16 are shown in the same plot.
taken under two distant exposures are enough to recover film response curve. In the above analysis we use 18 exposure stops from $1 / 4000$ to 30 . We also analyzed the effect of the number of the images used in radiance calculations. In Figure 3.18 reconstructed log exposure values of randomly selected 1000 pixels can be seen for different number of exposures used.

If we consider recovered radiance values which are calculated by using 18 images as a ground truth, from Figure 3.18 we can see that from 18, 9 and 5 images generates nearly the same result where 3 images significantly deviates from the ideal values. Hence, we can deduce that 5 images with 5 distant exposure values can be used to recover desired scene radiance values. Calculated RMS errors with respect to Figure 3.18 can be seen in Table 3.2.

### 3.3.3. Analytical BRDF Models

Analytical BRDF models are used in computer graphics since they are easy to implement and can be used to render realistic light - matter interactions. In order to define these models the notation summarized in Figure 3.19 can be used. Important parameters and their meaning can be seen in Table 3.3. The following BRDFs are well known BRDF


Figure 3.18. On the top recovered log radiance values, at the bottom recovered decimal radiance values for 1000 randomly selected samples from the scene. The radiance plots are generated by using 18, 9, 5 and 3 images. The plot generated with the whole 18 images taken at different exposure times is our ground truth. The plots generated with 9 and 5 exposure images closely match the ground truth.

Table 3.2. Radiance value can be calculated with different number of images. The following table shows RMS error for calculated radiance by using different number of images with respect to the ground truth value calculated by using all 18 images.

| Number of Images | Log Radiance RMS Error | Radiance RMS Error |
| :---: | :---: | :---: |
| 9 Images | 0.0270 | 0.4249 |
| 5 Images | 0.0283 | 0.7687 |
| 3 Images | 0.3140 | 5.1460 |

models which are widely used in computer graphics literature ((Pharr and Humphreys, 2010),(Jensen, 2001),(Dutre et al., 2006)). Here we aim to briefly list some of these analytical BRDF models.


Figure 3.19. BRDF notation

Lambert's model: This model (Lambert, 1760) is the idealized model for diffuse material reflections. Most analytical BRDF models assume that the surface reflection obeys dichromatic reflection model (Shafer, 1985) and diffuse component of the reflection is pure Lambertian. Lamberts model assumes that for an illumination direction the BRDF is constant and equal to Equation 3.17 for every outgoing light direction.

$$
\begin{equation*}
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{\rho_{d}}{\pi} \tag{3.17}
\end{equation*}
$$

Phong model: Phong model (Phong, 1975) is a popular model which is widely used in computer graphics applications. It can be modeled as Equation 3.18. $\mathbf{r}$ is the unit

Table 3.3. Table of notation used in representing BRDFs

| Symbol | Meaning |
| :---: | :--- |
| $\boldsymbol{\omega}_{i}, \boldsymbol{\omega}_{\boldsymbol{o}}$ | Unit-length vectors for incoming and outgoing directions |
| $f\left(\boldsymbol{\omega}_{i}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)$ | Bidirectional reflectance distribution function (BRDF) |
| $\rho_{d}, \rho_{s}$ | Diffuse and specular reflection parameters |
| $p$ | Corresponding point on sample surface |
| $\mathbf{n}$ | Unit-length surface normal vector |
| $\mathbf{h}$ | Unit-length halfway vector |
| $\mathbf{r}$ | Unit-length vector in the specular reflection direction |
| $\omega_{i x}, \omega_{i y}, \omega_{i z}$ | $x, y$ and $z$ components of the unit direction vector $\boldsymbol{\omega}_{\boldsymbol{i}}$ |
| $\omega_{o x}, \omega_{o y}, \omega_{o z}$ | $x, y$ and $z$ components of the unit direction vector $\boldsymbol{\omega}_{\boldsymbol{o}}$ |
| $\theta_{i}, \theta_{o}, \theta_{h}$ | $\theta$ angles of vectors $\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}$ and $\mathbf{h}$ (ref. Figure 3.19) |
| $\phi_{i}, \phi_{o}, \phi_{h}$ | $\phi$ angles of vectors $\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}$ and $h$ (ref. Figure 3.5) |
| $\boldsymbol{\Omega}_{+}$ | Unit hemisphere above the surface point $\mathbf{p}$ |
| $F\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{h}\right)$ | Fresnel reflectance for incident angle between $\boldsymbol{\omega}_{\boldsymbol{o}}$ and $\mathbf{h}$ |
| $L_{i}\left(\boldsymbol{\omega}_{\boldsymbol{i}}\right)$ | Incident radiance function |
| $L_{o}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right)$ | Outgoing radiance function |
| $L_{e}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right)$ | Emitted radiance function |

vector in the direction of the specular reflection with respect to $\omega_{i}$ (see Equation 3.19). In Figure 3.20 the effect of change in model parameter $m$ can be seen.

$$
\begin{equation*}
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{\rho_{d}}{\pi}+\rho_{s}\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{r}\right)^{m} . \tag{3.18}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{r}=2\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right) \mathbf{n}-\boldsymbol{\omega}_{\boldsymbol{i}} \tag{3.19}
\end{equation*}
$$

Ward model: Ward model (Ward, 1992) is defined by Equation 3.20. This model is able to capture anisotropic material reflection properties for $\alpha_{x} \neq \alpha_{y}$. If $\alpha_{x}=\alpha_{y}$ this equation simplifies to isotropic version in Equation 3.21. Variation of surface reflection properties for varying $\alpha_{x}$ and $\alpha_{y}$ values can be seen in Figure 3.21.

$$
\begin{gather*}
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{\rho_{d}}{\pi}+\rho_{s} \frac{\exp \left[\tan ^{2} \theta_{h}\left(\cos ^{2} \phi_{h} / \alpha_{x}^{2}+\sin ^{2} \phi_{h} / \alpha_{y}^{2}\right)\right]}{4 \pi \alpha_{x} \alpha_{y} \sqrt{\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right)\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{n}\right)}} .  \tag{3.20}\\
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{\rho_{d}}{\pi}+\rho_{s} \frac{\exp \left[\tan ^{2} \theta_{h} / \alpha^{2}\right]}{4 \pi \alpha^{2} \sqrt{\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right)\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{n}\right)}} . \tag{3.21}
\end{gather*}
$$

Blinn-Phong model: Unlike Phong model, this model (Blinn, 1977) uses halfway vector $\mathbf{h}$ (see Equation 3.22) instead of specular reflection vector $\mathbf{r}$ and can be given as


Figure 3.20. Phong BRDF model can capture specularity. Higher lobe exponent $m$ creates more specularity in renders. Diffuse and specular reflection parameters (i.e. $\rho_{d}$ and $\rho_{s}$ ) are taken equal.

Equation 3.23 .

$$
\begin{gather*}
\boldsymbol{h}=\frac{\boldsymbol{\omega}_{\boldsymbol{i}}+\boldsymbol{\omega}_{\boldsymbol{o}}}{\left\|\boldsymbol{\omega}_{\boldsymbol{i}}+\boldsymbol{\omega}_{\boldsymbol{o}}\right\|}  \tag{3.22}\\
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{\rho_{d}}{\pi}+\rho_{s}(\mathbf{h} \cdot \mathbf{n})^{m} . \tag{3.23}
\end{gather*}
$$

Lafortune model: Lafortune model (Lafortune et al., 1997) is defined by Equation 3.24.

$$
\begin{equation*}
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{\rho_{d}}{\pi}+\rho_{s} \frac{m+2}{2 \pi} \frac{\left[C_{x y}\left(\omega_{i x} \omega_{o x}+\omega_{i y} \omega_{o y}\right)+C_{z} \omega_{i z} \omega_{o z}\right]^{m}}{\left[\max \left(\left|C_{z}\right|,\left|C_{x y}\right|\right)\right]^{m}} . \tag{3.24}
\end{equation*}
$$

Cook-Torrance model: This model (Cook and Torrance, 1982) includes a microfacet model. Microfacet models assume that the surface is composed of a random collection of small smooth planar facets. Moreover this model also includes the Fresnel effect. Model equation can be given as Equation 3.25.

$$
\begin{equation*}
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{\rho_{d}}{\pi}+\rho_{s} \frac{F\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{h}\right)}{\pi} \frac{D(\mathbf{h}) G}{\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right)\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{n}\right)} . \tag{3.25}
\end{equation*}
$$

In Equation 3.25, $D(\mathbf{h})$ is the distribution function which specifies the distribution of the microfacets. Various functions can be used as microfacet distribution function. The only restriction is that, in order to obtain a physically plausible analytical model (i.e. it satisfies two properties reciprocity and energy conservation), corresponding microfacet distribution function should be normalized such that it satisfies equation 3.26.


(d) $\alpha_{x}=0.1, \alpha_{y}=1$

(e) $\alpha_{x}=0.1, \alpha_{y}=0.1$

(f) $\alpha_{x}=0.1, \alpha_{y}=0.01$

(g) $\alpha_{x}=0.01, \alpha_{y}=1$

(h) $\alpha_{x}=0.01, \alpha_{y}=0.1$

(i) $\alpha_{x}=0.01, \alpha_{y}=0.01$

Figure 3.21. Ward BRDF model captures anisotropic surface properties if $\alpha_{x} \neq \alpha_{y}$. The behaviour of the model for varying $\alpha_{x}$ and $\alpha_{y}$ values can be seen in the figures (a) to (i).

In applications, $D(\mathbf{h})$ is chosen to be either Blinn Microfacet Distribution in Equation 3.27 or Beckmann Microfacet Distribution in Equation 3.28.

$$
\begin{gather*}
\int_{0}^{2 \pi} \int_{0}^{\pi / 2} D(\mathbf{h}) \cos \theta_{h} \sin \theta_{h} d \theta_{h} d \phi_{h}=1 .  \tag{3.26}\\
D(\mathbf{h})=\frac{m+2}{2 \pi}(\mathbf{h} \cdot \mathbf{n})^{m}  \tag{3.27}\\
D(\mathbf{h})=\frac{1}{m^{2} \cos ^{4} \theta_{h}} e^{-\left(\frac{\tan \theta_{h}}{m}\right)^{2}} \tag{3.28}
\end{gather*}
$$

$G$ in Equation 3.29 is the geometry term which captures masking and self-shadowing generated by microfacets. It is defined as Equation 3.29.

$$
\begin{equation*}
G=\min \left\{1, \frac{2(\mathbf{h} \cdot \mathbf{n})\left(\boldsymbol{\omega}_{o} \cdot \mathbf{n}\right)}{\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right)}, \frac{2(\mathbf{h} \cdot \mathbf{n})\left(\boldsymbol{\omega}_{i} \cdot \mathbf{n}\right)}{\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right)}\right\} \tag{3.29}
\end{equation*}
$$

$F\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right)$ in Equation 3.25 is the Fresnel reflectance term. This term can be approximated by Schlick's formula (Christophe, 1994) given by Equation 3.30. $f_{0}$ in equation is the normal reflectance. Normal reflectance is the measure of reflection amount if the illumination direction is in the direction of surface normal.

$$
\begin{equation*}
F\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right)=f_{0}+\left(1-f_{0}\right)\left(1-\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right)\right)^{5} \tag{3.30}
\end{equation*}
$$

Ashikhmin-Shirley model: Isotropic version of Ashikhmin-Shirley model can be written as Equation 3.31.

$$
\begin{equation*}
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{\rho_{d}}{\pi}+\rho_{s} \frac{m+1}{8 \pi} \frac{F\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{h}\right)(\mathbf{n} \cdot \mathbf{h})^{m}}{\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{h}\right) \max \left\{\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right),\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{n}\right)\right\}} \tag{3.31}
\end{equation*}
$$

### 3.3.4. Rendering Equation

The rendering equation (Kajiya, 1986), constitutes the mathematical basis for all global illumination algorithms. It can be used to compute the outgoing radiance at any surface location assuming that the media is non-participating. For a sample point on a surface, general form of the rendering equation is defined as Equation 3.32

$$
\begin{equation*}
L_{o}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right)=L_{e}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right)+L_{r}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right) . \tag{3.32}
\end{equation*}
$$

Where $L_{o}$ is the outgoing radiance, $L_{e}$ is the emitted radiance and $L_{r}$ is the reflected radiance. If the open form of $L_{r}$ is written, Equation 3.33 an be written.

$$
\begin{equation*}
L_{o}\left(\boldsymbol{\omega}_{\boldsymbol{o}}\right)=L_{e}\left(\boldsymbol{\omega}_{o}\right)+\int_{\boldsymbol{\Omega}_{+}} L_{i}\left(\boldsymbol{\omega}_{\boldsymbol{i}}\right) f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right) d \boldsymbol{\omega}_{i} \tag{3.33}
\end{equation*}
$$

If the aim is to perform a physically based rendering, the chosen BRDF $f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)$ in Equation 3.33 should be physically plausible, which requires Helmholtz reciprocity (Equation 3.7) and energy conservation (Equation 3.8). Rendering equation is basically mathematical formulation of energy transport in a scene. Equation 3.33 can be interpreted as; pick up the self- emitted radiance, take into account all possible cosine factors and possible BRDF values along the path, perform the necessary integration at each surface point to finally arrive at the original radiance value (Dutré et al., 2004).

In (Edwards et al., 2006), it is shown that a microfacet function which is described with the halfway vector satisfies Equation 3.34 to conserve energy.

$$
\begin{equation*}
f\left(\boldsymbol{\omega}_{i}, \boldsymbol{\omega}_{o}\right) \leq \frac{F\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right) D(\mathbf{h})}{4\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right)\left(\boldsymbol{\omega}_{i} \cdot \mathbf{n}\right)} . \tag{3.34}
\end{equation*}
$$

The factor $\left(\boldsymbol{\omega}_{i} \cdot \mathbf{n}\right)$ in the denominator part of equation makes the inequality non-reciprocal. By making the modification proposed in (László et al., 1999), Equation 3.35 can be written.

$$
\begin{equation*}
f\left(\boldsymbol{\omega}_{i}, \boldsymbol{\omega}_{o}\right)=\frac{F\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right) D(\mathbf{h})}{4\left(\boldsymbol{\omega}_{o} \cdot \mathbf{h}\right) \max \left\{\left(\boldsymbol{\omega}_{o} \cdot \mathbf{n}\right),\left(\boldsymbol{\omega}_{i} \cdot \mathbf{n}\right)\right\}} \tag{3.35}
\end{equation*}
$$

If a normalized microfacet distribution function is chosen (Equation 3.27 or Equation 3.28), Equation 3.35 becomes a physically plausible BRDF model. In (Kurt et al., 2010), it is stated that this model does not have a good fitting performance. Hence they proposed a modified version of this model (Equation 3.36) which is better in data fitting than existing analytical BRDF models.

$$
\begin{equation*}
f\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)=\frac{F\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{h}\right) D(\mathbf{h})}{4\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{h}\right)\left(\left(\boldsymbol{\omega}_{\boldsymbol{o}} \cdot \mathbf{n}\right)\left(\boldsymbol{\omega}_{\boldsymbol{i}} \cdot \mathbf{n}\right)\right)^{\alpha}} . \tag{3.36}
\end{equation*}
$$

In (Kurt et al., 2010) Beckmann microfacet distribution function is used. They showed that if $m$ in Equation 3.28 and $\alpha$ in Equation 3.36 is chosen properly, the model is physically plausible as well. This can also be shown by plotting albedo function for different values of $m$ and $\alpha$. Albedo is simply computed by integrating the BRDF over the outgoing hemisphere $\Omega_{+}$for given incident light directions. It can be seen for Kurt et al.'s model in (Kurt et al., 2010) that albedo is below 1 which assures plausibility (Figure 3.22).


Figure 3.22. Albedo functions of Kurt et al.'s data fitting BRDF model (see Equation 3.36 ) for different $m$ and $\alpha$ values. If $m$ and $\alpha$ are correctly chosen, this model is physically plausible. Fresnel term is chosen as $f_{0}=1.0$.

### 3.4. Estimating Material BRDFs

In 2.3.2 we have introduced a system designed for dense 3D reconstruction of surfaces. We aim to use the same system for estimation of the BRDFs of the scanned objects. The proposed system consists of a projector and a digital camera. Since the both system components are calibrated we can deduce the incoming and reflected light ray directions for every reconstructed surface point. BRDF is defined with these directions but they should be represented with respect to the surface normal. Hence we should find surface normal directions of each reconstructed surface point.

### 3.4.1. Estimation of Surface Normals in a Point Cloud Data

The scanner system proposed in 2.3.2 is able to generate dense point cloud data of scanned objects. The generated point clouds closely approximate the scanned surface geometry. We use the surface normal estimation technique in (Mitra et al., 2004) which is a robust method for estimating surface normals in a noisy point cloud data. If we consider the sample scene in Figure 3.23a, we can obtain a dense point cloud of the reconstructed surface (Figure 3.23b).


Figure 3.23. (a) The experiment scene of a plastic toy truck. (b) Reconstruction of the plastic truck object. (c) A selected surface patch.

We select a sample surface area over the reconstructed surface such as Figure 3.23c. Surface normals at each point in the point cloud can be found by locally applying total least square line method (Mitra et al., 2004). Given a point cloud we can represent the set of points as $p_{i}(\ni 1 \leq i \leq k)$ where $k$ is the number of points in the point cloud. We want to find a line such that the square distances from the points $p_{i}$ 's to the line is minimized. We can define the line as in Equation 3.37.

$$
\begin{equation*}
a^{T} x=c \ni a^{T} a=1 . \tag{3.37}
\end{equation*}
$$

The sum of square distances can be calculated by using the function $f(\cdot)$ which can be defined as Equation 3.39. The aim is to find $a$ and $c$ which minimize $f(\cdot)$. This is a quadratic optimization problem and the system of equations in Equation 3.40 should be solved.

$$
\begin{align*}
f(a, c) & =\frac{1}{2 k} \sum_{i=1}^{k}\left(a^{T} p_{i}-c\right)^{2}, \\
& =\frac{1}{2} a^{T}\left(\frac{1}{k} \sum_{i=1}^{k} p_{i} p_{i}^{T}\right) a-c \bar{p}^{T} a+\frac{1}{2} c^{2},  \tag{3.38}\\
\text { Where } \bar{p} & =\frac{1}{k} \sum_{i=1}^{k} p_{i} .
\end{align*}
$$

$$
\begin{align*}
\frac{\partial}{\partial a} f(a, c)=\lambda a & \Rightarrow\left(\frac{1}{k} \sum_{i=1}^{k} p_{i} p_{i}^{T}\right) a-c \bar{p}=\lambda a  \tag{3.39}\\
\frac{\partial}{\partial c} f(a, c)=0 & \Rightarrow-\bar{p}^{T} a+c=0
\end{align*}
$$

$\lambda$ is a Lagrangian multiplier, and from Equation 3.40 we can deduce that $c=\bar{p}^{T} a$ and Equation 3.41 can be written. The problem becomes an eigenvalue - eigenvector problem and $a$ is the eigenvector corresponding to the smallest eigenvalue of $M$ and it is the normal we are seeking.

$$
\begin{align*}
\left(\frac{1}{k} \sum_{i=1}^{k} p_{i} p_{i}^{T}-\bar{p} \bar{p}^{T}\right) a & =\lambda a  \tag{3.40}\\
M a & =\lambda a \tag{3.41}
\end{align*}
$$

Detailed eigen analysis and error bound analysis can be found in (Mitra et al., 2004). The normals are calculated in a small neighborhood of each point in the cloud. The selection of neighborhood size important and it drastically effects the normal estimation. As in (Mitra et al., 2004) we empirically estimate the neighborhood ( $r$ ) size. If it is too small the estimated normal directions are distorted. The effect of neighborhood size can be seen in Figure 3.24.

We use surface normal information to identify BRDF measurements of a given surface since BRDF is defined by incoming and outgoing light directions with respect to the surface normal.

### 3.4.2. Estimation of Surface Irradiance

The objects in our experiments are illuminated with an Epson HD projector. Hence the surface irradiance of the objects in the experiments are directly related with the projector power and the projection area which is given as Figure 3.25 for the correspnding projector.

By using the projection distance in the specs and the distance of infinitesimal surface area to the projector, the corresponding surface irradiance can be calculated. It is important to consider the foreshortening which is known from the surface normal and the illumination directions. Hence, we can calculate the foreshortening $\left(\omega_{i} \cdot \mathbf{n}\right)$ for each pixel.



Figure 3.24. Aligned top view of the point cloud shown in Figure 3.23c. The selection of neighborhood size important. (a) If it is too small the estimated normal directions are distorted (b) If $r$ is selected properly the computed normal directions are better aligned.


Figure 3.25. Specifications of Epson EH-TW-5200. It has a luminous flux of 2000 lumens (lm). The projection area with the corresponding distance can be seen.

### 3.4.3. Estimation of Surface Radiance

To measure the material BRDF we need not only the surface irradiance but also the radiance in the direction of the viewer. We show that the scene radiance map can be calculated by using HDRI technique as we mention in Section 3.3.2. This method gives us a relative radiance information. In (Debevec and Malik, 2008) it is recommended to find absolute radiance values by using a luminaire with known radiance. The known radiance value of the luminaire (in our case the projector) is used to scale the cameras response function. Once the cameras response function is scaled it can be used to find absolute radiance (actually image irradiance $E$ in Figure 3.13 ) values.

By following HDRI procedure we take the pictures of the projector with different exposure times hence we can generate the radiance map of the projectors light. The setup looks like Figure 3.26. Sample images of varying exposure times can be seen in Figure 3.27.

We show the relation between scene radiance $L$ and image irradiance $E$ in Equation 3.16. Since we know the approximate distance between projector and camera in the setup given in Figure 3.26 and the area subtended by the camera lens we can estimate the corresponding scale factor for the camera response function. Since we use the same luminaire (i.e. the projector) we can use the scale factor to scale estimated scene radiance $L$ for each point in the experiments.

To sum up, we have all necessary components which can be used to represent


Figure 3.26. As proposed in (Debevec and Malik, 2008) the scale factor can be found by hdr imaging a luminaire with known specs. We use our projector for this purpose


Figure 3.27. HDR takes of the projector light at varying exposure times.
the reflectance properties of sample surfaces in BRDF format. This will give us a partial information about the materials complete BRDF.

### 3.4.4. Generating BRDF Measurement Data Set

One of our main assumptions is that we assume the material reflections to be isotropic (i.e. the surface reflection properties do not vary if we rotate $\omega_{i}$ and $\omega_{o}$ around the surface normal $\boldsymbol{n}$ ). Since we assume isotropy we do not consider objects made of anisotropic fabrics like the teddies in Figure 2.44. Instead we consider plastic and rough metal surfaces and assume isotropy for these materials.

In Figures 3.28a and 3.28d two sample experiment scenes can be seen. Corresponding reconstructed 3D point clouds are in Figures 3.28b and 3.28e. We aim to reconstruct the surface reflection properties of blue plastic part of the toy truck in Figure 3.28a
and the rough metal surface of the teapot in Figure 3.28d. We have selected sample areas, Figures 3.28 c and 3.28 f , on the surfaces in which both specular and diffuse reflection can be observed. We will use these sample surface patches to identify the reflection properties of the surfaces. Hence, our second assumption is that the selected sample surfaces exhibit both diffuse and specular reflections.


Figure 3.28. 3.28a-3.28d Two experimental scenes in which a plastic toy truck and a rough steel teapot can be seen. 3.28b-3.28e Corresponding reconstructions of the objects. $3.28 \mathrm{c}-3.28 \mathrm{f}$ Sample areas selected over the reconstruction results. These areas do have both specular and diffuse reflection characteristics.

The calibrated system together with surface normals gives us $\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)$ pairs for each surface point under our consideration (see Figure 3.29). Since we also know how to measure irradiance and radiance of a surface point we can calculate BRDF of the surface material at the corresponding points by using Equation 3.5.

We estimated the surface normal directions, n, by using the method in Section 3.4.1. The estimated surface normals can be seen in Figure 3.30. Incoming and outgoing


Figure 3.29. The calibrated system together with surface normals gives us $\left(\boldsymbol{\omega}_{i}, \omega_{o}\right)$ pairs for each surface point under our consideration.

Table 3.4. Tabluated Sample BRDF measurements from the blue plastic material in Figure 3.28c.

| $\theta_{i}$ | $\theta_{o}$ | $\phi$ | RED | GREEN | BLUE |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 16.7638 | 19.6249 | 180.0000 | 0.6683 | 1.2225 | 1.8604 |
| 16.7638 | 19.6249 | 180.0000 | 0.6683 | 1.2225 | 1.8604 |
| 16.6173 | 19.7420 | 179.9877 | 0.4140 | 0.8000 | 1.4043 |
| 16.6779 | 19.8566 | 180.0017 | 0.3143 | 0.7402 | 1.4052 |
| 16.6779 | 19.8566 | 180.0017 | 0.3143 | 0.7402 | 1.4052 |
| 16.4868 | 19.8830 | 179.9882 | 0.2992 | 0.6837 | 1.2637 |
| 16.4868 | 19.8830 | 179.9882 | 0.2992 | 0.6837 | 1.2637 |
| 16.4082 | 19.9830 | 179.9844 | 0.3795 | 0.7327 | 1.2719 |
| 16.4082 | 19.9830 | 179.9844 | 0.3795 | 0.7327 | 1.2719 |
| 20.0558 | 16.2212 | 179.9396 | 0.1038 | 0.3446 | 0.8686 |

light directions, $\omega_{i}$ and $\omega_{o}$ can be found with respect to surface normal $\mathbf{n}$.
The directions can be represented as unit vectors in spherical coordinates. A unit vector is represented by two angles, elevation $\theta$ and azimuth $\phi$, in spherical coordinate system (recall Figure 3.5). Since the materials are assumed to be isotropic, the azimuth angles of incoming and outgoing vectors can be given as the difference of them (i.e. $\left.\phi=\left|\phi_{i}-\phi_{o}\right|\right)$. The isotropic BRDF measurements can be tabulated by using three angles, $\theta_{i} \theta_{o}$ and $\phi$, and BRDF values for three color channels. In Tables 3.4 and 3.5, some of our BRDF measurements for blue plastic and rough metal can be seen respectively.

To perform a complete BRDF measurement every possible incoming and outgoing light directions and their combinations should be considered. In our study the directions $\omega_{i}$ and $\omega_{o}$ calculated over the selected surface areas subtend only a small part of direc-


Figure 3.30. We estimated the surface normal directions, $n$, by using the method in Section 3.4.1. Top - The surface normals estimated from the sample surface selected from the reconstruction of plastic truck toy. Bottom - The surface normals estimated from the sample surface selected from the reconstruction of rough steel teapot.

Table 3.5. Tabluated Sample BRDF measurements from the rough steel material in Figure 3.28f.

| $\theta_{i}$ | $\theta_{o}$ | $\phi$ | RED | GREEN | BLUE |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 23.4248 | 19.3939 | 179.8367 | 5.9788 | 6.0188 | 5.9108 |
| 23.4248 | 19.3939 | 179.9202 | 5.9788 | 6.0188 | 5.9108 |
| 23.9001 | 19.6714 | 179.7009 | 4.4958 | 4.5535 | 4.5140 |
| 24.3216 | 19.9254 | 179.4475 | 3.8607 | 4.0301 | 3.8184 |
| 24.4471 | 20.0176 | 179.3658 | 4.5079 | 4.7198 | 4.4124 |
| 24.4471 | 20.0176 | 179.3658 | 4.5079 | 4.7198 | 4.4124 |
| 24.7140 | 20.0037 | 179.2479 | 3.8747 | 3.9278 | 3.8436 |
| 23.8069 | 19.0410 | 179.9410 | 3.8868 | 3.9072 | 3.8736 |
| 23.8069 | 19.0410 | 179.9410 | 3.8868 | 3.9072 | 3.8736 |
| 24.1501 | 19.2543 | 179.7359 | 3.8329 | 3.8486 | 3.8618 |

tion space. In Figure 3.31 we show all incoming and outgoing directions for sample rough steel surface in Figure 3.28f on the unit hemisphere. The tabulated BRDF data of a complete measurement can be used directly in rendering processes as we show in Figure 3.11. But the data we measure is not sufficient to be used in a rendering process. Instead we prefer to use an analytical BRDF model which can be used to fit measured data and can be used in rendering processes.

### 3.4.5. Fitting an Analytical BRDF Model to Measured BRDF Data

Since we assume that the surfaces have isotropic nature and we have limited measurement data we choose the simple Phong BRDF Model which can represent simple isotropic material reflections. As proposed in (Ngan et al., 2005), (Kurt et al., 2010) and also in (Ozan and Gümüştekin, 2010) the desired BRDF parameters can be found by minimising the functional given in Equation 3.42. $R\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)$ represents the measured BRDF value at given $\boldsymbol{\omega}_{i}$ and $\boldsymbol{\omega}_{o}$, where $M\left(\boldsymbol{\omega}_{i}, \boldsymbol{\omega}_{\boldsymbol{o}} ; \mathbf{p}\right)$ represents the model BRDF value at the same incoming and outgoing light directions together with the model parameters given by vector $\mathbf{p}$.

$$
\begin{equation*}
E(\mathbf{p})=\sqrt{\frac{\Sigma w\left[R\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right) \cos \theta_{i}-M\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}} ; \mathbf{p}\right) \cos \theta_{i}\right]^{2}}{\Sigma w}} . \tag{3.42}
\end{equation*}
$$

Estimated Phong model parameters for the two sample materials are given in Table 3.6. Also in Figure 3.32 we show the measured and analytical BRDF model values for


Figure 3.31. In our study the directions $\omega_{i}$ and $\omega_{o}$ calculated over the selected surface areas subtend only a small part of direction space. 3.31a $\omega_{i}$ directions calculated for each point on a sample surface shown on the hemisphere. 3.31b $\omega_{o}$ directions calculated for each point on a sample surface shown on the hemisphere.
$\left(\boldsymbol{\omega}_{\boldsymbol{i}}, \boldsymbol{\omega}_{\boldsymbol{o}}\right)$ pairs. In the plots the y axis represents the BRDF values where the x axis is the angle between $\omega_{o}$ and $\mathbf{r}$. Even there is some amount of noise in BRDF measurements, it can be seen from the plots that BRDF drastically increases near the specular reflection direction as the angle between $\omega_{o}$ and $r$ gets smaller.

In order to visualise the material reflection Phong model can be used in ray tracing with estimated parameters. In Figure 3.33 we show rendered results where the estimated model parameters in Table 3.6 are used.

Table 3.6. Estimated Phong Model parameters for Blue Plastic and Rough Steel. $\rho_{d} \mathrm{~S}$ and $\rho_{s} \mathrm{~s}$ represent diffuse and specular parameters for each color channel. $m$ identifies the specular lobe.

| Material | $\rho_{d}^{R}$ | $\rho_{d}^{G}$ | $\rho_{d}^{B}$ | $\rho_{s}^{R}$ | $\rho_{s}^{G}$ | $\rho_{s}^{B}$ | $m$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Blue Plastic | 0.1040 | 0.7825 | 2.0697 | 0.0293 | 0.0465 | 0.0612 | $1.1512 \mathrm{e}+02$ |
| Rough Steel | 0.1052 | 0.1230 | 0.1316 | 0.2815 | 0.2871 | 0.2849 | $1.1760 \mathrm{e}+02$ |


(a)

(b)

Figure 3.32. 3.32a Measured and correspoinding fitted Phong BRDF samples shown for Blue Plastic material. 3.32b Measured and correspoinding fitted Phong BRDF samples shown for Rough Steel material. The x-axis represent the angle between $\omega_{o}$ and $\mathbf{r}$. The $y$-axis is the value of the sample BRDF


Figure 3.33. Synthetically rendered scenes with Phong BRDF model. BRDF parameters are estimated for blue plastic and rough steel material and used to render the scenes. In 3.33a, 3.33b estimated parameters for The Blue Plastic material are used. In 3.33c, 3.33d estimated parameters for The Rough Steel material are used.

## CHAPTER 4

## CONCLUSION

In this thesis our aim is to reconstruct geometrical and reflection properties of surfaces. We used various image based methods to achieve our goals.

We have seperated our work into two main chapters. In Chapter 2 we mainly focused on the reconstruction of surface geometry. We have presented the well-known stereo imaging methods. These methods basically use two images of the same scene taken from different angles and uses it to generate the depth map of a scene. We experimented that these methods are prone to fail if there exist specular reflections.

Specular reflection is a reflection property of an object and can be identified and somehow reconstructed by using passive and active imaging methods. We figured out that once the specular reflection is identified it can be removed to some extend. In Section 2.2 we have shown that by using dichromatic based methods we can identify and eliminate specularity passively. Also in same section we used polarization imaging methods to identify and eliminate specularity actively. Removing speculartiy has significantly enhanced the performance of stereo imaging methods and increased the quality of depth maps.

In Chapter 2 we also introduced structured light based scanning systems which are capable of generating more complete point clouds of surfaces compared to stereo imaging methods especially for surfaces having limited amount of surface features ( e.g. textureless and plain colored object surfaces examined in Section 3.4). Firstly we proposed three laser scanners with a pair of line lasers and also a novel method to calibrate these systems. Secondly we proposed a digital camera - projector based scanning system. This system can generate a very dense reconstruction of a scene. The calibration method we used for the projector based scanning system calibrates both camera and the projector in the same manner that we could obtain intrinsic and extrinsic parameters for both devices.

In Chapter 3 we tried to reconstruct surface reflection properties by using projector based scanning system. The calibration parameters of the projector-camera based structured light scanning system makes it possible to write illumination and viewing directions for each reconstructed surface point in a scene. We used the projector also to simply illuminate the scene. Known specs of the luminaire, i.e. the projector, and HDRI techniques made it possible for us to calculate both radiance and irradiance values at a reconstructed surface point.

We have adapted the gathered radiometric and geometric information to BRDF measurement format which is yet the most convenient way of storing and manipulating surface reflectance data. While doing this our main assumption was that the considered surface have both diffuse and sufficient amount of specular reflectance data. The obtained BRDF data is not a complete BRDF measurement since we do not cover every possible incoming and outgoing light directions. But we figured that analytical BRDF models can be used to fit to the measured data.

Our second assumption was that the material reflection is a simple isotropic reflection since it is not possible to capture anisotropy with the proposed system. Hence we did not consider objects made of anisotropic fabrics like teddies in the experiments instead we only considered blue plastic and rough steel materials and considered their reflections properties. We preferred to use Phong analytical BRDF model for data fitting since it is a basic and adequate model which can capture specular reflection. We have presented the reconstruction results with synthetic images rendered by using the estimated Phong model parameters. As a result, by using the projector based structured light system we were able to reconstruct physical properties of surfaces by using the previously reconstructed geometrical properties.

In future research, the projector based setup could be used to measure material BRDF together with the polarization information. Embedding polarization information in the BRDF would add a new dimension to computer graphics literature.

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## APPENDIX A

## LINEAR REGRESSION AND LEVENBERG-MARQUARDT METHODS

Two numerical analysis methods, Linear Regression and Levenberg-Marquardt, mentioned in Section 2.2 . can be briefly explained as follows. For more detailed information one can refer to (Chong and Żak, 2001).

## A.1. Linear Regression

The first method to solve the corresponding equation is Linear Regression Method. The linear system in Equation A. 1 can be solved by this method in Equation A.2.

$$
\text { Such that, } \mathbf{I}=\left[\begin{array}{c}
\mathbf{I} \quad=\quad \mathbf{A x},  \tag{A.1}\\
I\left(\theta_{f}^{1}, \phi\right) \\
I\left(\theta_{f}^{2}, \phi\right) \\
\vdots \\
I\left(\theta_{f}^{N}, \phi\right)
\end{array}\right] \quad \text {, and, } \mathbf{A}=\left[\begin{array}{c}
a_{1}^{T} \\
a_{2}^{T} \\
\vdots \\
a_{N}^{T}
\end{array}\right] .
$$

$$
\begin{equation*}
\mathbf{x}=\left(\mathbf{A}^{T} \mathbf{A}\right)^{-1} \mathbf{A}^{T} \mathbf{I} . \tag{A.2}
\end{equation*}
$$

If the components of vector $\mathbf{x}$ which is found by this method is used with Equation A.3, the parameters of TRS can be found as well. We use $I_{\text {min }}$ values in the proposed method.

$$
\begin{align*}
I_{\max } & =x_{1}+\sqrt{x_{2}^{2}+x_{3}^{2}}, \\
I_{\min } & =x_{1}-\sqrt{x_{2}^{2}+x_{3}^{2}}, \\
\phi & =\frac{1}{2} \arctan \frac{x_{3}}{x_{2}} . \tag{A.3}
\end{align*}
$$

## A.2. Levenberg - Marquardt

Unlike Linear Regression Method, Levenberg - Marquardt method is an iterative method. The value of vector $x$ can be found by Equation A.4.

$$
\begin{equation*}
\mathbf{x}^{k+1}=\mathbf{x}^{k}-\left(\mathbf{J}(\mathbf{x})^{T} \mathbf{J}(\mathbf{x})+\mu_{k} \mathbf{I}_{\mathbf{3}}\right)^{-1} \mathbf{J}(\mathbf{x})^{T} \mathbf{r}(\mathbf{x}) . \tag{A.4}
\end{equation*}
$$

$\mathbf{I}_{3}$ is an identity matrix of size $3 \times 3 . \mu_{k}$ is a user defined parameter which should take a positive value. $\mathbf{r}(\mathbf{x})$ can be written as Equation A. 5 by using Equation A.1.J ( $\mathbf{x}$ ) is the Jacobian matrix and can be found by using Equation A.6. The Jacobian matrix $\mathbf{J}(\mathbf{x})$, is of size Nx 3 where $N \geq 3$.

$$
\begin{gather*}
\mathbf{r}(\mathbf{x})=\mathbf{I}-\mathbf{A} \mathbf{x}  \tag{A.5}\\
\mathbf{J}(\mathbf{x})=\left[\begin{array}{ccc}
\frac{\partial r_{1}}{\partial x_{1}}(\mathbf{x}) & \frac{\partial r_{1}}{\partial x_{2}}(\mathbf{x}) & \frac{\partial r_{1}}{\partial x_{3}}(\mathbf{x}) \\
\vdots & \vdots & \vdots \\
\frac{\partial r_{N}}{\partial x_{1}}(\mathbf{x}) & \frac{\partial r_{N}}{\partial x_{2}}(\mathbf{x}) & \frac{\partial r_{N}}{\partial x_{3}}(\mathbf{x})
\end{array}\right] . \tag{A.6}
\end{gather*}
$$

Further details about these methods can be found in (Chong and Żak, 2001).

## APPENDIX B

## CAMERA CALIBRATION BY USING HOMOGRAPHIES

Zhang's method is a convenient way to estimate camera calibration parameters from 2D views of a simple calibration pattern. In this work we use a 20 by 14 checkerboard pattern. The pattern can be simply obtained by a conventional digital printer. We also put the pattern between two rectangular Plexiglas frame to make it stable. The method uses homographies between the world coordinates and image coordinates of feature points on the pattern.

We can relate the 3D coordinate of a scene point ( $\mathbf{P}$ ) with its corresponding image coordinate (p) by Equation B.1.

$$
\begin{equation*}
\mu \mathbf{p}=\mathbf{A}[\mathbf{R T}] \mathbf{P} \tag{B.1}
\end{equation*}
$$

$\mu$ is an arbitrary scale factor. $\mathbf{A}$ is the camera intrinsic parameter matrix and can be given as Equation B.2. ( $\mathbf{R}, \mathbf{T}$ ) pair represents camera extrinsic parameters with a rotation and translation matrix respectively. A list of camera calibration parameters can be seen in Table B.1.

$$
\mathbf{A}=\left[\begin{array}{ccc}
\alpha & \gamma & u_{o}  \tag{B.2}\\
0 & \beta & v_{o} \\
0 & 0 & 1
\end{array}\right]
$$

The radial lens distortion is an important artifact which should be handled. The method makes it possible to embed the distortion parameters $\left(k_{1}, k_{2}\right)$ into the system and can estimate them together with the intrinsic and extrinsic camera parameters. The lens distortion can simply be modeled as Equation B.3.

$$
\begin{align*}
u & =u_{d}+\left(u_{d}-u_{o}\right)\left[k_{1}\left(x^{2}+y^{2}\right)+k_{2}\left(x^{2}+y^{2}\right)^{2}\right] \\
v & =v_{d}+\left(v_{d}-v_{o}\right)\left[k_{1}\left(x^{2}+y^{2}\right)+k_{2}\left(x^{2}+y^{2}\right)^{2}\right] . \tag{B.3}
\end{align*}
$$

$(u, v)$ is the ideal (undistorted) pixel coordinate where $\left(u_{d}, v_{d}\right)$ is the observed (distorted) pixel coordinate. $(x, y)$ represents ideal image coordinates. System parameters, including


Figure B.1. We have used five different views of the calibration pattern for camera calibration. The calibration pattern is a 20 by 14 checkerboard pattern.
distortion parameters, are estimated in an optimization procedure of choice where the objective function is defined as Equation B.4.

$$
\begin{equation*}
\sum_{i=1}^{n} \sum_{j=1}^{m}\left\|\mathbf{p}_{i j}-\operatorname{Pr}\left(\mathbf{A}, \mathbf{R}_{i}, \mathbf{T}_{i}, k_{1}, k_{2}, \mathbf{P}_{j}\right)\right\|^{2} \tag{B.4}
\end{equation*}
$$

$n$ is the number of images used for calibration. $m$ represents number of feature points on the calibration pattern. In this work we use five different views of the calibration pattern ( $n=5$ ) and there are 247 feature (corner) points ( $m=247$ ) on the calibration pattern (see Figure B.1). $\mathbf{p}_{i j}$ is the $j_{t h}$ feature point on $i_{t h}$ view. Function $\operatorname{Pr}($.$) projects the 3D$ coordinate of $j_{t h}$ feature point in the reference plane to pixel coordinate with the radial distortion. The feature point extraction is performed with sub-pixel accuracy.

Table B.1. List of Camera Calibration Parameters. Since the projector can be modeled as a camera, these parameters applies for the projector calibration as well.

| Symbol | Description |
| :---: | :--- |
| $\mathbf{A}$ | Intrinsic parameter matrix |
| $u_{o}, v_{o}$ | The principle point in pixel coordinates |
| $\alpha, \beta$ | Corresponding scale factors of $u, v$ axes |
| $\gamma$ | Skewness with respect to $u, v$ axes |
| $\mathbf{R}$ | Rotation matrix |
| $\mathbf{T}$ | Translation matrix |
| $u, v$ | Ideal (undistorted) pixel coordinates |
| $u_{d}, v_{d}$ | Observed (distorted) pixel coordinates |
| $x, y$ | Corresponding image coordinates wrt. $u, v$ |

## APPENDIX C

## RECONSTRUCTING 3D COORDINATES BY USING SVD

If the image projection equation C .1 is written for multiple views the intrinsic paramater matrix $\mathbf{A}$ is the same for each view. But $\mathbf{R}$ and $\mathbf{t}$ matrices (rotation and translation matrices respectively) varies for different viewing directions. Hence the projection matrix for any of the views can be written as Equation C.1.

$$
\begin{equation*}
\mathbf{Q}_{n}=\mathbf{A}\left[\mathbf{R}_{n} \mathbf{t}_{n}\right](n=1,2,3,4,5) \tag{C.1}
\end{equation*}
$$

For a single view projection equation can be written as Equation C.2. Two linear relations in Equations C. 3 and C. 4 can be directly derived from Equation C.1. Superscript $m$ in $\mathbf{Q}^{m}$ represents the $m^{\text {th }}$ column of the matrix.

$$
\begin{align*}
{[u, v, 1]^{T} } & =\mathbf{Q}[X, Y, Z, 1]^{T} \\
\mathbf{x} & =\mathbf{Q X} \tag{C.2}
\end{align*}
$$

$$
\begin{align*}
v\left(\mathbf{Q}^{3} \mathbf{X}\right)-\left(\mathbf{Q}^{2} \mathbf{X}\right) & =0  \tag{C.3}\\
u\left(\mathbf{Q}^{3} \mathbf{X}\right)-\left(\mathbf{Q}^{1} \mathbf{X}\right) & =0 \tag{C.4}
\end{align*}
$$

For stereo image pairs Equations C. 3 and C. 4 can be written in matrix form as Equation C.5. Subscript $n$ in $\mathbf{Q}_{n}^{m}$ represents the corresponding view. $X$ in the equation can be found by SVD of $\mathbf{F}$. It can be written in the form $\mathbf{F}=\mathbf{U D V}^{T}$. IN SVD, eigenvalues of $\mathbf{F}^{\mathbf{T}} \mathbf{F}$ are the columns of $\mathbf{V}$. The smallest eigenvalue, the last column of $\mathbf{V}$, gives the $X$ which solves Equation C.5.

$$
\begin{array}{r}
{\left[\begin{array}{ll}
v_{1} \mathbf{Q}_{1}^{3} & -\mathbf{Q}_{1}^{2} \\
u_{1} \mathbf{Q}_{1}^{3} & -\mathbf{Q}_{1}^{1} \\
v_{2} \mathbf{Q}_{2}^{3} & -\mathbf{Q}_{2}^{2} \\
u_{2} \mathbf{Q}_{2}^{3} & -\mathbf{Q}_{2}^{1}
\end{array}\right] \mathbf{X}=0} \\
\mathbf{F} \mathbf{X} \tag{C.5}
\end{array}
$$

## APPENDIX D

## NONLINEAR LEAST SQUARES ALGORITHM (NLSA)

$$
\begin{equation*}
f(x ; \mathbf{a})=a_{1} e^{-\left(x-a_{2}\right) / 2 a_{3}^{2}} \tag{D.1}
\end{equation*}
$$

To obtain a subpixel accuracy Gaussian function can be fitted to the laser profile. In the Equation D.1, $a_{1}$ is the amplitude of the Gaussian which in our problem corresponds to the pixel intensity. $a_{2}$ is the position of the peak point, which is the value we aimed to find and lastly $a_{3}$ corresponds to the standard deviation which identifies the width of the bell shape of the gausssian. Nonlinear least-squares algorithm (Chong and ŻZak, 2001) is implemented as the following to solve this problem:

$$
\begin{equation*}
\operatorname{minimize}\left(\sum_{i=1}^{m}\left(r_{i}\left(x_{i} ; \mathbf{a}\right)\right)^{2}\right) \tag{D.2}
\end{equation*}
$$

Where $m$ is the number of measurements and $r_{i}$ can be defined as the difference between $i^{\text {th }}$ measured value $y_{i}$ and the fitted function value $f\left(x_{i}, \mathbf{y}\right)$. Hence $r_{i}$ can be written as:

$$
\begin{equation*}
r_{i}\left(x_{i}, \mathbf{a}\right)=y_{i}-f\left(x_{i} ; a_{1}, a_{2}, \cdots, a_{n}\right) \tag{D.3}
\end{equation*}
$$

In our problem, the number of variables $n$ is 3 (considering Equation D.1). $\mathbf{r}$ can be considered a small change in the function $f(x ; \mathbf{a})$. Hence the linearized estimate for the small changes in the function can also be written as D.4.

$$
\begin{equation*}
r_{i}\left(x_{i}, \mathbf{a}\right)=\left.\sum_{j=1}^{n} \frac{\partial f}{\partial a_{j}} d a_{j}\right|_{x_{i} ; \mathbf{a}} \tag{D.4}
\end{equation*}
$$

If the Jacobian matrix $\mathbf{J}$ is defined as D.5, Equation D. 4 can be written as Equation D. 6

$$
\begin{gather*}
\mathbf{J}=\left[\begin{array}{cccc}
\left.\frac{\partial f}{\partial a_{1}}\right|_{x_{1} ; \mathbf{a}} & \left.\frac{\partial f}{\partial a_{2}}\right|_{x_{1} ; \mathbf{a}} & \cdots & \frac{\partial f}{\partial a_{n}} \\
\left.\frac{\partial f}{\partial a_{1}}\right|_{x_{2} ; \mathbf{a}} & \left.\frac{\partial f}{\partial a_{2}}\right|_{x_{2} ; \mathbf{a}} & \cdots & \left.\frac{\partial f}{\partial a_{n}}\right|_{x_{2} ; \mathbf{a}} \\
\left.\frac{\partial f}{\partial a_{1}}\right|_{x_{m} ; \mathbf{a}} & \left.\frac{\partial f}{\partial a_{2}}\right|_{x_{m} ; \mathbf{a}} & \cdots & \left.\frac{\partial f}{\partial a_{n}}\right|_{x_{m} ; \mathbf{a}}
\end{array}\right]  \tag{D.5}\\
r_{i}=\sum_{j=1}^{n} J_{i j} d a_{j} . \tag{D.6}
\end{gather*}
$$

If the matrix notation is used by using Equations D. 5 and D.6, da can be rearranged as: After constructing an $m \times n$ Jacobian matrix $\mathbf{J}$ from the terms:

$$
\begin{equation*}
J_{i j}=\left.\frac{\partial f}{\partial a_{j}}\right|_{x_{i} ; \mathbf{a}} \tag{D.7}
\end{equation*}
$$

The displacement vector $d \mathbf{a}$ can be found as:

$$
\begin{equation*}
d \mathbf{a}=\left(\mathbf{J}^{T} \mathbf{J}\right)^{-1}(\mathbf{J r}) . \tag{D.8}
\end{equation*}
$$

In our problem, $m=9$, i.e. the Gaussian fit is performed by using nine neighboring pixels in the same row, where the mid pixel is the pixel found by dynamic programming. The original pixel intensity value at the coordinate where we find a path pixel in the binary path image is taken as initial $a_{1}$, the column position is taken as initial $a_{2}$ and initial $a_{3}$ is taken as 2.0 (in Equation D.1). $y_{i}$ values are taken as corresponding nine neighboring pixels in the original image. $d \mathbf{a}$ value is used to find the $a_{j}$ estimates in an iterative manner. At each iteration small $d a_{j}$ variations are added to each $a_{j}$ and by updating $\mathbf{r}$ and $\mathbf{J}$ at each iteration step optimal values are achieved after few iterations.

## APPENDIX E

## RANDOMIZED HOUGH TRANSFORM (RHT)

## Randomized Hough Transform (RHT)

```
while more points are needed from point cloud \(P\) do
    Randomly pick three points \(p_{1}, p_{2}, p_{3}\) from \(P\)
    if \(p_{1}, p_{2}\) are \(p_{3}\) satisfy the distance criterion then
                Calculate plane \((\rho, \theta, \beta)\) spanned by \(p_{1}, p_{2}, p_{3}\)
                Increment corresponding cell \(A(\rho, \theta, \beta)\) in the accumulator space
    else
            continue
    end if
end while
Return target triplet \(\left(\rho_{t}, \theta_{t}, \beta_{t}\right)\) from \(M A X(A(\rho, \theta, \beta))\)
for all points \(p_{i}\) in point cloud \(P\) do
    if \(f\left(\rho_{t}, \theta_{t}, \beta_{t}, p_{i}\right)\) is smaller than \(T\) then
        \(p_{i}\) is assigned as a plane point
    else
            \(p_{i}\) is assigned as a non-plane point
    end if
end for
```

In the above algorithm, $P$ represents the corresponding point cloud in which a plane is searched for. The key idea of RHT is to randomly pick point triplets (i.e. $\left(p_{1}, p_{2}, p_{3}\right)$ ) and parameterize the plane which is defined by the three chosen points. The distribution of plane parameter triplets (i.e. $(\rho, \theta, \beta)$ ) concentrates around a point (i.e. $\left(\rho_{t}, \theta_{t}, \beta_{t}\right)$ ) in parameter space and this point represents a plane equation. Once the parameters are found, the last step is to find the points which are close enough to the plane defined in Equation 2.25.

## APPENDIX F

## NELDER MEAD ALGORITHM (NMA)

The NMA can be described as follows:

## STEP 1: Initialize:

Order vertices according to the objective function values (Figure F.1a).
$f\left(\mathbf{x}_{\mathbf{1}}\right) \leq f\left(\mathbf{x}_{\mathbf{2}}\right) \leq \cdots \leq f\left(\mathbf{x}_{\mathbf{N}+\mathbf{1}}\right)$

## STEP 2: Calculate COG:

$\mathrm{x}_{\mathrm{o}}$ which is the center of gravity of $\left(\mathrm{x}_{\mathbf{1}}, \mathrm{x}_{\mathbf{2}}, \cdots, \mathrm{x}_{\mathrm{N}}\right)$ excluding $\mathrm{x}_{\mathrm{N}+\mathbf{1}}$

## STEP 3: Reflection:

Compute reflected point $\mathbf{x}_{\mathbf{R}}=\mathbf{x}_{\mathbf{o}}+\alpha\left(\mathbf{x}_{\mathbf{o}}-\mathbf{x}_{\mathbf{N}+\mathbf{1}}\right)$ (Figure F.1b)
If $f\left(\mathbf{x}_{\mathbf{1}}\right) \leq f\left(\mathbf{x}_{\mathbf{R}}\right) \leq f\left(\mathbf{x}_{\mathbf{N}}\right)$
Then replace $\mathrm{x}_{\mathrm{N}+1}$ with $\mathrm{x}_{\mathrm{R}}$

## STEP 4: Expansion:

If $\mathrm{x}_{\mathrm{R}}<\mathrm{x}_{1}$
Then compute expanded point $\mathbf{x}_{\mathbf{E}}=\mathbf{x}_{\mathbf{o}}+\gamma\left(\mathbf{x}_{\mathbf{o}}-\mathbf{x}_{\mathbf{N}+\mathbf{1}}\right)($ Figure F.1c)
If $\mathrm{x}_{\mathrm{E}}<\mathrm{x}_{\mathrm{R}}$
Then replace $\mathrm{x}_{\mathrm{N}+1}$ with $\mathrm{x}_{\mathrm{E}}$ and goto STEP 1
Else replace $\mathrm{x}_{\mathrm{N}+1}$ with $\mathrm{x}_{\mathrm{R}}$ and goto STEP 1
STEP 5: Contraction:
Since $\mathrm{x}_{\mathrm{R}} \geq \mathrm{x}_{\mathrm{N}}$
Compute contracted point $\mathbf{x}_{\mathbf{C}}=\mathbf{x}_{\mathbf{N}+\mathbf{1}}+\rho\left(\mathbf{x}_{\mathbf{o}}-\mathbf{x}_{\mathbf{N}+\mathbf{1}}\right)$ (Figure F.1d)
If $\mathrm{x}_{\mathrm{C}}<\mathrm{x}_{\mathrm{N}+\mathbf{1}}$
Then replace $\mathbf{x}_{\mathbf{N}+1}$ with $\mathbf{x}_{\mathrm{C}}$ and goto STEP 1
Else goto STEP 6
STEP 6: Multiple Contraction:
Compute contracted points $\mathbf{x}_{\mathbf{i}}=\mathbf{x}_{\mathbf{1}}+\sigma\left(\mathbf{x}_{\mathbf{i}}-\mathbf{x}_{\mathbf{1}}\right)$
For $i \in\{2, \cdots, N+1\}$ (Figure F.1e)
Else goto STEP 1


Figure F.1. Actions in Nelder-Mead Algorithm.

## APPENDIX G

## SURFACE RECONSTRUCTION FROM UNORGANIZED POINT CLOUDS BY CALCULATING SIGNED DISTANCE FUNCTION ON A 3D COMPUTATIONAL GRID

In order to visualize unorganized 3D point clouds the method proposed in (Ozan and Gümüştekin, 2011c) can be used. The method uses the signed distance function. The signed distance function is implicitly calculated on a computational grid and the structured result on the grid is triangulated with a proper method.

## G.1. Eikonal Equation

The eikonal equation is given as Equation G.1. The equation can be used to solve problems in optimal control, path planning, geometrical optics, computer vision and also computer graphics. With the given boundary condition in Equation G.2, the system becomes a hyperbolic partial differential equation (Zhao, 2002).

$$
\begin{equation*}
|\nabla u(\mathbf{x})|=f(\mathbf{x}), \mathbf{x} \in \mathbf{R}^{n} \tag{G.1}
\end{equation*}
$$

$$
\begin{equation*}
u(\mathbf{x})=\phi(\mathbf{x}), \mathbf{x} \in \boldsymbol{\Gamma} \subset \mathbf{R}^{n} \tag{G.2}
\end{equation*}
$$

## G.2. Signed Distance Function

Signed distance function $d(\mathbf{x})=(\mathbf{x}, \mathcal{S})$ holds implicit information about unorganized data set $\mathcal{S}$. In this work the data set $\mathcal{S}$ is an unorganized point cloud generated from 3D laser scans and holding 3D coordinate information, i.e. $\mathcal{S} \in \mathbf{R}^{3}$. The distance function, $d(\mathbf{x})$, of point cloud, $\mathcal{S}$, is the solution of the Eikonal equation given in Equation G.3.

In this work, to approximate close enough to the exact solution, distance function is chosen as $d(\mathbf{x})=\epsilon$ where $\epsilon$ is a small value. Hence the distance function becomes the $\epsilon$-offset of the manifold which is represented by the unorganized point cloud.

$$
\begin{equation*}
|\nabla d(\mathbf{x})|=1, d(\mathbf{x})=0, \mathbf{x} \in \mathcal{S} . \tag{G.3}
\end{equation*}
$$

## G.3. Fast Sweeping Method

Fast sweeping method (Zhao, 2005) is a numerical calculation method for solving Equations G. 1 and G.3. The method is a finite difference method which can be used to solve multidimensional problems. For simplicity the method is explained in 2D but the it can be easily extended to three or higher dimensions.

If the problem is considered in 2D, the unorganized point cloud representing a closed manifold and the $\epsilon$-offset surface surrounding that manifold can be represented as Figure G.1. In the following derivations $\mathbf{x}_{i, j}$ represents nodes of 2 D grid and $h$ is the grid size. In this work grid size is equal in each dimension. $u_{i, j}^{h}$ represents the numerical solution at grid point $\mathbf{x}_{i, j}$. Equation G. 3 can be rewritten as Equation G. 5 by using finite differencing. (. $)^{+}$represents Heaviside Step Function.


Figure G.1. A closed manifold represented by an unorganized point cloud.

$$
\begin{align*}
{\left[\left(u_{i, j}^{h}-u_{x \min }^{h}\right)^{+}\right]^{2}+\left[\left(u_{i, j}^{h}-u_{y m i n}^{h}\right)^{+}\right]^{2} } & =h^{2}  \tag{G.4}\\
u_{x \min }^{h} & =\min \left(u_{i-1, j}^{h}, u_{i+1, j}^{h}\right) \\
u_{y m i n}^{h} & =\min \left(u_{i, j-1}^{h}, u_{i, j+1}^{h}\right)
\end{align*}
$$

The solution procedure starts with initializing grid nodes. The grid areas containing the elements which satisfies the condition $u(\mathbf{x})=0, \mathbf{x} \in \mathcal{S}$ are shown as gray in Figure G.1. The nodes of gray grid areas are initialized with the Euclidean distance of nodes to the data set. The remaining empty nodes are initialized with a temporary value. This value is chosen at least as big as the distance between most distant two points in the data set.

The node values approximate to continues with Gauss-Seidel iterations with different directions (Zhao, 2005). The number of iterations are $2^{n}+1$ ( $2^{n}$ different directions +1 randomly chosen direction) for an $n$ dimensional system. For example in this work $2^{3}+1=9$ sweeps are sufficient to calculate solution at grid nodes.

For each sweeping iteration; we can assume that $u_{i, j}^{\text {old }}$ is the previous solution value and $u_{i, j}^{\text {new }}$ is the new accepted solution value at grid point $\mathbf{x}_{i, j}$. According to Equation G.5, $\bar{u}$ can be calculated by looking at $u_{i \pm 1, j}^{h}$ and $u_{i, j \pm 1}^{h}$ values. Hence the Equation which is to be solved for each grid point can be rewritten as Equation G.6. The unique solution for this equation becomes Equation G.7.

$$
\begin{gather*}
{\left[\left(x-a_{1}\right)^{+}\right]^{2}+\left[\left(x-a_{2}\right)^{+}\right]^{2}=h^{2}}  \tag{G.5}\\
a_{1}=u_{x m i n}^{h}, a_{2}=u_{y m i n}^{h}  \tag{G.6}\\
\bar{x}= \begin{cases}\min (a, b)+h, & \left|a_{1}-a_{2}\right| \geq h \\
\frac{a_{1}+a_{2}+\sqrt{2 h^{2}-\left(a_{1}-a_{2}\right)^{2}}}{2}, & \left|a_{1}-a_{2}\right|<h\end{cases} \tag{G.7}
\end{gather*}
$$

But the solution for higher dimensional problems can be found by using the systematic method described in (Zhao, 2002). According to this method for an n dimensional problem Equation G. 6 can be rewritten as Equation G.8. In this notation $a_{1} \leq a_{2} \leq \cdots \leq$ $a_{n}$ is considered.

$$
\begin{equation*}
\left[\left(x-a_{1}\right)^{+}\right]^{2}+\left[\left(x-a_{2}\right)^{+}\right]^{2}+\cdots+\left[\left(x-a_{n}\right)^{+}\right]^{2}=h^{2} . \tag{G.8}
\end{equation*}
$$

If there exist an integer $p$ such that $1 \leq p \leq n$; and $\bar{x}$ is the explicit solution to the Equation G.9; $\bar{x}$ and $p$ can be found in a recursive manner. For our 3D problem this method works as follows:

$$
\begin{gather*}
{\left[\left(x-a_{1}\right)^{+}\right]^{2}+\left[\left(x-a_{2}\right)^{+}\right]^{2}+\cdots+\left[\left(x-a_{p}\right)^{+}\right]^{2}=h^{2}}  \tag{G.9}\\
\text { ve } a_{p} \leq \bar{x} \leq a_{p+1}
\end{gather*}
$$

It starts by assuming $p=1$, if $\tilde{x}=a_{1}+h \leq a_{2}$ then $\bar{x}=\tilde{x}$. Otherwise $\tilde{x}>a_{2}$ and it is the solution to the Equation G.10. The explicit solution can be found by Equation G.11. If $\tilde{x} \leq a_{3}$ then $\bar{x}=\tilde{x}$.

$$
\begin{align*}
& {\left[\left(x-a_{1}\right)^{+}\right]^{2}+\left[\left(x-a_{2}\right)^{+}\right]^{2}=h^{2},}  \tag{G.10}\\
& \tilde{x}=\frac{a_{1}+a_{2}+\sqrt{2 h^{2}-\left(a_{1}-a_{2}\right)^{2}}}{2} . \tag{G.11}
\end{align*}
$$

Otherwise if $\tilde{x}>a_{3}$ then $\tilde{x}$ is the solution to Equation G.12, and the explicit solution can be found by Equation G.13.

$$
\begin{gather*}
{\left[\left(x-a_{1}\right)^{+}\right]^{2}+\left[\left(x-a_{2}\right)^{+}\right]^{2}+\left[\left(x-a_{3}\right)^{+}\right]^{2}=h^{2},}  \tag{G.12}\\
\bar{x}=\frac{K+\sqrt{3 h^{2}-2\left(K^{2}-L\right)}}{3}  \tag{G.13}\\
K=\sum_{i=1}^{3} a_{i}, L=a_{1} a_{2}+a_{1} a_{3}+a_{2} a_{3} .
\end{gather*}
$$

## G.4. Calculating $\epsilon$-offset

By using the method described above, we obtain a grid structure where each grid point has the value which shows the distance to the unorganized data set $\mathcal{S}$. Next the grid points are checked and labeled according to a preferred distance value $\epsilon$. The grid cells where the $\epsilon$-offset surface of the original data set resides can be found. In the next step marching cubes algorithm is going to be used to triangulate the implicit surface data.

## G.5. Marching Cubes Method

This method is widely used in visualising discrete 3D data. Since the method depends on look-up-tables it is fast and can be used in real time applications as well. Method was first introduced in (Lorensen and Cline, 1987). The main method has a look-up-table for different surface configurations but it may give faulty results for relatively complex surface and subsurface structures. In (Nielson, 2003) this drawback is partially compensated by enlarging the look-up-table such that it can handle additional more complex surface structures. We use the method in (Lorensen and Cline, 1987) which is sufficient. By using this method the $\epsilon$-offset of the original surface is obtained as a triangle mesh.

## G.6. Optimising the Data

The marching cube method described in the previous section creates a triangle mesh from the implicit data. But if the method is blindly applied, the obtained data consists of a vast amount of repeated vertex data. Hence we need an optimization.

Optimising the data is also important to store the data and enable it to be read and manipulated by 3D manipulation and rendering softwares efficiently. A quick sort algorithm (Hoare, 1961) is used to order data and remove repeated vertices. This method has a best case complexity of $O(n \log n)$ for $n$ data.

To show the effect of $\epsilon$ size, bunny data is chosen from Stanford University's scanning repository (Levoy et al., 2005). For a small number $h$ reconstruction results where $\epsilon$ is taken as $3 h$ and $10 h$ can be seen in Figure G.2.

## G.7. Rendering the Data

To render the data we can use an open source rendering and 3D object manipulation software named Blender (Crowder, 1999). Like all common 3D rendering and object manipulation softwares Blender can also read and process Wavefront Object File Format (.obj). Hence the results are saved in .obj format. Three sample objects are laser scanned with the turntable scanner system proposed in (Ozan and Gümüştekin, 2014a). The original objects and their rendered results can be seen in Figure G.3.


Figure G.2. (a) If a smaller $\epsilon$ is chosen the resulting surface converges to the original surface here $\epsilon=3 h$ ).(b) If a smaller $\epsilon$ is chosen the resulting surface converges to the original surface here $\epsilon=3 h$ ).

(a)

(c)

(e)

(b)

(d)

(f)

Figure G.3. Three objects scanned with the turntable scanner system in (Ozan and Gümüştekin, 2014a) and their reconstruction results can be seen left to right.

## APPENDIX H

## RECOVERING CAMERA RESPONSE FUNCTION

The algorithm is inherited from (Debevec and Malik, 2008). Camera response function $f($.$) can be defined as Equation H.1. It defines the relation between the pixel$ intensity value with radiance times exposure. $i$ is pixel index where $j$ is exposure index. A function $g=\ln f^{-1}$ can be derived as Equation H.2.

$$
\begin{align*}
Z_{i j} & =f\left(E_{i} \Delta t_{j}\right)  \tag{H.1}\\
g\left(Z_{i j}\right) & =\ln E_{i}+\ln \Delta t_{j} . \tag{H.2}
\end{align*}
$$

$g$ can be recovered in a weighted least squared error sense. Minimizing objective function in Equation H. 3 ensures solution satisfying Equation H. 2 .

$$
\begin{align*}
\mathcal{O} & =\sum_{i}^{N} w\left(Z_{i j}\right)\left[g\left(Z_{i j}\right)-\ln E_{i}-\ln \Delta t_{j}\right]^{2} \\
& +\lambda \sum_{z=Z_{\min }+1}^{Z_{\max }-1}\left[w(z) g^{\prime \prime}(z)\right]^{2} \tag{H.3}
\end{align*}
$$

Adding second derivative of $g$ ensures smoothness. It can be approximated as $g^{\prime \prime}(z)=g(z-1)-2 g(z)+g(z+1)$. $\lambda$ weights the smoothness term. Simple weighting function $w($.$) can be defined as Equation H.5. Also Z_{\max }=255$ and $Z_{\min }=0$ in the standard 8-bit images used in our study.

$$
\begin{gather*}
w(z)=\left\{\begin{array}{r}
z-Z_{\min } \text { for } z \leq \frac{1}{2}\left(Z_{\min }+Z_{\max }\right) \\
Z_{\max }-z \text { for } z>\frac{1}{2}\left(Z_{\min }+Z_{\max }\right)
\end{array}\right.  \tag{H.5}\\
\begin{aligned}
\ln E_{i} & =g\left(Z_{i j}\right)-\ln \Delta t_{j}
\end{aligned} \\
=\frac{\sum_{j=1}^{P} w\left(Z_{i j}\right)\left(g\left(Z_{i j}\right)-\ln \Delta t_{j}\right)}{\sum_{j=1}^{P} w\left(Z_{i j}\right)} \tag{H.6}
\end{gather*}
$$

## VITA

Şükrü Ozan was born in İzmir, Turkey, in March 16th, 1979. After his graduation from İzmir Atatürk High School he attended Electrical and Electronics Engineering Department at Middle East Technical University. Soon after acquiring his BSc degree from METU in 2002, he started the MSc program in Electrical and Electronics Department at İzmir Institute of Technology (IZTECH). In 2006, he started his PhD studies at the same department in IZTECH.

He worked as a research assistant at Electrical and Electronics Engineering department in IZTECH from 2004 to 2011. Meanwhile in 2007, he founded a Google Certified Partner Company named AdresGezgini with two of his colleagues at İzmir Technology Development Region.

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